Introductory Statistics

Tatiana Tyukina
tt51@leicester.ac.uk

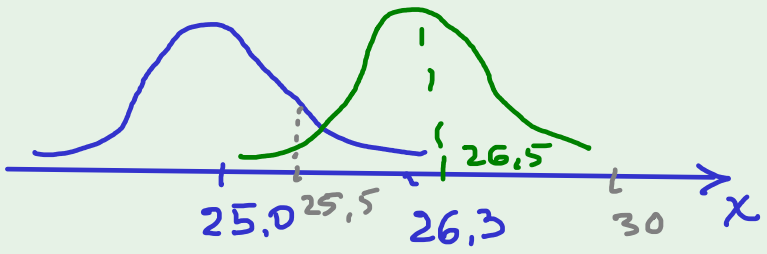
Topic 3 - Hypothesis Testing
3.2 - Hypothesis Testing Procedure

- **Topic 0: Introduction**
- **Topic 1: Descriptive Statistics**
- **Topic 2: Estimators: Point estimation, Interval estimation**
  - 2.1 **Point estimation**,
  - 2.2 **Properties of estimators**,
  - 2.3 **Interval estimation**,
  - 2.4 **Inference based on Normal Distribution**
- **Topic 3: Hypothesis Testing**
  - 3.1 **Inference for two populations**
  - 3.2 Hypothesis Testing Procedure
- Topic 4: Goodness of Fit: The χ^2 test
- Topic 5: Bayesian Estimation

HYPOTHESIS TESTING: GENERAL IDEA

EXAMPLE

A car manufacturer is looking for additives that might increase car's performance. As a pilot study, they send thirty cars fuelled with a new additive on a road.
Without the additive, those same cars are known to have average fuel consumption $\mu_0 = 25.0$ mpg with a standard deviation of $\sigma_0 = 2.4$ mpg. The fuel consumption is assumed to be normally distributed.
Suppose it turns out that the thirty cars average $\bar{x} = 26.3$ mpg with the additive.
Can the company claim that the additive has significant effect on mileage increase.



HYPOTHESIS TESTING: DEFINITIONS

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*The **null hypothesis**, denoted by **H0**, is usually the nullification of a claim. Unless evidence from the data indicates otherwise, the null hypothesis is **assumed** to be true.*

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*The **test statistic**, denoted by **TS**, is a function of the sample measurements upon which the statistical decision, to reject or not reject the null hypothesis, will be based.*

DEFINITION

*A **rejection region** (or a **critical region**) is the region (denoted by **RR**) that specifies the values of the observed test statistic for which the null hypothesis will be rejected. This is the range of values of the test statistic that corresponds to the rejection of **H0** at some fixed level of significance, α.*

HYPOTHESIS TESTING: DEFINITIONS

Conclusion: If the value of the observed test statistic falls in the rejection region, the null hypothesis is rejected and we will conclude that there is enough evidence to decide that the alternative hypothesis is true. If the TS do not fall in the rejection region, we conclude that we cannot reject the null hypothesis.

Failure to reject the null hypothesis does not necessarily mean that the null hypothesis is true.

HYPOTHESIS TESTING: DEFINITIONS

For example, if F_θ is a distribution function with parameter θ then null hypotheses about θ can be stated as
(a) H0 : $\theta = \theta_0$ (θ_0 is a constant, for example, 0)
(b) H0 : $\theta \leq \theta_0$ (or $\theta \geq \theta_0$)

and this can be tested against one of the alternatives:
(a) HA : $\theta \neq \theta_0$ (two-tailed), or $\theta < \theta_0$ (lower-), or $\theta > \theta_0$ (upper-tailed)
(b) HA : $\theta > \theta_0$ (or $\theta < \theta_0$, respectively)

DEFINITION

A hypothesis that, when true, completely specifies the population distribution is called a ***simple hypothesis****; one that does not is called a* ***composite hypothesis***

HYPOTHESIS TESTING: DEFINITIONS

EXAMPLE

In a coin-tossing experiment, let p be the probability of heads. We start with the claim that the coin is fair.

$H0$: $p = \frac{1}{2}$ $\quad p = \frac{1}{2}$ $\quad p \geq \frac{1}{2}$

HA : $p \neq \frac{1}{2}$ $\quad p > \frac{1}{2}$ $\quad p < \frac{1}{2}$

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EXAMPLE

A pharmaceutical company claims that drug A is 80% effective (i.e. p=0.8). We conduct an experiment, clinical trials to test this claim.

$H0$: $p = 0.8$ $\quad p = 0,8$ $\quad p \geq 0,8$

HA : $p \neq 0,8$ $\quad p > 0,8$ $\quad p < 0,8$

HYPOTHESIS TESTING: DEFINITIONS

The test statistic is a function of a random sample. Thus, the test statistic itself is a random variable whose distribution is known under the null hypothesis.

DEFINITION

The value of a test statistic when specific sample values are substituted is called the ***observed test statistic*** *or simply* ***test statistic*** *(TS).*

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EXAMPLE

Consider the hypothesis H0: $\mu = \mu_0$ versus HA:$\mu \neq \mu_0$, where μ_0 is known (or guessed).
Assume that the population is normal with a known variance σ^2. Consider $\bar{X}$, an unbiased estimator of μ based on the random sample, $X_1, ..., X_n$. Then

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

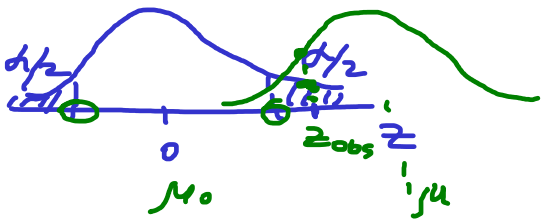
is a function of the random sample $X_1, ..., X_n$, and has a known distribution (standard normal), under H0.
If $x_1, x_2, ..., x_n$ are specific sample values, then $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$ is the *observed test (sample) statistic*.

HYPOTHESIS TESTING: PROCEDURE

DEFINITION

*A **rejection region** (or a **critical region**) is the region (denoted by **RR**) that specifies the values of the observed test statistic for which the null hypothesis will be rejected. This is the range of values of the test statistic that corresponds to the rejection of **H0** at some fixed level of significance, α.*

The probability distribution of the TS is known and does not depend on the parameters of the population!



HYPOTHESIS TESTING: PROCEDURE FOR ONE SAMPLE MEAN

$X_1, X_2, ...X_n$ from Normal Distribution $N(\mu, \sigma^2)$:

State Hypotheses

$$H0 : \mu = \mu_0$$

$$HA : \mu \neq \mu_0 \text{ (if two tailed);}$$

$$HA : \mu \lessgtr \mu_0 \text{ (if lower/upper tailed)}$$

Test Statistics for the mean:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$$

if σ is known:

$$z_{TS} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

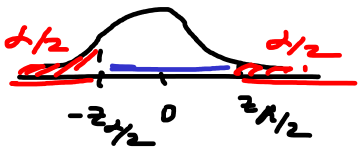
or, if σ is unknown:

$$z_{TS} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

continued ...

HYPOTHESIS TESTING: PROCEDURE FOR ONE SAMPLE MEAN

Find Rejection Region for α



$$z_{\alpha/2}: \; P(|Z| \geq z_{\alpha/2}) = \alpha/2 \;\; \text{(if two tailed)}$$

$$z_{\alpha}: \; P(Z \leq \underline{-z_{\alpha}}) = \alpha \text{ or } P(Z \geq \underline{z_{\alpha}}) = \alpha \text{ (if lower or upper tailed)}$$

Conclusion: If

$$z_{TS} \leq -z_{\alpha/2} \text{ or } z_{TS} \geq z_{\alpha/2} \text{ (if two tailed)}$$

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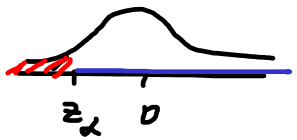

reject H0 at the α level of significance; otherwise, do not reject

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$$TS: Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1) \quad \Rightarrow \quad z_{TS} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{26.3 - 25.0}{2.4/\sqrt{30}} = 2.97$$

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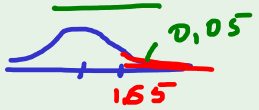
$$RR: z_\alpha = z_{critical} \Leftrightarrow P_{TS}(z > z_{critical}) = 0.05 \Rightarrow \quad z_\alpha = z_{critical} = \underline{1.65}$$



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$$z_{TS} = 2.97 > z_\alpha \qquad \Rightarrow$$

Conclusion: at 5% significance level the company can claim that the additive provides increase in petrol mileage.

HYPOTHESIS TESTING: DEFINITIONS

In a statistical test, it is impossible to establish the truth of a hypothesis with 100% certainty. There are two possible types of errors.

DEFINITION

*A **type I error** is made if H0 is rejected when in fact H0 is true. The probability of type I error is denoted by α. That is,*

$$P(\textit{rejecting } H0 | H0 \textit{ is true}) = \alpha.$$

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DEFINITION

*A **type II error** is made if H0 is accepted (not rejected) when in fact HA is true. The probability of a type II error is denoted by β. That is,*

$$P(\textit{not rejecting } H0 | H0 \textit{ is false}) = \beta.$$

HYPOTHESIS TESTING: TYPES OF ERROR

It is desirable that a test should have $\alpha = \beta = 0$ (this can be achieved only in trivial cases), or at least we prefer to use a test that minimizes both types of errors.

Statistical Decision and Error Probabilities

Statistical Decision	True State of Null Hypothesis	
	H0 True	H0 False
Do not reject H0	Correct decision	**Type II error** (β)
Reject H0	**Type I error** (α)	Correct decision

Statistical Decision	True State of Null Hypothesis	
	H0 True	H0 False
Do not reject H0	True Negative	**False Negative**(β)
Reject H0	**False Positive** (α)	True Positive

Illustration of Type I and Type II test errors

https://en.wikipedia.org/wiki/Type_I_and_type_II_errors

HYPOTHESIS TESTING: TYPES OF ERROR

It can be shown that there is a relation between the sample size, α (Type I error) and β (type II error)

Once we are given the sample size n, an α, a simple alternative HA, and a test statistic, we have no control over β and it is exactly determined.

In other words, for a given sample size and test statistic, any effort to lower β will increase in α and vice versa.

Also increasing the sample size n, we can decrease β for the same α to an acceptable level.

HYPOTHESIS TESTING: TYPES OF ERROR

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DEFINITION

*The **specificity** of a test is the probability that the null hypothesis is not rejected given that it is true:* $1 - \alpha$

For example, "Interpreting a covid-19 test result"
`https://www.bmj.com/content/bmj/369/bmj.m1808.full.pdf`

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State H0 and HA, and write the probability for Type I and Type II errors.
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Solution:

$$H0: \ \mu = \mu_0$$

$$HA: \ \mu > \mu_0$$

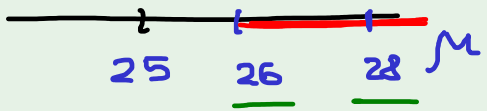
$$\alpha = P(\bar{X} \in RR | \mu = \mu_0)$$

$$\beta = P(\bar{X} \notin RR | \mu > \mu_0)$$

$\alpha = 1\%$

$\beta = 44\%$

$TS = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = z \sim N(0,1)$



$\alpha = P(\bar{x} \geq 26 \mid \mu = 25) = P\left(\frac{\bar{x} - \mu_0}{\sigma_0/\sqrt{n}} \geq \frac{26 - \mu_0}{\sigma_0/\sqrt{n}}\right) = P(z \geq 2.282) = 0.011$

$\beta = P(\bar{x} < 26 \mid \mu = 28) = P\left(\frac{\bar{x} - 28}{\sigma_0/\sqrt{n}} < \frac{26 - 28}{\sigma_0/\sqrt{n}}\right) = P(z < -0.152) =$

$1 - P(z < 0.152) = 0.44$

HYPOTHESIS TESTING: p-VALUE

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Corresponding to an observed value of a test statistic, the ***p-value*** *(or attained significance level) is the lowest level of significance at which the null hypothesis would have been rejected.*

Steps to find p-value

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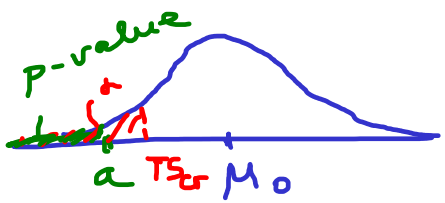
3. The p-value is given by

$$p - values = \begin{cases} P(TS < a|H_0) & \text{if lower tail test} \\ P(TS > a|H_0) & \text{if upper tail test} \\ P(|TS| > |a||H_0) & \text{if two-tailed test} \end{cases}$$



HYPOTHESIS TESTING: p-VALUE

Reporting test result as p - value

1. Choose the maximum value of α that you are willing to tolerate.
2. If the p-value of the test is less than the maximum value of α, reject H0.

> One common mistake in using the p-value is to declare a result as "significant" if the p value is less than 0.05.

EXAMPLE

In previous example:

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$$p - value : \; P(TS > \overset{z_{TS}}{a} | H_0) = P(TS > 2.97) = 1 - 0.9985 = 0.0015 = 0{,}15\%$$

p-value is 0.15%, the result is consistent with the previous conclusion, we can reject H0 hypothesis at 5% significance level.

HYPOTHESIS TESTING: PROCEDURE

Steps in Hypothesis testing

1. State the alternative hypothesis, HA (what is believed to be true).

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3. Decide on a level of significance α.

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1. State the alternative hypothesis, HA (what is believed to be true).
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3. Decide on a level of significance α.
4. Choose appropriate TS and compute the observed test statistic.

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5. Using the distribution of TS and α, determine the rejection region(s) (RR).

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6. **Conclusion**: If the observed test statistic falls in the RR, reject H0 and conclude that based on the sample information, we are $(1-\alpha)100\%$ confident that HA is true. Otherwise, conclude that there is not sufficient evidence to reject H0. In all the applied problems, interpret the meaning of your decision.

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7. State any assumptions you made in testing the given hypothesis.

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4. Choose appropriate TS and compute the observed test statistic.
5. Using the distribution of TS and α, determine the rejection region(s) (RR).
6. **Conclusion**: If the observed test statistic falls in the RR, reject H0 and conclude that based on the sample information, we are $(1-\alpha)100\%$ confident that HA is true. Otherwise, conclude that there is not sufficient evidence to reject H0. In all the applied problems, interpret the meaning of your decision.
7. State any assumptions you made in testing the given hypothesis.
8. Compute the p-value from the null distribution of the test statistic and interpret it.

HYPOTHESIS TESTING: FOR A SINGLE SAMPLE

EXAMPLE

The mean annual salary of classroom teachers is £30.4 thousand. A random sample of 90 classroom teachers in a City was collected. Assume that the standard deviation of annual salaries for all classroom teachers in this City is £9.2 thousand.
At the 5% significance level, do the data provide sufficient evidence to conclude that the mean annual salary of classroom teachers is greater than the national average salary £29.01 thousand?

Solution:

1) Hypotheses: $H0: \mu = 29.01$ vs $HA: \mu > 29.01$

2) TS: $TS = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$ $\quad z_{TS} = \frac{\bar{X} - 29.01}{9.2/\sqrt{90}} = \frac{30.4 - 29.01}{9.2/\sqrt{90}} = 1.43$

3) RR: $P(z > z_{cr}) = 0.05 \Rightarrow P(z > 1.65) = 0.05$, $z_{cr} = 1.65$

4) Conclusion: z_{TS} vs $z_{cr} \Rightarrow 1.43 < 1.65$ $\quad z_{TS} \notin RR \Rightarrow$
we do not reject H0

5) p-value: $P(TS > z_{TS}) = P(TS > 1.43) = 1 - 0.924 \approx 0.076 \approx 7.6\%$

HYPOTHESIS TESTING:FOR A SINGLE SAMPLE

EXAMPLE

We need to analyse the height of wheat plants in the field in different irrigation (watering) regime. Assume that the height has normal distribution with unknown variance.

35 plants from different parts of the field and found that the average height is 102 cm and the sample variance is 16. It was shown that in usual regime the mean height of the plants is 100 cm.

We want to test the hypothesis that the change of irrigation regime has significant influence on the height of plants at 5% significance level.

Solution:

1) Hypotheses: $H_0: \mu = \mu_0 = 100$ cm , $H_a: \mu > \mu_0$, $\alpha = 0{,}05$

2) TS: $z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \sim N(0,1)$, $\bar{x} = 102$, $s = 4$, $n = 35 \Rightarrow z_{obs} = 2{,}958$

3) RR: $P(z > z_{crit}) = 0{,}05$, $z_{crit} = 1{,}65$ or $1{,}645$

4) Conclusion: $z_{obs} > z_{crit} \Rightarrow z_{obs} \in RR \Rightarrow$ reject $H_0 \Rightarrow$

5) p-value: $P(z > z_{obs}) = 0{,}0015 \approx 0{,}1\% \ll 5\%$

HYPOTHESIS TESTING: FOR A SINGLE SAMPLE

EXAMPLE

A company would like to assess fatigue in steel plant workers due to heat stress. A random sample of 25 casting workers had a mean post-work heart rate of 78.3 beats per minute (bpm) with the sample standard deviation of 11.2 bpm.
At the 5% significance level, do the data provide sufficient evidence to conclude that the mean post-work heart rate for casting workers exceeds the normal resting heart rate of 72 bpm?
Assume that the population post-work heart rates for casting workers is normal.

Solution:

1) Hypotheses: $H_0: \mu = \mu_0 = 72$, $H_a: \mu > \mu_0$, $\alpha = 0{,}05$

2) TS: $T = \frac{\bar{x} - \mu}{s/\sqrt{n}} \sim t_{n-1}$, $\bar{x} = 78.3$, $s = 11.2$, $n = 25 \Rightarrow t_{obs} = 2{,}813$

3) RR: $P(t > t_{crit}) = 0{,}05$, $t_{crit} = t_{0{,}95,24} = 1{,}711$

4) Conclusion: $t_{obs} > t_{crit} \Rightarrow$ reject $H_0 \Rightarrow$ there is an increase in a heart rate.

5) p-value: $P(t > t_{obs}) = 0{,}005 \approx 0{,}5\% << 5\%$

HYPOTHESIS TESTING: FOR A SINGLE SAMPLE

EXAMPLE

A machine in a certain factory must be repaired if it produces more than 12% defectives among the large lot of items it produces in a week. A random sample of 175 items from a week's production contains 35 defectives, and it is decided that the machine must be repaired.

Does the sample evidence support this decision? Use $\alpha = 0.02$.

Compute the p-value.

Solution:

1) Hypotheses: $H_0: p \le p_0 = 0{,}12$ $\quad H_a: p > p_0$, $\alpha = 0{,}02$

2) TS: $\frac{X}{n} \cdot n > 5$, $\left(1 - \frac{X}{n}\right) \cdot n > 5 \Rightarrow z = \dfrac{\frac{X}{n} - p_0}{\sqrt{\frac{X}{n}\left(1 - \frac{X}{n}\right) \cdot \frac{1}{n}}} \sim N(0,1) \Rightarrow \frac{X}{n} = \frac{35}{175}$

$z_{obs} = 2{,}667$

3) RR: $P(z > z_{crit}) = 0{,}02 \Rightarrow z_{crit} = 2{,}054$

4) Conclusion: $z_{obs} > z_{crit} \Rightarrow$ reject $H_0 \Rightarrow$ the machine must be repaired

5) p-value: $P(z > z_{obs}) = 0{,}0038 \approx 0{,}4\% < 2\%$

HYPOTHESIS TESTING: FOR A SINGLE SAMPLE

Suppose that $X_1, X_2, ..., X_n$ is a random sample of Bernoulli random variables where n is too small to use the approximation by normal distribution.

For testing $H0: \ p = p_0$ the critical region should defined by using the exact binomial distribution:

$$P(X = k) = \binom{n}{k} p_0^k (1 - p_0)^{n-k}$$

Rejection region:

$$P(X \leq k_1) + P(X \geq k_2) = \alpha$$

H_a

Example: Suppose H0: p =0.75 is to be tested against H1: p < 0.75 using a random sample of size n =7. Find the rejection region if the significance level is 10%.

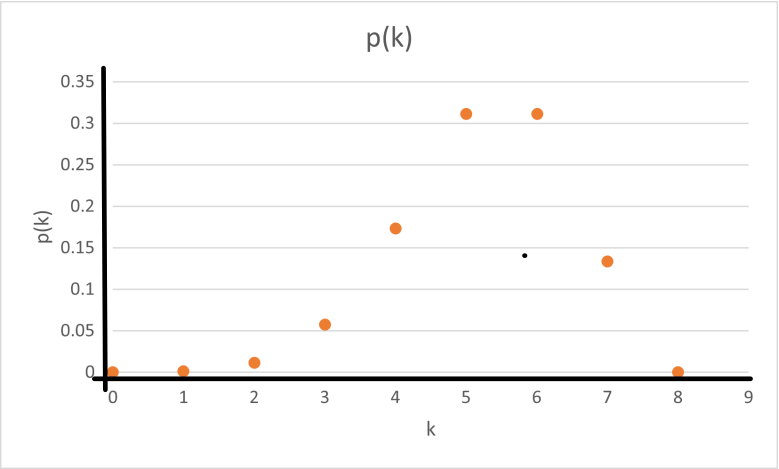
$H_0: p = p_0 = 0{,}75$

$H_a: p < p_0$

$$P(X=k) = \binom{n}{k} p_0^k (1-p_0)^{1-k} = \binom{7}{k} 0{,}75^k \cdot 0{,}25^{1-k}$$

RR: $P(X < k_{crit}) \leq 0{,}1$, $k_{crit} = 3$ $\qquad P(X<3) \leq 0{,}1$

k	p(k)	P(x≤k)
0	6.10352E-05	6.10352E-05
1	0.001281738	0.001342773
2	0.011535645	0.012878418
3	0.057678223	0.070556641
4	0.173034668	0.243591309
5	0.311462402	0.555053711
6	0.311462402	0.866516113
7	0.133483887	1



$p_{obs} < \frac{3}{7}$

HYPOTHESIS TESTING: FOR A SINGLE SAMPLE

EXAMPLE

How long sporting events last is quite variable. This variability can cause problems for TV broadcasters, since the amount of commercials and commentator blather varies with the length of the event. Assume that the lengths for a random sample of 16 middle-round contests at the 2008 Wimbledon Championships in women's tennis has sample standard deviation 27.25.

Assuming that match lengths are normally distributed, test the hypothesis that standard deviation of a match length is no more than 25 mins using $\alpha = 0.05$. Compute the p-value.

Solution:

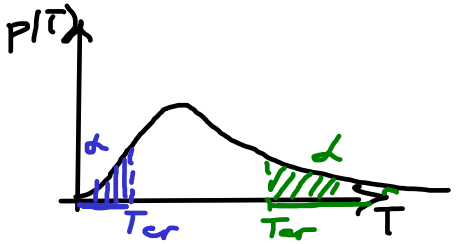
1) Hypotheses: $H_0: \sigma^2 \leq \sigma_0^2 = 25^2$ $\quad H_a: \sigma^2 > \sigma_0^2$

2) TS: $T = \frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$

$$\Rightarrow t_{obs} = \frac{15 \cdot 27.25^2}{25^2} = 17.822$$

continued ...

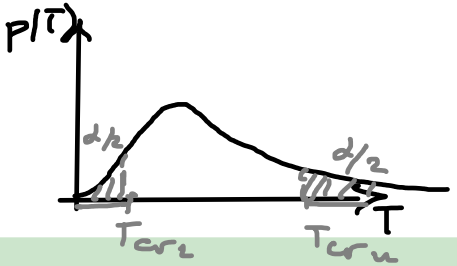
HYPOTHESIS TESTING: FOR A SINGLE SAMPLE



$H_a: \sigma^2 < \sigma_0^2$

$H_a: \sigma^2 > \sigma_0^2$

$H_a: \sigma^2 \neq \sigma_0^2$



EXAMPLE

3) RR: $P(T > t_{crit}) = \alpha = 0.05$ $\quad t_{crit} = \chi^2_{0.95,15} = 25.00$

4) Conclusion: $t_{obs} = 17.822 < t_{crit} \Rightarrow$ do not reject H_0

5) p-value: $P(T > t_{obs}) = 27\% > 5\%$

HYPOTHESIS TESTING: FOR TWO SAMPLES

EXAMPLE

In a salary equity study of faculty at a certain university, sample salaries of 50 male assistant professors and 50 female assistant professors yielded the following basic statistics.

	Sample Mean Salary	Sample Standard Deviation	
Male AssistProf	\$46400	360	1
Female AssistProf	\$46000	220	2

Test the hypothesis that the mean salary of male assistant professors is more than the mean salary of female assistant professors at this university. Use $\alpha = 0.05$.

Solution:

1) Hypotheses:

$$H_0: \mu_1 = \mu_2 \qquad \Longleftrightarrow \qquad H_0: \mu_1 - \mu_2 = 0$$

$$H_a: \mu_1 > \mu_2 \qquad\qquad H_a: \mu_1 - \mu_2 > 0$$

2) TS:

$$T = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim N(0,1) \Rightarrow t_{obs} = \frac{46400 - 46000}{\sqrt{\frac{360^2}{50} + \frac{220^2}{50}}} = 6.704$$

continued ...

HYPOTHESIS TESTING: FOR TWO SAMPLES

EXAMPLE

3) RR: $P(T > t_{crit}) = 0{,}05$, $t_{crit} = 1{,}645$

4) Conclusion: $t_{obs} > t_{crit} \Rightarrow$ reject $H_0 \Rightarrow \mu_1 > \mu_2$

5) p-value: $P(T > t_{obs}) = P(T > 6.7) = 0!$

HYPOTHESIS TESTING: FOR TWO SAMPLE

EXAMPLE

Assume that two populations are normally distributed with unknown and unequal variances. Two independent samples were drawn from these populations and the data obtained resulted in the following basic statistics:

	Sample Size	Sample Mean	Sample StDev
1	18	20.17	4.3
2	12	19.23	3.8

Test at the 5% significance level whether the two population means are different.

Solution:

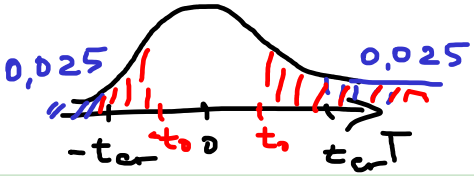
1) Hypotheses: $H_0: \mu_1 = \mu_2$, $H_a: \mu_1 \neq \mu_2$

2) TS: $T = \dfrac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} \sim t_\nu$ $\quad \nu = \dfrac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{(s_1^2/n_1)^2}{n_1 - 1} + \dfrac{(s_2^2/n_2)^2}{n_2 - 1}} = 25.685$

$T \sim t_{25}$, $t_{obs} = 0.629$

continued ...

HYPOTHESIS TESTING: FOR TWO SAMPLES



EXAMPLE

3) RR: $P(|T| > t_{crit}) = 0{,}05$

$t_{crit} = t_{0{,}975,25} = 2{,}06$

4) Conclusion: $t_{obs} \notin RR \Rightarrow$ do not reject H_0

5) p-value: $p(|T| > t_{obs}) = 2 \cdot 0.267 \approx 53\%$

HYPOTHESIS TESTING: FOR TWO SAMPLES

$X_1, X_2, ...X_n$ from Normal Distribution $N(\mu, \sigma^2)$ and σ is known:
State Hypotheses

$$H0 : \sigma_1^2 = \sigma_2^2$$

$$HA : \sigma_1^2 \neq \sigma_2^2 \text{ (if two tailed);}$$

$$HA : \sigma_1^2 > \sigma_2^2 \text{ or } \sigma_1^2 < \sigma_2^2 \text{ (if lower/upper tailed)}$$

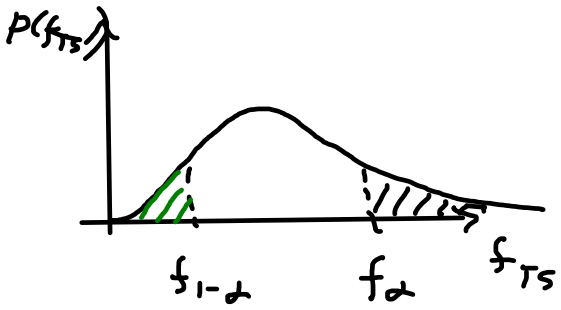
Test Statistics for the mean:

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F_{n_1-1,n_2-1}$$

$$f_{TS} = \frac{S_1^2}{S_2^2} \qquad \text{or} \qquad f_{TS} = \frac{s_1^2}{s_2^2}$$

continued ...

HYPOTHESIS TESTING: FOR TWO SAMPLES



Find Rejection Region for α

$$f_{critical}: \; P(F \geq f_{\alpha/2,n_1-1,n_2-1}) = \alpha/2$$

$$\text{and } P(F \leq f_{1-\alpha/2,n_1-1,n_2-1}) = \alpha/2 \;\; \text{(if two tailed)}$$

$$f_\alpha: \; P(F \leq f_{1-\alpha,n_1-1,n_2-1}) = \alpha$$

$$\text{or } f_{1-\alpha}: P(F \geq f_{\alpha,n_1-1,n_2-1}) = \alpha \text{ (if lower or upper tailed)}$$

Conclusion: If

$$f_{TS} \leq f_{1-\alpha/2} \text{ or } f_{TS} \geq f_{\alpha/2} \text{ (if two tailed)}$$

$$f_{TS} \leq f_{1-\alpha} \text{ or } f_{TS} \geq f_\alpha \text{ (if lower or upper tailed)}$$

reject H0 at the α level of significance; otherwise, do not reject

HYPOTHESIS TESTING: FOR TWO SAMPLES

EXAMPLE

Consider two independent random samples $X_1, ..., X_{n_1}$ from $N(\mu_1, \sigma_1^2)$ distribution and $Y_1, ..., Y_{n_2}$ from $N(\mu_2, \sigma_2^2)$ distribution.
Test $H0: \ \sigma_1^2 = \sigma_2^2$ versus $HA: \ \sigma_1^2 \neq \sigma_2^2$ for $\alpha = 0.20$ using the following basic statistics:

	Sample Size	Sample Mean	Sample Variance
1	25	410	95
2	16	390	300

Solution:

continued ...

HYPOTHESIS TESTING: FOR TWO SAMPLES

EXAMPLE

Consider two independent random samples $X_1, ..., X_{n_1}$ from $N(\mu_1, \sigma_1^2)$ distribution and $Y_1, ..., Y_{n_2}$ from $N(\mu_2, \sigma_2^2)$ distribution.
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Solution:
1) Hypotheses: $H0: \ \sigma_1^2 = \sigma_2^2$ versus $HA: \ \sigma_1^2 \neq \sigma_2^2$ - two sided

continued ...

HYPOTHESIS TESTING: FOR TWO SAMPLES

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Solution:
1) Hypotheses: $H0: \ \sigma_1^2 = \sigma_2^2$ versus $HA: \ \sigma_1^2 \neq \sigma_2^2$ - two sided
2) TS:

$$f_{TS} = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F_{n_1-1, n_2-1}$$

$$f_{TS} = \frac{S_1^2}{S_2^2} \Rightarrow f_{TS} = \frac{95}{300} = 0.317$$

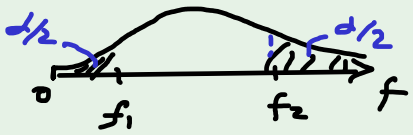
continued ...

HYPOTHESIS TESTING: FOR TWO SAMPLES

EXAMPLE

$$f_{TS} = 0.317$$

3) RR:



$$F_{0.1,24,15} = 1.90 \quad \Rightarrow \; F > 1.90$$

$$\text{and} \;\; F_{0.9,24,15} = 1/F_{0.1,15,24} = 1/1.78 = 0.56 \quad \Rightarrow \; F < 0.56$$

HYPOTHESIS TESTING: FOR TWO SAMPLES

EXAMPLE

$$f_{TS} = 0.317$$

3) RR:

$$F_{0.1,24,15} = 1.90 \quad \Rightarrow \; F > 1.90$$

$$\text{and } \; F_{0.9,24,15} = 1/F_{0.1,15,24} = 1/1.78 = 0.56 \quad \Rightarrow \; F < 0.56$$

4) Conclusion: $f_{TS} < 0.56$ hence, do reject H0;
there is evidence that the population variances are not equal.

5) p-value: $P(F <= 0.317) \cdot 2 = 0.00595 \cdot 2 \approx 0.012$

Hypothesis Testing: for two dependent samples

Definition

*Two samples of data are **dependent** when each score in one sample is paired with a specific score in the other sample. In short, these types of samples are related to each other.*

Examples:

HYPOTHESIS TESTING: FOR TWO DEPENDENT SAMPLES

DEFINITION

*Two samples of data are **dependent** when each score in one sample is paired with a specific score in the other sample. In short, these types of samples are related to each other.*

Examples:

- A group may be measured twice such as in a pretest-posttest situation (weight before and after following a certain diet program; level of radiation before entering a room and after)

HYPOTHESIS TESTING: FOR TWO DEPENDENT SAMPLES

DEFINITION

*Two samples of data are **dependent** when each score in one sample is paired with a specific score in the other sample. In short, these types of samples are related to each other.*

Examples:

- A group may be measured twice such as in a pretest-posttest situation (weight before and after following a certain diet program; level of radiation before entering a room and after)
- observation in one sample is matched with an observation in the second sample (blood test of the same group of people done in two laboratories)

HYPOTHESIS TESTING: FOR TWO DEPENDENT SAMPLES

Procedure to test the significance of the difference between two population means when the samples are dependent:

1. calculate for each pair of scores the difference, $D_i = X_{1,i} - X_{2,i}$, $i = 1, 2, ..., n$, between the two scores.
   Let $\mu_D = E(D_i)$, the expected value of D_i.

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   Let $\mu_D = E(D_i)$, the expected value of D_i.
2. Because $D_1, ..., D_n$ are i.i.d. random variables, if $d_1, ..., d_n$ are the observed values of $D_1, ..., D_n$

$$\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$$

$$s_d^2 = \frac{1}{n-1}\sum_{i=1}^{n}(d_i - \bar{d})^2$$

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$$s_d^2 = \frac{1}{n-1}\sum_{i=1}^{n}(d_i - \bar{d})^2$$
3. Now the testing will proceed as in the case of a single sample.

HYPOTHESIS TESTING: FOR TWO DEPENDENT SAMPLES

EXAMPLE

A new diet and exercise program has been advertised as remarkable way to reduce blood glucose levels in diabetic patients.
Ten randomly selected diabetic patients are put on the program, and the results after 1 month are given by the following table:

Before	268	225	252	192	307	228	246	298	231	185
After	106	186	223	110	203	101	211	176	194	203

Do the data provide sufficient evidence to support the claim that the new program reduces blood glucose level in diabetic patients? Use $\alpha = 0.05$

Solution:

Before	268	225	252	192	307	228	246	298	231	185
After	106	186	223	110	203	101	211	176	194	203
D	-162	-39	-29	-82	-104	-127	-35	-122	-37	18

From the table, the mean of the differences is $\bar{d} = 71.9$ and the standard deviation $s_d = 56.2$.

continued ...

HYPOTHESIS TESTING: FOR TWO DEPENDENT SAMPLES

EXAMPLE

The mean of the differences is $\bar{d} = -71.9$ and the standard deviation $s_d = 56.2$.
1) Hypotheses: $H0: \ \mu_d = 0$ versus $HA: \ \mu_d < 0$ - one sided
2) TS:

$$t_{TS} = \frac{\bar{d} - 0}{s_d/\sqrt{n}} \sim t_{n-1} \ \Rightarrow \ t_{obs} = \frac{-71.9}{56.2} = -4.046 \quad t_9$$

3) RR:

$$t_{0.05,9} = -1.833 \quad \Rightarrow t < -1.833$$

4) Conclusion: $t_{TS} = -4.046 < -1.833$, hence, reject H0;
the sample evidence suggests that the new diet and exercise program is effective.

5) p-value: $P(t <= -4.045) = 0.00145 \approx 0.001$ << 5%

QUESTIONS TO TAKE HOME

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6. Which tests statistics should be used for hypothesis test about two population means on the basis of small sample, which assumptions need to be satisfied; consider two cases;
7. Define notion of the polled sample mean;
8. Explain in which situations we have two dependent samples.