THE UNIVERSITY OF HONG KONG
DEPARTMENT OF STATISTICS AND ACTUARIAL SCIENCE

STAT3602 Statistical Inference

(2020-2021 First Semester)

Example Class 4

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Exercise 1

Let $X_1, \ldots, X_n$ be i.i.d. random variables with density function $f(x \mid \theta) = \theta e^{-\theta x}, x > 0$ where $\theta \in (0, \infty)$ is an unknown parameter. Consider testing the null hypothesis $H_0 : \theta \leq 1$ against $H_1 : \theta > 1$. Denote the corresponding range of θ as Θ_0 and Θ_1. Suppose we give prior guess:

$$\pi(\theta) \sim \text{Gamma}(\alpha, \beta)$$

with density

$$\frac{1}{\Gamma(\alpha)} \beta^\alpha \theta^{\alpha-1} e^{-\theta\beta}$$

And define the loss function as $L(\theta, a_j) = \mathbf{1}(\theta \in \Theta_{1-j})$. What is the Bayesian decision rule?

Exercise 2

Given the value of λ, the number X_i of transactions made by customer i at an online store in a year has a Poisson(λ) distribution, with X_i independent of X_j for $i \neq j$. The value of λ is unknown. Our prior distribution for λ is a gamma (5,1) distribution. We observe the numbers of transactions in a year for 45 customers and $\sum_{i=1}^{45} = 182$. (density of $\text{Gamma}(\alpha, \beta) = \frac{1}{\Gamma(\alpha)} \beta^\alpha x^{\alpha-1} e^{-\beta x}$)

- Find the 95% equal-tailed posterior interval of λ.
- Find an expression for the posterior predictive probability that a customer makes m transactions in a year.

Exercise 3

Suppose $X_1, ..., X_n$ are independent, identically distributed from the normal distribution $N(\mu, 1/\tau)$. The normal variance as well as the normal mean is unknown. Consider the prior in which τ has a Gamma distribution with parameters $\alpha > 0, \beta > 0$ and, conditionally on τ, μ has distribution $N(v, 1/(k\tau))$ for some constants $k > 0, v \in \mathbb{R}$. What is the posterior distribution, i.e. $\pi(\tau, \mu \mid x)$?