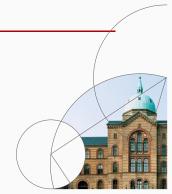
CENTER FOR ECONOMIC BEHAVIOR & INEQUALITY



# BabyMAKRO

Spring 2023

Jeppe Druedahl



#### Introduction

- MAKRO: New model developed by DREAM and now in use by the Danish Finance Ministry.
  - 1. Framework for business cycle forecasts
  - 2. Medium and long-run projections
  - 3. Evaluations of potential economic policies and shocks
- Today BabyMAKRO: Smaller-but-still-big model in the same style for students to use in classes and theses (bachelor + master).
  - 1. **Foundations:** Me (help from Peter Stephensen + Martin Bonde).
  - Alpha-version: Bachelor thesis students fall 2022. (Andreas Marius Laursen, Hans Christian Jul Lehmann, Mathias Held Berg, Nicholas Stampe Meier, Olivier Ding, Ufuk Yasin)
  - 3. **Beta-version**: RAs Anders Buch Jürs + Andreas Marius Laursen.
  - 4. Further versions: More student involvement. You?

[still work-in-progress]

#### Plan

- 1. Classes of Macro Models
- 2. MAKRO
- 3. BabyMAKRO
- 4. Solution method
- 5. Calibration
- 6. Government spending shock
- 7. Conclusion

**Classes of Macro Models** 

### Reminder: Consumption behavior (chapter 16)

1. Maximize utility s.t. intertemporal budget constraint:

utility: 
$$u(C_1) + \frac{u(C_2)}{1+\phi}$$

budget constraint: 
$$C_1 + \frac{C_2}{1+r} = V_1 + Y_1^d + \frac{(1+g)Y_1^d}{1+r}$$

- ⇒ derive qualitative (signed) properties of optimal micro-behavior
- 2. Aggregate consumption function:  $C_1 = C(Y_1^d, g, r, V_1) + (-1) +$
- 3. Estimate equation on aggregate data
- 4. Put equation into model

Note: Similar for investment (chapter 15).

#### Classes of Macro Models

#### 1. Old-style Keynesian macro-models (1950s-):

Structure: Aggregate equations »similar« to those in micro-theory

Estimation: Equation-by-equation on aggregate data

In teaching: AS-AD models

In practice: ADAM + MONA + SMEC

(»Den økonomiske genopretning 1976-1993«, Jørgen Rosted, 2021)

#### 2. Micro-founded macro-models (1970s-):

**Structure:** Exactly the equations in micro-theory (in general equilibrium)

**Estimation:** Calibration vs. moment-matching vs. full-system

In teaching: Dynamic Stochastic General Equilibrium Models (DSGE)

RBC: Real Business Cycle (1980s-)

RANK: Representative Agent New Keynesian (1990s-) HANK: Heterogeneous Agent New Keynesian (2010s-)

In practice: DREAM + DSGE at Nationalbanken + MAKRO

Blanchard: On the Need for (At Least) Five Classes of Macro Models

# MAKRO

### Structure of small open economy

#### Agents:

- 1. Unconstrainted households (»Ricardian«): One for each cohort
- 2. Hand-to-mouth households (»HtM«): One for each cohort
- 3. Firms (production, price setting, multiple sectors incl. housing)
- 4. Central bank (fixed exchange-rate)
- 5. Government (detailed sub-accounts)
- 6. Global foreign economy (exogenous)
- **Expectations:** Perfect foresight
- Market clearing: Walras + sticky prices + search-and-match
- Mathematically: Non-linear equation system
  - 1. Behavioral equations in terms of first order conditions
  - 2. Accounting identities
  - 3. Market clearing
- Code: <a href="https://github.com/DREAM-DK/MAKRO">https://github.com/DREAM-DK/MAKRO</a>

### **Empirical strategy**

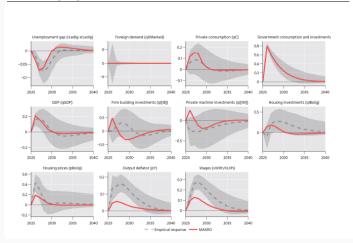
- Levels: Weights in production and utility functions. (directly observable in data, but changes over time...)
- 2. Long-run relationships: Substitution elasticities.
- 3. **Short-run dynamics:** Adjustment cost parameters. (especially focus on *convergence speed*)

#### **Documentation:**

Matching af impuls responser og øvrige kortsigtsmomenter (2021)

### Impulse-response functions (IRFs)

Figur 1 Stød til offentlige udgifter



Also: Foreign demand, foreign interest rate, labor supply, oil price

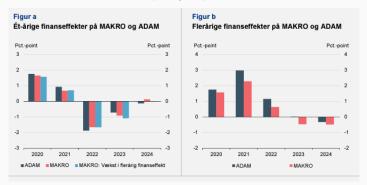
#### **Micro-moments**

Tabel 2 Yderligere relevant empiri til vurdering af MAKROs kortsigtsegenskaber

Analyse/moment	MAKRO og konsensus	Relevant litteratur
MPC ud af midlertidig/kortsigtet ind- komst, første år	MAKRO: ca. 0,45 Litteratur: 0,4-0,6	Jørgensen & Kuchler (2017), Crawley & Kuchler (2020), Kreiner et al (2019), ADAM, SMEC
MPC ud af boligprisstigninger/for- mue, første år	MAKRO: ca. 0,05 Litteratur: 0,03-0,06	Hviid & Kuchler (2017), Andersen & Leth-Petersen (2021)
Fortrængning af tvungen pensions- opsparing for 30-55 årige, år 1 [år 10]	MAKRO: Ca. 0,35-0,55 [0,10-0,35] Litteratur: 0-0,5 [0-0,5]	Arnberg & Barslund (2012), Chetty m.fl. (2014), Andersen, Hansen & Ku- chler (2021)
Rentefølsomhed, husholdningers <u>boligværdi</u> (stød til beskatning på aktie- og kapitalindkomst). Gns. 10 års-effekt.	MAKRO: knap -0,1 Litteratur: (-)0,25 – (-)0,18	Gruber, Jensen & Kleven (2021)*
Rentefølsomhed, husholdningers <u>for-mue</u> (stød til beskatning på aktie- og kapitalindkomst). 8-års effekt. [Lang- sigtet elasticitet]	MAKRO: ca. 0,2 [0,5] Litteratur: 0,2 – 0,4 [0,5 - 1]	Jakobsen, Jakobsen, Kleven & Zucman (2020)**

#### In practice: Finanseffekt

»Finanseffekt«: Is fiscal policy expansive or contractive wrt. GDP?



Kilde: Økonomisk Redegørelse, Marts 2023

# **BabyMAKRO**

### Small open economy in discrete time (annual freq., t)

- Demographics: Overlapping generations (age, a)
- Households: Hands-to-mouth ( $\lambda$ ) and unconstrained (1 −  $\lambda$ ) wrt. consumption-saving + supply labor exogenously
- Foreign economy: Fixed nominal rate of return + import goods at fixed prices + demand curve for the domestic export good.

#### Production:

- 1. Production firms rent capital + labor to produce the domestic output good.
- 2. Repacking firms combine imported goods with the domestic output good to produce a consumption good, an investment good, and an export good.
- 3. Capital is rented from a *capital agency*, which purchases the investment good to accumulate capital subject to adjustment costs.
- 4. Labor is rented from a *labor agency*, which post vacancies a search-and-match labor market to purchase labor from the households.
- Price of domestic output good: Price adjustment costs.
- Wage: Ad hoc bargaining.
- **Central bank** + **government**: Fixed exchange rate + taxation.

### Foreign economy

- Nominal interest rate:  $r_t^{hh}$
- Armington demand of the domestic exported good:

$$X_t = \gamma^X X_{t-1} + (1 - \gamma^X) \chi_t \left( \frac{P_t^X}{P_t^F} \right)^{-\sigma^F}.$$

■ Import goods:  $\bullet_t^M$  at prices  $P_t^{\bullet}$  for  $\bullet \in \{C, G, I, X\}$ 

### Demographic structure and population

- Life-span: #, hereof working #work
- Number of households: N<sub>a</sub>
- Mortality rate:  $\zeta_a$  (controlled by  $\zeta$ )

$$egin{aligned} \mathcal{N}_{a} &= egin{cases} 1 & & ext{if } a = 0 \ (1 - \zeta_{a-1}) \mathcal{N}_{a-1} & & ext{if } a > 0 \end{cases} \ \zeta_{a} &= egin{cases} 0 & & ext{if } a < \#_{ ext{work}} \ \left(rac{a+1-\#_{ ext{work}}}{\#-\#_{ ext{work}}}
ight)^{\zeta} & & ext{if } a < \#-1 \ 1 & & ext{if } a = \#-1 \end{cases} \ \mathcal{N} &= \sum_{a=0}^{\#-1} \mathcal{N}_{a} \ \mathcal{N}_{ ext{work}} &= \sum_{a=0}^{\#-1} 1_{\{a < \#_{ ext{work}}\}} \mathcal{N}_{a}. \end{aligned}$$

#### Labor market flows

- Employed and unemployed:  $L_{a,t}$  and  $U_{a,t} = N_a L_{a,t}$
- Job-separation and finding rate:  $\delta_a^L$  and  $m_t^s$
- **Searchers** (everybody search = exogenous labor supply):

$$S_{a,t} = \begin{cases} 1 & \text{if } a = 0 \\ (1 - \zeta_{a-1}) \left[ U_{a-1,t-1} + \delta_a^L L_{a-1,t-1} \right] & \text{if } a < \#_{\mathsf{work}} \\ 0 & \text{if } a \ge \#_{\mathsf{work}}. \end{cases}$$

Employed (before and after matching):

$$\underline{L}_{a,t} = \begin{cases} 0 & \text{if } a = 0 \\ (1 - \zeta_{a-1})(1 - \delta_a^L)L_{a-1,t-1} & \text{if } a < \#_{\text{work}} \\ 0 & \text{if } a \geq \#_{\text{work}} \end{cases}$$
 
$$L_{a,t} = \underline{L}_{a,t} + m_t^s S_{a,t}.$$

### **Search-and-matching**

- Vacancies: v<sub>t</sub>
- Searchers:  $S_t = \sum_{a=0}^{\#-1} S_{a,t}$
- **Matches** by *matching function*:

$$\mathcal{M}_t = \frac{S_t v_t}{\left(S_t^{\frac{1}{\sigma^m}} + v_t^{\frac{1}{\sigma^m}}\right)^{\sigma^m}}.$$

- Job-finding rate:  $m_t^s = \frac{\mathcal{M}_t}{S_t}$ .
- **Job-filling rate:**  $m_t^{\mathsf{v}} = \frac{\mathcal{M}_t}{\mathsf{v}_t}$ .

### Wage bargaining

Unmodeled wage bargaining mechanism give

$$W_t = W_{ss} \left( \frac{L_t}{L_{ss}} \right)^{\epsilon_w},$$

where the wage is increasing in the labor demand for  $\epsilon_w > 0$ .

Note: Theoretically there is a bargaining set

1. Upper limit: Firm will fire worker

2. Lower limit: Household will leave firm

Often: Nash-bargaining = »surplus split from bargaining power«

#### Income

- 1. Post-tax labor income:  $(1 \tau_t) W_t \frac{L_{a,t}}{N_a}$
- 2. Post-tax unemployment benefits:  $(1-\tau_t) W^U W_{ss} \frac{U_{a,t}}{N_a}$
- 3. Post-tax retirement benefits:  $(1 \tau_t) W^R W_{ss} \frac{N_a (L_{a,t} + U_{a,t})}{N_a}$
- 4. Equally divided inheritance:  $\frac{A_q^t}{N}$
- The age specific income is

$$\begin{split} &\operatorname{inc}_{a,t} = \left(1 - au_t\right) W_t rac{L_{a,t}}{N_a} + \left(1 - au_t\right) W^U W_{\operatorname{ss}} rac{U_{a,t}}{N_a} \\ &+ \left(1 - au_t\right) W^R W_{\operatorname{ss}} \mathbf{1}_{\left\{a \geq \#_{\operatorname{work}}
ight\}} + rac{A_t^q}{N}. \end{split}$$

### Hand-to-mouth households (HtM)

Consume all income:

$$C_{a,t}^{\mathsf{HtM}} = \frac{\mathsf{inc}_{a,t}}{P_{c}^{C}}.$$

where the price of consumption goods is  $P_t^{C}$ .

No savings:

$$A_{a,t}^{\mathsf{HtM}}=0.$$

## Unconstrained household (Ricardian) I

Utility from consumption:

$$\frac{\left(C_{a,t}^{R}\right)^{1-\sigma}}{1-\sigma}, \ \sigma > 0, \sigma \neq 1.$$

Utility from bequest:

$$\zeta_{\partial}\mu^{A^{q}}\frac{\left(\frac{A_{s,t}^{R}}{P_{t}^{C}}\right)^{1-\sigma}}{1-\sigma}, \ \mu^{A^{q}}>0.$$

Discounting of future utility:

$$\beta(1-\zeta_a), \ \beta>0.$$

• Budget constraint:  $A_{-1,t}^R = 0$  and

$$A_{a,t}^R = (1 + r_t^{hh})A_{a-1,t-1}^R + \mathsf{inc}_{a,t} - P_t^C C_{a,t}^R.$$

### Unconstrained household (Ricardian) II

- Cohort: t<sub>0</sub> (time of first decision)
- Full problem

$$V_{t_0} = \max_{\left\{C_{a,t}^R\right\}_{a=0}^{\#-1}} \sum_{a=0}^{\#-1} \left( \Pi_{j=1}^a \beta (1-\zeta_{j-1}) \right) \left[ \frac{\left(C_{a,t}^R\right)^{1-\sigma}}{1-\sigma} + \zeta_a \mu^{A^q} \frac{\left(\frac{A_{a,t}^R}{P_t^C}\right)^{1-\sigma}}{1-\sigma} \right]$$

s.t.

$$t = t_0 + a$$

$$A_{-1,t}^{R}=0$$

$$A_{a,t}^{R} = (1 + r_{t}^{hh})A_{a-1,t-1}^{R} + \mathrm{inc}_{a,t} - P_{t}^{C}C_{a,t}^{R}.$$

### Unconstrained household (Ricardian) III

#### First order conditions:

$$C_{a,t}^{R} = \begin{cases} \left( \zeta_{a} \mu^{A^{q}} \left( \frac{A_{\#-1,t}^{R}}{P_{t}^{C}} \right)^{-\sigma} \right)^{-\frac{1}{\sigma}} & \text{if } a = \#-1 \\ \left( \beta (1 - \zeta_{a}) \frac{1 + r_{Sh}^{hh}}{1 + \pi_{Sh}^{hh}} \left( C_{a+1,ss}^{R} \right)^{-\sigma} + \zeta_{a} \mu^{A^{q}} \left( \frac{A_{a,ss}^{R}}{P_{ss}^{C}} \right)^{-\sigma} \right)^{-\frac{1}{\sigma}} & \text{elif } t = T - 1 \\ \left( \beta (1 - \zeta_{a}) \frac{1 + r_{t+1}^{hh}}{1 + \pi_{t+1}^{hh}} \left( C_{a+1,t+1}^{R} \right)^{-\sigma} + \zeta_{a} \mu^{A^{q}} \left( \frac{A_{a,t}^{R}}{P_{t}^{C}} \right)^{-\sigma} \right)^{-\frac{1}{\sigma}} & \text{else} \end{cases}$$

## Household aggregation

Age:

$$\begin{split} C_{a,t} &= \lambda C_{a,t}^{\mathsf{HtM}} + (1 - \lambda) C_{a,t}^R \\ A_{a,t} &= \lambda A_{a,t}^{\mathsf{HtM}} + (1 - \lambda) A_{a,t}^R, \end{split}$$

Total:

$$C_t = \sum_{a=0}^{\#-1} N_a C_{a,t}$$
  $A_t = \sum_{a=0}^{\#-1} N_a C_{a,t}.$ 

Bequests:

$$A_{t}^{q} = \left(1 + r_{t}^{hh}\right) \sum_{a=0}^{\#-1} \zeta_{a} N_{a} A_{a,t}.$$

#### Production firms I

- Exogenous technology: Γ<sub>t</sub>
- Capital:  $K_{t-1}$ , at rental price  $r_t^K$
- Labor:  $\ell_t$ , at rental price  $r_t^{\ell}$
- Output:  $Y_t$ , with a CES technology, sold at  $P_t^{Y,0}$
- Profit maximization with prices taken as given

$$\Pi_t = \max_{K_{t-1}, \ell_t} P_t^{Y,0} Y_t - r_t^K K_{t-1} - r_t^\ell \ell_t$$
s.t.

$$Y_t = \Gamma_t \left( \left( \mu^K \right)^{\frac{1}{\sigma^Y}} K_{t-1}^{\frac{\sigma^Y - 1}{\sigma^Y}} + \left( 1 - \mu^K \right)^{\frac{1}{\sigma^Y}} \ell_t^{\frac{\sigma^Y - 1}{\sigma^Y}} \right)^{\frac{\sigma^Y}{\sigma^Y - 1}},$$

where 
$$\mu^K$$
,  $\sigma^Y$ ,  $\sigma^Y \neq 1$ .

#### Production firms II

• Free entry implies zero profit:

$$P_t^{Y,0} = \frac{1}{\Gamma_t} \left( \mu^K \left( r_t^K \right)^{1-\sigma^Y} + \left( 1 - \mu^K \right) \left( r_t^\ell \right)^{1-\sigma^Y} \right)^{\frac{1}{1-\sigma^Y}}.$$

First order condition for capital-labor ratio:

$$\frac{K_{t-1}}{\ell_t} = \frac{\mu_K}{1 - \mu_K} \left(\frac{r_t^\ell}{r_t^K}\right)^{\sigma^Y}$$

### Capital agency I

- Investment good:  $I_t$ , at price,  $P_t^I$ .
- **Capital:**  $K_t$ , rented out at rental rate  $r_{t+1}^K$  in the following period.
- Adjustment costs: Effective investment is ι<sub>t</sub>.
- Required internal rate of return:  $r^{\text{firm}}$ .
- Profit maximization:

$$\begin{split} V_0^{\mathsf{capital}}\left(\mathcal{K}_{t-1}\right) &= \max_{\{\mathcal{K}_t\}} \sum_{t=0}^{\infty} \left(1 + r^{\mathsf{firm}}\right)^{-t} \left[r_t^K \mathcal{K}_{t-1} - P_t^I \left(\iota_t + \Psi(\iota_t, \mathcal{K}_{t-1})\right)\right] \\ &\text{s.t.} \\ I_t &= \iota_t + \Psi(\iota_t, \mathcal{K}_{t-1}) \\ \mathcal{K}_t &= (1 - \delta^K) \mathcal{K}_{t-1} + \iota_t. \end{split}$$

### Capital agency II

Functional form:

$$\Psi(\iota_t, K_{t-1}) = \frac{\Psi_0}{2} \left( \frac{\iota_t}{K_{t-1}} - \delta^K \right)^2 K_{t-1}.$$

First order condition:

$$\begin{split} 0 &= - \, P_t^I \big( 1 + \Psi_\iota \left( \iota_t, K_{t-1} \right) \big) \\ &+ \frac{r_{t+1}^k + P_{t+1}^I \big( 1 - \delta^K \big) \big( 1 + \Psi_\iota \big( \iota_{t+1}, K_t \big) \big) - P_{t+1}^I \Psi_K \left( \iota_{t+1}, K_t \right)}{1 + r^{\text{firm}}} \end{split}$$

### Labor agency I

- Post vacancies:  $v_t$  at cost  $\kappa^L$  (in units of labor).
- Labor: Hires  $L_t$  and rent out labor at rental price  $r_t^\ell$
- **Exogenous match destruction:**  $\delta_t^L$  (implied by  $\delta_{a,t}^L$  and  $L_{a,t-1}$ )
- Exogenous wage: W<sub>t</sub>
- Profit maximization:

$$\begin{split} V_0^{\mathsf{labor}}(L_{t-1}) &= \max_{\left\{v_t\right\}} \sum_{t=0}^{\infty} \left(1 + r^{\mathsf{firm}}\right)^{-t} \left[r_t^\ell \ell_t - W_t L_t\right] \\ \text{s.t.} \\ L_t &= m_t^{\mathsf{v}} v_t + \left(1 - \delta_t^L\right) L_{t-1} \\ \ell_t &= L_t - \kappa^L v_t. \end{split}$$

#### Labor agency II

First order condition:

$$r_t^\ell = \frac{1}{1 - \frac{\kappa^L}{m_t^{\mathsf{v}}}} \left[ W_t - r_{t+1}^\ell \frac{1 - \delta_{t+1}^L}{1 + r^{\mathsf{firm}}} \frac{\kappa^L}{m_{t+1}^{\mathsf{v}}} \right]$$

#### Price setting I

• Input price:  $P_t^{Y,0}$ 

Output of differentiated goods: y<sub>t</sub> at p<sub>t</sub><sup>Y</sup>

■ Demand schedule:  $y_t = \left(\frac{p_t^Y}{P_t^Y}\right)^{-\sigma_D} Y_t$ 

• **Aggregate:**  $Y_t$  with price index  $P_t^Y$ 

Profit maximization:

$$\begin{split} V_t^{\text{intermediary}} &= \max_{\left\{ p_t^Y \right\}} \left( p_t^Y - P_t^{Y,0} \right) y_t - \vartheta_t + \frac{1}{1 + r^{\text{firm}}} V_{t+1}^{\text{intermediary}} \\ &\text{s.t.} \\ \vartheta_t &= \frac{\gamma}{2} \left[ \frac{p_t^Y / p_{t-1}^Y}{p_{t-1}^Y / p_{t-2}^Y} - 1 \right]^2 P_t^Y Y_t \\ y_t &= \left( \frac{p_t^Y}{P_t^Y} \right)^{-\sigma_D} Y_t. \end{split}$$

### Price setting II

- Assumption: Symmetric firms
- First order condition (New Keynesian Phillips Curve):

$$\begin{split} P_{t}^{Y} = & (1+\theta)P_{t}^{Y,0} - \eta \left(\frac{P_{t}^{Y}/P_{t-1}^{Y}}{P_{t-1}^{Y}/P_{t-2}^{Y}} - 1\right) \frac{P_{t}^{Y}/P_{t-1}^{Y}}{P_{t-1}^{Y}/P_{t-2}^{Y}} P_{t}^{Y} \\ & + \frac{2\eta}{1 + r^{\text{firm}}} \frac{Y_{t+1}}{Y_{t}} \left(\frac{P_{t+1}^{Y}/P_{t}^{Y}}{P_{t}^{Y}/P_{t-1}^{Y}} - 1\right) \frac{P_{t+1}^{Y}/P_{t}^{Y}}{P_{t}^{Y}/P_{t-1}^{Y}} P_{t+1}^{Y} \\ & \theta \equiv \frac{1}{\sigma_{D} - 1} \\ & \eta \equiv \theta \gamma. \end{split}$$

### Repacking firms

- Output goods:  $\bullet_t^Y$  at prices  $P_t^{\bullet}$  for  $\bullet \in \{C, G, I, X\}$
- Domestic input good: Y<sub>t</sub> at price P<sub>t</sub><sup>Y</sup>
- Foreign input goods:  $\bullet_t^M$  at prices  $P_t^{\bullet}$  for  $\bullet \in \{C, G, I, X\}$
- Profit maximization with CES production technology implies

$$\begin{split} P_t^{\bullet} &= \left(\mu^{M,\bullet} \left(P_t^{M,\bullet}\right)^{1-\sigma^{\bullet}} + \left(1-\mu^{M,\bullet}\right) \left(P_t^{Y}\right)^{1-\sigma^{\bullet}}\right)^{\frac{1}{1-\sigma^{\bullet}}} \\ \bullet_t^{M} &= \mu^{M,\bullet} \left(\frac{P_t^{\bullet}}{P_t^{M,\bullet}}\right)^{\sigma^{\bullet}} \bullet_t \\ \bullet_t^{Y} &= \left(1-\mu^{M,\bullet}\right) \left(\frac{P_t^{\bullet}}{P_t^{Y}}\right)^{\sigma^{\bullet}} \bullet_t, \end{split}$$
 for  $\bullet \in \{C, G, I, X\}$ 

#### Government

- Interest rate: r<sup>B</sup>
- Government consumption: *G<sub>t</sub>*
- Unemployment insurance expenses:  $E_t^U = W_U W_{ss} U_t$
- Retirement benefits expenses:  $E^R = W_R W_{ss} (N N_{work})$
- Tax base:  $T_t = W_t L_t + E_t^U + E^R$
- Budget constraint:

$$B_t = (1 + r^B)B_{t-1} + P_t^G G_t + E_t^U + E^R - \tau_t T_t.$$

Tax policy:

$$\begin{split} \tau_t &= \tau_{ss} + \varepsilon_B \frac{\tilde{B}_t - B_{ss}}{T_t}, \\ \tilde{B}_t &= (1 + r^B) B_{t-1} + P_t^G G_t + E_t^U + E^R - \tau_{ss} T_t. \end{split}$$

### Goods market clearing

1. Demand for domestically produced goods:

$$Y_t = \sum_{\bullet \in \{C,G,I,X\}} \bullet_t^Y.$$

2. Imports add up:

$$M_t = \sum_{\bullet \in \{C,G,I,X\}} \bullet_t^M.$$

# [Current account and net foreign asset position]

Not necessary to specify

Not specified yet



Solution method

## Targets and unknowns

- **Goal:** Find the *equilibrium path* in the economy.
- **Equilibrium path:** A set of paths for all variables where
  - 1. Optimal firm and household behavior in terms of FOCs.
  - 2. Accounting identities.
  - 3. Market clearing.
- **Target equations:** Must be zero on the equilibrium path.
- Unknown variables:
  - 1. Chosen by model-builder.
  - 2. All other variables must be derived from these.
  - 3. Target equations can be evaluated.
- Ordered series of blocks:

**Inputs:** Unknown and exogenous variables or outputs of previous blocks **Outputs:** Variables and errors in target equations

• **Truncation:** Assume back in steady state after *T* periods.

## **Exogenous variables**

- 1.  $\Gamma_t$ , technology
- 2.  $G_t$ , public spending
- 3.  $\chi_t$ , foreign demand shifter (»market size«)
- 4.  $P_t^{M,C}$ , import price of *private* consumption good
- 5.  $P_t^{M,G}$ , import price of *public* consumption good
- 6.  $P_t^{M,I}$ , import price of *investment* good
- 7.  $P_t^{M,X}$ , import price of *export* good
- 8.  $P_t^F$ , foreign price level
- 9.  $r_t^{hh}$ , foreign interest rate

## Unknowns in practice

- 1.  $A_t^q$ , inheritance flow (T unknowns)
- 2.  $A_t^{\mathrm{death}}$ , wealth of households at a=#-1 ( T unknowns)
- 3.  $K_t$ , capital (T unknowns)
- 4.  $L_t$ , labor supply (T unknowns)
- 5.  $r_t^K$ , rental price for capital (T unknowns)
- 6.  $P_t^Y$ , price of domestic output (T unknowns)
- $\Rightarrow$  all other variables in model can be derived and target equations can be evaluated (see code)

# Solving numerically

- Solve with Newton's method:
  - 1. x is a  $6 \times T$  vector containing all 6 unknown in all T periods.
  - 2. f is a  $6 \times T$  vector containing all 6 target in all T periods.
  - 3.  $\mathcal{J}$  is the Jacobian of f (the derivative of f) eval. in steady state.

Compute next guess,  $x_{n+1}$ , as

$$x_{n+1}=x_n-\frac{f(x_n)}{\mathcal{J}_n}.$$

Initial guess: Steady state

Converges to solution,  $f(x^*) = 0$ , as n grows.

- Implementation: In Python
- Course: Introduction to Programming and Numerical Analysis

**Calibration** 

## Steady state

#### Goals:

- 1. National account shares and ratios.
- 2. Unemployment level and labor market flows.
- 3. Life-cycle profiles of income, consumption and savings.

Simplify: Zero inflation, zero growth and constant demography.

#### Notebooks:

- 0 steady state aggregate.ipynb
- 0 steady state households.ipynb
- Status: Very ad hoc still.

### **Dynamics**

- Substitution elasticities: Use those from MAKRO.
- Impulse-responses: Match to data independently.
- Status: Not done  $\rightarrow$  example of government spending shock.



### Impulse-responses

#### An impulse response:

- The economy starts at steady state.
- 2. Some exogenous variables temporarily deviate from steady state.
- 3. The impulse responses: How variables respond to the shock.

### ■ Shock to government spending, G<sub>t</sub>:

$$G_t = G_{ss} + shock_t$$
  $shock_t = egin{cases} G_{ss} \cdot size \cdot persistence^{t-T_{start}} & ext{if } T_{start} \leq t < T_{end} \ 0 & ext{else} \end{cases}$ 

size = 0.01 (initial deviation from steady state at 1 percent) shock starts in  $T_{start} = 0$  and ends in  $T_{end} = 50$ . persistence = 0.8 (shock fades 20 percent per year)

### Initial effects I

- Repacking firms: Demand for domestically produced goods Y ↑ and imported goods M ↑.
- **Production firms:** Increases inputs to increase production  $Y \uparrow$ . Capital  $K \uparrow$  (limited by adjustment costs) and labor  $\ell \uparrow$ .
- Households: More income, greater consumption C↑.
   Real wage W↑ due to increased labor demand.
   Hand-to-mouth C<sup>HtM</sup>↑: Increase proportional to increase in W.
   Ricardian C<sup>R</sup>↓: Due to consumption smoothing.
- Prices: Higher input prices causes higher prices P↑:
   W↑ and r<sup>K</sup>↑ drives up marginal costs P<sup>Y,0</sup>↑.
   Output prices P<sup>Y</sup>↑ (limited by adjustment costs).
   Causes higher repacking prices P<sup>•</sup>↑ for ∈ {C, I, G, X}.
- Foreign economy: Exports  $X \downarrow$  due to increasing prices  $P^X \uparrow$ .

### Initial effects II

Labor agency: Employment L↑ and rents ℓ↑ to production firms.
 Job vacancies v↑ to meet higher labor demand.
 Number of matches M↑, »gross« labor L↑.
 Labor ℓ↑ (limited by adjustment costs).

Capital agency: Investments I ↑ to accumulate capital K ↑.
 Expensive investments: P<sup>I</sup> ↑

Greater return:  $r^K \uparrow$ 

### Transition path

#### Government:

Government spending  $G \downarrow$  by assumption.

Tax rate  $\tau \uparrow$  to finance growing government debt  $B \uparrow$ .

### Repacking firms:

Demand for domestic goods  $Y \downarrow$  and imports  $M \downarrow$ . Prices  $P \downarrow$ .

#### Production firms:

Drop in production inputs, labor  $\ell \downarrow$  and capital  $K \downarrow$ .

#### Households:

 $\ell \downarrow$  causes wages  $W \downarrow$ . Higher taxes from  $\tau \uparrow$ .

Leads to less disposable income and thus  $C\downarrow$ .

#### Foreign economy:

Exports  $X \uparrow$  due to falling prices  $P^X \downarrow$ .



**Conclusion** 

## Insights and takeaways

#### Your takeaways:

- 1. How a micro-founded macro-model is structured
- 2. Despite their complexity you can (soon) work with such models
- 3. Requires both analytical and numerical skills to master
- Open source: github.com/JeppeDruedahl/BabyMAKRO Will you participate in improving it?
- Related courses:

Macro III

Introduction to Programming and Numerical Analysis
Advanced Macroeconomics: Heterogenous Agent Models

### **Tasks**

- 1. Clarify steady state calibration.
- 2. Perform impulse-response matching.
- 3. Improve detailed accounting of main shocks.
- 4. Add technology growth, population growth, and trend inflation.
- 5. Add financial flows accounts wrt. to the foreign economy.
- 6. Add more government with taxes and spending.
- 7. Add endogenous labor supply.
- 8. Add multiple sectors and input-output structure.
- 9. Make the model quarterly.