Coursework 2 Answer

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Contents

1 Question a)

1.1 Question a) i)

Recall that, for a European call option as

$$g\left(t, S_t\right)=S_t e^{-q(T-t)} \Phi\left(d_1\right)-K e^{-\rho(T-t)} \Phi\left(d_2\right)$$

where

$$\begin{aligned} d_1 &= \frac{\log \left(\frac{S_t}{K}\right)+\left(\rho-q+\frac{1}{2} \sigma^2\right)(T-t)}{\sigma \sqrt{T-t}} \\ d_2 &= d_1-\sigma \sqrt{T-t} \end{aligned}$$

and we have $S_0=5.35$, $K=5.65$, $\rho=5.4\%$, $T=\frac{9}{12}=0.75$, $\sigma=0.3$. In addition, since the stock is non-dividend paying, we have $q=0$.

Hence, we have

$$\begin{aligned} d_1 &= \frac{\log \left(\frac{5.35}{5.65}\right)+\left(0.054+\frac{1}{2} \times 0.3^2\right) \times 0.75}{0.3 \sqrt{0.75}} \\ &= 0.07578 \\ d_2 &= d_1-0.3 \sqrt{0.75} \\ &= -0.1840 \end{aligned}$$

subtitute d_1 and d_2 into the formula, we have

$$\begin{aligned} g(0, S_0) &= S_0 \Phi(d_1)-K e^{-\rho T} \Phi(d_2) \\ &= 5.35 \Phi(0.07578)-5.65 e^{-0.054 \times 0.75} \Phi(-0.1840) \\ &= 0.5198 \end{aligned}$$

1.2 Question a) ii)

With Monte-Carlo simulation, we have European call option:

$$\begin{aligned} g\left(t, S_t\right) &= e^{-\rho(T-t)} \mathbb{E}\left[\left(S_T-K\right)_{+} \mid \mathcal{F}_t\right] \\ &\approx e^{-\rho(T-t)} \frac{1}{M} \sum_{i=1}^M\left(S_t e^{a(T-t)+\sigma z_i \sqrt{T-t}}-K\right)_{+} \end{aligned}$$

Using MATLAB, which shown in appendix 1, we can have the following solution

- For M = 1000, the call option price is: 0.53721
- For M = 10000, the call option price is: 0.5059
- For M = 100000, the call option price is: 0.52095
- For M = 1000000, the call option price is: 0.51886
- For M = 10000000, the call option price is: 0.51948
- For M = 100000000, the call option price is: 0.51989

1.3 Question a) iii)

As we calculated in Question a) i), we expect the call option price to be 0.5198.

As the error is decreasing as M increases, we can see that the Monte-Carlo simulation is converging to the true value, which is shown in Figure 1.

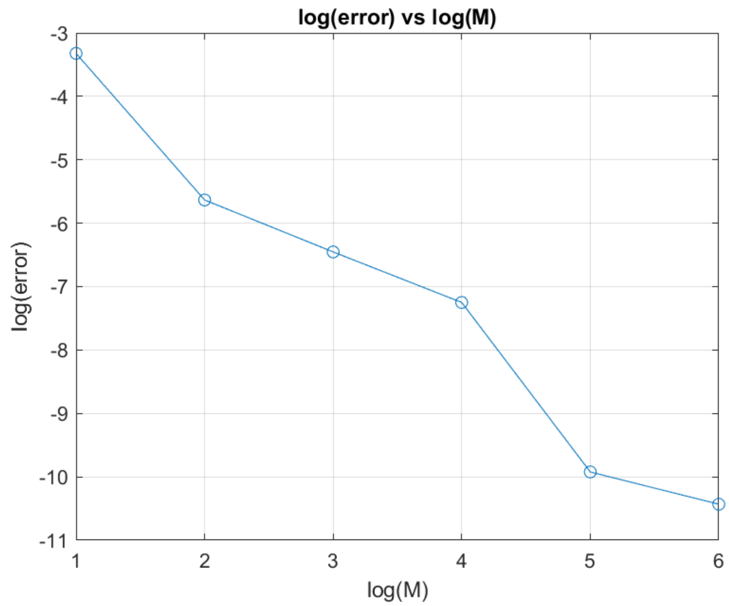
Hence we conclude that the as M increases, the Monte-Carlo simulation is more accurate. In addition while M increases, the running time of the simulation also increases.

2 Question b)

by adjusting the code in Question a) ii) to match exotic option payoff $C = \max\left(8\cos\left(S_T\right) - 5.65, 0\right)$, which is shown in appendix 2, we have

- For M = 1000, the exotic option price is: 0.50235
- For M = 10000, the exotic option price is: 0.49115
- For M = 100000, the exotic option price is: 0.48023
- For M = 1000000, the exotic option price is: 0.48059
- For M = 10000000, the exotic option price is: 0.48092
- For M = 100000000, the exotic option price is: 0.48067

Figure 1: Plot of the call option price against M



3 Question c)

3.1 Question c) i)

We have $S_0 = 5.35$, $K = 5.65$, $\rho = 5.4\%$, $T = \frac{12}{12} = 1$, $\sigma = 0.3$.

Recall that,

$$
\begin{aligned}
\sigma_G &= \frac{\sigma}{\sqrt{3}} \\
&= 0.1732 \\
b &= \frac{1}{2}\left(\rho - \frac{\sigma_G^2}{2}\right) \\
&= \frac{1}{2}\left(0.0054 - \frac{0.1732^2}{2}\right) \\
&= 0.0195 \\
d_1 &= \frac{\log\left(\frac{S_t}{K}\right) + \left(b + \frac{\sigma_G^2}{2}\right)(T-t)}{\sigma_G\sqrt{T-t}} \\
&= -0.1158 \\
d_2 &= d_1 - \sigma_G\sqrt{T-t} \\
&= -0.1158 - 0.1732\sqrt{1} \\
&= -0.2890
\end{aligned}
$$

Hence, we can subtitute the values into the formula,

$$
\begin{aligned}
g(0, S_0) &= S_0 e^{(b-\rho)T}\Phi(d_1) - K e^{-\rho T}\Phi(d_2) \\
&= 5.35 \times e^{(0.0195-0.054)\times 1}\Phi(-0.1158) - 5.65 \times e^{-0.054\times 1}\Phi(-0.2890) \\
&= 0.2782
\end{aligned}
$$

3.2 Question c) ii)

For the Monte-Carlo simulation, we used a matrix sampling M points at a time. Then do the calculation n times for simulating Asian call option. The code is shown in appendix 3,

Using MATLAB we have the following solution

- For M = 1000, the asian option price is: 0.28874
- For M = 10000, the asian option price is: 0.2827
- For M = 100000, the asian option price is: 0.28446
- For M = 1000000, the asian option price is: 0.28123
- For M = 10000000, the asian option price is: 0.28159

Appendix

A MATLAB Code

Listing 1: Question a)ii)

```
S0 = 5.35;
K = 5.65;
r = 0.054;
T = 0.75;
sigma = 0.3;

Ms = [1000, 10000, 100000, 1000000, 10000000, 100000000];

gt = 0.5198;

for i = 1:length(Ms)
    M = Ms(i);
    callPrice = MonteCarloCallPrice(S0, K, r, T, sigma, M);
    disp(['For M = ', num2str(M), ', the call option price ...
        is: ', num2str(callPrice)]);
end

% plot error
errors = zeros(length(Ms), 1);
for i = 1:length(Ms)
    M = Ms(i);
    callPrice = MonteCarloCallPrice(S0, K, r, T, sigma, M);
    errors(i) = abs(callPrice - gt);
end

plot(1:length(Ms), log(errors), 'o-');
xlabel('log(M)');
ylabel('log(error)');
title('log(error) vs log(M)');
grid on;
saveas(gcf, 'a_2.png');


function callPrice = MonteCarloCallPrice(S0, K, r, T, ...
    sigma, M)
    % calculate a
    a = r - 0.5 * sigma^2;
```

```
    % init callValues
    callValues = zeros(M, 1);

    % simulate M times
    for i = 1:M
        % generate a random number from standard Brownian ...
            motion
        z = randn;

        % calculate ST
        ST = S0 * exp(a * T + sigma * z * sqrt(T));

        % calculate call value
        callValues(i) = max(ST - K, 0);
    end

    % calculate call price
    callPrice = exp(-r * T) * mean(callValues);
end
```

Listing 2: Question b)

```
S0 = 5.35;
K = 5.65;
r = 0.054;
T = 0.75;
sigma = 0.3;

Ms = [1000, 10000, 100000, 1000000, 10000000, 100000000];

gt = 0.5198;

for i = 1:length(Ms)
    M = Ms(i);
    callPrice = MonteCarloCallPrice(S0, K, r, T, sigma, M);
    disp(['For M = ', num2str(M), ', the exotic option ...
        price is: ', num2str(callPrice)]);
end

function callPrice = MonteCarloCallPrice(S0, K, r, T, ...
    sigma, M)
    % calculate a
    a = r - 0.5 * sigma^2;
```

```
    % init callValues
    callValues = zeros(M, 1);

    % simulate M times
    for i = 1:M
        % generate a random number from standard Brownian ...
            motion.
        z = randn;

        % calculate ST
        ST = S0 * exp(a * T + sigma * z * sqrt(T));

        % calculate call value
        callValues(i) = max(8 * cos(ST) - 5.65, 0);
    end

    % calculate call price
    callPrice = exp(-r * T) * mean(callValues);
end
```

Listing 3: Question c)ii)

```
S0 = 5.35;
K = 5.65;
r = 0.054;
T = 1;
sigma = 0.3;

Ms = [1000, 10000, 100000, 1000000, 10000000];
n = 100;

for i = 1:length(Ms)
    M = Ms(i);
    asianOptionPrice = AsianCallOptionMonteCarlo(S0, K, r, ...
        T, sigma, M, n);
    disp(['For M = ', num2str(M), ', the asian option ...
        price is: ', num2str(asianOptionPrice)]);
end

function asianOptionPrice = AsianCallOptionMonteCarlo(S0, ...
    K, r, T, sigma, M, n)
    dt = T / n;

    % generate n time points
```

```
    t = linspace(dt, T, n);

    % generate standard Brownian motion.
    Z = randn(n, M) * sqrt(dt);

    % calculate all stock prices
    S = S0 * exp(cumsum((r - 0.5 * sigma^2) * dt + sigma * ...
        Z, 1));

    % calculate each path's geometric mean
    A_T = exp(mean(log(S), 1));

    % calculate call value
    callValues = max(A_T - K, 0);

    % calculate call price
    asianOptionPrice = exp(-r * T) * mean(callValues);
end
```