# MA3071 Financial Mathematics - DLI
# Year 2023-2024

### Coursework 3

## INSTRUCTIONS AND DEADLINE:

Please electronically submit one piece of written or typed work per person as a single file via *Blackboard* by **December 20, 2023, at 16:00 (UK) / 23:59 (China)**.

You can use this page as a cover page and write your name, student ID, and signature below.

**Name:**

**Student ID:**

**Signature:**

## MARKING CRITERIA:

>> Coursework is marked out of 100 points, with the number of marks for each main question indicated at the beginning of each.

>> Clearly justify and explain your answers. You are expected to use MATLAB for calculations. A printout without full explanations of the formulas and reasoning will result in a deduction of marks.

>> You are required to submit a single PDF file containing justifications, explanations, and codes for each question. Include your MATLAB code in the appendix of your answers, ensuring that it is properly commented. You can copy or screenshot your codes into the PDF file without providing the code files in any other format.

>> You can submit your answers up to 3 attempts when submitting via Blackboard. Only the last attempt of your submission will be assessed. Email submissions won't be accepted.

Please note: *Any numerical results should be rounded to four decimal places.*

## Question [100 marks]

An investor wants to invest in a portfolio consisting of the following stocks selected from the London Stock Exchange (LSE).

Table 1: Ten stocks selected from LSE

| **Symbol** | **Company name** |
|---|---|
| AHT.L | Ashtead Group plc |
| CCH.L | Coca-Cola HBC AG |
| FRAS.L | Frasers Group Plc |
| MNG.L | M&G plc |
| RMV.L | Rightmove plc |
| RR.L | Rolls-Royce Holdings plc |
| SDR.L | Schroders plc |
| SHEL.L | Shell plc |
| STJ.L | St. James's Place plc |
| TSCO.L | Tesco plc |

The spreadsheet 'Historical Prices.xlsx' lists weekly historical data for the closing prices of these 10 stocks from 01/Aug/2022 to 25/Sep/2023 (a total of 61 weeks).

Let $P_i(0)$ denote the initial price of the $i^{th}$ stock, and let $P_i(t)$ denote the closing price of the $i^{th}$ stock during the $t^{th}$ week. Therefore, the historical weekly rates of return for each stock are calculated as follows:

$$R_i(t) = \frac{P_i(t) - P_i(t-1)}{P_i(t-1)}, \quad i = 1, \cdots, 10, \quad t = 1, \cdots, 60$$

Then, the expected return and variance of return for each stock as well as their covariance can be estimated by

$$\begin{aligned}
\mathbb{E}[R_i] &= \frac{1}{60} \sum_{t=1}^{60} R_i(t), \quad i = 1, \cdots, 10 \\
\sigma_i^2 &= \frac{1}{60} \sum_{t=1}^{60} (R_i(t) - \mathbb{E}[R_i])^2, \quad i = 1, \cdots, 10 \\
\sigma_{ij} &= \frac{1}{60} \sum_{t=1}^{60} (R_i(t) - \mathbb{E}[R_i])(R_j(t) - \mathbb{E}[R_j]), \quad i \neq j
\end{aligned}$$

Assuming that short selling is allowed in this market, answer the following questions:

a) **[25 marks]** Find the weights, expected return, and variance for the minimum variance portfolio of the 10 stocks.

b) **[25 marks]** Determine the minimum variance portfolio with an expected return of 0.015 and calculate its variance.

c) **[25 marks]** Given that the utility function is $u = \mathbb{E}_p - 0.5\sigma_p^2$, identify the maximum utility portfolio.

d) **[25 marks]** Find the maximum return portfolio with a certain level of risk such that $\sigma_p^2 = 0.004$ and determine its expected return.

**Note**: You are not expected to use the matrix form when determining the optimal portfolio weights.

## Hints

You may implement the portfolio optimization problem in MATLAB using either of the following methods.

Consider the example on slide 15 of Section 6, where the minimum variance portfolio is $(x_1, x_2) = (5/7, 2/7)$ with $\sigma_p^2 = 3/700$ and $\mathbb{E}[R_p] = 4/35$.

i) Method 1: Lagrange Multipliers.

```
syms x1 x2 mu
Var_p = 0.01*x1^2 + 0.04*x2^2 + 2*x1*x2*0.1*0.2*(-0.5);
g = x1 + x2 - 1 == 0; % constraint x1 + x2 = 1
W = Var_p - mu * lhs(g); % Lagrange function
dL_dx1 = diff(W,x1) == 0; % derivative of L with respect to x1
dL_dx2 = diff(W,x2) == 0; % derivative of L with respect to x2
dL_dmu = diff(W,mu) == 0; % derivative of L with respect to mu
system = [dL_dx1; dL_dx2; dL_dmu]; % build the system of equations
[x1_val, x2_val, mu_val] = solve(system, [x1 x2 mu], 'Real', true) ...
    % solve the system of equations and display the results
results_numeric = double([x1_val, x2_val, mu_val]);
Var_p=0.01*results_numeric(1)^2 + 0.04*results_numeric(2)^2 ...
    + 2*results_numeric(1)*results_numeric(2)*0.1*0.2*(-0.5)
```

ii) Method 2: Solver for quadratic objective functions with linear constraints.

```
V = [0.01 0.1*0.2*(-0.5);0.1*0.2*(-0.5) 0.04];% variance-covariance ...
    matrix
H = 2*V;
f = zeros(2,1);
Aeq = [1 1];
beq = 1; % constraint x1 + x2 = 1
options=optimset('Display','none');
[X, fval] = quadprog(H,f,[],[],Aeq,beq,[],[],[],options);% solve ...
    the optimization
x1=X(1)
x2=X(2)
Var_p=fval
```