ML Week

0x01a Introduction

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2-5 novembre 2015

Definition

Supervised

Unsupervised

Reinforcement

Curse of Dimensionality

Addition rule: independent events

$$Pr(A \cup B) = Pr(A) + Pr(B)$$

Addition rule: dependent events

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

Multiplication rule: independent events

$$Pr(A \cap B) = Pr(A) Pr(B)$$

Multiplication rule: dependent events

$$Pr(A \cap B) = Pr(A \mid B) Pr(B)$$

Conditional probability

$$Pr(A \mid B) = \frac{Pr(A \cap B)}{Pr(B)}$$

Conditional probability

$$\cup_i A_i = A \wedge A_i \cap A_j = \emptyset \implies$$

$$P(A_1 \mid B) = \frac{\Pr(B \mid A_1) \Pr(A_1)}{\sum_{i} \Pr(B \mid A_1) \Pr(A_1) + \dots + \Pr(B \mid A_k) \Pr(A_k)}$$

What is Statistics

- 1 Identify a question or problem.
- 2 Collect relevant data on the topic.
- 3 Analyze the data.
- 4 Form a conclusion.

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Sadly, sometimes people forget 1.

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- 4 Form a conclusion.

Statistics is about making 2–4 efficient, rigorous, and meaningful.

OpenIntro Statistics, 2nd edition, D. Diez, C. Barr, M. Çetinkaya-Rundel, 2013.

(Exercise: Is this the same question as the last slide?)

- ① Define the question of interest
- 2 Get the data
- 3 Clean the data
- 4 Explore the data
- 6 Fit statistical models
- 6 Communicate the results
- Make your analysis reproducible

(Exercise: Is this the same question as the last slide?)

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What the public thinks.

(Exercise: Is this the same question as the last slide?)

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Where we spend most of our time.

(Exercise: Is this the same question as the last slide?)

- 1 Define the question of interest
- 2 Get the data
- 3 Clean the data
- 4 Explore the data
- 5 Fit statistical models
- 6 Communicate the results
- 7 Make your analysis reproducible

The easiest part to forget.

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http://simplystatistics.org/2015/03/17/
data-science-done-well-looks-easy-and-that-is-a-big-problem-f
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Anecdote

Some properties of anecdote:

- is data
- haphazardly collected
- is generally not representative
- sometimes result of selective retention
- does not accumulate to be representative
- might be true (by chance)
- is ok to use as hypothesis, but be clear that hypothesis is anecdote

Study Types

- Observational
- Experimental

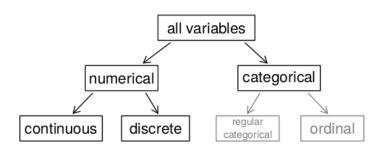
Study Types

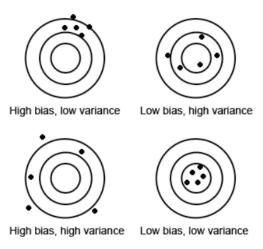
- Observational
- Experimental

What can go wrong?

- Forgetting that association ≠ causation
- Not random
- Confounding variables

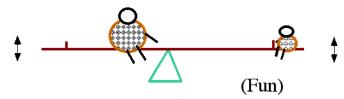
Variable types



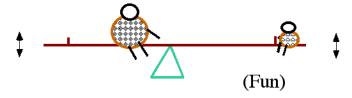


- · Weighted and unweighted
- Centroid to physicists

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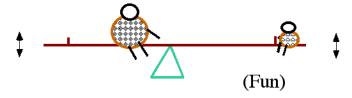
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$$\mu = E(X) = \sum w_i x_i = \mathbf{w} \cdot \mathbf{x}$$

http://telescopes.stardate.org/images/research/
teeter-totter/TT4.gif

- Weighted and unweighted
- · Centroid to physicists



$$\mu = E(X) = \sum \Pr(X = x_i)x_i$$

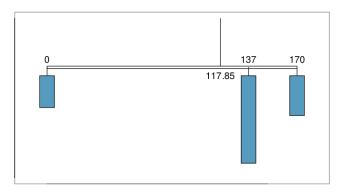
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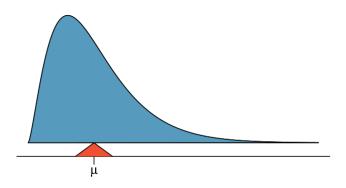
$$\mu = E(X) = \int x f(x) \, \mathrm{d}x$$

http://telescopes.stardate.org/images/research/
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Population statistics

Deviation is distance from mean.

Population statistics

Variance is mean square of deviations

Population statistics

Standard deviation is square root of variance

Population statistics

$$s^2 = \frac{(\overline{X} - X_1)^2 + \cdots (\overline{X} - X_n)^2}{n - 1}$$

Population statistics

$$\sigma^2 = \frac{(\overline{X} - X_1)^2 + \cdots (\overline{X} - X_n)^2}{n}$$

Population statistics

$$Var(X) = \sigma^2 = (\overline{X} - X_1)^2 \Pr(X = X_1) + \cdots + (\overline{X} - X_n)^2 \Pr(X = X_n)$$

Given a distribution X, the

probability distribution function (pdf) (continuous) or **probability mass function (pmf)** is the probability that the variate has value x:

$$Pr(a \le X \le b)$$
or
 $Pr(X = a)$

Given a distribution X, the

cumulative probability function (cdf) is the probability that the variate is less than *x*:

$$\Pr(X \leqslant x) = \int_{-\infty}^{x} pdf(x) dx \text{ or } \sum_{i \leqslant x} \Pr(X = x)$$

Given a distribution X, the

percent point function (ppf) is the inverse of the cdf. Given a probability, what's x? Also called the **inverse distribution**.

Given a distribution X, the

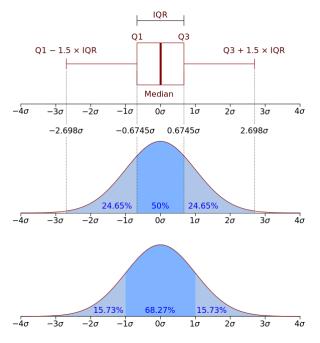
survival function (sf) is the probability that the variate takes a value greater than *x*:

$$ss(x) = Pr(X > x) = 1 - cdf(x)$$

Given a distribution X, the

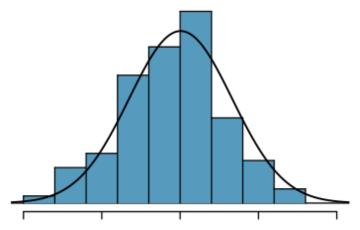
inverse survival function (isf) is the inverse of the survival function:

$$isf(\alpha) = ppf(1 - \alpha)$$



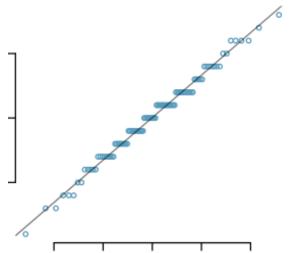
Evaluating Normal Approximations

Easy technique 1: visually compare to normal plot.



Evaluating Normal Approximations

Easy technique 2: normal probability plot.

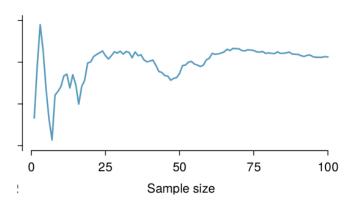


Also known as a quantile-quantile plot. Copyright 2015 Jeff Abrahamson, for private use by course students only



Running mean. Sequence of partial sums (divided by number in sum).

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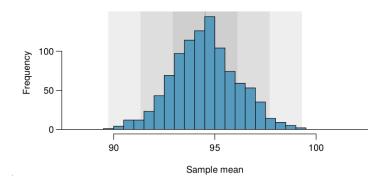
Sampling variation. Change of \overline{x} from one sample to the next.

Running mean. Sequence of partial sums (divided by number in sum).

Sampling variation. Change of \overline{x} from one sample to the next.

Sampling distribution. The distribution of possible point samples of a fixed size from a given population.

Sampling distribution



Confidence intervals

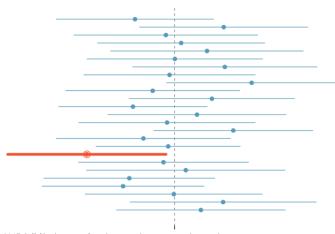
Sample *n* points, choose an interval around the sample mean.

A 95% confidence interval means if we sample repeatedly, about 95% of the samples will contain the population mean.

Confidence intervals

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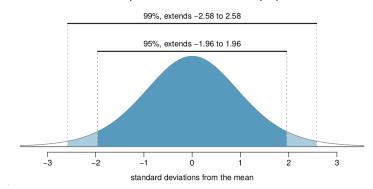
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Questions?

purple.com/talk-feedback