Center for Economic Behavior & Inequality

BabyMAKRO

Autumn 2023

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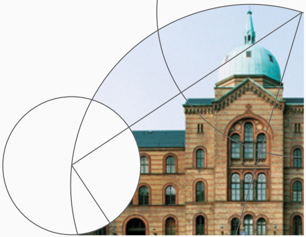


Introduction

- **MAKRO:** New model developed by *DREAM* and now in use by the *Danish Finance Ministry.*
  1. Framework for business cycle forecasts
  2. Medium and long-run projections
  3. Evaluations of potential economic policies and shocks
- Today **BabyMAKRO:** Smaller-but-still-big model in the same style for students to use in *classes and theses* (bachelor + master).
  1. **Foundations:** Me (help from Peter Stephensen + Martin Bonde).
  2. **Alpha-version:** Bachelor thesis students fall 2022.
     (Andreas Marius Laursen, Hans Christian Jul Lehmann, Mathias Held Berg, Nicholas Stampe Meier, Olivier Ding, Ufuk Yasin)
  3. **Beta-version**: RAs Anders Buch Jürs + Andreas Marius Laursen + Martin Andreas Kildemark.
  4. **Further versions:** More student involvement. *You?*

Plan

1. Classes of Macro Models
2. MAKRO
3. BabyMAKRO
4. Solution method
5. Calibration
6. Government spending shock
7. Conclusion

Classes of Macro Models

Reminder: Consumption behavior (chapter 16)

1. **Maximize utility s.t. intertemporal budget constraint:**

   utility: $u(C_1) + \frac{u(C_2)}{1+\phi}$

   budget constraint: $C_1 + \frac{C_2}{1+r} = V_1 + Y_1^d + \frac{(1+g)Y_1^d}{1+r}$

   $\Rightarrow$ *derive qualitative (signed) properties of optimal micro-behavior*

2. **Aggregate consumption function:** $C_1 = C(\underset{+}{Y_1^d}, \underset{+}{g}, \underset{(-)}{r}, \underset{+}{V_1})$
3. **Estimate equation on aggregate data**
4. **Put equation into model**

Note: Similar for investment (chapter 15).

Classes of Macro Models

1. **Old-style Keynesian macro-models** (1950s-):

   Structure: Aggregate equations »similar« to those in micro-theory
   Estimation: Equation-by-equation on aggregate data
   In teaching: AS-AD models
   In practice: ADAM + MONA + SMEC
   (»Den økonomiske genopretning 1976-1993«, Jørgen Rosted, 2021)

2. **Micro-founded macro-models** (1970s-):

   Structure: Exactly the equations in micro-theory (in general equilibrium)
   Estimation: Calibration vs. moment-matching vs. full-system
   In teaching: *Dynamic Stochastic General Equilibrium Models* (DSGE)
   RBC: Real Business Cycle (1980s-)
   RANK: Representative Agent New Keynesian (1990s-)
   HANK: Heterogeneous Agent New Keynesian (2010s-)
   In practice: DREAM + DSGE at Nationalbanken + MAKRO

Blanchard: On the Need for (At Least) Five Classes of Macro Models

MAKRO

Structure of small open economy

- **Agents:**
  1. **Unconstrainted households** (»Ricardian«): One for each cohort
  2. **Hand-to-mouth households** (»HtM«): One for each cohort
  3. **Firms** (production, price setting, multiple sectors incl. housing)
  4. **Central bank** (fixed exchange-rate)
  5. **Government** (detailed sub-accounts)
  6. **Global foreign economy** (exogenous)
- **Expectations:** Perfect foresight
- **Market clearing:** Walras + sticky prices + search-and-match
- **Mathematically:** *Non-linear equation system*
  1. Behavioral equations in terms of first order conditions
  2. Accounting identities
  3. Market clearing
- **Code:** https://github.com/DREAM-DK/MAKRO

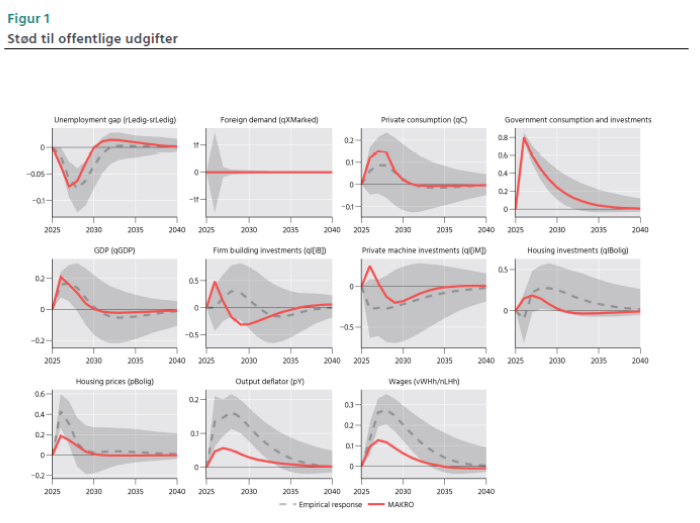
Empirical strategy

1. **Levels:** Weights in production and utility functions.
   (directly observable in data, but changes over time...)
2. **Long-run relationships:** Substitution elasticities.
3. **Short-run dynamics:** Adjustment cost parameters.
   (especially focus on *convergence speed*)

Documentation:

Matching af impuls responser og øvrige kortsigtsmomenter (2021)

Impulse-response functions (IRFs)



Also: Foreign demand, foreign interest rate, labor supply, oil price

Micro-moments

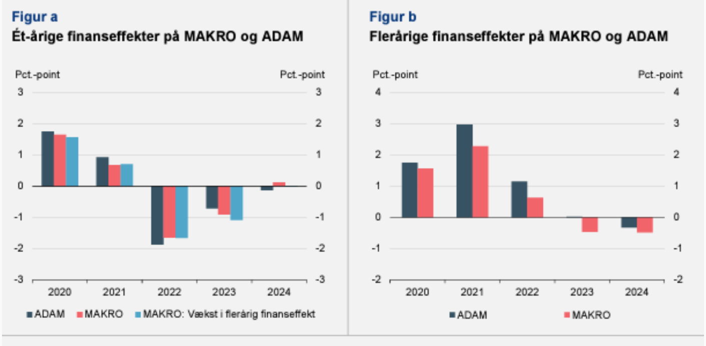
Tabel 2

Yderligere relevant empiri til vurdering af MAKROs kortsigtsegenskaber

Analyse/moment	MAKRO og konsensus	Relevant litteratur
MPC ud af midlertidig/kortsigtet indkomst, første år	MAKRO: ca. 0,45 Litteratur: 0,4-0,6	Jørgensen & Kuchler (2017), Crawley & Kuchler (2020), Kreiner et al (2019), ADAM, SMEC
MPC ud af boligprisstigninger/formue, første år	MAKRO: ca. 0,05 Litteratur: 0,03-0,06	Hviid & Kuchler (2017), Andersen & Leth-Petersen (2021)
Fortrængning af tvungen pensionsopsparing for 30-55 årige, år 1 [år 10]	MAKRO: Ca. 0,35-0,55 [0,10-0,35] Litteratur: 0-0,5 [0-0,5]	Arnberg & Barslund (2012), Chetty m.fl. (2014), Andersen, Hansen & Kuchler (2021)
Rentefølsomhed, husholdningers <u>boligværdi</u> (stød til beskatning på aktie- og kapitalindkomst). Gns. 10 års-effekt.	MAKRO: knap -0,1 Litteratur: (-)0,25 – (-)0,18	Gruber, Jensen & Kleven (2021)*
Rentefølsomhed, husholdningers <u>formue</u> (stød til beskatning på aktie- og kapitalindkomst). 8-års effekt. [Langsigtet elasticitet]	MAKRO: ca. 0,2 [0,5] Litteratur: 0,2 – 0,4 [0,5 - 1]	Jakobsen, Jakobsen, Kleven & Zucman (2020)**

In practice: Finanseffekt

»Finanseffekt«: Is fiscal policy expansive or contractive wrt. GDP?



Kilde: Økonomisk Redegørelse, Marts 2023

BabyMAKRO

Small open economy in discrete time (annual freq., t)

- **Demographics:** Overlapping generations (age, a)
- **Households:** Hands-to-mouth (λ) and unconstrained ($1-\lambda$) wrt. consumption-saving + supply labor exogenously
- **Foreign economy:** Fixed nominal rate of return + import goods at fixed prices + demand curve for the domestic export good.
- **Production:**
  1. *Production firms* rent *capital* + *labor* to produce the domestic output good.
  2. *Repacking firms* combine imported goods with the domestic output good to produce a consumption good, an investment good, and an export good.
  3. Capital is rented from a *capital agency*, which purchases the investment good to accumulate capital subject to adjustment costs.
  4. Labor is rented from a *labor agency*, which post vacancies a search-and-match labor market to purchase labor from the households.
- **Price of domestic output good:** Price adjustment costs.
- **Wage:** Ad hoc bargaining.
- **Central bank + government:** Fixed exchange rate + taxation.

Foreign economy

- **Nominal interest rate:** r_t^{hh}
- **Armington demand of the domestic exported good:**

$$X_t = \gamma^X X_{t-1} + (1-\gamma^X)\chi_t \left(\frac{P_t^X}{P_t^F}\right)^{-\sigma^F}.$$

- **Import goods:** $\bullet_t^M$ at prices $P_t^\bullet$ for $\bullet \in \{C, G, I, X\}$

Demographic structure and population

- **Life-span:** $\#$, hereof working $\#_{\text{work}}$
- **Number of households:** N_a
- **Mortality rate:** ζ_a (controlled by ζ)

$$N_a = \begin{cases} 1 & \text{if } a = 0 \\ (1-\zeta_{a-1})N_{a-1} & \text{if } a > 0 \end{cases}$$

$$\zeta_a = \begin{cases} 0 & \text{if } a < \#_{\text{work}} \\ \left(\frac{a+1-\#_{\text{work}}}{\#-\#_{\text{work}}}\right)^{\zeta} & \text{if } a < \# - 1 \\ 1 & \text{if } a = \# - 1 \end{cases}$$

$$N = \sum_{a=0}^{\#-1} N_a$$

$$N_{\text{work}} = \sum_{a=0}^{\#-1} 1_{\{a<\#_{\text{work}}\}} N_a.$$

Labor market flows

- **Employed and unemployed:** $L_{a,t}$ and $U_{a,t} = N_a - L_{a,t}$
- **Job-separation and finding rate:** δ_a^L and m_t^s
- **Searchers** (everybody search = exogenous labor supply):

$$S_{a,t} = \begin{cases} 1 & \text{if } a = 0 \\ (1-\zeta_{a-1})\left[U_{a-1,t-1} + \delta_a^L L_{a-1,t-1}\right] & \text{if } a < \#_{\text{work}} \\ 0 & \text{if } a \geq \#_{\text{work}}. \end{cases}$$

- **Employed** (before and after matching):

$$\underline{L}_{a,t} = \begin{cases} 0 & \text{if } a = 0 \\ (1-\zeta_{a-1})(1-\delta_a^L)L_{a-1,t-1} & \text{if } a < \#_{\text{work}} \\ 0 & \text{if } a \geq \#_{\text{work}} \end{cases}$$

$$L_{a,t} = \underline{L}_{a,t} + m_t^s S_{a,t}.$$

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$$L_{a,t} = \underline{L}_{a,t} + m_t^s S_{a,t}.$$

Search-and-matching

- **Vacancies:** v_t
- **Searchers:** $S_t = \sum_{a=0}^{\#-1} S_{a,t}$
- **Matches** by *matching function*:

$$\mathcal{M}_t = \frac{S_t v_t}{\left(S_t^{\frac{1}{\sigma^m}} + v_t^{\frac{1}{\sigma^m}}\right)^{\sigma^m}}.$$

- **Job-finding rate:** $m_t^s = \frac{\mathcal{M}_t}{S_t}$.
- **Job-filling rate:** $m_t^v = \frac{\mathcal{M}_t}{v_t}$.

Effective employment

- **Human capital:** Gained form work experience.

$$
\begin{aligned}
H_{a,t} =& 1 + \rho_1 \cdot x_{a,t} - \rho_2 \cdot x_{a,t}^2 \\
x_{a,t} =& \begin{cases} 0 & \text{if } a = 0 \\ x_{a-1,t-1} + \left(\frac{L_{a-1,t-1}}{N_{a-1}}\right)^{\Phi} \left(\frac{L_{a-1,ss}}{N_{a-1}}\right)^{1-\Phi} & \text{else} \end{cases}
\end{aligned}
$$

- **Effective employment:**

$$
L_{a,t}^H = L_{a,t} H_{a,t}
$$

Wage bargaining

- **Unmodeled wage bargaining mechanism** give

$$W_t = W_{ss} \left(\frac{L_t}{L_{ss}} \right)^{\epsilon_w},$$

where the wage is increasing in the labor demand for $\epsilon_w > 0$.

- **Note:** Theoretically there is a *bargaining set*
    1. Upper limit: Firm will fire worker
    2. Lower limit: Household will leave firm

  Often: Nash-bargaining = »surplus split from bargaining power«

Income

1. **Post-tax labor income:** $(1-\tau_t)\, W_t \frac{L^H_{a,t}}{N_a}$
2. **Post-tax unemployment benefits:** $(1-\tau_t)\, W^U W_{ss} \frac{U_{a,t}}{N_a}$
3. **Post-tax retirement benefits**: $(1-\tau_t)\, W^R W_{ss} \frac{N_a-(L_{a,t}+U_{a,t})}{N_a}$
4. **Equally divided inheritance**: $\frac{A^q_t}{N}$

- The age specific income is

$$\text{inc}_{a,t} = (1-\tau_t)\, W_t \frac{L^H_{a,t}}{N_a} + (1-\tau_t)\, W^U W_{ss} \frac{U_{a,t}}{N_a} + (1-\tau_t)\, W^R W_{ss} 1_{\{a \geq \#_{\text{work}}\}} + \frac{A^q_t}{N}.$$

Hand-to-mouth households (HtM)

- **Consume all income:**

$$C_{a,t}^{\text{HtM}} = \frac{\text{inc}_{a,t}}{P_t^C}.$$

where the price of consumption goods is P_t^C.

- **No savings:**

$$A_{a,t}^{\text{HtM}} = 0.$$

Unconstrained household (Ricardian) I

- **Utility from consumption:**

$$\frac{\left(C^R_{a,t}\right)^{1-\sigma}}{1-\sigma}, \;\; \sigma > 0, \sigma \neq 1.$$

- **Utility from bequest:**

$$\zeta_a \mu^{A^q} \frac{\left(\frac{A^R_{a,t}}{P^C_t}\right)^{1-\sigma}}{1-\sigma}, \;\; \mu^{A^q} > 0.$$

- **Discounting of future utility:**

$$\beta(1-\zeta_a), \;\; \beta > 0.$$

- **Budget constraint:** $A^R_{-1,t} = 0$ and

$$A^R_{a,t} = (1 + r^{hh}_t) A^R_{a-1,t-1} + \mathrm{inc}_{a,t} - P^C_t C^R_{a,t}.$$

Unconstrained household (Ricardian) II

- **Cohort:** t_0 (time of first decision)
- **Full problem**

$$V_{t_0} = \max_{\left\{C^R_{a,t}\right\}_{a=0}^{\#-1}} \sum_{a=0}^{\#-1} \left(\Pi_{j=1}^{a} \beta(1-\zeta_{j-1})\right) \left[\frac{\left(C^R_{a,t}\right)^{1-\sigma}}{1-\sigma} + \zeta_a \mu^{A^q} \frac{\left(\frac{A^R_{a,t}}{P^C_t}\right)^{1-\sigma}}{1-\sigma} \right]$$

s.t.

$$t = t_0 + a$$

$$A^R_{-1,t} = 0$$

$$A^R_{a,t} = (1 + r^{hh}_t) A^R_{a-1,t-1} + \mathrm{inc}_{a,t} - P^C_t C^R_{a,t}.$$

Unconstrained household (Ricardian) III

- **First order conditions:**

$$C^R_{a,t} = \begin{cases} \left(\zeta_a \mu^{A^q} \left(\frac{A^R_{\#-1,t}}{P^C_t}\right)^{-\sigma}\right)^{-\frac{1}{\sigma}} & \text{if } a = \# - 1 \\ \left(\beta(1-\zeta_a)\frac{1+r^{hh}_{ss}}{1+\pi^{hh}_{ss}}\left(C^{R,}_{a+1,ss}\right)^{-\sigma} + \zeta_a \mu^{A^q}\left(\frac{A^R_{a,ss}}{P^C_{ss}}\right)^{-\sigma}\right)^{-\frac{1}{\sigma}} & \text{elif } t = T-1 \\ \left(\beta(1-\zeta_a)\frac{1+r^{hh}_{t+1}}{1+\pi^{hh}_{t+1}}\left(C^{R,}_{a+1,t+1}\right)^{-\sigma} + \zeta_a \mu^{A^q}\left(\frac{A^R_{a,t}}{P^C_t}\right)^{-\sigma}\right)^{-\frac{1}{\sigma}} & \text{else} \end{cases}.$$

Household aggregation

- **Age:**

$$C_{a,t} = \lambda C_{a,t}^{\text{HtM}} + (1-\lambda) C_{a,t}^{R}$$
$$A_{a,t} = \lambda A_{a,t}^{\text{HtM}} + (1-\lambda) A_{a,t}^{R},$$

- **Total:**

$$C_t = \sum_{a=0}^{\#-1} N_a C_{a,t}$$
$$A_t = \sum_{a=0}^{\#-1} N_a C_{a,t}.$$

- **Bequests:**

$$A_t^q = \left(1 + r_t^{hh}\right) \sum_{a=0}^{\#-1} \zeta_a N_a A_{a,t}.$$

Production firms I

- **Exogenous technology:** Γ_t
- **Capital:** K_{t-1}, at rental price r_t^K
- **Labor:** ℓ_t, at rental price r_t^ℓ
- **Output:** Y_t, with a CES technology, sold at $P_t^{Y,0}$
- **Profit maximization** with prices taken as given

$$\Pi_t = \max_{K_{t-1},\ell_t} P_t^{Y,0} Y_t - r_t^K K_{t-1} - r_t^\ell \ell_t$$

s.t.

$$Y_t = \Gamma_t \left(\left(\mu^K\right)^{\frac{1}{\sigma^Y}} K_{t-1}^{\frac{\sigma^Y - 1}{\sigma^Y}} + \left(1 - \mu^K\right)^{\frac{1}{\sigma^Y}} \ell_t^{\frac{\sigma^Y - 1}{\sigma^Y}} \right)^{\frac{\sigma^Y}{\sigma^Y - 1}},$$

where μ^K, σ^Y, $\sigma^Y \neq 1$.

Production firms II

- **Free entry implies zero profit:**

$$P_t^{Y,0} = \frac{1}{\Gamma_t} \left(\mu^K \left(r_t^K \right)^{1-\sigma^Y} + \left(1 - \mu^K \right) \left(r_t^\ell \right)^{1-\sigma^Y} \right)^{\frac{1}{1-\sigma^Y}} .$$

- **First order condition for capital-labor ratio:**

$$\frac{K_{t-1}}{\ell_t} = \frac{\mu_K}{1-\mu_K} \left(\frac{r_t^\ell}{r_t^K} \right)^{\sigma^Y}$$

- **Investment good:** I_t, at price, P^I_t.
- **Capital:** K_t, rented out at rental rate r^K_{t+1} in the following period.
- **Adjustment costs:** Effective investment is ι_t.
- **Required internal rate of return:** r^{firm}.
- **Profit maximization:**

$$V_0^{\text{capital}}\left(K_{t-1}\right) = \max_{\{K_t\}} \sum_{t=0}^{\infty} \left(1 + r^{\text{firm}}\right)^{-t} \left[r^K_t K_{t-1} - P^I_t \left(\iota_t + \Psi(\iota_t, K_{t-1})\right)\right]$$

s.t.

$$I_t = \iota_t + \Psi(\iota_t, K_{t-1})$$
$$K_t = (1 - \delta^K)K_{t-1} + \iota_t.$$

Capital agency II

- **Functional form:**

$$\Psi(\iota_t, K_{t-1}) = \frac{\Psi_0}{2}\left(\frac{\iota_t}{K_{t-1}} - \delta^K\right)^2 K_{t-1}.$$

- **First order condition:**

$$\begin{aligned} 0 = & - P^I_t \left(1 + \Psi_\iota\left(\iota_t, K_{t-1}\right)\right) \\ & + \frac{r^k_{t+1} + P^I_{t+1}(1-\delta^K)(1+\Psi_\iota(\iota_{t+1}, K_t)) - P^I_{t+1}\Psi_K\left(\iota_{t+1}, K_t\right)}{1 + r^{\text{firm}}} \end{aligned}$$

Labor agency I

- **Post vacancies**: v_t at cost κ^L (in units of labor).
- **Labor:** Hires L_t^H and rent out labor at rental price r_t^ℓ
- **Exogenous match destruction:** δ_t^L (implied by $\delta_{a,t}^L$ and $L_{a,t-1}^H$)
- **Exogenous wage:** W_t
- **Profit maximization:**

$$V_0^{\text{labor}}(L_{t-1}) = \max_{\{L_t\}} \sum_{t=0}^{\infty} \left(1 + r^{\text{firm}}\right)^{-t} \left[r_t^\ell \ell_t - W_t H_t L_t\right]$$

s.t.

$$L_t = \left(1 - \delta_t^L\right) L_{t-1} + m_t^v v_t$$

$$\ell_t = H_t L_t - \kappa^L v_t.$$

Labor agency II

- **First order condition:**

$$r^{\ell}_{t} = \frac{1}{H_t - \frac{\kappa^L}{m^v_t}} \left[H_t - r^{\ell}_{t+1} \frac{1-\delta^L_{t+1}}{1+r^{\text{firm}}} \frac{\kappa^L}{m^v_{t+1}} \right]$$

Price setting I

- **Input price:** $P_t^{Y,0}$
- **Output of differentiated goods:** y_t at p_t^Y
- **Demand schedule:** $y_t = \left(\frac{p_t^Y}{P_t^Y}\right)^{-\sigma_D} Y_t$
- **Aggregate:** Y_t with price index P_t^Y
- **Profit maximization:**

$$V_t^{\text{intermediary}} = \max_{\{p_t^Y\}} \left(p_t^Y - P_t^{Y,0}\right) y_t - \vartheta_t + \frac{1}{1+r^{\text{firm}}} V_{t+1}^{\text{intermediary}}$$

s.t.

$$\vartheta_t = \frac{\gamma}{2}\left[\frac{p_t^Y/p_{t-1}^Y}{p_{t-1}^Y/p_{t-2}^Y} - 1\right]^2 P_t^Y Y_t$$

$$y_t = \left(\frac{p_t^Y}{P_t^Y}\right)^{-\sigma_D} Y_t.$$

Price setting II

- **Assumption:** Symmetric firms
- **First order condition** (New Keynesian Phillips Curve):

$$
\begin{aligned}
P_t^Y = &(1+\theta)P_t^{Y,0} - \eta\left(\frac{P_t^Y/P_{t-1}^Y}{P_{t-1}^Y/P_{t-2}^Y} - 1\right)\frac{P_t^Y/P_{t-1}^Y}{P_{t-1}^Y/P_{t-2}^Y}P_t^Y \\
&+ \frac{2\eta}{1+r^{\text{firm}}}\frac{Y_{t+1}}{Y_t}\left(\frac{P_{t+1}^Y/P_t^Y}{P_t^Y/P_{t-1}^Y} - 1\right)\frac{P_{t+1}^Y/P_t^Y}{P_t^Y/P_{t-1}^Y}P_{t+1}^Y
\end{aligned}
$$

$$
\begin{aligned}
\theta &\equiv \frac{1}{\sigma_D - 1} \\
\eta &\equiv \theta\gamma.
\end{aligned}
$$

Repacking firms

- **Output goods:** $\bullet_t^Y$ at prices $P_t^\bullet$ for $\bullet \in \{C, G, I, X\}$
- **Domestic input good:** Y_t at price P_t^Y
- **Foreign input goods:** $\bullet_t^M$ at prices $P_t^\bullet$ for $\bullet \in \{C, G, I, X\}$
- **Profit maximization with CES** production technology implies

$$P_t^\bullet = \left(\mu^{M,\bullet}\left(P_t^{M,\bullet}\right)^{1-\sigma^\bullet} + \left(1-\mu^{M,\bullet}\right)\left(P_t^Y\right)^{1-\sigma^\bullet}\right)^{\frac{1}{1-\sigma^\bullet}}$$

$$\bullet_t^M = \mu^{M,\bullet}\left(\frac{P_t^\bullet}{P_t^{M,\bullet}}\right)^{\sigma^\bullet}\bullet_t$$

$$\bullet_t^Y = \left(1-\mu^{M,\bullet}\right)\left(\frac{P_t^\bullet}{P_t^Y}\right)^{\sigma^\bullet}\bullet_t,$$

for $\bullet \in \{C, G, I, X\}$

Government

- **Interest rate:** r^B
- **Government consumption:** G_t
- **Unemployment insurance expenses:** $E_t^U = W_U W_{ss} U_t$
- **Retirement benefits expenses:** $E^R = W_R W_{ss}(N - N_{\text{work}})$
- **Tax base:** $T_t = W_t L_t + E_t^U + E^R$
- **Budget constraint:**

$$B_t = (1 + r^B)B_{t-1} + P_t^G G_t + E_t^U + E^R - \tau_t T_t.$$

- **Tax policy:**

$$\tau_t = \tau_{ss} + \varepsilon_B \frac{\tilde{B}_t - B_{ss}}{T_t},$$
$$\tilde{B}_t = (1 + r^B)B_{t-1} + P_t^G G_t + E_t^U + E^R - \tau_{ss} T_t.$$

Goods market clearing

1. **Demand for domestically produced goods:**

$$Y_t = \sum_{\bullet \in \{C,G,I,X\}} \bullet_t^Y.$$

2. **Imports add up:**

$$M_t = \sum_{\bullet \in \{C,G,I,X\}} \bullet_t^M.$$

[Current account and net foreign asset position]

Not necessary to specify

Not specified yet

Solution method

Targets and unknowns

- **Goal:** Find the *equilibrium path* in the economy.
- **Equilibrium path:** A set of paths for all variables where
  1. Optimal firm and household behavior in terms of FOCs.
  2. Accounting identities.
  3. Market clearing.
- **Target equations:** Must *be zero on the equilibrium path*.
- **Unknown variables:**
  1. Chosen by model-builder.
  2. All other variables must be derived from these.
  3. Target equations can be evaluated.
- **Ordered series of *blocks*:**

  Inputs: Unknown and exogenous variables or outputs of previous blocks
  Outputs: Variables and errors in target equations
- **Truncation:** Assume back in steady state after T periods.

Exogenous variables

1. Γ_t, technology
2. G_t, public spending
3. χ_t, *foreign* demand shifter (»market size«)
4. $P_t^{M,C}$, import price of *private* consumption good
5. $P_t^{M,G}$, import price of *public* consumption good
6. $P_t^{M,I}$, import price of *investment* good
7. $P_t^{M,X}$, import price of *export* good
8. P_t^F, *foreign* price level
9. r_t^{hh}, *foreign* interest rate

Unknowns in practice

1. A_t^q, inheritance flow (T unknowns)
2. A_t^{death}, wealth of households at $a = \# - 1$ (T unknowns)
3. K_t, capital (T unknowns)
4. L_t, labor supply (T unknowns)
5. r_t^K, rental price for capital (T unknowns)
6. P_t^Y, price of domestic output (T unknowns)

$\Rightarrow$ *all other variables in model can be derived and target equations can be evaluated (see code)*

Solving numerically

- **Solve with Newton's method**:
  1. x is a $6 \times T$ vector containing all 6 unknown in all T periods.
  2. f is a $6 \times T$ vector containing all 6 target in all T periods.
  3. $\mathcal{J}$ is the Jacobian of f (the derivative of f) eval. in steady state.

  Compute next guess, x_{n+1}, as

  $$x_{n+1} = x_n - \frac{f(x_n)}{\mathcal{J}_n}.$$

  Initial guess: Steady state

  Converges to solution, $f(x^*) = 0$, as n grows.

- **Implementation:** In Python using a quasi-Newton solver (broyden)
- **Course:** Introduction to Programming and Numerical Analysis

Calibration

Steady state

- **Goals:**
  1. National account shares and ratios.
  2. Unemployment level and labor market flows.
  3. Life-cycle profiles of income, consumption and savings.

  Simplify: Zero inflation, zero growth and constant demography.
- **Notebooks:**

  0b - steady state - data.ipynb

  0c - steady state - aggregate.ipynb

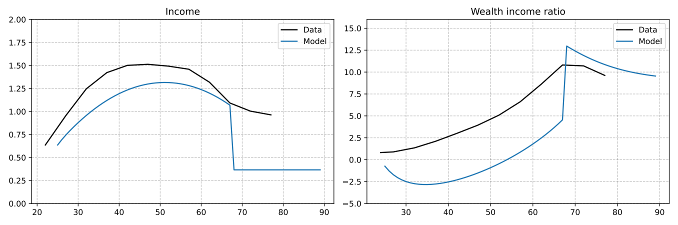
  0d - steady state - households.ipynb

Aggregates

Table 1: Comparison of Aggregate Steady State Ratios

	Model	Data
M/Y	0.46	0.46
X/Y	0.60	0.52
C/Y	0.36	0.47
G/Y	0.25	0.25
I/Y	0.26	0.22
K/Y	2.56	5.44
Employment to population ratio	0.72	0.74
Unemployment rate	1.67	3.93

Life-cycle


Income
Data
Model
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
20
30
40
50
60
70
80
90
Wealth income ratio
Data
Model
15.0
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
30
40
50
60
70
80
90

Dynamics

- **Substitution elasticities:** Use those from MAKRO.
- **Impulse-responses:** Match to data independently.

Government spending shock

Impulse-responses

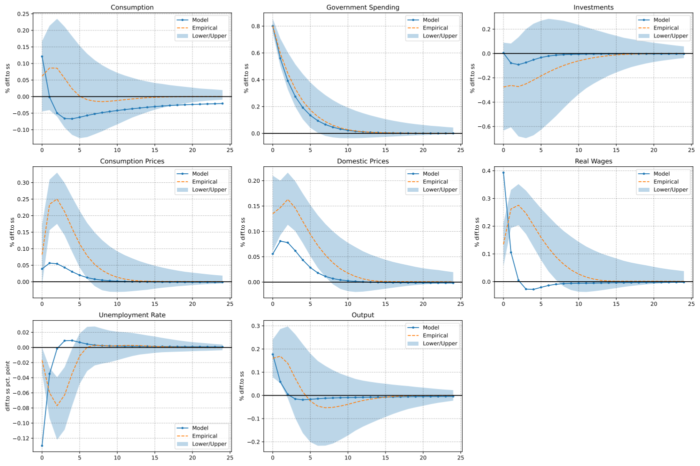
- **An *impulse response*:**
  1. The economy starts at steady state.
  2. Some exogenous variables *temporarily* deviate from steady state.
  3. The impulse responses: How variables respond to the shock.
- **Shock to government spending**, G_t:

$$G_t = G_{ss} + shock_t$$

$$shock_t = \begin{cases} G_{ss} \cdot size \cdot persistence^{t-T_{start}} & \text{if} \, T_{start} \leq t < T_{end} \\ 0 & \text{else} \end{cases}$$

$size = 0.01$ (initial deviation from steady state at 1 percent)
shock starts in $T_{start} = 0$ and ends in $T_{end} = 50$.
$persistence = 0.8$ (shock fades 20 percent per year)

Impulse-responses model vs data



Initial effects I

- **Repacking firms:** Demand for domestically produced goods $Y\uparrow$ and imported goods $M\uparrow$.
- **Production firms:** Increases inputs to increase production $Y\uparrow$.
  Capital $K\uparrow$ (limited by adjustment costs) and labor $\ell\uparrow$.
- **Households:** More income, greater consumption $C\uparrow$.
  Real wage $W\uparrow$ due to increased labor demand.
  Hand-to-mouth $C^{HtM}\uparrow$: Increase proportional to increase in W.
  Ricardian $C^{R}\downarrow$: Due to consumption smoothing.
- **Prices:** Higher input prices causes higher prices $P\uparrow$:
  $W\uparrow$ and $r^{K}\uparrow$ drives up marginal costs $P^{Y,0}\uparrow$.
  Output prices $P^{Y}\uparrow$ (limited by adjustment costs).
  Causes higher repacking prices $P^{\bullet}\uparrow$ for $\bullet\in\{C,I,G,X\}$.
- **Foreign economy:** Exports $X\downarrow$ due to increasing prices $P^{X}\uparrow$.

Initial effects II

- **Labor agency:** Employment $L\uparrow$ and rents $\ell\uparrow$ to production firms.

  Job vacancies $v\uparrow$ to meet higher labor demand.

  Number of matches $\mathcal{M}\uparrow$, »gross« labor $L\uparrow$.

  Labor $\ell\uparrow$ (limited by adjustment costs).

- **Capital agency:** Investments $I\uparrow$ to accumulate capital $K\uparrow$.

  Expensive investments: $P^I\uparrow$

  Greater return: $r^K\uparrow$

Transition path

- **Government:**

  Government spending $G\downarrow$ by assumption.

  Tax rate $\tau\uparrow$ to finance growing government debt $B\uparrow$.

- **Repacking firms:**

  Demand for domestic goods $Y\downarrow$ and imports $M\downarrow$.

  Prices $P\downarrow$.

- **Production firms:**

  Drop in production inputs, labor $\ell\downarrow$ and capital $K\downarrow$.

- **Households:**

  $\ell\downarrow$ causes wages $W\downarrow$.

  Higher taxes from $\tau\uparrow$.

  Leads to less disposable income and thus $C\downarrow$.

- **Foreign economy:**

  Exports $X\uparrow$ due to falling prices $P^X\downarrow$.

Conclusion

Insights and takeaways

- **Your takeaways:**
  1. How a micro-founded macro-model is structured
  2. Despite their complexity you can (soon) work with such models
  3. Requires both analytical and numerical skills to master
- **Open source:** github.com/JeppeDruedahl/BabyMAKRO
  Will you participate in improving it?
- **Related courses:**
  Macro III
  Introduction to Programming and Numerical Analysis
  Advanced Macroeconomics: Heterogenous Agent Models

Tasks

1. Better steady state calibration.
2. Better impulse-response matching.
3. Add technology growth, population growth, and trend inflation.
4. Add financial flows accounts wrt. to the foreign economy.
5. Add more government actions wrt. with taxes and spending.
6. Add endogenous labor supply.
7. Add multiple sectors and an input-output structure.
8. Make the model quarterly