A nēw prediction of wavelength selection in radial viscous fingering involving normal and tangential stresses.

Mathias Nagel[1, a)] and François Gallaire[1]

Laboratory of Fluid Mechanics and Instabilities - EPFL Lausanne, Switzerland

(Dated: 9 January 2015)

We reconsider the radial Saffman-Taylor instability, when a fluid injected from a point source displaces another fluid with a higher viscosity in a Hele-Shaw cell, where the fluids are confined between two neighboring flat plates. The advancing fluid front is unstable and forms fingers along the circumference. The so-called Brinkman equations is used to describe the flow field, which also takes into account viscous stresses in the plane and not only viscous stresses due to the confining plates like the Darcy equation. The dispersion relation agrees better with the experimental results than the classical linear stability analysis of radial fingering in Hele-Shaw cells that uses Darcy's law as a model for the fluid motion.

PACS numbers: 47.15.gp

Keywords: Saffman-Taylor instability, Hele-Shaw cell, Brinkman equation

[a)]Electronic mail: mathias.nagel@epfl.ch

I. INTRODUCTION

Viscous fingering, also called Saffman-Taylor[1] instability, is considered as an archetype of pattern forming instability (see Couder 2000[2] for an insightful review). It has also been widely studied in the context of industrial research, such as petroleum extraction in particular. The phenomenon belongs to the broad family of instabilities of growth in Laplacian fields, which includes solidification, aggregation, etc... It was first studied by Saffman and Taylor[1] in 1958, who observed the formation of patterns upon injection of a fluid into a channel filled with another more viscous fluid. Saffman and Taylor[1] studied the formation of fingers in shallow rectangular channels and observed that the formation of fingers was dependent on the ratio of channel height to channel width. The extension to radial geometry, depicted in figure **??**a, dates back to Bataille[3], Wilson[4] and Paterson[5].

REFERENCES

[1] Saffman, P. G., Taylor, G. I. 1958 The Penetration of a Fluid into a Porous Medium or Hele-Shaw Cell Containing a More Viscous Liquid. *Proc. Roy. Soc. A* **245** (1242) 312-329

[2] Couder Y. 2000 Viscous fingering as an archetype of growth patterns *In Perspectives in fluid dynamics, edited by Batchelor, G. K., Moffatt H. K. and Worster, M. G.* 53-104

[3] Bataille, J. 1968 Stabilité d'un déplacement radial non miscible *Revue Inst. Pétrole* **23** 1349-1364

[4] Wilson, S.D.R 1975 A note on the measurement of dynamic contact angles. *Journal of Colloid and Interface Science* **51**(3)

[5] Paterson, L. 1981 Radial fingering in a Hele Shaw cell. *J. Fluid Mech.* **113** 513-529

[6] Maxworthy, T. 1989 Experimental study of interface instability in a Hele-Shaw cell. *Phys. Rev. A*, **39** (11) 5863-5866

[7] Chuoke, R. L., van Meurs, P. and van der Pol, C. 1959 The instability of slow immiscible viscous liquid-liquid displacements in permeable media *Petrol. Trans. AIME* **216** 188-194

[8] Dai, W.-S., Shelley, M. J A numerical study of the effect of surface tension and noise on an expanding Hele-Shaw bubble *Phys. Fluids* **5**(9) 2131-2146

[9] Paterson, L. 1985 Fingering with miscible fluids in a Hele Shaw cell. *Phys. Fluids*, **28** (1) 26-30

[10]Kim, H., Funada, T., Joseph, D. D., Homsy, G. M. Viscous potential flow analysis of radial fingering in a Hele-Shaw cell. *Phys. Fluids* **21** 074106

[11]Ben Amar M. and Boudaoud A. 2009 Suction in Darcy and Stokes interfacial flows: Maximum growth rate versus minimum dissipation. *Eur. Phys. J. Special Topics* **166** 83–88

[12]Logvinov, O. A., Ivashnyov, O. E., Smirnov, N. N. 2010 Evaluation of viscous fingering width in Hele-Shaw flows. *Acta Astronautica* **67** 53-59

[13]J. Fernandez, J., Kurowski P., Limat L. and Petitjeans P. 2001 Wavelength selection of fingering instability inside Hele-Shaw cells *Phys. Fluids* **13** 3120-3125

[14]Carlès P., Huang Z., Carbone G. and Rosenblatt C. 2006 Rayleigh-Taylor Instability for Immiscible Fluids of Arbitrary Viscosities: A Magnetic Levitation Investigation and Theoretical Model *Phys. Rev. Lett.* **96** 104501

[15]Boos W. and Thess, A. 1997 Thermocapillary flow in a Hele-Shaw cell *J. Fluid Mech.* **352** 305-320

[16]Gallaire F., Meliga P., Laure P. and Baroud, C. 2013 Marangoni-induced force on a drop in a Hele-Shaw cell *submitted*

[17]Park, C.-W., Homsy, G. M. 1984 Two-phase displacement in Hele Shaw cells: theory. *J. Fluid Mech.* **139** 291-308