Solutions Lecture 11
Intelligent System Programming (ISP)

Exercise 1

Let x and y denote the amount of vodka and orange juice in cl., respectively. We have

1) We want to maximize $(40x / (x + y))\,(x + y) - 3x - y$, since this is the profit generated by one serving of the drink.
2) $10 \leq x + y \leq 20$, since the drink must be between 10 and 20 cl. in total,
3) $15 \leq 40x / (x + y) \leq 20$, since the volume percent alcohol must be between 15 and 20,
4) $x \geq 0$ and $y \geq 0$, since we cannot have a negative amount of vodka and orange juice,

From 1) we get the objective

Maximize $37x - y$

Constraint 2) formulated on standard form becomes:

$-x - y \leq -10$ and $x + y \leq 20$

Constraint 3) formulated on standard form becomes:

$15 \leq 40x / (x + y) \Leftrightarrow 15(x + y) \leq 40x \Leftrightarrow -25x + 15y \leq 0$ and
$40x / (x + y) \leq 20 \Leftrightarrow 40x \leq 20(x + y) \Leftrightarrow 20x - 20y \leq 0$

Constraint 4) is already on standard form.

Exercise 2

Let $x_1 = y_1 - y_2$, $x_2 = -y_3$, and $x_3 = y_4$

We then transform *Minimize* $x_1 - x_2$ to *Maximize* $-(x_1 - x_2) \Leftrightarrow$ *Maximize* $-((y_1 - y_2) - (-y_3)) \Leftrightarrow$
Maximize $-y_1 + y_2 - y_3$

Moreover, we have:

1) $2x_1 + x_2 = 3 \Leftrightarrow 2(y_1 - y_2) + (-y_3) = 3 \Leftrightarrow 2y_1 - 2y_2 - y_3 = 3 \Leftrightarrow 2y_1 - 2y_2 - y_3 \leq 3$, $-2y_1 + 2y_2 + y_3 \leq -3$
2) $2x_2 + x_3 \geq 7 \Leftrightarrow 2(-y_3) + y_4 \geq 7 \Leftrightarrow -2y_3 + y_4 \geq 7 \Leftrightarrow 2y_3 - y_4 \leq -7$

Thus, the complete equivalent transformed LP problem on standard form is:

Maximize $-y_1 + y_2 - y_3$

Subject to

$2y_1 - 2y_2 - y_3 \leq 3$
$-2y_1 + 2y_2 + y_3 \leq -3$
$2y_3 - y_4 \leq -7$

$y_1 \geq 0, y_2 \geq 0, y_3 \geq 0, y_4 \geq 0$

Exercise 3 (Adapted from C83 2.1)

Initial dictionary

	Max increase of x_2
$x_4 = 4 - x_1 - x_2 - 2x_3$	4
$x_5 = 5 - 2x_1 - 3x_3$	inf
$x_6 = 7 - 2x_1 - x_2 - 3x_3$	7
$z = 3x_1 + 2x_2 + 4x_3$	

(obs. here we have chosen x_2 as entering variable rather than x_1 that has largest coefficient simply because this leads to less fractional values in the resulting dictionary)

First dictionary: increase x_2 to 4

	Max increase of x_1
$x_2 = 4 - x_1 - 2x_3 - x_4$	4
$x_5 = 5 - 2x_1 - 3x_3$	5/2
$x_6 = 7 - 2x_1 - (4 - x_1 - 2x_3 - x_4) - 3x_3$	
$\quad = 3 - x_1 - x_3 + x_4$	3
$z = 3x_1 + 2(4 - x_1 - 2x_3 - x_4) + 4x_3$	
$\quad = 8 + x_1 - 2x_4$	

Second (optimal) dictionary: increase x_1 to 5/2

$x_2 = 4 - (5/2 - 3/2x_3\ 1/2x_5) - 2x_3 - x_4$
$\quad = 3/2 - 1/2x_3 - x_4 + 1/2x_5$
$x_1 = 5/2 - 3/2x_3\ 1/2x_5$
$x_6 = 3 - (5/2 - 3/2x_3\ 1/2x_5) - x_3 + x_4$
$\quad = 1/2 + 1/2x_3 + x_4 + 1/2x_5$
$z = 8 + (5/2 - 3/2x_3\ 1/2x_5) - 2x_4$
$\quad = 21/2 - 3/2x_3 - 2x_4 - 1/2x_5$