CENTER FOR ECONOMIC BEHAVIOR & INEQUALITY



## **BabyMAKRO**

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#### Plan

- 1. MAKRO
- 2. BabyMAKRO
- 3. BabyMAKRO
- 4. Solution method
- 5. Calibration
- 6. Government spending
- 7. Conclusion

#### Introduction

TBA

## Reminder: Consumption behavior (chapter 16)

1. Maximize utility:

utility: 
$$u(C_1) + \frac{u(C_2)}{1+\phi}$$

budget constraint: 
$$C_1 + \frac{C_2}{1+r} = V_1 + Y_1^d + \frac{(1+g)Y_1^d}{1+r}$$

- ⇒ derive properties of optimal micro-behavior
- 2. Aggregate consumption function:  $C_1 = C(Y_1^d, g, r, V_1) + (-1) + (-1) + (-1)$
- 3. Estimate equation on aggregate data
- 4. Put equation into model

**Note:** Similar for investment (chapter 15).

#### Classes of Macro Models

#### 1. Old-style Keynesian macro-models (1950s-):

Structure: Aggregate equations »similar« to those in micro-theory

Estimation: Equation-by-equation on aggregate data

In teaching: AS-AD models

In practice: ADAM + MONA + SMEC

(»Den økonomiske genopretning 1976-1993«, Jørgen Rosted, 2021)

#### 2. Micro-founded macro-models (1970s-):

**Structure:** Exactly the equations in micro-theory (in general equilibrium)

**Estimation:** Calibration vs. moment-matching vs. full-system

In teaching: Dynamic Stochastic General Equilibrium Models (DSGE)

RBC: Real Business Cycle (1980s-)

RANK: Representative Agent New Keynesian (1990s-) HANK: Heterogeneous Agent New Keynesian (2010s-)

In practice: DREAM + DSGE at Nationalbanken + MAKRO

Blanchard: On the Need for (At Least) Five Classes of Macro Models

# MAKRO

## Structure of small open economy

#### Agents:

- 1. Unconstrainted households (»Ricardian«): One for each cohort
- 2. Hand-too-mouth households (»HtM«): One for each cohort
- 3. Firms (production, price setting, multiple sectors)
- 4. Central bank (fixed exchange-rate)
- 5. Government
- 6. Global foreign economy (exogenous)
- Expectations: Perfect foresight
- Market clearing: Walras + sticky prices + search-and-match
- Mathematically: Non-linear equation system
- Code: <a href="https://github.com/DREAM-DK/MAKRO">https://github.com/DREAM-DK/MAKRO</a>

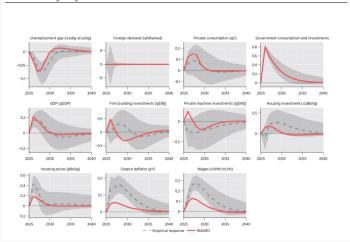
## **Empirical strategy**

- Levels: Weights in production and utility functions (directly observable in data, but changes over time...)
- 2. Long-run relationships: Substitution elasticities
- Short-run dynamics: Adjustment cost parameters (especially focus on convergence speed)

**Documentation:** Matching af impuls responser og øvrige kortsigtsmomenter: MAKRO ift. empirien (2021)

## Impulse-response functions (IRFs)

Figur 1 Stød til offentlige udgifter



Also: Foreign demand, foreign interest rate, labor supply, oil price

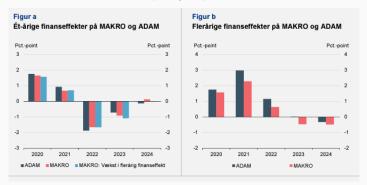
#### **Micro-moments**

Tabel 2 Yderligere relevant empiri til vurdering af MAKROs kortsigtsegenskaber

Analyse/moment	MAKRO og konsensus	Relevant litteratur
MPC ud af midlertidig/kortsigtet ind- komst, første år	MAKRO: ca. 0,45 Litteratur: 0,4-0,6	Jørgensen & Kuchler (2017), Crawley & Kuchler (2020), Kreiner et al (2019), ADAM, SMEC
MPC ud af boligprisstigninger/for- mue, første år	MAKRO: ca. 0,05 Litteratur: 0,03-0,06	Hviid & Kuchler (2017), Andersen & Leth-Petersen (2021)
Fortrængning af tvungen pensions- opsparing for 30-55 årige, år 1 [år 10]	MAKRO: Ca. 0,35-0,55 [0,10-0,35] Litteratur: 0-0,5 [0-0,5]	Arnberg & Barslund (2012), Chetty m.fl. (2014), Andersen, Hansen & Ku- chler (2021)
Rentefølsomhed, husholdningers <u>boliqværdi</u> (stød til beskatning på aktie- og kapitalindkomst). Gns. 10 års-ef- fekt.	MAKRO: knap -0,1 Litteratur: (-)0,25 – (-)0,18	Gruber, Jensen & Kleven (2021)*
Rentefølsomhed, husholdningers <u>for-mue</u> (stød til beskatning på aktie- og kapitalindkomst). 8-års effekt. [Lang- sigtet elasticitet]	MAKRO: ca. 0,2 [0,5] Litteratur: 0,2 - 0,4 [0,5 - 1]	Jakobsen, Jakobsen, Kleven & Zucman (2020)**

#### In practice: Finanseffekt

»Finanseffekt«: Is fiscal policy expansive or contractive wrt. GDP?



Kilde: Økonomisk Redegørelse, Marts 2023

BabyMAKRO

#### Verbal overview

TBA

# BabyMAKRO

#### **Notation**

- Small open economy with a fixed exchange rate and overlapping generations.
- Households: Unconstrained or hands-too-mouth wrt.
   consumption-saving + supply labor exogenously
- Foreign economy: Fixed nominal rate of return + import goods at fixed prices + demand curve for the domestic export good.

#### Production:

- Production firms rent capital and labor to produce the domestic output good.
- 2. Price adjustments for domestic good is infrequent
- Repacking firms combine imported goods with the domestic output good to produce a consumption good, an investment good, and an export good.
- 4. Capital is rented from a *capital agency*, which purchases the investment good to accumulate capital subject to adjustment costs.
- Labor is rented from a *labor agency*, which post vacancies a search-and-match labor market to purchase labor from the

#### **Notation**

- TBA
- Notation: a denotes age, t denotes time
- The aggregate job-separation rate incl. mortality is

$$\delta_t^L = \frac{L_{t-1} - \underline{L}_t}{L_{t-1}}.$$

## Foreign economy

- Nominal interest rate: r
- Armington demand of the domestic exported good:

$$X_t = \gamma^X X_{t-1} + (1 - \gamma^X) \chi_t \left( \frac{P_t^X}{P_t^F} \right)^{-\sigma^F}.$$

■ Import goods:  $\bullet_t^M$  at prices  $P_t^{\bullet}$  for  $\bullet \in \{C, G, I, X\}$ 

## Demographic structure and population

- **Life-span:** #, hereof working, #<sub>work</sub>
- Number of households: N<sub>a</sub>
- Mortality rate:  $\zeta_a$  (controlled by  $\zeta$ )
- Demographic structure and population:

$$\begin{split} N_{a} &= \begin{cases} 1 & \text{if } a = 0 \\ (1 - \zeta_{a-1}) N_{a-1} & \text{if } a > 0 \end{cases} \\ \zeta_{a} &= \begin{cases} 0 & \text{if } a < \#_{\text{work}} \\ \left(\frac{a+1-\#_{\text{work}}}{\#-\#_{\text{work}}}\right)^{\zeta} & \text{if } a < \#-1 \\ 1 & \text{if } a = \#-1 \end{cases} \\ N &= \sum_{a=0}^{\#-1} N_{a} \end{split}$$

#### Labor market flows

■ Employed: L<sub>a,t</sub>

• Unemployed:  $U_{a,t} = N_a - L_{a,t}$ 

• Job-separation rate:  $\delta_a^L$ 

• Job-finding rate:  $m_t^s$ 

• **Searchers** (everybody search = exogenous labor supply):

$$S_{a,t} = \begin{cases} 1 & \text{if } a = 0 \\ (1 - \zeta_{a-1}) \left[ U_{a-1,t-1} + \delta_a^L L_{a-1,t-1} \right] & \text{if } a < \#_{\text{work}} \\ 0 & \text{if } a \ge \#_{\text{work}} \end{cases}$$

Employed (before and after matching):

$$\underline{L}_{a,t} = \begin{cases} 0 & \text{if } a = 0 \\ (1 - \zeta_{a-1})(1 - \delta_a^L)L_{a-1,t-1} & \text{if } a < \#_{\text{work}} \\ 0 & \text{if } a \ge \#_{\text{work}} \end{cases}$$

$$L_{a,t} = \underline{L}_{a,t} + m_t^s S_{a,t}.$$

## Search and matching

- Vacancies: v<sub>t</sub>
- Searchers:  $\sum_{a} S_{a,t}$
- Matches:

$$\mathcal{M}_t = rac{S_t v_t}{\left(S_t^{rac{1}{\sigma^m}} + v_t^{rac{1}{\sigma^m}}
ight)^{\sigma^m}}$$

- Job-filling rate:  $m_t^v = \frac{\mathcal{M}_t}{v_t}$
- Job-finding rate:  $m_t^s = \frac{\mathcal{M}_t}{S_t}$ .

## Wage bargaining

The wage, W<sub>t</sub>, is determined by an unmodeled bargaining mechanism, and it is set such that it is increasing in the labor demand:

$$W_t = W_{ss} \left(\frac{L_t}{L_{ss}}\right)^{\epsilon_w}.$$

#### Income

- The model has two types of households. A share of  $\lambda$  of households is hands-to-mouth and a share of  $1-\lambda$  households is unconstrained (Ricardian). All households have four sources of income:
- 1. Post-tax labor income:  $(1-\tau_t)\,W_t rac{L_{a,t}}{N_a}$
- 2. Post-tax unemployment benefits:  $(1-\tau_t)\,W^UW_{\rm ss}\frac{U_{a,t}}{N_a}$
- 3. Post-tax retirement benefits:  $(1 \tau_t) W^R W_{ss} \frac{N_s (L_{s,t} + U_{s,t})}{N_s}$
- 4. Equally divided inheritance:  $\frac{A_t^q}{N}$
- The age specific income is

$$egin{aligned} &\operatorname{inc}_{a,t} = \left(1 - au_t
ight) W_t rac{L_{a,t}}{N_a} + \left(1 - au_t
ight) W^U W_{\operatorname{ss}} rac{U_{a,t}}{N_a} \ &+ \left(1 - au_t
ight) W^R W_{\operatorname{ss}} \mathbf{1}_{\left\{a \geq \#_{\operatorname{work}}
ight\}} + rac{A_t^q}{N}, \end{aligned}$$

and the price of consumption goods is  $P_t^{\mathcal{C}}$ .

## Hand-to-mouth households (HtM)

Consumer all income:

$$C_{a,t}^{\mathsf{HtM}} = \frac{\mathsf{inc}_{a,t}}{P_t^{C}}$$

No savings:

$$A_{a,t}^{\mathsf{HtM}} = 0$$

## Unconstrained household (Ricardian) I

Utility from consumption:

$$\frac{\left(C_{\mathsf{a},t}^R\right)^{1-\sigma}}{1-\sigma}$$

Utility from wealth:

$$\zeta_{a}\mu^{A^{q}}\frac{\left(\frac{A_{a,t}^{R}}{P_{t}^{C}}\right)^{1-\sigma}}{1-\sigma}$$

Discounting:

$$\beta(1-\zeta_a)$$

Budget constraint:

$$\begin{aligned} A_{-1,t}^R &= 0 \\ A_{a,t}^R &= (1 + r_t^{hh}) A_{a-1,t-1}^R + \mathrm{inc}_{a,t} - P_t^C C_{a,t}^R. \end{aligned}$$

## Unconstrained household (Ricardian) II

Utility from consumption:

$$\frac{\left(C_{a,t}^R\right)^{1-\sigma}}{1-\sigma}$$

Utility from wealth:

$$\zeta_{a}\mu^{A^{q}}\frac{\left(\frac{A_{a,t}^{R}}{P_{t}^{C}}\right)^{1-\sigma}}{1-\sigma}$$

Discounting:

$$\beta(1-\zeta_a)$$

Budget constraint:

$$A_{-1,t}^{R} = 0$$

$$A_{a,t}^{R} = (1 + r_{t}^{hh})A_{a-1,t-1}^{R} + inc_{a,t} - P_{t}^{C}C_{a,t}^{R}.$$

## Unconstrained household (Ricardian) II

- Cohort: t<sub>0</sub>
- Full problem

$$V_{t_0} = \max_{\left\{C_{a,t}^R\right\}_{a=0}^{\#-1}} \sum_{a=0}^{\#-1} \left( \Pi_{j=0}^{a-1} \beta(1-\zeta_j) \right) \left[ \frac{\left(C_{a,t}^R\right)^{1-\sigma}}{1-\sigma} + \zeta_a \mu^{A^q} \frac{\left(\frac{A_{a,t}^R}{P_t^C}\right)^{1-\sigma}}{1-\sigma} \right]$$

s.t.

$$t = t_0 + a$$

$$A_{-1,t}^{R}=0$$

$$A_{a,t}^R = (1 + r_t^{hh}) A_{a-1,t-1}^R + \mathrm{inc}_{a,t} - P_t^C C_{a,t}^R.$$

## Unconstrained household (Ricardian) III

First order condition:

$$C_{a,t}^{R} = \begin{cases} \left( \zeta_{a} \mu^{A^{q}} \left( \frac{A_{\#-1,t}^{R}}{P_{t}^{C}} \right)^{-\sigma} \right)^{-\frac{1}{\sigma}} & \text{if } a = \#-1 \\ \left( \beta (1 - \zeta_{a}) \frac{1 + r_{ss}^{hh}}{1 + \pi_{ss}^{hh}} \left( C_{a+1,ss}^{R} \right)^{-\sigma} + \zeta_{a} \mu^{A^{q}} \left( \frac{A_{a,ss}^{R}}{P_{ss}^{C}} \right)^{-\sigma} \right)^{-\frac{1}{\sigma}} & \text{elif } t = T - 1 \\ \left( \beta (1 - \zeta_{a}) \frac{1 + r_{t+1}^{hh}}{1 + \pi_{t+1}^{hh}} \left( C_{a+1,t+1}^{R} \right)^{-\sigma} + \zeta_{a} \mu^{A^{q}} \left( \frac{A_{a,t}^{R}}{P_{t}^{C}} \right)^{-\sigma} \right)^{-\frac{1}{\sigma}} & \text{else} \end{cases}$$

Bequests:

$$A_t^q = (1 + r^{hh}) \sum_{a=0}^{\#-1} \zeta_a N_a A_{a,t}.$$

#### Production firms I

• Capital:  $K_{t-1}$ , at rental price  $r_t^K$ 

• Labor:  $\ell_t$ , at rental price  $r_t^\ell$ 

• Output:  $Y_t$ , with a CES technology, sold at  $P_t^{Y,0}$ 

• Profit maximization with prices taken as given

$$\Pi_{t} = \max_{X_{i}, X_{j}} P_{t}^{Y, 0} Y_{t} - r_{t}^{K} K_{t-1} - r_{t}^{\ell} \ell_{t}$$

s.t.

$$\begin{split} Y_t &= \Gamma \left( \left( \mu^K \right)^{\frac{1}{\sigma^Y}} K_{t-1}^{\frac{\sigma^Y-1}{\sigma^Y}} + \left( 1 - \mu^K \right)^{\frac{1}{\sigma^Y}} \ell_t^{\frac{\sigma^Y-1}{\sigma^Y}} \right)^{\frac{\sigma^Y}{\sigma^Y-1}} \end{split}$$
 where  $\mu^K$ ,  $\sigma^Y$ ,  $\Gamma > 0$ ,  $\sigma^Y \neq 1$ .

#### **Production firms**

• Free entry implies zero profit:

$$P_t^{Y,0} = \frac{1}{\Gamma} \left( \mu^K \left( r_t^K \right)^{1-\sigma^Y} + \left( 1 - \mu^K \right) \left( r_t^\ell \right)^{1-\sigma^Y} \right)^{\frac{1}{1-\sigma^Y}}.$$

First order condition for capital-labor ratio:

$$\frac{K_{t-1}}{\ell_t} = \frac{\mu_K}{1 - \mu_K} \left(\frac{r_t^\ell}{r_t^K}\right)^{\sigma^r}$$

## Capital agency I

- Investment good:  $I_t$ , at price,  $P_t^I$
- Capital:  $K_t$ , rented out at rental rate  $r_{t+1}^K$  in the following period.
- Adjustment costs: Effective investment is  $\iota_t$ .
- Required internal rate of return: r<sup>firm</sup>.
- Profit maximization:

$$\begin{split} V_0^{\mathsf{capital}}\left(\mathcal{K}_{t-1}\right) &= \max_{\left\{\mathcal{K}_t\right\}} \sum_{t=0}^{\infty} \left(1 + r^{\mathsf{firm}}\right)^{-t} \left[r_t^K \mathcal{K}_{t-1} - P_t^I \left(\iota_t + \Psi(\iota_t, \mathcal{K}_{t-1})\right)\right] \\ \text{s.t.} \\ I_t &= \iota_t + \Psi(\iota_t, \mathcal{K}_{t-1}) \\ \mathcal{K}_t &= (1 - \delta^K) \mathcal{K}_{t-1} + \iota_t. \end{split}$$

## Capital agency II

Functional form:

$$\Psi(\iota_t, K_{t-1}) = \frac{\Psi_0}{2} \left( \frac{\iota_t}{K_{t-1}} - \delta^K \right)^2 K_{t-1}.$$

First order condition:

$$\begin{split} 0 &= - \, P_t^I \big( 1 + \Psi_\iota \left( \iota_t, K_{t-1} \right) \big) \\ &+ \frac{r_{t+1}^k + P_{t+1}^I \big( 1 - \delta^K \big) \big( 1 + \Psi_\iota \big( \iota_{t+1}, K_t \big) \big) - P_{t+1}^I \Psi_K \left( \iota_{t+1}, K_t \right)}{1 + r^{\text{firm}}} \end{split}$$

## Labor agency I

- Post vacancies:  $v_t$  at cost  $\kappa^L$  (in units of labor).
- Labor: Hires  $L_t$  and rent out labor at rental price  $r_t^\ell$
- **Exogenous match destruction:**  $\delta_t^L$  (implied by  $\delta_{a,t}^L$  and  $L_{a,t-1}$ )
- Exogenous wage: W<sub>t</sub>
- Profit maximization:

$$\begin{split} V_0^{\text{labor}}(L_{t-1}) &= \max_{\left\{v_t\right\}} \sum_{t=0}^{\infty} \left(1 + r^{\text{firm}}\right)^{-t} \left[r_t^\ell \ell_t - W_t L_t\right] \\ \text{s.t.} \\ L_t &= m_t^{\text{v}} v_t + \left(1 - \delta_t^L\right) L_{t-1} \\ \ell_t &= L_t - \kappa^L v_t. \end{split}$$

#### Labor agency II

First order condition:

$$r_t^\ell = \frac{1}{1 - \frac{\kappa^L}{m_t^V}} \left[ W_t - r_{t+1}^\ell \frac{1 - \delta_{t+1}^L}{1 + r^{\mathsf{firm}}} \frac{\kappa^L}{m_{t+1}^V} \right]$$

## Phillips curve

• Input price:  $P_t^{Y,0}$ 

Output price: p<sub>t</sub><sup>Y</sup> for differentiated goods

- **Demand schedule:**  $y_t = \left(\frac{p_t^Y}{P_t^Y}\right)^{-\sigma_D} Y_t$ , where  $P_t^Y$  and  $Y_t$  are aggregates
- Profit maximization:

$$\begin{split} V_t^{\text{intermediary}} &= \max_{\left\{p_t^Y\right\}} \left(p_t^Y - P_t^{Y,0}\right) y_t - g_t + \frac{1}{1 + r^{\text{firm}}} V_{t+1}^{\text{intermediary}} \\ &\text{s.t.} \\ g_t &= \frac{\gamma}{2} \left[\frac{p_t^Y/p_{t-1}^Y}{p_{t-1}^Y/p_{t-2}^Y} - 1\right]^2 P_t^Y Y_t \\ y_t &= \left(\frac{p_t^Y}{p_t^Y}\right)^{-\sigma_D} Y_t. \end{split}$$

#### Phillips curve

- Assumption: Symmetric firms
- First order condition:

$$\begin{split} P_{t}^{Y} = & (1+\theta)P_{t}^{Y,0} - \eta \left(\frac{P_{t}^{Y}/P_{t-1}^{Y}}{P_{t-1}^{Y}/P_{t-2}^{Y}} - 1\right) \frac{P_{t}^{Y}/P_{t-1}^{Y}}{P_{t-1}^{Y}/P_{t-2}^{Y}} P_{t}^{Y} \\ & + \frac{2\eta}{1 + r^{\text{firm}}} \frac{Y_{t+1}}{Y_{t}} \left(\frac{P_{t+1}^{Y}/P_{t}^{Y}}{P_{t}^{Y}/P_{t-1}^{Y}} - 1\right) \frac{P_{t+1}^{Y}/P_{t}^{Y}}{P_{t}^{Y}/P_{t-1}^{Y}} P_{t+1}^{Y} \\ & \theta \equiv \frac{1}{\sigma_{D} - 1} \\ & \eta \equiv \theta \gamma. \end{split}$$

## Repacking firms

- **Output goods:**  $\bullet_t^Y$  at prices  $P_t^{\bullet}$  for  $\bullet \in \{C, G, I, X\}$  private consumption,  $C_t$ , public consumption,  $G_t$ , investment,  $I_t$ , or exports,  $X_t$ .
- Domestic input good: Y<sub>t</sub> at price P<sub>t</sub><sup>Y</sup>
- Foreign input goods:  $\bullet_t^M$  at prices  $P_t^{\bullet}$  for  $\bullet \in \{C, G, I, X\}$
- Profit maximization with CES production technology implies

$$\begin{split} P_t^{\bullet} &= \left(\mu^{M,\bullet} \left(P_t^{M,\bullet}\right)^{1-\sigma^{\bullet}} + \left(1-\mu^{M,\bullet}\right) \left(P_t^{Y}\right)^{1-\sigma^{\bullet}}\right)^{\frac{1}{1-\sigma^{\bullet}}} \\ \bullet_t^{M} &= \mu^{M,\bullet} \left(\frac{P_t^{\bullet}}{P_t^{M,\bullet}}\right)^{\sigma^{\bullet}} \bullet_t \\ \bullet_t^{Y} &= \left(1-\mu^{M,\bullet}\right) \left(\frac{P_t^{\bullet}}{P_t^{Y}}\right)^{\sigma^{\bullet}} \bullet_t \,. \end{split}$$
 for  $\bullet \in \{C, G, I, X\}$ 

#### Government

- Interest rate: r<sup>B</sup>
- Government consumption: *G<sub>t</sub>*
- Unemployment insurance:  $E_t^U = W_U W_{ss} U_t$
- Retirement benefits:  $E^R = W_R W_{ss} 1_{\{a \ge \#_{work}\}}$
- Tax base:  $T_t = W_t L_t + E_t^U + E^R$
- Budget constraint:

$$B_t = (1 + r^B)B_{t-1} + P_t^G G_t + E_t^U + E^R - \tau_t T_t.$$

Tax policy

$$\begin{split} \tau_t &= \tau_{ss} + \varepsilon_B \frac{\tilde{B}_t - B_{ss}}{T_t}, \\ \tilde{B}_t &= (1 + r^B) B_{t-1} + P_t^G G_t + E_t^U + E^R - \tau_{ss} T_t \end{split}$$

# Goods market clearing

1. Demand for domestically produced goods:

$$Y_t = \sum_{\bullet \in \{C,G,I,X\}} \bullet_t^Y.$$

1. Imports add up:

$$M_t = \sum_{\bullet \in \{C,G,I,X\}} \bullet_t^M.$$

# Current account and net foreign position

Not specified yet

Solution method

# Targets and unknowns

- Goal: Find the equilibrium path in the economy.
- Equilibrium path: A set of paths for all variables, which satisfies
  - 1. Optimal firm and household behavior in terms of FOCs.
  - 2. All accounting identities,
  - 3. Implies market clearing,
- **Target equations:** Must be zero on the equilibrium path.
- Unknown variables:
  - 1. Chosen by model-builder
  - 2. All other variables must be derived from these
  - 3. Target equations can be evaluated
- Truncation: Assume back in steady state after T periods

# **Exogenous variables**

- 1.  $\Gamma_t$ , technology
- 2.  $G_t$ , public spending
- 3.  $\chi_t$ , foreign demand shifter (»market size«)
- 4.  $P_t^{M,C}$ , import price of *private consumption* component good
- 5.  $P_t^{M,G}$ , import price of *public consumption* component good
- 6.  $P_t^{M,I}$ , import price of *investment* component good
- 7.  $P_t^{M,X}$ , import price of export component good
- 8.  $P_t^F$ , foreign price level
- 9.  $r_t^{hh}$ , foreign interest rate

## Unknowns in practice

- 1.  $A_t^q$ , inheritance flow (T unknowns)
- 2.  $A_t^{\text{death}}$ , wealth of households at a=#-1 ( T unknowns)
- 3.  $K_t$ , capital (T unknowns)
- 4.  $L_t$ , labor supply (T unknowns)
- 5.  $r_t^K$ , rental price for capital (T unknowns)
- 6.  $P_t^Y$ , price of domestic output (T unknowns)

# Solving numerically

- Solve with Newton's method: Successive approximations of the root.
  - x is a 6 × T vector containing all 6 unknown in all T periods.
  - f is a  $6 \times T$  vector containing all 6 target in all T periods.
  - $\mathcal{J}$  is the Jacobian of f (the derivative of f).
  - with  $x_n$  being the n'th guess, compute next guess,  $x_{n+1}$ , as

$$x_{n+1}=x_n-\frac{f(x_n)}{\mathcal{J}_n}.$$

• Converges to solution,  $f(x^*) = 0$ , (if it exists) as n grows.

**Calibration** 

## Steady state

- Steady state: When all variables are constant over time, the equilibrium path is a steady state.
- To solve for steady state:
  - Choose values for a selection of the endogenous variables.
  - Derive the rest from closed form expressions, or solve sub-systems numerically.

# **Dynamics**

TBA



## Impulse-responses

- An impulse response:
  - The economy starts at steady state.
  - Some exogenous variables temporarily deviate from steady state.
  - The impulse responses: How variables respond to the shock.
- Shock to government spending, *G*<sub>t</sub>:

$$G_t = G_{ss} + Shock_t$$
 
$$Shock_t = \begin{cases} G_{ss} \cdot Size \cdot Persistence^{t-T_{start}} & \text{if } T_{start} \leq t < T_{end} \\ 0 & \text{else} \end{cases}$$

- Size = 1.01: Initial deviation from steady state at 1 pct.
- Shocks starts in  $T_{start} = 0$  and ends in  $T_{end} = 50$ .
- Persistence = 0.8.
  - Shock fades as t grows.
  - Higher values → longer convergence.

## Initial effects I

- Repacking firms: Demand for domestic goods Y ↑ and imports M ↑.
- Production firms: Increases inputs to increase production Y ↑:
  - Capital  $K \uparrow$  (limited by adjustment costs) and labor  $\ell \uparrow$  .
- **Households:** More income, greater consumption  $C \uparrow$ :
  - Real wage  $W \uparrow$  due to increased labor demand (indexed to  $P^{C}$ ).
  - Hand-to-mouth  $C^{HtM}$  ↑: Increase proportional to increase in W.
  - Ricardian  $C^R \downarrow$ : Due to consumption smoothing.
- **Prices:** Higher input prices causes higher prices  $P \uparrow$ :
  - $W \uparrow$  and  $r^K \uparrow$  drives up marginal costs  $P^{Y,0} \uparrow$ .
  - Output prices  $P^Y \uparrow$  (limited by adjustment costs).
  - Output prices 7 | (initited by adjustment costs).
  - Causes higher repacking prices  $P^{\bullet} \uparrow$  for  $\bullet \in \{C, I, G, X\}$ .
- Foreign economy: Exports  $X \downarrow$  due to increasing prices  $P^X \uparrow$ .

## Initial effects II

- **Labor agency:** Employs more  $L \uparrow$  and rents  $\ell \uparrow$  to production firms.
  - Job vacancies  $v \uparrow$  to meet higher labor demand.
  - Number of matches  $\mathcal{M} \uparrow$ , »gross« labor  $L \uparrow$ .
  - Labor ℓ↑ (limited by adjustment costs).
- Capital agency: Investments I↑ to accumulate capital K↑.
  - Expensive investments: P<sup>I</sup> ↑
  - Greater return:  $r^K \uparrow$

## Transition path

#### Government:

- Government spending  $G \downarrow$  by assumption.
- Tax rate  $\tau \uparrow$  to finance growing government debt  $B \uparrow$ .

## Repacking firms:

- Demand for domestic goods  $Y \downarrow$  and imports  $M \downarrow$ .
- Prices  $P \downarrow$ .

#### Production firms:

■ Drop in production inputs, labor  $\ell \downarrow$  and capital  $K \downarrow$ .

#### Households:

- $\ell \downarrow$  causes wages  $W \downarrow$ .
- Higher taxes from  $\tau \uparrow$ .
- Leads to less disposable income and thus  $C \downarrow$ .

#### Foreign economy:

■ Exports  $X \uparrow$  due to falling prices  $P^X \downarrow$ .



**Conclusion** 

# Insights and takeaways

TBA