Problem 1

a)

Line through the origin $(0;0;0)$ $\perp$ to the plane $2x - y - z = 4$

According the Component equation simplified for a Plane:

$$Ax + By + Cz = 0, \text{ where}$$

$$D = A\underset{0}{\underset{\shortparallel}{x_0}} + B\underset{0}{\underset{\shortparallel}{y_0}} + C\underset{0}{\underset{\shortparallel}{z_0}}$$

$A = 2$ ; $B = -1$ ; $C = -1$ So parametric equation for the line $\perp$ to the plane through $(0,0,0)$ is

$$x = 2t, \quad y = -t, \quad z = -t$$

This line intersects the plane $3x - 5y + 2z = 6$ when $t$ is the solution of

$$3\cdot(2t) - 5\cdot(-t) + 2\cdot(-t) = 6$$

$$6t + 5t - 2t = 6$$

$$9t = 6$$

$$t = \frac{6}{9} = \frac{2}{3}$$

$$\boxed{t = \frac{2}{3}}$$

So, $x = 2\cdot\frac{2}{3} = \frac{4}{3}$ the point of intersection is $\boxed{\left(\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}\right)}$

$y = -\frac{2}{3}$

$z = -\left(\frac{2}{3}\right)$

b) $v = a_1 i + a_2 j + a_3 k$ $\quad a_1 - ?$
$a_2 - ?$
$a_3 - ?$

$|v| = \sqrt{a_1^2 + a_2^2 + a_3^2} = 2$

$v \underbrace{\parallel}_{\text{parallel}}$ to line of intersection of
$x + 2y + z - 1 = 0$
$x - y + 2z + 7 = 0$

$\Downarrow$

$$v = n_1 \times n_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = [4 - (-1)]i +$$

$$+ [(2-1)]j + [(-1) - 2]k =$$

$$= 5i - 1j - 3k$$

$|v| = \sqrt{25 + 1 + 9} = \sqrt{35}$

Unit vector is $\frac{v}{|v|} = \frac{1}{\sqrt{35}}(5i - j - 3k)$

Desired vector ~~is~~ of Magnitude 2 is

$$2 \cdot \left(\frac{v}{|v|}\right) = \boxed{\frac{2}{\sqrt{35}}(5i - j - 3k)}$$

c) $r(t) = (e^t \cos t)i + (e^t \sin t)j$

a is the acceleration. So, $a(t) = r''(t)$

$r'(t) = (e^t \cdot \cos t - e^t \cdot \sin t)i + (e^t \sin t + e^t \cos t)j$

$r''(t) = (-2e^t \sin t)i + (2e^t \cos t)j$

Let $\theta$ be the angle between $\underline{r}$ and $\underline{a}$

Then $\theta = \cos^{-1}\left[\dfrac{r \cdot a}{|r| \cdot |a|}\right]$

$r \cdot a = -2e^{2t} \cos t \cdot \sin t + 2e^{2t} \sin t \cos t =$
$= 0$

$\theta = \cos^{-1}\left(\dfrac{0}{|r| \cdot |a|}\right) = \cos^{-1} 0 = \dfrac{\pi}{2}$ for all $t$

↓ it is not function of $t$ (time!), so it never changes.

d) $r(t) = t\,i + t^2 j + t^3 k$ at $(1,1,1)$

① Normal plane : $T$ (unit tangent) $\perp$ to Normal plane.

$$v(t) = r'(t) = i + 2t\,j + 3t^2 k$$

$$|v| = \sqrt{1+4t^2+9t^4}$$

At $(1,1,1)$ $\quad r(t) = \underbrace{t}_{=1}\,i + \underbrace{t^2}_{=1}\,j + \underbrace{t^3}_{=1}\,k$

So $t = 1$ at $(1,1,1)$

$$|v(t=1)| = \sqrt{1+4+9} = \sqrt{14}$$

$$T(1) = \frac{1}{\sqrt{14}}\,i + \frac{2}{\sqrt{14}}\,j + \frac{3}{\sqrt{14}}\,k$$

So component equation of the Normal Plane is

$$A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$$

$$\frac{1}{\sqrt{14}}(x-1) + \frac{2}{\sqrt{14}}(y-1) + \frac{3}{\sqrt{14}}(z-1) = 0$$

$$\boxed{x + 2y + 3z = 6}$$

② Rectifying plane : $N$ (principal normal) $\perp$ to Rectifying plane

$$a = \underbrace{\frac{d^2 s}{dt^2}}_{\frac{d|v|}{dt}}\,T + \overbrace{\underbrace{K}_{\text{curvature}}\left(\frac{ds}{dt}\right)^2}^{K|v|^2}\,N$$

$$a = 2j + 6t\,K \quad , \quad a(1) = 2j + 6K$$

$$v(1) \times a(1) = \begin{vmatrix} i & j & K \\ 1 & 2 & 3 \\ 0 & 2 & 6 \end{vmatrix} = 6i - 6j + 2K$$

$$|v(1) \times a(1)| = \sqrt{36+36+4} = \sqrt{76}$$

Vector Formula for Curvature

$$K = \frac{|v \times a|}{|v|^3} \quad , \text{ so}$$

- $$K(1) = \frac{\sqrt{76}}{(\sqrt{14})^3} = \frac{\sqrt{19}}{7\sqrt{14}}$$

$$\frac{ds}{dt} = \underbrace{|v(t)|}_{= \sqrt{1+4t^2+9t^4}} \Rightarrow \left.\frac{d^2 s}{dt^2}\right|_{t=1} = \frac{1}{2}\left(1+4t^2+9t^4\right)^{-1/2}\left(8t + 36t^3\right)\Big|_{t=1} = \frac{22}{\sqrt{14}}$$

$$2j + 6K = \frac{22}{\sqrt{14}} \cdot \underbrace{\left[\frac{1}{\sqrt{14}} \cdot (i + 2j + 3K)\right]}_{T} +$$

$$+ \underbrace{\frac{\sqrt{19}}{7\sqrt{14}}}_{= K} \cdot \underbrace{(\sqrt{14})^2}_{= |v|^2} \cdot N$$

$$N = \frac{\sqrt{14}}{2\sqrt{19}} \cdot \left[-\frac{11}{7} i - \frac{8}{7} j + \frac{9}{7} K\right]$$

Rectif. Plane : $-\frac{11}{7}(x-1) - \frac{8}{7}(y-1) + \frac{9}{7}(z-1) = 0$

R.P $\Rightarrow$ $\boxed{11x + 8y - 9z = 10}$

③ Osculating Plane : B (Binormal) $\perp$ to Osculating Plane.

$$B(1) = T(1) \times N(1) =$$

$$= \frac{\sqrt{14}}{2\sqrt{19}} \cdot \frac{1}{\sqrt{14}} \cdot \frac{1}{7} \cdot \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -11 & -8 & 9 \end{vmatrix} = \frac{1}{\sqrt{19}}(3i - 3j + k)$$

Os. P : $3(x-1) - 3(y-1) + (z-1) = 0$

$$\boxed{3x - 3y + z = 1}$$