EE307 Homework 4

Name: 吉辰卿 ID: 11911303

Question 1-2：

EE 307 HW4

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1. solution:

because: $A = \vec{A_r} + \vec{A_\theta} + \vec{A_\phi} = \frac{\mu I_0 l e^{-jkr}}{4\pi r}\cos\theta \, \vec{a_r} + \left(\frac{-\mu I_0 l e^{-jkr}}{4\pi r}\sin\theta\right)\vec{a_\theta}$

so: for H field: $H_A = \frac{1}{\mu}\nabla\times\vec{A} = \frac{1}{\mu}\left[0\,\vec{a_r} + 0\,\vec{a_\theta} + \frac{\vec{a_\phi}}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]\right]$

so: $H_A = \frac{\vec{a_\phi}}{\mu r}\left[\frac{jk\mu I_0 l e^{-jkr}}{4\pi}\sin\theta + \frac{\mu I_0 l e^{-jkr}}{4\pi r}\sin\theta\right]$

$= \frac{jkI_0 l\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{-jkr}\,\vec{a_\phi}$

Therefore: $H_A = \frac{jkI_0 l\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{-jkr}\,\vec{a_\phi}$ so, $H_r = H_\theta = 0$

$H_\phi = \frac{jkI_0 l\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{-jkr}$

Then, For E field: $E_A = -\nabla\phi_e - j\omega A = -j\omega A - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot A)$

So: $\nabla\cdot A = \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(\frac{\mu I_0 l\, r\, e^{-jkr}}{4\pi}\cos\theta\right) + \frac{1}{r\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\frac{-\mu I_0 l e^{-jkr}}{4\pi r}\sin^2\theta\right)$

$= \frac{1}{r^2}\left(\frac{\mu I_0 l e^{-jkr}\cos\theta}{4\pi} + \frac{-jkr\mu I_0 l e^{-jkr}\cos\theta}{4\pi}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{-\mu I_0 l e^{-jkr}(1-\cos 2\theta)}{8\pi r}\right)$

$= \frac{1}{r^2}\left(\frac{\mu I_0 l e^{-jkr}\cos\theta}{4\pi} + \frac{-jkr\mu I_0 l e^{-jkr}\cos\theta}{4\pi}\right) + \frac{1}{r\sin\theta}\,\frac{-\mu I_0 l e^{-jkr}\sin 2\theta}{4\pi r}$

$= \frac{1}{r^2}\left(\frac{\mu I_0 l e^{-jkr}\cos\theta}{4\pi} + \frac{-jkr\mu I_0 l e^{-jkr}\cos\theta}{4\pi}\right) + \frac{-\mu I_0 l e^{-jkr}\cdot\cos\theta}{2\pi r^2} = \nabla\cdot A$

So: $\nabla(\nabla\cdot A) = \vec{a_r}\frac{\partial(\nabla\cdot A)}{\partial r} + \vec{a_\theta}\cdot\frac{1}{r}\frac{\partial(\nabla\cdot A)}{\partial\theta} + \vec{a_\phi}\cdot\frac{1}{r\sin\theta}\cdot\frac{\partial(\nabla\cdot A)}{\partial\phi}$

$= \vec{a_r}\frac{\partial\left[\frac{\mu I_0 l\cos\theta}{4\pi}\left(\frac{e^{-jkr}}{r^2} + \frac{-jkr e^{-jkr}}{r^2} + \frac{-2e^{-jkr}}{r^2}\right)\right]}{\partial r}$

$+ \frac{1}{r}\left[\frac{1}{r^2}\frac{-\mu I_0 l e^{-jkr}\sin\theta}{4\pi} + \frac{1}{r^2}\frac{+jkr\mu I_0 l e^{-jkr}\sin\theta}{4\pi} + \frac{\mu I_0 l e^{-jkr}\sin\theta}{2\pi r^2}\right]\vec{a_\theta}$

$$= \frac{\mu I_0 l\cos\theta}{4\pi}\left(\frac{-jke^{-jkr}}{r^2} - \frac{2e^{-jkr}}{r^3} + \frac{jke^{-jkr}}{r} - \frac{k^2e^{-jkr}}{r} + \frac{jk2e^{-jkr}}{r^2} + \frac{4e^{-jkr}}{r}\right)\vec{a_r}$$

$$+ \frac{1}{r^2}\frac{\mu I_0 l e^{-jkr}\sin\theta}{4\pi}\left[jkr - 1 + 2r\right]\vec{a_\theta}$$

and: $\eta = \frac{w\mu}{k} = \sqrt{\frac{\mu}{\varepsilon}}$

So: $E_A = -jwA - j\frac{1}{w\mu\varepsilon}\nabla(\nabla\cdot A) = -jw\left[\frac{\mu I_0 l e^{-jkr}}{4\pi r}\cos\theta\,\vec{a_r} - \frac{\mu I_0 l e^{-jkr}}{4\pi r}\sin\theta\,\vec{a_\theta}\right]$

$$-j\frac{1}{w\mu\varepsilon}\left[\frac{\mu I_0 l\cos\theta}{4\pi}\left(\frac{-jke^{-jkr}}{r^2} - \frac{2e^{-jkr}}{r^3} + \frac{jke^{-jkr}}{r} - \frac{k^2e^{-jkr}}{r} + \frac{jk2e^{-jkr}}{r^2} + \frac{4e^{-jkr}}{r}\right)\vec{a_r} + \frac{1}{r^2}\frac{\mu I_0 l e^{-jkr}\sin\theta}{4\pi}\left[jkr+1\right]\vec{a_\theta}\right]$$

$$= \frac{w\mu I_0 l\cos\theta}{2\pi r^2\cdot k}\left[\frac{1}{jkr}+1\right]e^{-jkr}\vec{a_r} + \frac{jw\mu k I_0 l\sin\theta}{4\pi kr}\left[1 - \frac{1}{(kr)^2} + \frac{1}{jkr}\right]e^{-jkr}$$

So: $E_A = \frac{\eta I_0 l\cos\theta}{2\pi r^2}\left[1+\frac{1}{jkr}\right]e^{-jkr}\vec{a_r} + \frac{j\eta k I_0 l\sin\theta}{4\pi r}\left[1+\frac{1}{jkr} - \frac{1}{(kr)^2}\right]e^{-jkr}\vec{a_\theta}$

and: $E_r = \eta\frac{I_0 l\cos\theta}{2\pi r^2}\left[1+\frac{1}{jkr}\right]e^{-jkr}$ and $E_\theta = j\eta\frac{kI_0 l\sin\theta}{4\pi r}\left[1+\frac{1}{jkr} - \frac{1}{(kr)^2}\right]e^{-jkr}$

$E_\phi = 0$

$\boxed{\text{2. solution}}$

because: $f = 10^6\,Hz$ so: $\lambda = \frac{c}{f} = \frac{3\times10^8}{10^6} = 300m$

so: $l = 1m = \frac{\lambda}{300}$ and because $l = \frac{\lambda}{300} < \frac{\lambda}{50}$

Therefore, we can assumed that this antenna is an Infinite simal dipole antenna

so. the average power density. $W_{av} = \frac{1}{2}Re(\vec{E}\times\vec{H^*}) = \vec{a_r}\cdot\frac{\eta}{2}\left|\frac{kI_0 l}{4\pi}\right|^2\frac{\sin^2\theta}{r^2}$

In this problem: $I_0 = 12A$, $r = 5\times10^3 m$ $\theta = 45°$ and $\eta = \sqrt{\frac{\mu}{\varepsilon}} \approx \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377$

~~so: $W_{av} = \frac{377}{2}$~~ $k = \frac{2\pi}{\lambda} = \frac{\pi}{150}$

So: $W_{av} = \frac{377}{2}\times\left|\frac{\frac{\pi}{150}\cdot 12A\times 1m}{4\pi}\right|^2\times\frac{\left(\frac{\sqrt{2}}{2}\right)^2}{25\times10^6} \approx 1.508\times10^{-9}\ W/m^2$

so, the average power density is $1.508\times10^{-9}\ w/m^2$.

Question 3

The radiation pattern for :

- **half wavelength dipole**

1. **result figure**

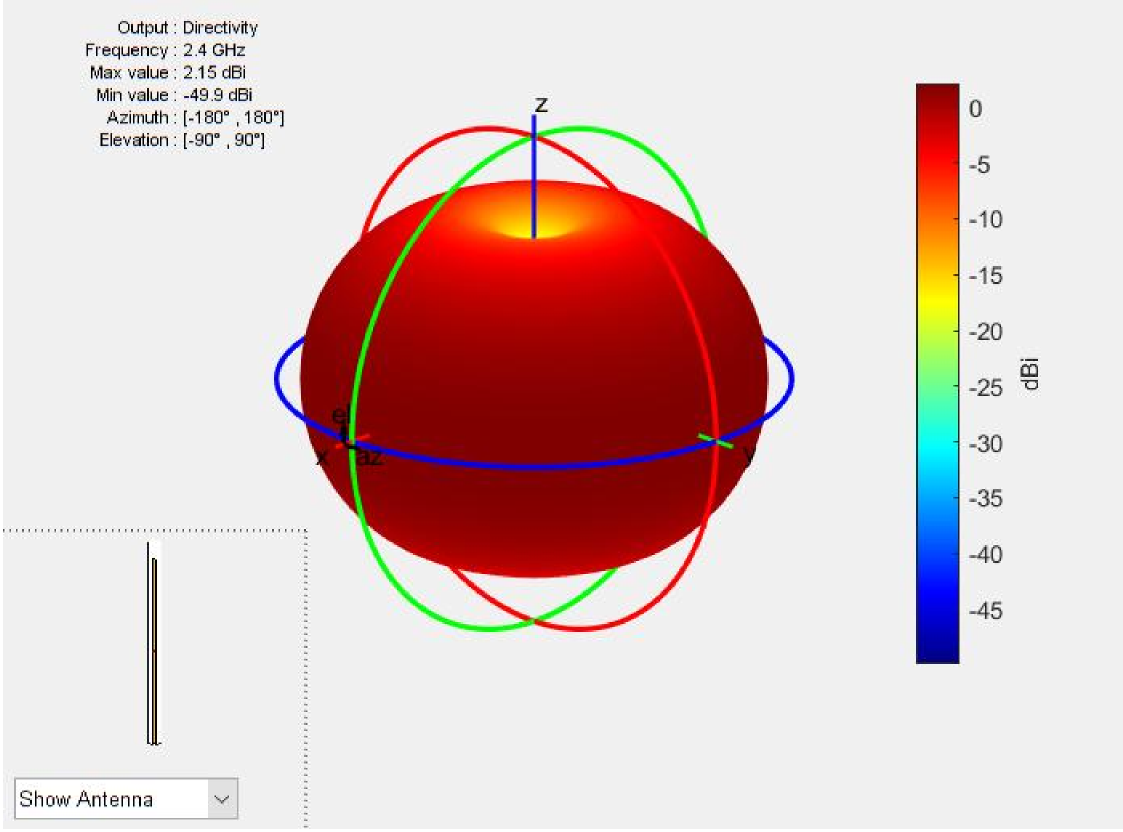


Figure 1 3D radiation pattern for half wavelength dipole

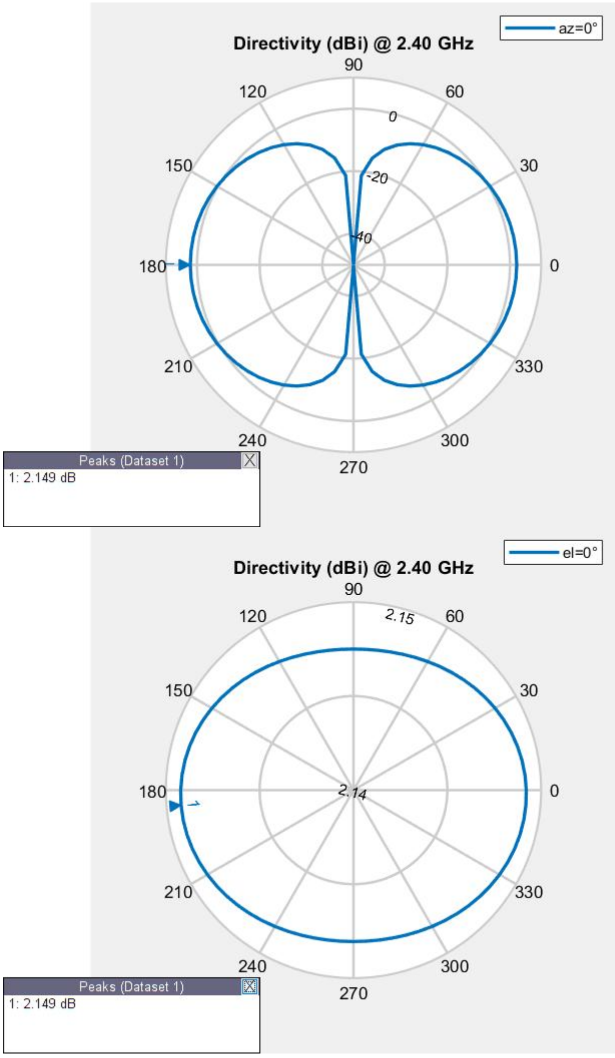


Figure 2 2D radiation pattern for half wavelength dipole

2. **MATLAB codes**

```
% Create a dipole antenna
% Generated by MATLAB(R) 9.10 and Antenna Toolbox 5.0.
% Generated on: 12-Mar-2022 00:56:42

%% Antenna Properties

antennaObject = design(dipole, 2400*1e6);
antennaObject.Length = 0.062;
% Show
figure;
show(antennaObject)

%% Antenna Analysis
% Define plot frequency
plotFrequency = 2400*1e6;
% Define frequency range
freqRange = (2160:24:2640)*1e6;
% sparameter
figure;
s = sparameters(antennaObject, freqRange);
rfplot(s)
% pattern
figure;
pattern(antennaObject, plotFrequency)
% azimuth
figure;
patternAzimuth(antennaObject, plotFrequency, 0, 'Azimuth', 0:5:360)
% elevation
figure;
patternElevation(antennaObject, plotFrequency,0,'Elevation',0:5:360)
```

- **quarter wavelength monopole (infinite ground)**

1. **Result figure**

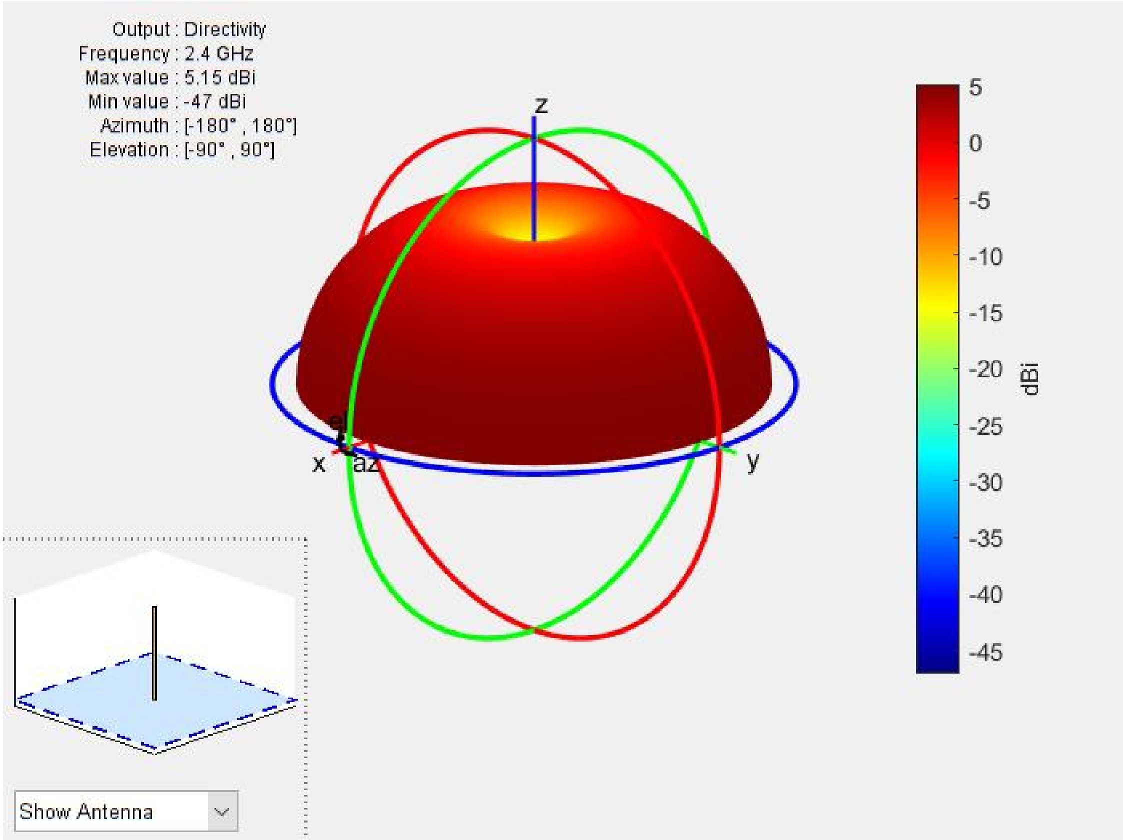


Figure 3 3D radiation pattern for quarter wavelength monopole

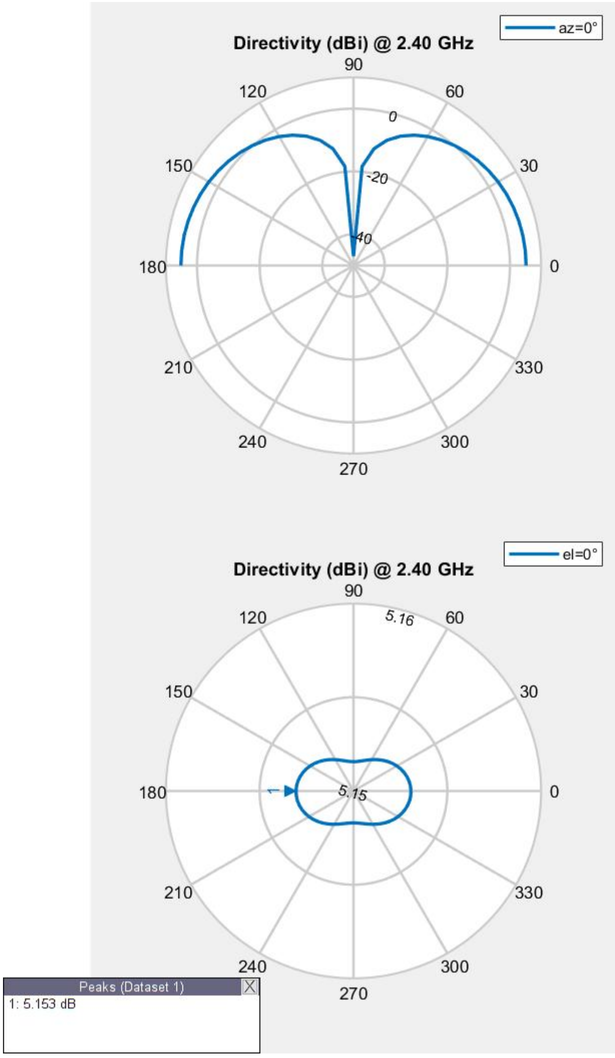


Figure 4 2D radiation pattern for quarter wavelength monopole

2. **MATLAB codes**

```
% Create a monopole antenna
% Generated by MATLAB(R) 9.10 and Antenna Toolbox 5.0.
% Generated on: 12-Mar-2022 01:15:12

%% Antenna Properties

antennaObject = design(monopole, 2400*1e6);
antennaObject.Height = 0.0307;
antennaObject.GroundPlaneLength = inf;
antennaObject.GroundPlaneWidth = inf;
% Show
figure;
show(antennaObject)

%% Antenna Analysis
% Define plot frequency
plotFrequency = 2400*1e6;
% Define frequency range
freqRange = (2160:24:2640)*1e6;
% pattern
figure;
pattern(antennaObject, plotFrequency)
% azimuth
figure;
patternAzimuth(antennaObject, plotFrequency, 0, 'Azimuth', 0:5:360)
% elevation
figure;
patternElevation(antennaObject, plotFrequency,0,'Elevation',0:5:360)
```

Theoretical Calculation (Calculate Directivity and HPBW for each antenna)

See the next two page below :

3. solution

calculate the directivity and HPBW for each antenna.

① half-wavelength dipole

because we know: for linear Antenna:

$$W_{av} = \vec{a_r} W_{av} = \vec{a_r} \cdot \eta \frac{|I_0|^2}{8\pi^2 r^2} \left[\frac{\cos(\frac{kl}{2}\cos\theta) - \cos(\frac{kl}{2})}{\sin\theta} \right]^2$$

and $U = r^2 W_{av} = \eta \cdot \frac{|I_0|^2}{8\pi^2} \left[\frac{\cos(\frac{kl}{2}\cos\theta) - \cos(\frac{kl}{2})}{\sin\theta} \right]^2$

so: $D_0 = D_{max} = D(\theta,\phi)_{max} = \frac{U(\theta,\phi)_{max}}{U_0} = \frac{4\pi U(\theta,\phi)_{max}}{P_{rad}}$

and: $P_{rad} = \int_0^{2\pi}\int_0^{\pi} \vec{a_r} \cdot W_{av} \cdot \vec{a_r} \cdot r^2 \sin\theta \, d\theta \, d\phi = \eta \cdot \frac{|I_0|^2}{4\pi} \int_0^{\pi} \frac{\left[\cos(\frac{kl}{2}\cos\theta) - \cos(\frac{kl}{2})\right]^2}{\sin\theta} d\theta$

$\therefore P_{rad} = \eta \cdot \frac{|I_0|^2}{4\pi} \{ C + \ln(kl) - C_i(kl) + \frac{1}{2}\sin(kl)\left[S_i(2kl) - 2S_i(kl)\right]$

$+ \frac{1}{2}\cos(kl)\left[C + \ln(kl/2) + C_i(2kl) - 2C_i(kl)\right] \}$ where $C = 0.5772$

and $C_i(x)$ and $S_i(x)$ are the cosine and sine integrals.

so: $D_0 = \frac{2F(\theta)|_{max}}{Q}$, and $F(\theta) = \left[\frac{\cos(\frac{kl}{2}\cos\theta) - \cos(\frac{kl}{2})}{\sin\theta} \right]^2$

$Q = \{ C + \ln(kl) - C_i(kl) + \frac{1}{2}\sin(kl)\left[S_i(2kl) - 2S_i(kl)\right] + \frac{1}{2}\cos(kl)\left[C + \ln(kl/2)\right.$

$\left. + C_i(2kl) - 2C_i(kl)\right] \}$

so. by calculator, we easily know that $Q \simeq 1.2195$

because $l = \frac{\lambda}{2}$ in this problem, $k = \frac{2\pi}{\lambda}$ so: $kl = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi \Rightarrow \frac{kl}{2} = \frac{\pi}{2}$

so: $F(\theta) = \left[\frac{\cos(\frac{\pi}{2}\cos\theta) - \cos(\frac{\pi}{2})}{\sin\theta} \right]^2$, it means that ~~we~~ when: $\theta = \frac{\pi}{2}$,

$F(\theta)_{max} = 1$ Therefore: $D_0 = \frac{2F(\theta)|_{max}}{Q} = \frac{2}{1.2195} \simeq 1.64 = D_{max}$

so: $D_{max} = D_0 = 1.64 = 2.15 dBi$

so, the directivity of $\frac{\lambda}{2}$ dipole is 2.15dBd

Then: because: $D_0 = \frac{2F(\theta)_{max}}{Q}$ and $F(\theta) = \left[\frac{\cos(\frac{kl}{2}\cos\theta) - \cos(\frac{kl}{2})}{\sin\theta} \right]^2 = \left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta} \right]^2$

because when: $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$, we can get $F(\theta)_{max} = 1$

and when: $F(\theta) = \frac{1}{2} \Rightarrow$ That means: $\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ ··· ①

so, by solving equation 1, we can get: $\theta = 51^\circ$ or 129° (在 $\frac{\pi}{2}$ 两侧)

so: because $\theta = 51^\circ$ or 129°, HPBW $= (129^\circ - 51^\circ) = 78^\circ$

∴ Therefore, for $\frac{\lambda}{2}$ dipole, the directivity is 2.15dBi and the HPBW for this antenna is 78°.

② for $\frac{\lambda}{4}$ monopole (infinite ground)

Solution:

∵ 由镜像原理，该$\frac{\lambda}{4}$单极子天线和其镜像构成了一个偶极子.

∴ 在地面上方，坡印亭矢量和偶极子天线是完全相同的。

但 Prad 只包围在了上半空间的半球面。因此，单极子的辐射功率(Prad)是偶极子相应值的一半。

so: for monopole antenna: $P_{rad}/_{mono} = \frac{1}{2} P_{rad}/_{dipole}$

Therefore: for monopole antenna: $D_0 = \frac{4\pi U(\theta,\phi)_{max}}{P_{rad}/_{mono}} = \frac{4\pi U(\theta,\phi)_{max}}{\frac{1}{2} P_{rad}/_{dipole}}$

so: 相同长度的单极子天线的 D_0 为相同长度偶极子天线的 D_0 的 2倍.
(辟极子等效)

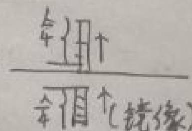
so, for dipole: $D_0 = \frac{2F(\theta)|_{max}}{Q}$ and $Q \simeq 1.2195$

$$F(\theta) = \left[\frac{\cos(\frac{kl}{2}\cos\theta) - \cos(\frac{kl}{2})}{\sin\theta} \right]^2$$

~~when $l=\frac{\lambda}{4}$ $k=\frac{2\pi}{\lambda}$ so $kl=\frac{\pi}{2}$ and $\frac{kl}{2}=\frac{\pi}{4}$~~

~~so: $F(\theta) = \left[\frac{\cos(\frac{\pi}{4}\cos\theta) - \frac{\sqrt{2}}{2}}{\sin\theta}\right]^2$~~

因为这个 monopole 可以建模为:



∴ 该$\frac{\lambda}{4}$ monopole antenna 相当于总长为$\frac{\lambda}{2}$的 dipole.

∴ 该$\frac{\lambda}{4}$ monopole antenna 的 D_0 为 $\frac{\lambda}{2}$ 的 dipole 的 D_0 的 2倍.

so. in the last problem: for $\frac{\lambda}{2}$ dipole: $D_0 = 1.64 = 2.15dBi = D_0/_{\frac{\lambda}{2}dipole}$

so: for $\frac{\lambda}{4}$ monopole antenna: $D_0 = \frac{4\pi U(\theta,\phi)_{max}}{P_{rad}/_{mono}} = \frac{4\pi U(\theta,\phi)_{max}}{\frac{1}{2}P_{rad}/_{dipole}} = 2D_0/_{\frac{\lambda}{2}dipole}$

so. $D_0 = 2D_0/_{\frac{\lambda}{2}dipole} = 3.28 \simeq 5.15dBi$

Therefore, for HPBW: $F(\theta) = \left[\frac{\cos(\frac{kl}{2})\cos\theta) - \cos(\frac{kl}{2})}{\sin\theta}\right]^2$ 因为$\frac{\lambda}{4}$ monopole antenna 可等效为$\frac{\lambda}{2}$ dipole. so, as the same in problem (1), when: $F(\theta) = \frac{1}{2}F(\theta)|_{max}$

we get: $\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta} = \frac{\sqrt{2}}{2}$ -- ① $\Rightarrow \theta = 51°$ or $129°$ and when $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$, $F(\theta)$ can get maximum value.

but θ is for the dipole antenna, so, for monopole antenna,

we can get: $\alpha = \frac{\theta}{2}$ -- ② so, when $F(\theta)$ is max: $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2} \Rightarrow \alpha = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

When $F(\theta)$ is half the $F(\theta)|_{max}$: $\theta = 51°$ or $129° \Rightarrow \alpha = \frac{\theta}{2} = 25.5°$ or $64.5°$

so: HPBW $= 64.5° - 25.5° = 39°$

Therefore: for monopole antenna: The directivity is 3.28 (5.15dBi) and the HPBW is 39°.