

# Análisis de Datos Multivariantes

## Práctica: Análisis de Componentes Principales (ACP)

### Conjunto de datos: calificaciones

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# 1 Datos. Conjunto de calificaciones de 6 materias

```
library(openxlsx)
Notas=read.xlsx("Ejemplo_notas.xlsx")
Notas
```

```
##      Etiqueta Mate Fis Qui Leng Geo His      Med.1      Med.2
## 1      A1 1      7  10  8      7  9  8 8.333333 8.000000
## 2      A1 2      3  4  5      6  6  4 4.000000 5.333333
## 3      A1 3      8  7  10     7  7  9 8.333333 7.666667
## 4      A1 4      4  6  4      6  7  6 4.666667 6.333333
## 5      A1 5      7  7  4      3  6  7 6.000000 5.333333
## 6      A1 6     10  9  10     9  8  7 9.666667 8.000000
## 7      A1 7      8  7  7      8  8  8 7.333333 8.000000
## 8      A1 8      9  7  9      9  10 8 8.333333 9.000000
## 9      A1 9      5  5  5      5  4  2 5.000000 3.666667
## 10     A1 10     5  6  6      6  5  6 5.666667 5.666667
## 11     A1 11     6  5  3      3  6  7 4.666667 5.333333
## 12     A1 12     3  6  3      6  3  7 4.000000 5.333333
## 13     A1 13     10  9  10     8  7  8 9.666667 7.666667
## 14     A1 14     3  3  4      1  2  5 3.333333 2.666667
## 15     A1 15     5  4  3      5  4  1 4.000000 3.333333
## 16     A1 16     8  10  10     8  7  9 9.333333 8.000000
## 17     A1 17     4  7  5      7  5  5 5.333333 5.666667
## 18     A1 18     4  5  3      4  5  5 4.000000 4.666667
## 19     A1 19     4  3  2      5  3  5 3.000000 4.333333
## 20     A1 20     7  7  4      7  5  3 6.000000 5.000000
## 21     A1 21     1  5  1      2  2  4 2.333333 2.666667
## 22     A1 22     4  3  2      2  4  2 3.000000 2.666667
## 23     A1 23     5  3  3      4  5  1 3.666667 3.333333
```

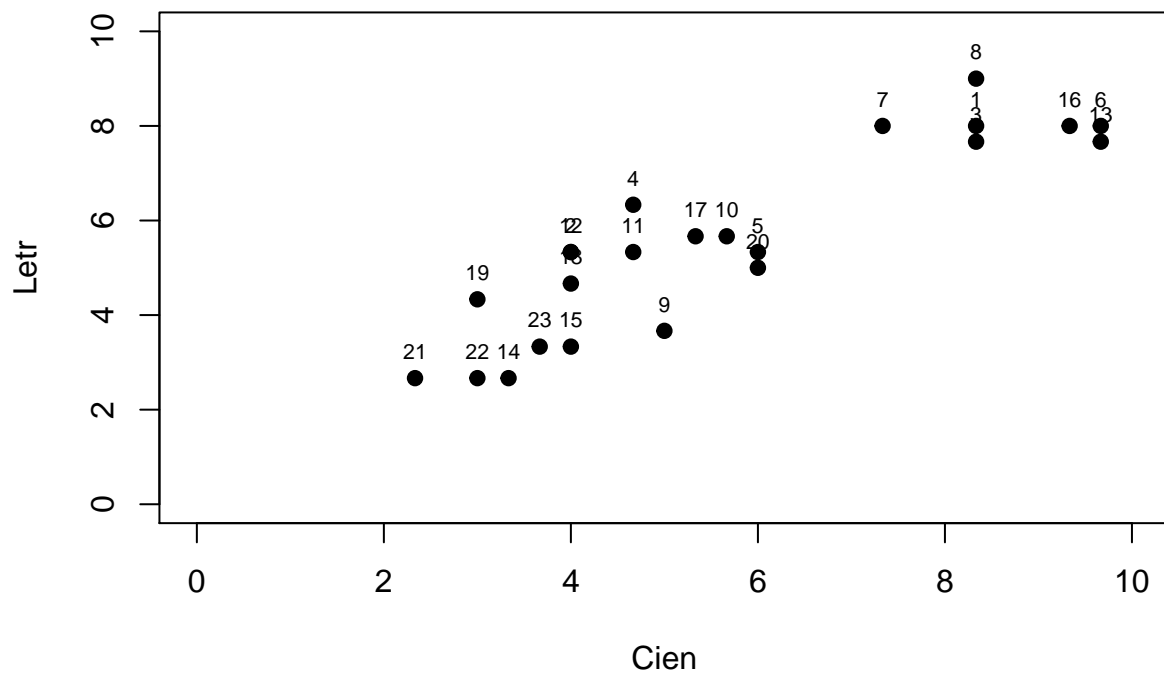
```
X=Notas[,c(2,3,4,5,6,7)]
X[1:10,]
```

```
##      Mate Fis Qui Leng Geo His
## 1      7  10  8      7  9  8
## 2      3  4  5      6  6  4
## 3      8  7  10     7  7  9
## 4      4  6  4      6  7  6
## 5      7  7  4      3  6  7
## 6     10  9  10     9  8  7
## 7      8  7  7      8  8  8
## 8      9  7  9      9  10  8
## 9      5  5  5      5  4  2
## 10     5  6  6      6  5  6
```

```
# Gráfico de las medias
```

```
Cien=(1/3)*(X[,1]+X[,2]+X[,3])
Letr=(1/3)*(X[,4]+X[,5]+X[,6])
Medias=cbind(Cien,Letr)

plot(Medias, xlim=c(0,10), ylim=c(0,10), pch=19, lty="solid", pin=c(2,5))
text(Medias, labels=c(1:23), cex=0.7, pos=3)
```



## 1.1 Matriz de distancias entre los datos originales

```
dist_ori=dist(X[1:10,], diag=T)
round(dist_ori,3)
```

##	1	2	3	4	5	6	7	8	9	10
## 1	0.000									
## 2	9.327	0.000								
## 3	4.359	9.274	0.000							
## 4	7.071	3.317	7.937	0.000						
## 5	7.141	6.633	7.616	4.583	0.000					
## 6	4.472	11.000	4.123	9.592	9.434	0.000				
## 7	3.606	7.874	3.464	5.916	6.325	4.359	0.000			
## 8	4.359	10.100	4.000	8.544	9.055	3.317	3.162	0.000		
## 9	10.149	3.742	10.000	5.385	6.481	11.091	8.832	11.136	0.000	
## 10	6.708	3.742	6.325	3.000	4.472	8.307	5.292	8.000	4.472	0.000

## 1.2 Matriz de distancias entre las medias

```
dist_Medias=dist(Medias[1:10,], diag=T)
round(dist_Medias,3)
```

##	1	2	3	4	5	6	7	8	9	10
## 1	0.000									
## 2	5.088	0.000								
## 3	0.333	4.922	0.000							
## 4	4.028	1.202	3.902	0.000						
## 5	3.543	2.000	3.300	1.667	0.000					
## 6	1.333	6.263	1.374	5.270	4.534	0.000				
## 7	1.000	4.269	1.054	3.145	2.981	2.333	0.000			
## 8	1.000	5.676	1.333	4.534	4.346	1.667	1.414	0.000		
## 9	5.467	1.944	5.207	2.687	1.944	6.368	4.922	6.289	0.000	
## 10	3.543	1.700	3.333	1.202	0.471	4.631	2.867	4.269	2.108	0.000

## 2 Análisis de Componentes Principales

### 2.1 Matriz de varianzas-covarianzas $\hat{\Sigma}$

```
Sigma=cov(X)
Sigma

##           Mate      Fis      Qui      Leng      Geo      His
## Mate 5.873518 3.772727 5.867589 3.887352 4.069170 3.371542
## Fis  3.772727 4.727273 5.000000 3.636364 3.227273 3.909091
## Qui  5.867589 5.000000 8.474308 5.254941 4.754941 4.994071
## Leng 3.887352 3.636364 5.254941 5.256917 3.575099 2.918972
## Geo  4.069170 3.227273 4.754941 3.575099 4.529644 3.237154
## His  3.371542 3.909091 4.994071 2.918972 3.237154 6.169960
```

#### 2.1.1 Autovalores-autovectores de $\hat{\Sigma}$

```
#Autovalores
lambdas=eigen(Sigma)$values
lambdas

## [1] 26.8885216  3.2253535  1.7496496  1.3493885  1.1054355  0.7132719

sum(lambdas)

## [1] 35.03162

#Autovectores
E=eigen(Sigma)$vectors
E

##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] -0.4136994 -0.3229029  0.60391376  0.08553725  0.3024214  0.5109789
## [2,] -0.3701437  0.1758847 -0.30610515  0.07917298  0.7968571 -0.3116221
## [3,] -0.5329247 -0.1010684  0.05174562  0.59896921 -0.4236522 -0.4060217
## [4,] -0.3753982 -0.3567364 -0.69766542 -0.16936565 -0.1788804  0.4294128
## [5,] -0.3562484 -0.1604058  0.22311308 -0.76299625 -0.1414297 -0.4420542
## [6,] -0.3741670  0.8376112  0.04892549 -0.12962601 -0.2051302  0.3116592

round(E*%t(E),3)

##           [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]      1    0    0    0    0    0
## [2,]      0    1    0    0    0    0
## [3,]      0    0    1    0    0    0
## [4,]      0    0    0    1    0    0
## [5,]      0    0    0    0    1    0
## [6,]      0    0    0    0    0    1
```

## 2.2 Puntuaciones de los alumnos respecto de las CP

```
# Puntuaciones de los alumnos respecto de las CP
X=as.matrix(X)
CP_X=X%*%E
colnames(CP_X)=c("1CP", "2CP", "3CP", "4CP", "5CP", "6CP")
round(CP_X[1:10,],3)
```

```
##          1CP      2CP      3CP      4CP      5CP      6CP
## 1  -19.688  1.450 -0.904 -2.907  2.530 -1.267
## 2  -11.273 -0.523 -1.806 -2.545 -0.766 -0.573
## 3  -19.719  1.556  0.324 -0.465 -0.327  0.563
## 4  -12.998  1.122 -1.545 -3.922  1.002 -0.098
## 5  -13.501  2.397  1.880 -2.445  3.179  0.589
## 6  -21.645 -1.287 -0.350 -0.978  1.782  0.755
## 7  -18.478  0.504 -0.354 -3.065  0.828  1.456
## 8  -21.045 -0.698  0.102 -3.476 -0.178  0.701
## 9  -10.634 -1.991 -0.750 -0.340  1.508 -0.031
## 10 -13.766  0.918 -1.283 -1.112  0.740  0.485
```

## 2.3 Matriz de distancias entre los datos originales

```
dist_ori=dist(X[1:10,], diag=T)
round(dist_ori,3)
```

```
##          1          2          3          4          5          6          7          8          9          10
## 1  0.000
## 2  9.327  0.000
## 3  4.359  9.274  0.000
## 4  7.071  3.317  7.937  0.000
## 5  7.141  6.633  7.616  4.583  0.000
## 6  4.472 11.000  4.123  9.592  9.434  0.000
## 7  3.606  7.874  3.464  5.916  6.325  4.359  0.000
## 8  4.359 10.100  4.000  8.544  9.055  3.317  3.162  0.000
## 9 10.149  3.742 10.000  5.385  6.481 11.091  8.832 11.136  0.000
## 10  6.708  3.742  6.325  3.000  4.472  8.307  5.292  8.000  4.472  0.000
```

## 2.4 Matriz de distancias entre las coordenadas en CP

```
dist_CP_X=dist(CP_X[1:10,], diag=T)
round(dist_CP_X,3)
```

##	1	2	3	4	5	6	7	8	9	10
## 1	0.000									
## 2	9.327	0.000								
## 3	4.359	9.274	0.000							
## 4	7.071	3.317	7.937	0.000						
## 5	7.141	6.633	7.616	4.583	0.000					
## 6	4.472	11.000	4.123	9.592	9.434	0.000				
## 7	3.606	7.874	3.464	5.916	6.325	4.359	0.000			
## 8	4.359	10.100	4.000	8.544	9.055	3.317	3.162	0.000		
## 9	10.149	3.742	10.000	5.385	6.481	11.091	8.832	11.136	0.000	
## 10	6.708	3.742	6.325	3.000	4.472	8.307	5.292	8.000	4.472	0.000

## 2.5 Matriz de distancias con (1CP,2CP)

```
dist_CP_1_2=dist(CP_X[1:10,1:2], diag=T)
round(dist_CP_1_2,3)
```

##	1	2	3	4	5	6	7	8	9	10
## 1	0.000									
## 2	8.643	0.000								
## 3	0.110	8.698	0.000							
## 4	6.698	2.384	6.734	0.000						
## 5	6.259	3.673	6.274	1.371	0.000					
## 6	3.365	10.401	3.434	8.976	8.939	0.000				
## 7	1.536	7.278	1.627	5.514	5.324	3.639	0.000			
## 8	2.541	9.774	2.615	8.250	8.154	0.841	2.835	0.000		
## 9	9.686	1.601	9.752	3.909	5.242	11.034	8.231	10.491	0.000	
## 10	5.946	2.879	5.987	0.794	1.503	8.182	4.730	7.457	4.273	0.000

## 2.6 Matriz de distancias con (1CP,2CP, 3CP)

```
dist_CP_1_2_3=dist(CP_X[1:10,1:3], diag=T)
round(dist_CP_1_2_3,3)
```

##	1	2	3	4	5	6	7	8	9	10
## 1	0.000									
## 2	8.690	0.000								
## 3	1.233	8.955	0.000							
## 4	6.728	2.398	6.989	0.000						
## 5	6.850	5.203	6.464	3.689	0.000					
## 6	3.410	10.502	3.500	9.055	9.212	0.000				
## 7	1.632	7.421	1.763	5.641	5.774	3.639	0.000			
## 8	2.733	9.958	2.625	8.413	8.346	0.955	2.872	0.000		
## 9	9.687	1.917	9.811	3.988	5.864	11.041	8.240	10.525	0.000	
## 10	5.959	2.926	6.200	0.836	3.502	8.235	4.821	7.584	4.307	0.000

### 2.6.1 Media y varianza - covarianza de las CP

```
# Medias de las CP
mediaCP_X=rep(0,6)
for (i in 1:6) {mediaCP_X[i]=mean(CP_X[,i])}
round(mediaCP_X,3)

## [1] -13.501  0.446 -0.522 -1.795  1.346  0.533
```

```
# Varianzas - covarianzas de la sCP
round(cov(CP_X),3)
```

```
##          1CP  2CP  3CP  4CP  5CP  6CP
## 1CP 26.889 0.000 0.00 0.000 0.000 0.000
## 2CP  0.000 3.225 0.00 0.000 0.000 0.000
## 3CP  0.000 0.000 1.75 0.000 0.000 0.000
## 4CP  0.000 0.000 0.00 1.349 0.000 0.000
## 5CP  0.000 0.000 0.00 0.000 1.105 0.000
## 6CP  0.000 0.000 0.00 0.000 0.000 0.713
```

### 2.6.2 Puntuaciones centradas (media 0)

```
uno_seis=rep(1,23)
CP_X_centradas=CP_X-uno_seis%%t(mediaCP_X)
round(CP_X_centradas[1:10,], 3)
```

```
##          1CP  2CP  3CP  4CP  5CP  6CP
## 1  -6.187  1.005 -0.382 -1.112  1.184 -1.800
## 2   2.228 -0.968 -1.284 -0.750 -2.112 -1.106
## 3  -6.218  1.110  0.846  1.330 -1.674  0.030
## 4   0.502  0.676 -1.023 -2.127 -0.344 -0.631
## 5  -0.001  1.952  2.402 -0.650  1.833  0.056
## 6  -8.145 -1.733  0.172  0.817  0.436  0.222
## 7  -4.977  0.059  0.168 -1.270 -0.518  0.924
## 8  -7.544 -1.144  0.624 -1.682 -1.525  0.168
## 9   2.866 -2.436 -0.228  1.455  0.161 -0.564
## 10 -0.265  0.472 -0.762  0.683 -0.606 -0.048
```

```
# Medias de las CP centradas
mediaCP_X_centradas=rep(0,6)
for (i in 1:6) {mediaCP_X_centradas[i]=mean(CP_X_centradas[,i])}
round(mediaCP_X_centradas,3)
```

```
## [1] 0 0 0 0 0 0
```



### 3 Análisis de Componentes Principales con R ( princomp(...) )

```
CP2=princomp(X)
CP2

## Call:
## princomp(x = X)
##
## Standard deviations:
##      Comp.1      Comp.2      Comp.3      Comp.4      Comp.5      Comp.6
## 5.0714352 1.7564512 1.2936684 1.1360984 1.0282865 0.8259904
##
## 6 variables and 23 observations.

summary(CP2,loading=T)

## Importance of components:
##
##              Comp.1      Comp.2      Comp.3      Comp.4
## Standard deviation  5.071435 1.75645117 1.29366836 1.13609836
## Proportion of Variance 0.767550 0.09206978 0.04994486 0.03851916
## Cumulative Proportion 0.767550 0.85961981 0.90956468 0.94808383
##
##              Comp.5      Comp.6
## Standard deviation  1.02828648 0.8259904
## Proportion of Variance 0.03155536 0.0203608
## Cumulative Proportion 0.97963920 1.0000000
##
## Loadings:
##      Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6
## Mate -0.414 -0.323  0.604      0.302 -0.511
## Fis  -0.370  0.176 -0.306      0.797  0.312
## Qui  -0.533 -0.101      0.599 -0.424  0.406
## Leng -0.375 -0.357 -0.698 -0.169 -0.179 -0.429
## Geo  -0.356 -0.160  0.223 -0.763 -0.141  0.442
## His  -0.374  0.838      -0.130 -0.205 -0.312
```

#### 3.1 Puntuaciones de los alumnos respecto de las CP

```
CP2_X=CP2$scores
round(CP2_X[1:10,],3)

##      Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6
## 1  -6.187  1.005 -0.382 -1.112  1.184  1.800
## 2   2.228 -0.968 -1.284 -0.750 -2.112  1.106
## 3  -6.218  1.110  0.846  1.330 -1.674 -0.030
## 4   0.502  0.676 -1.023 -2.127 -0.344  0.631
## 5  -0.001  1.952  2.402 -0.650  1.833 -0.056
## 6  -8.145 -1.733  0.172  0.817  0.436 -0.222
## 7  -4.977  0.059  0.168 -1.270 -0.518 -0.924
## 8  -7.544 -1.144  0.624 -1.682 -1.525 -0.168
## 9   2.866 -2.436 -0.228  1.455  0.161  0.564
## 10 -0.265  0.472 -0.762  0.683 -0.606  0.048
```

## 3.2 Distancias en el espacio de CP - espacio original

```
# En el de CP las distancias coinciden con las del espacio original  
round(dist(CP2_X[c(1:10)],,diag=T),3)
```

```
##      1      2      3      4      5      6      7      8      9     10  
## 1  0.000  
## 2  9.327  0.000  
## 3  4.359  9.274  0.000  
## 4  7.071  3.317  7.937  0.000  
## 5  7.141  6.633  7.616  4.583  0.000  
## 6  4.472 11.000  4.123  9.592  9.434  0.000  
## 7  3.606  7.874  3.464  5.916  6.325  4.359  0.000  
## 8  4.359 10.100  4.000  8.544  9.055  3.317  3.162  0.000  
## 9 10.149  3.742 10.000  5.385  6.481 11.091  8.832 11.136  0.000  
## 10 6.708  3.742  6.325  3.000  4.472  8.307  5.292  8.000  4.472  0.000
```

### 3.2.1 Usando sólo una CP

```
Y_r1=CP2_X[,c(1)]  
round(Y_r1,3)
```

```
##      1      2      3      4      5      6      7      8      9     10  
## -6.187  2.228 -6.218  0.502 -0.001 -8.145 -4.977 -7.544  2.866 -0.265  
##      11     12     13     14     15     16     17     18     19     20  
##  1.686  2.500 -7.787  6.059  4.677 -7.704  0.310  3.243  4.853  0.351  
##      21     22     23  
##  7.743  6.745  5.066
```

```
var(Y_r1)
```

```
## [1] 26.88852
```

```
# Matriz de distancias  
round(dist(Y_r1[c(1:10)],,diag=T),3)
```

```
##      1      2      3      4      5      6      7      8      9     10  
## 1  0.000  
## 2  8.415  0.000  
## 3  0.031  8.446  0.000  
## 4  6.690  1.726  6.720  0.000  
## 5  6.187  2.229  6.217  0.503  0.000  
## 6  1.957 10.372  1.926  8.647  8.144  0.000  
## 7  1.211  7.205  1.241  5.479  4.976  3.168  0.000  
## 8  1.357  9.772  1.326  8.047  7.544  0.600  2.567  0.000  
## 9  9.054  0.639  9.085  2.364  2.867 11.011  7.843 10.411  0.000  
## 10 5.923  2.493  5.953  0.767  0.264  7.880  4.712  7.279  3.131  0.000
```

### 3.2.2 Usando 2 CP

```
Y_r2=CP2_X[,c(1,2)]  
round(Y_r2[c(1:10),],3)
```

```
##      Comp.1 Comp.2  
## 1  -6.187  1.005  
## 2   2.228 -0.968  
## 3  -6.218  1.110  
## 4   0.502  0.676  
## 5  -0.001  1.952  
## 6  -8.145 -1.733  
## 7  -4.977  0.059  
## 8  -7.544 -1.144  
## 9   2.866 -2.436  
## 10 -0.265  0.472
```

```
round(var(Y_r2),3)
```

```
##           Comp.1 Comp.2  
## Comp.1 26.889  0.000  
## Comp.2  0.000  3.225
```

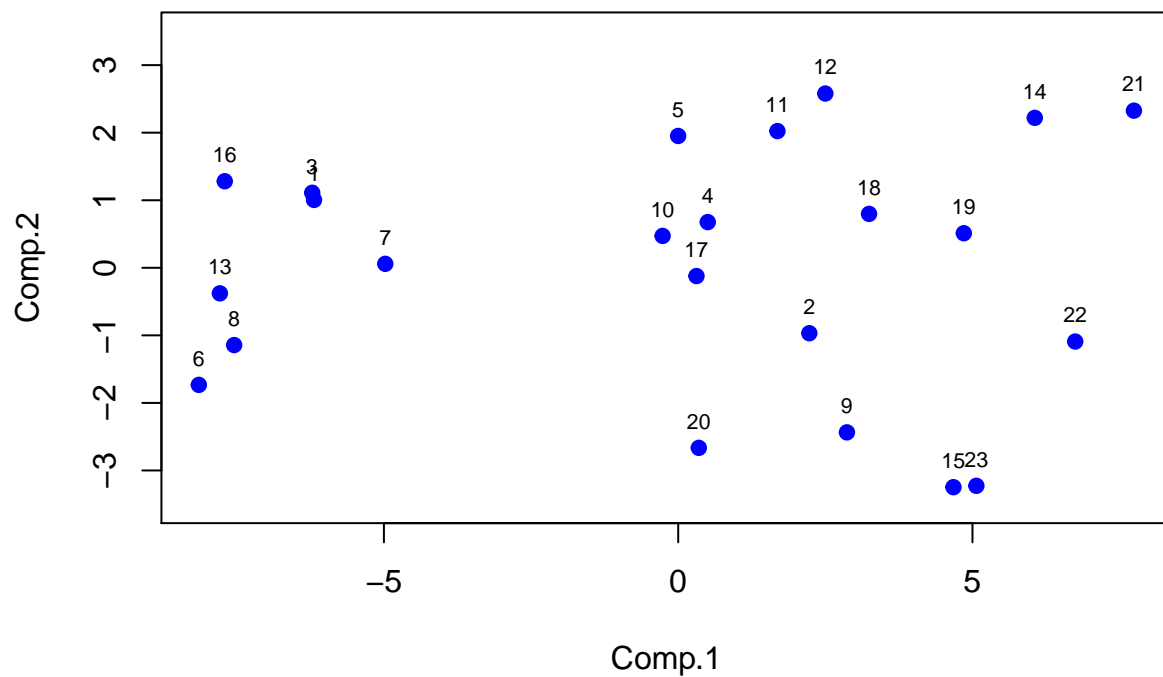
```
# Matriz de distancias
```

```
round(dist(Y_r2[c(1:10),],diag=T),3)
```

```
##           1      2      3      4      5      6      7      8      9      10  
## 1  0.000  
## 2  8.643  0.000  
## 3  0.110  8.698  0.000  
## 4  6.698  2.384  6.734  0.000  
## 5  6.259  3.673  6.274  1.371  0.000  
## 6  3.365 10.401  3.434  8.976  8.939  0.000  
## 7  1.536  7.278  1.627  5.514  5.324  3.639  0.000  
## 8  2.541  9.774  2.615  8.250  8.154  0.841  2.835  0.000  
## 9  9.686  1.601  9.752  3.909  5.242 11.034  8.231 10.491  0.000  
## 10 5.946  2.879  5.987  0.794  1.503  8.182  4.730  7.457  4.273  0.000
```

```
# Representación gráfica
```

```
{plot(Y_r2, col= "blue", , ylim=c(-3.5,3.5), pch = 19, cex = 1, lty = "solid", lwd = 1)  
text(Y_r2, labels=c(1:23), cex= 0.7, pos=3)}
```



### 3.2.3 Usando 3, 4, 5 y 6 CP

```
Y_r3=CP2_X[,c(1:3)]
round(dist(Y_r3[c(1:10),],diag=T),3)
```

##	1	2	3	4	5	6	7	8	9	10
## 1	0.000									
## 2	8.690	0.000								
## 3	1.233	8.955	0.000							
## 4	6.728	2.398	6.989	0.000						
## 5	6.850	5.203	6.464	3.689	0.000					
## 6	3.410	10.502	3.500	9.055	9.212	0.000				
## 7	1.632	7.421	1.763	5.641	5.774	3.639	0.000			
## 8	2.733	9.958	2.625	8.413	8.346	0.955	2.872	0.000		
## 9	9.687	1.917	9.811	3.988	5.864	11.041	8.240	10.525	0.000	
## 10	5.959	2.926	6.200	0.836	3.502	8.235	4.821	7.584	4.307	0.000

```
Y_r4=CP2_X[,c(1:4)]
round(dist(Y_r4[c(1:10),],diag=T),3)
```

```
##      1      2      3      4      5      6      7      8      9     10
## 1  0.000
## 2  8.698  0.000
## 3  2.736  9.193  0.000
## 4  6.804  2.766  7.797  0.000
## 5  6.865  5.204  6.760  3.974  0.000
## 6  3.918 10.618  3.537  9.522  9.328  0.000
## 7  1.639  7.439  3.141  5.706  5.807  4.195  0.000
## 8  2.791 10.002  3.995  8.424  8.409  2.675  2.901  0.000
## 9 10.021  2.922  9.812  5.361  6.231 11.059  8.679 10.983  0.000
## 10 6.223  3.258  6.233  2.931  3.747  8.237  5.201  7.944  4.375  0.000
```

```
Y_r5=CP2_X[,c(1:5)]
round(dist(Y_r5[c(1:10),],diag=T),3)
```

```
##      1      2      3      4      5      6      7      8      9     10
## 1  0.000
## 2  9.302  0.000
## 3  3.956  9.204  0.000
## 4  6.974  3.282  7.910  0.000
## 5  6.896  6.531  7.616  4.531  0.000
## 6  3.989 10.920  4.119  9.554  9.433  0.000
## 7  2.363  7.608  3.347  5.708  6.265  4.302  0.000
## 8  3.890 10.019  3.998  8.507  9.055  3.316  3.071  0.000
## 9 10.073  3.702  9.982  5.385  6.451 11.063  8.706 11.111  0.000
## 10 6.475  3.589  6.324  2.943  4.471  8.302  5.202  7.997  4.442  0.000
```

```
Y_r6=CP2_X[,c(1:6)]
round(dist(Y_r6[c(1:10),],diag=T),3)
```

```
##      1      2      3      4      5      6      7      8      9     10
## 1  0.000
## 2  9.327  0.000
## 3  4.359  9.274  0.000
## 4  7.071  3.317  7.937  0.000
## 5  7.141  6.633  7.616  4.583  0.000
## 6  4.472 11.000  4.123  9.592  9.434  0.000
## 7  3.606  7.874  3.464  5.916  6.325  4.359  0.000
## 8  4.359 10.100  4.000  8.544  9.055  3.317  3.162  0.000
## 9 10.149  3.742 10.000  5.385  6.481 11.091  8.832 11.136  0.000
## 10 6.708  3.742  6.325  3.000  4.472  8.307  5.292  8.000  4.472  0.000
```