Exercises Lecture 13
Intelligent System Programming (ISP)

Exercise 1 (adapted from C83 5.2)

Consider the LP problem

Maximize $-x_1 - 2x_2$

Subject to

$$\begin{aligned} -3x_1 + x_2 &\leq -1 \\ x_1 - x_2 &\leq 1 \\ -2x_1 + 7x_2 &\leq 6 \\ 9x_1 - 4x_2 &\leq 6 \\ -5x_1 + 2x_2 &\leq 6 \\ x_1, x_2 &\geq 0 \end{aligned}$$

1) Make at least three linear combinations of constrains that prove upper bounds on the optimal value (they each must involve different sets of constraints)
2) Write the dual problem
3) Solve the dual problem using simplex
4) Write the dual and primal solution from the optimal dual dictionary
5) Check feasibility and optimality of the primal solution found using the method described on the slide with title “Checking Solutions” of the third lecture on LP.

Exercise 2

Give an example of an LP problem where the dual is unbounded and the primal is infeasible.

Exercise 3

Go back to the first lecture on LP and consider the first simplex example we went through on the problem:

Maximize $5x_1 + 4x_2 + 3x_3$

Subject to

$$\begin{aligned} 2x_1 + 3x_2 + x_3 &\leq 5 \\ 4x_1 + x_2 + 2x_3 &\leq 11 \\ 3x_1 + 4x_2 + 2x_3 &\leq 8 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

1) For each dictionary computed by simplex on this example (just take numbers from the slides)
   a. Write the dual of the dictionary as $y_i = -\bar{c}_{n+i} \quad (i = 1, \dots, m)$, where z in the dictionary is expressed as $z' + \sum_{k=1}^{n+m} \bar{c}_k x_k$
   b. Compute the linear combination of the constraints that these dual values correspond to

2) What is the relation between z' and these linear combinations?
3) Given that your answer to 2) holds in general, explain why these dual solutions for suboptimal dictionaries must be infeasible in the dual system.

Mandatory assignment (adapted from C83 5.10)

Construct an example showing that the conclusion of C83 Theorem 5.5 may fail if the hypothesis that the LP problem has a nondegenerate basic optimal solution is omitted.