Solutions Lecture 13
Intelligent System Programming (ISP)

Exercise 1 (adapted from C83 5.2)

1) Let c_i denote constraint number i.

Combination 1, $c_3 + c_4$: $7x_1 + 3x_2 \leq 12$

Combination 2, $2c_1 + c_4$: $3x_1 - 2x_2 \leq 4$

Combination 3, $c_1 + 2c_2$: $-x_1 - x_2 \leq 1$

2) The Dual Problem

Minimize $-y_1 + y_2 + 6y_3 + 6y_4 + 6y_5$

Subject to

$$-3y_1 + y_2 - 2y_3 + 9y_4 - 5y_5 \geq -1$$

$$y_1 - y_2 + 7y_3 - 4y_4 + 2y_5 \geq -2$$

$$y_1, y_2, y_3, y_4 \geq 0$$

3) The Dual Problem on standard form

Maximize $y_1 - y_2 - 6y_3 - 6y_4 - 6y_5$

Subject to

$$3y_1 - y_2 + 2y_3 - 9y_4 + 5y_5 \leq 1$$

$$-y_1 + y_2 - 7y_3 + 4y_4 - 2y_5 \leq 2$$

$$y_1, y_2, y_3, y_4 \geq 0$$

Initial dictionary (slack form of dual on standard form)

	Maximum increase y_1
$y_6 = 1 - 3y_1 + y_2 - 2y_3 + 9y_4 - 5y_5$	1/3
$y_7 = 2 + y_1 - y_2 + 7y_3 - 4y_4 + 2y_5$	∞
$z = y_1 - y_2 - 6y_3 - 6y_4 - 6y_5$	

First dictionary (optimal)

$y_1 = 1/3 + 1/3y_2 - 2/3y_3 + 3y_4 - 5/3y_5 - 1/3y_6$
$y_7 = 2 + (1/3 + 1/3y_2 - 2/3y_3 + 3y_4 - 5/3y_5 - 1/3y_6) - y_2 + 7y_3 - 4y_4 + 2y_5$
$= 7/3 - 2/3y_2 + 19/3y_3 - y_4 + 1/3y_5 - 1/3y_6$
$z = (1/3 + 1/3y_2 - 2/3y_3 + 3y_4 - 5/3y_5 - 1/3y_6) - y_2 - 6y_3 - 6y_4 - 6y_5$
$= 1/3 - 2/3y_2 - 20/3y_3 - 3y_4 - 23/3y_5 - 1/3y_6$

4)

Dual solution: $y_1 = 1/3$, $y_2 = 0$, $y_3 = 0$, $y_4 = 0$, $y_5 = 0$
Primal solution: $x_1 = 1/3$, $x_2 = 0$

5)

Checking primal feasibility:

$-3*1/3 + 0 \leq 0$	ok
$1/3 - 0 \leq 1$	ok
$-2*1/3 + 0 \leq 6$	ok
$9*1/3 - 0 \leq 6$	ok
$-5*1/3 + 0 \leq 0$	ok

Primal objective value: $-1/3 + 0 = -1/3$

Checking dual feasibility

$-3*1/3 + 0 - 0 + 0 - 0 \geq -1$	ok
$1/3 - 0 + 0 - 0 + 0 \geq -1$	ok

Dual objective value: $-1/3 - 0 - 0 - 0 - 0 - 0 = -1/3$

Dual objective value = Primal objective value ok

Exercise 2

Unbounded dual (make a geometric interpretation of the problem if in doubt)

Minimize $-y_1$
Subject to $y_1 - y_2 \geq 0$
$y_1, y_2 \geq 0$

Dual on standard form

Maximize y_1
Subject to $-y_1 + y_2 \leq 0$
$y_1, y_2 \geq 0$

Primal (dual of dual)

Minimize $\quad 0$

Subject to $\quad -x_1 \geq 1$

$\quad\quad\quad\quad\quad x_1 \geq 0$

$\quad\quad\quad\quad x_1, x_2 \geq 0$

The sum of constraint 1 and 2 is $0 \geq 1$ which is impossible. So Primal is infeasible.

Exercise 3

1) Again let c_i denote constraint i

Initial dictionary:

$y_1 = 0$, $y_2 = 0$, $y_3 = 0$

$0c_1 + 0c_2 + 0c_3 \Leftrightarrow 0x_1 + 0x_2 + 0x_3 \leq 0$

First dictionary:

$y_1 = 5/2$, $y_2 = 0$, $y_3 = 0$

$5/2c_1 + 0c_2 + 0c_3 \Leftrightarrow 10/2x_1 + 15/2x_2 + 5/2x_3 \leq 25/2$

Second dictionary (optimal):

$y_1 = 1$, $y_2 = 0$, $y_3 = 1$

$1c_1 + 0c_2 + 1c_3 \Leftrightarrow 5x_1 + 7x_2 + 3x_3 \leq 13$

2) z' is equal to the right side ("the bound") of the linear combinations

3) They must be since $z' < z^*$ for suboptimal dictionaries while any dual feasible liner combination of the constraints satisfy that the computed bound is larger than or equal to z^*.