天线与电波传播
ANTENNAS AND WAVE PROPAGATION

LECTURE 4

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Dipole/Monopole Antenna

Use vector potential to find far-field characteristics

Infinitesimal dipole ($L \le \lambda / 50$)

Short dipole ($\lambda / 50 < L \le \lambda / 10$)

Linear dipole

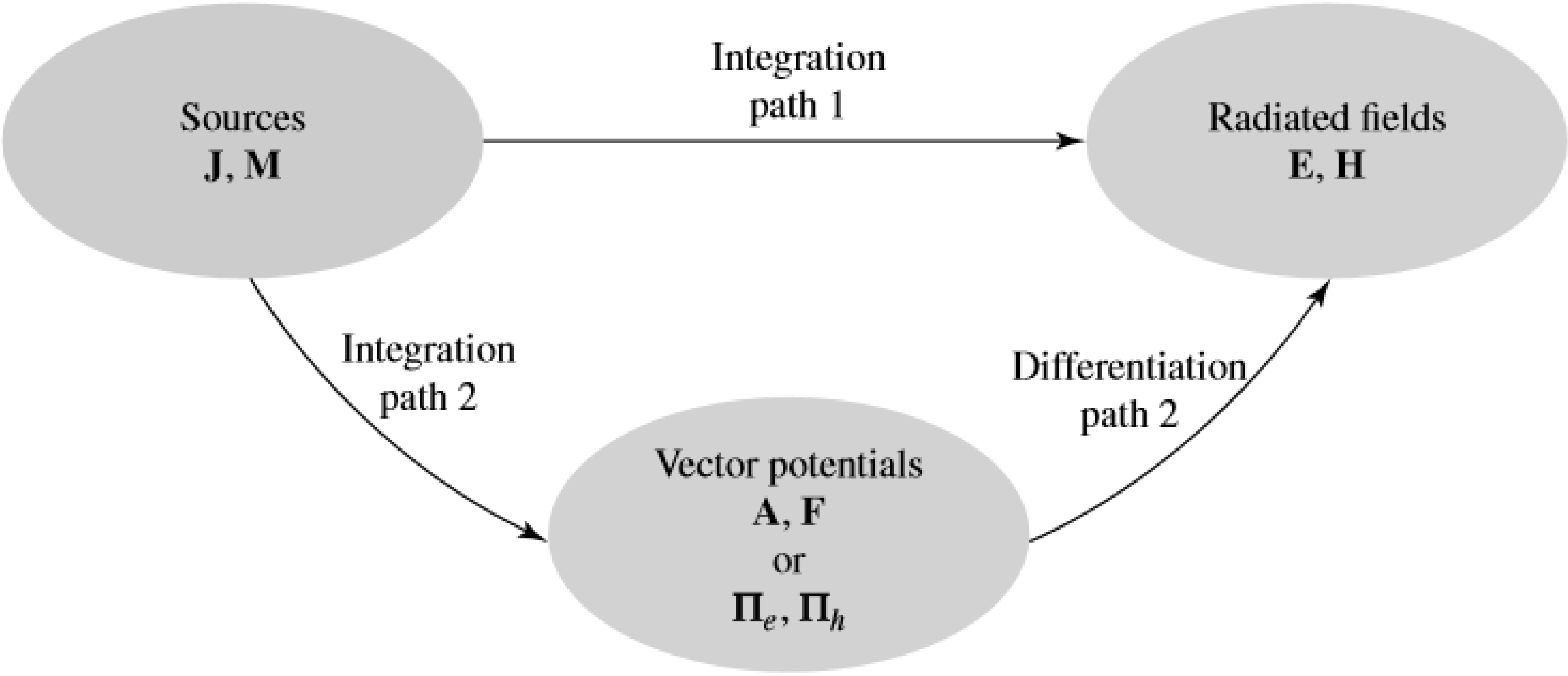
- current distribution and side lobes
- wavelength vs radiation pattern/directivity etc.)

Balun

Monopole (Method of images)

- Wavelength vs radiation pattern/directivity
- Effects of ground plane size

Vector Potential



- (Chapter 3, Fig. 3.1, Balanis Book)

Far-Field Radiation Characteristics

- Specify electric and/or magnetic current densities *J*, *M* (Chapter 3, Fig. 3.1, Balanis Book)
- Determine **vector potential** components A_θ, A_ϕ and/or F_θ, F_ϕ (3-46)–(3-54)
- Find far-zone E and H radiated fields (E_θ, E_ϕ; H_θ, H_ϕ) using (3-58a)–(3-58b)

Far-Field Radiation Characteristics

- Find Radiation Power Density (W_{rad} , W/m^2) or Radiation Power Intensity (U_{rad}, W/Sr) at all directions through integration
- Find Directivity

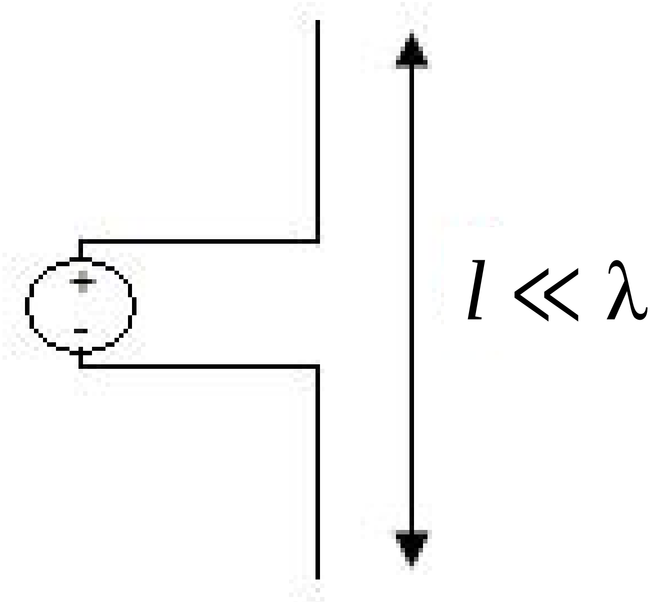
$$D_0 = D_{\max} = D(\theta,\phi)|_{\max} = \frac{U(\theta,\phi)|_{\max}}{U_0} = \frac{4\pi U(\theta,\phi)|_{\max}}{P_{\mathrm{rad}}}$$

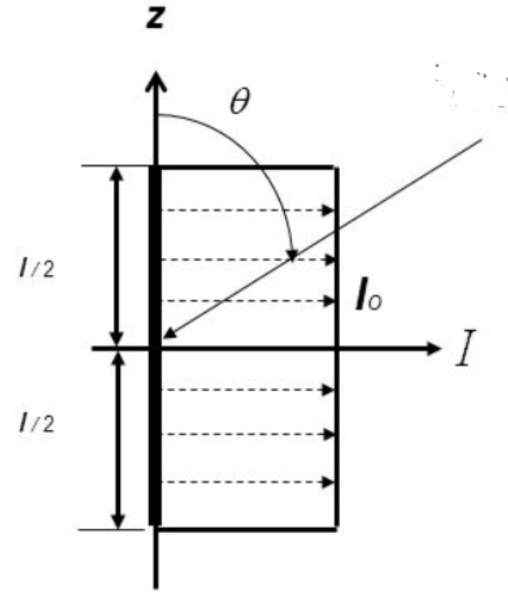
- Form normalized power amplitude pattern:

$$P_n(\theta,\phi) = \frac{U(\theta,\phi)}{U_{\max}}$$

Infinitesimal Dipole Antenna

- very small $l \ll \lambda$
- very thin $a \ll \lambda$





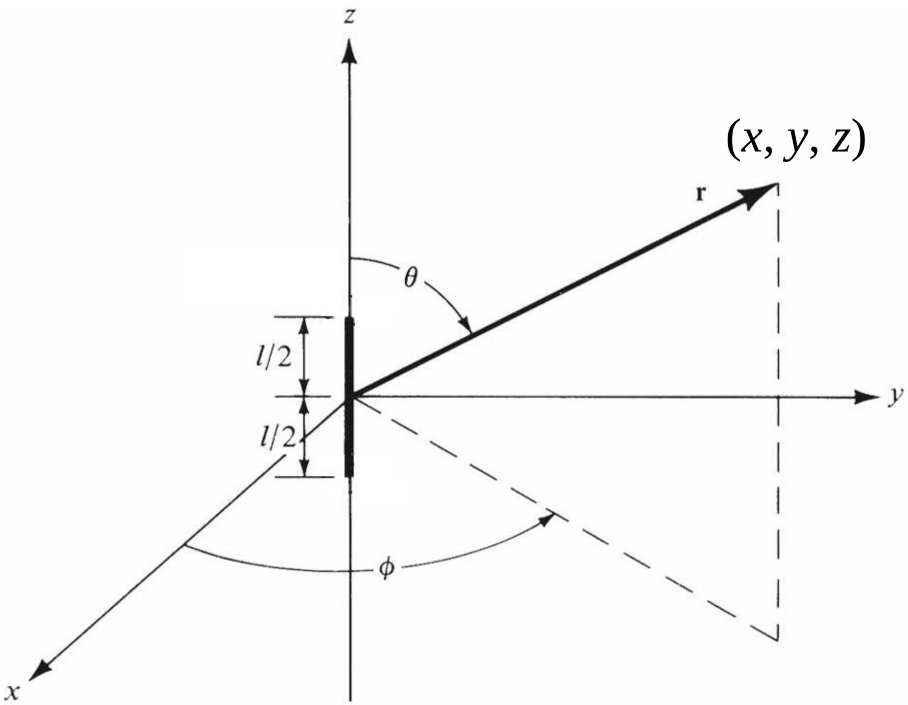
$$\mathbf{I}(z') = \hat{\mathbf{a}}_z I_o$$

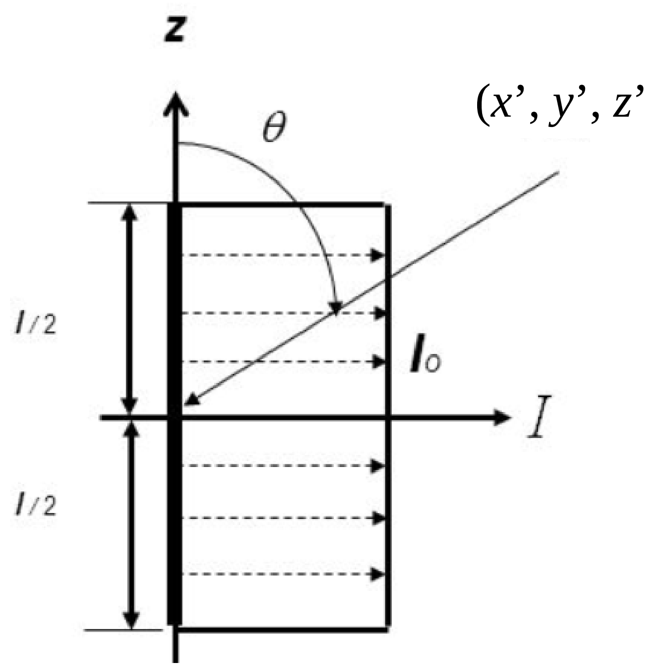
Constant current

Infinitesimal Dipole Antenna

Find vector potential at an observation point (x, y, z)

A point on the source (x', y', z')



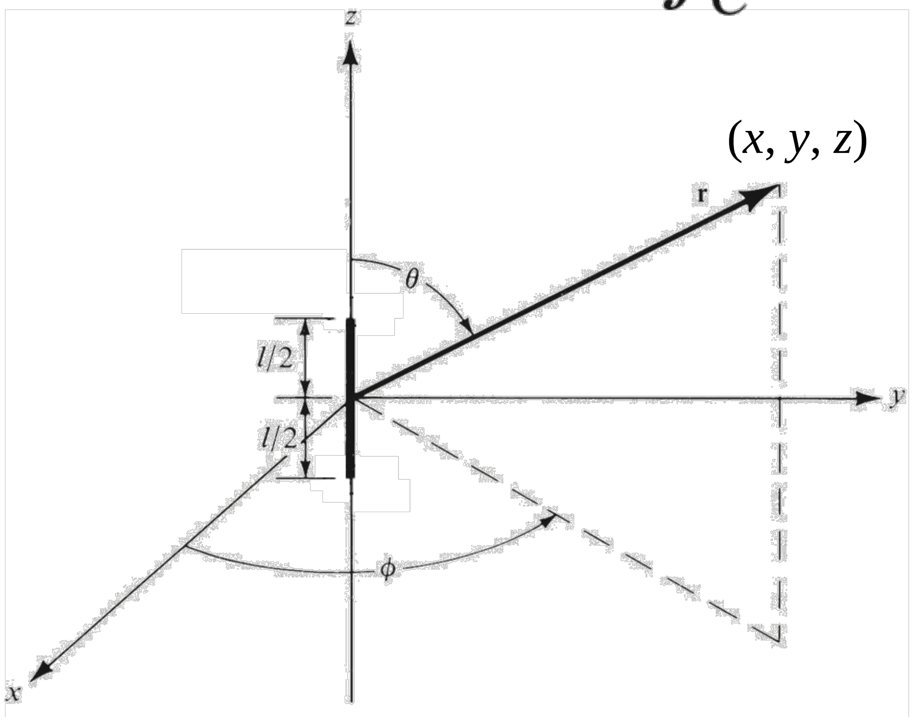


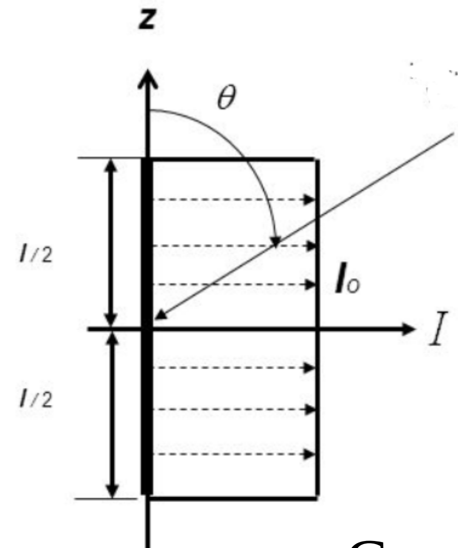
$$\mathbf{I}(z') = \hat{\mathbf{a}}_z I_o$$

Constant current

Infinitesimal Dipole Antenna Vector Potential

$$\mathbf{A}(x,y,z) = \frac{\mu}{4\pi}\int_C \mathbf{I}_e(x',y',z')\frac{e^{-jkR}}{R}\,dl'$$





$$\mathbf{I}(z') = \hat{\mathbf{a}}_z I_o$$

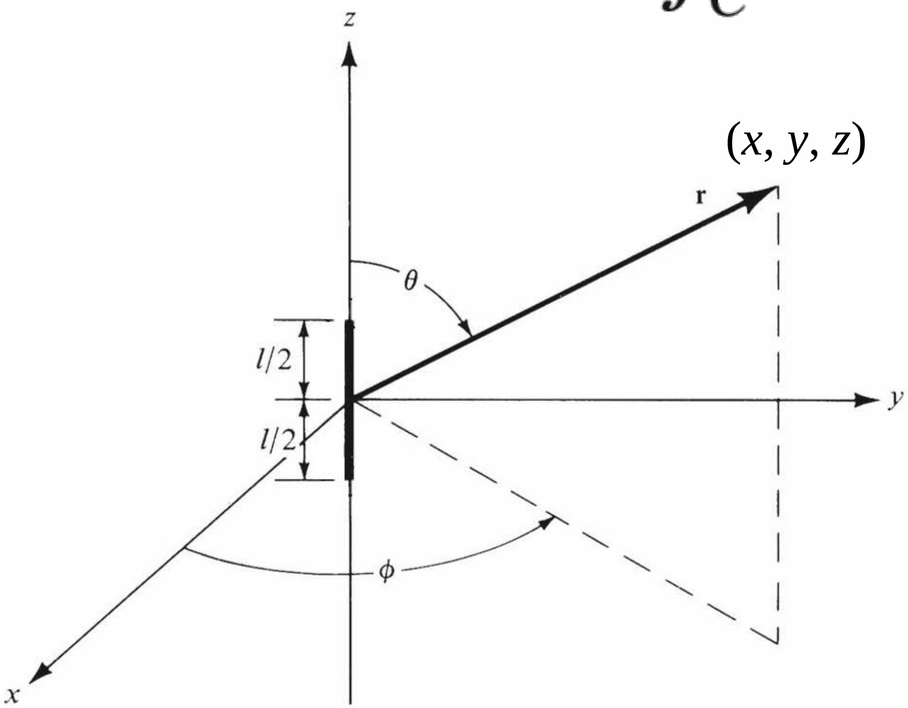
Constant current

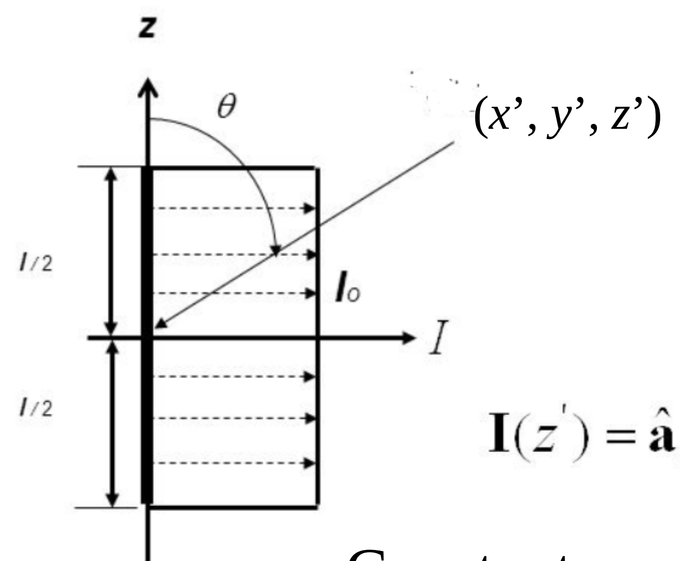
Infinitesimal Dipole Antenna
Vector Potential

$$\mathbf{I}_e(x', y', z') = \hat{\mathbf{a}}_z I_0$$

$$x' = y' = z' = 0 \text{ (infinitesimal dipole)}$$

$$\mathbf{A}(x, y, z) = \frac{\mu}{4\pi} \int_C \mathbf{I}_e(x', y', z') \frac{e^{-jkR}}{R}\, dl'$$





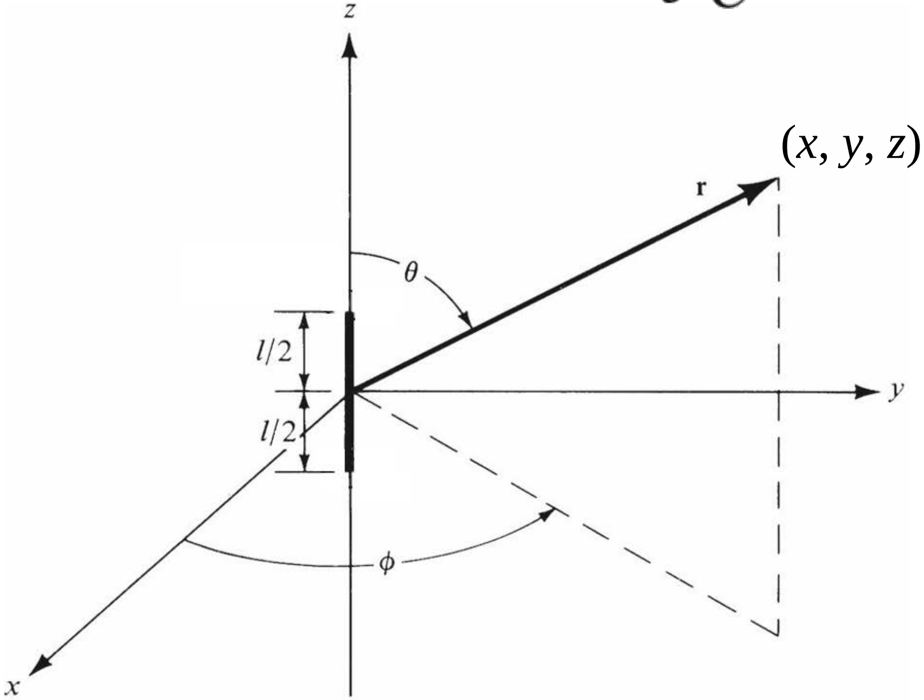
$$\mathbf{I}(z') = \hat{\mathbf{a}}_z I_o$$

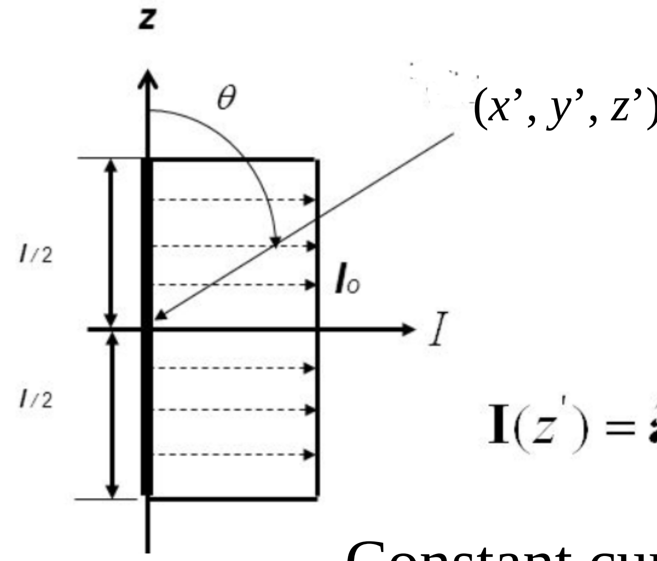
Constant current

Infinitesimal Dipole Antenna Vector Potential

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} = \sqrt{x^2+y^2+z^2}$$
$$= r = \text{constant}$$

$$\mathbf{A}(x,y,z) = \frac{\mu}{4\pi}\int_C \mathbf{I}_e(x',y',z')\frac{e^{-jkR}}{R}\,dl'$$





$$\mathbf{I}(z') = \hat{\mathbf{a}}_z I_o$$

Constant current

Infinitesimal Dipole Antenna Vector Potential

$$\mathbf{A}(x,y,z) = \frac{\mu}{4\pi}\int_C \mathbf{I}_e(x',y',z')\frac{e^{-jkR}}{R}\,dl'$$

$$\boxed{\mathbf{A}(x,y,z) = \hat{\mathbf{a}}_z \frac{\mu I_0}{4\pi r} e^{-jkr}\int_{-l/2}^{+l/2} dz' = \hat{\mathbf{a}}_z \frac{\mu I_0 l}{4\pi r} e^{-jkr}}$$

Convert to Spherical coordinates

$$A_r = A_z\cos\theta = \frac{\mu I_0 l e^{-jkr}}{4\pi r}\cos\theta \qquad A_\theta = -A_z\sin\theta = -\frac{\mu I_0 l e^{-jkr}}{4\pi r}\sin\theta \qquad A_\phi = 0$$

Infinitesimal Dipole Antenna Vector Potential

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H field

E field

$$\mathbf{H}_A = \frac{1}{\mu}\nabla \times \mathbf{A}$$

$$\mathbf{E}_A = -\nabla\phi_e - j\omega\mathbf{A} = -j\omega\mathbf{A} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot\mathbf{A})$$

$$H_r = H_\theta = 0$$

$$H_\phi = j\frac{kI_0 l \sin\theta}{4\pi r}\left[1 + \frac{1}{jkr}\right]e^{-jkr}$$

$$E_r = \eta\frac{I_0 l \cos\theta}{2\pi r^2}\left[1 + \frac{1}{jkr}\right]e^{-jkr}$$

$$E_\theta = j\eta\frac{kI_0 l \sin\theta}{4\pi r}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right]e^{-jkr}$$

$$E_\phi = 0$$

Infinitesimal Dipole Antenna

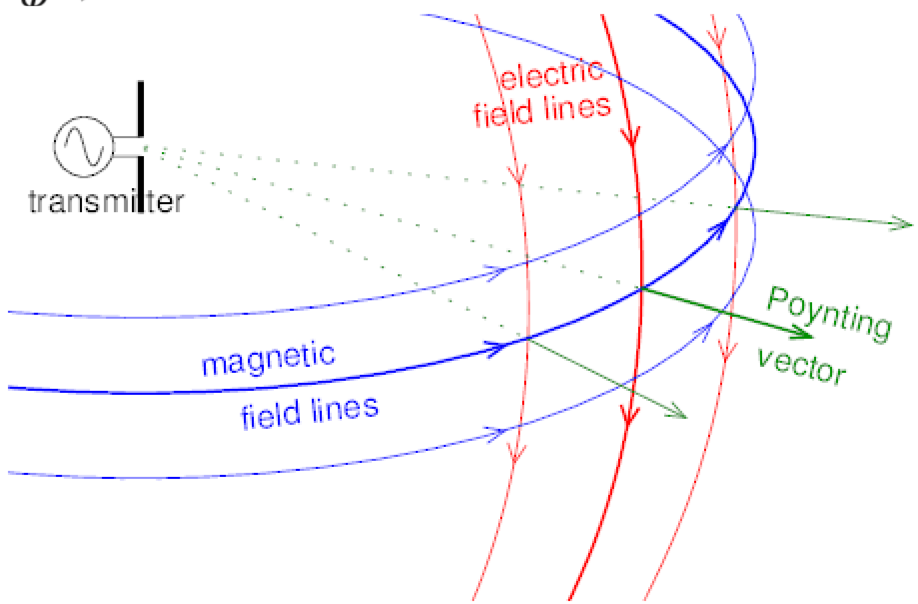
Power Density (Poynting Vector)

$$\mathbf{W} = \frac{1}{2}(\mathbf{E}\times\mathbf{H}^*) = \frac{1}{2}(\hat{\mathbf{a}}_r E_r + \hat{\mathbf{a}}_\theta E_\theta)\times(\hat{\mathbf{a}}_\phi H_\phi^*)$$

$$= \frac{1}{2}(\hat{\mathbf{a}}_r E_\theta H_\phi{}^* - \hat{\mathbf{a}}_\theta E_r H_\phi{}^*)$$

$$W_r = \frac{\eta}{8}\left|\frac{I_0 l}{\lambda}\right|^2 \frac{\sin^2\theta}{r^2}\left[1 - j\frac{1}{(kr)^3}\right]$$

$$W_\theta = j\eta\frac{k|I_0 l|^2\cos\theta\sin\theta}{16\pi^2 r^3}\left[1 + \frac{1}{(kr)^2}\right]$$



The transverse component W_θ of the power density does not contribute to the integral.
(Pure imaginary, no real radiated power!)

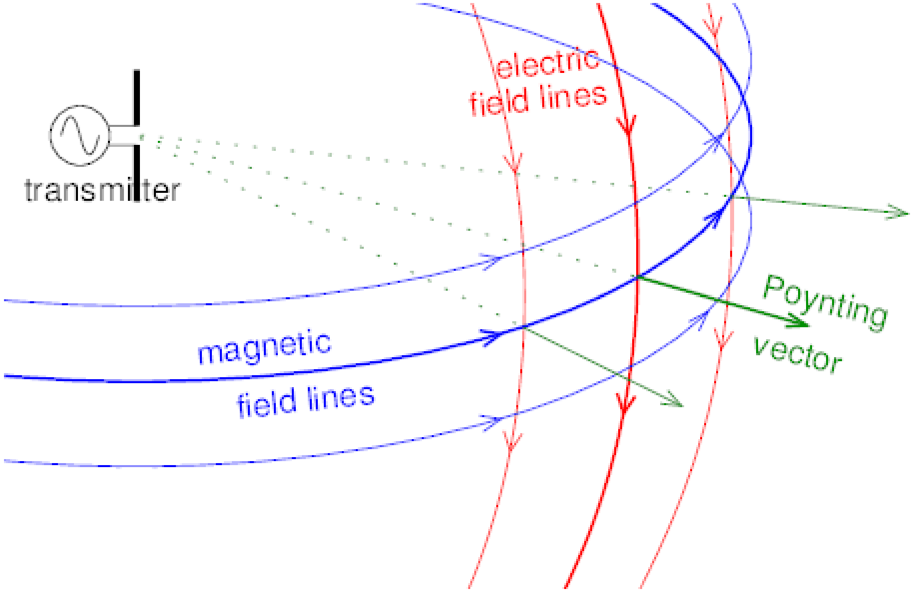
Infinitesimal Dipole Antenna

Radiated Power

$$P = \oiint_S \mathbf{W} \cdot d\mathbf{s} = \int_0^{2\pi} \int_0^{\pi} (\hat{\mathbf{a}}_r W_r + \hat{\mathbf{a}}_\theta W_\theta) \cdot \hat{\mathbf{a}}_r r^2 \sin\theta \, d\theta \, d\phi$$

$$P = \int_0^{2\pi} \int_0^{\pi} W_r r^2 \sin\theta \, d\theta \, d\phi$$

$$= \eta \frac{\pi}{3} \left| \frac{I_0 l}{\lambda} \right|^2 \left[1 - j\frac{1}{(kr)^3} \right]$$



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Infinitesimal Dipole Antenna

Radiated Power

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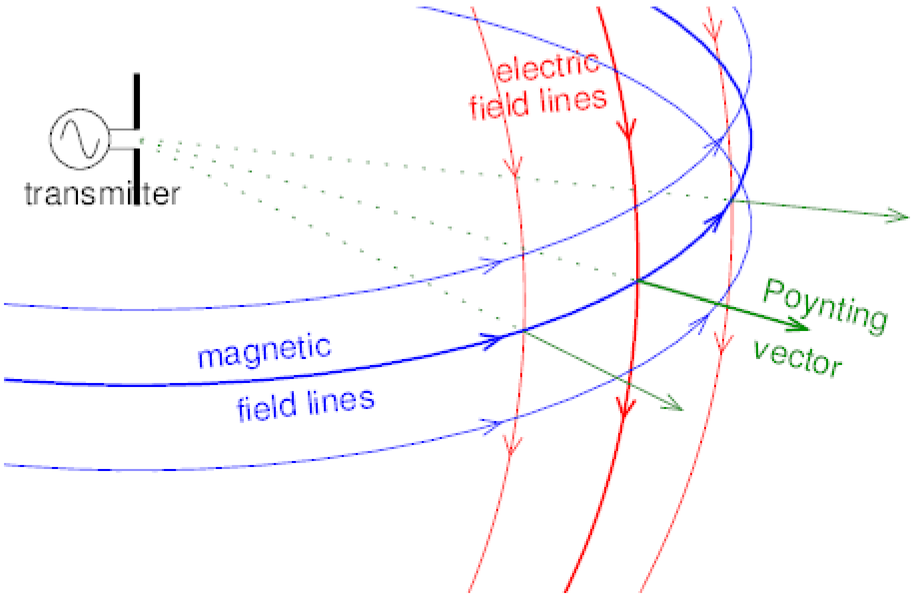
$$P = \int_0^{2\pi} \int_0^{\pi} W_r r^2 \sin\theta \, d\theta \, d\phi$$



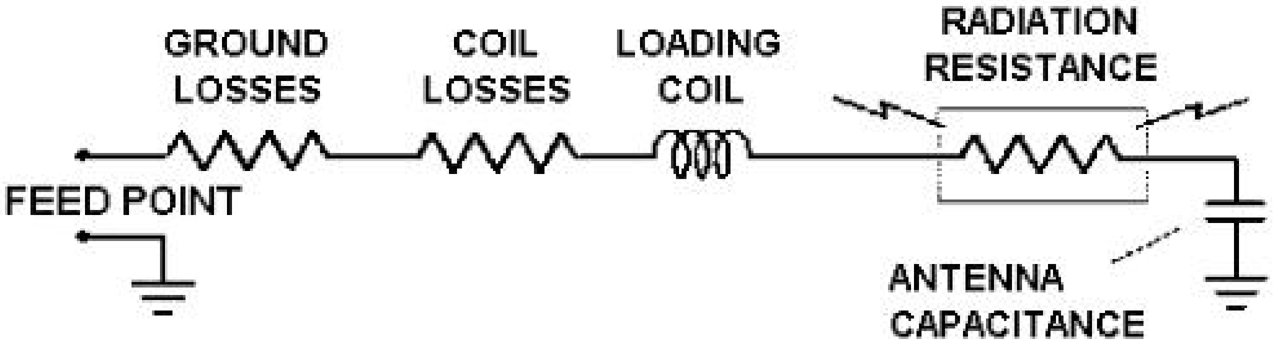
$$= \eta \frac{\pi}{3} \left| \frac{I_0 l}{\lambda} \right|^2 \left[1 - j \frac{1}{(kr)^3} \right]$$

P_{rad}

The transverse component W_θ of the power density does not contribute to the integral.
(Pure imaginary, no real radiated power!)

Radiation Resistance

Antenna System Equivalent Circuit



An electrically short antenna looks like a small, lossy capacitor.

To make an electrically short antenna resonant, a loading coil is inserted in series with the antenna. The coil is built to have an inductive reactance equal and opposite to the capacitive reactance of the short antenna, so the combination of reactances cancels.

$$X = X_L + X_C = \omega L - \frac{1}{\omega C}$$

Radiation Resistance

an antenna's feedpoint resistance (R_{in})

$$R_{in} = R_R + R_L$$

R_R: **radiation resistance** caused by **the radiation of electromagnetic waves** from the antenna

R_L: **loss resistance** (also called ohmic resistance) caused by **ordinary electrical resistance** in the antenna, or **energy lost to nearby objects**, such as the earth, which dissipate RF energy as heat.

Radiation Resistance

R_L is the equivalent **resistance**

a **fictitious resistance** that would dissipate the **same amount** of **power** as if it is **radiated** by the **antenna**.

When substituted in series with the **antenna** will consume the **same power** as is **actually radiated**.

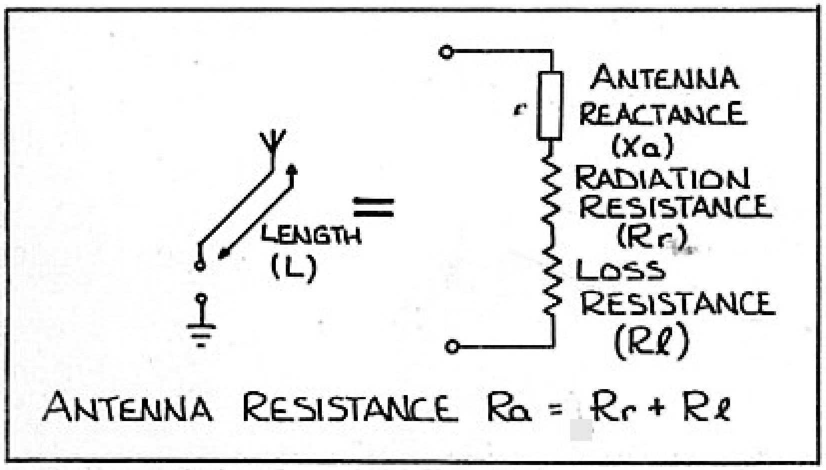


Figure 1 — Equivalent Antenna Electrical Circuit

Radiation Resistance

If radiated power is

$$P = |I|^2 R_r$$

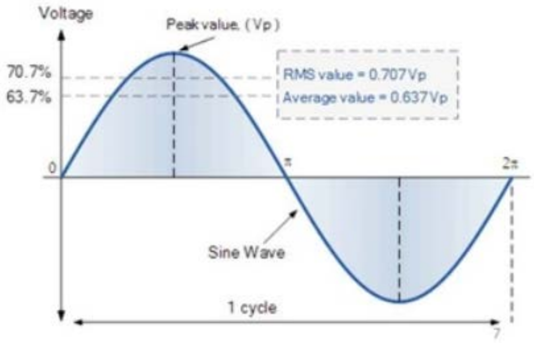
where *I* is the electric current flowing into the feeds of the antenna. Usually *I* is the time average of the current (**root mean square value, RMS**)

Radiation resistance (effective resistance)

$$R_r = \frac{P}{|I|^2}$$

Radiation Resistance

root mean square of sine or cos wave



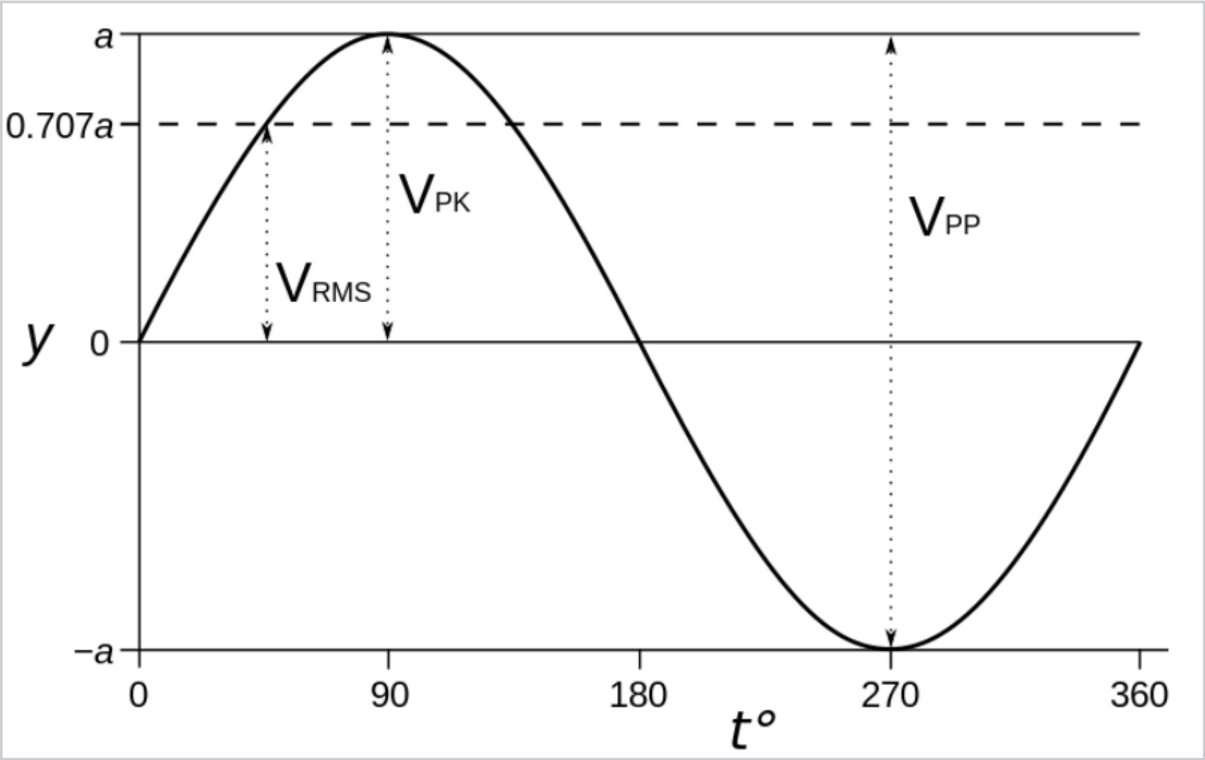
$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)dt}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 cos^2(\omega t + \varphi)dt} = \frac{V_m}{\sqrt{2}}$$

$=0.707Vm$

Radiation Resistance

root mean square of sine or cos wave



Graph of a sine wave's voltage vs. time (in degrees), showing RMS, peak (PK), and peak-to-peak (PP) voltages.

Radiation Resistance (Infinitesimal Dipole Antenna)

Radiated Power

$$\boxed{P_{\text{rad}} = \eta\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2 = \frac{1}{2}|I_0|^2 R_r}$$

R_r is the radiation resistance

$$\boxed{R_r = \eta\left(\frac{2\pi}{3}\right)\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(\frac{l}{\lambda}\right)^2}$$

Radiation Resistance (Infinitesimal Dipole Antenna)

Example 4.1

Find the radiation resistance of an infinitesimal dipole whose overall length is $l = \lambda/50$.

Solution: Using (4-19)

$$R_r = 80\pi^2 \left(\frac{l}{\lambda}\right)^2 = 80\pi^2 \left(\frac{1}{50}\right)^2 = 0.316 \text{ ohms}$$

Since the radiation resistance of an infinitesimal dipole is about 0.3 ohms, it will present a very large mismatch when connected to practical transmission lines, many of which have characteristic impedances of 50 or 75 ohms. The reflection efficiency (e_r) and hence the overall efficiency (e_0) will be very small.

Infinitesimal Dipole Antenna

time average power density

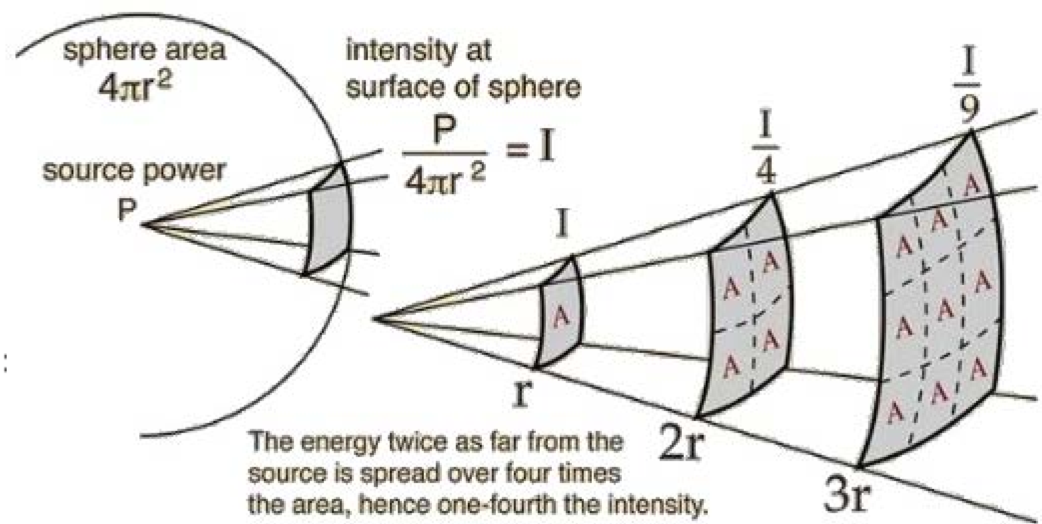
$$\mathbf{W}_{\mathrm{av}} = \frac{1}{2}\mathrm{Re}(\mathbf{E} \times \mathbf{H}^*) = \hat{\mathbf{a}}_r \frac{1}{2\eta}|E_\theta|^2 = \hat{\mathbf{a}}_r \frac{\eta}{2}\left|\frac{kI_0 l}{4\pi}\right|^2 \frac{\sin^2\theta}{r^2}$$

power intensity

$$U = r^2 W_{\mathrm{av}}$$

$$= \frac{\eta}{2}\left(\frac{kI_0 l}{4\pi}\right)^2 \sin^2\theta$$

- The intensity follows a $1/r^2$ relationship



Infinitesimal Dipole Antenna

maximum power intensity

$$U = r^2\, W_{av} = \frac{\eta}{2}\left(\frac{kI_0 l}{4\pi}\right)^2 \sin^2\theta \Rightarrow U_{max} = \frac{\eta}{2}\left(\frac{kI_0 l}{4\pi}\right)^2$$

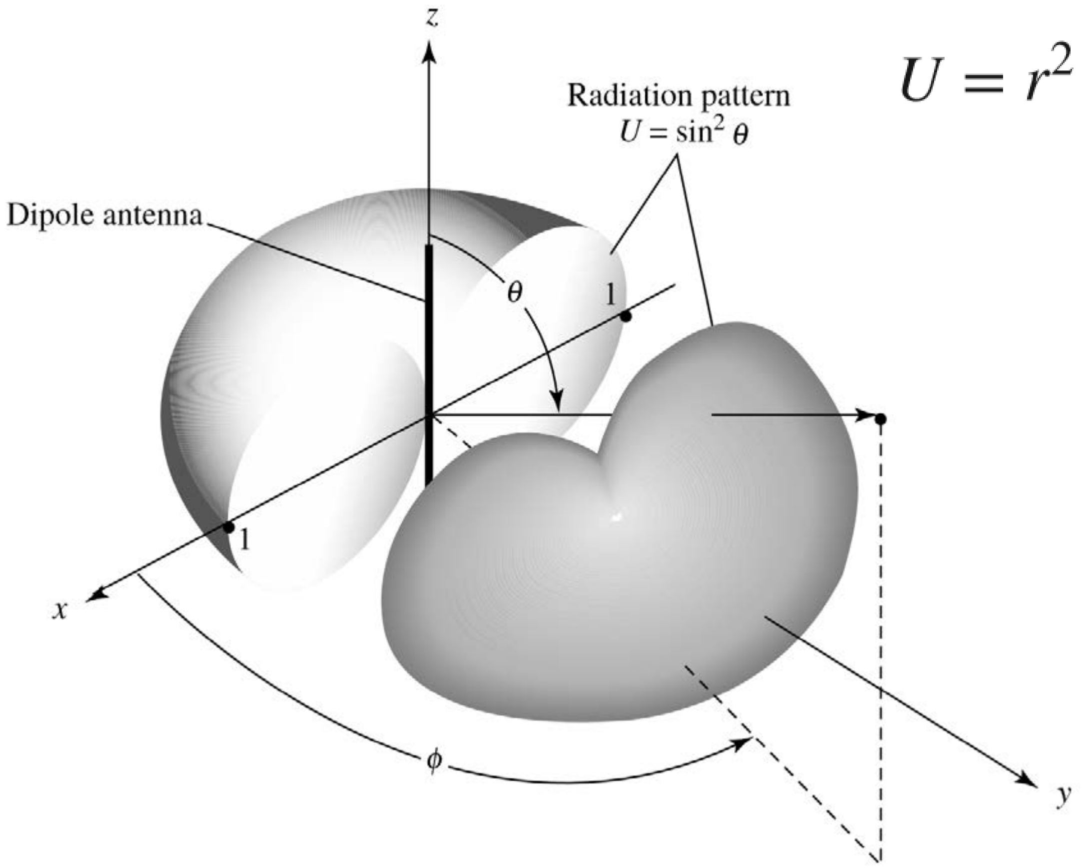
directivity

$$\boxed{D_0 = 4\pi\frac{U_{max}}{P_{rad}} = \frac{3}{2}}$$

$$P_{rad} = \eta\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2$$

Infinitesimal Dipole Antenna

Radiation pattern

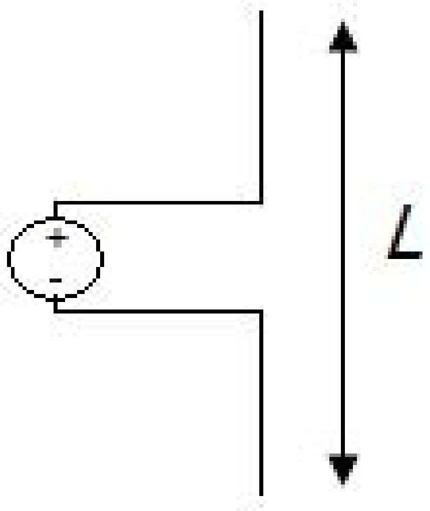


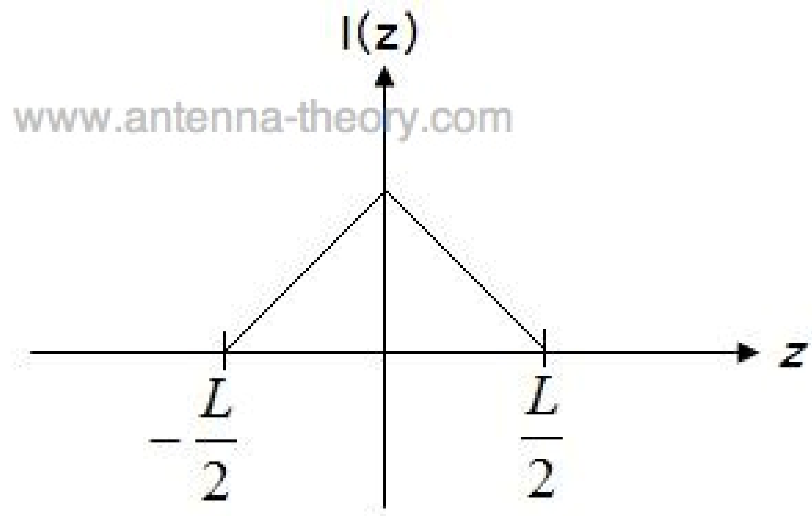
$$U = r^2 W_{av} = \frac{\eta}{2}\left(\frac{kI_0 l}{4\pi}\right)^2 \sin^2\theta$$

Short Dipole Antenna

the simplest of all antennas.

simply an open-circuited wire, fed at its center as shown (left), current distribution (right)





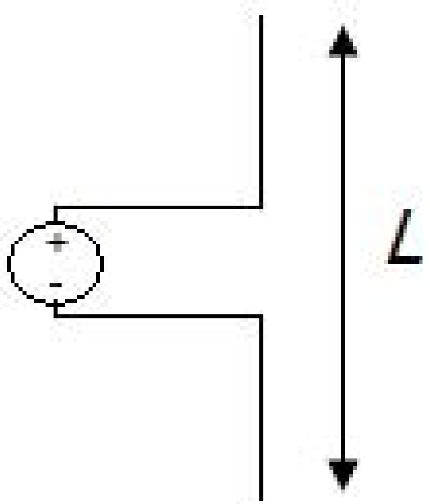
$\lambda/50 < L \leq \lambda/10$

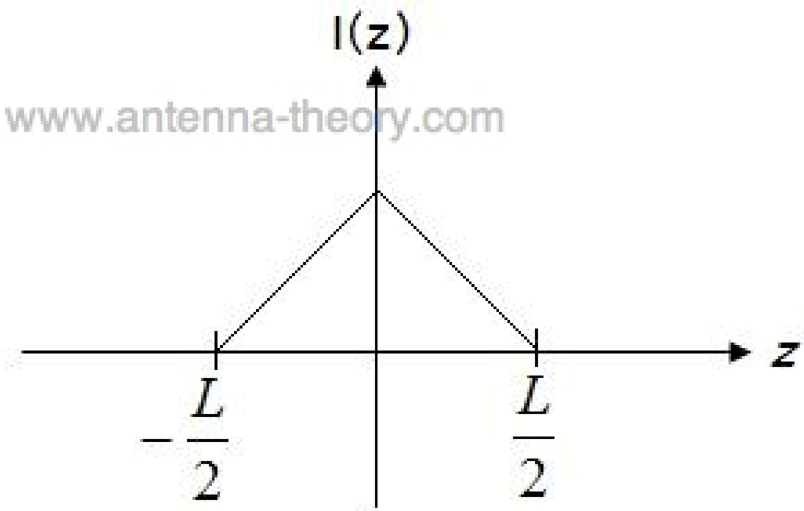
this is the amplitude of the current distribution; it is oscillating in time sinusoidally at frequency *f*.

Short Dipole Antenna

the simplest of all antennas.

simply an open-circuited wire, fed at its center as shown (left), current distribution (right)





$$\boxed{\lambda /50 < L \leq \lambda /10}$$

$$\mathbf{I}_e(x', y', z') = \begin{cases} \hat{\mathbf{a}}_z I_0 \left(1 - \frac{2}{l} z'\right), & 0 \leq z' \leq l/2 \\ \hat{\mathbf{a}}_z I_0 \left(1 + \frac{2}{l} z'\right), & -l/2 \leq z' \leq 0 \end{cases}$$

Short Dipole Antenna

TABLE 4.1 Summary of Procedure to Determine the Far-Field Radiation Characteristics of an Antenna

1. Specify electric and/or magnetic current densities **J**, **M** [physical or equivalent (see Chapter 3, Figure 3.1)]
2. Determine vector potential components A_θ, A_ϕ and/or F_θ, F_ϕ using (3-46)–(3-54) in far field
3. Find far-zone **E** and **H** radiated fields (E_θ, E_ϕ; H_θ, H_ϕ) using (3-58a)–(3-58b)
4. Form either

   a.

$$\mathbf{W}_{\text{rad}}(r,\theta,\phi) = \mathbf{W}_{\text{av}}(r,\theta,\phi) = \frac{1}{2}\text{Re}[\mathbf{E}\times\mathbf{H}^*]$$

$$\simeq \frac{1}{2}\text{Re}\,[(\hat{\mathbf{a}}_\theta E_\theta + \hat{\mathbf{a}}_\phi E_\phi)\times(\hat{\mathbf{a}}_\theta H_\theta^* + \hat{\mathbf{a}}_\phi H_\phi^*)]$$

$$\mathbf{W}_{\text{rad}}(r,\theta,\phi) = \hat{\mathbf{a}}_r \frac{1}{2}\left[\frac{|E_\theta|^2 + |E_\phi|^2}{\eta}\right] = \hat{\mathbf{a}}_r \frac{1}{r^2}|f(\theta,\phi)|^2$$

or

   b. $U(\theta,\phi) = r^2\, W_{\text{rad}}(r,\theta,\phi) = |f(\theta,\phi)|^2$

5. Determine either

   a. $P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} W_{\text{rad}}(r,\theta,\phi) r^2 \sin\theta\, d\theta\, d\phi$

or

   b. $P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi$

6. Find directivity using

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_0} = \frac{4\pi U(\theta,\phi)}{P_{\text{rad}}}$$

$$D_0 = D_{\max} = D(\theta,\phi)|_{\max} = \frac{U(\theta,\phi)|_{\max}}{U_0} = \frac{4\pi U(\theta,\phi)|_{\max}}{P_{\text{rad}}}$$

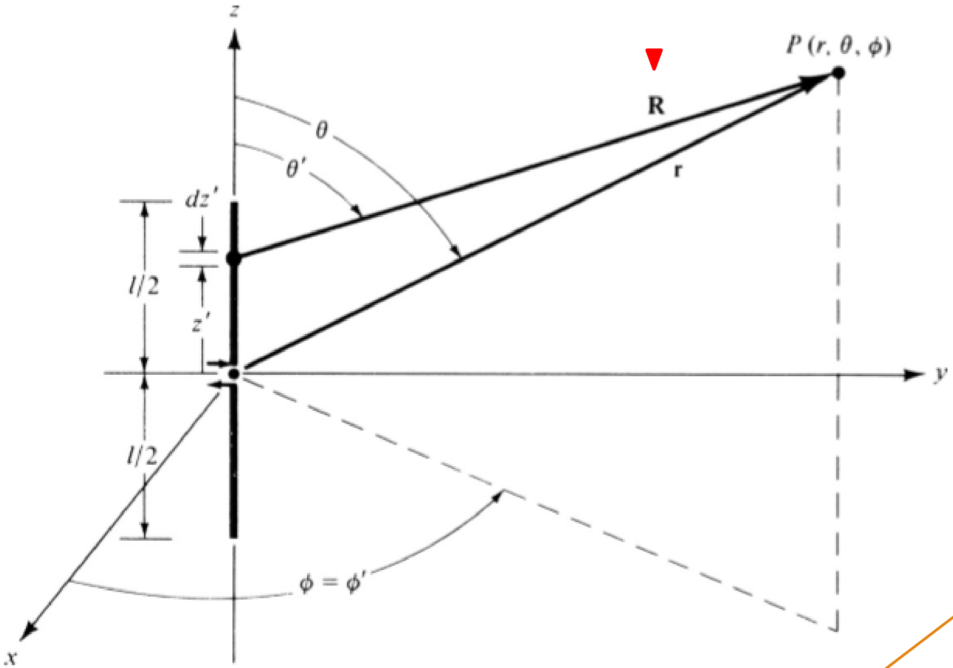
7. Form *normalized* power amplitude pattern:

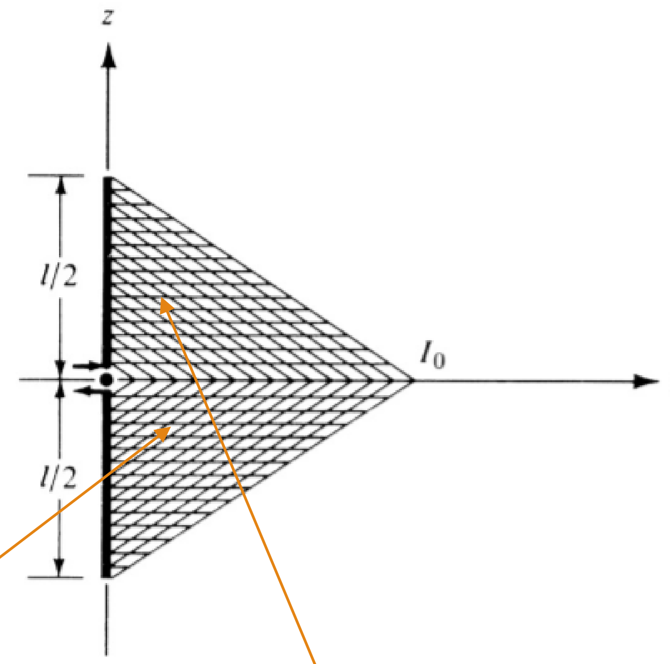
$$P_n(\theta,\phi) = \frac{U(\theta,\phi)}{U_{\max}}$$

8. Determine radiation and input resistance:

$$R_r = \frac{2P_{\text{rad}}}{|I_0|^2}; \qquad R_{in} = \frac{R_r}{\sin^2\left(\frac{kl}{2}\right)}$$

Short Dipole Antenna

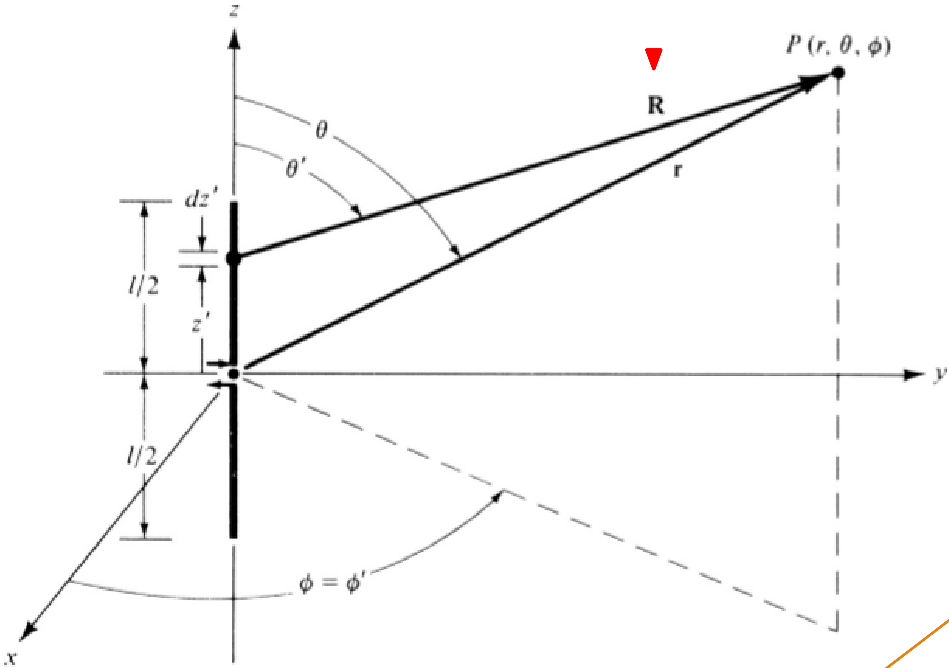


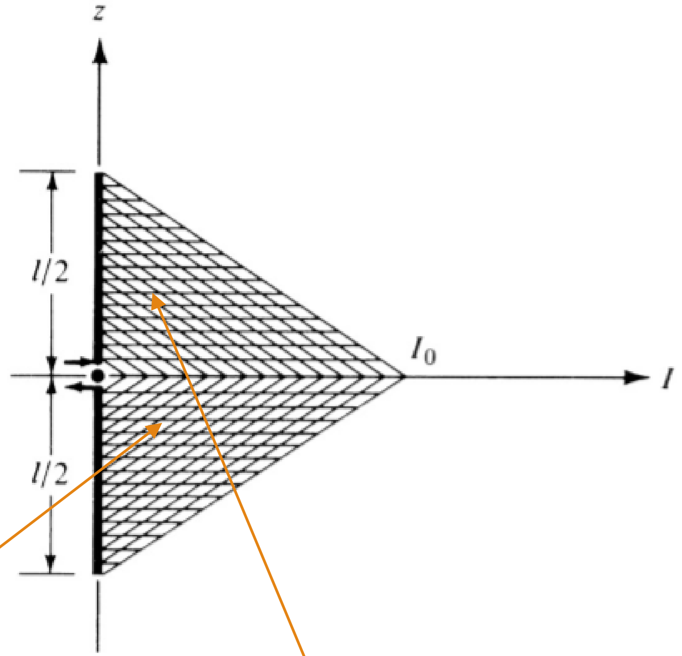


Vector Potential

$$\mathbf{A}(x,y,z) = \frac{\mu}{4\pi}\left[\hat{\mathbf{a}}_z \int_{-l/2}^{0} I_0\left(1+\frac{2}{l}z'\right)\frac{e^{-jkR}}{R}\,dz' + \hat{\mathbf{a}}_z \int_{0}^{l/2} I_0\left(1-\frac{2}{l}z'\right)\frac{e^{-jkR}}{R}\,dz'\right]$$

Short Dipole Antenna





Vector Potential

$$\mathbf{A}(x,y,z) = \frac{\mu}{4\pi}\left[\hat{\mathbf{a}}_z \int_{-l/2}^{0} I_0\left(1+\frac{2}{l}z'\right)\frac{e^{-jkR}}{R}\,dz' + \hat{\mathbf{a}}_z \int_{0}^{l/2} I_0\left(1-\frac{2}{l}z'\right)\frac{e^{-jkR}}{R}\,dz'\right]$$

R ≃ r for small dipole $\lambda/50 < l \leq \lambda/10$

$$\boxed{\mathbf{A} = \hat{\mathbf{a}}_z A_z = \hat{\mathbf{a}}_z \frac{1}{2}\left[\frac{\mu I_0 l e^{-jkr}}{4\pi r}\right]}$$

Short Dipole Antenna

Fields solution

Vector Potential

$$\mathbf{A} = \hat{\mathbf{a}}_z A_z = \hat{\mathbf{a}}_z \frac{1}{2}\left[\frac{\mu I_0 l e^{-jkr}}{4\pi r}\right]$$

Ref. P. 18

$$\left.\begin{aligned} E_\theta &\simeq j\eta \frac{kI_0 l e^{-jkr}}{8\pi r}\sin\theta \\ E_r &\simeq E_\phi = H_r = H_\theta = 0 \\ H_\phi &\simeq j\frac{kI_0 l e^{-jkr}}{8\pi r}\sin\theta \end{aligned}\right\} \quad kr \gg 1$$

Short Dipole Antenna

Radiation resistance (1/4 of infinitesimal dipole)

$$\boxed{R_r = \frac{2P_{\text{rad}}}{|I_0|^2} = 20\pi^2 \left(\frac{l}{\lambda}\right)^2}$$

Input impedance of short dipole

$$Z = R_r + R_{loss} + jX$$

R_{loss} and X depend on the radius a of the dipole, the conductivity σ and permeability of the conductor μ of the dipole and frequency f

$$R_{loss} = \frac{L}{6\pi a}\sqrt{\frac{\pi f \mu}{2\sigma}} \qquad X = \frac{-120\,\lambda}{\pi L}\left(\ln\left(\frac{L}{2a}\right) - 1\right)$$

$$\left.\begin{aligned} E_\theta &\simeq j\eta \frac{kI_0 l e^{-jkr}}{8\pi r}\sin\theta \\ E_r &\simeq E_\phi = H_r = H_\theta = 0 \\ H_\phi &\simeq j\frac{kI_0 l e^{-jkr}}{8\pi r}\sin\theta \end{aligned}\right\} \quad kr \gg 1$$

Short Dipole Antenna

in the far-field, only the E_θ and H_ϕ fields are nonzero.

these fields are orthogonal and in-phase.

the fields are perpendicular to the direction of propagation $\hat{r}$ (away from the antenna).

ratio of the E-field to the H-field is given by the intrinsic impedance of free space.

$$\eta = Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi 10^{-7}}{8.854e-12}} \approx 120\pi \approx 377 \text{ Ohms}$$

Short Dipole Antenna

- the fields die off as $1/r$, indicates the power falls of as

$$P(r) \propto \frac{1}{r^2}$$

- the fields are **proportional to** antenna length L, (a longer dipole will radiate more power as long as the short dipole assumption is valid, i.e., $\lambda/50 < L \leq \lambda/10$)

- the fields are **proportional to** the **current amplitude** I_0 of **current** on short dipole

$$\left.\begin{aligned} E_\theta &\simeq j\eta \frac{kI_0 l e^{-jkr}}{8\pi r} \sin\theta \\ E_r &\simeq E_\phi = H_r = H_\theta = 0 \\ H_\phi &\simeq j \frac{kI_0 l e^{-jkr}}{8\pi r} \sin\theta \end{aligned}\right\} \quad kr \gg 1$$

Short Dipole Antenna

$$\left.\begin{aligned} E_\theta &\simeq j\eta \frac{kI_0 l e^{-jkr}}{8\pi r}\sin\theta \\ E_r &\simeq E_\phi = H_r = H_\theta = 0 \\ H_\phi &\simeq j\frac{kI_0 l e^{-jkr}}{8\pi r}\sin\theta \end{aligned}\right\} \quad kr \gg 1$$

The exponential term:

$$e^{-jkr}$$

describes the phase-variation of the wave versus distance. The parameter k is known as the **wavenumber**.

$$k = \frac{2\pi}{\lambda} = 2\pi f\sqrt{\mu\varepsilon} = 2\pi f\sqrt{\mu_r\varepsilon_r\mu_0\varepsilon_0}$$

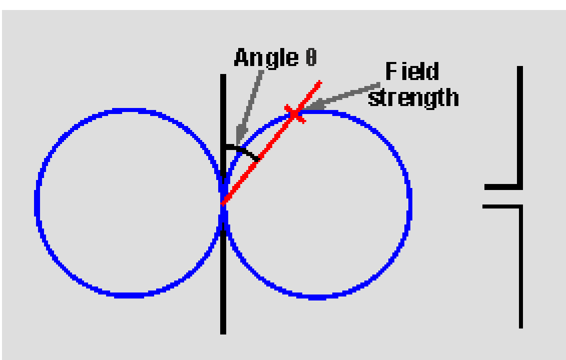
Note also that the fields are oscillating in time at a frequency f in addition to the above spatial variation.

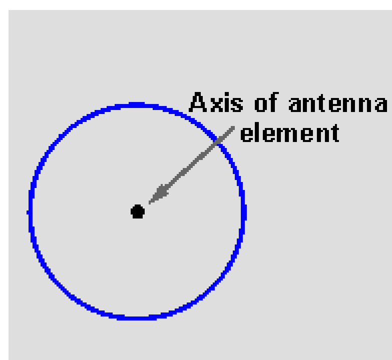
$$
\left.\begin{aligned}
E_\theta &\simeq j\eta \frac{kI_0 l e^{-jkr}}{8\pi r} \sin\theta \\
E_r &\simeq E_\phi = H_r = H_\theta = 0 \\
H_\phi &\simeq j\frac{kI_0 l e^{-jkr}}{8\pi r} \sin\theta
\end{aligned}\right\} \quad kr \gg 1
$$

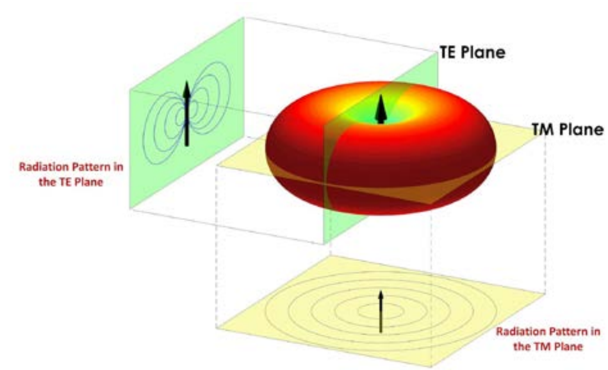
Short Dipole Antenna

the **spatial variation** of the **fields** as a function of **direction** from the antenna are given by radiation pattern $\sin\theta$ for short dipole. (**same as infinitesimal dipole**)

for a vertical antenna oriented along the z-axis, the radiation will be maximum in the x-y plane. Theoretically, **no radiation along the z-axis** far from the antenna.





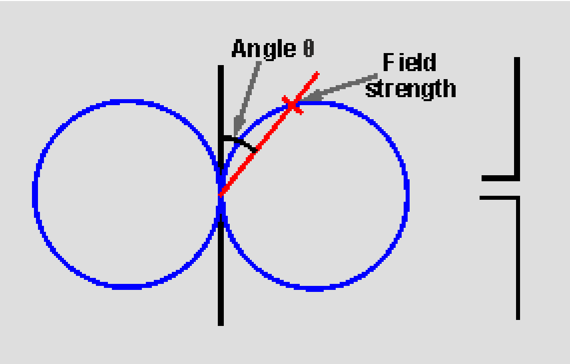


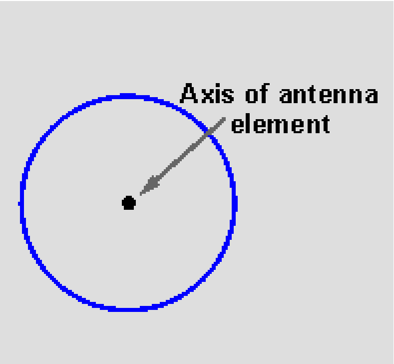
Short Dipole Antenna

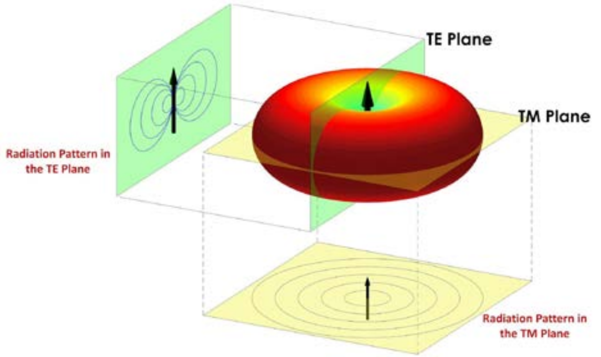
the directivity D = 1.5 (1.76 dB)

fields of the short dipole antenna are only a function of the polar angle, they have no azimuthal variation (omnidirectional).

the Half-Power Beamwidth is 90 degrees.

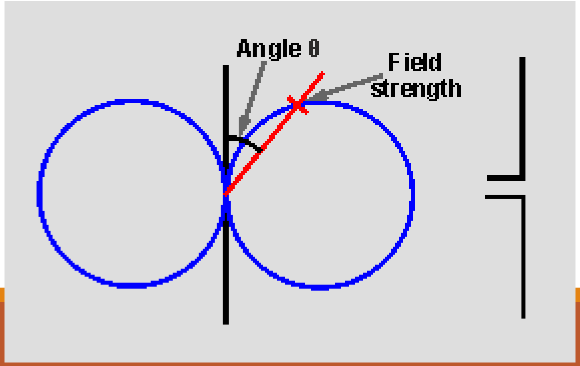


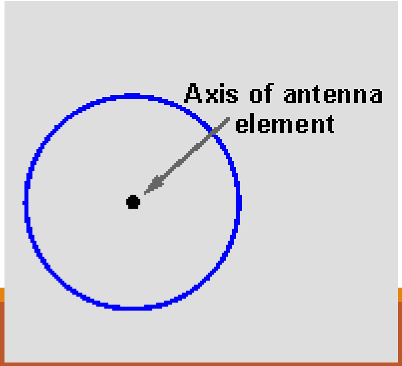


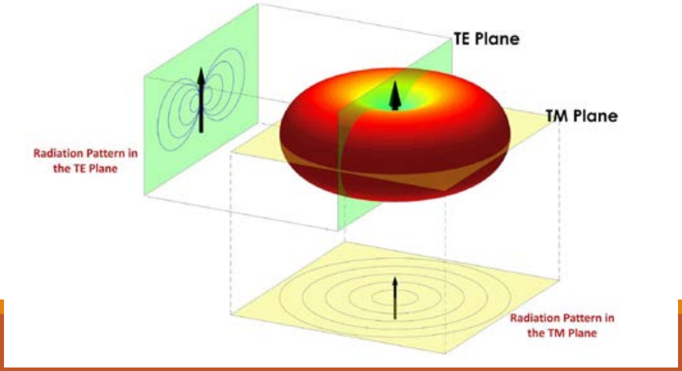


Short Dipole Antenna

Antenna Type	Typical Directivity	Typical Directivity (dB)
Short Dipole Antenna	1.5	1.76
Half-Wave Dipole Antenna	1.64	2.15
Patch (Microstrip) Antenna	3.2-6.3	5-8
Horn Antenna	10-100	10-20
Dish Antenna	10-10,000	10-40

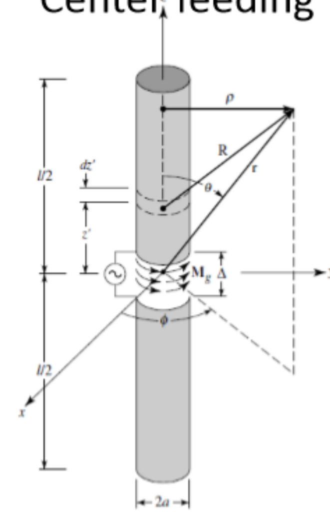




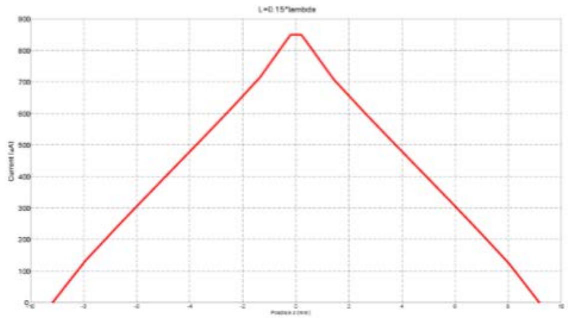


Linear Antenna (dipole) Current Distribution

Center feeding

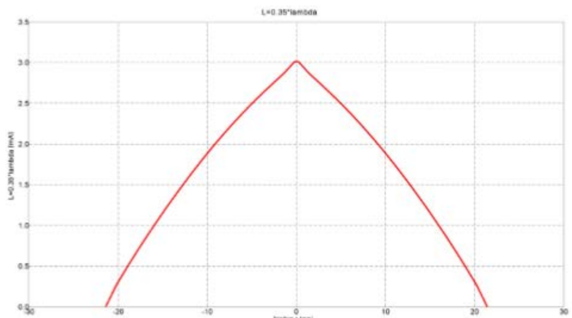


Whole dipole length

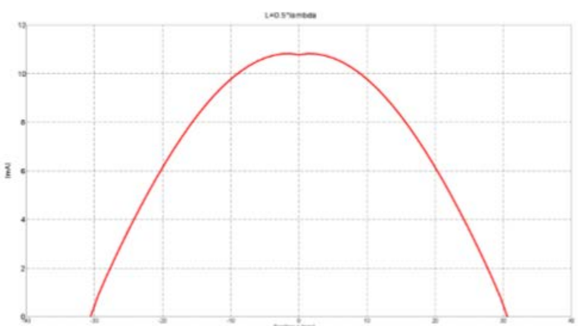


$\boldsymbol{L = 0.15\lambda}$

small dipole



$\boldsymbol{L = 0.35\lambda}$



$\boldsymbol{L = 0.5\lambda}$

half-wavelength dipole

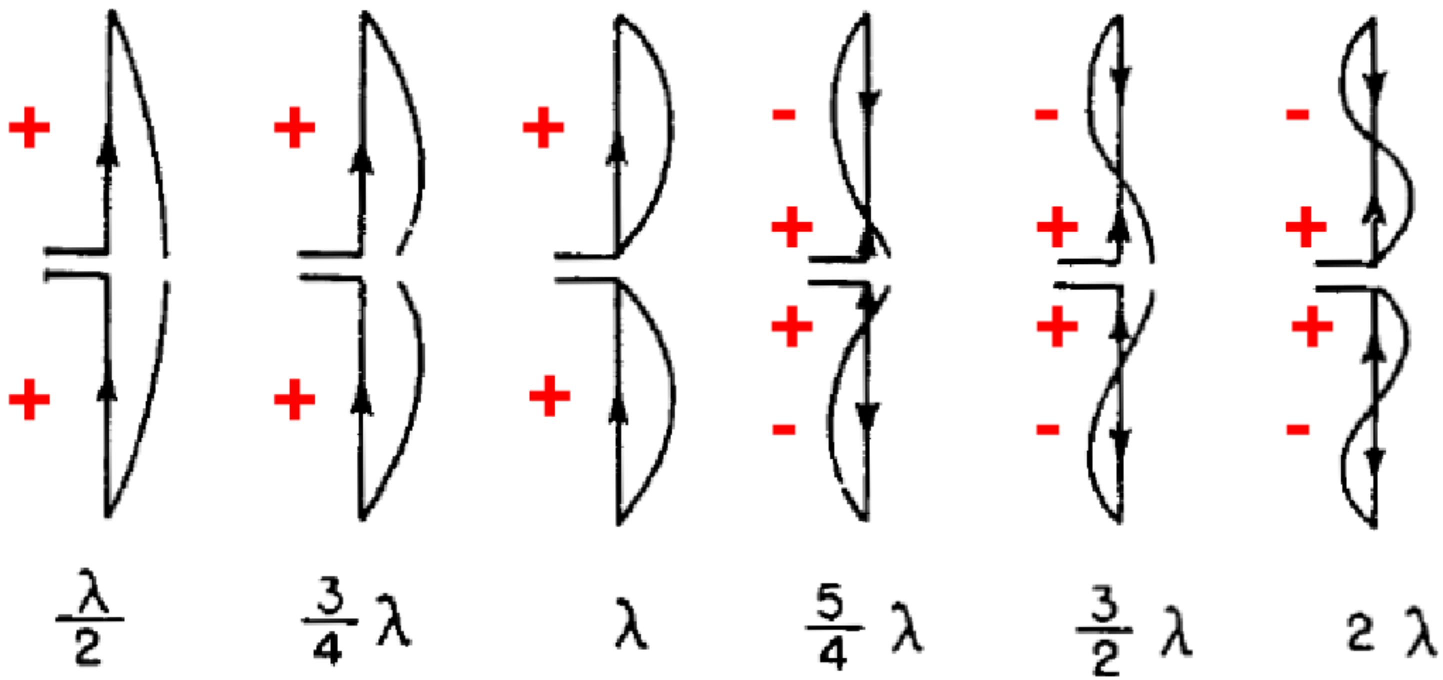
Triangular distribution

$$I(z) = \mathrm{I}_0\left(1 - \frac{2|z|}{l}\right)$$

Sine distribution

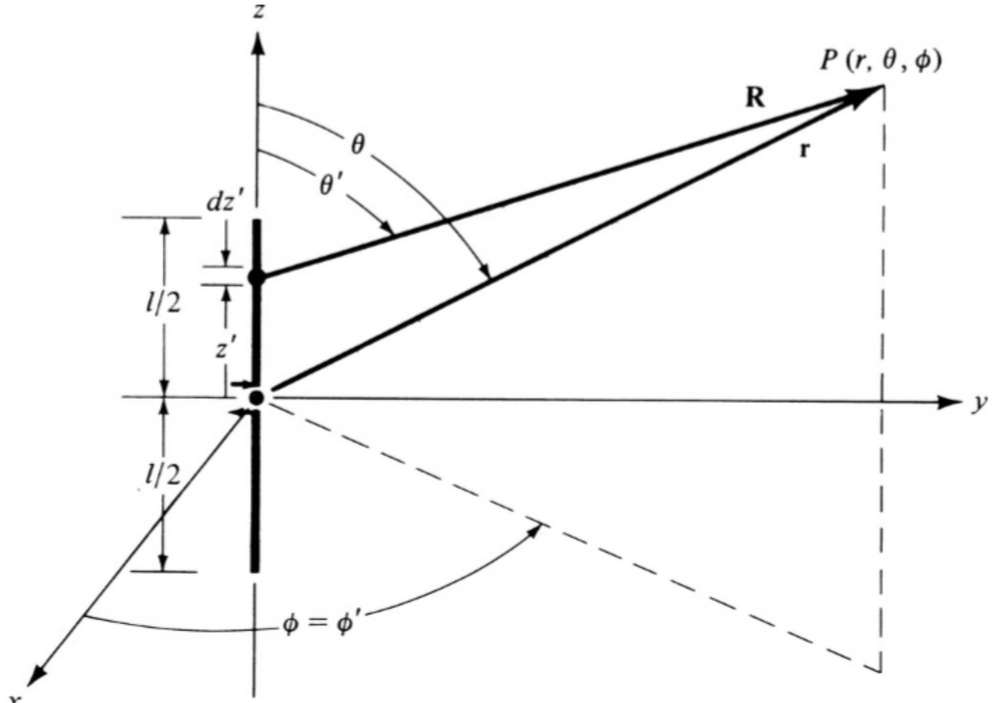
$$I(z) = \mathrm{I}_0 \sin\left[k\left(\frac{l}{2} - |z|\right)\right]$$

Linear Antenna (dipole) Current Distribution



$L > \lambda$ → out-of phase currents → sidelobes

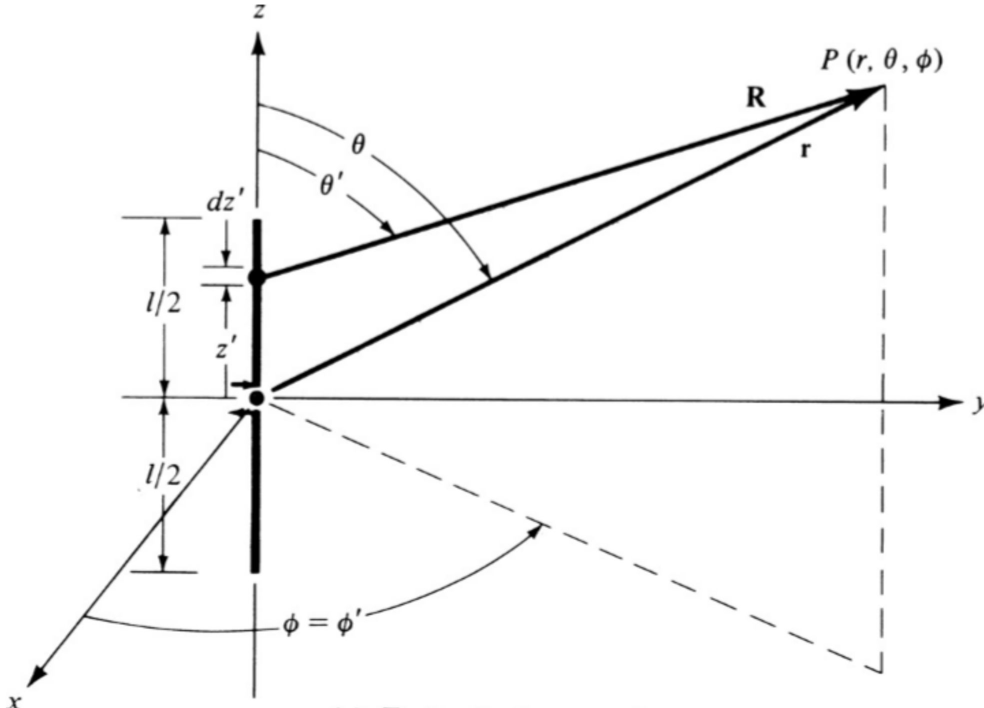
Linear Antenna (dipole) Vector Potential



(a) Finite dipole geometry

$$\mathbf{A}(x, y, z) = \frac{\mu}{4\pi} \int_C \mathbf{I}_e(x', y', z') \frac{e^{-jkR}}{R} \, dl'$$

Linear Antenna (dipole) Vector Potential



(a) Finite dipole geometry

$$r^2 = x^2 + y^2 + z^2$$

$$z = r\cos\theta$$

$$\mathbf{A}(x, y, z) = \frac{\mu}{4\pi}\int_C \mathbf{I}_e(x', y', z')\frac{e^{-jkR}}{R}\,dl'$$

Thin wire

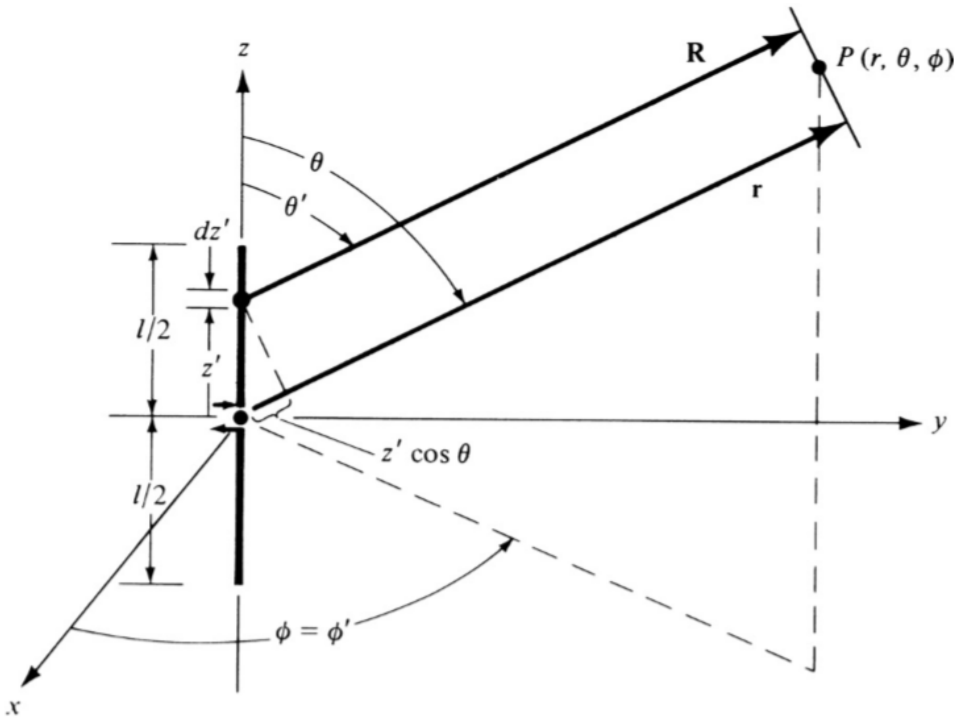
$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$= \sqrt{x^2 + y^2 + (z-z')^2}$$

$$= \sqrt{(x^2 + y^2 + z^2) + (-2zz' + z'^2)}$$

$$= \sqrt{r^2 + (-2rz'\cos\theta + z'^2)}$$

Linear Antenna (dipole) Vector Potential



$$\mathbf{A}(x, y, z) = \frac{\mu}{4\pi} \int_C \mathbf{I}_e(x', y', z') \frac{e^{-jkR}}{R}\, dl'$$

Thin wire

$$R = \sqrt{r^2 + (-2rz' \cos\theta + z'^2)}$$

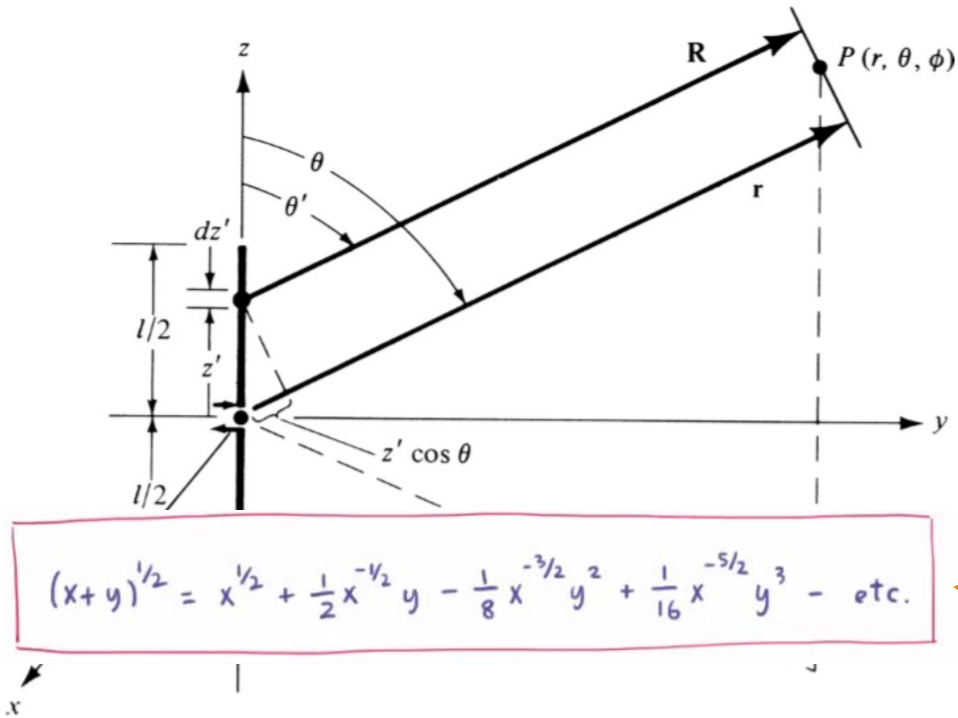
Binomial Expansion

$$R = r - z' \cos\theta + \frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right) + \frac{1}{r^2}\left(\frac{z'^3}{2}\cos\theta\sin^2\theta\right) + \cdots$$

$$\boxed{R \simeq r - z' \cos\theta}$$

higher order terms become less significant provided $r \gg z'$

Linear Antenna (dipole)
Vector Potential



$$(x+y)^{1/2} = x^{1/2} + \frac{1}{2}x^{-1/2}y - \frac{1}{8}x^{-3/2}y^2 + \frac{1}{16}x^{-5/2}y^3 - \text{etc.}$$

$$\boxed{\mathbf{A}(x,y,z) = \frac{\mu}{4\pi}\int_C \mathbf{I}_e(x',y',z')\frac{e^{-jkR}}{R}\,dl'}$$

Thin wire

$$R = \sqrt{r^2 + (-2rz'\cos\theta + z'^2)}$$

Binomial Expansion

$$R = r - z'\cos\theta + \frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right) + \frac{1}{r^2}\left(\frac{z'^3}{2}\cos\theta\sin^2\theta\right) + \cdots$$

$$\boxed{R \simeq r - z'\cos\theta}$$

higher order terms become less significant provided $r \gg z'$

Linear Antenna (dipole)
Vector Potential

Far field approximations, i.e. $r \geq 2\left(\frac{l^2}{\lambda}\right)$

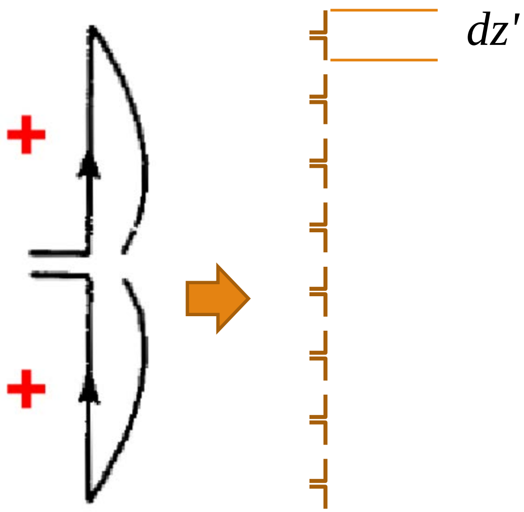
$$\boxed{\begin{array}{ll} R \simeq r - z' \cos\theta & \text{for phase terms} \\ R \simeq r & \text{for amplitude terms} \end{array}}$$

Vector Potential has an analytical (closed form) solution

Linear Antenna (dipole) Decomposition

- A **standard approach** used to calculate the far field for an **arbitrary wire antenna**.
- Based on the solution for **the field of the infinitesimal dipole (Hertzian Dipole)**.
- The finite length dipole is **subdivided into an infinite number of infinitesimal dipoles** of length *dz'*.
- Each such dipole produces the **elementary far field given by infinitesimal dipole E and H field**.

Linear Antenna (dipole) Decomposition



$$dE_\theta \simeq j\eta \frac{kI_e(x', y', z')e^{-jkR}}{4\pi R} \sin\theta \; dz'$$

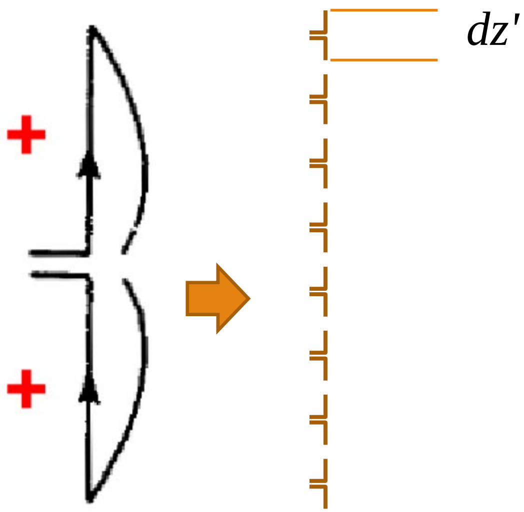
$$dE_r \simeq dE_\phi = dH_r = dH_\theta = 0$$

$$dH_\phi \simeq j \frac{kI_e(x', y', z')e^{-jkR}}{4\pi R} \sin\theta \; dz'$$

infinitesimal dipole

infinitesimal dipole E and H field

Linear Antenna (dipole) Decomposition



$$dE_\theta \simeq j\eta \frac{kI_e(x',y',z')e^{-jkR}}{4\pi R}\sin\theta\, dz'$$

$$dE_r \simeq dE_\phi = dH_r = dH_\theta = 0$$

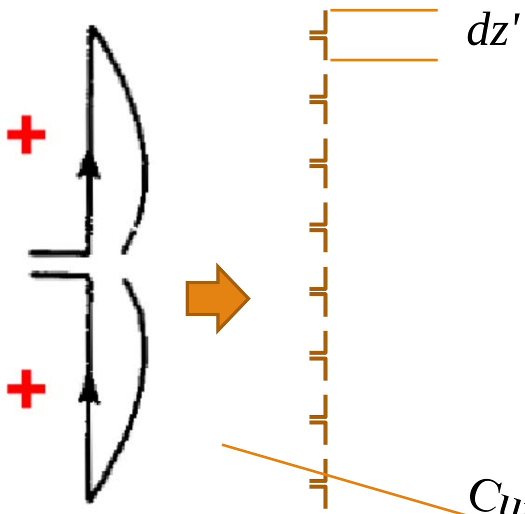
$$dH_\phi \simeq j\frac{kI_e(x',y',z')e^{-jkR}}{4\pi R}\sin\theta\, dz'$$

$$E_\theta = \int_{-l/2}^{+l/2} dE_\theta = j\eta\frac{ke^{-jkr}}{4\pi r}\sin\theta\left[\int_{-l/2}^{+l/2} I_e(x',y',z')e^{jkz'\cos\theta}\, dz'\right]$$

Linear Antenna (dipole) Decomposition

$$R \simeq r - z' \cos\theta \qquad \text{for phase terms}$$

$$R \simeq r \qquad \text{for amplitude terms}$$



$$dE_\theta \simeq j\eta \frac{k I_e(x', y', z') e^{-jkR}}{4\pi R} \sin\theta \, dz'$$

$$dE_r \simeq dE_\phi = dH_r = dH_\theta = 0$$

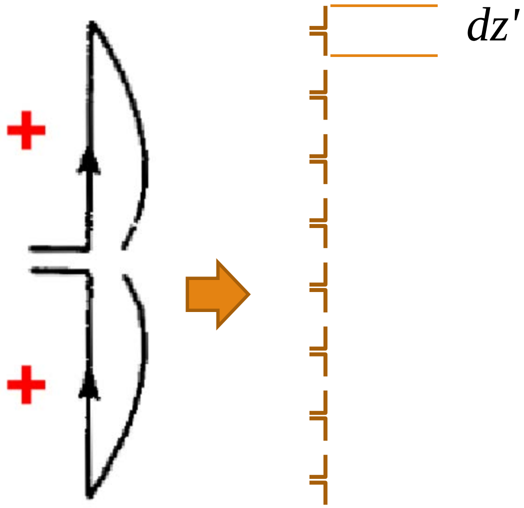
$$dH_\phi \simeq j \frac{k I_e(x', y', z') e^{-jkR}}{4\pi R} \sin\theta \, dz'$$

Current distribution

$$E_\theta = \int_{-l/2}^{+l/2} dE_\theta = j\eta \frac{k e^{-jkr}}{4\pi r} \sin\theta \left[\int_{-l/2}^{+l/2} I_e(x', y', z') e^{jkz' \cos\theta} \, dz' \right]$$

Linear Antenna (dipole) Decomposition

Sum (Integral) of all small infinitesimal dipole antennas



$$E_\theta \simeq j\eta \frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right)-\cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]$$

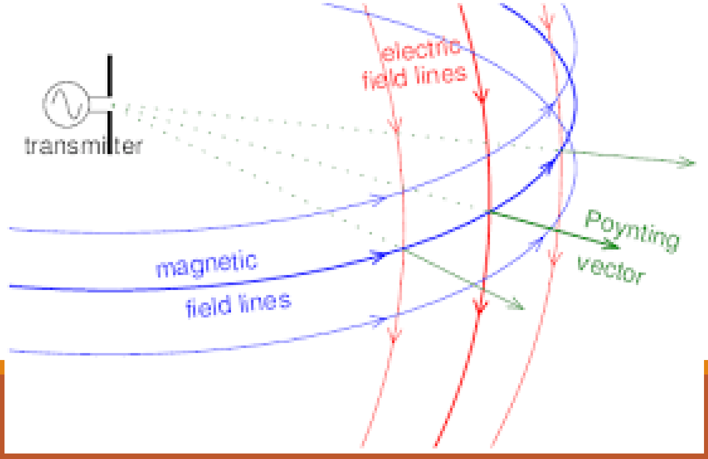
$$H_\phi \simeq \frac{E_\theta}{\eta} \simeq j\frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right)-\cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]$$

Linear Antenna (dipole) Radiation Density

at a given direction

$$\mathbf{W}_{\text{av}} = \frac{1}{2}\text{Re}[\mathbf{E} \times \mathbf{H}^*] = \frac{1}{2}\text{Re}[\hat{\mathbf{a}}_\theta E_\theta \times \hat{\mathbf{a}}_\phi H_\phi^*] = \frac{1}{2}\text{Re}\left[\hat{\mathbf{a}}_\theta E_\theta \times \hat{\mathbf{a}}_\phi \frac{E_\theta^*}{\eta}\right]$$

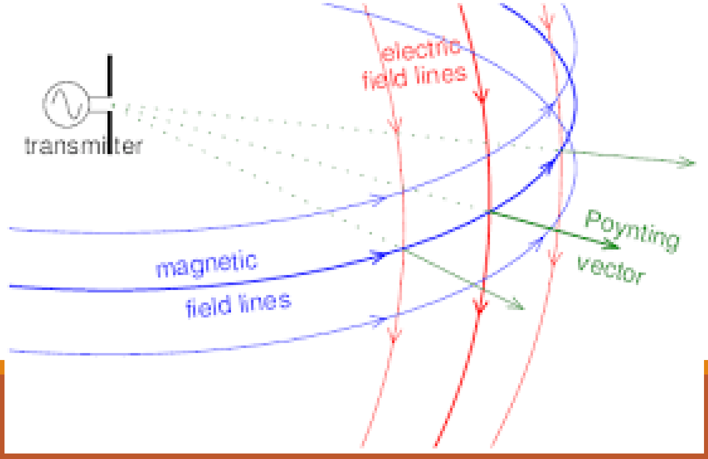
$$\mathbf{W}_{\text{av}} = \hat{\mathbf{a}}_r W_{\text{av}} = \hat{\mathbf{a}}_r \frac{1}{2\eta}|E_\theta|^2 = \hat{\mathbf{a}}_r \eta \frac{|I_0|^2}{8\pi^2 r^2}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]^2$$



Linear Antenna (dipole) Radiation Intensity

at a given direction

$$U = r^2\, W_{\mathrm{av}} = \eta \frac{|I_0|^2}{8\pi^2} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]^2$$



Linear Antenna (dipole) Directivity

$$D_0 = D_{\max} = D(\theta, \phi)|_{\max} = \frac{U(\theta, \phi)|_{\max}}{U_0} = \frac{4\pi U(\theta, \phi)|_{\max}}{P_{\text{rad}}}$$

Total power radiated Prad

$$P_{\text{rad}} = \oiint_S \mathbf{W}_{\text{av}} \cdot d\mathbf{s} = \int_0^{2\pi} \int_0^{\pi} \hat{\mathbf{a}}_r W_{\text{av}} \cdot \hat{\mathbf{a}}_r r^2 \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi} \int_0^{\pi} W_{\text{av}} r^2 \sin\theta \, d\theta \, d\phi$$

$$= \eta \frac{|I_0|^2}{4\pi} \int_0^{\pi} \frac{\left[\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)\right]^2}{\sin\theta} \, d\theta$$

$$= \eta \frac{|I_0|^2}{4\pi} \{C + \ln(kl) - C_i(kl) + \tfrac{1}{2}\sin(kl)[S_i(2kl) - 2S_i(kl)]$$

$$+ \tfrac{1}{2}\cos(kl)[C + \ln(kl/2) + C_i(2kl) - 2C_i(kl)]\}$$

where $C = 0.5772$ (Euler's constant) and $Ci(x)$ and $Si(x)$ are the cosine and sine integrals (see Appendix III)

Linear Antenna (dipole) Directivity

Finite length dipole

$$D_0 = \frac{2F(\theta)|_{\max}}{Q}$$

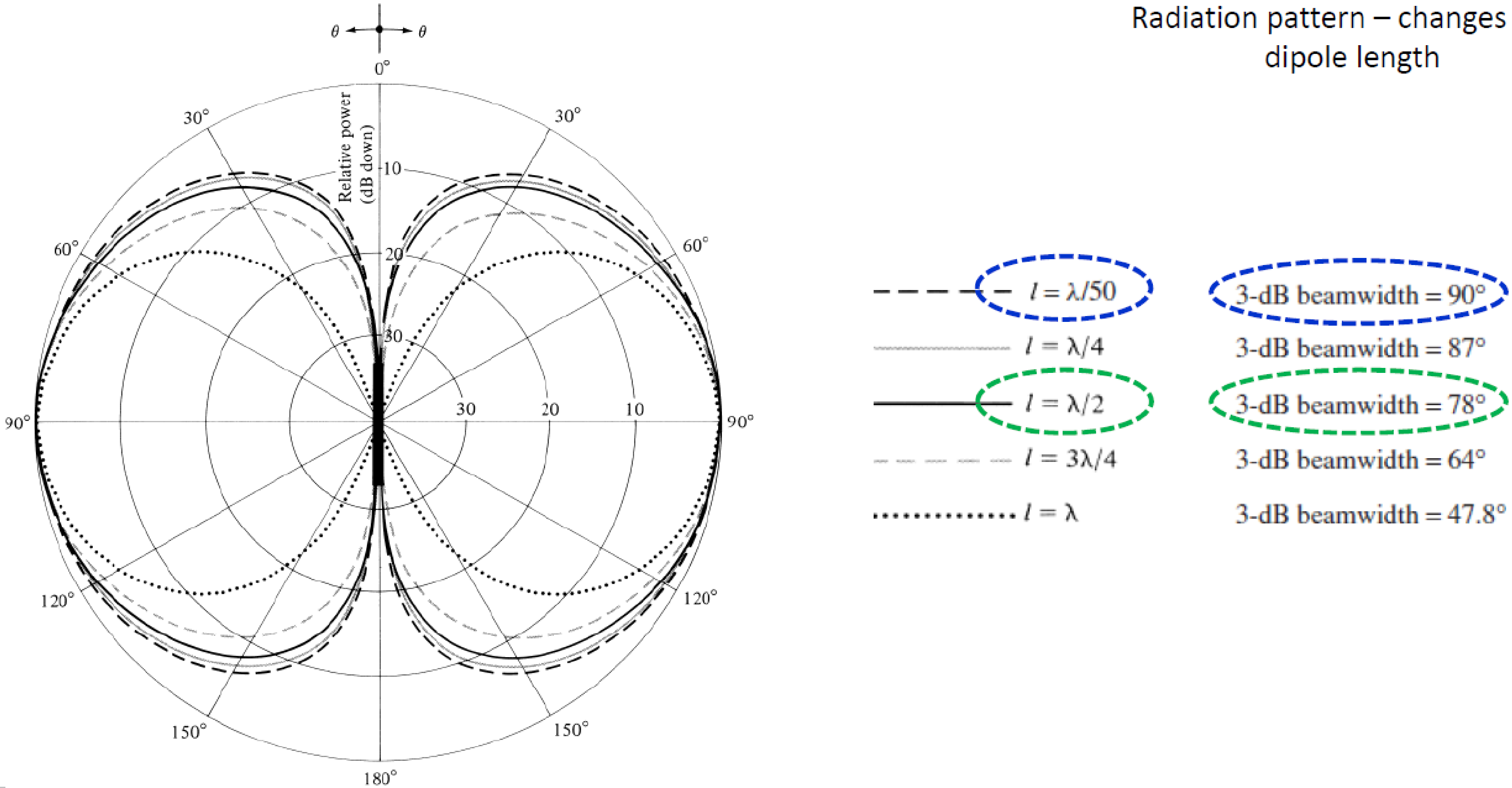
$$F(\theta, \phi) = F(\theta) = \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]^2$$

where $C = 0.5772$ (Euler's constant) and $Ci(x)$ and $Si(x)$ are the cosine and sine integrals (see Appendix III)

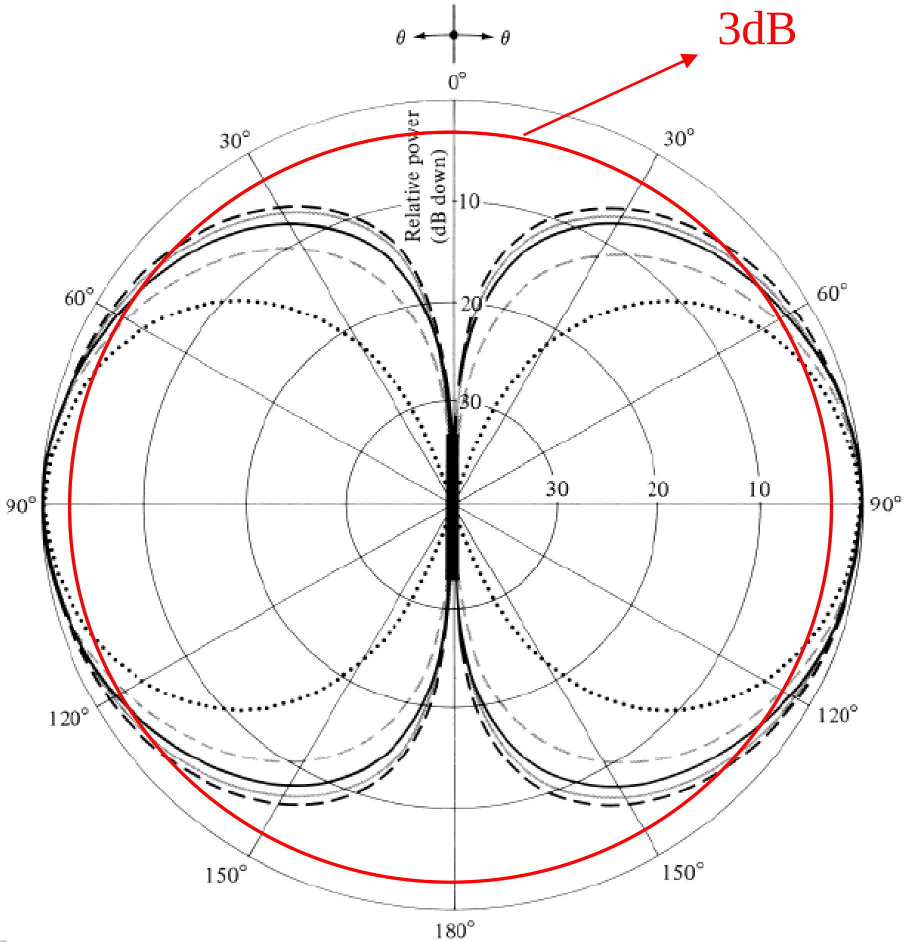
$$Q = \{C + \ln(kl) - C_i(kl) + \frac{1}{2}\sin(kl)[S_i(2kl) - 2S_i(kl)]$$
$$+ \frac{1}{2}\cos(kl)[C + \ln(kl/2) + C_i(2kl) - 2C_i(kl)]\}$$

Linear Antenna (dipole) Radiation Pattern

Radiation pattern – changes with dipole length



Linear Antenna (dipole) Radiation Pattern

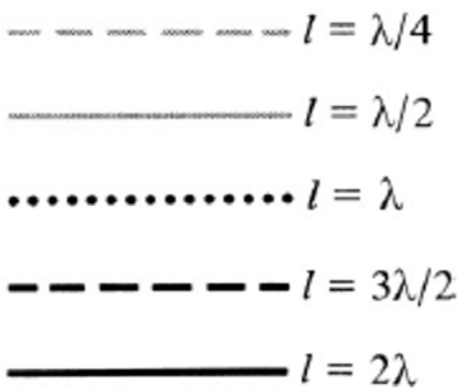


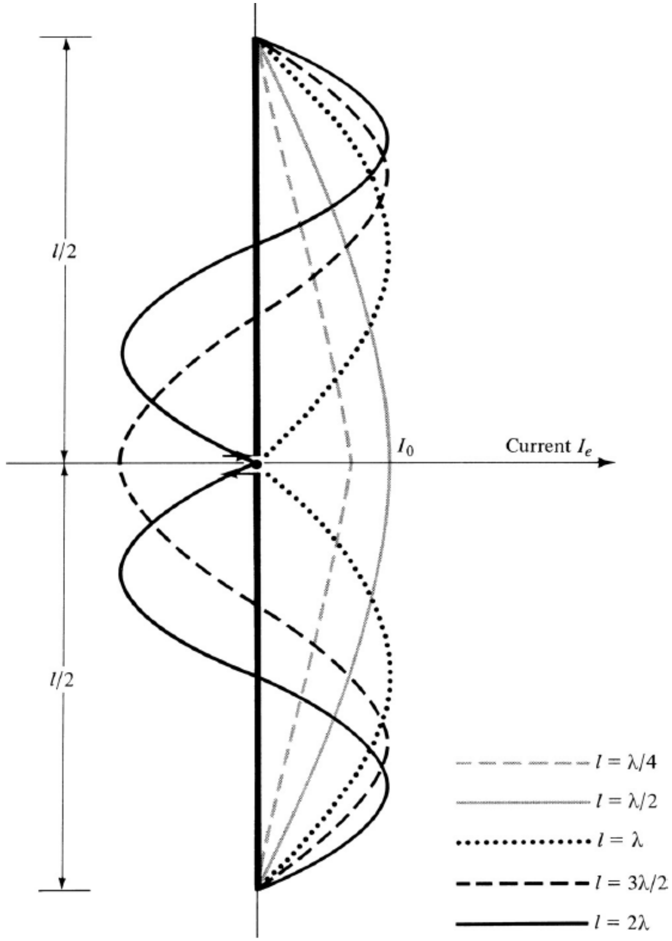
Radiation pattern – changes with dipole length

$l = \lambda/50$	3-dB beamwidth = 90°
$l = \lambda/4$	3-dB beamwidth = 87°
$l = \lambda/2$	3-dB beamwidth = 78°
$l = 3\lambda/4$	3-dB beamwidth = 64°
$l = \lambda$	3-dB beamwidth = 47.8°

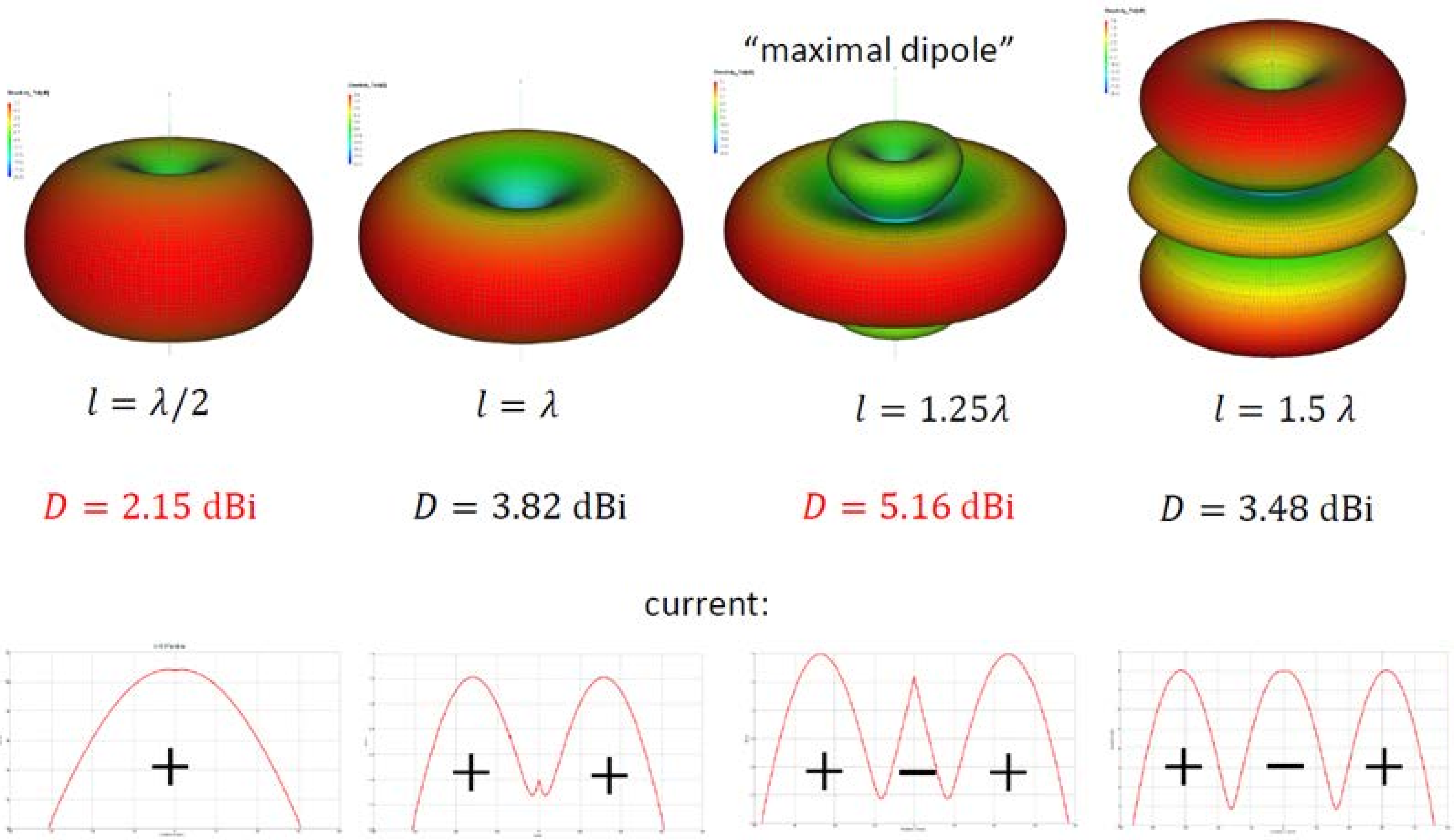
Linear Antenna (dipole) Radiation Pattern

Current distributions along the length of a linear wire antenna





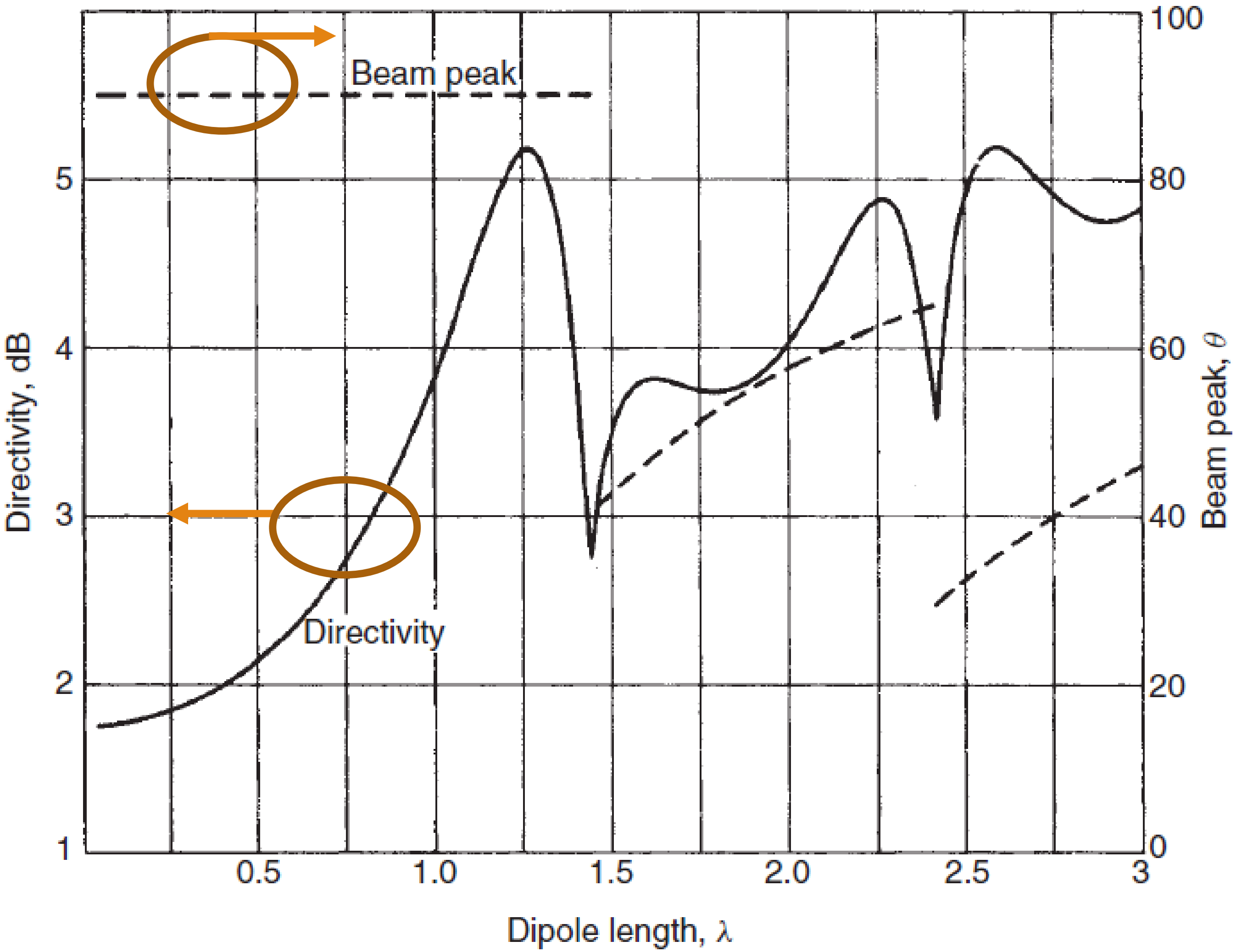
Linear Antenna (dipole) Radiation Pattern



"One of the goals of antenna design is to place lobes at the desired angles."

For dipole antenna, length of 1.25λ, directivity is maximum

Linear Antenna (dipole) Directivity

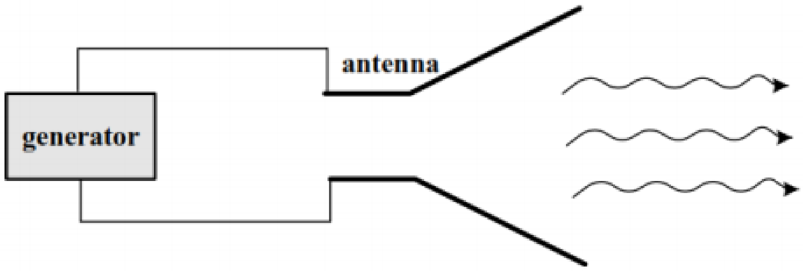


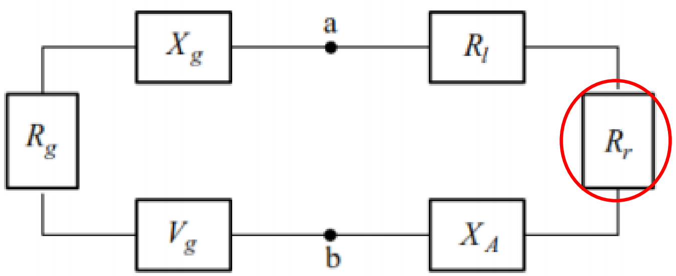
Linear Antenna (dipole) Radiation Resistance

$$R_r = \frac{2P_{\text{rad}}}{|I_0|^2} = \frac{\eta}{2\pi}\{C + \ln(kl) - C_i(kl)$$
$$+ \frac{1}{2}\sin(kl) \times [S_i(2kl) - 2S_i(kl)]$$
$$+ \frac{1}{2}\cos(kl) \times [C + \ln(kl/2) + C_i(2kl) - 2C_i(kl)]\}$$

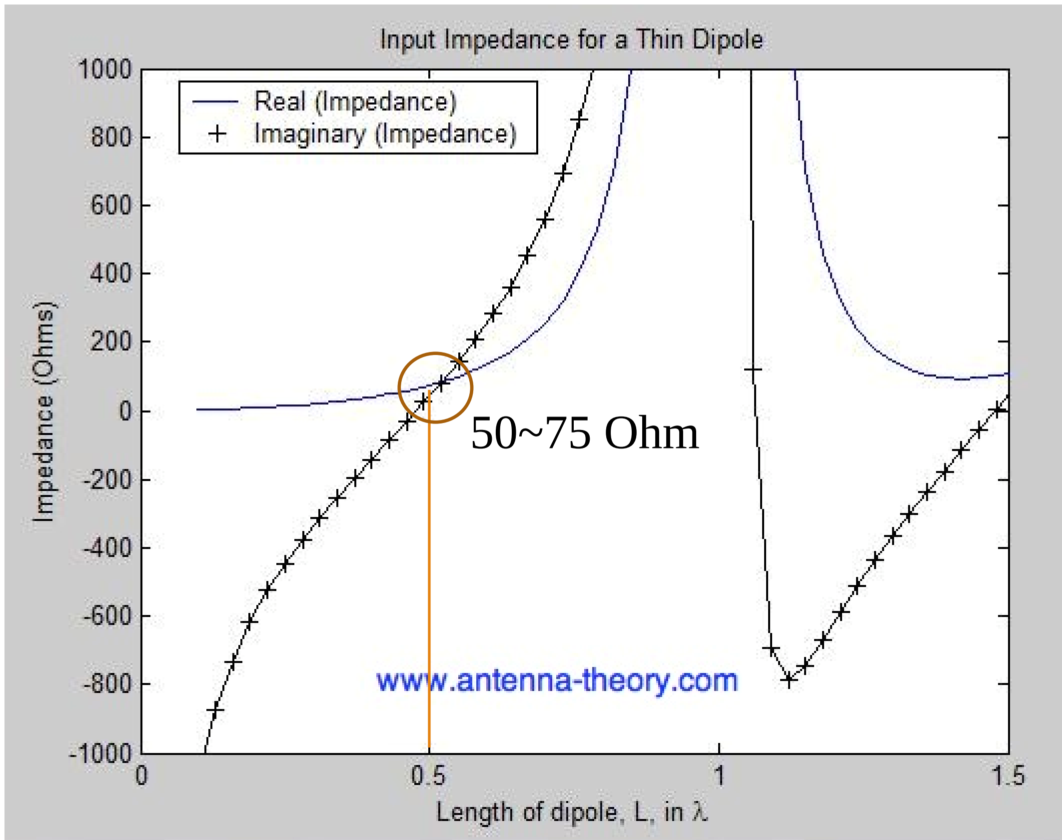
$$P_{\text{rad}} = \frac{1}{2}|I_0|^2 R_r$$

where $C = 0.5772$ (Euler's constant) and $Ci(x)$ and $Si(x)$ are the cosine and sine integrals (see Appendix III)





Linear Antenna (dipole) Input Impedance

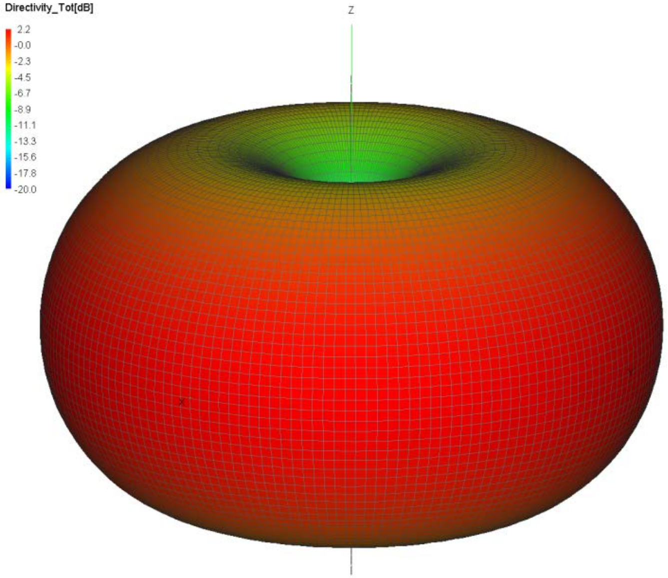


Input impedance as a function of the length (L) of a dipole antenna.

Linear Antenna (dipole)

The $\lambda/2$ dipole

- Its radiation pattern is omnidirectional in the H-plane, which is required by many applications (including mobile communications).
- Its directivity (2.15 dBi) is reasonable – larger than short dipoles
- The input impedance is not sensitive to the radius and is about 73 Ω, which is well matched with a standard transmission line of characteristic impedance 75 or 50 Ω (with $VSWR < 2$). This is probably the most important and unique reason.
- dBd – decibels above $\lambda/2$ dipole, 0 dBd = 2.15 dBi

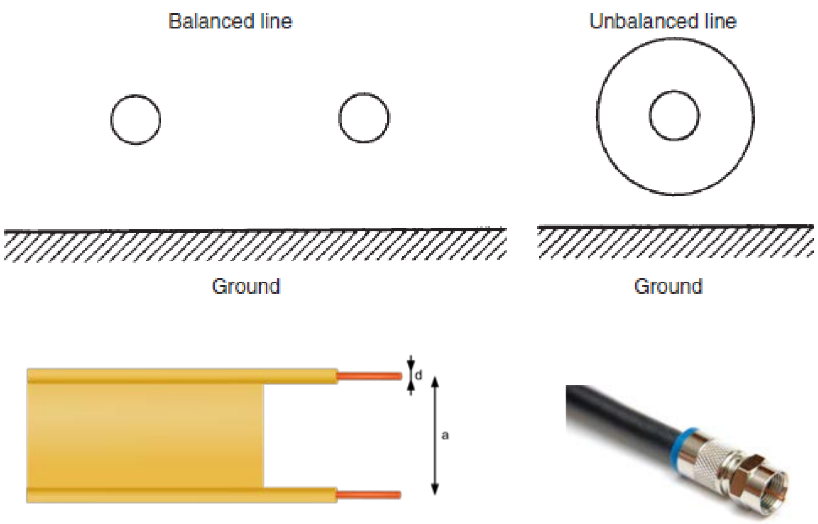


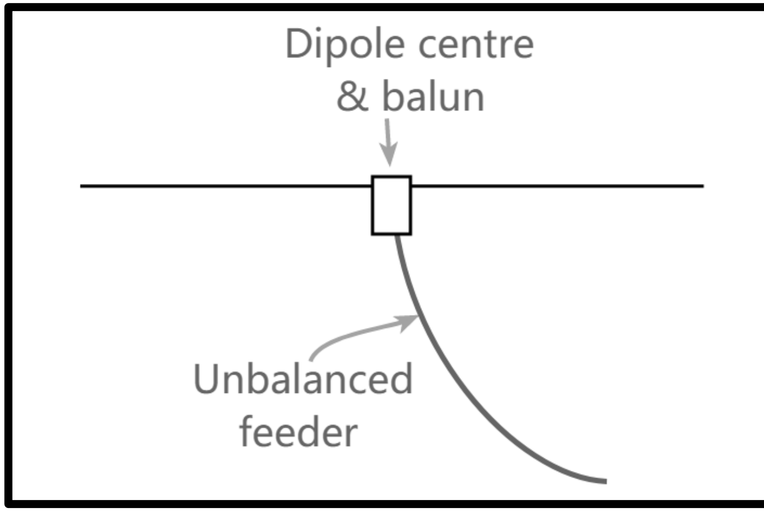
Video – Dipole Antenna 3’56’’

Feed a Dipole: Balun

A **Balun** is used to "balance" unbalanced systems - i.e. those where power flows from an unbalanced line to a balanced line (hence, balun derives from *bal*ance to *un*balanced).

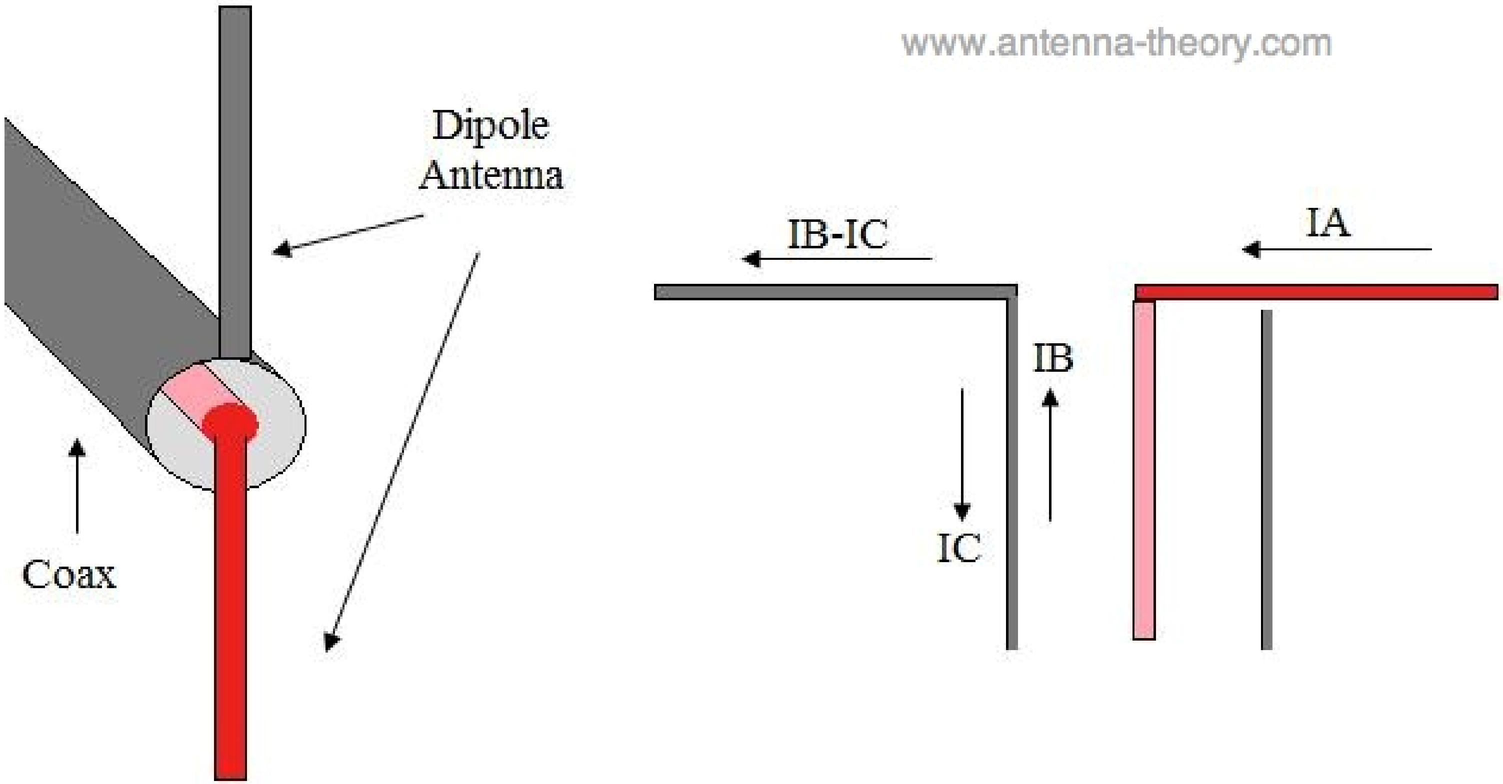
Feed a Dipole: Balun





Note: Voltage to the ground is different

Balun

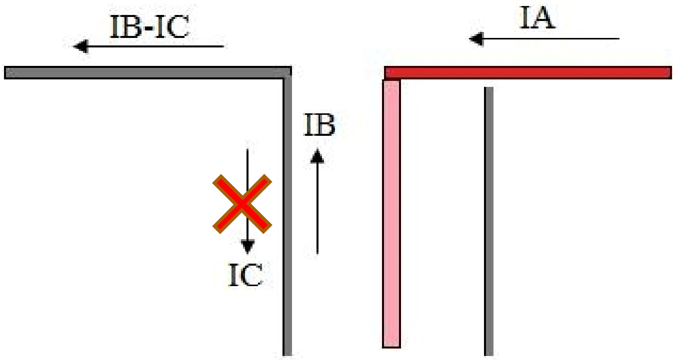


(a) Physical Model

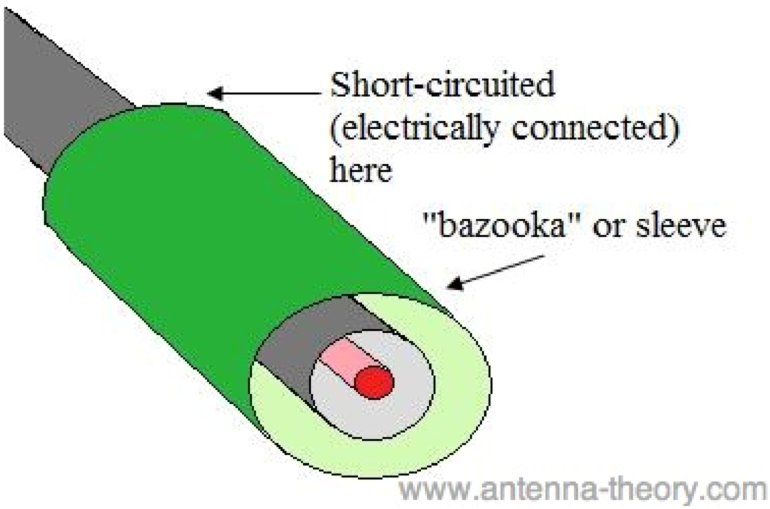
(b) Equivalent View Showing
Current Paths

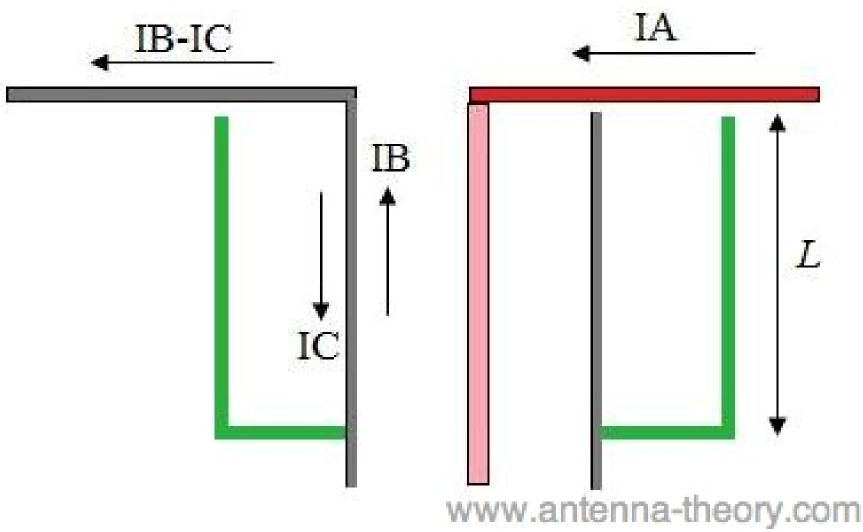
Balun

A balun forces an unbalanced transmission line to properly feed a balanced component. this would be done by forcing *IC* to be zero somehow - this is often called choking the current or a current choke.



Bazooka or Sleeve Balun.



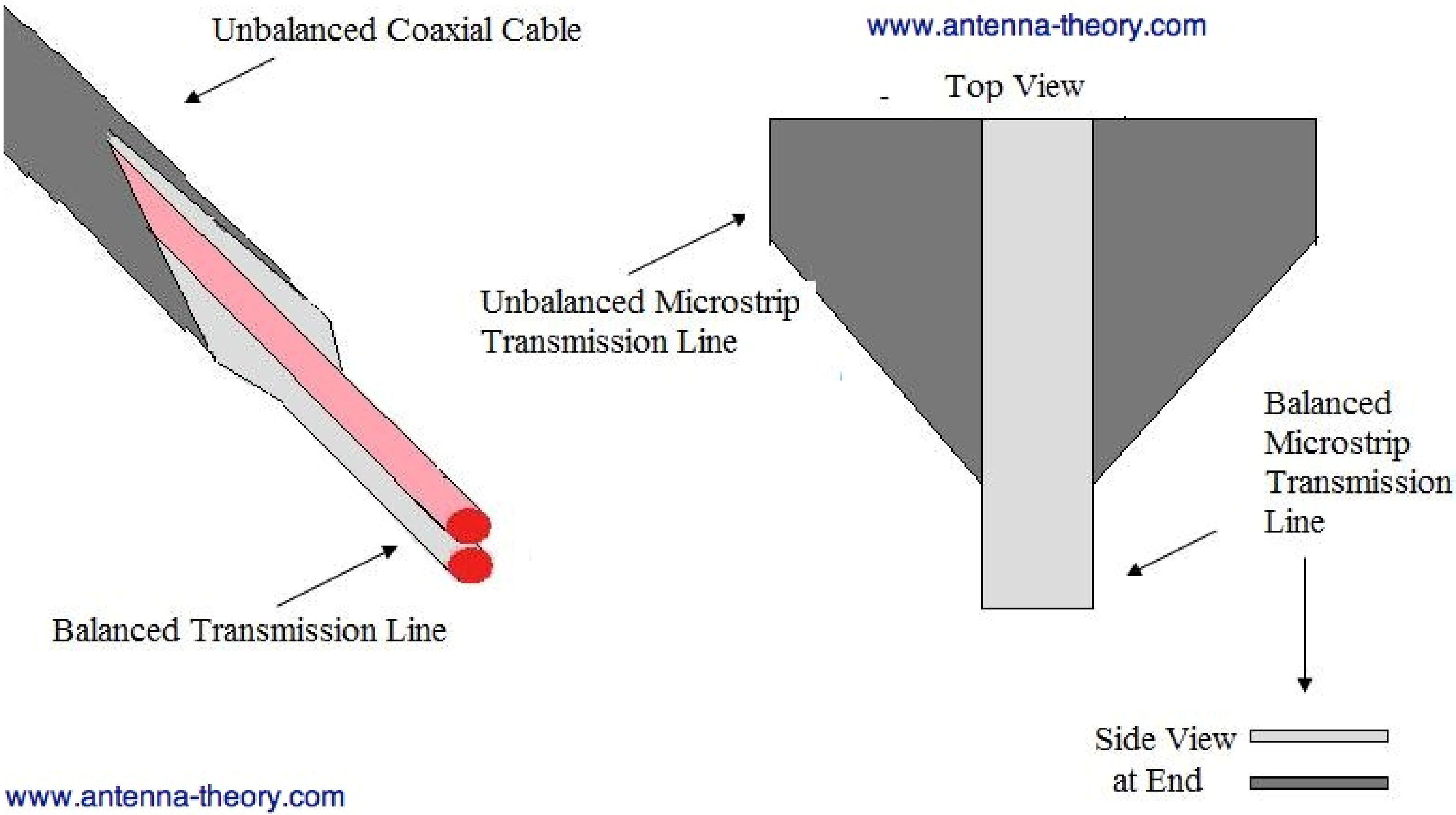


$\lambda_0/4$ parallel line – very large impedance:

- Suppression of the currents IC on the outer shield
- No interference with the antenna input impedance

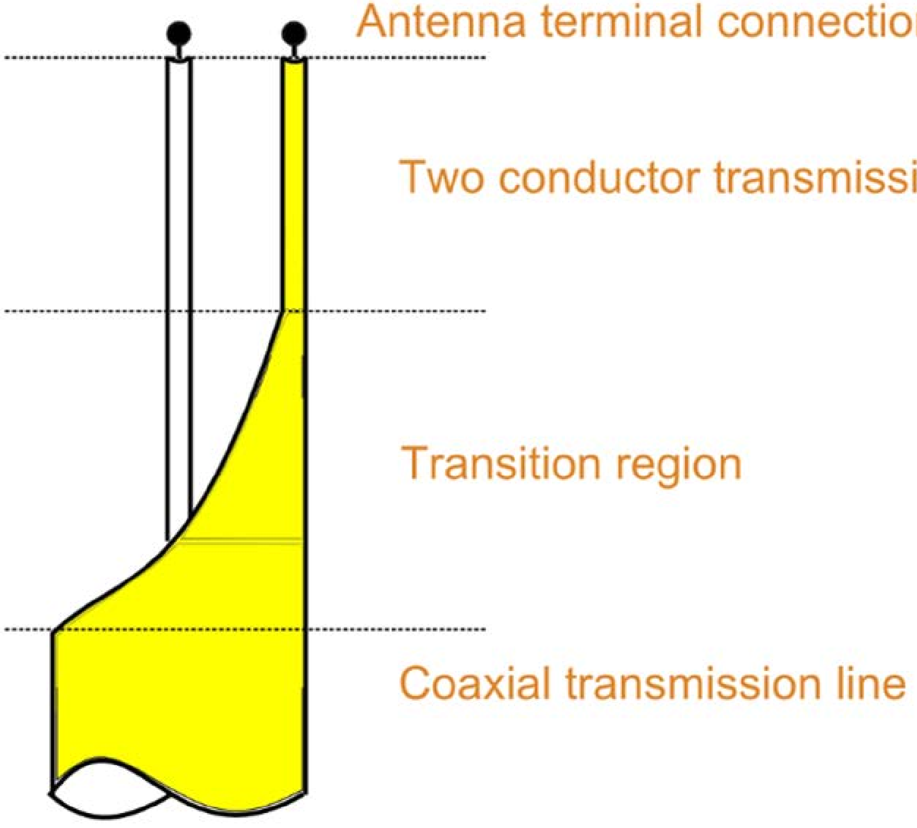
"Balun transforms the balanced input impedance of the dipole to the unbalanced impedance of the coaxial line such that there is no net current on the outer conductor of the coax."

Tapered Balun.



(a) Coaxial Cable Tapered Balun (b) Microstrip Transmission Line Tapered Balun

Tapered Balun.



For the **tapered transition** to work, it should **vary slowly**. The term “slowly” in electronics always mean “**slow relative to a wavelength**”. Hence, for the desired frequency of operation, the transition should take place over **at least a few wavelengths in length**.

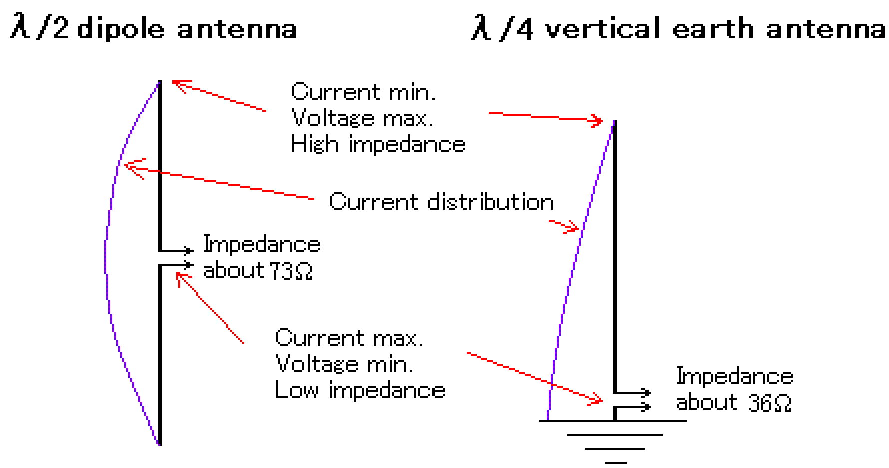
Linear Antenna (Monopole)

A **monopole antenna** is a class of radio antenna consisting of a **straight rod-shaped conductor**, often mounted perpendicularly over some type of **conductive surface**, called a ***ground plane***.

Mast radiator monopole antenna used for broadcasting. AM radio station WARE, Warren, Massachusetts, US.

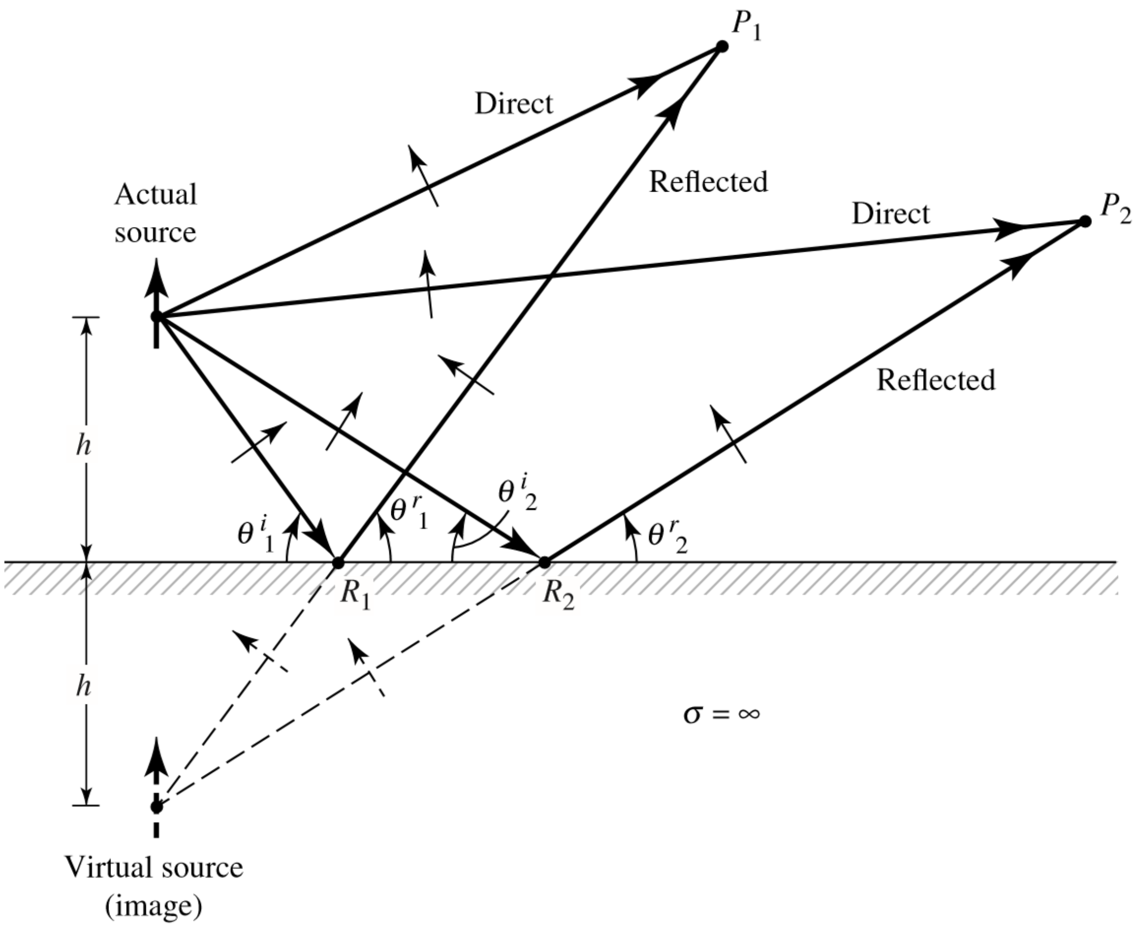
Linear Antenna (Monopole)

- The **driving signal** (from the transmitter) or **output signal** to the receiver **between** the **lower end of the monopole** and **the ground plane**.
- One side of the antenna feedline attached to **the lower end of the monopole**, and the other side attached to the **ground plane** (often the Earth).
- This contrasts with a **dipole antenna:**
    - Consists of two identical rod conductors
    - **signal** from the transmitter applied **between the two halves of the antenna**.



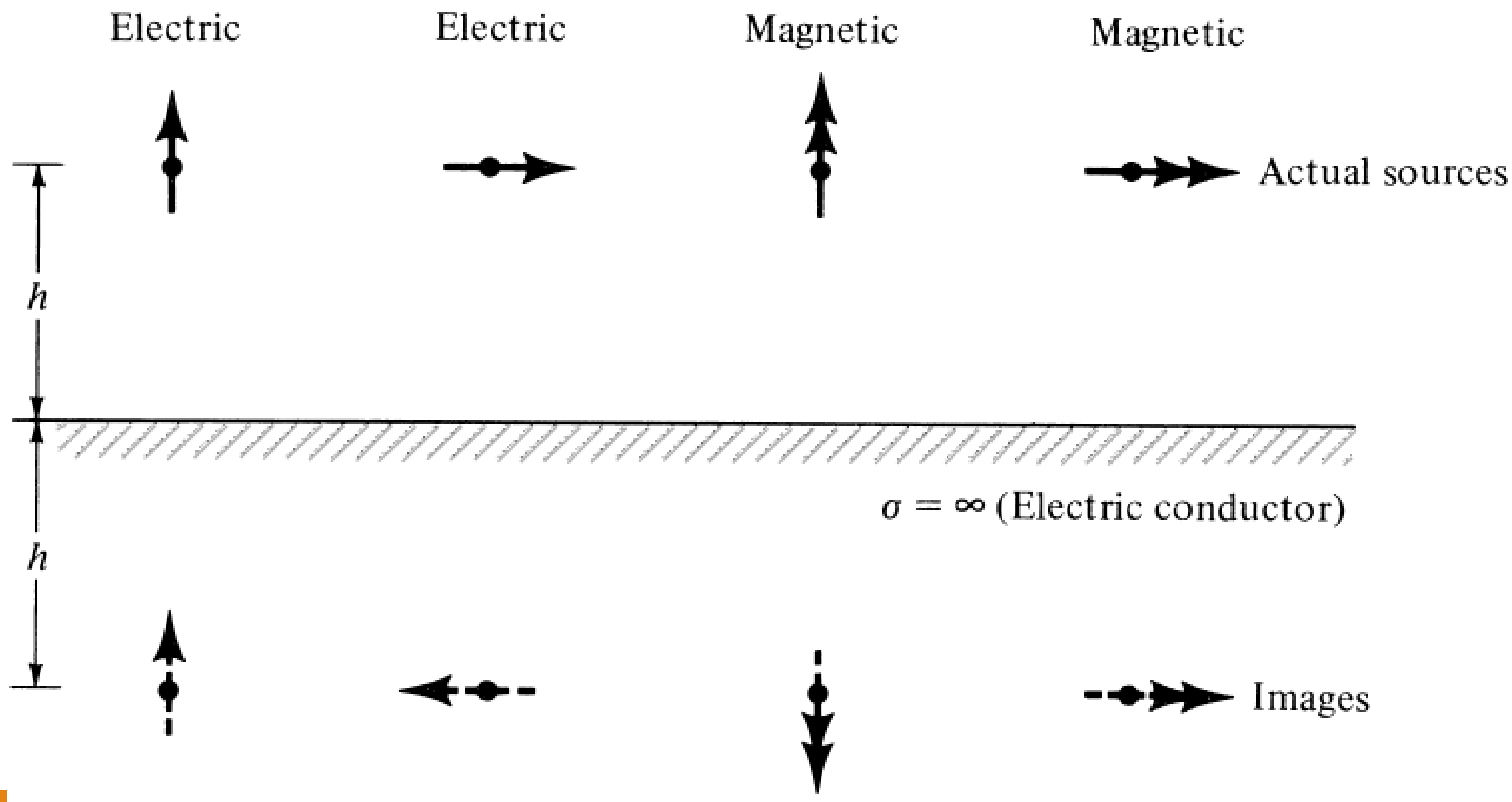
Linear Antenna (Monopole)

vertical current above PEC ground • Method of images



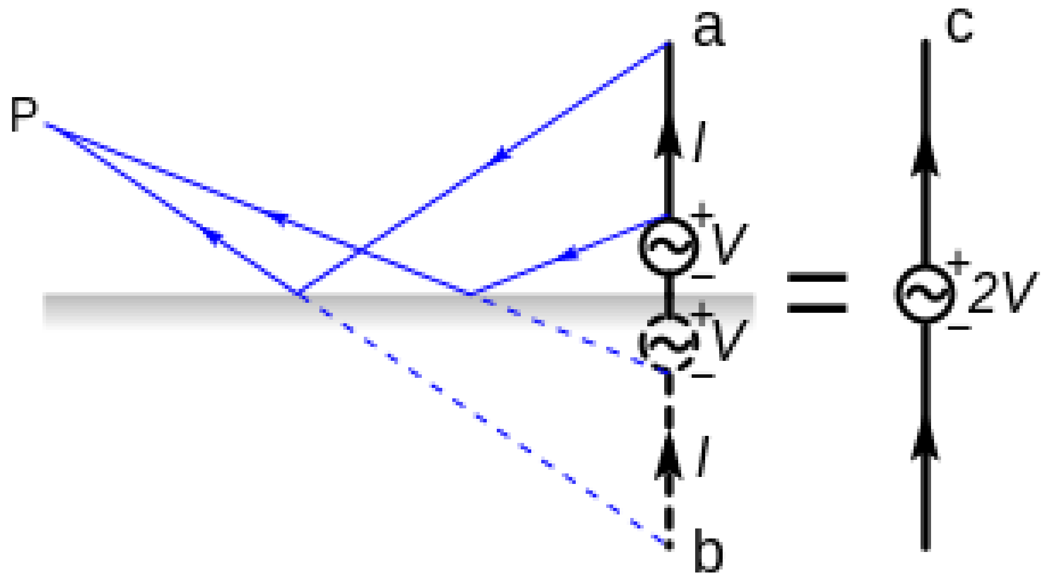
Linear Antenna (Monopole)

vertical current above PEC ground • Method of images



Linear Antenna (Monopole)

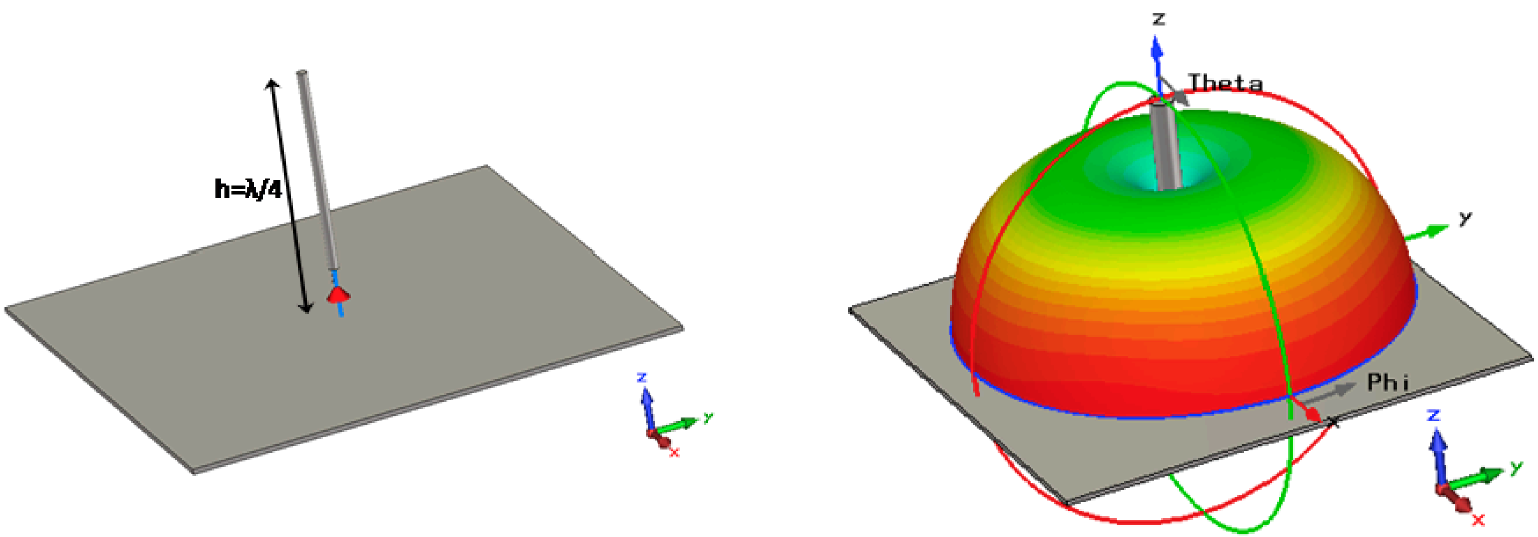
vertical current above PEC ground • Method of images



the monopole antenna has the **same radiation pattern over perfect ground** as **a dipole** in free space with **twice the voltage** above the ground plane

The monopole antenna fields **below the ground plane** are **zero**.

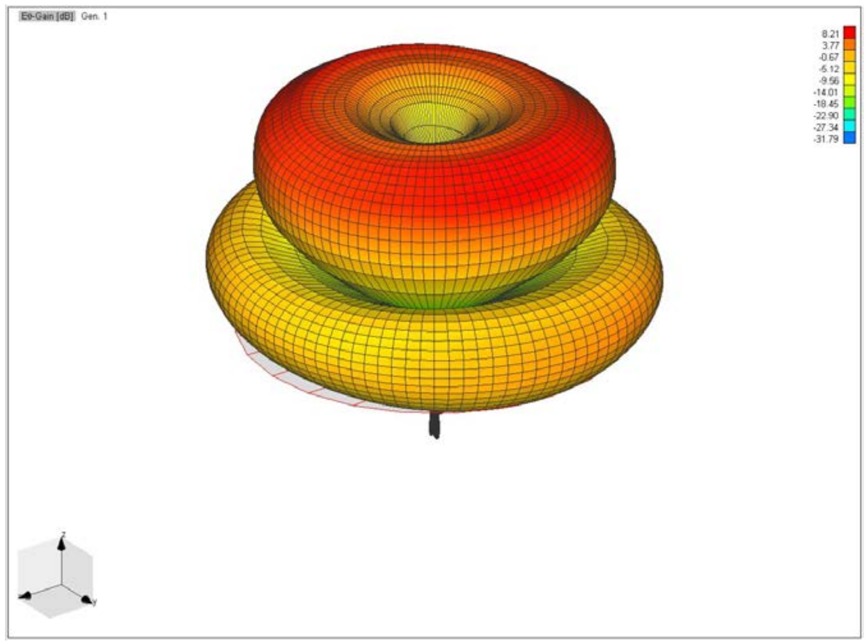
Linear Antenna (Monopole)



(a) (b)

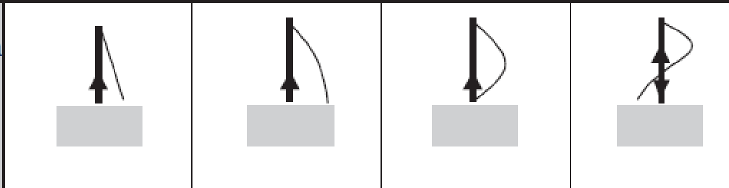
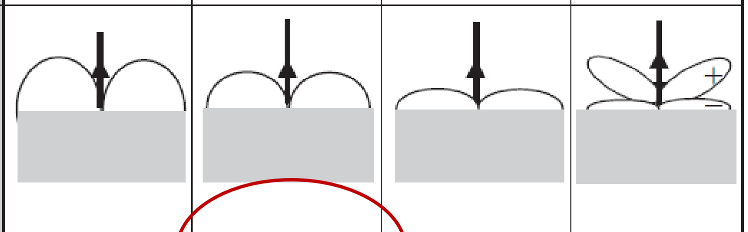
Monopole antennas **up to 1/4** wavelength long have a **single "lobe"**, with **field strength declining monotonically** from a **maximum in the horizontal direction** to **zero in the vertical direction**.

Linear Antenna (Monopole)



Radiation pattern of 3/2 wavelength monopole. **Longer monopoles (> 1/4 wavelength)** have more complicated patterns with **several conical "lobes"** (radiation maxima) **directed at angles into the sky.**

Linear Antenna (Monopole)

Monopole length l	$\lambda/20$	$\lambda/4$	$\lambda/2$	3/4
Current distribution				
Radiation pattern				
Directivity	3.0 or 4.76 dBi	3.28 or 5.15 dBi	4.8 or 6.8 dBi	About 4.6
HPBW	45°	39°	23.5°	NA
Input impedance	R: very small ($\sim 1\Omega$) jX: capacitive	R: $\sim$ 37 Ω jX: $\sim$ 0 Ω	R: very large jX: $\sim$ 0 Ω for thin dipole	R: $\sim$50 Ω jX: $\sim$ 0 Ω for thin dipole
Note	jX sensitive to the radius	$R+jX$ not sensitive to the radius	$R+jX$ sensitive to the radius	$R+jX$ sensitive to the radius

radiates only into **the space above the ground plane** (half the space of a dipole antenna)

a gain of twice (3 dBi over) **the gain of a similar dipole** antenna

a radiation resistance half that of a **dipole**

For infinite ground plane

$$Z_{monopole} = \frac{1}{2} Z_{dipole} = 36.5 + j21.25\ \Omega$$

Antenna can be tuned by shortening the radiator.

Linear Antenna (Monopole)

Using arms instead of real ground, R_{in} could be made close to 50Ω and thus matched to coaxial cable.

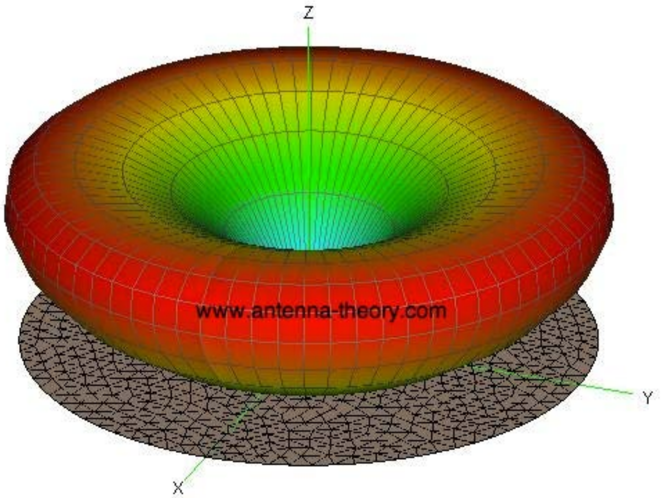
VHF ground plane antenna, a type of monopole antenna used at high frequencies. The three conductors projecting downward are the ground plane

Linear Antenna (Monopole)

Effects of a Finite Size Ground Plane on the Pattern

radiation pattern radiates in a “skewed” direction, away from the horizontal plane

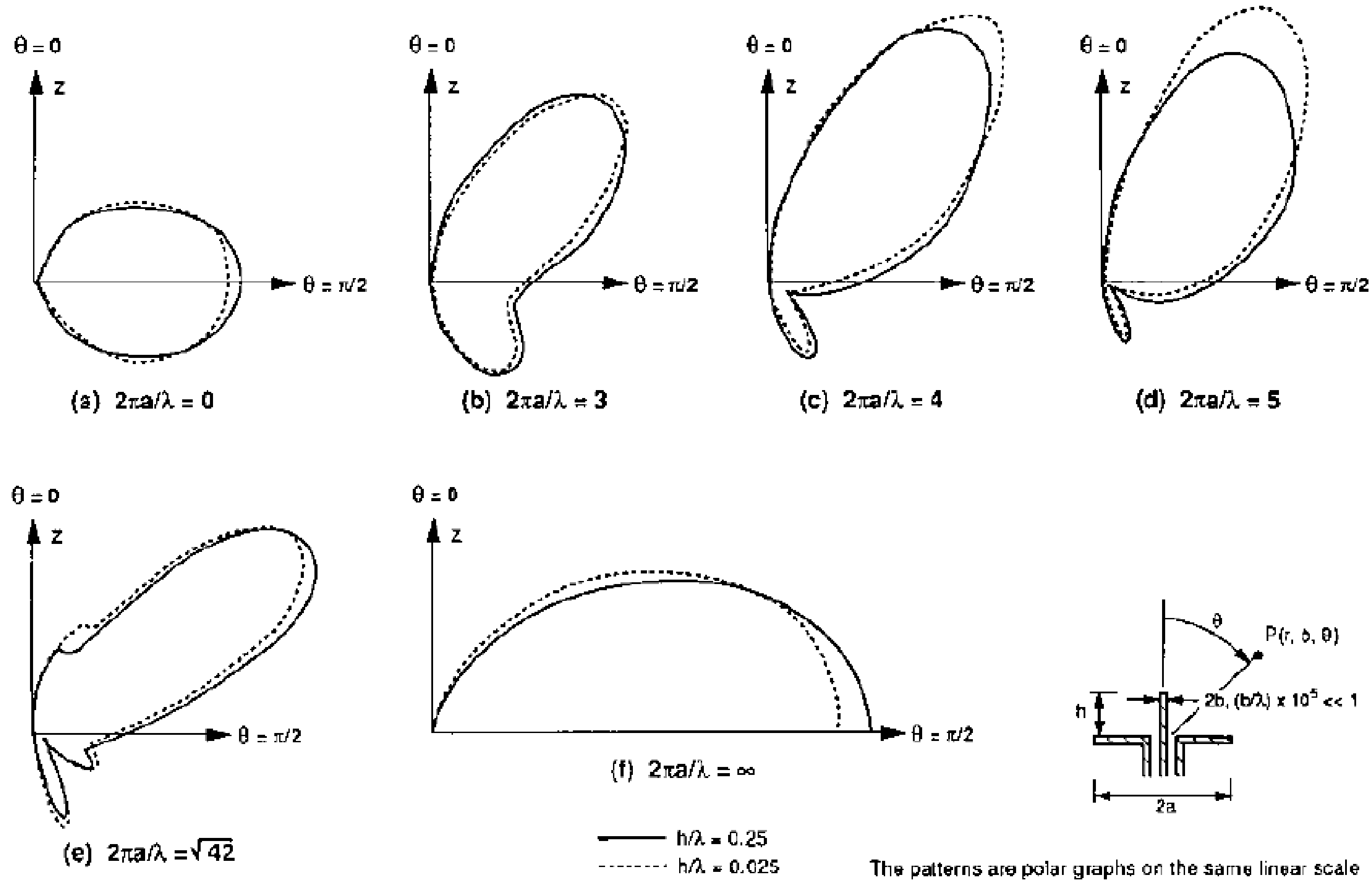
An example of the radiation pattern for a quarter-wavelength monopole antenna (oriented in the +z-direction) on a ground plane with a diameter of 3 wavelengths



Linear Antenna (Monopole)

Effects of a Finite Size Ground Plane

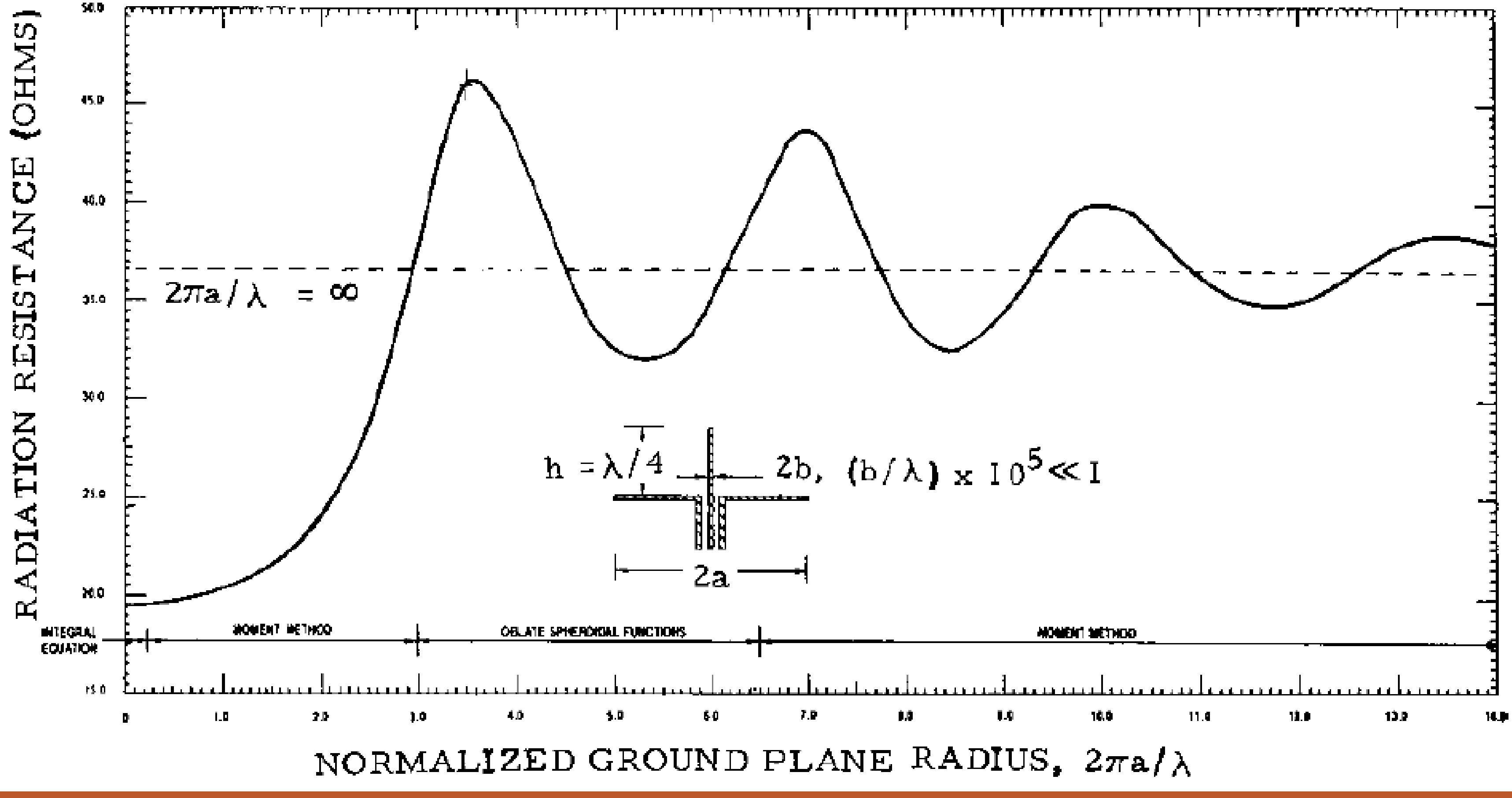
λ



In general, the large the ground plane is, the lower this direction of maximum radiation; as the ground plane approaches infinite size, the radiation pattern approaches a maximum in the x-y plane.

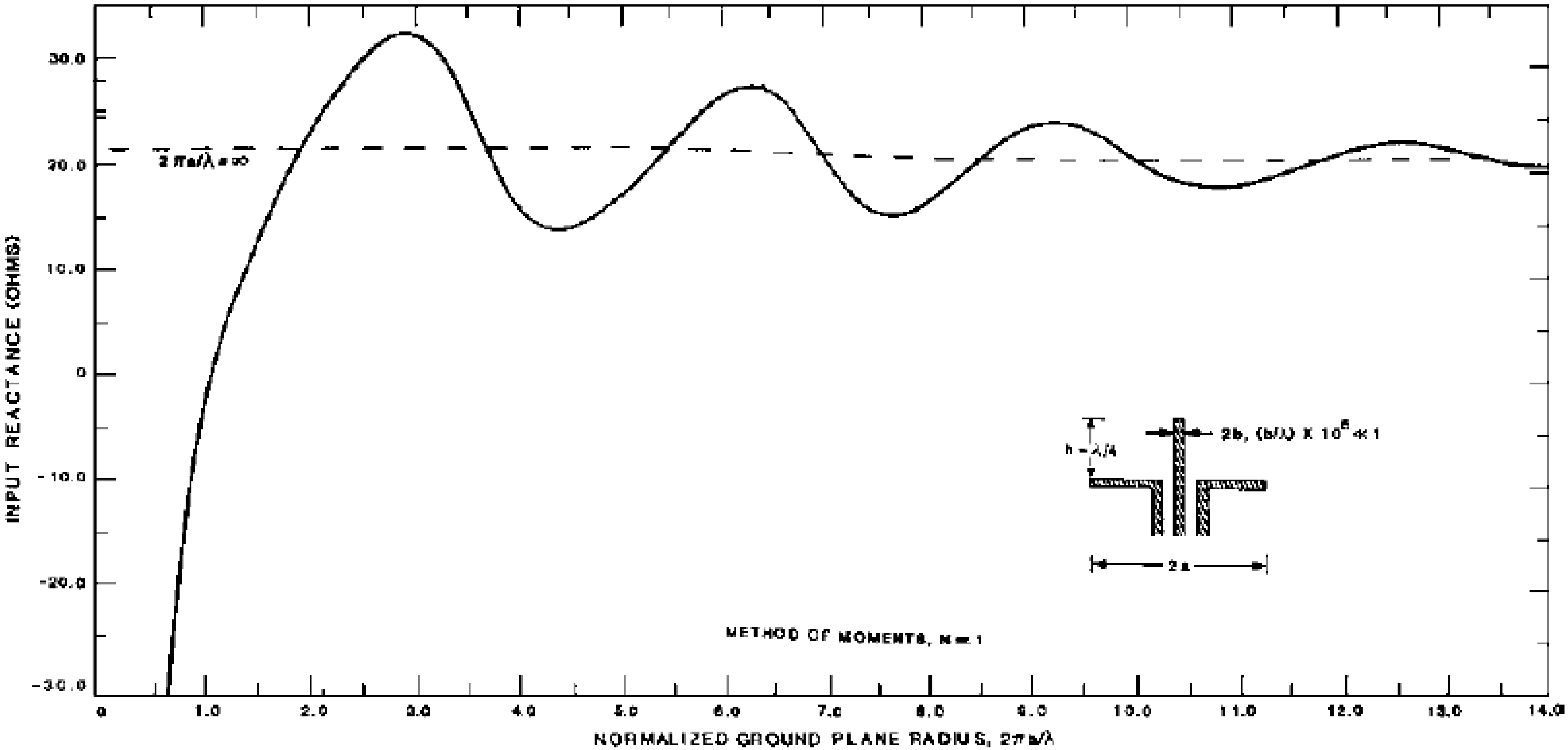
Linear Antenna (Monopole)

Effects of a Finite Size Ground Plane



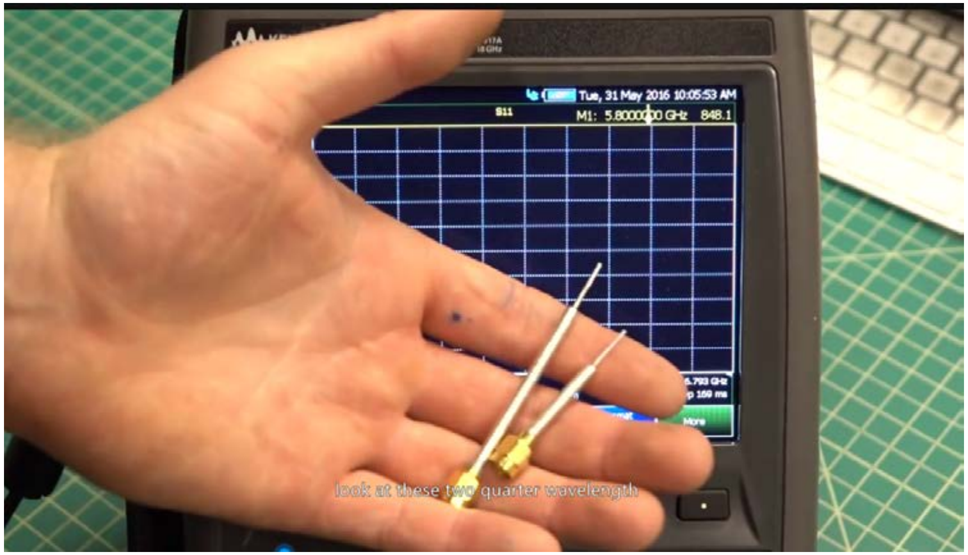
Linear Antenna (Monopole)

Effects of a Finite Size Ground Plane



Linear Antenna (Monopole)

Monopole Antenna Test video 5’42



Homework

Derive H and E field from Vector Potential on page 18

A 1-m–long dipole is excited by a 1-MHz current with an amplitude of 12 A. What is the average power density radiated by the dipole at a distance of 5 km in a direction that is 45◦ from the dipole axis?

Plot radiation pattern for

- half wavelength dipole
- quarter wavelength monopole (infinite ground)

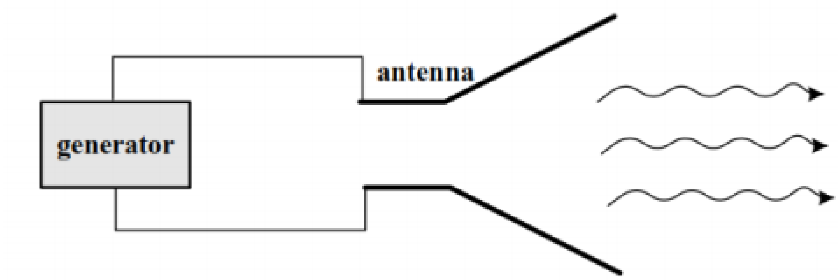
Calculate directivity and HPBW for each antenna

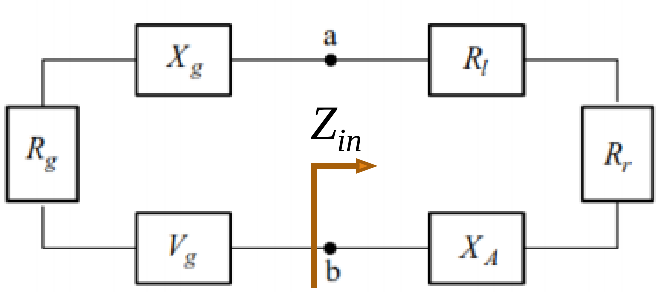
Input Resistance

Assume lossless (R_l=0) and the imaginary part of the antenna impedance

$$X_A = \frac{\eta}{4\pi}\left\{2S_i(kl) + \cos(kl)[2S_i(kl) - S_i(2kl)]\right.$$
$$\left. - \sin(kl)\left[2C_i(kl) - C_i(2kl) - C_i\left(\frac{2ka^2}{l}\right)\right]\right\}$$

where C = 0.5772 (Euler's constant) and $Ci(x)$ and $Si(x)$ are the cosine and sine integrals (see Appendix III)





$$\frac{|I_{in}|^2}{2}R_{in} = \frac{|I_0|^2}{2}R_r$$

$$R_{in} = \left[\frac{I_0}{I_{in}}\right]^2 R_r$$

Infinitesimal Dipole Antenna Vector Potential

Spherical Coordinates

$$\nabla\psi = \hat{\mathbf{a}}_r \frac{\partial\psi}{\partial r} + \hat{\mathbf{a}}_\theta \frac{1}{r}\frac{\partial\psi}{\partial\theta} + \hat{\mathbf{a}}_\phi \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\phi}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\mathbf{A} = \frac{\hat{\mathbf{a}}_r}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right] + \frac{\hat{\mathbf{a}}_\theta}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right] + \frac{\hat{\mathbf{a}}_\phi}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]$$