天线与电波传播
ANTENNAS AND WAVE PROPAGATION

LECTURE 5

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Last Week

Use vector potential to find far-field characteristics

Infinitesimal dipole (L $\leq \lambda/50$ L

Short dipole ($\lambda/50 < L \leq \lambda/10$)

Linear dipole

- current distribution and side lobes
- wavelength vs radiation pattern/directivity etc.)

Balun

Monopole (Method of images)

- Wavelength vs radiation pattern/directivity
- Effects of ground plane size

In general, the large the ground plane is, the lower this direction of maximum radiation; as the ground plane approaches infinite size, the radiation pattern approaches a maximum in the x-y plane.

This Week

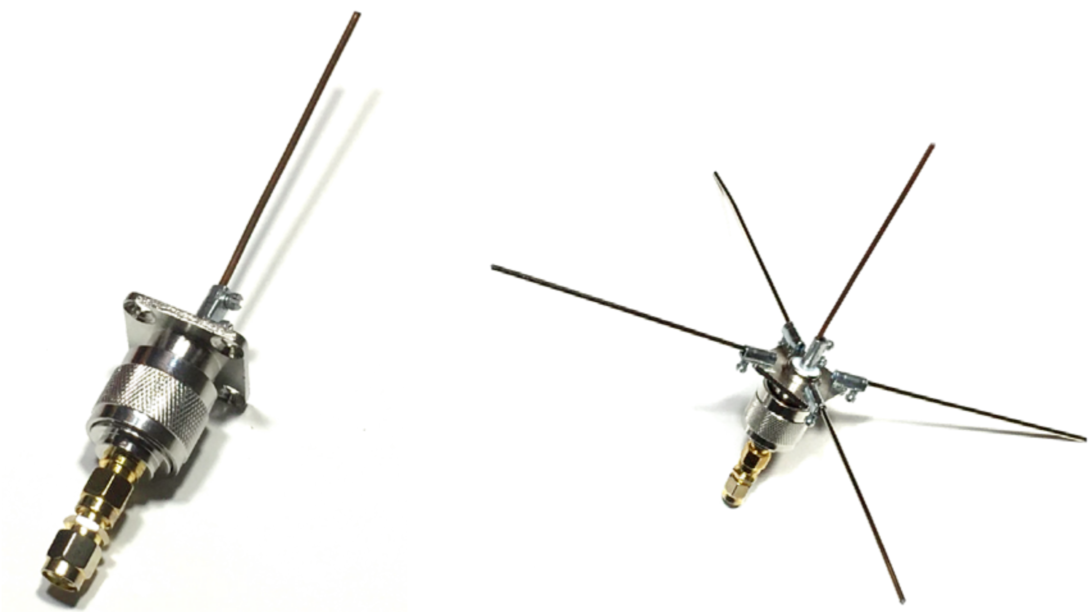
Linear antenna

Polarization

Friis Transmission equation

Radar Cross Section (RCS)

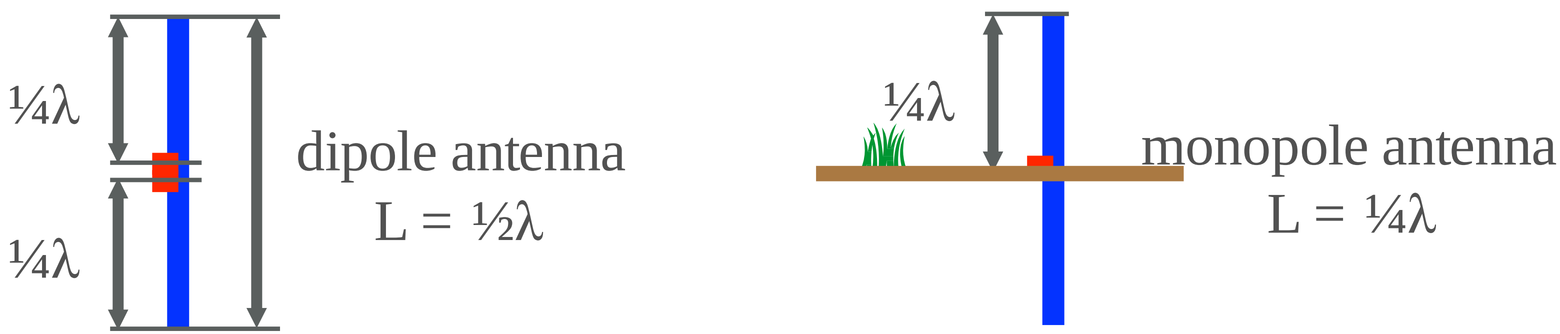
Linear Antenna (Monopole)



Mast radiator monopole antenna used for broadcasting. AM radio station WARE, Warren, Massachusetts, US.

1/4 Wavelength Monopole Antenna

- A ½λ dipole antenna has two elements (each element is L = ¼λ), but a monopole antenna has only one element (L = ¼λ).

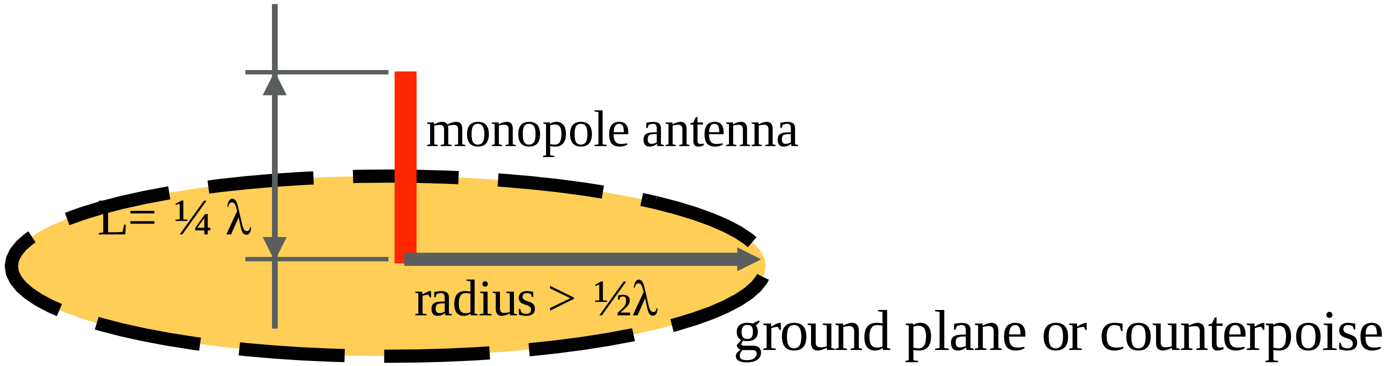


- relies on a good conducting plane for its operation. The plane is used as a mirror to create a "second imaginary" element.
- The ground (earth) is a conductor and RF energy is reflected from the ground.
- The monopole antenna performance can be improved by using a plane made of a better conductive material instead of using the earth itself.

Ground Plane / Counterpoise

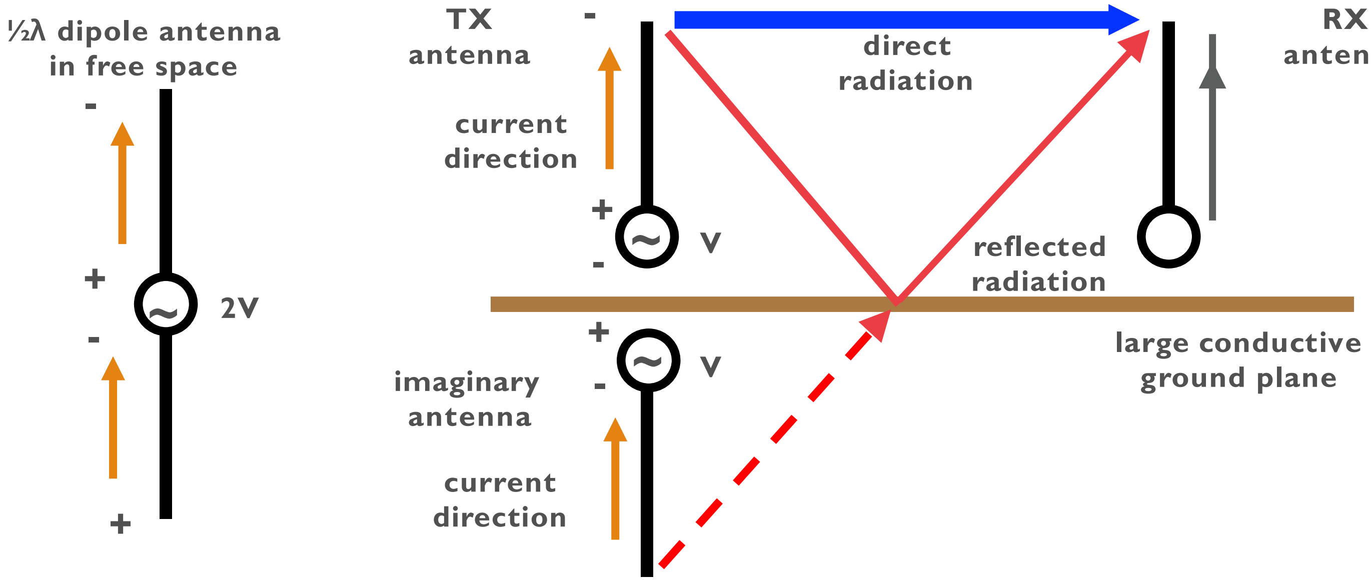
- This conducting plane is called the ground plane or counterpoise (network of suspended horizontal wires or cables or a metal screen).

- The ground plane must have a radius greater than ½ wavelength from the base of the monopole antenna for better efficiency.



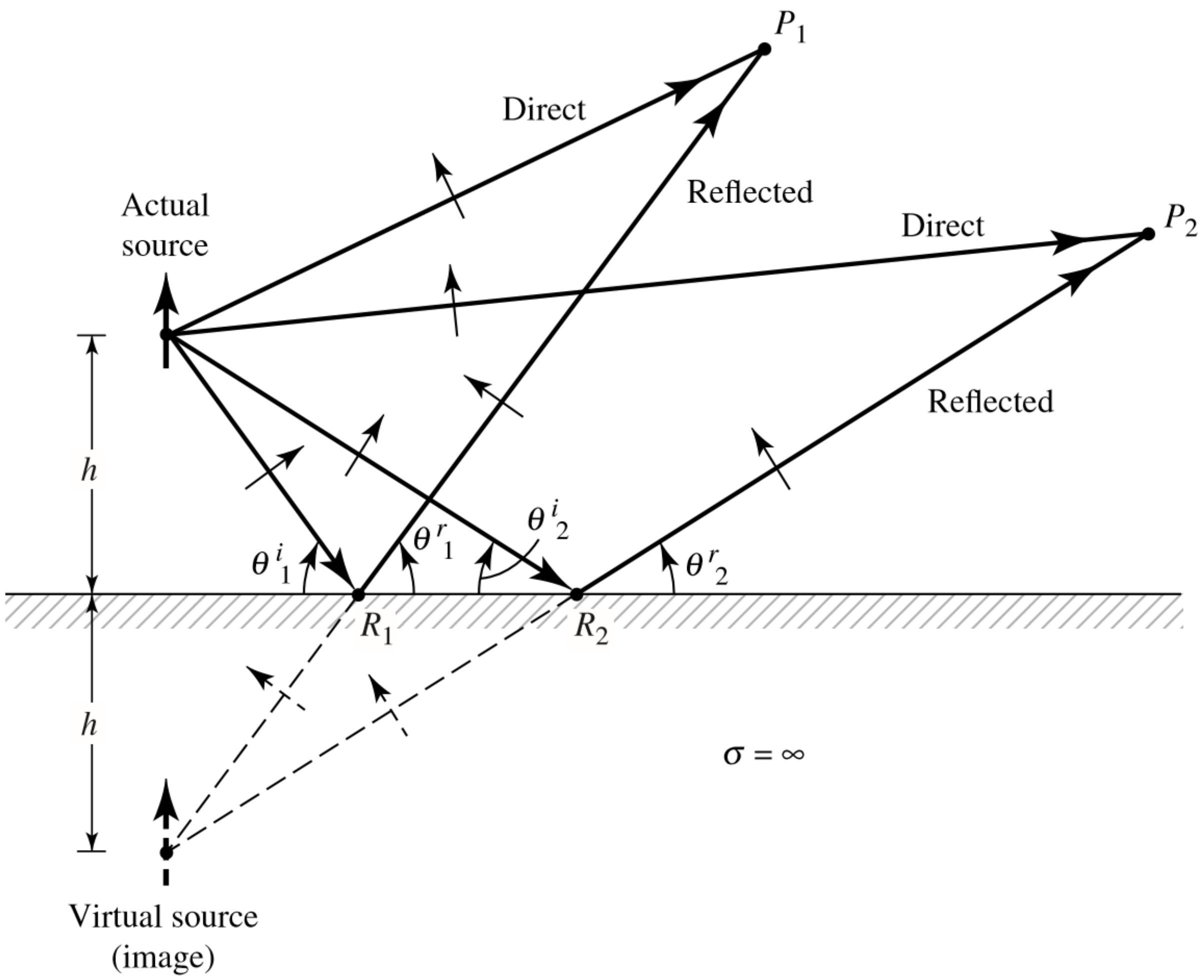
Ground Plane / Counterpoise

- The receiving antenna (RX) gets energy from **the direct path AND from the reflected path** which is in phase. A ¼λ monopole antenna will have twice the gain (3 dB) of a ½λ dipole antenna (in free space).



Linear Antenna (Monopole)

vertical current above PEC ground • Method of images



Linear Antenna (Monopole)

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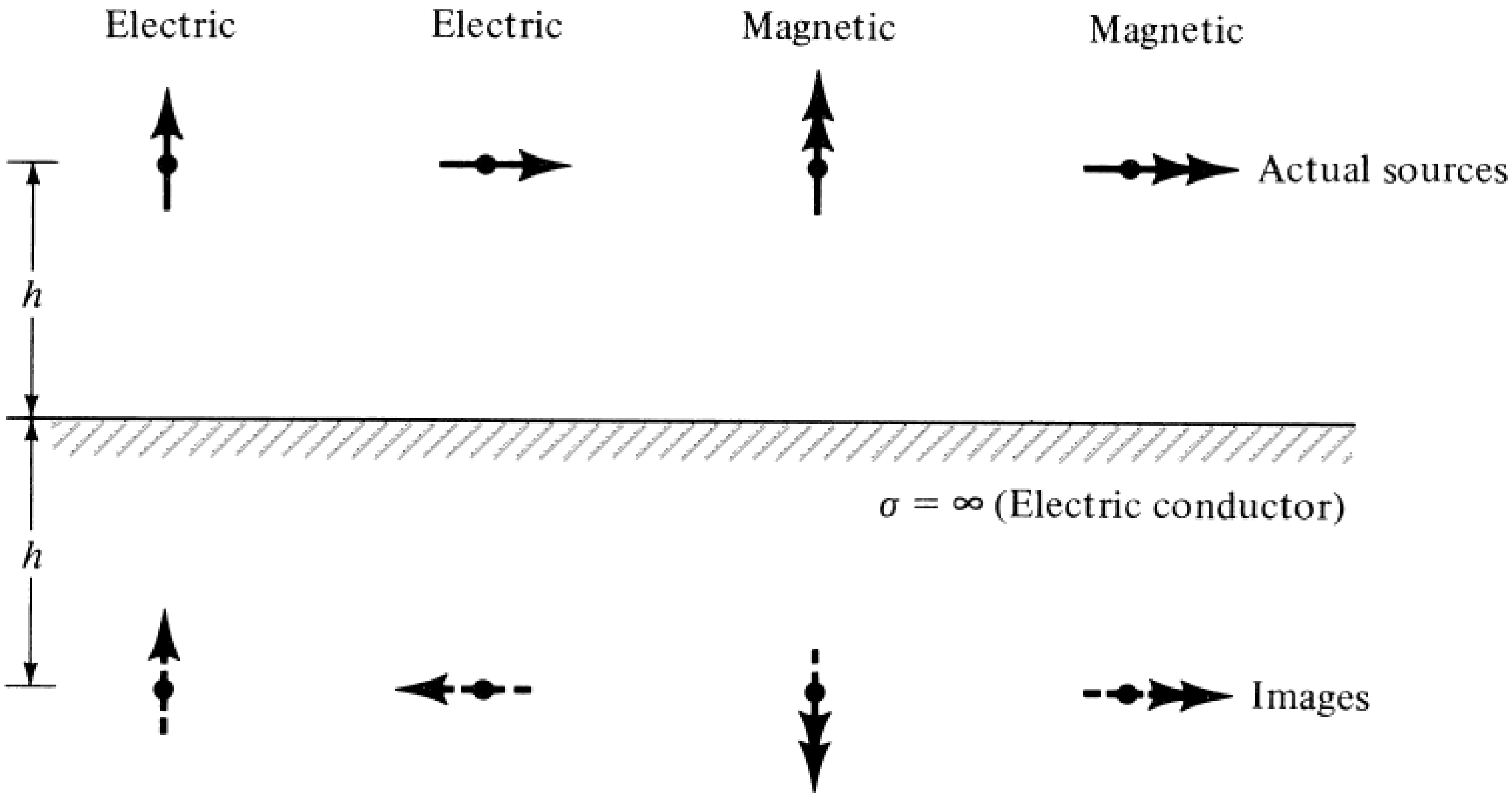
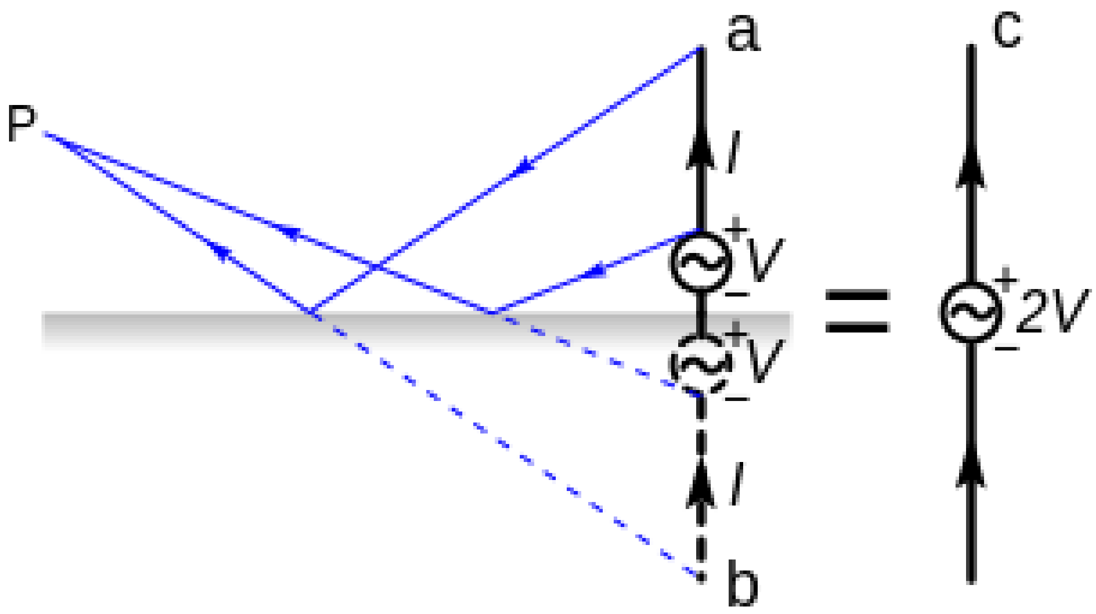


Linear Antenna (Monopole)

vertical current above PEC ground • Method of images



the monopole antenna has the same radiation pattern over perfect ground as a dipole in free space with twice the voltage above the ground plane

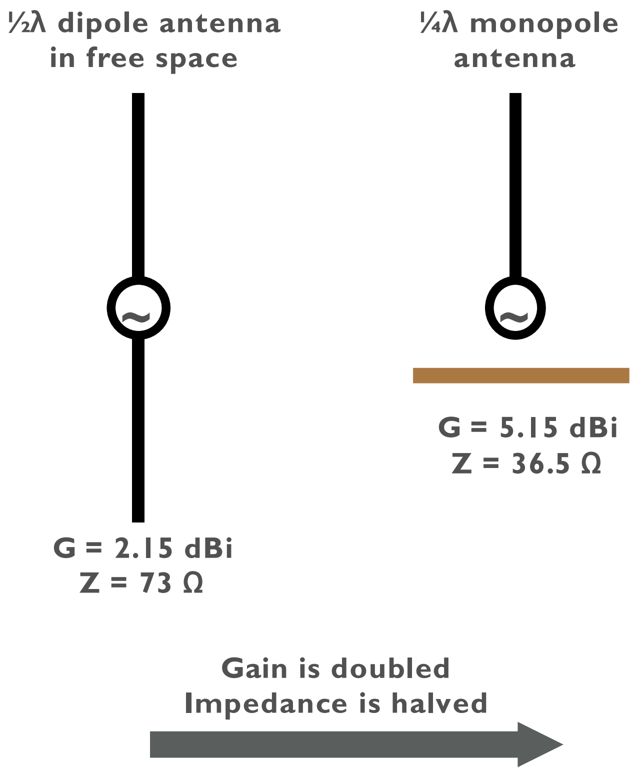
The monopole antenna fields below the ground plane are zero.

Ground Plane / Counterpoise

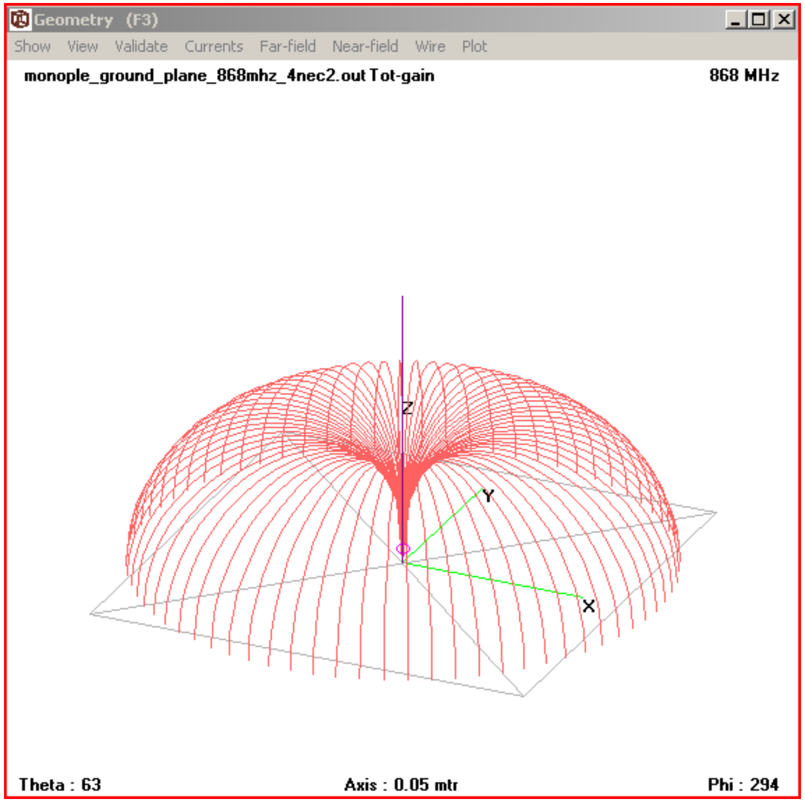
- A ½λ dipole antenna has a gain (G) of 1.64 and an impedance (Z) of 73 Ω at its centre, which is the radiation resistance.

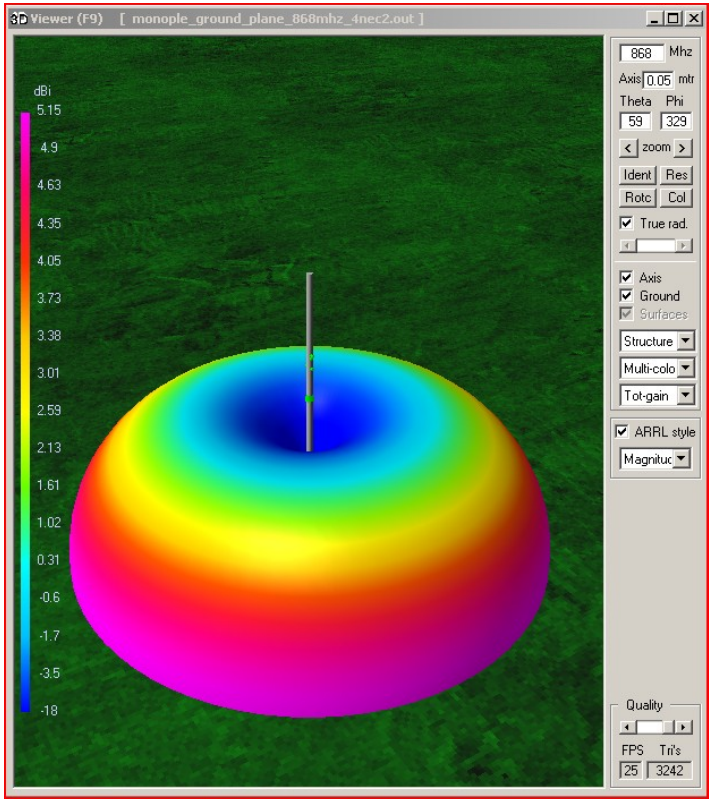
- By the way: $G = 10 \log_{10}(1.64) = 2.15$ dBi which is the same as 0 dBd

- A ¼λ monopole antenna has a gain (G) of 2.15 + 3 = 5.15 dBi (or 3 dBd) and a radiation resistance of 0.5 x 73 = 36.5 Ω when positioned over a large conductive ground plane.



Antenna Modelling 4nec2





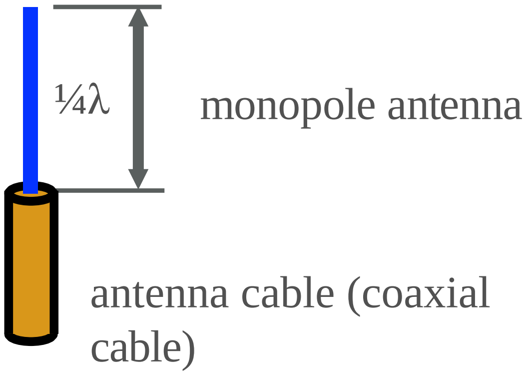
¼λ monopole antenna over perfect ground.

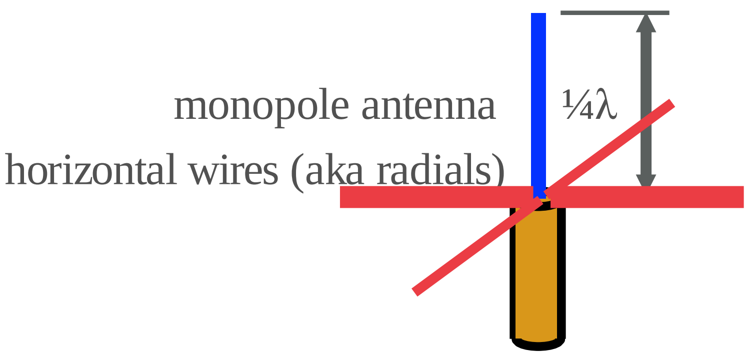
4nec2 is a completely free Nec2, Nec4 and windows based tool for creating, viewing, optimizing and checking 2D and 3D style antenna geometry structures and generate, display and/or compare near/far-field radiation patterns for both the starting and experienced antenna modeler. (Numerical Electromagnetics Code, or NEC)

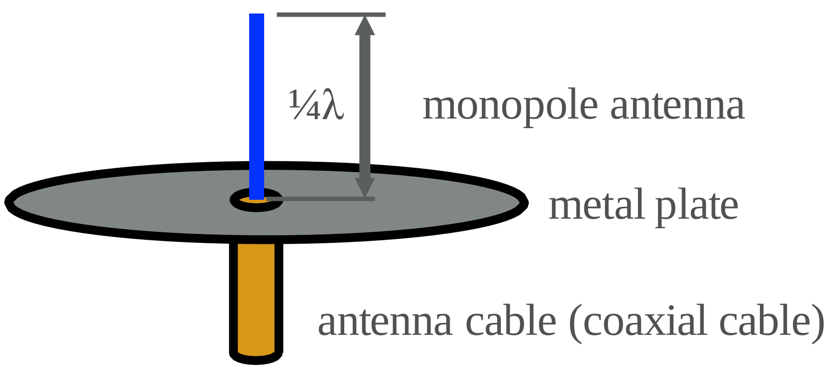
Monopole Antenna

- By default a monopole antenna will perform poorly without the use of a ground plane.

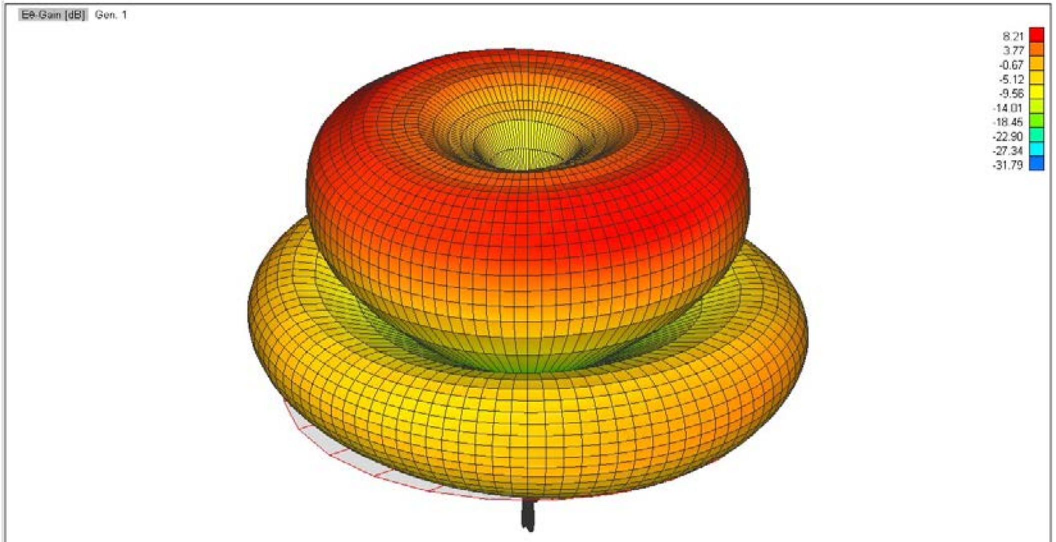
- A ground plane can be a network of horizontal wires or a metal plate.







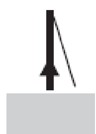
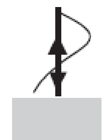
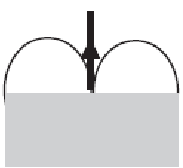
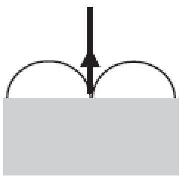
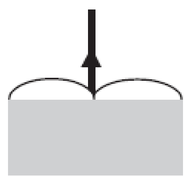
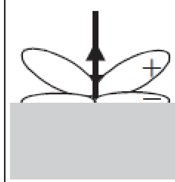
Linear Antenna (Monopole)



Radiation pattern of 3/2 wavelength monopole.

Monopole antennas **up to 1/4 wavelength** long have **a single "lobe",** with field strength declining monotonically from a maximum in the horizontal direction, but longer monopoles have more complicated patterns with several conical "lobes" (radiation maxima) directed at angles into the sky.

Linear Antenna (Monopole)

Monopole length l	$\lambda/20$	$\lambda/4$	$\lambda/2$	3/4
Current distribution				
Radiation pattern				
Directivity	3.0 or 4.76 dBi	3.28 or 5.15 dBi	4.8 or 6.8 dBi	About 4.6
HPBW	45°	39°	23.5°	NA
Input impedance	R: very small ($\sim 1\Omega$) jX: capacitive	R: $\sim 37\ \Omega$ jX: $\sim 0\ \Omega$	R: very large jX: $\sim 0\ \Omega$ for thin dipole	R: $\sim 50\ \Omega$ jX: $\sim 0\ \Omega$ for thin dipole
Note	jX sensitive to the radius	$R+jX$ not sensitive to the radius	$R+jX$ sensitive to the radius	$R+jX$ sensitive to the radius

It radiates only into the space above the ground plane (half the space of a dipole antenna), a monopole antenna will have a gain of twice (3 dBi over) the gain of a similar dipole antenna, and a radiation resistance half that of a dipole

For infinite ground plane

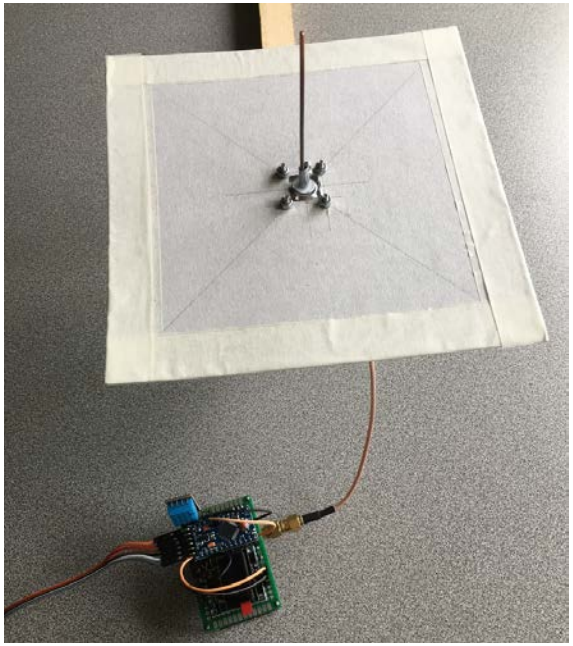
$$Z_{monopole} = \frac{1}{2} Z_{dipole} = 36.5 + j21.25\ \Omega$$

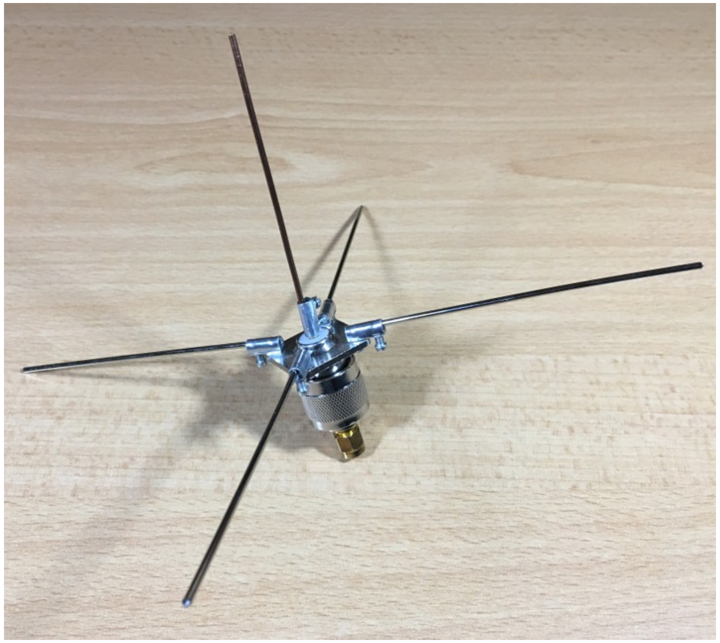
antenna can be tuned by shortening the radiator.

Using arms instead of real ground, R_{in} could be made close to 50Ω and thus matched to coaxial cable.

Metal Plate Vs Radials

- using a metal plate (radius > ½λ) or radials as the ground plane produces approximately the same result.





Metal Plate Vs Radials

- using a metal plate (radius > ½λ) or radials as the ground plane produces approximately the same result.

- Using radials is often preferred:
  The radials are lighter than a metal plate.
  The radials are cheaper than a metal plate.
  The radials are more resistant to weather conditions (wind, rain).

- When using a metal plate or radials as a ground plane, in both cases the impedance is around 75Ω instead of 50Ω. How to fix this?

- When radials are used, bend it to a certain angle and the impedance will be 50Ω and the VSWR will remain below 2.

Linear Antenna (Monopole)

VHF ground plane antenna
a type of monopole antenna used at high frequencies.
The three conductors projecting downward are the ground plane.
The downward projecting conductor creates desired impedance.

VHF: 30 MHz to 300 MHz
Wavelength: 10M to 1M

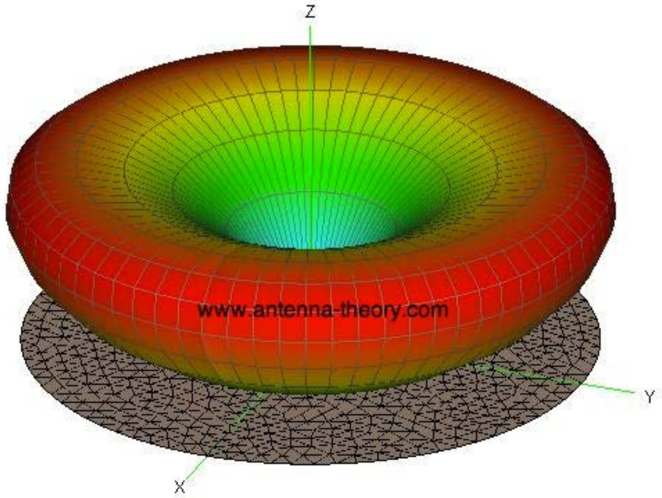
Television and FM radio broadcasting, as well as ship and aircraft communications, disaster prevention and administration radio,

Linear Antenna (Monopole)

Effects of a Finite Size Ground Plane on the Monopole Antenna

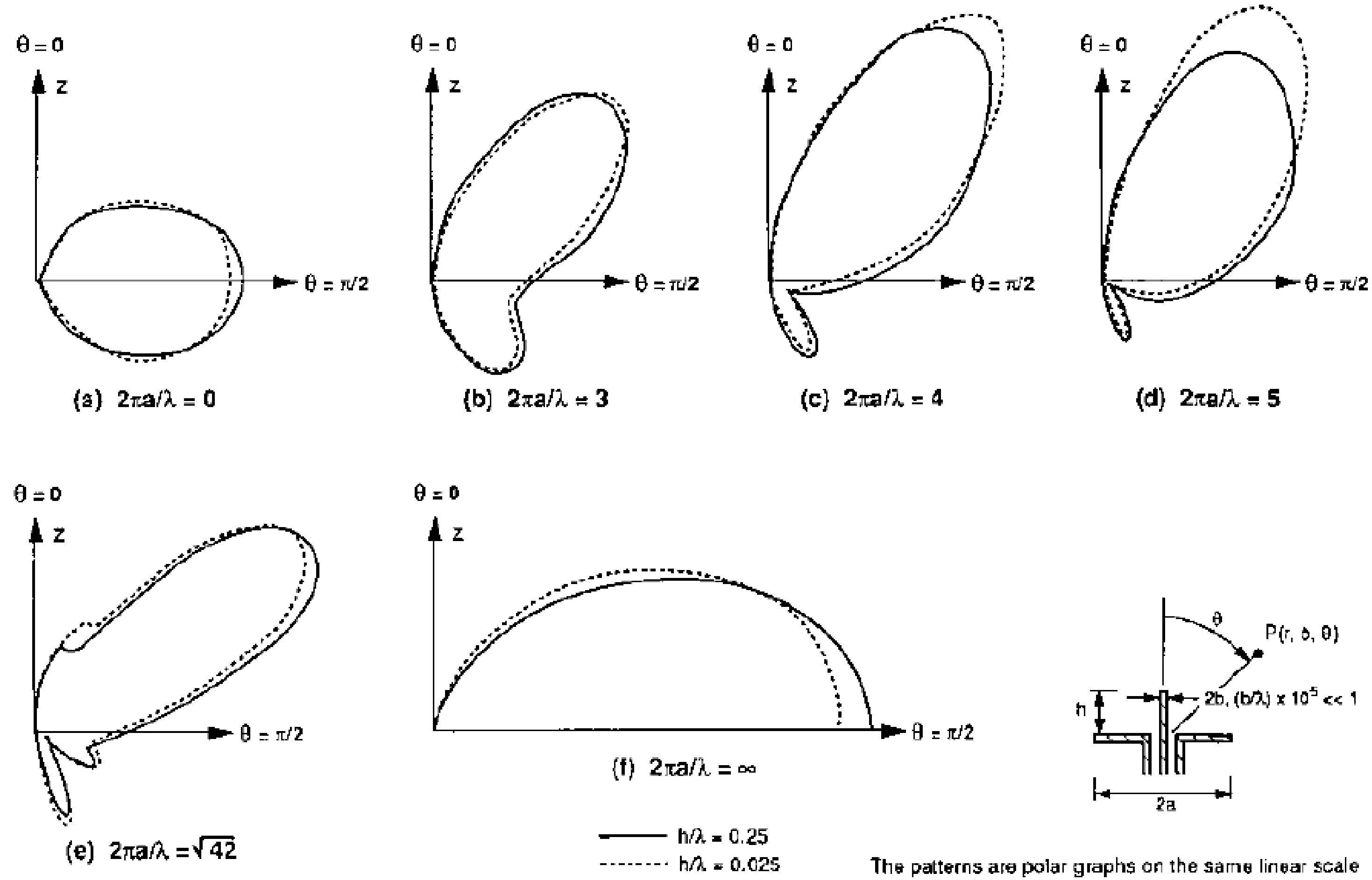
radiation pattern radiates in a “skewed” direction, away from the horizontal plane

An example of the radiation pattern for a quarter-wavelength monopole antenna (oriented in the +z-direction) on a ground plane with a diameter of 3 wavelengths



Linear Antenna (Monopole)

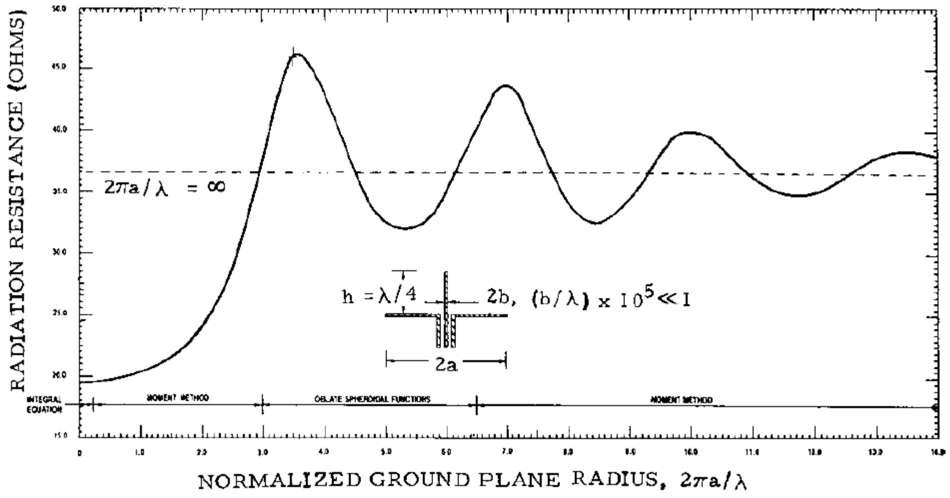
Effects of a Finite Size Ground Plane

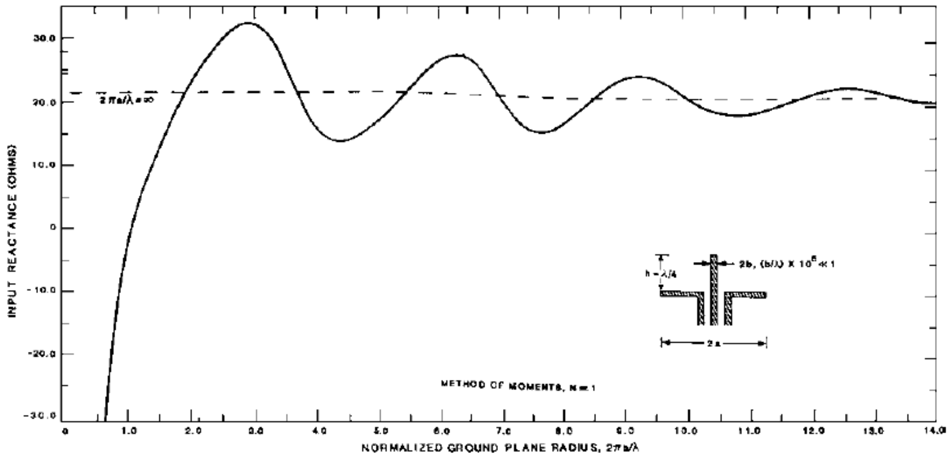


In general, the large the ground plane is, the lower this direction of maximum radiation; as the ground plane approaches infinite size, the radiation pattern approaches a maximum in the x-y plane.

Linear Antenna (Monopole)

Effects of a Finite Size Ground Plane





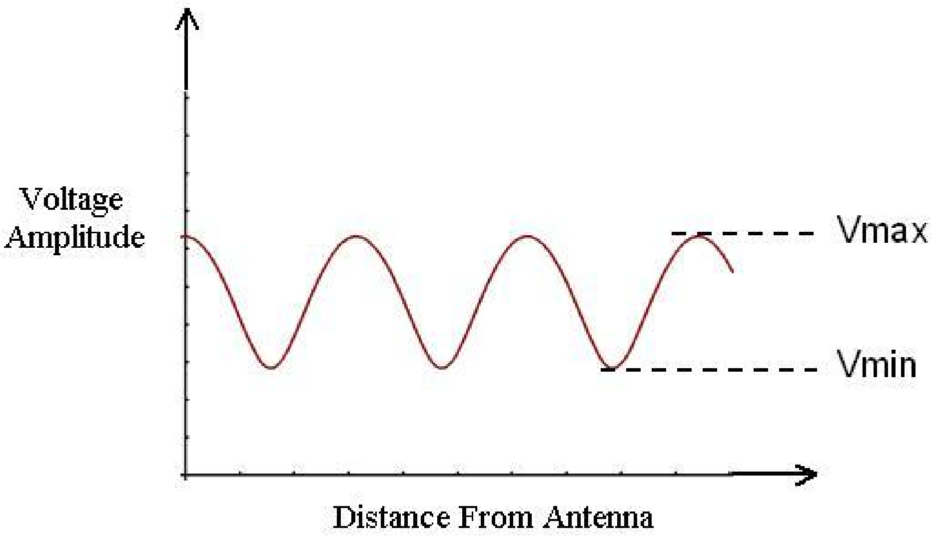
For infinite ground plane

$$Z_{monopole} = \frac{1}{2} Z_{dipole} = 36.5 + j21.25\ \Omega$$

Linear Antenna (Monopole)

video

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$



$$\Gamma = \frac{E^-}{E^+}$$

VSWR is the ratio of the peak amplitude of a standing wave to the minimum amplitude of a standing wave

Linear Antenna (Monopole)

TABLE: VSWR AND TRANSMITTED POWER

VSWR	$\|\Gamma\|^2 \times 100\%$	$\|T\|^2 \times 100\%$
1.0	0.0	100.0
1.1	0.2	99.8
1.2	0.8	99.2
1.5	4.0	96.0
2.0	11.1	88.9
3.0	25.0	75.0
4.0	36.0	64.0
5.0	44.4	55.6
5.83	50.0	50.0
10.0	66.9	33.1

Infinitesimal Dipole
(Balanis, Table 4.3, pp.217-218)

	Infinitesimal Dipole ($l \leq \lambda/50$)	
Normalized power pattern	$U = (E_{\theta n})^2 = C_0 \sin^2\theta$	(4-29)
Radiation resistance R_r	$R_r = \eta\left(\frac{2\pi}{3}\right)\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(\frac{l}{\lambda}\right)^2$	(4-19)
Input resistance R_{in}	$R_{in} = R_r = \eta\left(\frac{2\pi}{3}\right)\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(\frac{l}{\lambda}\right)^2$	(4-19)
Wave impedance Z_w	$Z_w = \frac{E_\theta}{H_\phi} \simeq \eta = 377$ ohms	
Directivity D_0	$D_0 = \frac{3}{2} = 1.761$ dB	(4-31)
Maximum effective area A_{em}	$A_{em} = \frac{3\lambda^2}{8\pi}$	(4-32)
Vector effective length ℓ_e	$\boldsymbol{\ell}_e = -\hat{\mathbf{a}}_\theta l \sin\theta$ $\lvert\boldsymbol{\ell}_e\rvert_{\max} = \lambda$	(2-92) Example 4.2
Half-power beamwidth	HPBW = 90°	(4-65)
Loss resistance R_L	$R_L = \frac{l}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{l}{2\pi b}\sqrt{\frac{\omega\mu_0}{2\sigma}}$	(2-90b)

Small Dipole
(Balanis, Table 4.3, pp.217-218)

	Small Dipole ($\lambda/50 < l \leq \lambda/10$)	
Normalized power pattern	$U = (E_{\theta n})^2 = C_1 \sin^2\theta$	(4-36a)
Radiation resistance R_r	$R_r = 20\pi^2 \left(\frac{l}{\lambda}\right)^2$	(4-37)
Input resistance R_{in}	$R_{in} = R_r = 20\pi^2 \left(\frac{l}{\lambda}\right)^2$	(4-37)
Wave impedance Z_w	$Z_w = \frac{E_\theta}{H_\phi} \simeq \eta = 377$ ohms	(4-36a), (4-36c)
Directivity D_0	$D_0 = \frac{3}{2} = 1.761$ dB	
Maximum effective area A_{em}	$A_{em} = \frac{3\lambda^2}{8\pi}$	
Vector effective length ℓ_e	$\boldsymbol{\ell}_e = -\hat{\mathbf{a}}_\theta \frac{l}{2}\sin\theta$	(2-92)
	$\lvert\boldsymbol{\ell}_e\rvert_{\max} = \frac{l}{2}$	(4-36a)
Half-power beamwidth	HPBW $= 90°$	(4-65)

Half Wavelength Dipole
(Balanis, Table 4.3, pp.217-218)

Half Wavelength Dipole
$(l = \lambda/2)$

Normalized power pattern	$U = (E_{\theta n})^2 = C_2 \left[\dfrac{\cos\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta} \right]^2 \simeq C_2 \sin^3\theta$	(4-87)
Radiation resistance R_r	$R_r = \dfrac{\eta}{4\pi} C_{in}(2\pi) \simeq 73$ ohms	(4-93)
Input resistance R_{in}	$R_{in} = R_r = \dfrac{\eta}{4\pi} C_{in}(2\pi) \simeq 73$ ohms	(4-79), (4-93)
Input impedance Z_{in}	$Z_{in} = 73 + j42.5$	(4-93a)
Wave impedance Z_w	$Z_w = \dfrac{E_\theta}{H_\phi} \simeq \eta = 377$ ohms	
Directivity D_0	$D_0 = \dfrac{4}{C_{in}(2\pi)} \simeq 1.643 = 2.156$ dB	(4-91)
Vector effective length ℓ_e	$\boldsymbol{\ell}_e = -\hat{\mathbf{a}}_\theta \dfrac{\lambda}{\pi} \dfrac{\cos\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta}$	(2-91)
	$\lvert\boldsymbol{\ell}_e\rvert_{\max} = \dfrac{\lambda}{\pi} = 0.3183\lambda$	(4-84)
Half-power beamwidth	HPBW $= 78^\circ$	(4-65)
Loss resistance R_L	$R_L = \dfrac{l}{2P}\sqrt{\dfrac{\omega\mu_0}{2\sigma}} = \dfrac{l}{4\pi b}\sqrt{\dfrac{\omega\mu_0}{2\sigma}}$	Example (2-13)

Quarter-Wavelength Monopole
(Balanis, Table 4.3, pp.217-218)

Quarter-Wavelength Monopole
$(l = \lambda/4)$

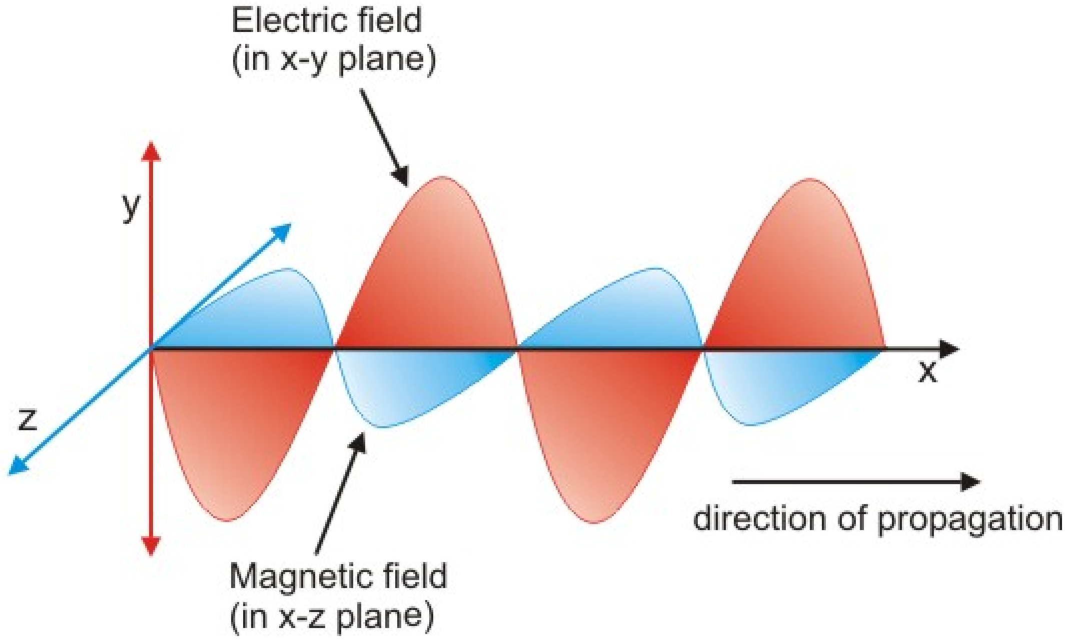
Normalized power pattern	$U = (E_{\theta n})^2 = C_2 \left[\dfrac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right]^2 \simeq C_2 \sin^3\theta$	(4-87)
Radiation resistance R_r	$R_r = \dfrac{\eta}{8\pi} C_{in}(2\pi) \simeq 36.5$ ohms	(4-106)
Input resistance R_{in}	$R_{in} = R_r = \dfrac{\eta}{8\pi} C_{in}(2\pi) \simeq 36.5$ ohms	(4-106)
Input impedance Z_{in}	$Z_{in} = 36.5 + j21.25$	(4-106)
Wave impedance Z_w	$Z_w = \dfrac{E_\theta}{H_\phi} \simeq \eta = 377$ ohms	
Directivity D_0	$D_0 = 3.286 = 5.167$ dB	
Vector effective length ℓ_e	$\boldsymbol{\ell}_e = -\hat{\mathbf{a}}_\theta \dfrac{\lambda}{\pi} \cos\left(\dfrac{\pi}{2}\cos\theta\right)$	(2-91)
	$\lvert\boldsymbol{\ell}_e\rvert_{\max} = \dfrac{\lambda}{\pi} = 0.3183\lambda$	(4-84)

Polarization

Orientation and magnitude of **field vectors** (usually E) at a given point and how it **varies with time**.

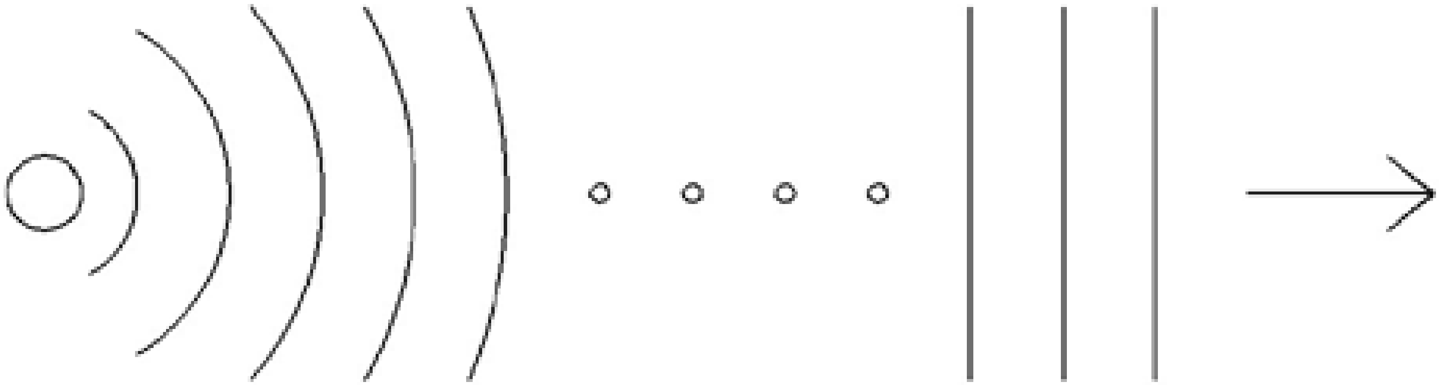
associated with TEM time-harmonic waves where the H vector relates to the E vector simply by

$$\mathbf{H} = \hat{\mathbf{r}} \times \mathbf{E} / \eta$$



Polarization

In antenna theory, we are concerned with the polarization of the field in the plane orthogonal to the direction of propagation



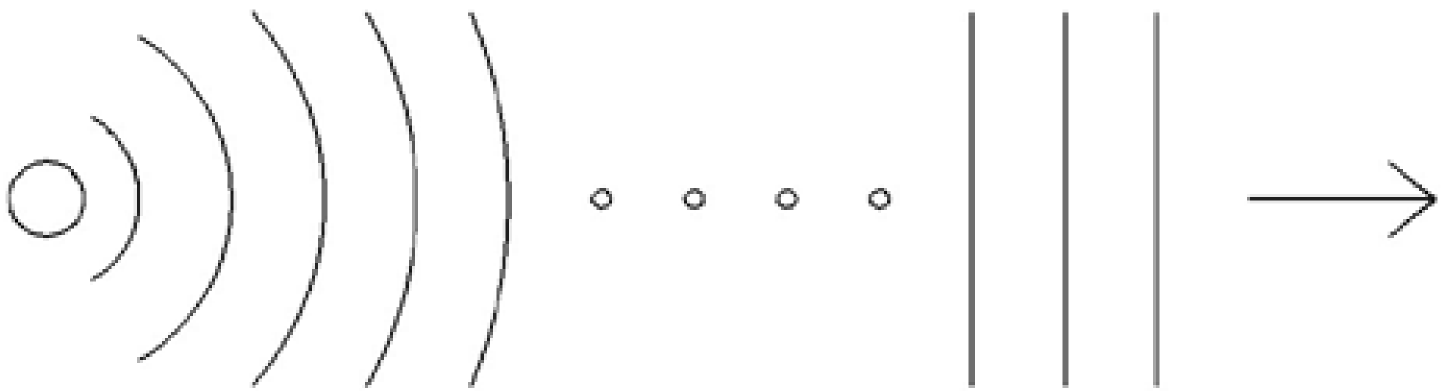
Polarization

In antenna theory, the **polarization of the field** in the plane orthogonal to the direction of propagation

the far field is a **quasi-TEM** field

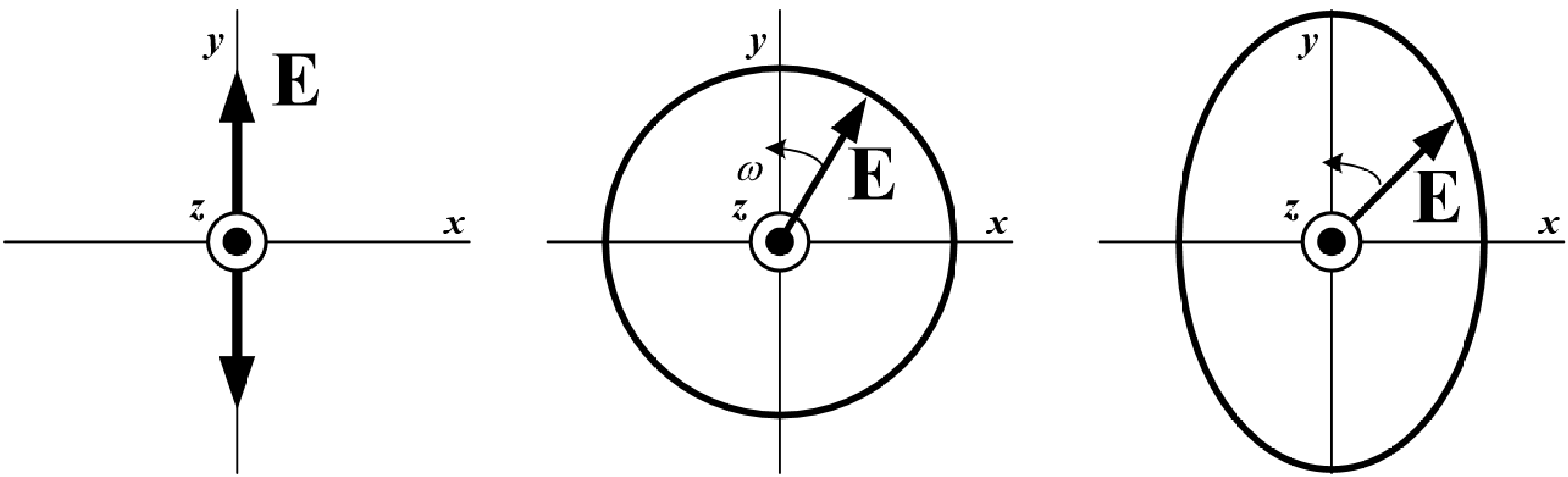
near **field** to the **far field**, EM radiation changes from spherical waves to plane waves

the longitudinal **field** components are negligible,



Polarization

The **polarization** is the locus traced by the extremity of the time-varying field vector at a fixed observation point.



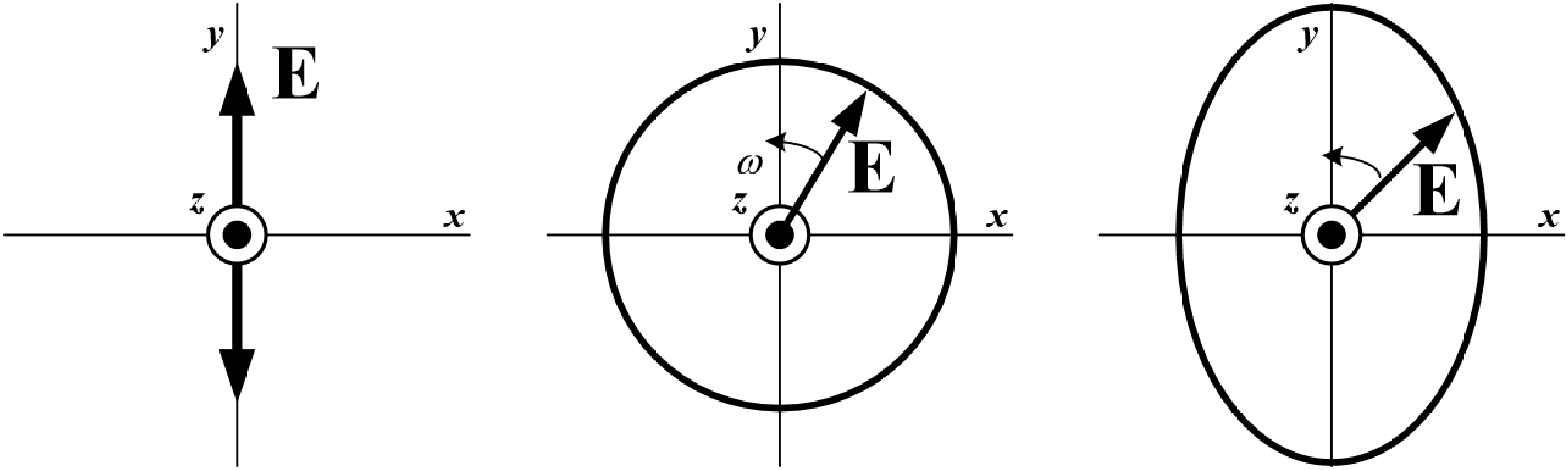
(a) linear polarization (b) circular polarization (c) elliptical polarization

Polarization

three types of **polarization** exist for harmonic fields: **linear**, **circular** and **elliptical**.

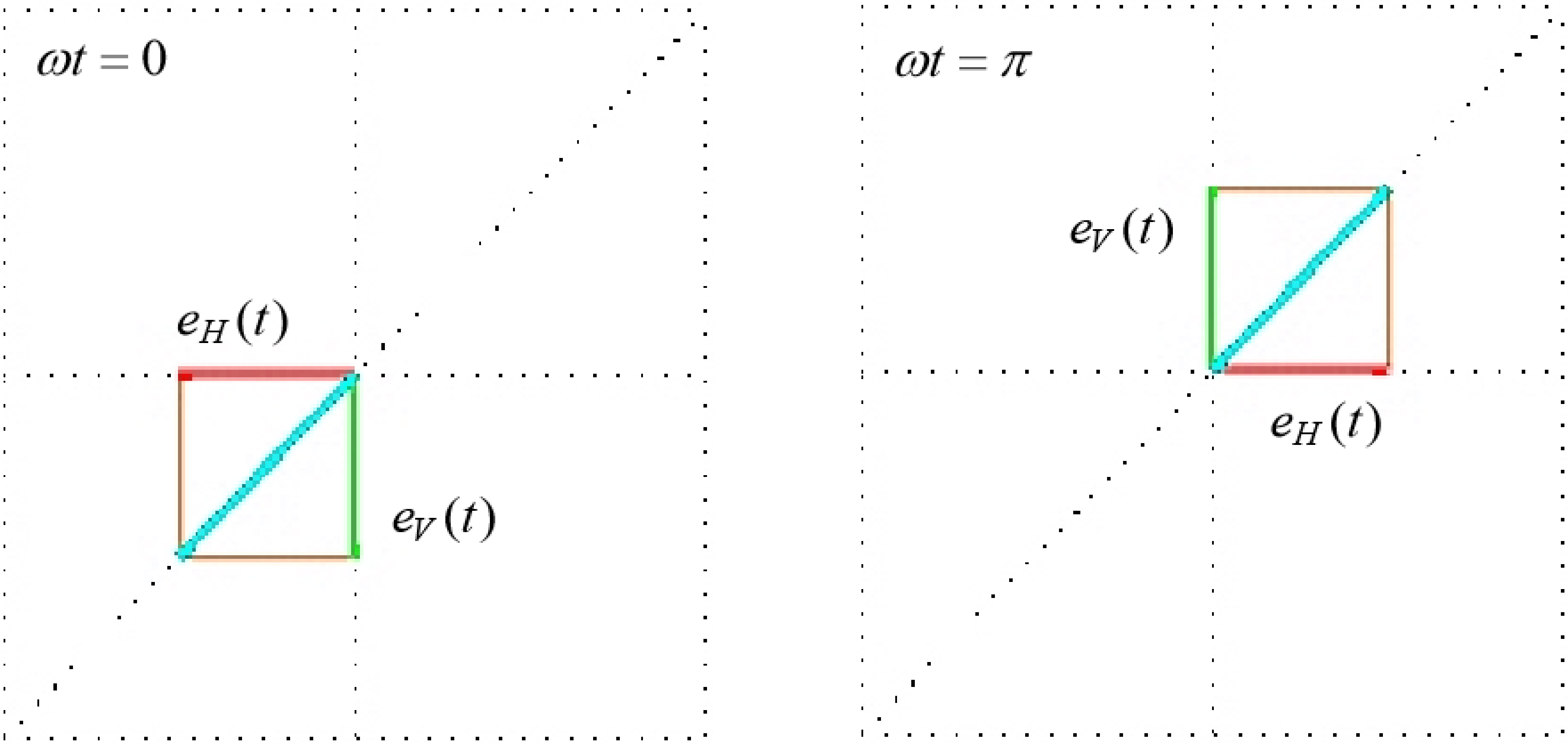
any polarization can be represented by **two orthogonal linear polarizations**, (E_x, E_y) or (E_H E_v)

the fields in general have **different magnitudes** and may be **out of phase** by an angle δ_L



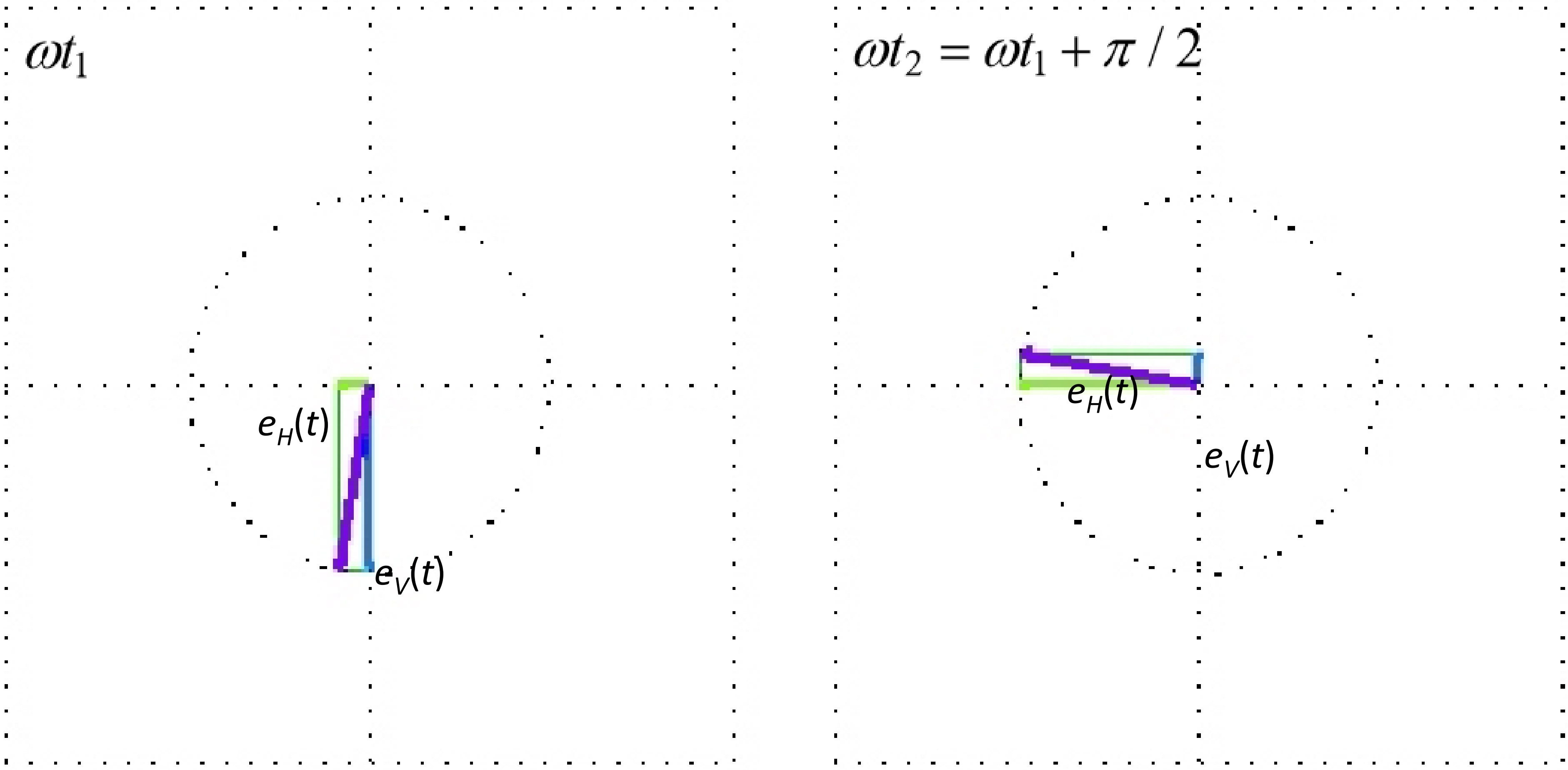
(a) linear polarization (b) circular polarization (c) elliptical polarization

Linear Polarization (δ_L=0 or nπ)



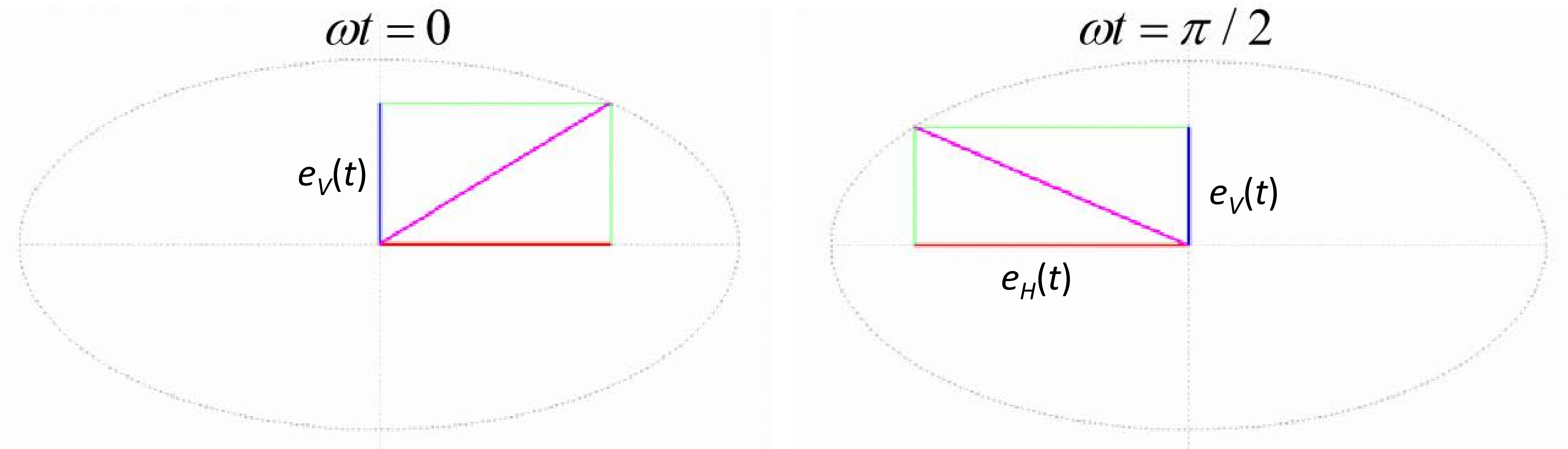
Animation: Linear Polarization

Circular Polarization ($\delta_L = \pi/2$, $|E_x| = |E_y|$)



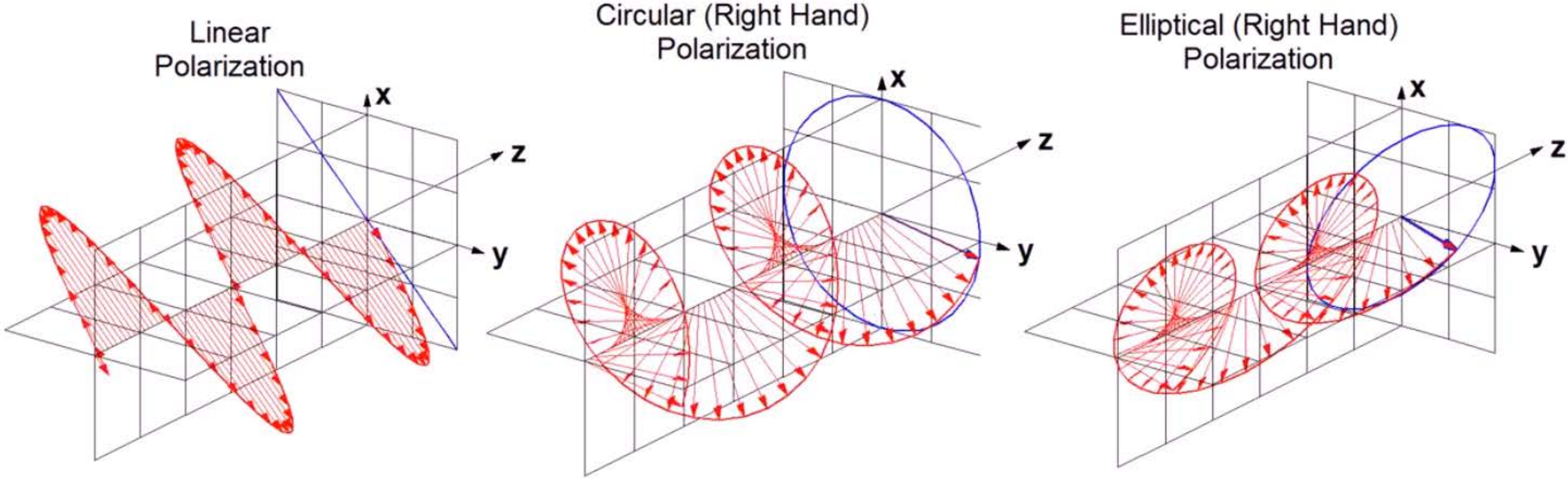
Animation: circular Polarization

Elliptical Polarization (Most General Case)



Animation: Counter-clockwise Elliptical Rotation

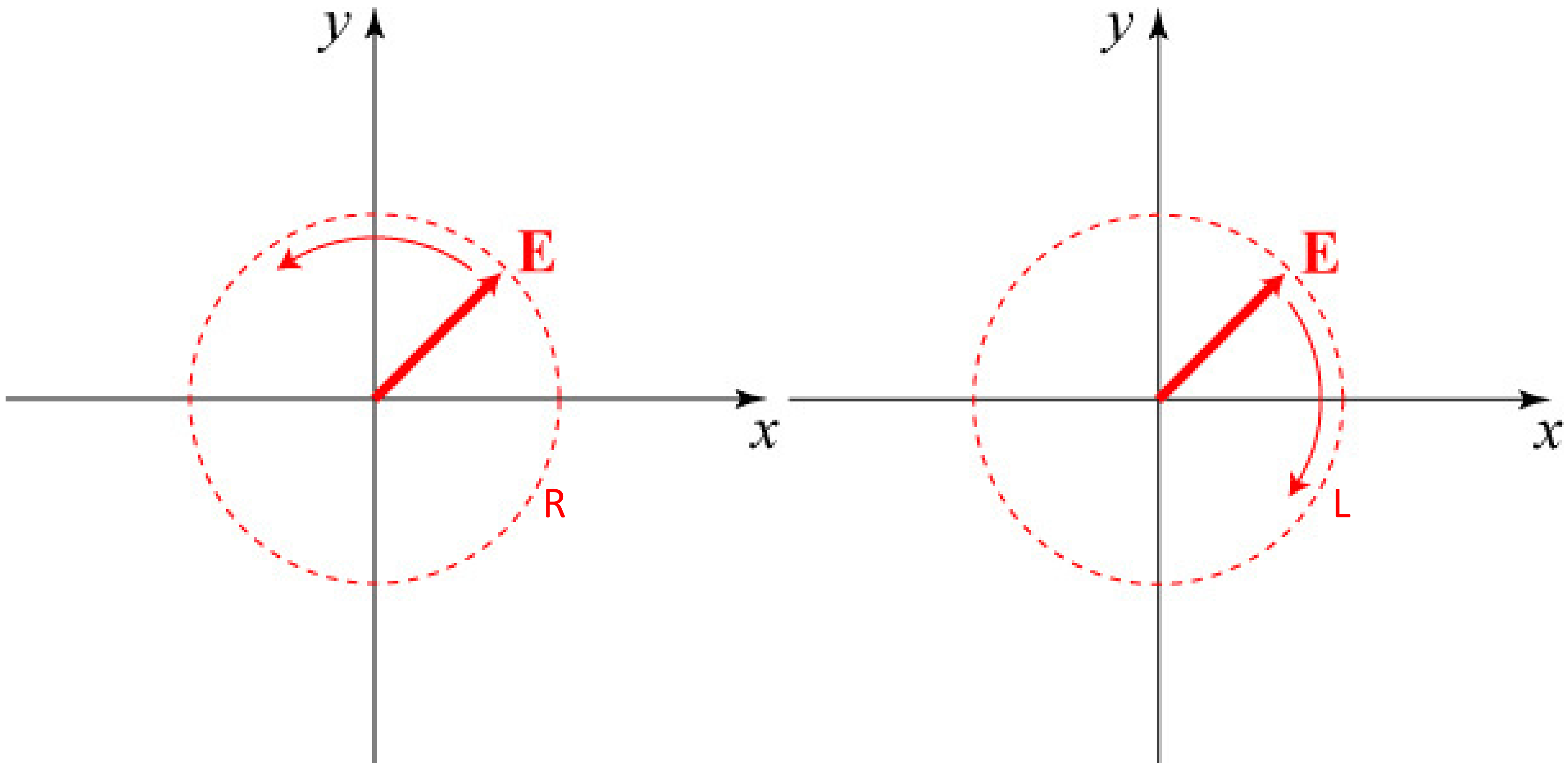
Polarization Animation



Animation: Linear, Circular and Elliptical Polarization Animation in a Single Shot

Polarization Characterization

It is also true that any type of polarization can be represented by a righthand circular and a left-hand circular polarizations (E_L, E_R).



Polarization Characterization

The polarization of any field can be represented by a set of two orthogonal linearly polarized fields.

Assume locally a far-field wave propagates along the z-axis.

The far-zone field vectors have only transverse components

Phasor form

$$\mathbf{e} = \mathbf{e}_x + \mathbf{e}_y \quad \Rightarrow \quad \mathbf{E} = \mathbf{E}_x + \mathbf{E}_y$$

$$\mathbf{e}_x = E_x \cos(\omega t - \beta z)\hat{\mathbf{x}}$$

$$\mathbf{e}_y = E_y \cos(\omega t - \beta z + \delta_L)\hat{\mathbf{y}}$$

Phasor form $\Rightarrow$

$$\mathbf{E}_x = E_x \hat{\mathbf{x}}$$

$$\mathbf{E}_y = E_y e^{j\delta_L} \hat{\mathbf{y}}$$

Polarization Characterization

At a fixed position (assume $z = 0$)

$$\mathbf{e}(t) = \hat{\mathbf{x}} \cdot E_x \cos \omega t + \hat{\mathbf{y}} \cdot E_y \cos(\omega t + \delta_L)$$

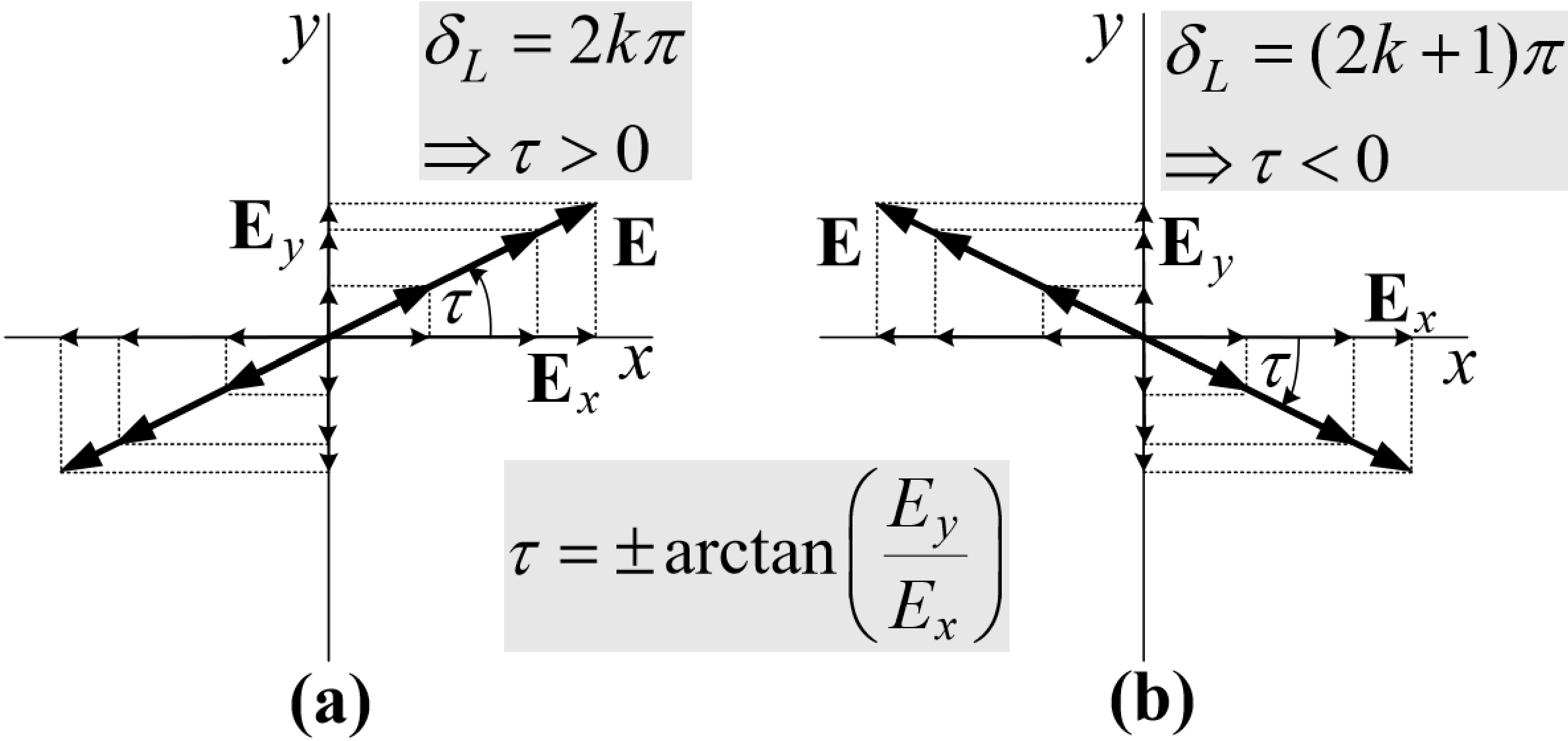
$$\Rightarrow \boxed{\mathbf{E} = \hat{\mathbf{x}} \cdot E_x + \hat{\mathbf{y}} \cdot E_y e^{j\delta_L}}$$

Field Polarization in Terms of Two Orthogonal Linearly Polarized Components

Case 1: Linear polarization: $\delta_L = n\pi, \quad n = 0, 1, 2, \ldots$

$$\mathbf{e}(t) = \hat{\mathbf{x}} \cdot E_x \cos(\omega t) + \hat{\mathbf{y}} \cdot E_y \cos(\omega t \pm n\pi)$$

$$\Rightarrow \boxed{\mathbf{E} = \hat{\mathbf{x}} \cdot E_x \pm \hat{\mathbf{y}} \cdot E_y}$$



$$\tau = \pm \arctan\left(\frac{E_y}{E_x}\right)$$

Case 2: Circular polarization:

$$E_x = E_y = E_m \quad \text{and} \quad \delta_L = \pm\left(\frac{\pi}{2} + n\pi\right), \quad n = 0,1,2,\ldots$$

$$\mathbf{e}(t) = \hat{\mathbf{x}}E_x\cos(\omega t) + \hat{\mathbf{y}}E_y\cos[\omega t \pm (\pi/2 + n\pi)]$$

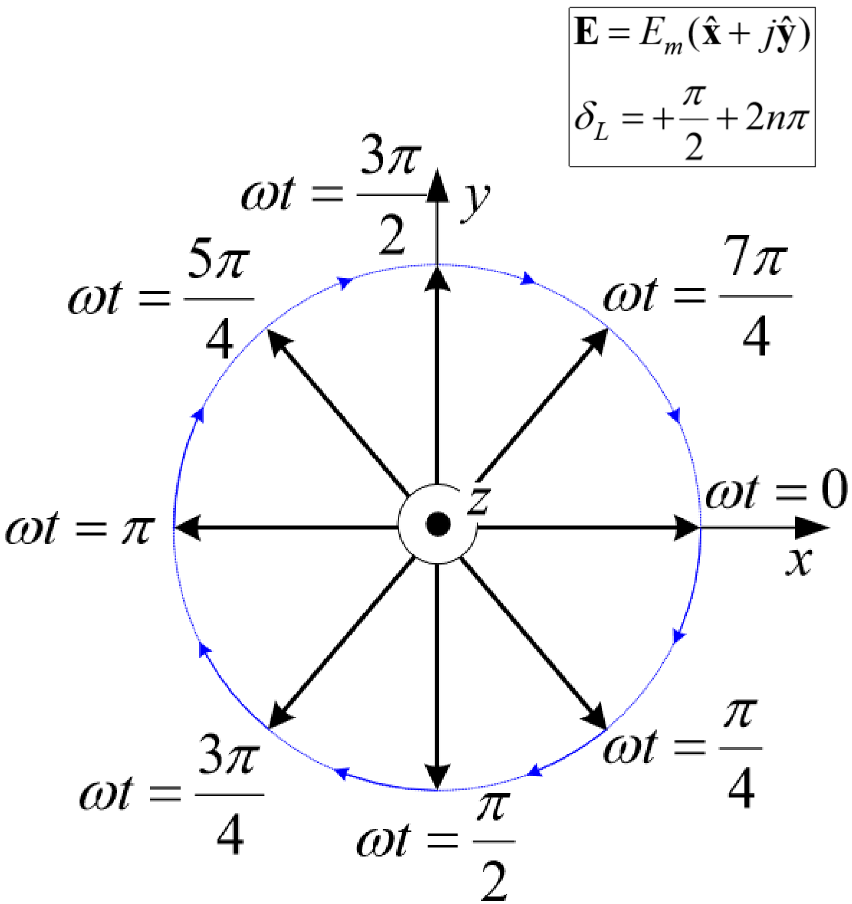
$$\Rightarrow \boxed{\mathbf{E} = E_m(\hat{\mathbf{x}} \pm j\hat{\mathbf{y}})}$$

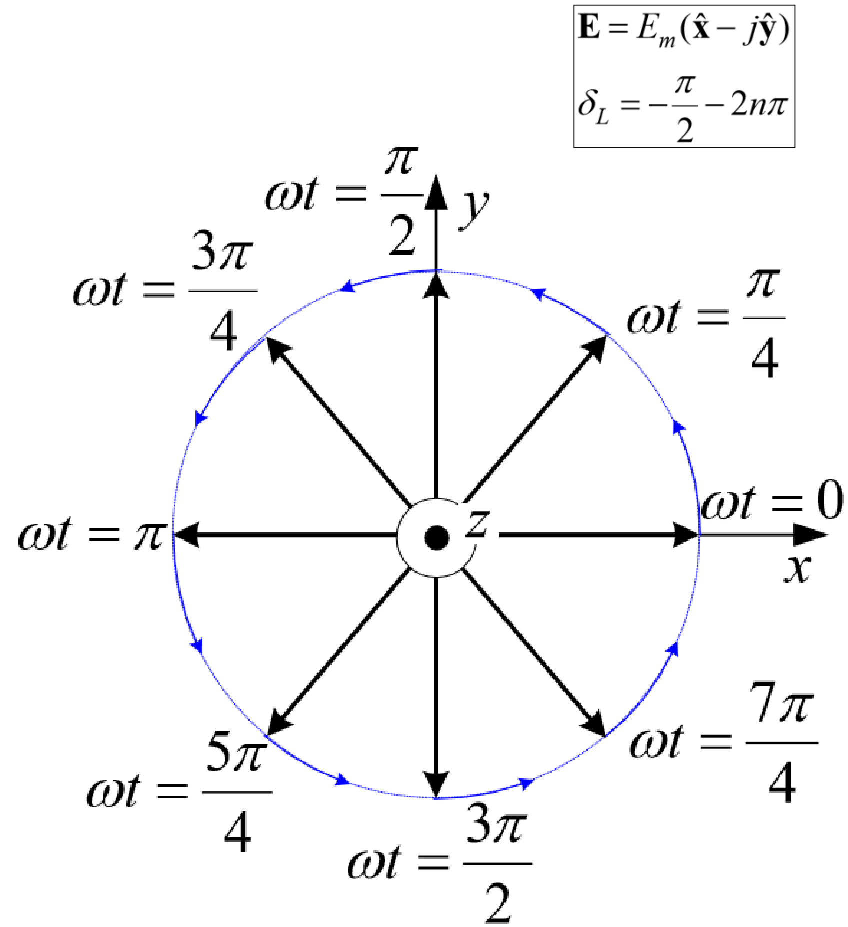
$$\boxed{\begin{aligned} &\mathbf{E} = E_m(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) \\ &\delta_L = +\frac{\pi}{2} + 2n\pi \end{aligned}}$$

$$\boxed{\begin{aligned} &\mathbf{E} = E_m(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) \\ &\delta_L = -\frac{\pi}{2} - 2n\pi \end{aligned}}$$

<u>**Case 2**</u>: Circular polarization:

wave propagates along $+\hat{\mathbf{z}}$



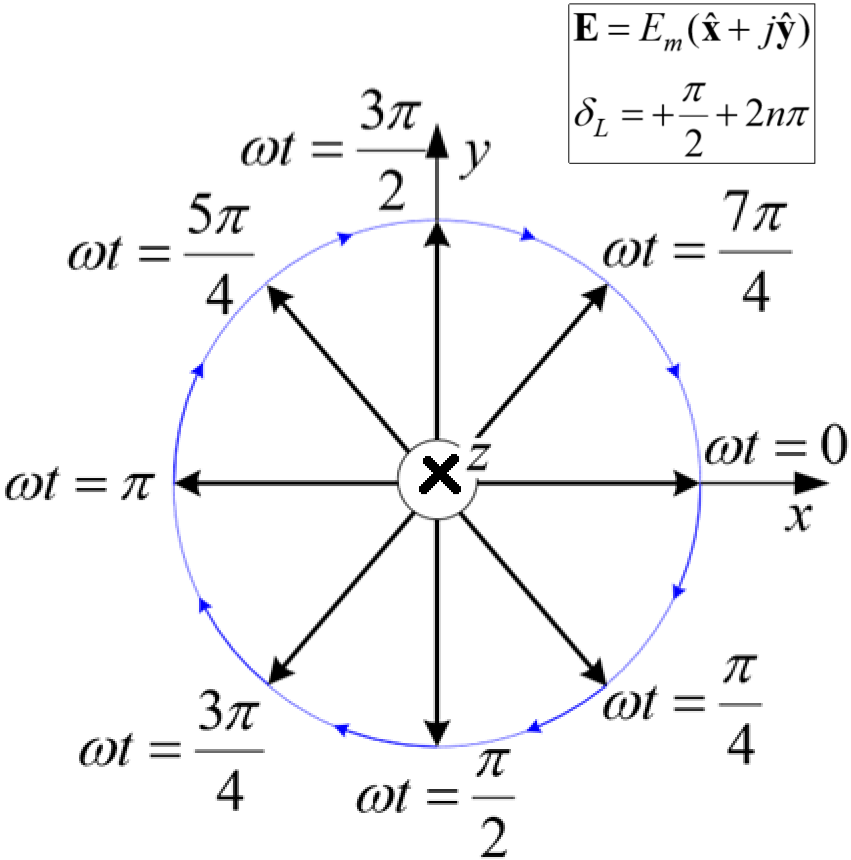


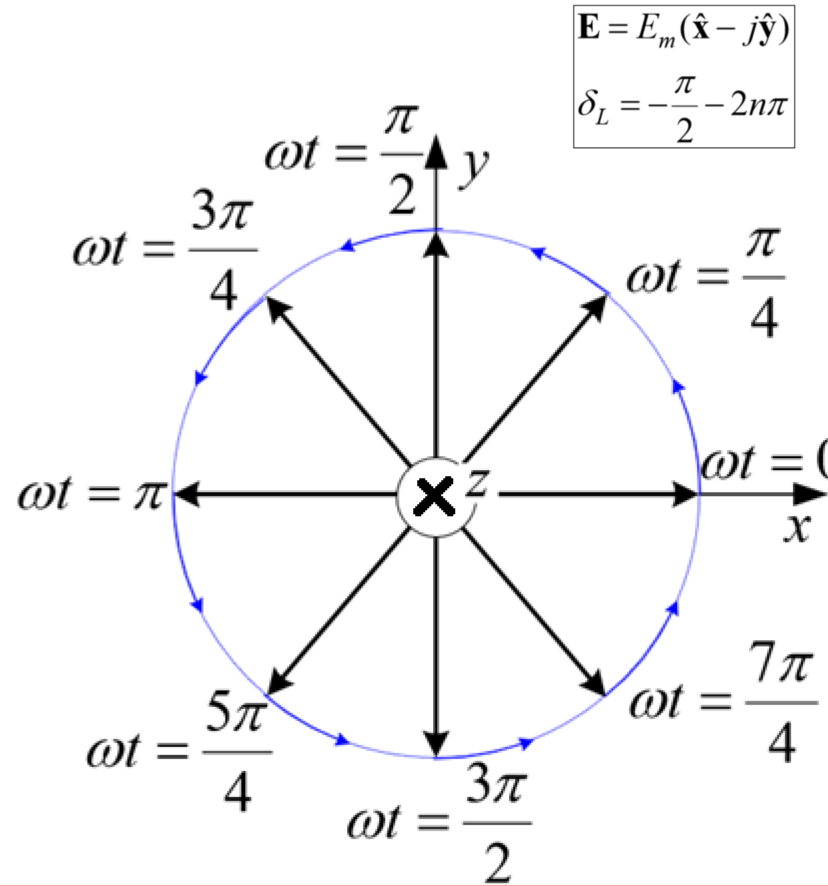
If $+\hat{\mathbf{z}}$ is the direction of propagation: ***counterclockwise (CCW)*** or ***left-hand*** **(LH)** polarization

If $+\hat{\mathbf{z}}$ is the direction of propagation: ***clockwise (CW)*** or ***right-hand*** **(RH)** polarization

Case 2: Circular polarization:

wave propagates along $-\hat{\mathbf{z}}$





If $-\hat{\mathbf{z}}$ is the direction of propagation: ***clockwise (CW)*** or ***right-hand (RH)*** polarization

If $-\hat{\mathbf{z}}$ is the direction of propagation: ***counterclockwise (CCW)*** or ***lefthand (LH)*** polarization

Case 2: Circular polarization:

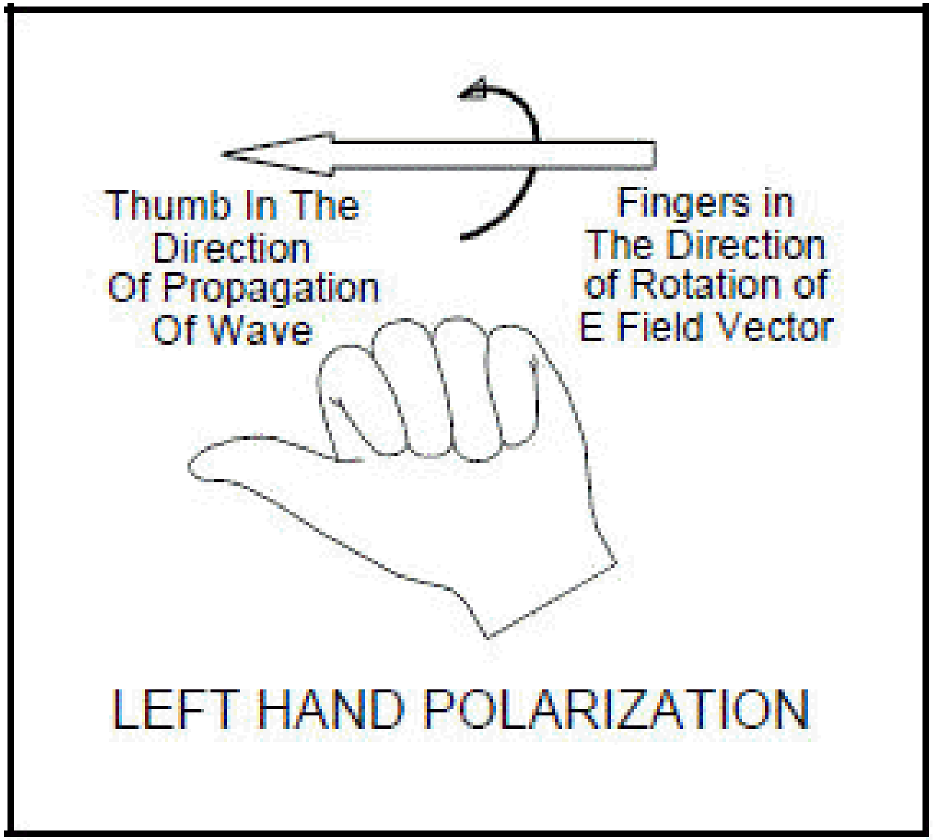


Figure 4. Left Hand Polarization

$$\boxed{\begin{aligned}\mathbf{E} &= E_m(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) \\ \delta_L &= +\frac{\pi}{2} + 2n\pi\end{aligned}}$$

Handedness from Source/Transmitter view

Using this convention, polarization is defined from the point of view of the **source.**

left or right handedness determined by pointing one's left or right thumb **away** from the source, in the **same** direction that the wave is propagating

matching the curling of one's fingers to the direction of the temporal rotation of the field at a given point in space.

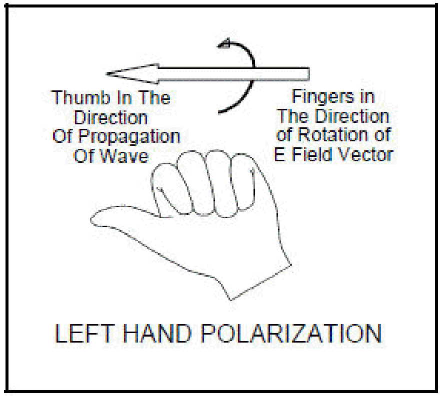
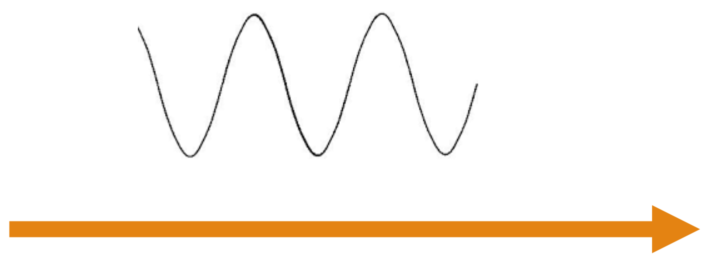


Figure 4. Left Hand Polarization

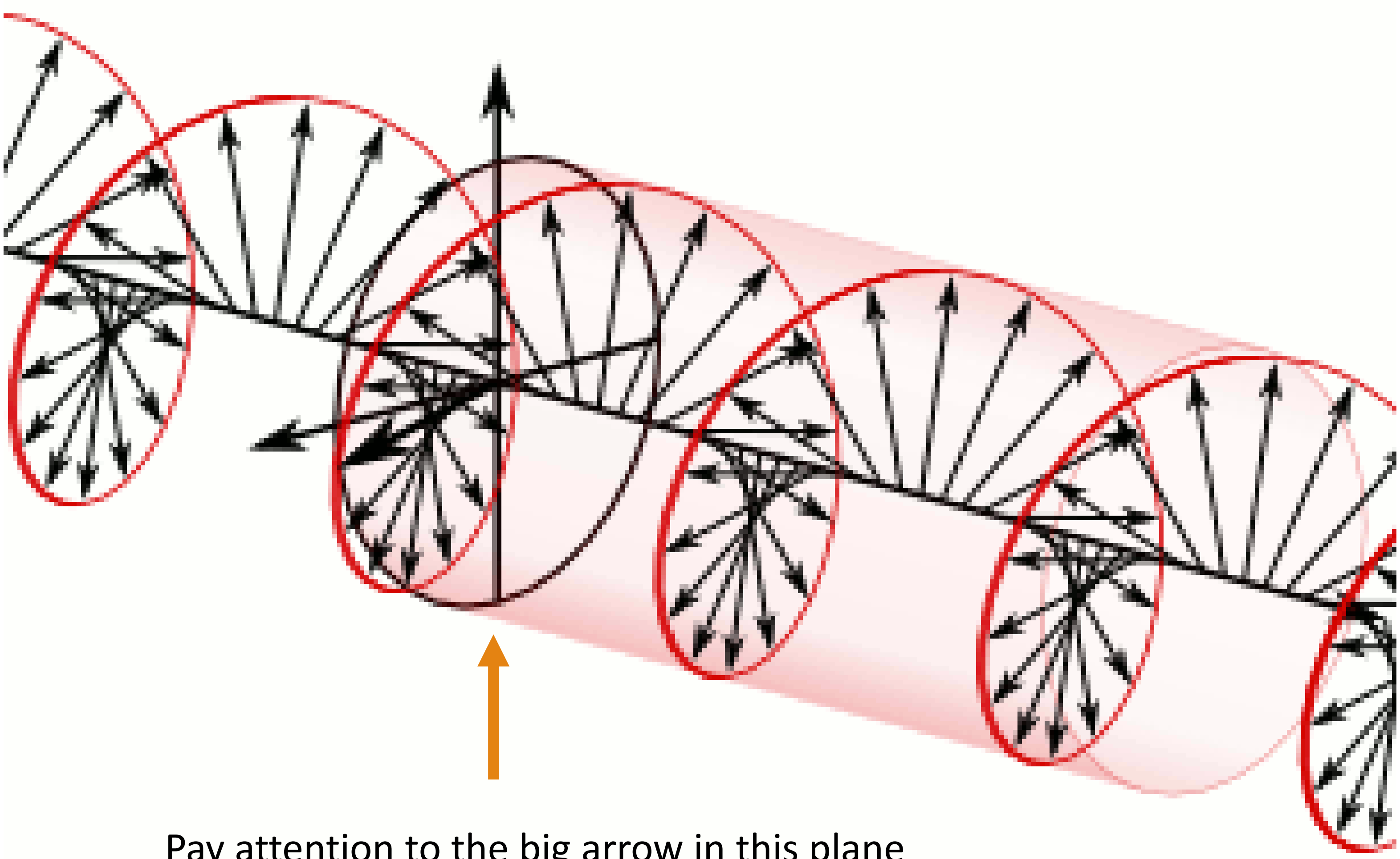
Handedness from Source/Transmitter view

Using this convention, polarization is defined from the point of view of the source.

When determining if the wave is clockwise or anti-clockwise circularly polarized, one again takes the point of view of the **source**, and while looking **away** from the source and in the **same** direction of the wave’s propagation, one observes the direction of the field’s temporal rotation.



Is this a left-hand circularly polarized (LHCP) or right-hand circularly polarized (RHCP) wave?



Pay attention to the big arrow in this plane

Case 3: Elliptic polarization

- the most general type of polarization
- obtained for any phase difference δ_L
- obtained for any ratio (E_x/E_y)
- linear and the circular polarizations are special cases
- In practice, however, the term elliptical polarization is used to indicate polarizations <u>other than linear or circular</u>.

Case 3: Elliptic polarization

$$\mathbf{e}(t) = \hat{\mathbf{x}} E_x \cos \omega t + \hat{\mathbf{y}} E_y \cos(\omega t + \delta_L)$$

$$\Rightarrow \mathbf{E} = \hat{\mathbf{x}} E_x + \hat{\mathbf{y}} E_y e^{j\delta_L}$$

It can be shown that the trace of the time-dependent vector is an ellipse

elliptical polarization can be right-handed or left-handed, depending on the relation between the direction of propagation and the sense of rotation.

Case 3: Elliptic polarization characterization parameters

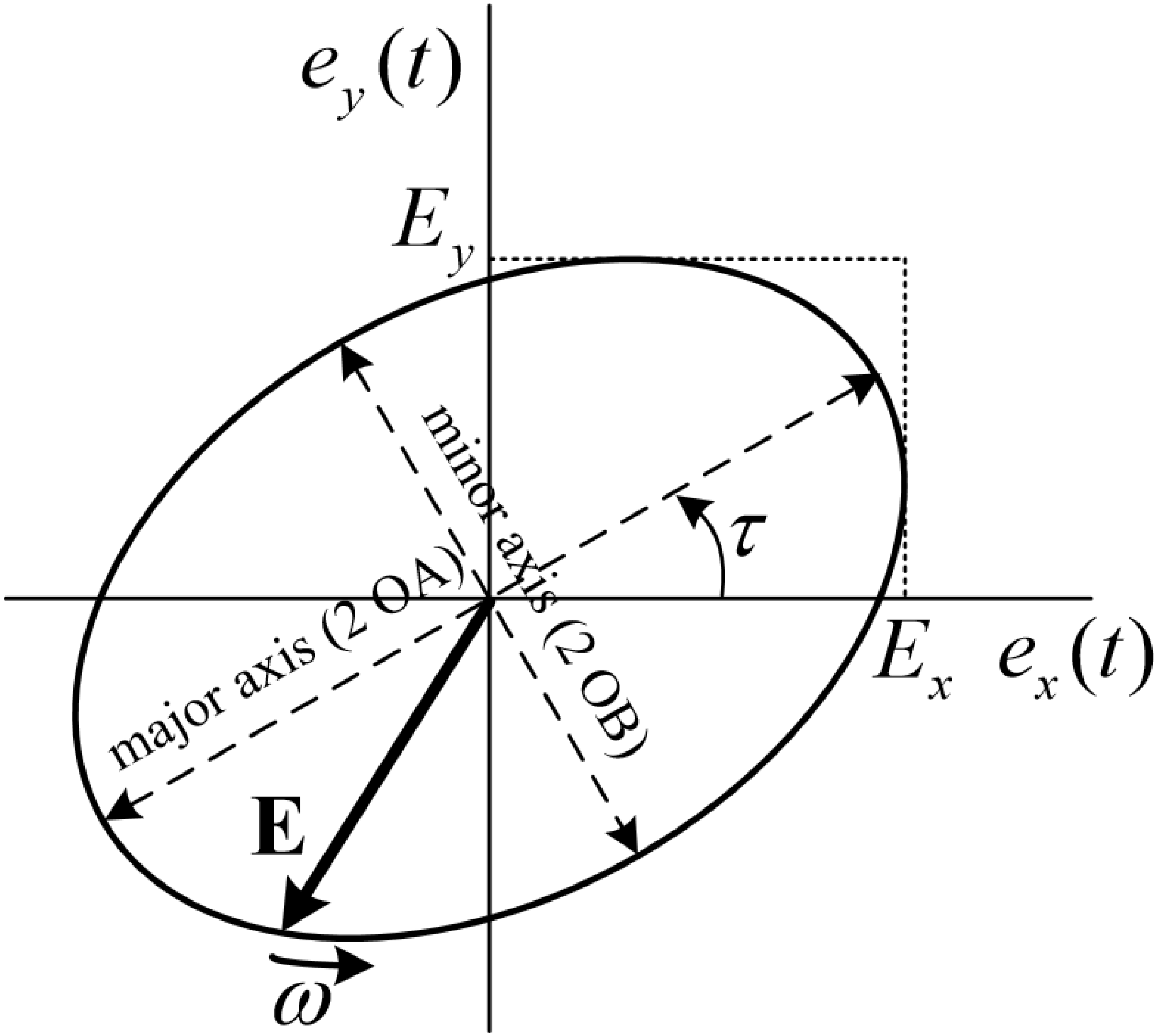
major axis (2 OA)

minor axis (2 OB)

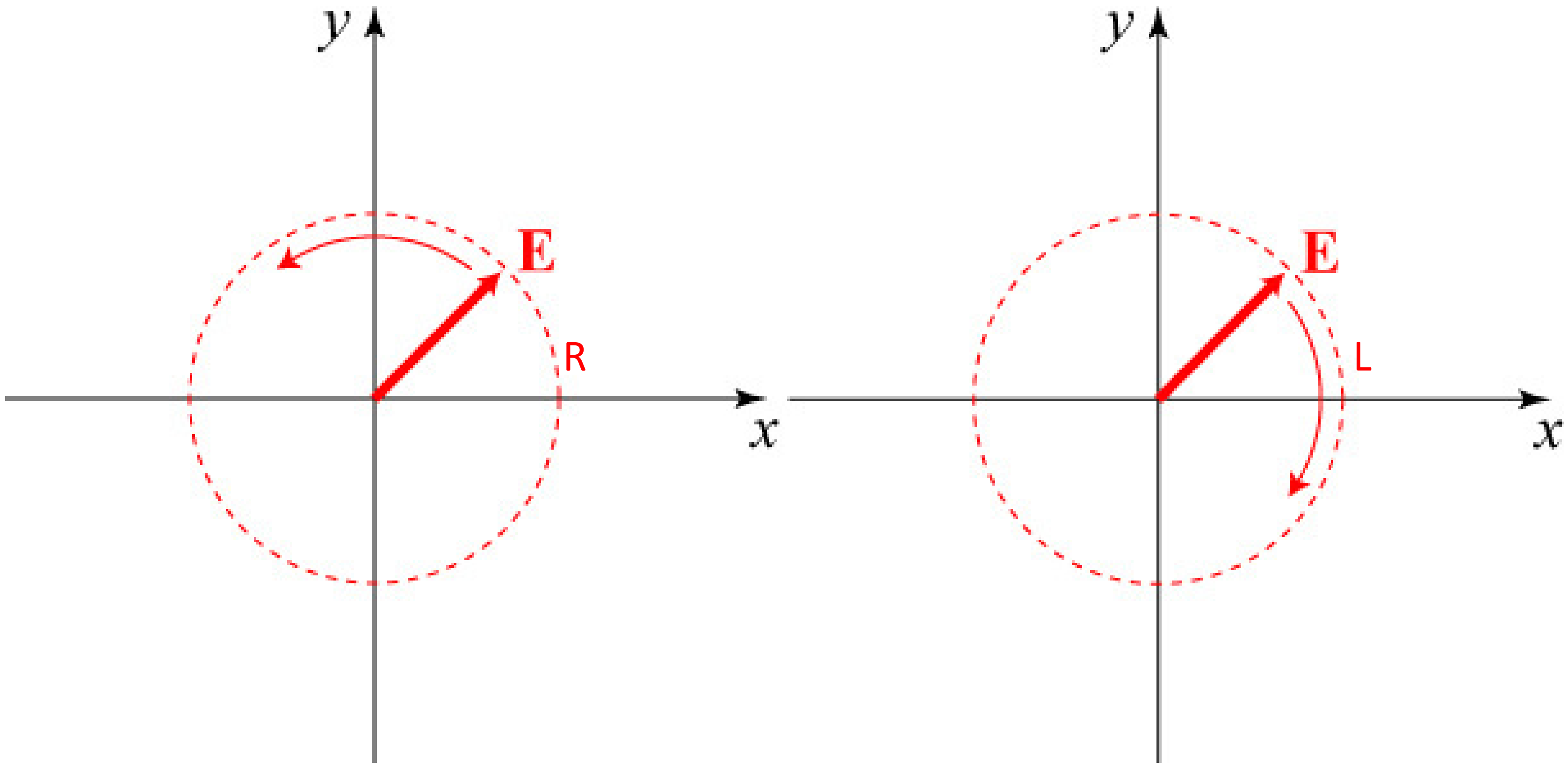
tilt angle τ

Axial ratio AR

AR=OA/OB

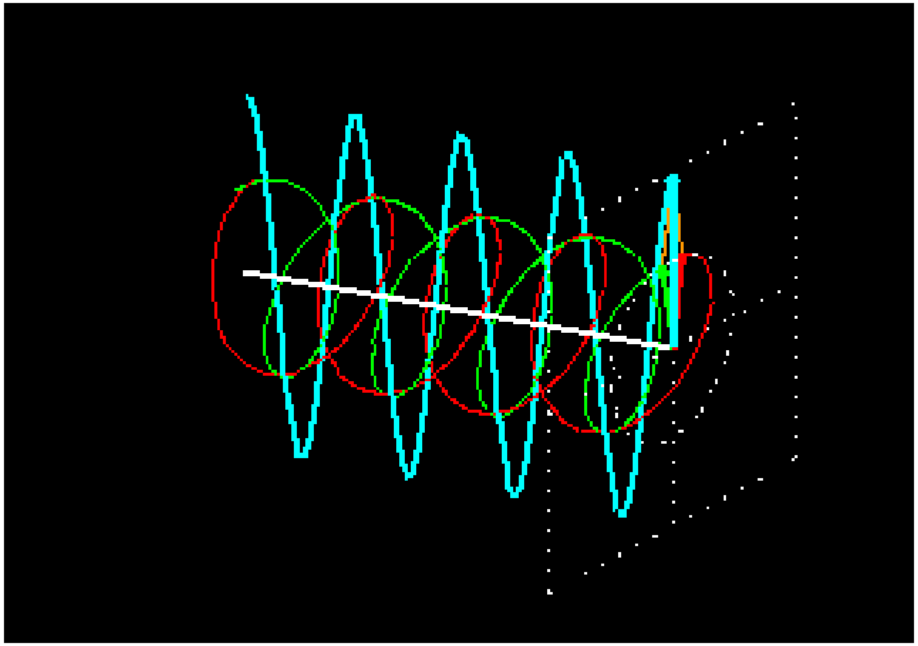


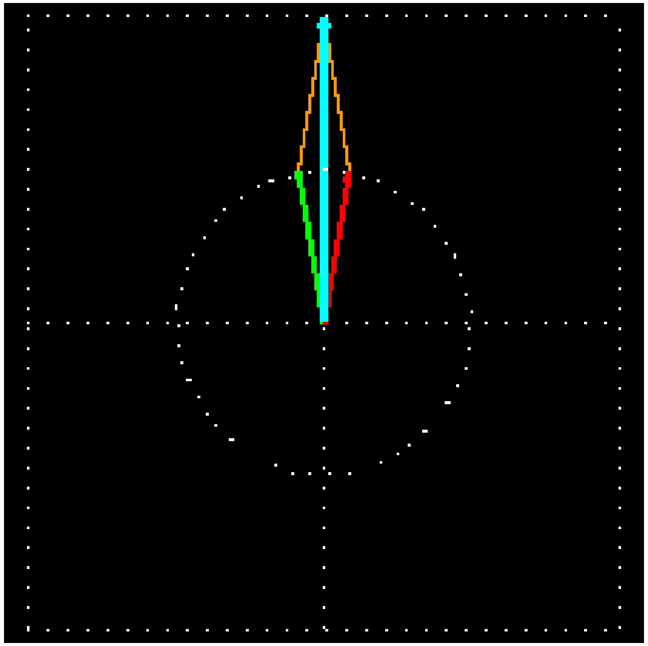
Polarization Characterization



Field Polarization in Terms of Two Circularly Polarized Components

Field Polarization in Terms of Two Circularly Polarized Components





Any linearly polarized light wave can be obtained as a superposition of a left circularly polarized and a right circularly polarized light wave, whose amplitudes are identical.

Field Polarization in Terms of Two Circularly Polarized Components

- less intuitive than linear polarized components
- but it is actually more useful in the calculation of the polarization ellipse parameters
- the total field phasor represented as the superposition of two circularly polarized waves, one right-handed and the other left-handed

Field Polarization in Terms of Two Circularly Polarized Components

Consider TEM Wave moving in -Z direction

$$\mathbf{E} = E_R(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) + E_L(\hat{\mathbf{x}} - j\hat{\mathbf{y}})$$

E_R and E_L are, in general, complex phasors

Relative phase difference $\delta_C = \phi_R - \phi_\mathrm{L}$

$$\mathbf{E} = e_R e^{j\delta_C}(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) + e_L(\hat{\mathbf{x}} - j\hat{\mathbf{y}})$$

e_R and e_L are real numbers

Linear and circular-component conversion

$$\mathbf{E} = \hat{\mathbf{x}}\underbrace{(E_R + E_L)}_{E_x} + \hat{\mathbf{y}}\underbrace{j(E_R - E_L)}_{E_y}$$

$$\Rightarrow \left|\begin{array}{l} E_x = E_R + E_L \\ E_y = j(E_R - E_L) \end{array}\right.$$

$$\Rightarrow \left|\begin{array}{l} E_R = 0.5(E_x - jE_y) \\ E_L = 0.5(E_x + jE_y). \end{array}\right.$$

Polarization Vector

the normalized phasor of the electric field vector. It is a complex-valued vector of unit magnitude, i.e.,

$$\hat{\boldsymbol{\rho}}_L \cdot \hat{\boldsymbol{\rho}}_L^* = 1$$

$$\boxed{\hat{\boldsymbol{\rho}}_L = \frac{\mathbf{E}}{E_m} = \hat{\mathbf{x}}\frac{E_x}{E_m} + \hat{\mathbf{y}}\frac{E_y}{E_m}e^{j\delta_L}, \quad E_m = \sqrt{E_x^2 + E_y^2}}$$

Polarization Vector

Case 1: Linear polarization (the polarization vector is real-valued.)

$$\hat{\boldsymbol{\rho}} = \hat{\mathbf{x}}\frac{E_x}{E_m} \pm \hat{\mathbf{y}}\frac{E_y}{E_m}, \quad E_m = \sqrt{E_x^2 + E_y^2}$$

Case 2: Circular polarization (the polarization vector is complex-valued. E_x=E_y)

$$\hat{\boldsymbol{\rho}}_L = \frac{1}{\sqrt{2}}\left(\hat{\mathbf{x}} \pm j\hat{\mathbf{y}}\right), \quad E_m = \sqrt{2}E_x = \sqrt{2}E_y$$

Polarization Ratio

the ratio of the phasors of the two orthogonal polarization components. In general, it is a complex number:

$$\boxed{\tilde{r}_L = r_L e^{\delta_L} = \frac{\tilde{E}_y}{\tilde{E}_x} = \frac{E_y e^{j\delta_L}}{E_x} \quad \text{or} \quad \tilde{r}_L = \frac{\tilde{E}_V}{\tilde{E}_H}}$$

Polarization Ratio

circular-component representation, the polarization ratio is defined as

$$\boxed{\tilde{r}_C = r_C e^{j\delta_C} = \frac{\tilde{E}_R}{\tilde{E}_L}}$$

the axial ratio of the polarization ellipse AR is

$$\boxed{AR = \left|\frac{r_C + 1}{r_C - 1}\right|}$$

Polarization Ratio

Tilt angle respect to the y (vertical) axis

$$\boxed{\tau_V = \delta_C / 2 + n\pi}, \; n = 0, \pm 1, \ldots$$

relation between the phase difference δ_C of the circular-component representation and the linear polarization ratio

$$\delta_C = \arctan\left(\frac{2r_L}{1 - r_L^2} \cos \delta_L \right)$$

Axial Ratio

$$AR = \frac{r_C + 1}{r_C - 1} = \sqrt{\frac{1 + r_L^2 + \sqrt{1 + r_L^4 + 2r_L^2 \cos(2\delta_L)}}{1 + r_L^2 - \sqrt{1 + r_L^4 + 2r_L^2 \cos(2\delta_L)}}}$$

r_c is circular polarization ratio,
r_L is linear polarization ratio

Antenna Polarization

polarization of a transmitting antenna:
the polarization of its radiated wave in the far zone.

polarization of a receiving antenna:
the polarization of a plane wave, incident from a given direction, which, for a given power flux density, results in maximum available power at the antenna terminals.

<u>By convention, the antenna polarization is defined by the polarization vector of the wave it transmits.</u> The antenna polarization vector is determined according to the **definition of antenna polarization** in a **transmitting mode**.

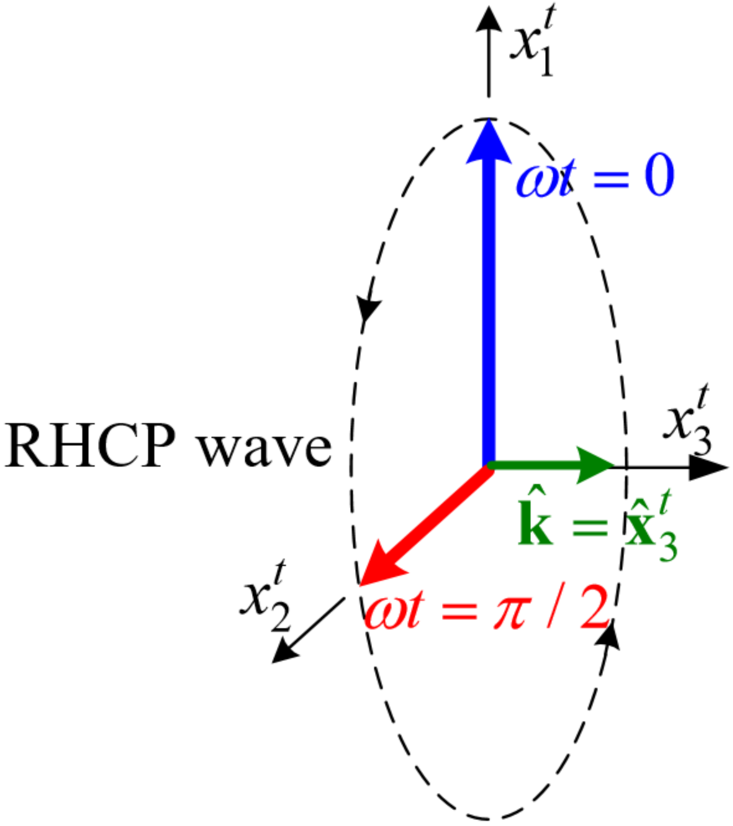
Antenna Polarization

polarization vector of a wave in the coordinate system of the transmitting antenna is represented by its complex conjugate in the coordinate system of the receiving antenna

$$\hat{\boldsymbol{\rho}}_w^r = (\hat{\boldsymbol{\rho}}_w^t)^*$$

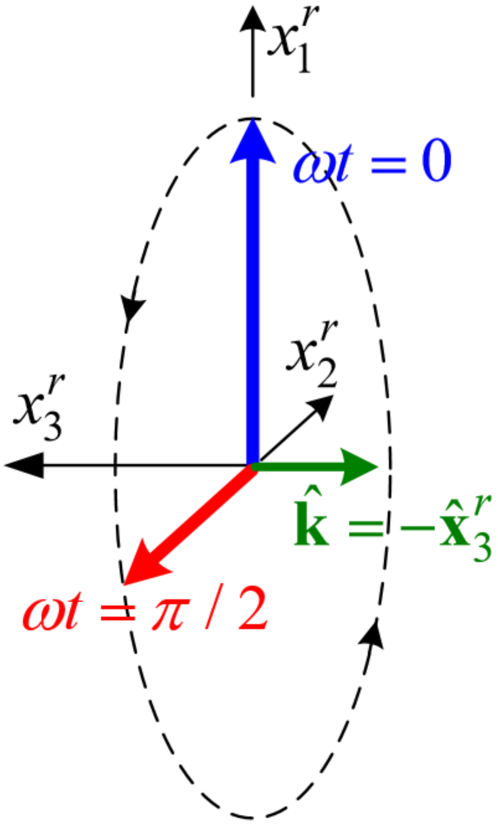
polarization vector for a linearly polarized wave is real. Conjugate is only important for circular and elliptical polarized waves

Antenna Polarization



$$\hat{\boldsymbol{\rho}}_w^t = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}}_1^t - j\hat{\mathbf{x}}_2^t)$$

Coordinate of transmission antenna



$$\hat{\boldsymbol{\rho}}_w^r = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}}_1^r + j\hat{\mathbf{x}}_2^r)$$

Coordinate of receiving antenna

Antenna Polarization

This is illustrated in the previous figure with a right-hand CP wave. Let the coordinate triplet (x_1^t, x_2^t, x_3^t) represent the coordinate system of the transmitting antenna while (x_1^r, x_2^r, x_3^r) represents that of the receiving antenna. In antenna analysis, the plane of polarization is usually given in spherical coordinates by $(\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2) \equiv (\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\varphi}})$ and the third axis obeys $\hat{\mathbf{x}}_1 \times \hat{\mathbf{x}}_2 = \hat{\mathbf{x}}_3$, i.e., $\hat{\mathbf{x}}_3 = \hat{\mathbf{r}}$. Since the transmitting and receiving antennas face each other, their coordinate systems are oriented so that $\hat{\mathbf{x}}_3^t = -\hat{\mathbf{x}}_3^r$ (i.e., $\hat{\mathbf{r}}^r = -\hat{\mathbf{r}}^t$). If we align the axes $\hat{\mathbf{x}}_1^t$ and $\hat{\mathbf{x}}_1^r$, then $\hat{\mathbf{x}}_2^t = -\hat{\mathbf{x}}_2^r$ must hold. This changes the sign in the imaginary part of the wave polarization vector.

Polarization Loss Factor (Polarization Efficiency)

polarization mismatch between receiving antenna and the incident wave is characterized by polarization loss factor (PLF)

$$\text{PLF} = |\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a|^2$$

polarization vector of the receiving antenna

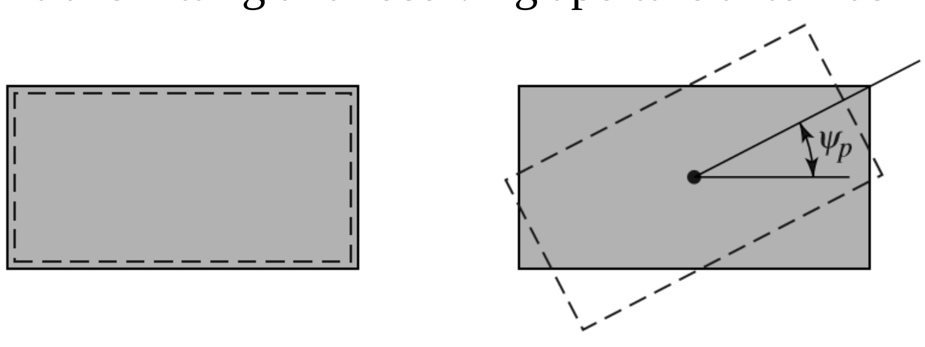
Incident wave polarization vector

If incident wave is $\mathbf{E}^i = E_m^i \hat{\boldsymbol{\rho}}_a^*$ the maximum possible received power at the antenna terminals is produced. (PLF=1)

In general $0 \leq \text{PLF} \leq 1$

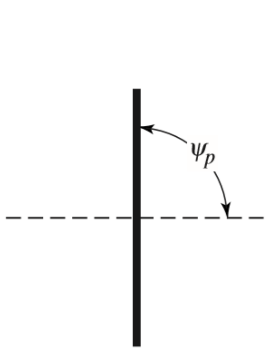
Polarization Loss Factor (Polarization Efficiency)

PLF for transmitting and receiving aperture antennas



$\text{PLF} = \lvert \hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a \rvert^2 = 1$	$\text{PLF} = \lvert \hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a \rvert^2 = \cos^2 \psi_p$	$\text{PLF} = \lvert \hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a \rvert^2 = 0$
(aligned)	(rotated)	(orthogonal)

PLF for transmitting and receiving linear wire antennas



$\text{PLF} = \lvert \hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a \rvert^2 = 1$	$\text{PLF} = \lvert \hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a \rvert^2 = \cos^2 \psi_p$	$\text{PLF} = \lvert \hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a \rvert^2 = 0$
(aligned)	(rotated)	(orthogonal)

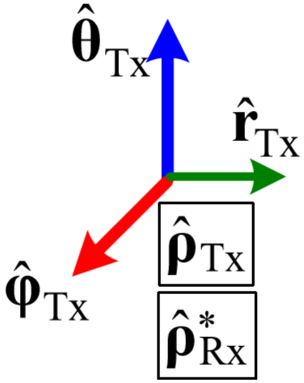
Polarization Loss Factor (Polarization Efficiency)

In a communication link, the PLF has to be expressed by the polarization vectors of the transmitting and receiving antennas

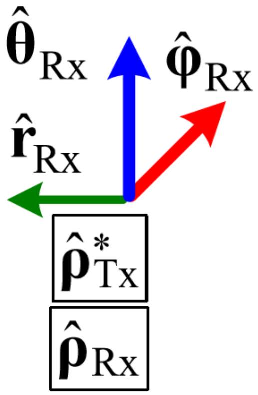
$$\hat{\boldsymbol{\rho}}_{\mathrm{Tx}} \text{ and } \hat{\boldsymbol{\rho}}_{\mathrm{Rx}}$$

PLF is then calculated as

$$\mathrm{PLF} = \left|\hat{\boldsymbol{\rho}}^*_{\mathrm{Tx}} \cdot \hat{\boldsymbol{\rho}}_{\mathrm{Rx}}\right|^2 \quad \text{or} \quad \mathrm{PLF} = \left|\hat{\boldsymbol{\rho}}_{\mathrm{Tx}} \cdot \hat{\boldsymbol{\rho}}^*_{\mathrm{Rx}}\right|^2$$



transmitting antenna



receiving antenna

Examples

Example 5.1. The electric field of a linearly polarized EM wave is

$$\mathbf{E}^i = \hat{\mathbf{x}} \cdot E_m(x, y)e^{-j\beta z}.$$

It is incident upon a linearly polarized antenna whose polarization is

$$\mathbf{E}_a = (\hat{\mathbf{x}} + \hat{\mathbf{y}}) \cdot E(r, \theta, \varphi).$$

Find the PLF.

$$\mathrm{PLF} = \left| \hat{\mathbf{x}} \cdot \frac{1}{\sqrt{2}} (\hat{\mathbf{x}} + \hat{\mathbf{y}}) \right|^2 = \frac{1}{2}$$

$$\mathrm{PLF}_{[\mathrm{dB}]} = 10 \log_{10} 0.5 = -3 \text{ dB}$$

Examples

Example 5.2. A transmitting antenna produces a far-zone field, which is RH circularly polarized. This field impinges upon a receiving antenna, whose polarization (in transmitting mode) is also RH circular. Determine the PLF.

RHCP

$$\mathbf{E}^{far} = E_m\left[\hat{\boldsymbol{\theta}}\cdot\cos\omega t + \hat{\boldsymbol{\varphi}}\cdot\cos(\omega t - \pi/2)\right]$$

Polarization vector for transmission and receiving antenna (transmission mode)

$$\hat{\boldsymbol{\rho}}_{\mathrm{Tx}} = \frac{\hat{\boldsymbol{\theta}} - j\hat{\boldsymbol{\varphi}}}{\sqrt{2}} \quad \text{and} \quad \hat{\boldsymbol{\rho}}_{\mathrm{Rx}} = \frac{(\hat{\boldsymbol{\theta}}' - j\hat{\boldsymbol{\varphi}}')}{\sqrt{2}}$$

Examples

Example 5.2. A transmitting antenna produces a far-zone field, which is RH circularly polarized. This field impinges upon a receiving antenna, whose polarization (in transmitting mode) is also RH circular. Determine the PLF.

$$\mathrm{PLF} = \left|\hat{\boldsymbol{\rho}}_{\mathrm{Tx}}^{*} \cdot \hat{\boldsymbol{\rho}}_{\mathrm{Rx}}\right|^2 = \frac{|(\hat{\boldsymbol{\theta}}' + j\hat{\boldsymbol{\varphi}}')(\hat{\boldsymbol{\theta}}' - j\hat{\boldsymbol{\varphi}}')|^2}{4} = 1$$

$$\mathrm{PLF}_{[\mathrm{dB}]} = 10\log_{10} 1 = 0\,.$$

There is no polarization loss.

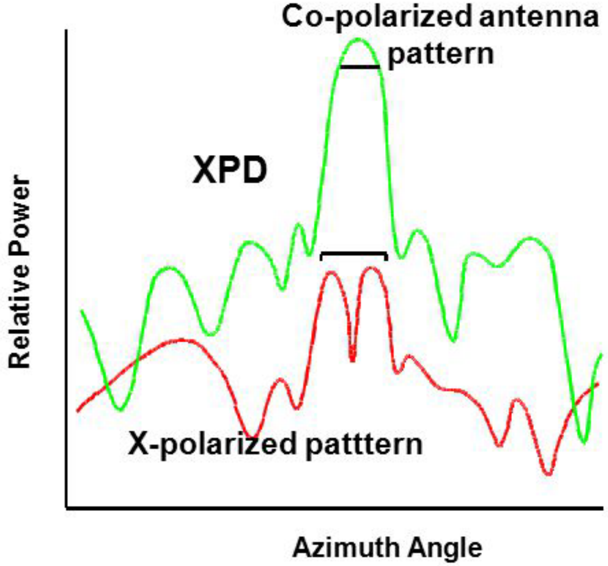
Cross Polarization Loss

video

Co-polarization vs. Cross-Polarization

Co-Polarization
The desired polarization (the *main polarization)* (COPOL)

Cross-Polarization
The undesired orthogonal polarization (CROSSPOL).



A well designed antenna will have **CROSSPOL** components at least **20 dB** below the **COPOL** in the main-beam region, and **5 to 10 dB below in the side lobe regions**.

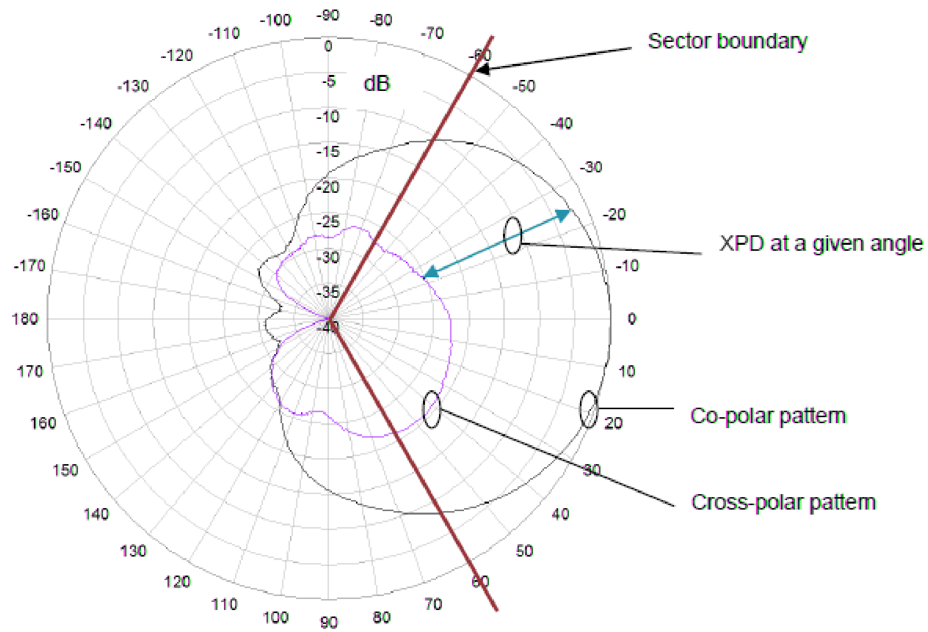
Cross-polar Discrimination

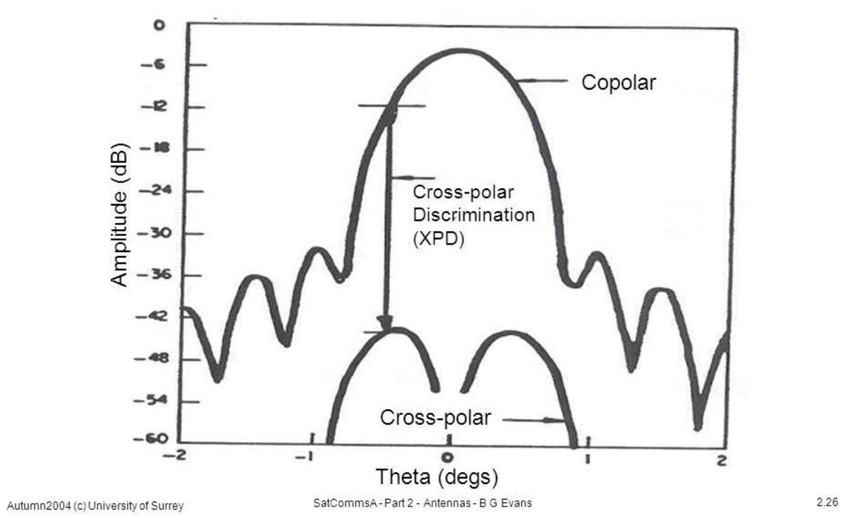
antenna's ability to **maintain radiated or received polarization purity** between horizontally and vertically polarized signals. This is called **cross-polar discrimination,** or XPD.

In transmit mode, XPD is the proportion of signal that is transmitted in the orthogonal polarization to that which is required. In receive mode, it is the antenna's ability to maintain the incident signal's polarization purity.

Cross-polar Discrimination

Definition of Cross Polar Discrimination
The cross polar discrimination is defined as a ratio of the copolar component of the specified polarization compared to the orthogonal cross-polar component over the sector or beamwidth angle. beamwidth of the co-polarized main beam.”





Friis Transmission Equation

essential in the analysis and design of wireless communication systems

relates the power fed to the transmitting antenna and the power received by the receiving antenna

antennas separated by a sufficiently large distance

i.e., they are in each other's far zones.

$$R \gg 2D_{\max}^2 / \lambda$$

Friis Transmission Equation

A transmitting antenna produces power density W_t in the direction (θ_t, φ_t)

$$W_t = \frac{P_t}{4\pi R^2} G_t(\theta_t, \varphi_t) = \frac{P_t}{4\pi R^2} e_t D_t(\theta_t, \varphi_t)$$

P_t: power of the transmitter
G_t: gain on given direction
e_t: radiation efficiency of the transmitting antenna
D_t: directivity

Friis Transmission Equation

power at the terminals of the receiving antenna

$$P_r = A_r W_t$$

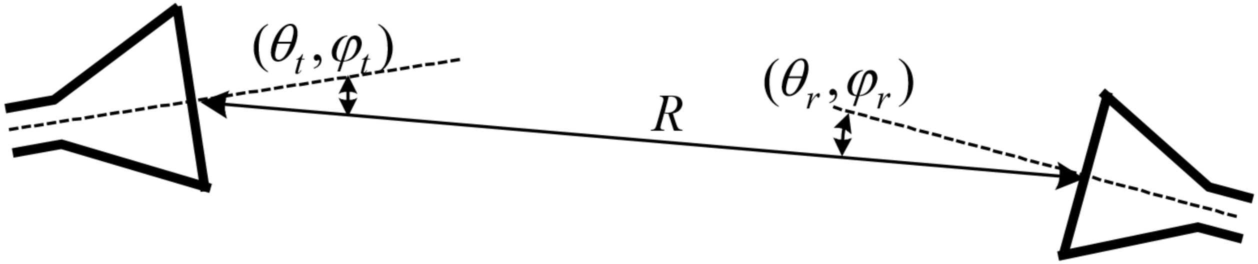
A_r: effective area of receiving antenna

Now include radiation efficiency of the receiving antenna e_r and polarization loss factor PLF

$$P_r = e_r \cdot \mathrm{PLF} \cdot A_r W_t = A_r W_t e_r \,|\, \hat{\boldsymbol{\rho}}_t^* \cdot \hat{\boldsymbol{\rho}}_r \,|^2 ,$$

$$\Rightarrow P_r = \underbrace{D_r(\theta_r, \varphi_r) \cdot \frac{\lambda^2}{4\pi}}_{A_r} \cdot W_t e_r \,|\, \hat{\boldsymbol{\rho}}_t^* \cdot \hat{\boldsymbol{\rho}}_r \,|^2 .$$

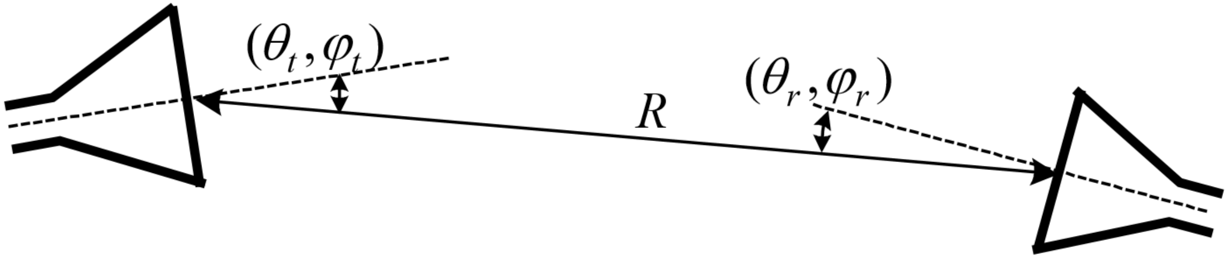
Friis Transmission Equation



received power

$$\Rightarrow P_r = D_r(\theta_r, \varphi_r) \cdot \frac{\lambda^2}{4\pi} \cdot \underbrace{\frac{P_t}{4\pi R^2} e_t D_t(\theta_t, \varphi_t)}_{W_t} \cdot e_r \, | \hat{\boldsymbol{\rho}}_t^* \cdot \hat{\boldsymbol{\rho}}_r |^2$$

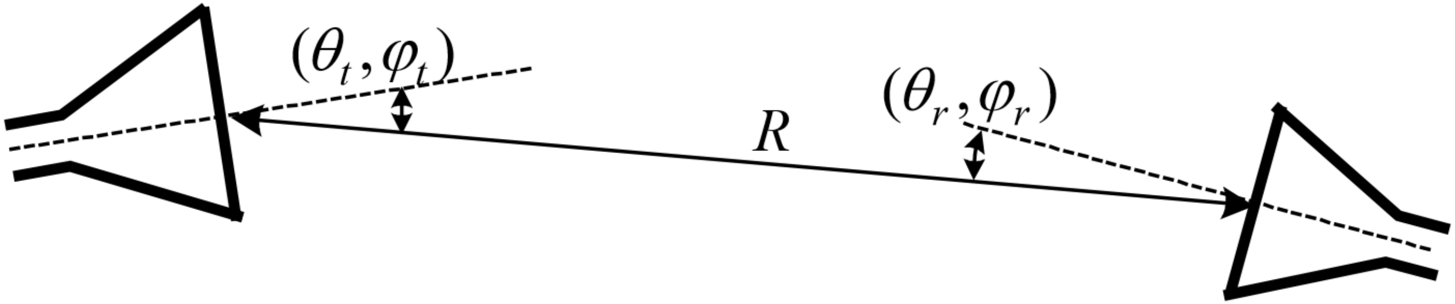
Friis Transmission Equation



ratio of the received to the transmitted power

$$\frac{P_r}{P_t} = e_t e_r \, |\, \hat{\boldsymbol{\rho}}_t^* \cdot \hat{\boldsymbol{\rho}}_r \,|^2 \left(\frac{\lambda}{4\pi R} \right)^2 D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r)$$

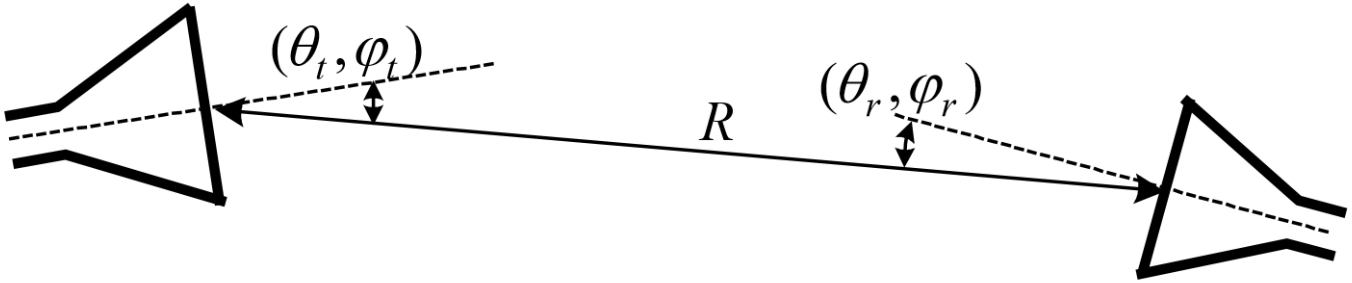
Friis Transmission Equation



Including impedance-mismatch loss

$$\frac{P_r}{P_t} = (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) e_t e_r \, |\hat{\boldsymbol{\rho}}_t^* \cdot \hat{\boldsymbol{\rho}}_r|^2 \left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r)$$

Friis Transmission Equation



For free-space loss factor impedance-matched and polarization-matched transmitting and receiving antennas

$$\frac{P_r}{P_t} = \boxed{\left(\frac{\lambda}{4\pi R}\right)^2} G_t(\theta_t, \varphi_t) G_r(\theta_r, \varphi_r)$$

free-space loss factor

Maximum Range of a Wireless Link

Friis' transmission equation is frequently used to calculate the maximum range at which a wireless link can operate

$$R_{\max}^2 = (1-|\Gamma_t|^2)(1-|\Gamma_r|^2)e_t e_r |\hat{\boldsymbol{\rho}}_t^* \cdot \hat{\boldsymbol{\rho}}_r|^2 \left(\frac{\lambda}{4\pi}\right)^2 \left(\frac{P_t}{P_{r\ \min}}\right) D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r)$$

Effective Isotropically Radiated Power (EIRP)

characterize a transmission system

$$EIRP = P_t G_t e_{\mathrm{TL}}, \ \mathrm{W}$$

or

$$EIRP = 4\pi U_{\max,t}, \ \mathrm{W}$$

e_{TL} the loss efficiency of the transmission line connecting the transmitter to the antenna

- a fictitious amount of power that an isotropic radiator would have to emit in order to produce the peak power density observed in the direction of the maximum radiation
- much greater than the actual power an antenna needs in order to achieve a given amount of radiation intensity in its direction of maximum radiation.

Radar Cross-section (RCS) or Echo Area

The RCS of a target σ is the equivalent area capturing that amount of power, which, when scattered isotropically, produces at the receiver an amount of power density, which is equal to that scattered by the target itself:

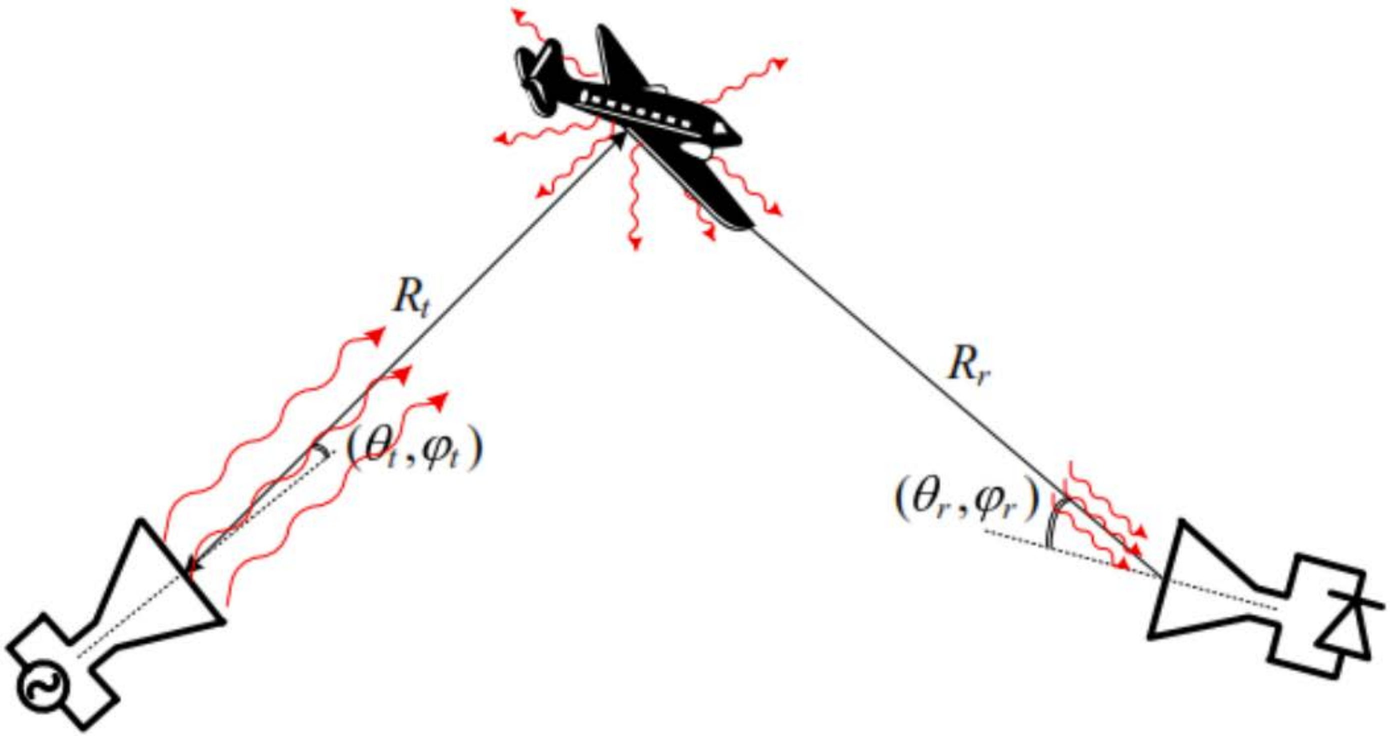
$$\sigma = \lim_{R\to\infty}\left[4\pi R^2 \frac{W_s}{W_i}\right] = \lim_{R\to\infty}\left[4\pi R^2 \frac{|\mathbf{E}_s|^2}{|\mathbf{E}_i|^2}\right], \text{ m}^2.$$

R is the distance from the target, m;
W_i is the incident power density, W/m^2;
W_s is the scattered power density at the receiver, W/m^2.

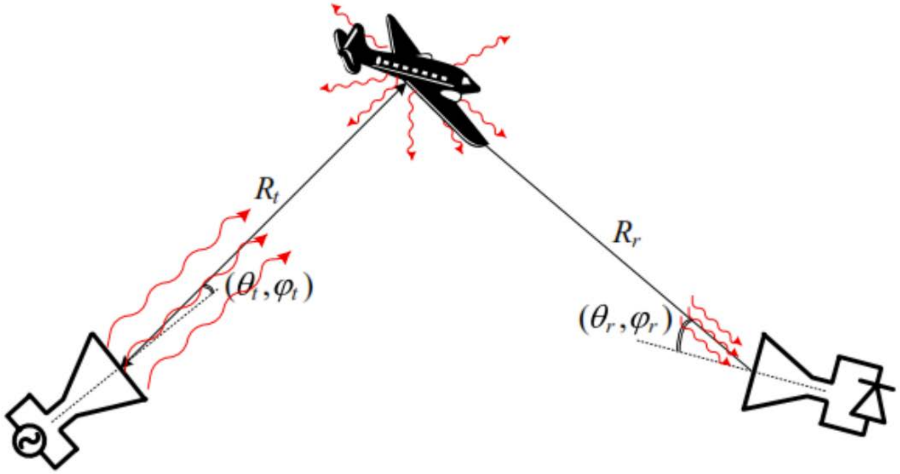
$$\lim_{R\to\infty}\left[\frac{\sigma W_i}{4\pi R^2}\right] = W_s(R)$$

Radar Range Equation

the ratio of the transmitted power (fed to the transmitting antenna) to the received power, after it has been scattered (reradiated) by a target of cross-section σ.



Radar Range Equation



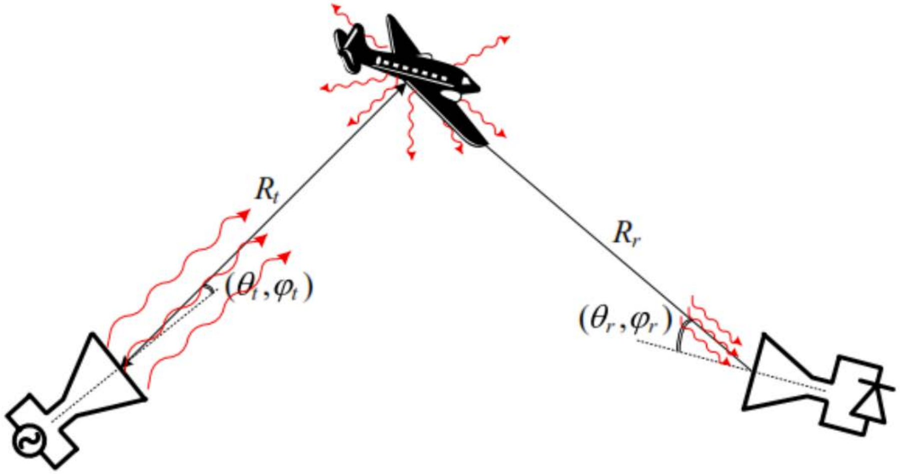
power density of the transmitted wave at the target location

$$W_t = \frac{P_t}{4\pi R_t^2} \cdot G_t(\theta_t, \varphi_t) = \frac{P_t}{4\pi R_t^2} \cdot e_t \cdot D_t(\theta_t, \varphi_t), \quad W/m^2$$

density of the reradiated (scattered) power at the receiving antenna

$$W_r = \frac{P_c}{4\pi R_r^2} = \frac{\sigma \cdot W_t}{4\pi R_r^2} = \sigma \cdot e_t \cdot \frac{P_t \cdot D_t(\theta_t, \varphi_t)}{(4\pi R_t R_r)^2}$$

Radar Range Equation



Re-arranging and including impedance mismatch losses as well as polarization losses, yields the complete radar range equation:

$$\frac{P_r}{P_t} = e_t e_r (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2)|\hat{\boldsymbol{\rho}}_t \cdot \hat{\boldsymbol{\rho}}_r|^2 \sigma \left(\frac{\lambda}{4\pi R_t R_r}\right)^2 \frac{D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r)}{4\pi}$$

For polarization matched loss-free antennas aligned for maximum directional radiation and reception

$$\frac{P_r}{P_t} = \sigma \left(\frac{\lambda}{4\pi R_t R_r}\right)^2 \frac{D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r)}{4\pi}$$

Radar Cross-section (RCS) or Echo Area

- trucks and jumbo jet airliners have large RCS, $\sigma > 100\ m^2$)
- RCS increases also due to sharp metallic or dielectric edges and corners
- stealth military aircraft
  - achieved by careful shaping and coating (with special materials) of the outer surface
  - the materials absorbing EM waves at the radar frequencies (usually S and X bands)
  - RCS smaller than $10^{-4}\ m^2$, comparable or smaller than the RCS of a penny

Boeing 747
RCS = $63 m^2$

F-117 Nighthawk
RCS = $0.003 m^2$

Radar Cross-section (RCS) or Echo Area

RCS of a target can be viewed as a comparison of the strength of the reflected signal from a target to the reflected signal from a perfectly smooth sphere of cross sectional area of 1 m^2

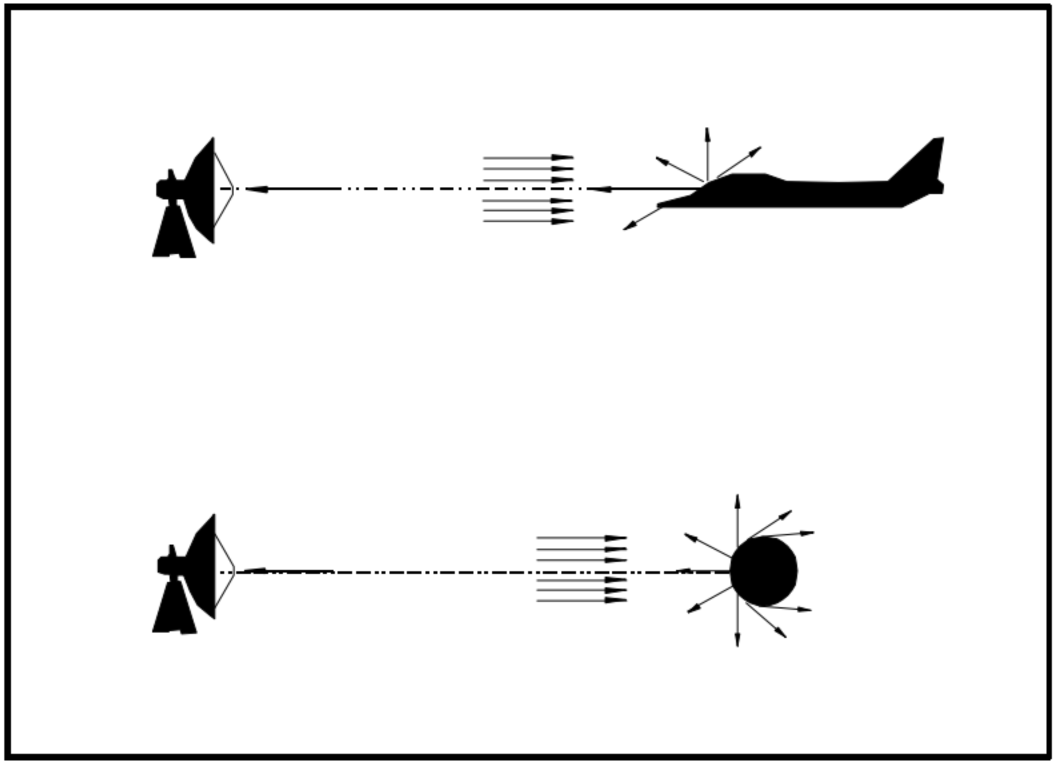


Figure 1. Concept of Radar Cross Section

Radar Cross-section (RCS) or Echo Area

RCS does not equal geometric area

The RCS of a sphere is independent of frequency if operating at sufficiently high frequencies where λ<<Range and λ << radius (r).

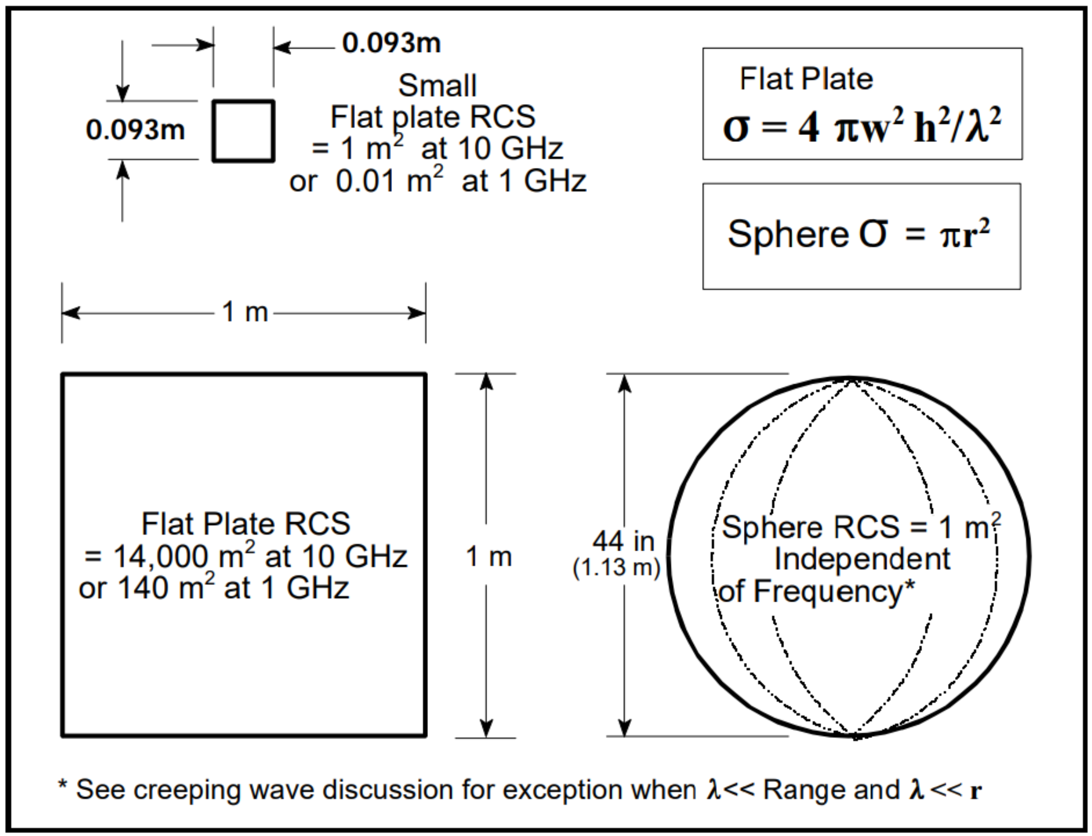


Figure 2. RCS vs Physical Geometry

Radar Cross-section (RCS) or Echo Area

a typical RCS plot of a jet aircraft.
an azimuth cut made at zero degrees elevation (on the aircraft horizon).

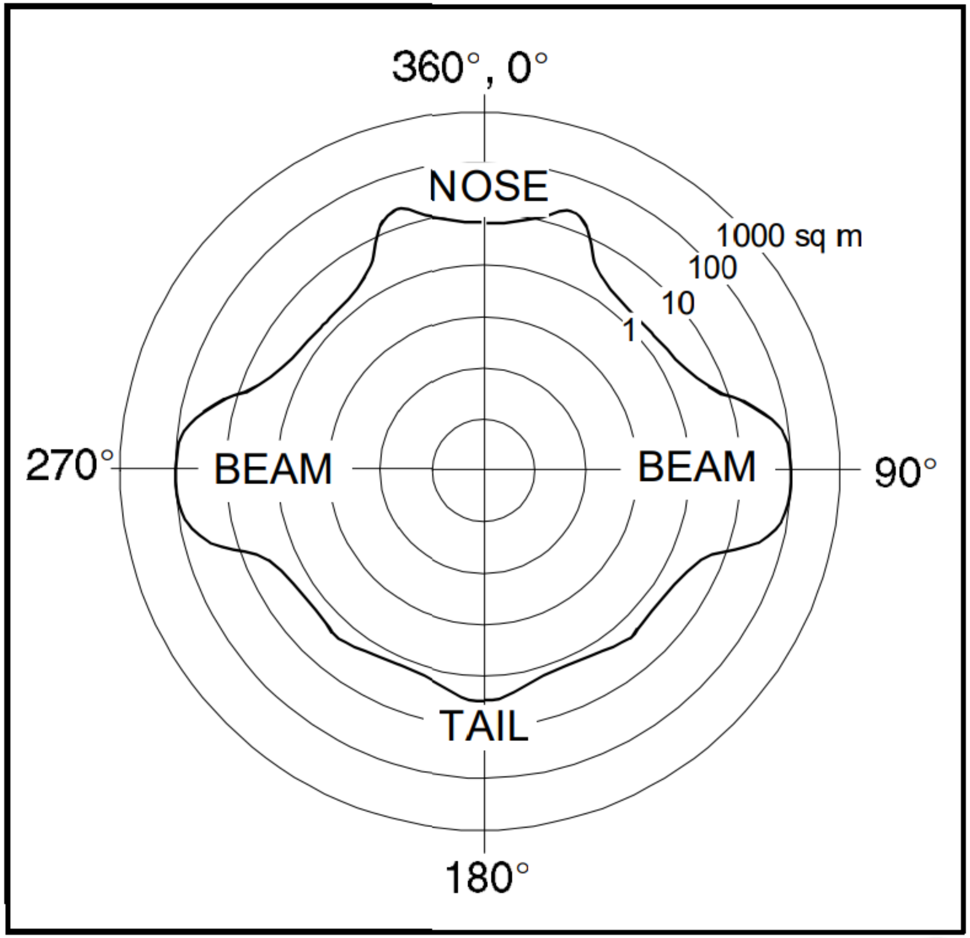
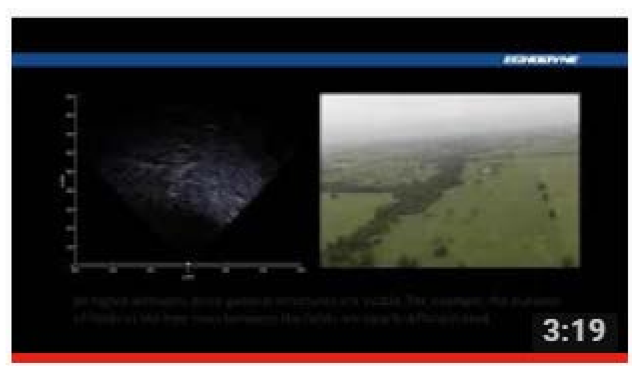


Figure 5. Typical Aircraft RCS

Radar Cross-section (RCS) or Echo Area

Missile: 0.5 sq m
Tactical Jet: 5 to 100 sq m;
Bomber: 10 to 1000 sq m;
Ships: 3,000 to 1,000,000 sq m.
RCS can also be expressed in decibels referenced to a square meter (dBsm) which equals 10 log (RCS in m^2).

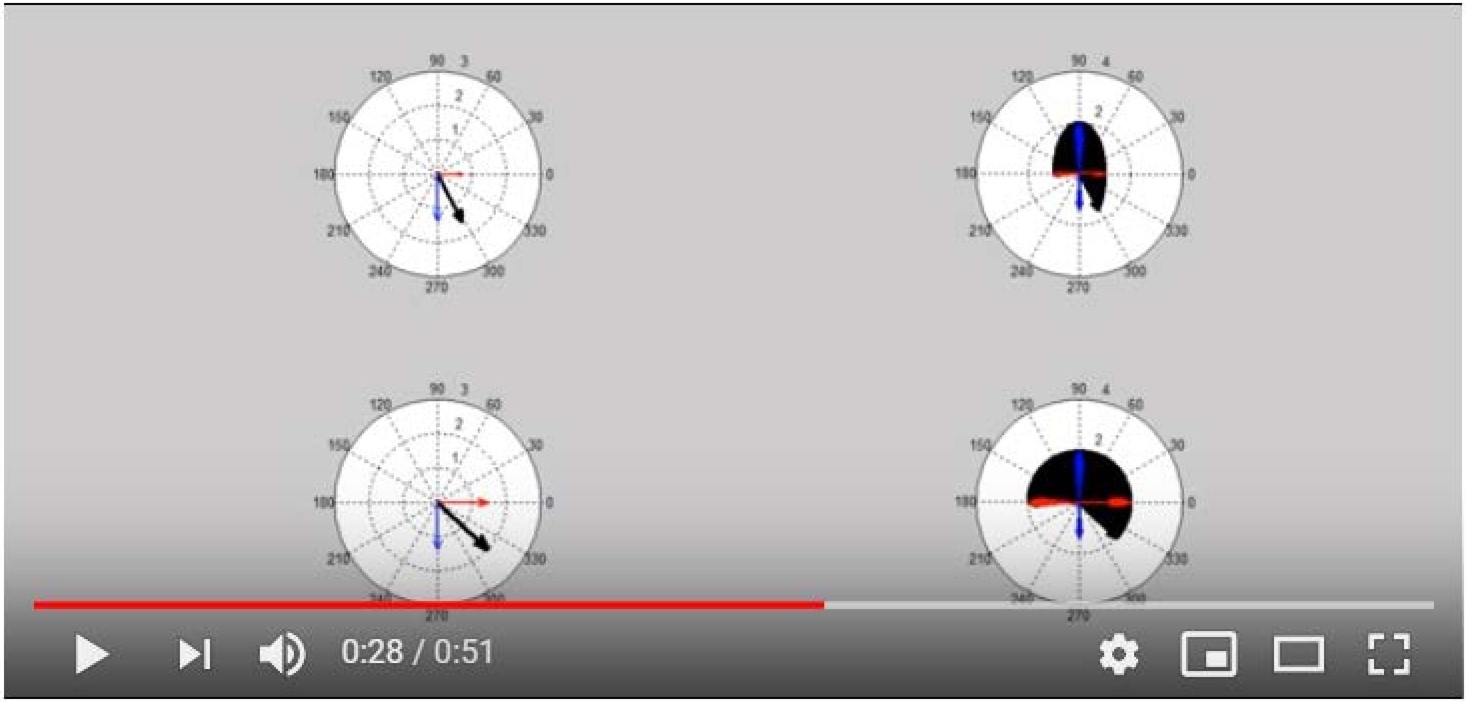


Homework [UG and PG]

Plot Left-hand polarization animation in 2D

Plot left-hand polarization vector Ex + Ey at from ωt= 0 to 6*pi on the same time axis

Plot animated elliptical polarization using LHCP and RHCP



Homework [PG]

An engineer is designing a communications link at 3 GHz where the receiver sensitivity is such that 1 μW of power is needed to overcome receiver noise. The receiving antenna gain is 8dB, the transmitter antenna gain is 10dB, the transmitting power level is 25 Watts. What is the largest distance between the two antennas allowing the communications link to work?