

MA3071-DLI, Financial Mathematics

Year 2023-2024

Typical theoretical and computational questions

- 1) Definition of European call option, European put option;
- 2) Long position, short position;
- 3) Option claim/payoff;
- 4) Profit of European call/put option holder;
- 5) Probability distribution of option Profit;
- 6) Definition of option price;
- 7) Discount factor;
- 8) Arbitrage;
- 9) Factors affecting option prices;
- 10) Short selling;
- 11) Conditional expectation;
- 12) Law of total expectation;
- 13) Markov property, when to use it?
- 14) Martingale property and its relationship with no-arbitrage;
- 15) Binomial tree (single-period, two-period, n -period);
- 16) Discounted asset price is a martingale, how to prove it?
- 17) q -probabilities, how to calculate them?
- 18) No arbitrage condition in Binomial tree models;
- 19) Formulas of arbitrage free option price in Binomial tree models;
- 20) Hedging Portfolio in Binomial tree models, how to identify it?
- 21) Time varying binomial tree models;
- 22) Definition of standard Brownian motion (SBM) and its properties;

- 23) Distribution of a standard Brownian motion;
- 24) Covariance between two Brownian motions;
- 25) Standard Brownian motion conditioning;
- 26) What conclusions we need to use to calculate $\mathbb{E}[f(t, B_t) | \mathcal{F}_s]$?
- 27) How to show a stochastic process is a martingale?
- 28) Definition of Ito integral/stochastic integral;
- 29) Properties of stochastic integral;
- 30) Distribution of stochastic integral;
- 31) Stochastic differential equation (SDE);
- 32) Ito process;
- 33) Definition of deterministic integral;
- 34) Properties of deterministic integral;
- 35) Distribution of deterministic integral;
- 36) Relationship between stochastic integral and deterministic integral;
- 37) If an Ito process consists of an initial value, a stochastic integral and a deterministic integral, what's the distribution?
- 38) Ito isometry;
- 39) Fubini theorem;
- 40) Classical Ito's Lemma;
- 41) General Ito's Lemma;
- 42) Multivariate/bivariate Ito's Lemma;
- 43) Ito rules;
- 44) How to find stochastic differential $df(t, B_t)$, $df(t, X_t)$ or $df(X_t, Z_t)$?
- 45) How to rewrite $f(t, B_t)$, $f(t, X_t)$ or $f(X_t, Z_t)$ as an integral form?
- 46) Ito martingale;
- 47) How to show a process is an Ito martingale?
- 48) Conditional Ito integral;

- 49) Conditional Fubini theorem;
- 50) How to calculate conditional expectations via Ito calculus?
- 51) Definition of Geometric Brownian motion (GBM);
- 52) When and how $e^{-\rho t} S_t$ can be a martingale?
- 53) No arbitrage condition for GBM;
- 54) Distribution of GBM;
- 55) Expectation and variance of GBM;
- 56) Geometric Brownian motion conditioning;
- 57) What conclusions we need to use to calculate $\mathbb{E}[f(S_T)|\mathcal{F}_t]$?
- 58) How to construct a delta-hedged portfolio $(-1, g'_{S_t})$ to eliminate risk?
- 59) Black-Scholes partial differential equation;
- 60) How to calculate prices of European options via Black-Scholes?
- 61) Put-call parity;
- 62) General solution of the Black-Scholes equation;
- 63) Linearity of solutions of the Black-Scholes equation;
- 64) Definitions of dynamic hedging and the Greeks;
- 65) How to calculate values of Greeks?
- 66) Hedging portfolio in Black-Scholes models, how to identify it?
- 67) Collect observations for B_t and S_t ;
- 68) Estimated expectations of B_t and S_t ;
- 69) Monte-Carlo simulations for $\mathbb{E}[f(t, B_t)|\mathcal{F}_s]$ and $\mathbb{E}[f(S_T)|\mathcal{F}_t]$;
- 70) Monte-Carlo simulation in option pricing;
- 71) Expected return and variance of return of a portfolio;
- 72) Opportunity set;
- 73) The purpose of portfolio optimization;
- 74) For the simplest portfolio consisting of two assets A_1, A_2 , what is the shape of the opportunity set in a $(\mathbb{E}[R], \sigma)$ -plane?

- 75) Efficient frontier;
- 76) Return vector r , weight vector X , one vector $\mathbf{1}$, variance-covariance matrix V ;
- 77) Minimum variance portfolio, and how to find it?
- 78) Lagrange multipliers and Lagrangian function;
- 79) Minimum variance portfolio for a given expected return, and how to find it?
- 80) Risk preference and utility function;
- 81) Maximum utility portfolio, and how to find it?
- 82) Capital allocation line (CAL) and its equation;
- 83) Slope of the CAL and the meaning;
- 84) Tangent portfolio, and how to find it?
- 85) Market equilibrium;
- 86) Capital market line (CML) and its equation;
- 87) Slope of the CML is called the market price of risk;
- 88) Market portfolio;
- 89) Market capitalisation;
- 90) Equation of the capital asset pricing model;
- 91) Definition of betas;
- 92) Calculate betas;