THE UNIVERSITY OF HONG KONG
DEPARTMENT OF STATISTICS AND ACTUARIAL SCIENCE

STAT3602 Statistical Inference

(2020-2021 First Semester)

Example Class 3

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Exercise 1

We are faced with a shipment of N manufactured items. An unknown number D of these items are defective. A sample of n is drawn without replacement and inspected. The number X of defective items is recorded

a. What is the distribution of X?

b. Now suppose D has a binomial prior distribution with parameters N and p; that is

$$P(D=d)=\left(\begin{array}{c} N \\ d \end{array}\right) p^d(1-p)^{N-d}, d=0,1,\ldots,N \tag{1}$$

Show the posterior distribution of D given $X=x$ is that of $x+Y$ where Y has a binomial distribution with parameters $N-n$ and p.

Exercise 2

Suppose $X_1,...,X_n$ have a Poisson distribution with mean $\theta(>0)$, and the prior distribution of is $\Gamma(\alpha,\beta)$, where $\alpha,\beta>0$, i.e.

$$\pi(\theta)=\frac{1}{\Gamma(\alpha)}\beta^\alpha\theta^{\alpha-1}e^{-\beta\theta}, \beta, \alpha>0 \tag{2}$$

a. Determine the posterior distribution of θ given the random sample $x_1,...,x_n$.

b. If we regard the problem of estimating θ based on a size-n sample of X as a statistical decision problem and we adopt the square loss function, then what is the Bayes rule for estimating θ?

Exercise 3

Suppose X has a Bernoulli distribution with parameter θ where θ has prior uniform distribution on $[0,1]$. A random sample of size n is taken.

a. What is the posterior distribution of θ?

b. Find the Bayes estimate of θ when the loss function is $(\theta-\hat{\theta})^2$

Exercise 4

Let $X_1, ..., X_n$ iid samples from $N(\theta, 1)$, suppose the prior distribution of θ is a standard normal distribution. Find the equal-tailed interval for θ of fixed posterior coverage probability 0.95.