More Examples for Lecture 2.

MA2032 Vector Calculus

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Vectors in the Plane

Example 1

Find the component form of the vector $\overrightarrow{\mathrm{OP}}$ where O is the origin and P is the midpoint of segment RS, where $R = (2, -1)$ and $S = (-4, 3)$.

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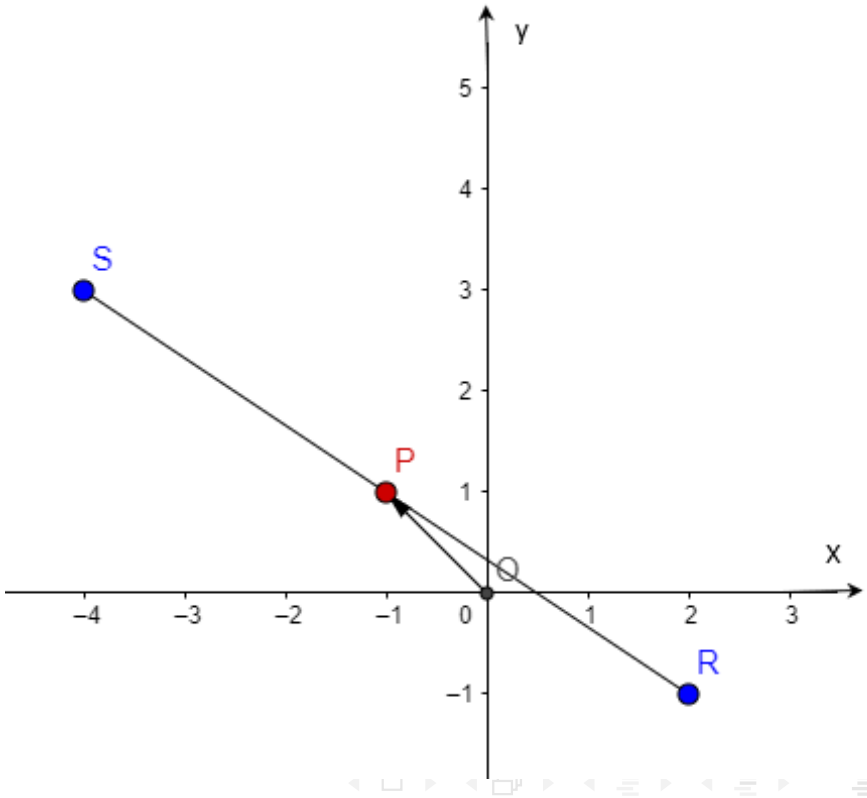
Solution:

$$\left\langle \frac{2+(-4)}{2} - 0, \frac{-1+3}{2} - 0 \right\rangle = \langle -1, 1 \rangle$$



Vectors in Space

Example 2

Express vector $\overrightarrow{\mathrm{AB}}$ in the form $v = \nu_1 i + \nu_2 j + \nu_3 k$ if A is the point $(-7, -8, 1)$ and B is the point $(-10, 8, 1)$.

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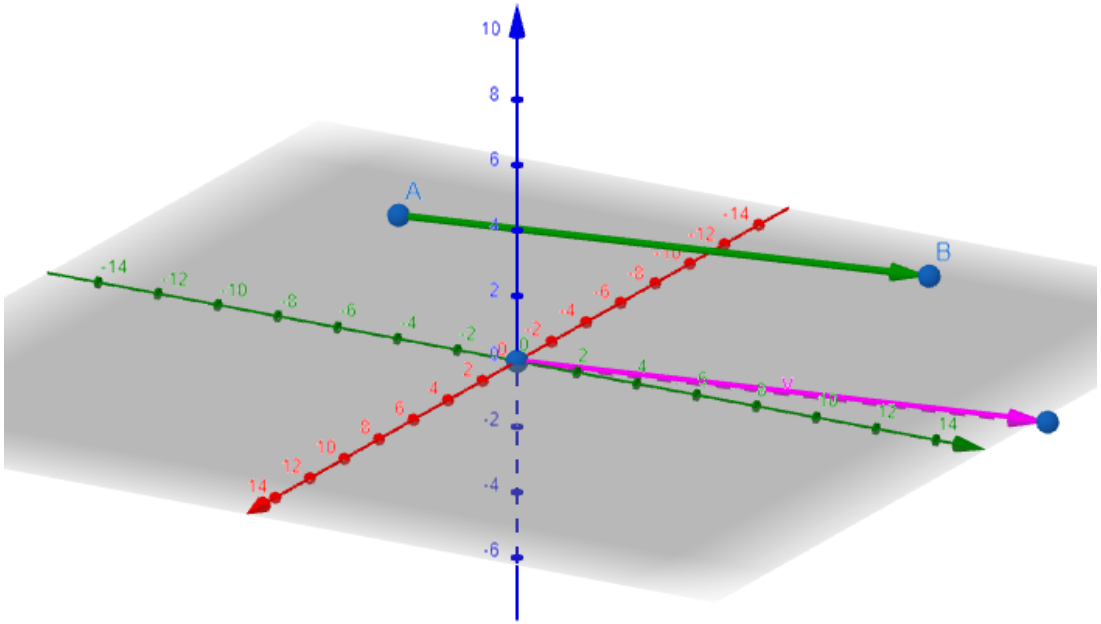
Solution:

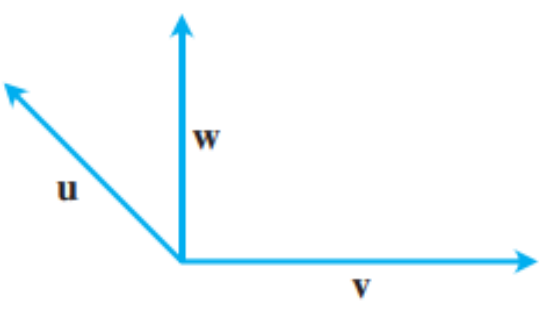
$$\overrightarrow{AB} = (-10-(-7))\mathbf{i} + (8-(-8))\mathbf{j} + (1-1)\mathbf{k} = -3\mathbf{i} + 16\mathbf{j}$$



Geometric Representations

Example 3

Copy vectors u, v, and w head to tail as needed to sketch the indicated vector.



a. $\mathbf{u} + \mathbf{v}$

b. $\mathbf{u} + \mathbf{v} + \mathbf{w}$

c. $\mathbf{u} - \mathbf{v}$

d. $\mathbf{u} - \mathbf{w}$

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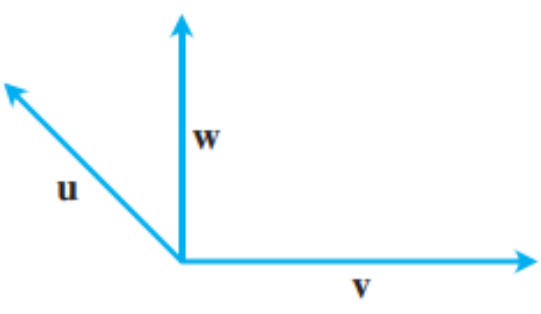

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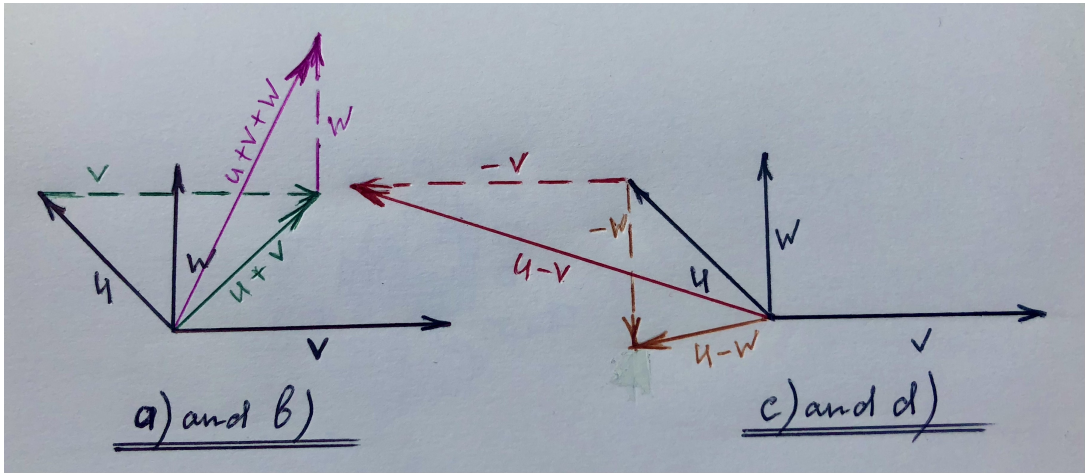

Length and Direction

Example 4

Find a vector of magnitude 3 in the direction opposite to the direction of $v = (1/2)i - (1/2)j - (1/2)k$.

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Solution:

$$|\mathbf{v}| = \sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}} = \frac{\sqrt{3}}{2}; \quad \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{1}{\sqrt{3}}\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j} - \frac{1}{\sqrt{3}}\mathbf{k} \Rightarrow \text{ the desired vector is } -3\left(\frac{1}{\sqrt{3}}\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j} - \frac{1}{\sqrt{3}}\mathbf{k}\right)$$
$$= -\sqrt{3}\mathbf{i} + \sqrt{3}\mathbf{j} + \sqrt{3}\mathbf{k}$$

Example 5

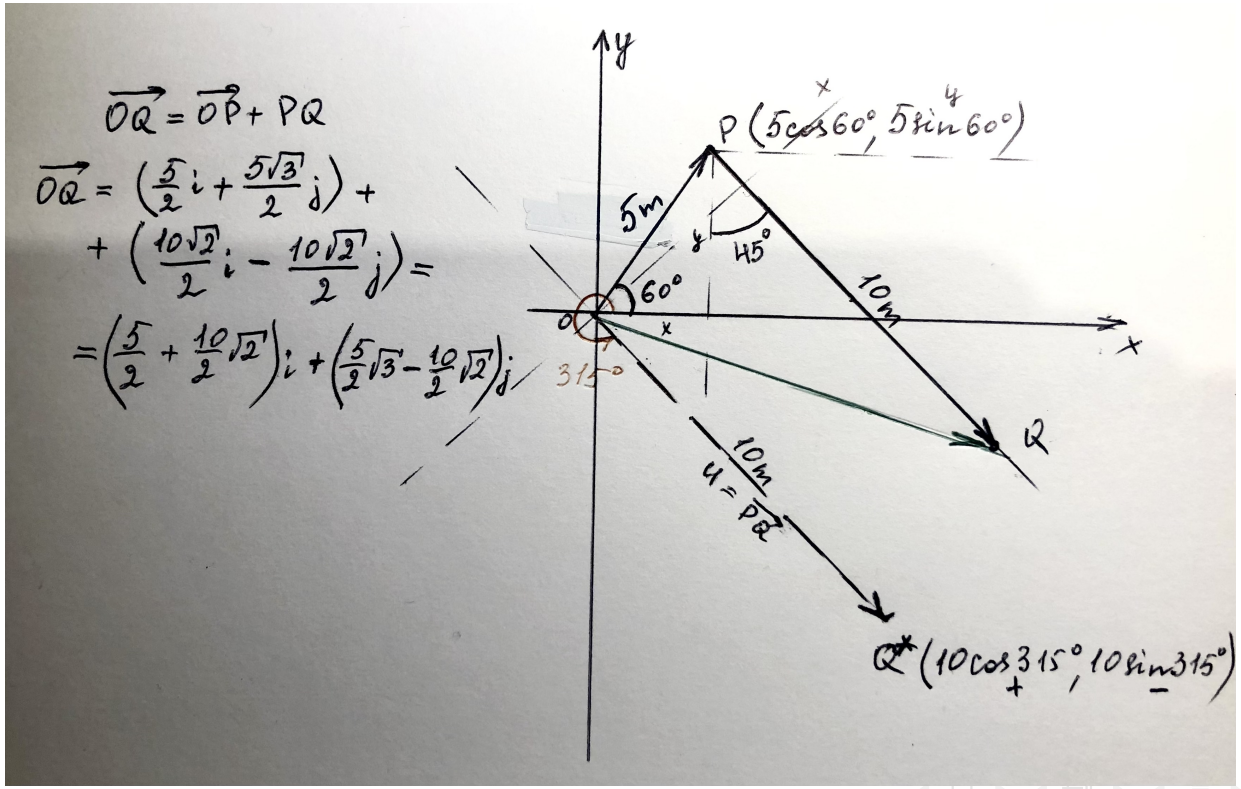
A bird flies from its nest 5 km in the direction 60° north of east, where it stops to rest on a tree. It then flies 10 km in the direction due southeast and lands atop a telephone pole. Place an xy-coordinate system so that the origin is the bird's nest, the x-axis points east, and the y-axis points north.

a) At what point is the tree located?

b) At what point is the telephone pole?

Solution:

(a) The tree is located at the tip of the vector $\overrightarrow{OP} = (5\cos 60°)\mathbf{i} + (5\sin 60°)\mathbf{j} = \frac{5}{2}\mathbf{i} + \frac{5\sqrt{3}}{2}\mathbf{j} \Rightarrow P = \left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$

(b) The telephone pole is located at the point Q, which is the tip of the vector $\overrightarrow{OP} + \overrightarrow{PQ}$

$$= \left(\frac{5}{2}\mathbf{i} + \frac{5\sqrt{3}}{2}\mathbf{j}\right) + (10\cos 315°)\mathbf{i} + (10\sin 315°)\mathbf{j} = \left(\frac{5}{2} + \frac{10\sqrt{2}}{2}\right)\mathbf{i} + \left(\frac{5\sqrt{3}}{2} - \frac{10\sqrt{2}}{2}\right)\mathbf{j} \Rightarrow Q = \left(\frac{5+10\sqrt{2}}{2}, \frac{5\sqrt{3}-10\sqrt{2}}{2}\right)$$