CENTER FOR ECONOMIC BEHAVIOR & INEQUALITY

4. Examples: I-HANK & HANK-SAM

IIES lectures

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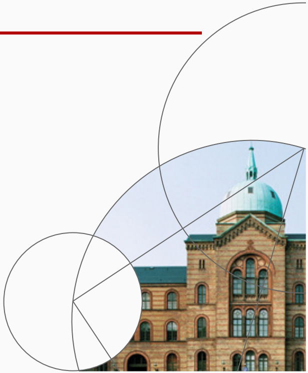


Introduction

Introduction

- **Further topics:**
  1. Policy analysis
  2. Life-cycle
  3. Endogenous idiosyncratic risk (time-varying)
  4. Discrete choices (with *taste shocks*)
  5. Bounded rationality (non-FIRE)
- **Examples:**
  1. **I-HANK:**

     Auclert, et. al. (2021), »Exchange Rates and Monetary Policy with Heterogeneous Agents: Sizing up the Real Income Channel«

     Druedahl et al. (2022),
     »The Transmission of Foreign Demand Shocks«
  2. **HANK-SAM:**

     Broer et. al. (2023),
     »Fiscal stimulus policies according to HANK-SAM«

Policies

Policies

1. **Policies as shocks:** *Choose ARMA(p,q) process and study IRFs* (also for utility, inequality and social welfare)
   Non-linear transition: Interaction with initial state and other shocks
2. **Policies from targets:** Make different policies comparable by achieving the *exact* same path of outcomes
3. **Policy rules:** Parameterize and study effects on IRFs to shocks
4. **Optimal policy with quadratic loss function** (McKay and Wolf, 2023)
   4.1 Easy to solve numerically with *ad hoc* loss function
   4.2 Harder to derive loss function from social welfare (Ramsey problem)
5. **Optimal policy:** Discretion, commitment, timeless perspective (Dávila and Schaab, 2023)

Life-cycle

Households with life-cycle

- **Age:** $h_{it} \in \{0, 1, \ldots, \#_h - 1\}$
- **Mortality:** $\delta\left(h_{it}\right) \in [0, 1]$, $\delta(\#_h - 1) = 1$, $\zeta(j) = \int \mathbf{1}\{h_{it} = j\} d\boldsymbol{D}_{ss}$
- **Income profile:** $\mathcal{Z}(z_{it}, h_{it+1})$
- **Household problem**, $\{r_t, w_t, q_t\} \to \{a_{it}, b_{it}, c_{it}, \ell_{it}\}$:

$$v_0(\beta_i, h_{it}, z_{it}, a_{it-1}) = \max_{c_{it}} u(c_t) + \beta_i(1 - \delta(h_{it}))\underline{v}_{t+1}\left(\beta_i, h_{it+1}, z_{it+1}, a_{it}\right)$$

$$\begin{aligned}
\text{s.t. } \ell_{it} &= z_{it} \\
a_{it} &= \begin{cases} (1 + r_t)q_t + w_t\ell_{it} - c_{it} + \Pi_t & \text{if } h_{it} = 0 \\ (1 + r_t)a_{it-1} + w_t\ell_{it} - c_{it} + \Pi_t & \text{else} \end{cases} \\
b_{it} &= \delta(h_{it})a_{it} \\
h_{it+1} &= \begin{cases} 0 & \text{with prob. } \delta(h_{it}) \\ h_{it} + 1 & \text{else} \end{cases} \\
z_{it+1} &\sim \mathcal{Z}(z_{it}, h_{it+1}),\ \mathbb{E}\left[z_{it}\right] = 1 \\
a_{it} &\geq 0
\end{aligned}$$

Equation system

$$\boldsymbol{H}\left(\boldsymbol{K}, \boldsymbol{q}, \boldsymbol{\Gamma}, \underline{\boldsymbol{D}}_0\right) = \begin{bmatrix} A_t - A_t^{hh} \\ \zeta(0) q_t - B_{t-1}^{hh} \\ \forall t \in \{0, 1, \ldots, T-1\} \end{bmatrix} = \mathbf{0}$$

where $K_{-1} = \int a_{t-1} d\underline{\boldsymbol{D}}_0$ and

$$\begin{aligned} L_t &= 1 \\ A_t &= K_t \\ r_t^K &= \alpha \Gamma_t (K_{t-1}/L_t)^{\alpha-1} \\ w_t &= (1-\alpha)\Gamma_t (K_{t-1}/L_t)^{\alpha} \\ \boldsymbol{D}_t &= \Pi_z' \underline{\boldsymbol{D}}_t \\ \underline{\boldsymbol{D}}_{t+1} &= \Lambda_t' \boldsymbol{D}_t \\ A_t^{hh} &= \boldsymbol{a}_t^{*\prime} \boldsymbol{D}_t \\ &\forall t \in \{0, 1, \ldots, T-1\} \end{aligned}$$

Endogenous idiosyncratic risk

Consumption problem

- **Recursive household problem:**

$$v_t(u_{it}, a_{it-1}) = \max_{c_{it}} u(c_{it}) + \beta \underline{v}_t (u_{it}, a_{it})$$

$$\text{s.t.}$$

$$a_{it} = (1 + r_t)a_{it-1} + y_{it} - c_{it}$$

$$y_{it} = (1 - \tau_t)w_t \begin{cases} 1 & \text{if } u_{it} = 0 \\ \phi \in (0,1) & \text{if } u_{it} = 1 \end{cases}$$

$$a_{it} \geq 0$$

- Working if $u_{it} = 0$, unemployed if $u_{it} = 1$
- **Solution method:** *Standard EGM*
- **Code:** *HANK-SAM-Simple**

External endogenous risk

- **Expectation step:**

$$\underline{v}_t(u_{it-1}, a_{it-1}) = \mathbb{E}\left[v_t(u_{it}, a_{it-1}) \,|\, u_{it-1}, a_{it-1}, \delta_t, \lambda_t^u\right]$$

s.t.

$$\pi_t(u_{it} \,|\, u_{it-1}) = \begin{cases} \delta_t & \text{if } u_{it} = 1 \text{ and } u_{it-1} = 0 \\ 1 - \delta_t & \text{if } u_{it} = 0 \text{ and } u_{it-1} = 0 \\ \lambda_t^u & \text{if } u_{it} = 0 \text{ and } u_{it-1} = 1 \\ 1 - \lambda_t^u & \text{if } u_{it} = 1 \text{ and } u_{it-1} = 1 \end{cases}$$

- **Stochastic transition matrix:** $\Pi_{t,z} = \Pi_z(\delta_t, \lambda_t^u)$
- **Envelope condition:** *Nothing changed*
- **Transition steps:**

$$\boldsymbol{D}_t = \Pi_{t,z}' \underline{\boldsymbol{D}}_t$$

$$\underline{\boldsymbol{D}}_{t+1} = \Lambda_t' \boldsymbol{D}_t$$

Internal endogenous risk

- **Expectation step:**

$$\underline{v}_t(u_{it-1}, a_{it-1} \mid s_{it}) = \mathbb{E}\left[v_t(u_{it}, a_{it-1}) \mid u_{it-1}, a_{it-1}, s_{it}, \delta_t, \lambda_t^{u,s}\right]$$

s.t.

$$\pi_t(u_{it} \mid u_{it-1}, s_{it}) = \begin{cases} \delta_t & \text{if } u_{it} = 1 \text{ and } u_{it-1} = 0 \\ 1-\delta_t & \text{if } u_{it} = 0 \text{ and } u_{it-1} = 0 \\ \lambda_t^{u,s} s_{it} & \text{if } u_{it} = 0 \text{ and } u_{it-1} = 1 \\ 1-\lambda_t^{u,s} s_{it} & \text{if } u_{it} = 1 \text{ and } u_{it-1} = 1 \end{cases}$$

- **Search decision:**
    1. Discrete search choice: $s_{it} \in \{0, 1\}$
    2. Search cost: λ if $s_{it} = 1$
    3. Taste shocks: $\varepsilon(s_{it}) \sim$ Extreme value (Iskhakov et. al., 2017)
- **See also:** Bardóczy (2021)

Discrete search decision

- **Standard logit formula:**

$$\underline{v}_t(u_{it-1}, a_{it-1}) = \max_{s_{it}\in\{0,1\}} \left\{\underline{v}_t(u_{it-1}, a_{it-1} \,|\, s_{it}) - \lambda \mathbf{1}_{s_{it}=1} + \sigma_\varepsilon \varepsilon\left(s_{it}\right)\right\}$$
$$= \sigma_\varepsilon \log\left(\exp \frac{\underline{v}_t(u_{it-1}, a_{it-1} \,|\, 0)}{\sigma_\varepsilon} + \exp \frac{\underline{v}_t(u_{it-1}, a_{it-1} \,|\, 1)}{\sigma_\varepsilon}\right)$$

- **Transition matrix:**

$$\Pi_{t,z} = \Pi_z \left(\left\{r_\tau, w_\tau, \tau_\tau, \delta_\tau, \lambda_\tau^{u,s}\right\}_{\tau \geq t}\right)$$

Envelope condition

- **Choice probabilities:**

$$P_t(s \mid u_{it-1}, a_{it-1}) = \frac{\exp \frac{\underline{v}_t(u_{it-1}, a_{it-1} \mid s)}{\sigma_\xi}}{\sum_{s' \in \{0,1\}} \exp \frac{\underline{v}_t(u_{it-1}, a_{it-1} \mid s')}{\sigma_\xi}}$$

- **Envelope condition:**

$$\begin{aligned}
\underline{v}_{a,t}(u_{t-1}, a_{t-1}) &= \sum_{s \in \{0,1\}} P_t(s \mid u_{it-1}, a_{it-1}) \pi_t(u_{it} \mid u_{it-1}, s) v_{a,t}(u_{it}, a_{it-1}) \\
&= \sum_{s \in \{0,1\}} P_t(s \mid u_{it-1}, a_{it-1}) \pi_t(u_{it} \mid u_{it-1}, s) c_t^*(u_{it}, a_{it-1})^{-\sigma}
\end{aligned}$$

- **Break of *monotonicity*** $\Rightarrow$ FOC still *necessary*, but not *sufficient*
    1. **Normally:** Savings $\uparrow \Rightarrow$ future consumption $\uparrow \Rightarrow$ marginal utility $\downarrow$
    2. **Now also:** Future search jump $\downarrow \Rightarrow$ future income $\downarrow$
       $\Rightarrow$ future consumption $\downarrow \Rightarrow$ marginal utility $\uparrow$

Upper envelope for given z^{i_z}

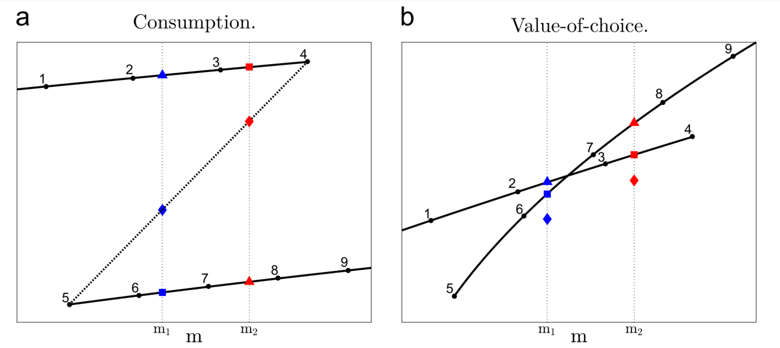
1. **Generate candidate points:** $\forall i_a \in \{0, 1, \ldots, \#_a - 1\}$

$$
\begin{aligned}
w^{i_a} &= \beta \underline{v}_{t+1}(z^{i_z}, a^{i_a}) \\
c^{i_a} &= u'^{-1}\left(\beta \underline{v}_{a,t+1}(z^{i_z}, a^{i_a})\right) \\
m^{i_a} &= a^{i_a} + c^{i_a} \\
v^{i_a} &= u\left(c^{i_a}\right) + w^{i_a}
\end{aligned}
$$

2. **Apply upper-envelope:** $\forall i_{a-} \in \{0, 1, \ldots, \#_a - 1\}$

$$
\begin{aligned}
c^*(a^{i_{a-}}) = \max_{j \in \{0,1,\ldots \#_a - 2\}} & u\left(c^{i_{a-}}\right) + w^{i_{a-}} \text{ s.t.} \\
m^{i_{a-}} &= (1 + r_t)a^{i_{a-}} + w_t z^{i_z} \in \left[m^j, m^{j+1}\right] \\
c^{i_{a-}} &= \min\left\{\text{interp } \left\{m^{i_a}\right\} \to \left\{c^{i_a}\right\} \text{ at } m^{i_{a-}}, m^{i_{a-}}\right\} \\
a^{i_{a-}} &= m^{i_{a-}} - c^{i_{a-}} \\
w^{i_{a-}} &= \text{interp } \left\{a^{i_a}\right\} \to \left\{w^{i_a}\right\} \text{ at } a^{i_{a-}}
\end{aligned}
$$

Illustration



1. **Numbering:** Different levels of end-of-period assets, a^{i_a}
2. **Problem:** Find the consumption function at m_1 and m_2
3. **Largest value-of-choice:** Denoted by the *triangles*

Source: Druedahl and Jørgensen (2017), *G^2EGM*

Example

- **Beg.-of-period value function:**

$$\underline{v}_{t+1}(a_t) = \sqrt{m_{t+1}} + \eta \max\left\{m_{t+1} - \underline{m}, 0\right\}$$
$$\text{where } m_{t+1} = (1+r)a_t + 1$$

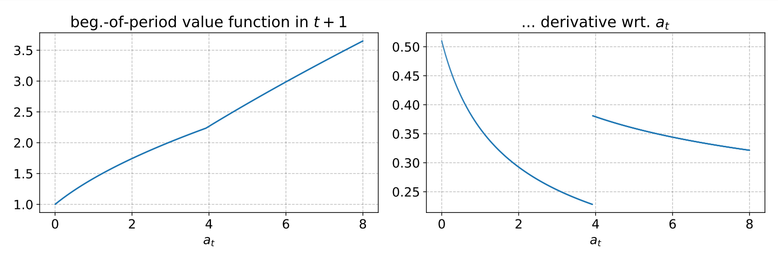
- **Derivative:**

$$\underline{v}_{a,t+1}(a_t) = \frac{1}{2}(1+r)m_{t+1}^{-\frac{1}{2}} + (1+r)\eta\mathbf{1}\left\{m_{t+1} > \underline{m}\right\}$$

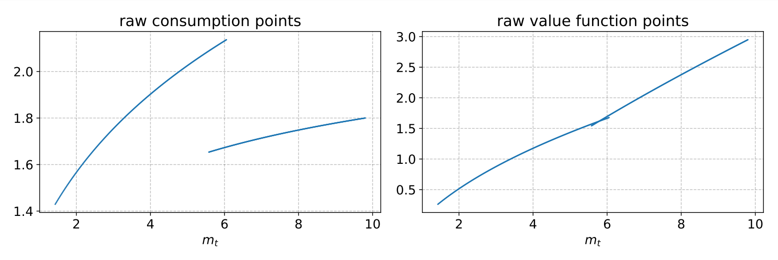
- **Budget constraint:**

$$a_t + c_t = (1+r)a_{t-1} + 1$$

Next-period values

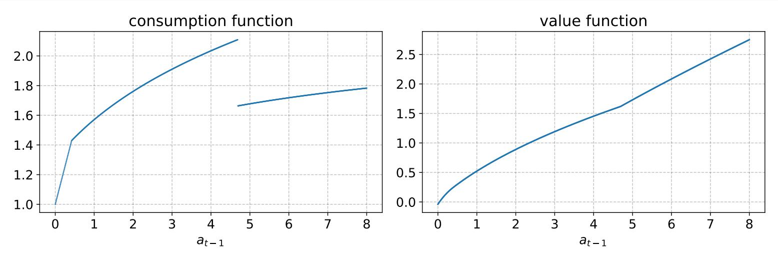


Raw values of c^{i_a} and v^{i_a}



Problem: Overlaps $\Rightarrow$ not a function m_t!

Result after upper envelope



General problem structure

- **General problem structure with *nesting*:**

$$\overline{v}_t\left(\overline{x}_t, d_t, e_t, m_t\right) = \max_{c_t \in [0, m_t]} u(c_t, d_t, e_t) + \beta \underline{v}_{t+1}\left(\underline{\Gamma}_t\left(\overline{x}_t, d_t, e_t, a_t\right)\right)$$

$$\text{with } a_t = m_t - c_t$$

$$v(x_t) = \max_{d_t \in \Omega^d(x_t)} \overline{v}_t\left(\overline{\Gamma}_t\left(x_t, d_t\right)\right)$$

$$\underline{v}_t\left(\underline{x}_t\right) = \max_{e_t \in \Omega^e(\underline{x}_t)} \mathbb{E}\left[v\left(\Gamma\left(\underline{x}_t, e_t\right)\right) \mid \underline{x}_t, e_t\right]$$

- **Finding c_t:** *EGM with upper envelope can (typically) still be used*
- **Finding d_t and e_t:**
  1. Combination of discrete and continuous choices
  2. Typically requires use of numerical optimizer or root-finder
- **Druedahl (2021),** »A Guide on Solving Non-Convex Consumption-Saving Models« (costly with extra states in $\overline{v}$)

Non-FIRE

Motivating example

- **FIRE:** *Full International Rational Expectations*
- **IKC:** $d\boldsymbol{Y} = \boldsymbol{M}^r d\boldsymbol{r} + \boldsymbol{M} d\boldsymbol{Y}$ where $M_{t,s} = \frac{\partial C_t}{\partial Y_s}$ and similar for $\boldsymbol{M}^r$
- **Myopic behavior:**
  1. Agents *never* thinks about the future
  2. Agents gradually observe current aggregate variables

$$\boldsymbol{M}^{\text{myopic}} = \begin{bmatrix} M_{0,0} & 0 & 0 & \cdots \\ M_{1,0} & M_{0,0} & 0 & \cdots \\ M_{2,0} & M_{1,0} & M_{0,0} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Consider $t = 1$:

- $M_{1,0} dY_0$: Effect from past shock observed
- $M_{0,0} dY_1$: Effect of unexpected change in period 1

Sticky expectations

- **Sticky expectations:** A fraction $1-\theta$ updates expectations each period (from Carroll et. al., 2020)

$$\boldsymbol{M}^{\text{sticky}} = \begin{bmatrix} M_{0,0} & (1-\theta)M_{0,1} & (1-\theta)M_{0,2} & \cdots \\ M_{1,0} & (1-\theta)M_{1,1}+\theta M_{0,0} & (1-\theta)M_{1,2}+\theta(1-\theta)M_{0,1} & \cdots \\ M_{2,0} & (1-\theta)M_{2,1}+\theta M_{1,0} & \vdots & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Consider $t=0$:

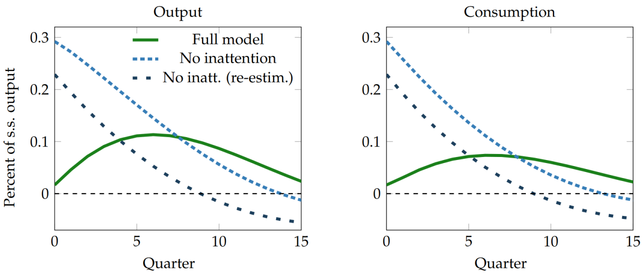
1. Non-updaters: $\theta M_{0,0} dY_0$
2. Updaters: $(1-\theta)\sum_{s\geq 0} M_{0,s} dY_s$

Consider $t=1$:

1. Ingoing updaters: $(1-\theta)\sum_{s\geq 0} M_{1,s} dY_s$
2. Ingoing non-updaters: $\theta\left(M_{1,0} dY_0 + M_{0,0} dY_1\right)$
3. New updaters: $\theta(1-\theta)\sum_{s\geq 1} M_{0,s} dY_{s+1}$

Hump-shaped response to monetary policy

Figure 4: Impulse responses with and without inattention

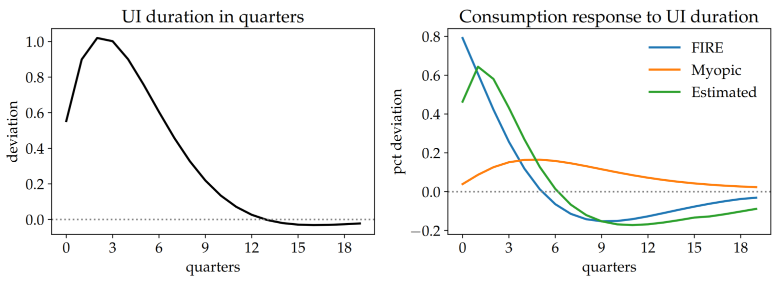


Note. This figure shows the general equilibrium paths of output and consumption in our estimated HA model with different assumptions on inattention. The solid green uses our baseline estimates of household inattention. The dashed blue line is the impulse response when the inattention parameter is set to $\theta = 0$, holding all other parameters fixed at their estimated value in table 2. The dotted dark blue line reestimates the model parameters without inattention.

Source: Auclert et. al. (2020), »Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model«

UI extensions might be less powerful

Figure 4: Partial-equilibrium Consumption Responses to an UI Extension



Source: Bardóczy and Guerreiro (2020), »Unemployment Insurance in Macroeconomic Stabilization with Imperfect Expectations«

I-HANK

International HANK

- **Baseline RANK model:** Gali and Monacelli (2005)

- **Exchange rate shocks:**
  Auclert, Rognlie, Souchier and Straub (2021),
  »Exchange Rates and Monetary Policy with Heterogeneous Agents: Sizing up the Real Income Channel«

- **Foreign demand shocks:**
  Druedahl, Ravn, Sunder-Plassmann, Sundram, Waldstrøm (2023),
  »The Transmission of Foreign Demand Shocks«

  Code: *I-HANK\Code** (fit slides)

Overview of model

- **Small-open-economy** $\Rightarrow$ *trading partner is exogenous*
- **Goods:** *Home* and *foreign*
- **Households:** Standard + CES demand
  (initially households only hold domestic stocks $\Rightarrow$ no reevaluation effects)
- **Production of home goods:** Flexible prices with mark-up μ
  1. Production: $Y_t = Z_t N_t$
  2. Wage from FOC: $W_t = \frac{1}{\mu} P_{H,t} Z_t$
  3. Dividends: $D_t = P_{H,t} Y_t - W_t N_t$
- **Unions:** Sticky wages $\Rightarrow$ NKWPC
- **Financial markets:** Floating exchange rate + UIP condition
- **Central bank:** Constant real rate rule

Household problem

- **Nominal exchange rate:** E_t (domestic per foreign currency)
- **Real exchange rate:** $Q_t = E_t \frac{P_t^*}{P_t}$ (depreciation $\equiv Q_t \uparrow$)
- **Domestic CES demand:**

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C^{hh}\left(\boldsymbol{Y}^{hh}\right)$$

$$C_{F,t} = \alpha\left(\frac{E_t P_t^*}{P_t}\right)^{-\eta} C^{hh}\left(\boldsymbol{Y}^{hh}\right)$$

$$P_t = \left[\alpha\left(E_t P_t^*\right)^{1-\eta} + (1-\alpha) P_{H,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

- **Real income:**

$$Y_t^{hh} = \frac{W_t N_t + D_t}{P_t} = \frac{P_{H,t} Y_t}{P_t} \Rightarrow$$

$$dY_t^{hh} = dY_t - \left(dP_t - dP_{H,t}\right) = dY_t - \frac{\alpha}{1-\alpha} dQ_t$$

Foreign economy and market clearing

- **Armington demand for home goods:**

$$C^*_{H,t} = \alpha \left(\frac{P_{H,t}}{E_t P^*_t} \right)^{-\gamma} C^*_t$$

- **Market clearing:**

$$Y_t = C_{H,t} + C^*_{H,t} \Rightarrow$$
$$dY_t = (1-\alpha) dC_t + \alpha dC^*_t + \frac{\alpha}{1-\alpha} \chi dQ_t$$

Composite trade elasticity: $\chi \equiv \eta(1-\alpha) + \gamma$

Real exchange rate shock

- **Real exchange rate shock:** $d\boldsymbol{Q}_t$
- **Consumption satisfies:**

$$d\boldsymbol{C} = \underbrace{\boldsymbol{M}d\boldsymbol{Y}}_{\text{multiplier}} - \underbrace{\frac{\alpha}{1-\alpha}\boldsymbol{M}d\boldsymbol{Q}}_{\text{real income}}$$

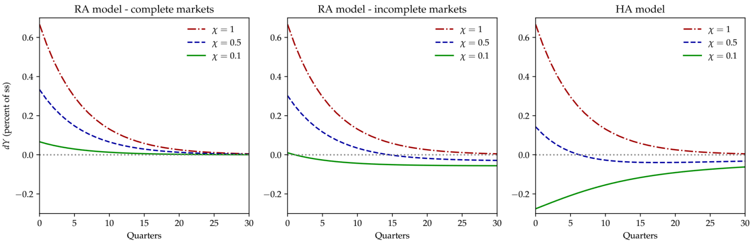
- **Intertemporal Keynesian Cross:**

$$\begin{aligned} d\boldsymbol{Y} = & \underbrace{\frac{\alpha}{1-\alpha}d\boldsymbol{Q}}_{\text{expenditure switching}} - \underbrace{\alpha\boldsymbol{M}d\boldsymbol{Q}}_{\text{real income}} + \underbrace{(1-\alpha)\boldsymbol{M}d\boldsymbol{Y}}_{\text{multiplier}} \\ &= \mathcal{M}\left(\frac{\alpha}{1-\alpha}\chi d\boldsymbol{Q} - \alpha\boldsymbol{M}d\boldsymbol{Q}\right) \end{aligned}$$

  - Expansion: For high χ
  - Contraction: For low χ

Contractionary depreciation

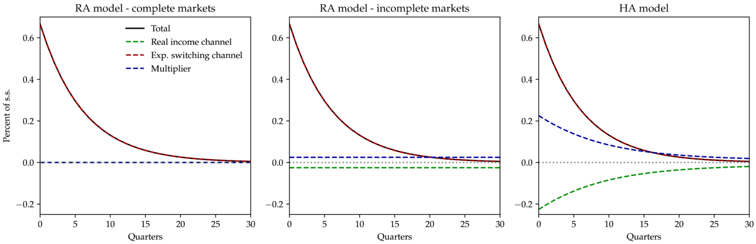
Figure 2: Effect of exchange rate shocks on output for various χ's



Note: impulse response in all three models to the shock to i_t^* displayed in Figure 1. χ is the trade elasticity (the sum of the import and export elasticity to the exchange rate). The HA model generates a contraction on impact for $\chi < \chi^* = 0.37$.

Equivalence and decomposition

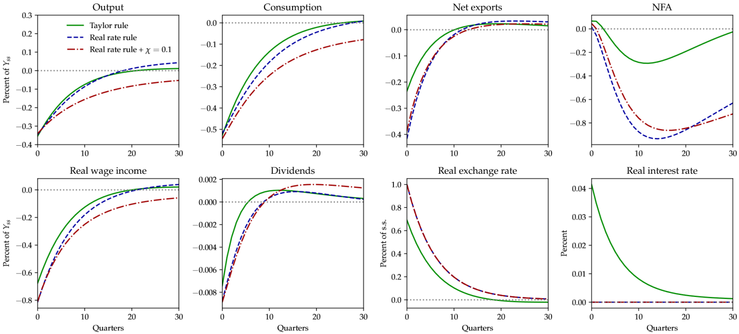
Figure 3: Exchange rate shock when $\chi = 1$ and its transmission channels



Note: impulse response in all three models to the shock to i_t^* displayed in Figure 1, with decomposition from proposition 2.

Quantitative model

Figure 9: Contractionary depreciations



Note: impulse response in the quantitative model to the shock to i_t^* displayed in Figure 1. The model with Taylor rule is our quantitative model; the one with real rate rule is our quantitative model without the Taylor rule; the model with real rate and $\chi = 0.1$ drops delayed substitution and allows households to immediately adjust their consumption baskets across countries.

Add-ons: Non-homothethic preferences, sticky prices , imperfect exchange rate pass-through, delayed substitution, dollar currency pricing, UIP deviations

Foreign demand shocks

The Transmission of Foreign Demand Shocks

Jeppe Druedahl Søren Hove Ravn Laura Sunder-Plassmann
Jacob Marott Sundram Nicolai Waldstrøm

September, 2023

Regional Keynesian Cross

The Regional Keynesian Cross

Proposition

The first-order response of employment $d\mathbf{L}_j$ to a monetary shock $d\mathbf{r}_j$ and tradable goods demand shock $d\mathbf{C}^T$ solves

$$d\mathbf{L}_j = \underbrace{\rho_j \left(\mathbf{M}_j^r d\mathbf{r}_j + \mathbf{M}_j d\mathbf{L}_j \right)}_{\text{Regional exposure}} + \underbrace{(1-\rho_j) d\mathbf{C}^T}_{\text{National exposure}} - \underbrace{\frac{\nu}{\eta}(1-\rho_j) \left(d\mathbf{L}_j - d\mathbf{C}^T \right)}_{\text{Expenditure switching}}$$

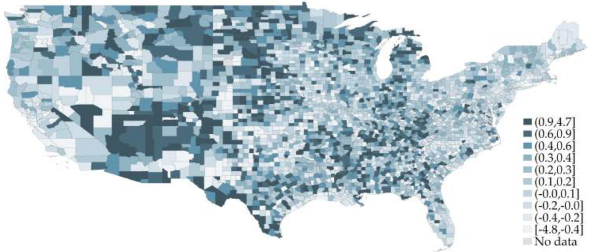
ν: elasticity of subs. between c^{NT} & c^T

η: elasticity of subs. between ℓ^{NT} & ℓ^T

Source: Bellifemine, Couturier, and Jamilov (2023), »The Regional Keynesian Cross«

Regional employment effects of MP

Figure 1: Regional Heterogeneity in the Effects of U.S. Monetary Policy

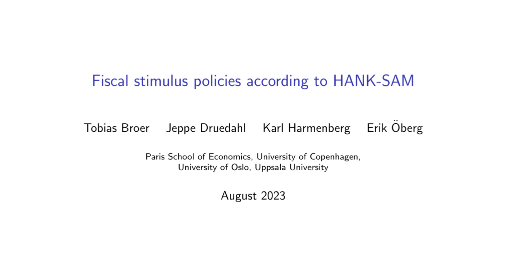


Note: This figure plots the 3-year ahead county-specific cumulative employment responses to a 1 standard deviation expansionary monetary policy shock $\beta_{j,36}$, estimated from the panel local projection (1). The coefficients are in percentage points and represent deviations from the (population weighted) average response.

HANK-SAM

My ongoing work

- **Zero liqduity:**
  Broer, Druedahl, Harmenberg and Öberg (2023),
  »The Unemployment-Risk Channel in Business-Cycle Fluctuations«
- **Positive liqudity:**
  Broer, Druedahl, Harmenberg and Öberg (2023),
  »Fiscal stimulus policies according to HANK-SAM«


Fiscal stimulus policies according to HANK-SAM

Tobias Broer Jeppe Druedahl Karl Harmenberg Erik Öberg

Paris School of Economics, University of Copenhagen,
University of Oslo, Uppsala University

August 2023

Code: *HANK-SAM-Fiscal**

Household problem

$$V_t^w(\beta_i, u_{it}, \mathrm{UI}_{it}, a_{it-1}) = \max_{c_{it}, a_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta_i \underline{V}_{t+1}^w(\beta_i, u_{it}, \mathrm{UI}_{it}, a_{it})$$

$$\text{s.t. } a_{it} + c_{it} = R_t^{\text{real}} a_{it-1} + (1-\tau_t) y_t(u_{it}) + \mathrm{Div}_t + T_t$$

$$a_{it} \geq 0$$

1. **Dividends and government transfers:** Div_t and T_t
2. **Real wage**: W
3. **Income tax:** τ_t
4. **Separation rate** for employed: δ_t
5. **Job-finding rate** for unemployed: $\lambda_t^{u,s} s(u_{it-1})$
   (where $s(u_{it-1})$ is exogenous search effectiveness)
6. US-style duration-dependent **UI system:**
   a) High replacement rate $\overline{\phi}_t$, first $\overline{u}_t$ months
   b) Low replacement rate $\underline{\phi}$, after $\overline{u}_t$ months
   c) Receive low UI immediately ($\mathrm{UI}_{it} = 0$) with probability $1 - \pi^{UI}$

Income process

- **Income** is

$$y_{it} = \begin{cases} W & \text{if } u_{it} = 0 \\ \underline{\phi} & \text{else if } \mathrm{UI}_{it} = 0 \\ \overline{\phi}_t \tilde{\mathrm{UI}}_{it} + \tilde{\mathrm{UI}}_{it} \underline{\phi} & \text{else} \end{cases}$$

where share of the month with UI is

$$\tilde{\mathrm{UI}}_{it} = \begin{cases} 0 & \text{if } u_{it} = 0 \text{ or } \mathrm{UI}_{it} = 0 \\ 1 & \text{else if } u_{it} < \overline{u}_t \\ 0 & \text{else if } u_{it} > \overline{u}_t + 1 \\ \overline{u}_t - (u_{it} - 1) \in [0, 1] & \text{else} \end{cases}$$

- **Note:** *Hereby $\overline{u}_t$ becomes a continuous variables*

Transition probabilities

- **Beginning-of-period value function:**

$$\underline{V}_t^w\left(u_{it-1}, a_{it-1}\right) = \mathbb{E}\left[u_{it}, a_{it-1} \mid u_{it-1}, a_{it-1}\right]$$

- **Grids:** $u_{it} \in \{0, 1, \ldots, \#_u - 1\}$ for $\#_u - 1$
- **Workers** with $u_{it-1} = 0$:

$$u_{it} = \begin{cases} 0 & \text{with } 1 - \delta_t \\ 1 & \text{with } \delta_t \end{cases}, \; \mathrm{UI}_{it} = \begin{cases} 1 & \text{with } \pi^{\mathrm{UI}} \\ 0 & \text{with } 1 - \pi^{\mathrm{UI}} \end{cases}$$

- **Unemployed** with $u_{it-1} = 1$:

$$u_{it} = \begin{cases} 0 & \text{with } \lambda_t^{u,s} \mathrm{s}(u_{it-1}) \\ \min\left\{u_{it-1} + 1, \#_u - 1\right\} & \text{with } 1 - \lambda_t^{u,s} s(u_{it-1}) \end{cases}, \; \mathrm{UI}_{it} = \mathrm{UI}_{it-1}$$

Hiring and firing

- **Job value:** Firm must pay continuation cost $\chi \sim G$. Functional form assumption gives **separation rate**, δ_t, with elasticity ψ

$$V_t^j = p_t^x z_t - (w_t - \text{retention subsidy}_t) + \beta^{\text{firm}} \mathbb{E}_t \left[(1 - \delta_{t+1})(V_{t+1}^j - \mu_{t+1}^j) \right]$$

$$\delta_t = \delta_{ss} \left(\frac{V_t^j}{V_{ss}^j} \right)^{-\psi}, \quad \mu_{t+1}^j \text{ is continuation cost}$$

- **Vacancy value:** Mass of F entrants draw entry cost $c \sim H$. Functional form assumption gives **entry**, ι_t, with elasticity ξ

$$V_t^v = -\kappa + \lambda_t^v (V_t^j + \text{hiring subidy}_t) + (1 - \lambda_t^v)(1 - \delta_{ss}) \beta^{\text{firm}} \mathbb{E}_t \left[V_{t+1}^v \right]$$

$$\iota_t = \iota_{ss} \left(\frac{V_t^v}{V_{ss}^v} \right)^{\xi}$$

Labor market dynamics

- **Vacancy dynamics:**

$$\tilde{v}_t = (1-\delta_{ss})v_{t-1} + \iota_t$$
$$v_t = (1-\lambda_t^v)\tilde{v}_t$$

- **Effective searchers and Unemployed:**

$$S_t = \int s(u_{it-1}) d\underline{\boldsymbol{D}}_t$$
$$u_t = u_{t-1} + \delta_t(1-u_{t-1}) - \lambda_t^{u,s} S_t$$

- **Tightness, $\theta_t = \frac{\tilde{v}_t}{u_t}$, and matching function:**

$$\lambda_t^v = A\theta_t^{-\alpha}$$
$$\lambda_t^{u,s} = A\theta_t^{1-\alpha}$$
$$\lambda_t^u = \frac{\lambda_t^{u,s} S_t}{u_{t-1}}$$

Standard New Keynesian block

- **Intermediate goods price:** P_t^X
- Dixit-Stiglitz **demand curve** $\Rightarrow$ **Phillips curve** relating marginal cost, $MC_t = P_t^X$, and **final goods price inflation**, $\Pi_t = P_t/P_{t-1}$,

$$1 - \epsilon + \epsilon P_t^X = \phi(\Pi_t - 1)\Pi_t - \phi\beta^{\text{firm}}\mathbb{E}_t\left[(\Pi_{t+1} - 1)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]$$

  with output $Y_t = (1 - u_t)Z_t$
- **Flexible price limit:** $\phi \to 0$
- **Taylor rule:**

$$R_t = R_{ss}\Pi_t^{\delta_\pi}$$

- **Fisher equation:**

$$R_t^{\text{real}} = R_{t-1}/\Pi_t$$

Government

- **Fiscal rule:**

$$\tau_t = \tau_{ss} + \omega q_{ss} \frac{B_{t-1} - B_{ss}}{Y_{ss}^{hh}}$$

where ω determines response of taxes to fluctuations in debt level

- **Government budget with long-term bonds:**

$$\begin{aligned} q_t B_t =& q_t \delta_q B_{t-1} + B_{t-1} \\ &+ (1-\tau_t)\left(\overline{\phi}_t \tilde{UI}_t^{hh} + \underline{\phi}\left(u_t - \tilde{UI}_t^{hh}\right)\right) w_t \\ &- \tau_t(1-u_t)w_t \\ &+ \text{retention subsidy}_t \cdot (1-u_t) \\ &+ \text{hiring subsidy}_t \cdot \lambda_t^v((1-\delta_{ss})v_{t-1} + \iota_t) \\ &+ \text{transfer}_t + G_t \end{aligned}$$

where UI_t^{hh} is share of households on UI

Equilibrium

1. **No arbitrage** requires

$$\frac{1+\delta_q q_{t+1}}{q_t} = R^{\text{real}}_{t+1}$$

2. **Asset market clearing:**

$$q_t B_t = \int a_t^\star(\beta_i, u_{it}, \text{UI}_{it}, a_{it-1}) d\boldsymbol{D}_t$$

Unknowns and targets

- **8 unknowns:** P_t^X, V_t^j, V_t^v, Π_t, u_t, $\tilde{v}_t$, $\widetilde{UI}_t^{hh}$, S_t
- **8 targets:**
  1. Bellman for V_t^j
  2. Bellman for V_t^v
  3. New Keynesian Philips Curve (NKPC)
  4. Taylor Rule
  5. Market clearing for assets
  6. $\tilde{v}_t = (1 - \delta_{ss}) v_{t-1} + \iota_t$
  7. $\widetilde{UI}_t^{hh} = \int \widetilde{UI}_t \, d\boldsymbol{D}_t$
  8. $u_t = \int \mathbf{1}\{u_{it} > 0\} \, d\boldsymbol{D}_t$

SAM paramters

Parameter	Value	Source / Target
Firm discount factor, β^{firm}	$0.98^{\frac{1}{12}}$	Standard
Matching function elasticity, α	0.60	Petrongolo and Pissarides (2001)
Separation rate, δ_{ss}	0.027	Data
Job-finding rate, λ^{u}_{ss}	0.31	Data
Tightness, θ_{ss}	0.60	Hagedorn and Manovskii (2008)
Search effectiveness, $\varphi(u_{it})$	(figure)	Eubank and Wiczer (2016)
Technology shock, persistence, ρ_Z	$0.907^{\frac{1}{3}}$	Data
Technology, standard deviation, σ_Z	0.01	Scaling
Separation elasticity, ψ	1.698	EU share of unemployment volatility
Entry elasticity, ξ	0.015	UE lag relative to EU
Wage level, w_{ss}	0.69	Unemployment var. w.r.t. TFP shock

Note: Adjustment costs are virtual

HA parameters

HA Parameters	Value	Source / Target
Discount factors, β_i^{12}	$\{0.00, 0.971\}$	Consumption drop during unemployment
... population shares	$\{0.38, 0.62\}$	Consumption drop at UI expiration
CRRA coefficient, σ	2	Standard
High UI, $\overline{\phi}$	0.76	Kekre (2022)
Low UI, $\underline{\phi}$	0.55	Kekre (2022)
UI duration, $\overline{u}$	6.0	Standard
UI prob, π^{UI}	0.48	UI recipients / unemployed = 39 percent
Degree of tax financing, ω	0.1	Auclert et. al. (2020)
Bond maturity, δ_q	$1 - 1/60$	Auclert et. al. (2020)
Value of bonds, $\frac{q_{ss} B_{ss}}{Y_{ss}^{hh}}$	2.12	Steady state interest rate of 2 percent

NK parameters

NK Parameters	Value	Source / Target
Substitution elasticity, ϵ_p	6	Standard
Rotemberg cost, φ	355	Standard
Taylor rule parameter, ϕ_π	1.5	Standard

Note: Adjustment costs are virtual

Representative Agent (RA)

- **Consumption from resource constraint:**

$$C_t^{\mathrm{RA}} = Y_t - G_t$$

- Replace **asset market clearing** with ***error in aggregate Euler-equation***:

$$\left(C_t^{\mathrm{RA}}\right)^{-\sigma} = \beta^{\mathrm{RA}} R_{t+1}^{\mathrm{real}} \left(C_{t+1}^{\mathrm{RA}}\right)^{-\sigma}$$

where

$$\beta_{ss}^{RA} = \frac{1}{R_{ss}^{\mathrm{real}}}$$

Main policy experiment

- **Baseline:** IRF of government spending, G_t, as *shock*
- **All other polices:** Consider each policy as a *new unknown* and the unemployment IRF from the baseline a *new target* to hit
  - Unconditional cash transfers, T_t
  - Unemployment insurance (UI) increases, $\overline{\phi}_t$
  - Unemployment insurance (UI) duration extensions, $\overline{u}_t$
  - Retention subsidy$_t$
  - Hiring subsidy$_t$
- **Comparison:** Different multipliers

$$\frac{\sum_{t=0}^{\infty} d\text{output}_t}{\sum_{t=0}^{\infty} d\text{taxes}_t}$$

Summary

Summary

- **Today:**
    1. Policies (shocks, targets, rule, optimal)
    2. Life-cycle (age, mortality, income profile)
    3. Endogenous idiosyncratic risk (external/internal)
    4. Discrete choices with taste shocks (upper envelope, non-convex)
    5. Bounded rationality (manipulation of Jacobian, myopic, sticky)
    6. I-HANK (real income channel, contractionary depreciation, exchange rate and foreign demand shocks, regional dynamics)
    7. HANK-SAM (labor market dynamics, fiscal policy)
- **DONE**