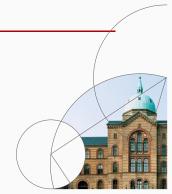
CENTER FOR ECONOMIC BEHAVIOR & INEQUALITY



BabyMAKRO

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Introduction

- MAKRO: New model developed by DREAM and now in use by the Danish Finance Ministry.
 - 1. Framework for business cycle forecasts
 - 2. Medium and long-run projections
 - 3. Evaluations of potential economic policies and shocks
- Today BabyMAKRO: Smaller-but-still-big model in the same style for students to use in classes and theses (bachelor + master).
 - 1. **Foundations:** Me (help from Peter Stephensen + Martin Bonde).
 - Alpha-version: Bachelor thesis students fall 2022. (Andreas Marius Laursen, Hans Christian Jul Lehmann, Mathias Held Berg, Nicholas Stampe Meier, Olivier Ding, Ufuk Yasin)
 - 3. **Beta-version**: RAs Anders Buch Jürs + Andreas Marius Laursen.
 - 4. Further versions: More student involvement. You?

[still work-in-progress]

Plan

- 1. Classes of Macro Models
- 2. MAKRO
- 3. BabyMAKRO
- 4. Solution method
- 5. Calibration
- 6. Government spending shock
- 7. Conclusion

Classes of Macro Models

Reminder: Consumption behavior (chapter 16)

1. Maximize utility s.t. intertemporal budget constraint:

utility:
$$u(C_1) + \frac{u(C_2)}{1+\phi}$$

budget constraint:
$$C_1 + \frac{C_2}{1+r} = V_1 + Y_1^d + \frac{(1+g)Y_1^d}{1+r}$$

- ⇒ derive qualitative (signed) properties of optimal micro-behavior
- 2. Aggregate consumption function: $C_1 = C(Y_1^d, g, r, V_1) + (-1) +$
- 3. Estimate equation on aggregate data
- 4. Put equation into model

Note: Similar for investment (chapter 15).

Classes of Macro Models

1. Old-style Keynesian macro-models (1950s-):

Structure: Aggregate equations »similar« to those in micro-theory

Estimation: Equation-by-equation on aggregate data

In teaching: AS-AD models

In practice: ADAM + MONA + SMEC

(»Den økonomiske genopretning 1976-1993«, Jørgen Rosted, 2021)

2. Micro-founded macro-models (1970s-):

Structure: Exactly the equations in micro-theory (in general equilibrium)

Estimation: Calibration vs. moment-matching vs. full-system

In teaching: Dynamic Stochastic General Equilibrium Models (DSGE)

RBC: Real Business Cycle (1980s-)

RANK: Representative Agent New Keynesian (1990s-) HANK: Heterogeneous Agent New Keynesian (2010s-)

In practice: DREAM + DSGE at Nationalbanken + MAKRO

Blanchard: On the Need for (At Least) Five Classes of Macro Models

MAKRO

Structure of small open economy

Agents:

- 1. Unconstrainted households (»Ricardian«): One for each cohort
- 2. Hand-to-mouth households (»HtM«): One for each cohort
- 3. Firms (production, price setting, multiple sectors incl. housing)
- 4. Central bank (fixed exchange-rate)
- 5. Government (detailed sub-accounts)
- 6. Global foreign economy (exogenous)
- **Expectations:** Perfect foresight
- Market clearing: Walras + sticky prices + search-and-match
- Mathematically: Non-linear equation system
 - 1. Behavioral equations in terms of first order conditions
 - 2. Accounting identities
 - 3. Market clearing
- Code: https://github.com/DREAM-DK/MAKRO

Empirical strategy

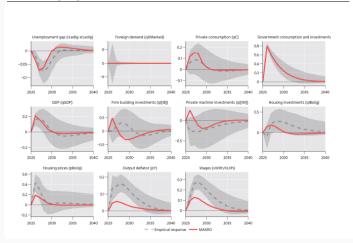
- Levels: Weights in production and utility functions. (directly observable in data, but changes over time...)
- 2. Long-run relationships: Substitution elasticities.
- 3. **Short-run dynamics:** Adjustment cost parameters. (especially focus on *convergence speed*)

Documentation:

Matching af impuls responser og øvrige kortsigtsmomenter (2021)

Impulse-response functions (IRFs)

Figur 1 Stød til offentlige udgifter



Also: Foreign demand, foreign interest rate, labor supply, oil price

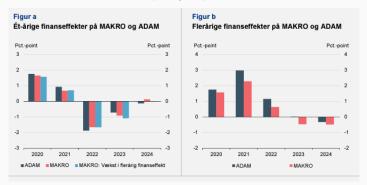
Micro-moments

Tabel 2 Yderligere relevant empiri til vurdering af MAKROs kortsigtsegenskaber

Analyse/moment	MAKRO og konsensus	Relevant litteratur
MPC ud af midlertidig/kortsigtet ind- komst, første år	MAKRO: ca. 0,45 Litteratur: 0,4-0,6	Jørgensen & Kuchler (2017), Crawley & Kuchler (2020), Kreiner et al (2019), ADAM, SMEC
MPC ud af boligprisstigninger/for- mue, første år	MAKRO: ca. 0,05 Litteratur: 0,03-0,06	Hviid & Kuchler (2017), Andersen & Leth-Petersen (2021)
Fortrængning af tvungen pensions- opsparing for 30-55 årige, år 1 [år 10]	MAKRO: Ca. 0,35-0,55 [0,10-0,35] Litteratur: 0-0,5 [0-0,5]	Arnberg & Barslund (2012), Chetty m.fl. (2014), Andersen, Hansen & Ku- chler (2021)
Rentefølsomhed, husholdningers <u>boligværdi</u> (stød til beskatning på aktie- og kapitalindkomst). Gns. 10 års-effekt.	MAKRO: knap -0,1 Litteratur: (-)0,25 – (-)0,18	Gruber, Jensen & Kleven (2021)*
Rentefølsomhed, husholdningers <u>for-mue</u> (stød til beskatning på aktie- og kapitalindkomst). 8-års effekt. [Lang- sigtet elasticitet]	MAKRO: ca. 0,2 [0,5] Litteratur: 0,2 – 0,4 [0,5 - 1]	Jakobsen, Jakobsen, Kleven & Zucman (2020)**

In practice: Finanseffekt

»Finanseffekt«: Is fiscal policy expansive or contractive wrt. GDP?



Kilde: Økonomisk Redegørelse, Marts 2023

BabyMAKRO

Small open economy in discrete time (annual freq., t)

- Demographics: Overlapping generations (age, a)
- Households: Hands-to-mouth (λ) and unconstrained (1 − λ) wrt. consumption-saving + supply labor exogenously
- Foreign economy: Fixed nominal rate of return + import goods at fixed prices + demand curve for the domestic export good.

Production:

- 1. Production firms rent capital + labor to produce the domestic output good.
- 2. Repacking firms combine imported goods with the domestic output good to produce a consumption good, an investment good, and an export good.
- 3. Capital is rented from a *capital agency*, which purchases the investment good to accumulate capital subject to adjustment costs.
- 4. Labor is rented from a *labor agency*, which post vacancies a search-and-match labor market to purchase labor from the households.
- Price of domestic output good: Price adjustment costs.
- Wage: Ad hoc bargaining.
- **Central bank** + **government**: Fixed exchange rate + taxation.

Foreign economy

- Nominal interest rate: r_t^{hh}
- Armington demand of the domestic exported good:

$$X_t = \gamma^X X_{t-1} + (1 - \gamma^X) \chi_t \left(\frac{P_t^X}{P_t^F} \right)^{-\sigma^F}.$$

■ Import goods: \bullet_t^M at prices P_t^{\bullet} for $\bullet \in \{C, G, I, X\}$

Demographic structure and population

- Life-span: #, hereof working #work
- Number of households: N_a
- Mortality rate: ζ_a (controlled by ζ)

$$egin{aligned} \mathcal{N}_{a} &= egin{cases} 1 & & ext{if } a = 0 \ (1 - \zeta_{a-1}) \mathcal{N}_{a-1} & & ext{if } a > 0 \end{cases} \ \zeta_{a} &= egin{cases} 0 & & ext{if } a < \#_{ ext{work}} \ \left(rac{a+1-\#_{ ext{work}}}{\#-\#_{ ext{work}}}
ight)^{\zeta} & & ext{if } a < \#-1 \ 1 & & ext{if } a = \#-1 \end{cases} \ \mathcal{N} &= \sum_{a=0}^{\#-1} \mathcal{N}_{a} \ \mathcal{N}_{ ext{work}} &= \sum_{a=0}^{\#-1} 1_{\{a < \#_{ ext{work}}\}} \mathcal{N}_{a}. \end{aligned}$$

Labor market flows

- Employed and unemployed: $L_{a,t}$ and $U_{a,t} = N_a L_{a,t}$
- Job-separation and finding rate: δ_a^L and m_t^s
- **Searchers** (everybody search = exogenous labor supply):

$$S_{a,t} = \begin{cases} 1 & \text{if } a = 0 \\ (1 - \zeta_{a-1}) \left[U_{a-1,t-1} + \delta_a^L L_{a-1,t-1} \right] & \text{if } a < \#_{\mathsf{work}} \\ 0 & \text{if } a \ge \#_{\mathsf{work}}. \end{cases}$$

Employed (before and after matching):

$$\underline{L}_{a,t} = \begin{cases} 0 & \text{if } a = 0 \\ (1 - \zeta_{a-1})(1 - \delta_a^L)L_{a-1,t-1} & \text{if } a < \#_{\text{work}} \\ 0 & \text{if } a \geq \#_{\text{work}} \end{cases}$$

$$L_{a,t} = \underline{L}_{a,t} + m_t^s S_{a,t}.$$

Search-and-matching

- Vacancies: v_t
- Searchers: $S_t = \sum_{a=0}^{\#-1} S_{a,t}$
- **Matches** by *matching function*:

$$\mathcal{M}_t = \frac{S_t v_t}{\left(S_t^{\frac{1}{\sigma^m}} + v_t^{\frac{1}{\sigma^m}}\right)^{\sigma^m}}.$$

- Job-finding rate: $m_t^s = \frac{\mathcal{M}_t}{S_t}$.
- **Job-filling rate:** $m_t^{\mathsf{v}} = \frac{\mathcal{M}_t}{\mathsf{v}_t}$.

Wage bargaining

Unmodeled wage bargaining mechanism give

$$W_t = W_{ss} \left(\frac{L_t}{L_{ss}} \right)^{\epsilon_w},$$

where the wage is increasing in the labor demand for $\epsilon_w > 0$.

Note: Theoretically there is a bargaining set

1. Upper limit: Firm will fire worker

2. Lower limit: Household will leave firm

Often: Nash-bargaining = »surplus split from bargaining power«

Income

- 1. Post-tax labor income: $(1 \tau_t) W_t \frac{L_{a,t}}{N_a}$
- 2. Post-tax unemployment benefits: $(1-\tau_t) W^U W_{ss} \frac{U_{a,t}}{N_a}$
- 3. Post-tax retirement benefits: $(1 \tau_t) W^R W_{ss} \frac{N_a (L_{a,t} + U_{a,t})}{N_a}$
- 4. Equally divided inheritance: $\frac{A_q^t}{N}$
- The age specific income is

$$\begin{split} &\operatorname{inc}_{a,t} = \left(1 - au_t\right) W_t rac{L_{a,t}}{N_a} + \left(1 - au_t\right) W^U W_{\operatorname{ss}} rac{U_{a,t}}{N_a} \\ &+ \left(1 - au_t\right) W^R W_{\operatorname{ss}} \mathbf{1}_{\left\{a \geq \#_{\operatorname{work}}
ight\}} + rac{A_t^q}{N}. \end{split}$$

Hand-to-mouth households (HtM)

Consume all income:

$$C_{a,t}^{\mathsf{HtM}} = \frac{\mathsf{inc}_{a,t}}{P_{c}^{C}}.$$

where the price of consumption goods is P_t^{C} .

No savings:

$$A_{a,t}^{\mathsf{HtM}}=0.$$

Unconstrained household (Ricardian) I

Utility from consumption:

$$\frac{\left(C_{a,t}^{R}\right)^{1-\sigma}}{1-\sigma}, \ \sigma > 0, \sigma \neq 1.$$

Utility from bequest:

$$\zeta_{\partial}\mu^{A^{q}}\frac{\left(\frac{A_{s,t}^{R}}{P_{t}^{C}}\right)^{1-\sigma}}{1-\sigma}, \ \mu^{A^{q}}>0.$$

Discounting of future utility:

$$\beta(1-\zeta_a), \ \beta>0.$$

• Budget constraint: $A_{-1,t}^R = 0$ and

$$A_{a,t}^R = (1 + r_t^{hh})A_{a-1,t-1}^R + \mathsf{inc}_{a,t} - P_t^C C_{a,t}^R.$$

Unconstrained household (Ricardian) II

- Cohort: t₀ (time of first decision)
- Full problem

$$V_{t_0} = \max_{\left\{C_{a,t}^R\right\}_{a=0}^{\#-1}} \sum_{a=0}^{\#-1} \left(\Pi_{j=1}^a \beta (1-\zeta_{j-1}) \right) \left[\frac{\left(C_{a,t}^R\right)^{1-\sigma}}{1-\sigma} + \zeta_a \mu^{A^q} \frac{\left(\frac{A_{a,t}^R}{P_t^C}\right)^{1-\sigma}}{1-\sigma} \right]$$

s.t.

$$t = t_0 + a$$

$$A_{-1,t}^{R}=0$$

$$A_{a,t}^{R} = (1 + r_{t}^{hh})A_{a-1,t-1}^{R} + \mathrm{inc}_{a,t} - P_{t}^{C}C_{a,t}^{R}.$$

Unconstrained household (Ricardian) III

First order conditions:

$$C_{a,t}^{R} = \begin{cases} \left(\zeta_{a} \mu^{A^{q}} \left(\frac{A_{\#-1,t}^{R}}{P_{t}^{C}} \right)^{-\sigma} \right)^{-\frac{1}{\sigma}} & \text{if } a = \#-1 \\ \left(\beta (1 - \zeta_{a}) \frac{1 + r_{Sh}^{hh}}{1 + \pi_{Sh}^{hh}} \left(C_{a+1,ss}^{R} \right)^{-\sigma} + \zeta_{a} \mu^{A^{q}} \left(\frac{A_{a,ss}^{R}}{P_{ss}^{C}} \right)^{-\sigma} \right)^{-\frac{1}{\sigma}} & \text{elif } t = T - 1 \\ \left(\beta (1 - \zeta_{a}) \frac{1 + r_{t+1}^{hh}}{1 + \pi_{t+1}^{hh}} \left(C_{a+1,t+1}^{R} \right)^{-\sigma} + \zeta_{a} \mu^{A^{q}} \left(\frac{A_{a,t}^{R}}{P_{t}^{C}} \right)^{-\sigma} \right)^{-\frac{1}{\sigma}} & \text{else} \end{cases}$$

Household aggregation

Age:

$$\begin{split} C_{a,t} &= \lambda C_{a,t}^{\mathsf{HtM}} + (1 - \lambda) C_{a,t}^R \\ A_{a,t} &= \lambda A_{a,t}^{\mathsf{HtM}} + (1 - \lambda) A_{a,t}^R, \end{split}$$

Total:

$$C_t = \sum_{a=0}^{\#-1} N_a C_{a,t}$$
 $A_t = \sum_{a=0}^{\#-1} N_a C_{a,t}.$

Bequests:

$$A_{t}^{q} = \left(1 + r_{t}^{hh}\right) \sum_{a=0}^{\#-1} \zeta_{a} N_{a} A_{a,t}.$$

Production firms I

- Exogenous technology: Γ_t
- Capital: K_{t-1} , at rental price r_t^K
- Labor: ℓ_t , at rental price r_t^{ℓ}
- Output: Y_t , with a CES technology, sold at $P_t^{Y,0}$
- Profit maximization with prices taken as given

$$\Pi_t = \max_{K_{t-1}, \ell_t} P_t^{Y,0} Y_t - r_t^K K_{t-1} - r_t^\ell \ell_t$$
s.t.

$$Y_t = \Gamma_t \left(\left(\mu^K \right)^{\frac{1}{\sigma^Y}} K_{t-1}^{\frac{\sigma^Y - 1}{\sigma^Y}} + \left(1 - \mu^K \right)^{\frac{1}{\sigma^Y}} \ell_t^{\frac{\sigma^Y - 1}{\sigma^Y}} \right)^{\frac{\sigma^Y}{\sigma^Y - 1}},$$

where
$$\mu^K$$
, σ^Y , $\sigma^Y \neq 1$.

Production firms II

• Free entry implies zero profit:

$$P_t^{Y,0} = \frac{1}{\Gamma_t} \left(\mu^K \left(r_t^K \right)^{1-\sigma^Y} + \left(1 - \mu^K \right) \left(r_t^\ell \right)^{1-\sigma^Y} \right)^{\frac{1}{1-\sigma^Y}}.$$

First order condition for capital-labor ratio:

$$\frac{K_{t-1}}{\ell_t} = \frac{\mu_K}{1 - \mu_K} \left(\frac{r_t^\ell}{r_t^K}\right)^{\sigma^Y}$$

Capital agency I

- Investment good: I_t , at price, P_t^I .
- **Capital:** K_t , rented out at rental rate r_{t+1}^K in the following period.
- Adjustment costs: Effective investment is ι_t.
- Required internal rate of return: r^{firm} .
- Profit maximization:

$$\begin{split} V_0^{\mathsf{capital}}\left(\mathcal{K}_{t-1}\right) &= \max_{\{\mathcal{K}_t\}} \sum_{t=0}^{\infty} \left(1 + r^{\mathsf{firm}}\right)^{-t} \left[r_t^K \mathcal{K}_{t-1} - P_t^I \left(\iota_t + \Psi(\iota_t, \mathcal{K}_{t-1})\right)\right] \\ &\text{s.t.} \\ I_t &= \iota_t + \Psi(\iota_t, \mathcal{K}_{t-1}) \\ \mathcal{K}_t &= (1 - \delta^K) \mathcal{K}_{t-1} + \iota_t. \end{split}$$

Capital agency II

Functional form:

$$\Psi(\iota_t, K_{t-1}) = \frac{\Psi_0}{2} \left(\frac{\iota_t}{K_{t-1}} - \delta^K \right)^2 K_{t-1}.$$

First order condition:

$$\begin{split} 0 &= - \, P_t^I \big(1 + \Psi_\iota \left(\iota_t, K_{t-1} \right) \big) \\ &+ \frac{r_{t+1}^k + P_{t+1}^I \big(1 - \delta^K \big) \big(1 + \Psi_\iota \big(\iota_{t+1}, K_t \big) \big) - P_{t+1}^I \Psi_K \left(\iota_{t+1}, K_t \right)}{1 + r^{\text{firm}}} \end{split}$$

Labor agency I

- Post vacancies: v_t at cost κ^L (in units of labor).
- Labor: Hires L_t and rent out labor at rental price r_t^ℓ
- **Exogenous match destruction:** δ_t^L (implied by $\delta_{a,t}^L$ and $L_{a,t-1}$)
- Exogenous wage: W_t
- Profit maximization:

$$\begin{split} V_0^{\mathsf{labor}}(L_{t-1}) &= \max_{\left\{v_t\right\}} \sum_{t=0}^{\infty} \left(1 + r^{\mathsf{firm}}\right)^{-t} \left[r_t^\ell \ell_t - W_t L_t\right] \\ \text{s.t.} \\ L_t &= m_t^{\mathsf{v}} v_t + \left(1 - \delta_t^L\right) L_{t-1} \\ \ell_t &= L_t - \kappa^L v_t. \end{split}$$

Labor agency II

First order condition:

$$r_t^\ell = \frac{1}{1 - \frac{\kappa^L}{m_t^{\mathsf{v}}}} \left[W_t - r_{t+1}^\ell \frac{1 - \delta_{t+1}^L}{1 + r^{\mathsf{firm}}} \frac{\kappa^L}{m_{t+1}^{\mathsf{v}}} \right]$$

Price setting I

• Input price: $P_t^{Y,0}$

Output of differentiated goods: y_t at p_t^Y

■ Demand schedule: $y_t = \left(\frac{p_t^Y}{P_t^Y}\right)^{-\sigma_D} Y_t$

• **Aggregate:** Y_t with price index P_t^Y

Profit maximization:

$$\begin{split} V_t^{\text{intermediary}} &= \max_{\left\{ p_t^Y \right\}} \left(p_t^Y - P_t^{Y,0} \right) y_t - \vartheta_t + \frac{1}{1 + r^{\text{firm}}} V_{t+1}^{\text{intermediary}} \\ &\text{s.t.} \\ \vartheta_t &= \frac{\gamma}{2} \left[\frac{p_t^Y / p_{t-1}^Y}{p_{t-1}^Y / p_{t-2}^Y} - 1 \right]^2 P_t^Y Y_t \\ y_t &= \left(\frac{p_t^Y}{P_t^Y} \right)^{-\sigma_D} Y_t. \end{split}$$

Price setting II

- Assumption: Symmetric firms
- First order condition (New Keynesian Phillips Curve):

$$\begin{split} P_{t}^{Y} = & (1+\theta)P_{t}^{Y,0} - \eta \left(\frac{P_{t}^{Y}/P_{t-1}^{Y}}{P_{t-1}^{Y}/P_{t-2}^{Y}} - 1\right) \frac{P_{t}^{Y}/P_{t-1}^{Y}}{P_{t-1}^{Y}/P_{t-2}^{Y}} P_{t}^{Y} \\ & + \frac{2\eta}{1 + r^{\text{firm}}} \frac{Y_{t+1}}{Y_{t}} \left(\frac{P_{t+1}^{Y}/P_{t}^{Y}}{P_{t}^{Y}/P_{t-1}^{Y}} - 1\right) \frac{P_{t+1}^{Y}/P_{t}^{Y}}{P_{t}^{Y}/P_{t-1}^{Y}} P_{t+1}^{Y} \\ & \theta \equiv \frac{1}{\sigma_{D} - 1} \\ & \eta \equiv \theta \gamma. \end{split}$$

Repacking firms

- Output goods: \bullet_t^Y at prices P_t^{\bullet} for $\bullet \in \{C, G, I, X\}$
- Domestic input good: Y_t at price P_t^Y
- Foreign input goods: \bullet_t^M at prices P_t^{\bullet} for $\bullet \in \{C, G, I, X\}$
- Profit maximization with CES production technology implies

$$\begin{split} P_t^{\bullet} &= \left(\mu^{M,\bullet} \left(P_t^{M,\bullet}\right)^{1-\sigma^{\bullet}} + \left(1-\mu^{M,\bullet}\right) \left(P_t^{Y}\right)^{1-\sigma^{\bullet}}\right)^{\frac{1}{1-\sigma^{\bullet}}} \\ \bullet_t^{M} &= \mu^{M,\bullet} \left(\frac{P_t^{\bullet}}{P_t^{M,\bullet}}\right)^{\sigma^{\bullet}} \bullet_t \\ \bullet_t^{Y} &= \left(1-\mu^{M,\bullet}\right) \left(\frac{P_t^{\bullet}}{P_t^{Y}}\right)^{\sigma^{\bullet}} \bullet_t, \end{split}$$
 for $\bullet \in \{C, G, I, X\}$

Government

- Interest rate: r^B
- Government consumption: *G_t*
- Unemployment insurance expenses: $E_t^U = W_U W_{ss} U_t$
- Retirement benefits expenses: $E^R = W_R W_{ss} (N N_{work})$
- Tax base: $T_t = W_t L_t + E_t^U + E^R$
- Budget constraint:

$$B_t = (1 + r^B)B_{t-1} + P_t^G G_t + E_t^U + E^R - \tau_t T_t.$$

Tax policy:

$$\begin{split} \tau_t &= \tau_{ss} + \varepsilon_B \frac{\tilde{B}_t - B_{ss}}{T_t}, \\ \tilde{B}_t &= (1 + r^B) B_{t-1} + P_t^G G_t + E_t^U + E^R - \tau_{ss} T_t. \end{split}$$

Goods market clearing

1. Demand for domestically produced goods:

$$Y_t = \sum_{\bullet \in \{C,G,I,X\}} \bullet_t^Y.$$

2. Imports add up:

$$M_t = \sum_{\bullet \in \{C,G,I,X\}} \bullet_t^M.$$

[Current account and net foreign asset position]

Not necessary to specify

Not specified yet



Solution method

Targets and unknowns

- **Goal:** Find the *equilibrium path* in the economy.
- **Equilibrium path:** A set of paths for all variables where
 - 1. Optimal firm and household behavior in terms of FOCs.
 - 2. Accounting identities.
 - 3. Market clearing.
- **Target equations:** Must be zero on the equilibrium path.
- Unknown variables:
 - 1. Chosen by model-builder.
 - 2. All other variables must be derived from these.
 - 3. Target equations can be evaluated.
- Ordered series of blocks:

Inputs: Unknown and exogenous variables or outputs of previous blocks **Outputs:** Variables and errors in target equations

• **Truncation:** Assume back in steady state after *T* periods.

Exogenous variables

- 1. Γ_t , technology
- 2. G_t , public spending
- 3. χ_t , foreign demand shifter (»market size«)
- 4. $P_t^{M,C}$, import price of *private* consumption good
- 5. $P_t^{M,G}$, import price of *public* consumption good
- 6. $P_t^{M,I}$, import price of *investment* good
- 7. $P_t^{M,X}$, import price of *export* good
- 8. P_t^F , foreign price level
- 9. r_t^{hh} , foreign interest rate

Unknowns in practice

- 1. A_t^q , inheritance flow (T unknowns)
- 2. A_t^{death} , wealth of households at a=#-1 (T unknowns)
- 3. K_t , capital (T unknowns)
- 4. L_t , labor supply (T unknowns)
- 5. r_t^K , rental price for capital (T unknowns)
- 6. P_t^Y , price of domestic output (T unknowns)
- \Rightarrow all other variables in model can be derived and target equations can be evaluated (see code)

Solving numerically

- Solve with Newton's method:
 - 1. x is a $6 \times T$ vector containing all 6 unknown in all T periods.
 - 2. f is a $6 \times T$ vector containing all 6 target in all T periods.
 - 3. \mathcal{J} is the Jacobian of f (the derivative of f) eval. in steady state.

Compute next guess, x_{n+1} , as

$$x_{n+1}=x_n-\frac{f(x_n)}{\mathcal{J}_n}.$$

Initial guess: Steady state

Converges to solution, $f(x^*) = 0$, as n grows.

- Implementation: In Python using a quasi-Newton solver (broyden)
- Course: Introduction to Programming and Numerical Analysis

Calibration

Steady state

Goals:

- 1. National account shares and ratios.
- 2. Unemployment level and labor market flows.
- 3. Life-cycle profiles of income, consumption and savings.

Simplify: Zero inflation, zero growth and constant demography.

Notebooks:

- 0b steady state data.ipynb
- Oc steady state aggregate.ipynb
- 0d steady state households.ipynb
- Status: Very ad hoc still.

Dynamics

- Substitution elasticities: Use those from MAKRO.
- Impulse-responses: Match to data independently.
- Status: Not done \rightarrow example of government spending shock.



Impulse-responses

An impulse response:

- The economy starts at steady state.
- 2. Some exogenous variables temporarily deviate from steady state.
- 3. The impulse responses: How variables respond to the shock.

■ Shock to government spending, G_t:

$$G_t = G_{ss} + shock_t$$
 $shock_t = egin{cases} G_{ss} \cdot size \cdot persistence^{t-T_{start}} & ext{if } T_{start} \leq t < T_{end} \ 0 & ext{else} \end{cases}$

size = 0.01 (initial deviation from steady state at 1 percent) shock starts in $T_{start} = 0$ and ends in $T_{end} = 50$. persistence = 0.8 (shock fades 20 percent per year)

Initial effects I

- Repacking firms: Demand for domestically produced goods Y ↑ and imported goods M ↑.
- **Production firms:** Increases inputs to increase production $Y \uparrow$. Capital $K \uparrow$ (limited by adjustment costs) and labor $\ell \uparrow$.
- Households: More income, greater consumption C↑.
 Real wage W↑ due to increased labor demand.
 Hand-to-mouth C^{HtM}↑: Increase proportional to increase in W.
 Ricardian C^R↓: Due to consumption smoothing.
- Prices: Higher input prices causes higher prices P↑:
 W↑ and r^K↑ drives up marginal costs P^{Y,0}↑.
 Output prices P^Y↑ (limited by adjustment costs).
 Causes higher repacking prices P[•]↑ for ∈ {C, I, G, X}.
- Foreign economy: Exports $X \downarrow$ due to increasing prices $P^X \uparrow$.

Initial effects II

Labor agency: Employment L↑ and rents ℓ↑ to production firms.
 Job vacancies v↑ to meet higher labor demand.
 Number of matches M↑, »gross« labor L↑.
 Labor ℓ↑ (limited by adjustment costs).

Capital agency: Investments I ↑ to accumulate capital K ↑.
 Expensive investments: P^I ↑

Greater return: $r^K \uparrow$

Transition path

Government:

Government spending $G \downarrow$ by assumption.

Tax rate $\tau \uparrow$ to finance growing government debt $B \uparrow$.

Repacking firms:

Demand for domestic goods $Y \downarrow$ and imports $M \downarrow$. Prices $P \downarrow$.

Production firms:

Drop in production inputs, labor $\ell \downarrow$ and capital $K \downarrow$.

Households:

 $\ell \downarrow$ causes wages $W \downarrow$. Higher taxes from $\tau \uparrow$.

Leads to less disposable income and thus $C\downarrow$.

Foreign economy:

Exports $X \uparrow$ due to falling prices $P^X \downarrow$.



Conclusion

Insights and takeaways

Your takeaways:

- 1. How a micro-founded macro-model is structured
- 2. Despite their complexity you can (soon) work with such models
- 3. Requires both analytical and numerical skills to master
- Open source: github.com/JeppeDruedahl/BabyMAKRO Will you participate in improving it?
- Related courses:

Macro III

Introduction to Programming and Numerical Analysis
Advanced Macroeconomics: Heterogenous Agent Models

Tasks

- 1. Clarify steady state calibration.
- 2. Perform impulse-response matching.
- 3. Improve detailed accounting of main shocks.
- 4. Add technology growth, population growth, and trend inflation.
- 5. Add financial flows accounts wrt. to the foreign economy.
- 6. Add more government with taxes and spending.
- 7. Add endogenous labor supply.
- 8. Add multiple sectors and input-output structure.
- 9. Make the model quarterly.