THE CANONICAL HANK MODEL WITH STICKY PRICES

1 Model

Households. The model has a continuum of infinitely lived households indexed by $i \in [0,1]$. Households are *ex post* heterogeneous in terms of their time-varying stochastic productivity, z_t, and their (end-of-period) savings, a_{t-1}. The distribution of households over idiosyncratic states is denoted $\underline{\boldsymbol{D}}_t$ before shocks are realized and $\boldsymbol{D}_t$ afterwards. Households supply labor, ℓ_t, chosen by a union, and choose consumption, c_t, on their own. Households are not allowed to borrow. The return on savings is r_t^a, the real wage is w_t, labor income is taxed with the rate $\tau_t \in [0,1]$, and households receive transfers, χ_t.

The household problem is

$$\begin{aligned} v_t(z_t, a_{t-1}) &= \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_t^{1+\nu}}{1+\nu} + \beta \mathbb{E}\left[v_{t+1}(z_{t+1}, a_t) \mid z_t, a_t\right] \\ \text{s.t. } a_t + c_t &= (1+r_t^a) a_{t-1} + (1-\tau_t) w_t \ell_t z_t + \chi_t \\ \log z_{t+1} &= \rho_z \log z_t + \psi_{t+1} \;\; , \psi_t \sim \mathcal{N}(\mu_\psi, \sigma_\psi),\ \mathbb{E}[z_t] = 1 \\ a_t &\geq 0 \end{aligned} \tag{1}$$

where β is the discount factor, σ is the inverse elasticity of substitution, φ controls the disutility of supplying labor and ν is the inverse of the Frish elasticity.

Aggregate quantities are

$$A_t^{hh} = \int a_t d\boldsymbol{D}_t \tag{2}$$

$$L_t^{hh} = \int \ell_t z_t d\boldsymbol{D}_t \tag{3}$$

$$C_t^{hh} = \int c_t d\boldsymbol{D}_t \tag{4}$$

Firms. A representative firm hires labor, L_t, to produce goods, with the production function

$$Y_t = \Gamma_t L_t \tag{5}$$

where Γ_t is the exogenous technology level. Profits are

$$\Pi_t = P_t Y_t - W_t L_t \tag{6}$$

where P_t is the price level and W_t is the wage level. The first order condition for labor implies that the real wage is exogenous

$$w_t \equiv W_t / P_t = \Gamma_t \tag{7}$$

Inflation rates for wages and price are given by

$$\pi_t^w \equiv W_t / W_{t-1} - 1 \tag{8}$$

$$\pi_t \equiv \frac{P_t}{P_{t-1}} - 1 = \frac{W_t / \Gamma_t}{W_{t-1} / \Gamma_{t-1}} - 1 = \frac{1 + \pi_t^w}{\Gamma_t / \Gamma_{t-1}} - 1 \tag{9}$$

Union. A union chooses the labor supply of each household and sets wages. Each household is chosen to supply the same amount of labor,

$$\ell_t = L_t^{hh} \tag{10}$$

Unspecified adjustment costs imply a *New Keynesian Wage Philips Curve*,

$$\pi_t^w = \kappa \left(\varphi \left(L_t^{hh} \right)^{\nu} - \frac{1}{\mu} \left(1 - \tau_t \right) w_t \left(C_t^{hh} \right)^{-\sigma} \right) + \beta \pi_{t+1}^w \tag{11}$$

where κ is the slope parameter and μ is a wage mark-up.

Central bank. The central bank follows a standard Taylor rule with persistence,

$$1 + i_t = (1 + i_{t-1})^{\rho_i} \left((1 + r_{ss}) (1 + \pi_t)^{\phi_\pi} \right)^{1-\rho_i} \tag{12}$$

where i_t is the nominal return from period t to period $t+1$, ϕ_π is the Taylor coefficient, and $\rho_i \in [0, 1)$ is persistence parameter.

The *ex ante* real interest rate is

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}} \tag{13}$$

Government. The government chooses spending, G_t, transfers, χ_t, and the labor income tax rate, τ_t. The total tax bill is

$$\mathcal{T}_t \equiv \tau_t w_t L_t^{hh} = \tau_t \Gamma_t L_t^{hh} = \tau_t Y_t \tag{14}$$

The government can finance its expenses with long-term bonds, B_t, with a geometrically declining payment stream of $1, \delta, \delta^2, \ldots$ for $\delta \in [0,1]$. The bond price is q_t.
The budget constraint for the government then is

$$q_t(B_t - \delta B_{t-1}) = B_{t-1} + G_t + \chi_t - \tau_t Y_t \tag{15}$$

Spending, G_t, and transfers, χ_t, are chosen exogenously. The labor income tax follows the rule

$$\tau_t = \tau_{ss} + \omega q_{ss} \frac{B_{t-1} - B_{ss}}{Y_{ss}} \tag{16}$$

where ω controls the sensitivity of the tax rate to public debt.

Market clearing. Arbitrage implies that all assets must give the same rate of return. A bond with a unit return bought in period t at price q_t can be sold in period $t+1$ for δq_{t+1}, so we specifically have

$$\frac{1 + \delta q_{t+1}}{q_t} = 1 + r_t \tag{17}$$

The *ex post* return on savings (all in government bonds) from period $t-1$ to t then is

$$1 + r_t^a = \frac{1 + \delta q_t}{q_{t-1}} \tag{18}$$

Market clearing implies

1. Asset market: $q_t B_t = A_t^{hh}$
2. Labor market: $L_t = L_t^{hh}$
3. Goods market: $Y_t = C_t^{hh} + G_t$

2. Calibration

The parameters and steady state government behavior are as follows:

1. **Preferences and abilities:** $\sigma = 2$, $\nu = 1.0$
2. **Income:** $\rho_z = 0.95$, $\sigma_\psi = 0.10$

3. **Production:** $\Gamma_{ss} = 1$
4. **Union:** $\kappa = 0.1$, $\mu = 1.2$
5. **Central bank:** $r_{ss} = 1.02^{\frac{1}{4}} - 1$, $\phi^{\pi} = 1.5$, $\rho_i = 0.90$
6. **Government:** $G_{ss} = 0.20$, $\chi_{ss} = 0$, $q_{ss}B_{ss} = 1.0$, $\delta = 0.8$, $\omega = 0.1$

We let β and φ be unspecified and adjust those to obtain the steady state we want.

3. Finding the steady state

1. Guess on β
2. Set Γ_{ss}, r_{ss}, G_{ss}, χ_{ss} and $q_{ss}B_{ss}$ as specified in the calibration
3. Set aggregate labor supply to, $L_{ss} = 1$
4. Set steady state inflation to zero, $\pi_{ss} = \pi_{ss}^{w} = 0$
5. Calculate the value of all other aggregate steady state variables
6. Solve for and simulate household behavior
7. Calculate $\varphi = \frac{\frac{1}{\mu}(1-\tau_{ss})w_{ss}\left(C_{ss}^{hh}\right)^{-\sigma}}{\left(L_{ss}^{hh}\right)^{\nu}}$
8. Check a remaining market clearing condition

4. Equation system

The model can be summarized by the following equation system

$$
\boldsymbol{H}(\boldsymbol{\pi^w}, \boldsymbol{L}, \boldsymbol{G}, \boldsymbol{\chi}, \boldsymbol{\Gamma}) = \begin{bmatrix}
w_t - \Gamma_t \\
1 + \pi_t - \frac{1+\pi_t^w}{\Gamma_t/\Gamma_{t-1}} \\
Y_t - \Gamma_t L_t \\
1 + i_t - (1+i_{t-1})^{\rho_i} \left((1+r_{ss}) \left(1+\pi_t\right)^{\phi_\pi} \right)^{1-\rho_i} \\
1 + r_t - \frac{1+i_t}{1+\pi_{t+1}} \\
\frac{1+\delta q_{t+1}}{q_t} - (1+r_t) \\
1 + r_t^a - \frac{1+\delta q_t}{q_{t-1}} \\
\tau_t - \left[\tau_{ss} + \omega q_{ss} \frac{B_{t-1} - B_{ss}}{Y_{ss}} \right] \\
q_t (B_t - \delta B_{t-1}) - [B_{t-1} + G_t + \chi_t - \tau_t Y_t] \\
q_t B_t - A_t^{hh} \\
\pi_t^w - \left[\kappa \left(\varphi \left(L_t^{hh}\right)^{\nu} - \frac{1}{\mu} (1-\tau_t) \, w_t \left(C_t^{hh}\right)^{-\sigma} \right) + \beta \pi_{t+1}^W \right]
\end{bmatrix} = \boldsymbol{0}
$$

A potential DAG is the one below.