Statistics for Machine Learning and Big Data

An Introduction

Part 3: regression

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7 April 2015

Regression

Linear models

Problem: We have a set of points $\{(x_i, y_i)\}$. Given a new x value, we'd like to predict $\hat{y}$.

Linear model: We'll assume there exists a linear relationship $y = \beta_0 + \beta_1 x$ that offers a good approximation to the data.

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We call x the **explanatory** or **predictor** variable.

We call y the **response** variable.

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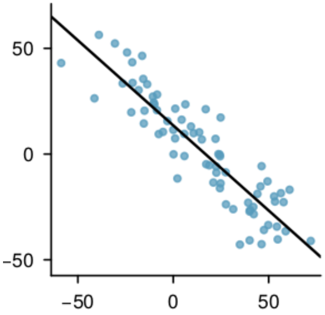

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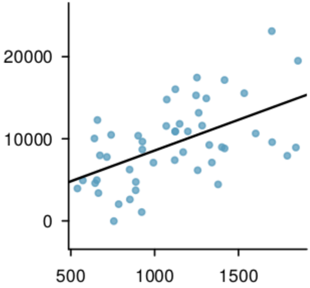

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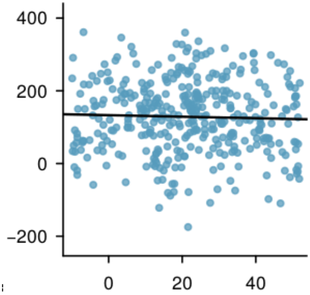

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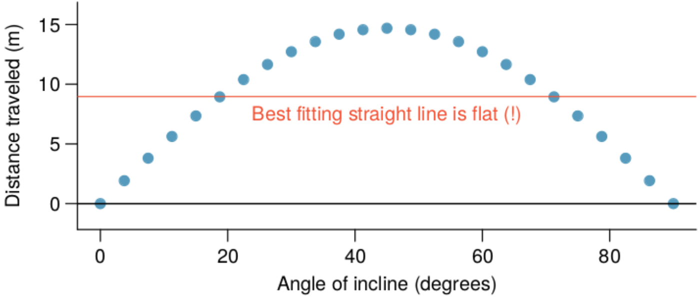


Residuals

What's left over.

$$\text{data} = \text{fit} + \text{residual}$$

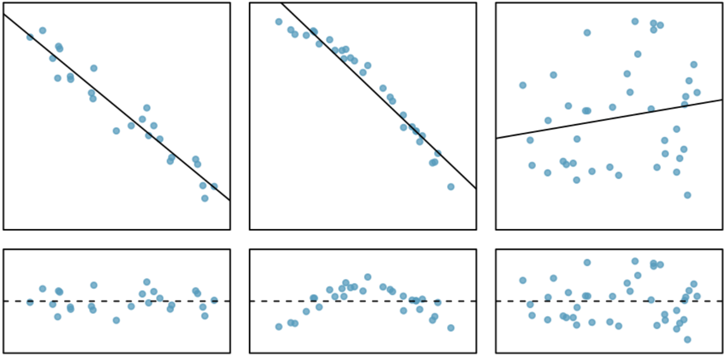
Residuals

What's left over.

$$y_i = \hat{y}_i + e_i$$

Residuals

What's left over.



Residuals

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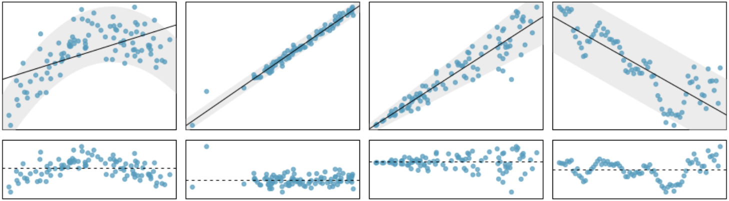


Residuals

What's left over.

Goal: small residuals.

$$\sum | e_i |$$

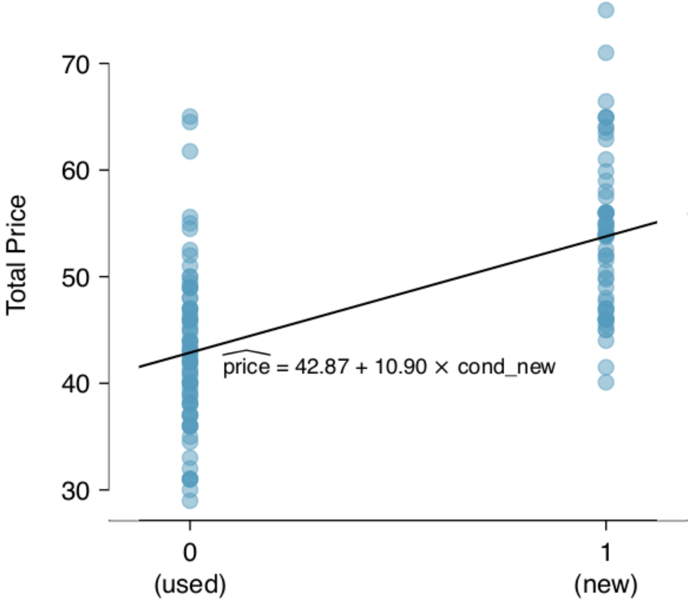
Residuals

What's left over.

Goal: small residuals.

$$\sum e_i^2$$

Categorical regression

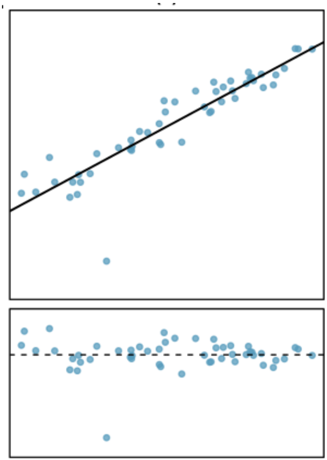


Outliers

High leverage points fall far from the regression line.

Influential points make their leverage known.

Outliers



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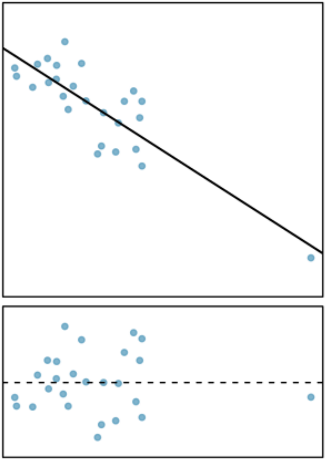

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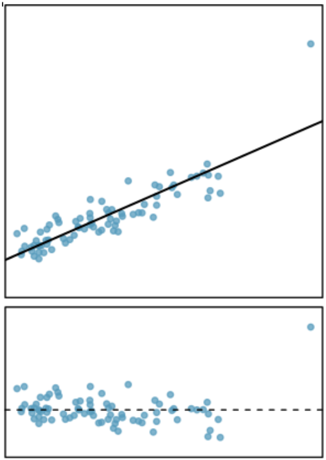

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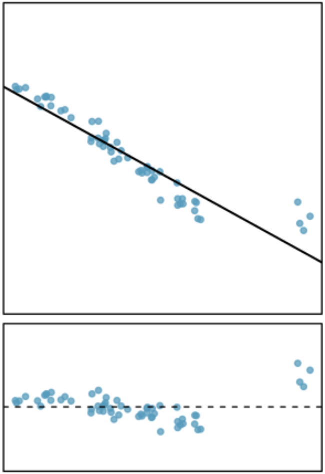

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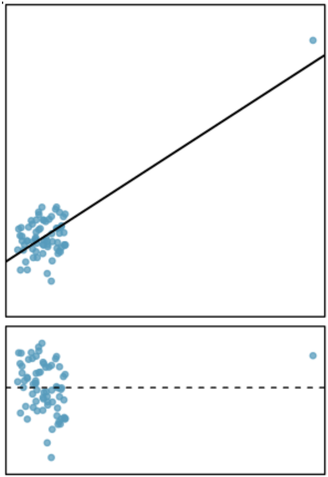

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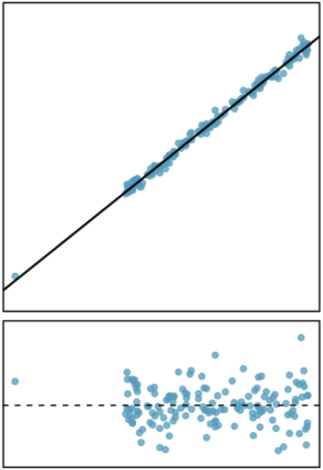
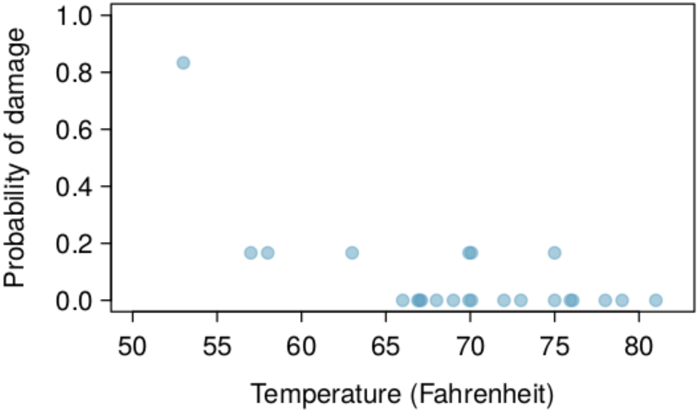


Don't ignore outliers.

Outliers



Probability of damage
1.0
0.8
0.6
0.4
0.2
0.0
50
55
60
65
70
75
80
Temperature (Fahrenheit)

Correlation

Population correlation.

$$\rho_{X,Y} = \mathbf{Corr}(X, Y) = \frac{\mathbf{Cov}(X, Y)}{\sigma_X \sigma_Y} = \frac{\mathbf{E}[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y}$$

Correlation

Sample correlation.

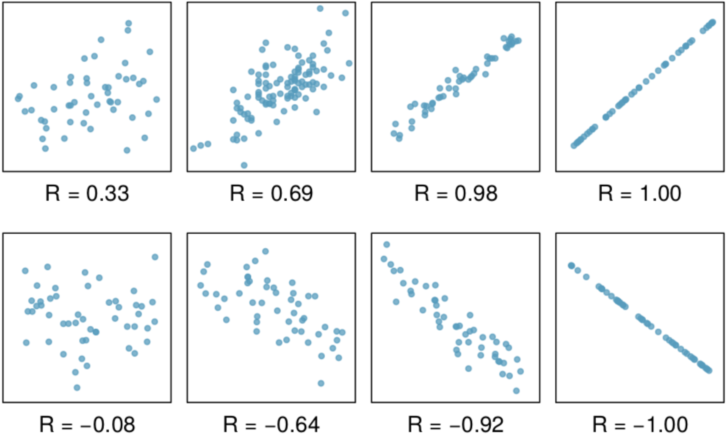
$$r_{x,y} = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \overline{x}}{s_x} \right) \left(\frac{y_i - \overline{y}}{s_y} \right)$$

Correlation

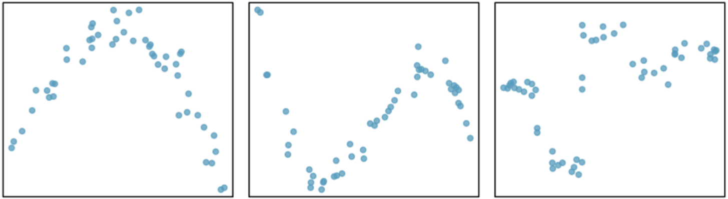
Coefficient of determination

$$r^2 = \frac{\mathbf{Var}\, e_i}{\mathbf{Var}\, y_i}$$

Correlation



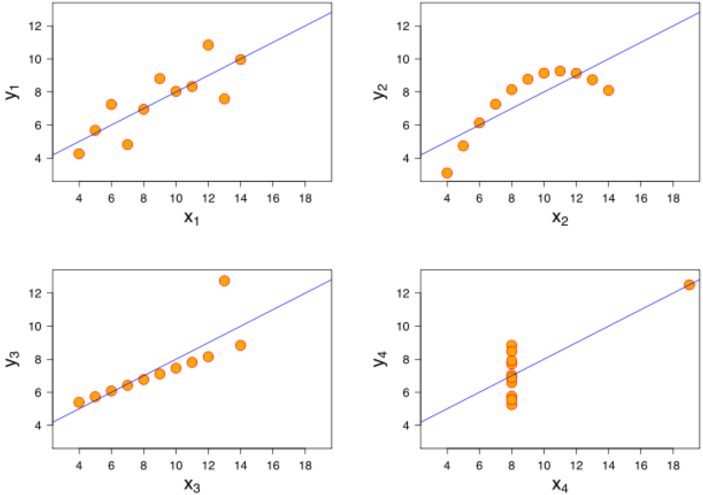
Correlation



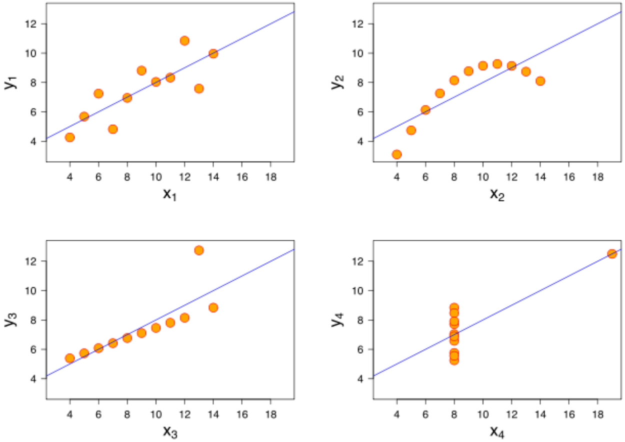
R = −0.23 R = 0.31 R = 0.50

Correlation

Anscombe's Quartet

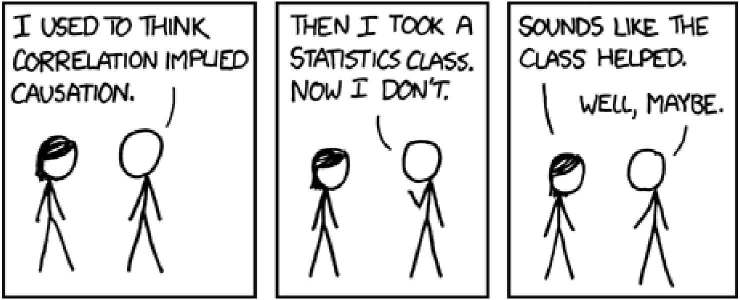


Anscombe's Quartet



http://en.wikipedia.org/wiki/File:Anscombe%27s_quartet_3.svg

Correlation does not imply causation



https://xkcd.com/552/

Correlation does not imply causation

Tufte:

"Empirically observed covariation is a necessary but not sufficient condition for causality."

http://en.wikipedia.org/wiki/Correlation_does_not_imply_causation

Correlation does not imply causation

Tufte:

"Correlation is not causation but it sure is a hint."

http://en.wikipedia.org/wiki/Correlation_does_not_imply_causation

Code lab: linear regression

For each data set:

Tasks:

- Visualise the data
- Compute mean, variance, standard deviation, correlation
- Compute linear regression and visualise
- Compute residuals and visualise
- Discuss what the data tells you

Linear regression and inference

Remember about inference?

- Standard error
- *p* values

Multiple regression

One response $\hat{y}$ but many predictors $x_1, \ldots, x_n$.

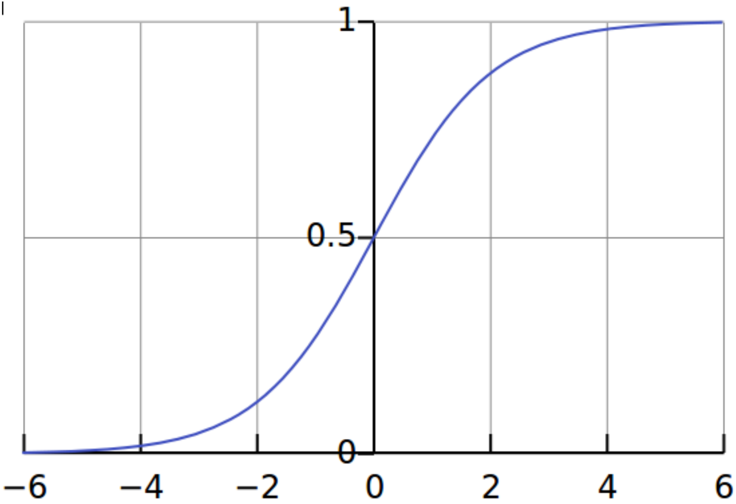
$$\hat{y} = \beta_0 + \beta_1 x_1 + \cdots + \beta_n x_n$$

Logistic regression

Also: *logit regression*, *logit model*

Dependent variable (y) is binary.

Logistic regression



Logistic regression

Note that $f(-x) = 1 - f(x)$.

And it satisfies this differential equation:

$$\frac{\mathrm{d}}{\mathrm{d}x} f(x) = f(x) \cdot (1 - f(x))$$

Integrating,

$$f(x) = \frac{e^x}{e^x + 1} = \frac{1}{1 + e^{-x}}$$

Fitting polynomials

- Under/over-fitting
- Test is not validation

Questions?

`purple.com/talk-feedback`

Break