1. In this assignment, you will evaluate the accuracy of Stirling's famous approximation

$$n! \approx n^n e^{-n}\sqrt{2\pi n}$$

Write a program to output a table of the form

n	n!	Stirling's approximation	Absolute error	Relative error

Judging from the table, does the accuracy increase or decrease with increasing n?

2. In this assignment, you will compute the number π using an iterative method. An equilateral regular polygon, inscribed in a circle of radius 1, has the perimeter nL_n, where n is the number of sides of the polygon, and L_n is the length of one side. This can serve as an approximation for the circle perimeter 2π. Therefore, $\pi \approx \frac{nL_n}{2}$. A polygon with twice as many sides, inscribed in the same circle, has the side length

$$L_{2n} = \sqrt{2 - \sqrt{4 - L_n^2}} \qquad (*)$$

(a) Write a program to interatively compute approximations for π using (*) and starting from $n = 6$ and $L_6 = 1$. Output a table of the form

n	L_n	Absolute error in approximating π

for $n = 6, 6 \times 2, 6 \times 4, \cdots, 6 \times 2^{21}$.

(b) Use the formula $b - \sqrt{b^2 - a} = \frac{a}{b + \sqrt{b^2 - a}}$ to derive a different form of equation (*).

(c) Modify your program using the new equation and repeat the computation to produce a new table.

(d) Compare the tables and explain the source of the difference.

3. The exact solution of the initial-value problem

$$\begin{cases} y'(t) = f(t,y) = y^2(t)e^{-t} \\ y(0) = 1 \end{cases}$$

is $\mathrm{y(t)} = \mathrm{e}^t$.

Solve the problem numerically on the interval $\mathrm{t} \in [0,1]$ using

(a) Euler's method

$$\mathrm{w}_{i+1} = w_i + hf(t_i, w_i)$$

(b) Second-order Taylor method

$$\mathrm{w}_{i+1} = w_i + hf(t_i, w_i) + \frac{h^2}{2}\left[\frac{\partial f}{\partial t} + \frac{\partial f}{\partial y}(t_i, w_i)\right]$$

(c) Midpoint method

$$\mathrm{w}_{i+1} = w_i + hf\left(t_i + \frac{h}{2}, w_i + \frac{h}{2}f(t_i, w_i)\right)$$

Take the step size $\mathrm{h} = 0.1$ and output the error at all steps of the computation.

4. Fit the data

$\mathrm{x}_i = 1,2,3,\cdots,10,$

$y_i = 34.6588, 40.3719, 14.6448, -14.2721, -13.3570, 24.8234,$
$75.2795, 103.5743, 97.4847, 78.2392$

with the discrete least squares polynomial of degree at most 3,4,5,6 and figure the data and the curves.