# Regression in R

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## Why a part on simple regression

- some machine learning concepts are based on regression
- I would like to remind you how simple regression in R works.
- I also want to show the constraints
- In a next step we will learn, how to coop with that

## Variables of the mtcars dataset

Help for the `mtcars` dataset:

```
?mtcars
```

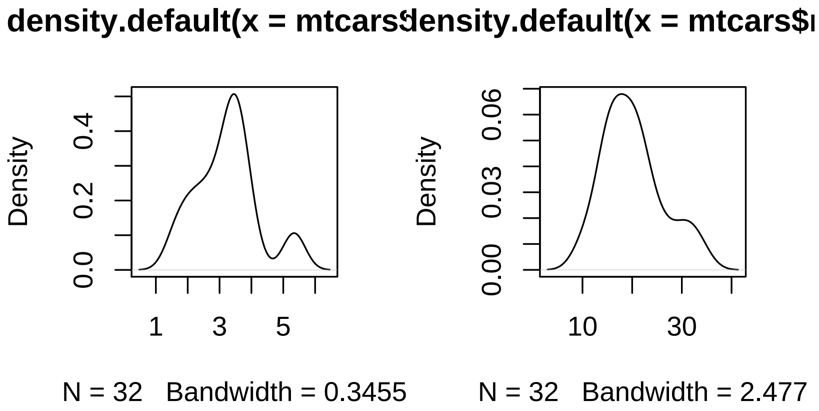
- mpg - Miles/(US) gallon
- cyl - Number of cylinders
- disp - Displacement (cu.in.)
- hp - Gross horsepower
- drat - Rear axle ratio
- wt - Weight (1000 lbs)
- qsec - 1/4 mile time
- vs - Engine (0 = V-shaped, 1 = straight)
- am - Transmission (0 = automatic, 1 = manual)
- gear - Number of forward gears
- carb - Number of carburetors

## Dataset mtcars

| | mpg | cyl | disp | hp | drat | wt | qsec | vs | ar |
|---|---|---|---|---|---|---|---|---|---|
| Mazda RX4 | 21.0 | 6 | 160.0 | 110 | 3.90 | 2.620 | 16.46 | 0 | |
| Mazda RX4 Wag | 21.0 | 6 | 160.0 | 110 | 3.90 | 2.875 | 17.02 | 0 | |
| Datsun 710 | 22.8 | 4 | 108.0 | 93 | 3.85 | 2.320 | 18.61 | 1 | |
| Hornet 4 Drive | 21.4 | 6 | 258.0 | 110 | 3.08 | 3.215 | 19.44 | 1 | |
| Hornet Sportabout | 18.7 | 8 | 360.0 | 175 | 3.15 | 3.440 | 17.02 | 0 | |
| Valiant | 18.1 | 6 | 225.0 | 105 | 2.76 | 3.460 | 20.22 | 1 | |
| Duster 360 | 14.3 | 8 | 360.0 | 245 | 3.21 | 3.570 | 15.84 | 0 | |
| Merc 240D | 24.4 | 4 | 146.7 | 62 | 3.69 | 3.190 | 20.00 | 1 | |
| Merc 230 | 22.8 | 4 | 140.8 | 95 | 3.92 | 3.150 | 22.90 | 1 | |
| Merc 280 | 19.2 | 6 | 167.6 | 123 | 3.92 | 3.440 | 18.30 | 1 | |
| Merc 280C | 17.8 | 6 | 167.6 | 123 | 3.92 | 3.440 | 18.90 | 1 | |
| Merc 450SE | 16.4 | 8 | 275.8 | 180 | 3.07 | 4.070 | 17.40 | 0 | |
| Merc 450SL | 17.3 | 8 | 275.8 | 180 | 3.07 | 3.730 | 17.60 | 0 | |
| Merc 450SLC | 15.2 | 8 | 275.8 | 180 | 3.07 | 3.780 | 18.00 | 0 | |
| Cadillac Fleetwood | 10.4 | 8 | 472.0 | 205 | 2.93 | 5.250 | 17.98 | 0 | |
| Lincoln Continental | 10.4 | 8 | 460.0 | 215 | 3.00 | 5.424 | 17.82 | 0 | |

## Distributions of two variables of mtcars

```
par(mfrow=c(1,2))
plot(density(mtcars$wt)); plot(density(mtcars$mpg))
```



## A simple regression model

### Dependent variable - miles per gallon (mpg)

### Independent variable - weight (wt)

```
m1 <- lm(mpg ~ wt,data=mtcars)
m1
```

```
##
## Call:
## lm(formula = mpg ~ wt, data = mtcars)
##
## Coefficients:
## (Intercept)           wt
##      37.285       -5.344
```

## Get the model summary

```
summary(m1)
```

```
##
## Call:
## lm(formula = mpg ~ wt, data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.5432 -2.3647 -0.1252  1.4096  6.8727
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  37.2851     1.8776  19.858  < 2e-16 ***
## wt           -5.3445     0.5591  -9.559 1.29e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
##
## Residual standard error: 3.046 on 30 degrees of freedom
```

## The model formula

### Model without intercept

```
m2 <- lm(mpg ~ - 1 + wt,data=mtcars)
summary(m2)$coefficients
```

```
##    Estimate Std. Error  t value     Pr(>|t|)
## wt 5.291624  0.5931801 8.920771 4.55314e-10
```

### Adding further variables

```
m3 <- lm(mpg ~ wt + cyl,data=mtcars)
summary(m3)$coefficients
```

```
##              Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) 39.686261  1.7149840 23.140893 3.043182e-20
## wt          -3.190972  0.7569065 -4.215808 2.220200e-04
## cyl         -1.507795  0.4146883 -3.635972 1.064282e-03
```

## The command as.formula

```
?as.formula
```

```
class(fo <- mpg ~ wt + cyl)
```

```
## [1] "formula"
```

```
m3 <- lm(fo,data=mtcars)
```

## The command `model.matrix`

```
?model.matrix
```

- See `Matrix::sparse.model.matrix` for increased efficiency on large dimension data.

## Further possibilities to specify the formula

### Interaction effect

```
# effect of cyl and interaction effect:
m3a<-lm(mpg~wt*cyl,data=mtcars)

# only interaction effect:
m3b<-lm(mpg~wt:cyl,data=mtcars)
```

### Take the logarithm

```
m3d<-lm(mpg~log(wt),data=mtcars)
```

## A model with interaction effect

-disp - Displacement (cu.in.)

```
m3d<-lm(mpg~wt*disp,data=mtcars)
m3dsum <- summary(m3d)
m3dsum$coefficients
```

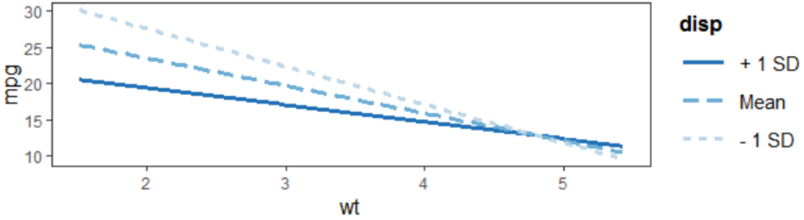
```
##                 Estimate  Std. Error   t value     Pr(>|t|)
## (Intercept) 44.08199770 3.123062627 14.114990 2.955567e-14
## wt          -6.49567966 1.313382622 -4.945763 3.216705e-05
## disp        -0.05635816 0.013238696 -4.257078 2.101721e-04
## wt:disp      0.01170542 0.003255102  3.596022 1.226988e-03
```

## Exploring interactions

```
install.packages("jtools")

library(jtools)
interact_plot(m3d, pred = "wt", modx = "disp")
```

- With a continuous moderator (in our case `disp`) you get three lines — 1 standard deviation above and below the mean and the mean itself.



## Example: object orientation

- m3 is now a special regression object
- Various functions can be applied to this object

```
predict(m3) # Prediction
resid(m3) # Residuals
```

```
##          Mazda RX4      Mazda RX4 Wag         Datsun 710     Horn
##           22.27914           21.46545           26.25203
## Hornet Sportabout            Valiant
##           16.64696           19.59873
```

```
##          Mazda RX4      Mazda RX4 Wag         Datsun 710     Horn
##         -1.2791447         -0.4654468         -3.4520262
## Hornet Sportabout            Valiant
##          2.0530424         -1.4987281
```

## Make model prediction

```
pre <- predict(m1)
head(mtcars$mpg)
```

```
## [1] 21.0 21.0 22.8 21.4 18.7 18.1
```

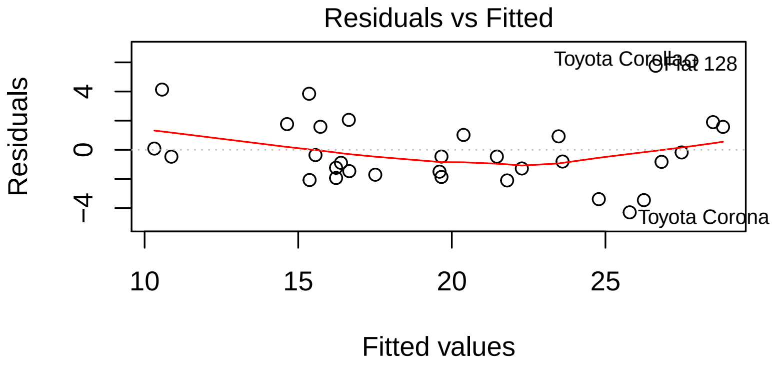
```
head(pre)
```

```
##         Mazda RX4     Mazda RX4 Wag        Datsun 710    Horn
##          23.28261          21.91977          24.88595
## Hornet Sportabout           Valiant
##          18.90014          18.79325
```

## Residual plot - model assumptions violated?

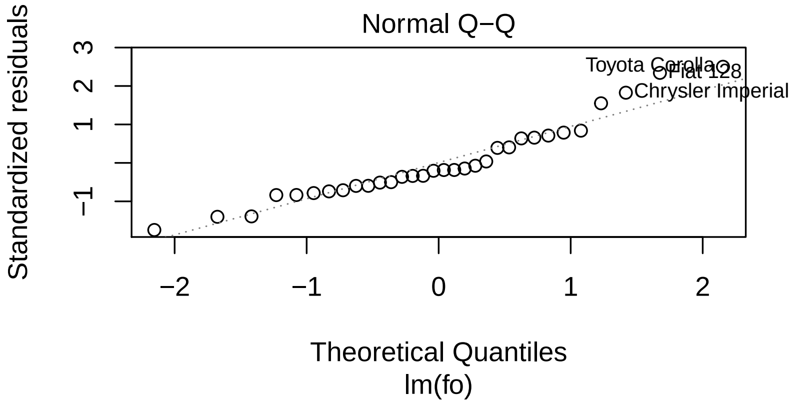
- the case if a deviation pattern from line

```
plot(m3,1)
```



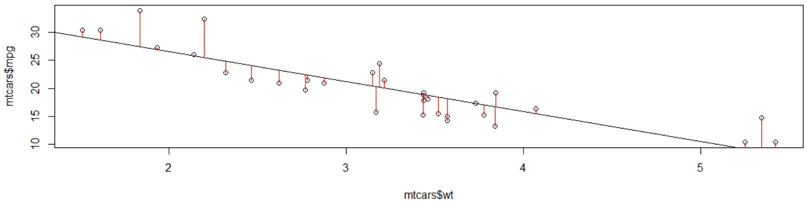
## Residual plot

```r
plot(m3,2)
```



## Regression diagnostic with base-R

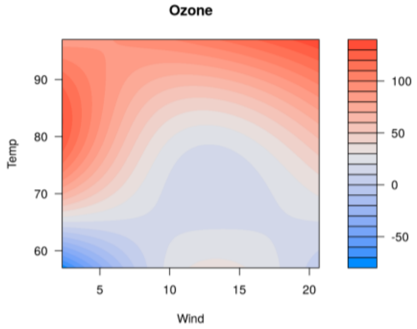
```
plot(mtcars$wt,mtcars$mpg)
abline(m1)
segments(mtcars$wt, mtcars$mpg, mtcars$wt, pre, col="red")
```

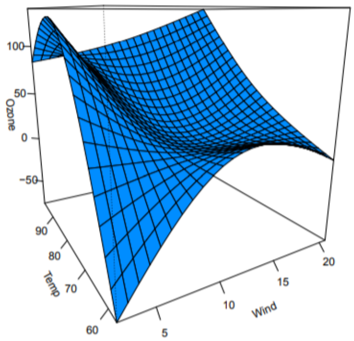


## THE VISREG-PACKAGE

```
install.packages("visreg")

library(visreg)
```

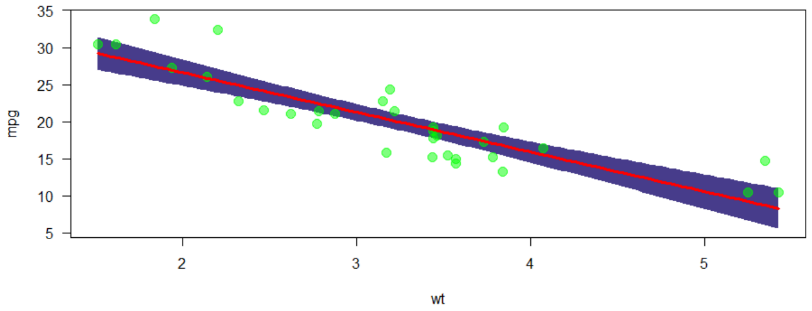




## The visreg-package

- The default-argument for type is conditional.
- Scatterplot of mpg and wt plus regression line and confidence bands

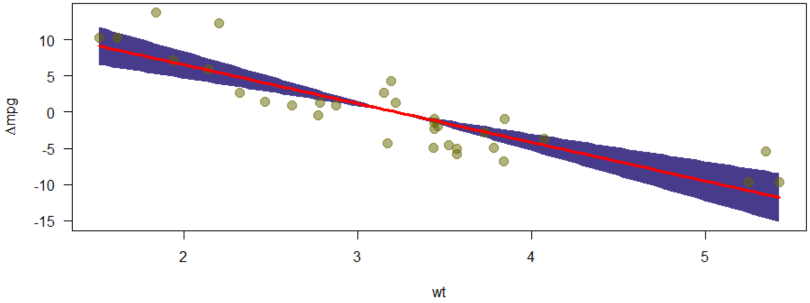
```
visreg(m1, "wt", type = "conditional")
```



## Visualisation with visreg

- Second argument - Specification covariate for visualisation
- plot shows the effect on the expected value of the response by moving the x variable away from a reference point on the x-axis (for numeric variables, the mean).

```r
visreg(m1, "wt", type = "contrast")
```



## Regression with factors

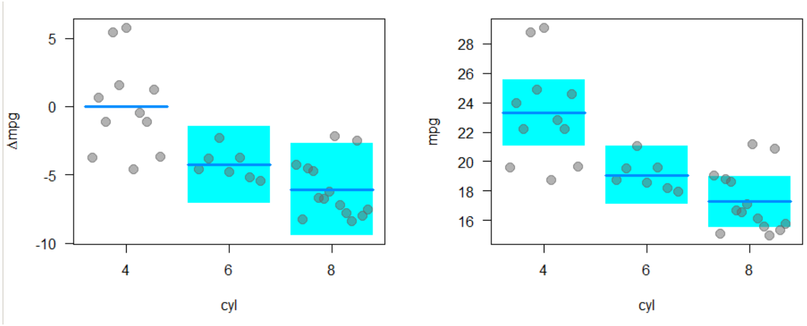
- The effects of factors can also be visualized with `visreg`:

```
mtcars$cyl <- as.factor(mtcars$cyl)
m4 <- lm(mpg ~ cyl + wt, data = mtcars)
# summary(m4)
```

```
##              Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) 33.990794  1.8877934 18.005569 6.257246e-17
## cyl6        -4.255582  1.3860728 -3.070244 4.717834e-03
## cyl8        -6.070860  1.6522878 -3.674214 9.991893e-04
## wt          -3.205613  0.7538957 -4.252065 2.130435e-04
```

## Effects of factors

```
par(mfrow=c(1,2))
visreg(m4, "cyl", type = "contrast")
visreg(m4, "cyl", type = "conditional")
```



## The command `model.matrix`

```
?model.matrix
```

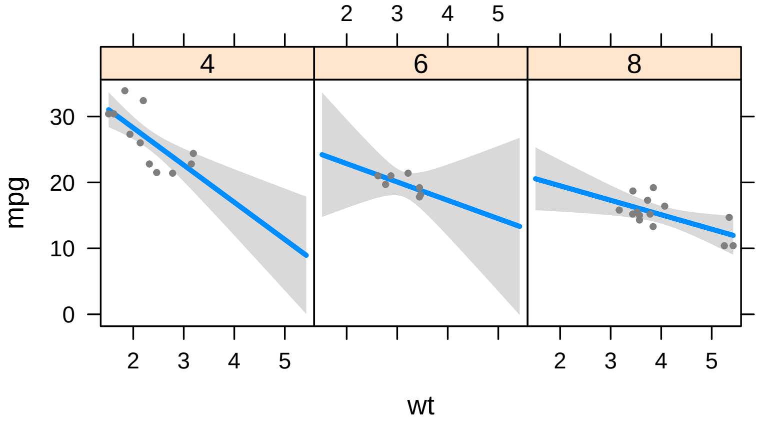
## The package visreg - Interactions

```
m5 <- lm(mpg ~ cyl*wt, data = mtcars)
# summary(m5)
```

```
##              Estimate Std. Error    t value     Pr(>|t|)
## (Intercept)  39.571196   3.193940 12.3894599 2.058359e-12
## cyl6        -11.162351   9.355346 -1.1931522 2.435843e-01
## cyl8        -15.703167   4.839464 -3.2448150 3.223216e-03
## wt           -5.647025   1.359498 -4.1537586 3.127578e-04
## cyl6:wt       2.866919   3.117330  0.9196716 3.661987e-01
## cyl8:wt       3.454587   1.627261  2.1229458 4.344037e-02
```

## Control of the graphic output with `layout`.

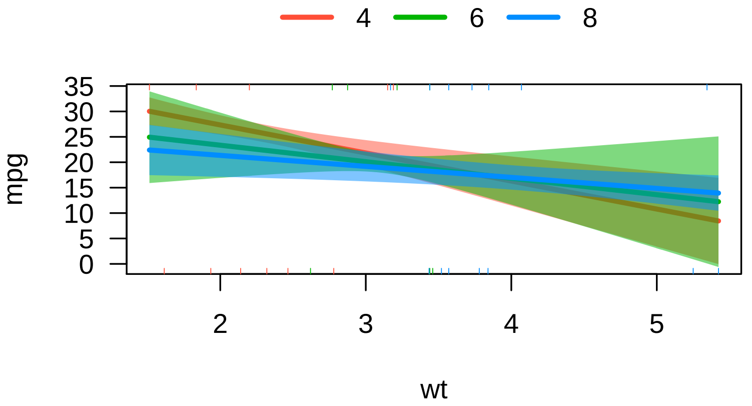
```
visreg(m5, "wt", by = "cyl",layout=c(3,1))
```



## The package visreg - Interactions overlay

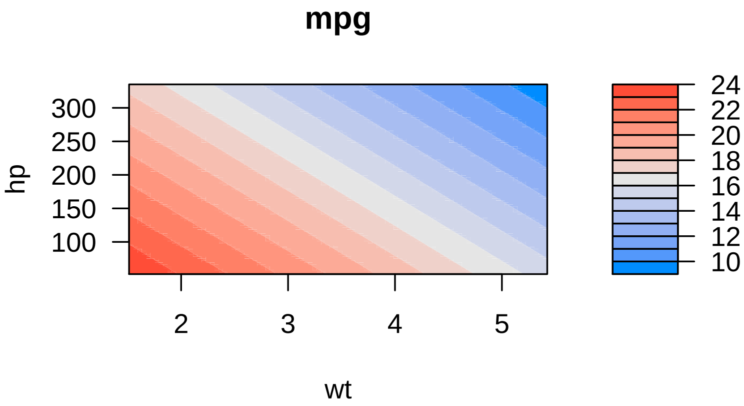
```
m6 <- lm(mpg ~ hp + wt * cyl, data = mtcars)

visreg(m6, "wt", by="cyl", overlay=TRUE, partial=FALSE)
```



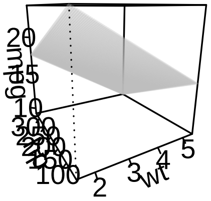
## The package visreg - visreg2d

```
visreg2d(m6, "wt", "hp", plot.type = "image")
```



## The package visreg - surface

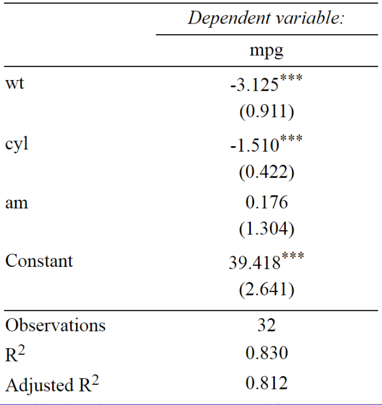
```
visreg2d(m6, "wt", "hp", plot.type = "persp")
```



## Producing nice table output with Package stargazer

```r
library(stargazer)
stargazer(m3, type="html")
```

### Example HTML output:



| | *Dependent variable:* |
|---|---|
| | mpg |
| wt | $-3.125^{***}$ |
| | (0.911) |
| cyl | $-1.510^{***}$ |
| | (0.422) |
| am | 0.176 |
| | (1.304) |
| Constant | $39.418^{***}$ |
| | (2.641) |
| Observations | 32 |
| $R^2$ | 0.830 |
| Adjusted $R^2$ | 0.812 |

## Exercise

- Install the package `AmesHousing` and create a processed Version of the Ames housing data with the variables `Sale_Price`, `Gr_Liv_Area` and `TotRms_AbvGrd`
- Create a suitable regression model with `Sale_Price` as dependent and `Gr_Liv_Area` and `TotRms_AbvGrd` as independent variables. What do you think?

## Multicollinearity

- As p increases we are more likely to capture multiple features that have some multicollinearity.
- When multicollinearity exists, we often see high variability in our coefficient terms.
- E.g. we have a correlation of 0.801 between `Gr_Liv_Area` and `TotRms_AbvGrd`
- Both variables are strongly correlated to the response variable (`Sale_Price`).

```
cor(ames_data[,c("Sale_Price","Gr_Liv_Area","TotRms_AbvGrd")])
```

```
##               Sale_Price Gr_Liv_Area TotRms_AbvGrd
## Sale_Price     1.0000000   0.7067799     0.4954744
## Gr_Liv_Area    0.7067799   1.0000000     0.8077721
## TotRms_AbvGrd  0.4954744   0.8077721     1.0000000
```

## Multicollinearity

```
ames_data <- AmesHousing::make_ames()

lm(Sale_Price ~ Gr_Liv_Area + TotRms_AbvGrd, data = ames_data)
```

```
##
## Call:
## lm(formula = Sale_Price ~ Gr_Liv_Area + TotRms_AbvGrd, data =
##
## Coefficients:
##   (Intercept)    Gr_Liv_Area  TotRms_AbvGrd
##       42767.6          139.4       -11025.9
```

- When we fit a model with both these variables we get a positive coefficient for `Gr_Liv_Area` but a negative coefficient for `TotRms_AbvGrd`, suggesting one has a positive impact to Sale_Price and the other a negative impact.

## Seperated models

- If we refit the model with each variable independently, they both show a positive impact.
- The Gr_Liv_Area effect is now smaller and the TotRms_AbvGrd is positive with a much larger magnitude.

```
lm(Sale_Price ~ Gr_Liv_Area, data = ames_data)$coefficients
```

```
## (Intercept) Gr_Liv_Area
##   13289.634     111.694
```

```
lm(Sale_Price ~ TotRms_AbvGrd, data = ames_data)$coefficients
```

```
##   (Intercept) TotRms_AbvGrd
##      18665.40      25163.83
```

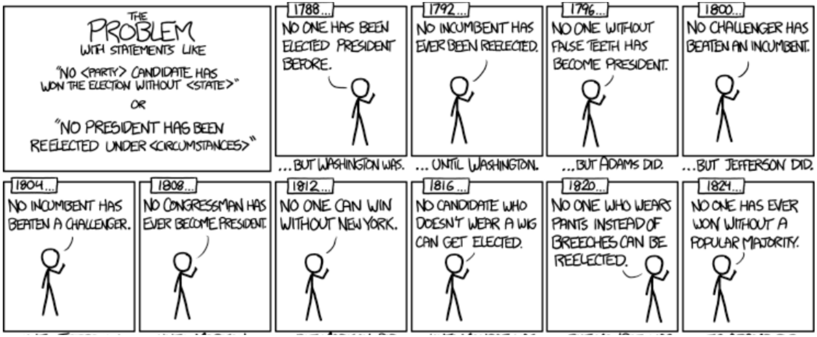
- This is a common result when collinearity exists.
- Coefficients for correlated features become over-inflated and can fluctuate significantly.

## Consequences

- One consequence of these large fluctuations in the coefficient terms is **overfitting**, which means we have high variance in the bias-variance tradeoff space.
- We can use tools such as variance inflaction factors (Myers, 1994) to identify and remove those strongly correlated variables, but it is not always clear which variable(s) to remove.
- Nor do we always wish to remove variables as this may be removing signal in our data.

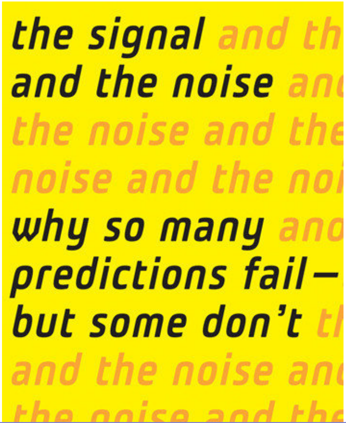
## The problem - Overfitting

- Our model doesn't generalize well from our training data to unseen data.

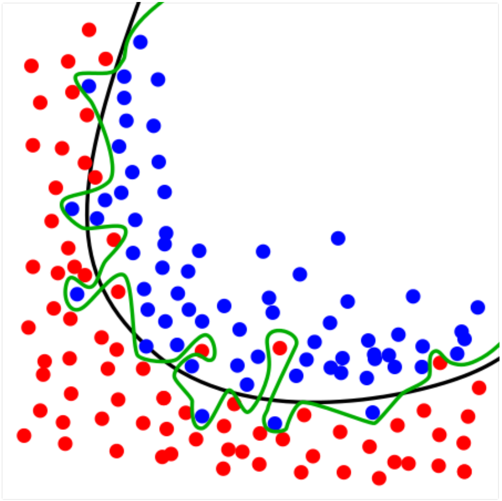

THE PROBLEM WITH STATEMENTS LIKE
"NO <PARTY> CANDIDATE HAS WON THE ELECTION WITHOUT <STATE>"
OR
"NO PRESIDENT HAS BEEN REELECTED UNDER <CIRCUMSTANCES>"
1788...
NO ONE HAS BEEN ELECTED PRESIDENT BEFORE.
...BUT WASHINGTON WAS.
1792...
NO INCUMBENT HAS EVER BEEN REELECTED.
...UNTIL WASHINGTON.
1796...
NO ONE WITHOUT FALSE TEETH HAS BECOME PRESIDENT.
...BUT ADAMS DID.
1800...
NO CHALLENGER HAS BEATEN AN INCUMBENT.
...BUT JEFFERSON DID.
1804...
NO INCUMBENT HAS BEATEN A CHALLENGER.
1808...
NO CONGRESSMAN HAS EVER BECOME PRESIDENT.
1812...
NO ONE CAN WIN WITHOUT NEW YORK.
1816...
NO CANDIDATE WHO DOESN'T WEAR A WIG CAN GET ELECTED.
1820...
NO ONE WHO WEARS PANTS INSTEAD OF BREECHES CAN BE REELECTED.
1824...
NO ONE HAS EVER WON WITHOUT A POPULAR MAJORITY.

## The Signal and the Noise

- In predictive modeling, you can think of the “signal” as the true underlying pattern that you wish to learn from the data.
- “Noise,” on the other hand, refers to the irrelevant information or randomness in a dataset.



## Overfitting.



The green line represents an overfitted model and the black line represents a regularized model. While the green line best follows the training data, it

## What can be done against overvitting

- Cross Validation
- Train with more data
- Remove features
- Regularization - e.g. ridge and lasso regression
- Ensembling - e.g. bagging and boosting

## Cross-validation

- Cross-validation is a powerful preventative measure against overfitting.
- Use your initial training data to generate multiple mini train-test splits. Use these splits to tune your model.

### Cross Validation in R

```
library(tidyverse)
library(caret)
```

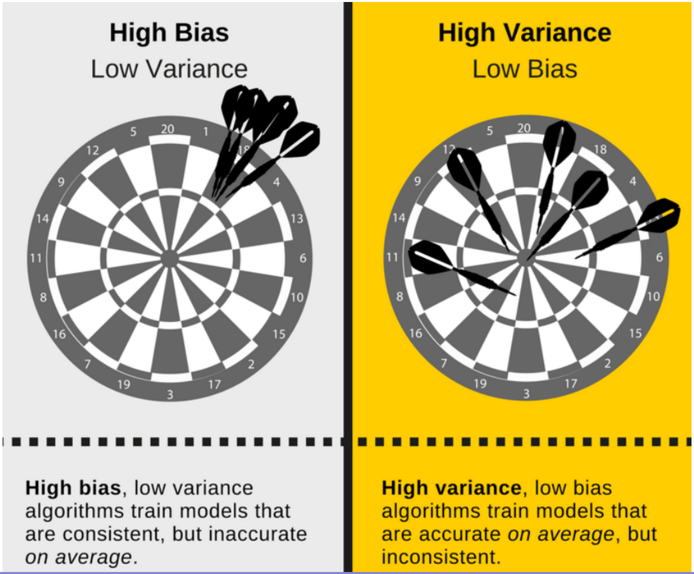
```
data("swiss")
```

```
training.samples <- swiss$Fertility %>%
createDataPartition(p = 0.8, list = FALSE)
train.data  <- swiss[training.samples, ]
test.data <- swiss[-training.samples, ]

# Build the model
model <- lm(Fertility ~., data = train.data)
# Make predictions and compute the R2, RMSE and MAE
predictions <- model %>% predict(test.data)
```

## K-FOLD CROSS VALIDATION

## The bias variance tradeoff



**High bias**, low variance algorithms train models that are consistent, but inaccurate *on average*.

**High variance**, low bias algorithms train models that are accurate *on average*, but inconsistent.

## Good literature for linear regression in R

### Useful PDF document:

J H Maindonald - **Using R for Data Analysis and Graphics Introduction, Code and Commentary**

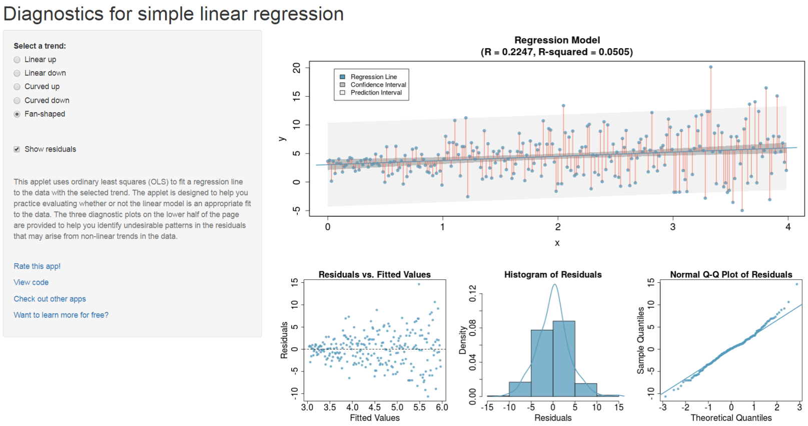
- Introduction to R
- Data analysis
- Statistical models
- Inference concepts
- Regression with one predictor
- Multiple linear regression
- Extending the linear model
- ...

## Links - linear regression

- Regression - **r-bloggers**
- The complete book of **Faraway**- very intuitive
- Good introduction on **Quick-R**
- **Multiple regression**
- **15 Types of Regression you should know**
- `ggeffects` - **Create Tidy Data Frames of Marginal Effects for 'ggplot' from Model Outputs**

## Shiny App - Diagnostics for simple linear regression

https://gallery.shinyapps.io/slr_diag/



Diagnostics for simple linear regression

**Select a trend:**

- Linear up
- Linear down
- Curved up
- Curved down
- [✓ Fan-shaped]

[✓ Show residuals]

This applet uses ordinary least squares (OLS) to fit a regression line to the data with the selected trend. The applet is designed to help you practice evaluating whether or not the linear model is an appropriate fit to the data. The three diagnostic plots on the lower half of the page are provided to help you identify undesirable patterns in the residuals that may arise from non-linear trends in the data.

Rate this app!

View code

Check out other apps

Want to learn more for free?

- Shiny App - Simple Linear Regression

## Resources

- machine learning iteration