THE UNIVERSITY OF HONG KONG
DEPARTMENT OF STATISTICS AND ACTUARIAL SCIENCE

STAT3602 Statistical Inference

Example Class 10 (2020-2021 1st semester)

Problem

1. Given a random sample $X_1, \ldots, X_n$ from the density function

$$f(x|\lambda) = \lambda^{-1} e^{-x/\lambda}, x > 0$$

   (a) Find the UMP size α test of $H_0 : \lambda \leq 1$ against $H_1 : \lambda > 1$;

   (b) Find the equation for the sample size n required for this test to have power 95% when $\lambda = 2$, illustrating your answer with a sketched graph.

2. Let $X_1, \ldots, X_n$ be an independent sample from a normal distribution with mean 0 and variance σ^2.

   (a) Find the UMP test of size α to test $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 = \sigma_1^2 > \sigma_0^2$.

   (b) Find the UMP test of size α to test $H_0 : \sigma^2 \leq \sigma_0^2$ against $H_1 : \sigma^2 > \sigma_0^2$.

   (c) Show that the UMPU test of size α to test $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$ is in the form $\varphi(\mathbf{X}) = \mathbf{1}\left\{\sum_{i=1}^n X_i^2 \notin [t_1, t_2]\right\}$ with t_1, t_2 satisfying

$$\begin{cases} 1 + \mathbb{P}\left(Y < \dfrac{t_1}{\sigma_0^2}\right) - \mathbb{P}\left(Y < \dfrac{t_2}{\sigma_0^2}\right) = \alpha, \quad \text{where } Y \sim \chi_n^2, \\ \dfrac{t_1 - t_2}{n(\ln t_1 - \ln t_2)} = \sigma_0^2. \end{cases}$$

3. A local councillor suspects that traffic conditions in his village A have become more hazardous than those of a neighbouring village B. He therefore records the numbers of traffic accidents N_A, N_B which occur in A and B over a fixed period of time respectively. Assuming N_A, N_B are independent Poisson random variables with parameters $\lambda, \beta\lambda$ respectively, the councillor wishes to test $H_0 : \beta \geq 1$ against $H_1 : \beta < 1$.

   (a) Derive a form of a UMPU test, of size α, for testing the above hypotheses.

   (b) What is the outcome of the test when $\alpha = 0.1$, $N_A = 7$, $N_B = 2$?