1. Solve the following linear systems by Jacobi method and Gauss-Seidel method. (the solutions are accurate to five decimal places)

(1) $\begin{pmatrix} 6 & 2 & -1 \\ 1 & 4 & -2 \\ -3 & 2 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$;

(2) $\begin{pmatrix} 1 & 0.8 & 0.8 \\ 0.8 & 1 & 0.8 \\ 0.8 & 0.8 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$;

(3) $\begin{pmatrix} 1 & 3 \\ -7 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$;

Question: (a) First analyze the convergence theoretically, and check it by programming.

(b) Compare their convergent rate.

(c) List a table to compare the two methods. (Including the solutions of the system, the number of iteration, the results of convergence).

2. The equation

$$x + \mathrm{e}^x = 0$$

has the root around $x \approx -0.6$.

(a) Implement the bisection algorithm to find a better approximation using the initial interval $[-1,0]$. Output the results in a table

n	a_n	b_n	c_n	$f(c_n)$

(perform 22 iterations.)

(b) Fixed-point iteration. Apply it to $\mathrm{x} = \mathrm{e}^x$ starting with $\mathrm{x}_0 = -1$.

(c) Newton's method with initial value $\mathrm{x}_0 = -1$.

The approximations are accurate to six significant places.