Scores

1. **Proof Question**(15 points)

Please prove that the 2-norm of matrix A is equal to its maximum singular value, i.e. $\sigma_1(A) = \max\limits_{||x||_2=1} ||Ax||_2$.

Scores

2. **Short-answer Question**(10 points)

Consider the function $f(x) = x_1^2 + x_2^2$, $x = (x_1, x_2) \in R^2$, and the sequence of iterative points $x^k = (1 + \frac{1}{2^k})(\cos k, \sin k)^T$, $k = 1, 2, \ldots,$. Please explain

(a) if the sequence $\{f(x^{k+1})\}$ is convergent, and if convergent, please analyze the rate of convergence;

(b) if the sequence $\{x^{k+1}\}$is convergent, and if convergent, please analyze the rate of convergence.

Scores

3. **Short-answer Question**(15 points)

Consider the following least-squares problem with a linear equality-constrained:

$$\min_{x \in R^n} ||Ax - b||_2^2, \quad s.t. \quad Gx = h,$$

where $A \in R^{m\times n}$, $rank(A) = n, G \in R^{p\times n}$ and $rank(G) = p$. Please write the Lagrangian function and the dual function, and derive the dual problem.

Scores

4. Short-answer Question(15 points)

Consider the following quadratic programming

$$\min_{x \in R^n} \quad x^T A x + 2b^T x$$

where $A \in S^n, b \in R^n$. In order to ensure the existence of the optimal solution to this problem, please state what properties of A, b need to satisfy and prove the corresponding conclusion?

Scores

5. Short-answer Question(20分)

Consider the following linear programming problem

$$\begin{aligned} \min_{x} \quad & c^T x \\ \text{s.t.} \quad & Ax = b \\ & x \geqslant 0 \end{aligned}$$

where A is row full rank. Please derive the dual problem of the linear programming problem. Please write the ADMM algorithm framework for solving the dual problem. (The specific forms of the optimal solution to the subproblems needs to be given in detail)

Scores

6. Short-answer Question(25 points)

For the LASSO problem

$$\min_{x \in R^n} \quad \frac{1}{2}||Ax - b||_2^2 + \mu||x||_1,$$

please write the augmented Lagrangian methods for the primal problem and its dual problem.