MA3071 Financial Mathematics - DLI
Year 2023-2024

Coursework 2

INSTRUCTIONS AND DEADLINE:

Please electronically submit one piece of written or typed work per person as a single file via *Blackboard* by **December 8, 2023, at 16:00 (UK) / 23:59 (China)**.

You can use this page as a cover page and write your name, student ID, and signature below.

Name:

Student ID:

Signature:

MARKING CRITERIA:

>> Coursework is marked out of 100 points, with the number of marks for each main question indicated at the beginning of each.

>> Clearly justify and explain your answers. You are expected to use MATLAB for calculations. A printout without full explanations of the formulas and reasoning will result in a deduction of marks.

>> You are required to submit a single PDF file containing justifications, explanations, and codes for each question. Include your MATLAB code in the appendix of your answers, ensuring that it is properly commented. You can copy or screenshot your codes into the PDF file without providing the code files in any other format.

>> You can submit your answers up to 3 attempts when submitting via Blackboard. Only the last attempt of your submission will be assessed. Email submissions won't be accepted.

Please note: *Any numerical results should be rounded to four decimal places.*

Question [100 marks]

Consider a continuous-time market with a fixed interest rate of 5.4% per annum. The current price of a non-dividend paying stock in this market is 5.35, and its future price is modelled by an SDE.

$$dS_t = 0.054 S_t dt + 0.3 S_t dB_t \tag{1}$$

where $\{B_t; t \geq 0\}$ is a standard Brownian motion.

a) For a European call option on this stock with a strike price of 5.65 and a maturity of 9 months, answer the following questions:

   i) **[5 marks]** Calculate the current arbitrage-free price of the call option using the Black-Scholes model.

   ii) **[10 marks]** Using the Monte-Carlo method with $M = 10^k$, $k = 3, 4, 5, 6, 7, 8$, calculate the current arbitrage-free price of the call option.

   iii) **[5 marks]** Compare the results you obtained in i) and ii), and comment on the errors and complexities of the Monte-Carlo method.

b) **[10 marks]** For an exotic option payoff $C = \max(8\cos(S_T) - 5.65, 0)$ and a 9-month term to maturity, estimate the current arbitrage-free option price using the Monte-Carlo method.

An Asian option is a special type of option contract. For Asian options the payoff is determined by the average underlying asset price over some pre-set period of time. This is different from the case of the usual European option and American option, where the payoff of the option contract depends on the price of the underlying instrument at exercise. Asian options are thus one of the basic forms of exotic options.

Consider a European-style (can only be exercised at maturity) Asian call option on the stock modelled by (1). In the continuous case, the payoff with the geometric average price of the underlying asset is defined by

$$C = \max(A_T - K, 0), \quad A_T = e^{\frac{1}{T}\int_0^T \log(S_t)dt}$$

where A_T denotes the geometric average price over the period $[0, T]$ and K is a fixed strike price.

c) Given that the interest rate is still 5.4% per annum, the time to maturity is 12 months, and $K = 5.65$, answer the following questions:

   i) **[10 marks]** Calculate the current arbitrage-free option price of the Asian call option with geometric average price using the Black-Scholes model which is given by

$$
\begin{aligned}
g(t, S_t) &= S_t e^{(b-\rho)(T-t)}\Phi(d_1) - K e^{-\rho(T-t)}\Phi(d_2) \\
b &= \frac{1}{2}\left(\rho - \frac{\sigma_G^2}{2}\right) \\
\sigma_G &= \frac{\sigma}{\sqrt{3}} \\
d_1 &= \frac{\log\left(\frac{S_t}{K}\right) + \left(b + \frac{\sigma_G^2}{2}\right)(T-t)}{\sigma_G\sqrt{T-t}} \\
d_2 &= d_1 - \sigma_G\sqrt{T-t}
\end{aligned}
$$

ii) **[60 marks]** Using the Monte-Carlo method, calculate the current arbitrage-free price of the Asian call option.

Hint: You can split the period $[0, T]$ into n periods such that

$$[t_0, t_1) \cup [t_1, t_2) \cup \cdots \cup [t_{n-1}, t_n]$$

where $0 = t_0 < t_1 < \cdots < t_{n-1} < t_n = T$. Then, letting $\Delta t_i = t_i - t_{i-1}$, and for a large n, the geometric average price A_T can be approximated as

$$A_T = e^{\frac{1}{T}\int_0^T \log(S_t)dt} \approx e^{\frac{1}{T}\sum_{i=1}^n \log\left(S_{t_i}\right)\Delta t_i}$$

You can then collect M sample paths of S_t, each consisting of n observations of $(S_{t_1}, \cdots, S_{t_n})$.

You also need to take care: S_t has independent increments over each period $[t_{i-1}, t_i)$.

Take $n = 100$ and choose $M = 10^k$, $k = 3, 4, 5, 6, 7$ for this question.