Exercises Lecture 8
Intelligent Systems Programming (ISP)

Exercise 1 (adapted from Mozart-OZ online tutorial)

A CSP model consists of a set of variables, a set of domains (possible values) for each variable and a set of constraints between the variables that limit the values that variables can take, all describing a specific problem.

The code of Professor Smart's safe is a sequence of 4 distinct nonzero digits, $C_1, \dots, C_4$ such that the following equations and inequalities are satisfied:

$C_1 \neq 1, \dots, C_4 \neq 4$

$C_4 - C_3 > 2$

$C_3 + C_1 \geq 5$

$C_1 * C_2 \geq 6$

$C_1 + C_2 \leq 7$

$C_3 \leq 5$

$C_4 > 4$

A CSP model for this problem is:

Variables: $\{C_1, C_2, C_3, C_4\}$

Domains: $\{[1 \dots 6], [1 \dots 6], [1 \dots 6], [1 \dots 6], [1 \dots 6]\}$

Constraints: $\{C_1 \neq 1, C_2 \neq 2, C_3 \neq 3, C_4 \neq 4, C_1 \neq C_2, C_1 \neq C_3, C_1 \neq C_4, C_2 \neq C_3, C_2 \neq C_4, C_3 \neq C_4, C_4 - C_3 > 2, C_3 + C_1 \geq 5, C_1 * C_2 \geq 6, C_1 + C_2 \leq 7, C_3 \leq 5, C_4 > 4\}$

Now what you have to do:

a) Use backtracking to find a solution, assume the order $\{C_1, C_2, C_3, C_4\}$ for the selection of the variables, draw the search tree and show in the failed leaves which constraint is the one that fails.
b) Use FC and MRV to find a solution, draw the search tree and show the new domains of the variables for each leave after the execution of FC. Make the CSP node consistent before you start the search.

Exercise 2 (adapted from Mozart-OZ online tutorial)

Betty, Chris, Donald, Fred, Gary, Mary, and Paul want to align in one row for taking a photo. Some of them have preferences next to whom they want to stand:

1. Betty wants to stand next to Gary and Mary.
2. Chris wants to stand next to Gary.
3. Fred wants to stand next to Donald.
4. Paul wants to stand next to Donald.

Be aware that 2 people can't stand in the same position at the same time, meaning that there is a different constraint between each pair of variables representing the position where two people stand.

We can define such a constraint using mathematical operators: $diff(X, Y)$: $X \neq Y$, where X and Y are variables.

A partial CSP model for this problem is:

Variables: $\{B, C, D, F, G, M, P\}$

Domains: $\{[1...7], [1...7], [1...7], [1...7], [1...7], [1...7], [1...7]\}$

Constraints: $\{B \neq C, B \neq D, B \neq F, B \neq G, B \neq M, B \neq P, C \neq D, C \neq F, C \neq G, C \neq M, C \neq P, D \neq F, D \neq G, D \neq M, D \neq P, F \neq G, F \neq M, F \neq P, M \neq G, M \neq P, G \neq P\}$

a) Define a constraint $adj(X, Y)$ representing the fact that for a possible value of X, the value of Y has to be next to it. Use logical and mathematical operators to do so.

b) As you may remember from the lecture, a constraint graph is a graph where each node represents a variable and each edge represents one of the binary constraints defined in the set of constraints. Draw a constraint graph for a CSP' just considering the adjacency constraints of this problem. Variables and domains are the same as defined above.

c) Add the adjacency constraints together with constraint $B=1$ to the CSP and run the **AC-3** algorithm in order to make it arc-consistent. Show the domains of the variables after the execution of the algorithm.

Exercise 3 (adapted from RN10 6.7)

Define the CSP model of the next problem definition i.e. the set of variables, possible values that those variables can take and the constraints according to the problem description below. A problem definition can be modeled in several different ways, so don't worry if your description doesn't match the one of your classmates. **DO NOT SOLVE IT**, this exercise is about modeling. Use the *adj* constraint defined in the previous exercise if needed.

Problem Definition:

In five houses, each painted with a different color, live 5 people of different nationalities, each of whom prefer a different brand of cigarette, different drink, and a different pet. Given the following facts, the question to answer is: "Where does the zebra live, and in which house do they drink water?"

- The English man lives in the red house.
- The Spaniard owns the dog.
- The Norwegian lives in the first house on the left.
- Kools are smoked in the yellow house.
- The man who smokes Chesterfields lives in the house next to the man with the fox.
- The Norwegian lives next to the blue house.
- The Winston smoker owns snails.
- The Lucky Strike smoker drinks orange juice.
- The Ukrainian drinks tea.
- The Japanese smokes Parliaments.
- Kools are smoked in the house next to the house where the horse is kept.
- Coffee is drunk in the green house.
- The green house is immediately to the right (your right) of the ivory house.
- Milk is drunk in the middle house.

Mandatory assignment

It is possible to reduce any CSP to a binary CSP. This means that any n-ary constraints can be expressed as an equivalent set of binary constraints. Even thought n-ary constraints are known to perform better in practice, this property is quite important for theoretical aspects of CSPs.

As previously mentioned in the lecture, a CSP is a triple (V, D, C) , where V is a set of variables, D is the set of possible values of the variables in V and C is the set of constraints over the set of variables V.

In a general view, it is possible to express each constraint from the set of constraints C as a subset of tuples of the Cartesian product of the values from the domain of the variables in a constraint. i.e

$V = \{v_1, v_2, v_3, v_4\}$

$D = \{\{\{1..5\}\}, \{\{1..5\}\}, \{\{1..5\}\}, \{1..5\}\}$

$C = \{c_1, c_2, c_3\}$, where:

$c_1(v_2, v_4) = \{(2,3), (4,1), (4,3)\}$

$c_2(v_1, v_2, v_3) = \{(1,3,2), (1,2,2), (1,5,4)\}$

$c_3(v_1, v_3, v_4) = \{(1,2,3), (4,3,5)\}$

Two of the three constrains in the CSP above are n-ary constraints, making it impossible to draw the CSP graph for it. It is then necessary to redefine the constraints and variables of the CSP in such a way that they fit in to a graph representation. To do so you might need to consider adding new constraints and redefining the set of variables.

Draw a graph that represents the CSP defined above.

Hint: You may consider associating variables to the constraints, in order to build binary relations among the variables.