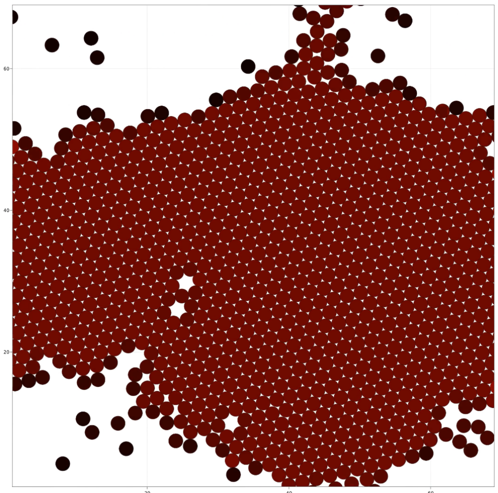

A Primer on Active Brownian Particles

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Active Brownian Particles (ABPs) are an idealised physical model of real-world *active matter* (AM) - that is objects like insects, birds or bacteria which are able to move of there own volition. AM theory is interesting to physicists partly due to it's applications to complex real-world behaviour e.g understanding bird flocks and computer graphics, but also because it deviates fundamentally from standard undergraduate physics (equilibrium statistical mechanics).

Keywords: *active-matter*, *active-Brownian-motion*

Code-repository: **Video:**

The Model

Keeping it simple, an ABP is a circle (in 2D) able to propel itself at some speed. Its heading direction changes randomly by increasing or decreasing its angle by small Gaussian (normally distributed) increments across time, in jargon this is called *rotational diffusion*. Sometimes an ABP may also have a random component directly applied to its position, as apposed to via an orientational change, such an ABP can be *translationally diffusive* as well (hence the Brownian part of its name). This means of random change follows the theory of Brownian motion, originally inspired from the actual erratic movements of pollen grains observed by Robert Brown in 1827, which was carefully constructed by Albert Einstein in 1905 [1]. The strengths of these random components are encoded with the rotational and translational diffusion coefficients D_r and D.

Most often interest lies within large groups of ABPs, in this case typically each particle will be identical in size, speed, and diffusion properties, although this is not required. The reason why groups are interesting is that each particle may interact with the others. At a simple level particles may collided with each other, bouncing off onto new trajectories, and at a complex level particles may change their self propulsion speed and direction dependent on what the others are doing. In fact there is no bound to what an "interaction" is, for example particles could change in size from small to large and back again periodically over time, eat other particles, or anything else!

The Maths

An ABP's movement can be described by differential equations split into translational E.q 1 and rotational E.q 2 parts, θ is the particles angle, t is time, $\boldsymbol{\eta}(t)$ and its coefficient give the translational diffusion, while $\eta_r(t)$ and its coefficient the rotational diffusion. These quantities are known as "White noise" meaning that any $\eta_r(t)$ is normally distributed with mean 0 and variance 1, but is totally statistically independent from any other $\eta_r(t')$. The same goes for $\boldsymbol{\eta}(t)$, except it is a vector of two components $[\eta_x, \eta_y]^T$ which must also have the same independence properties in relation to each other and themselves through time.

$$\frac{d\boldsymbol{r}(t)}{dt} = v_0 \begin{bmatrix} \cos(\theta(t)) \\ \sin(\theta(t)) \end{bmatrix} + \sqrt{2D}\boldsymbol{\eta}(t) \tag{1}$$

$$\frac{d\theta(t)}{dt} = \sqrt{2D_r}\eta_r(t) \tag{2}$$

For a group we may include a collision force term in E.q 1, say $k\boldsymbol{F}_{col}$. It may seem a bit complicated but listing 1 is all that is required to simulate an assembly of N ABPs each following equations 1 and 2 *plus collisions*, and *periodic boundary conditions* where particles "wrap around" as they go outside a box of given length.

One particular phenomenon of ABPs seen both in real life experiments as well as computer simulations is *motility induced phase separation* (MIPS), simply put MIPS is a mechanism by which ABPs confined at a high enough density will spontaneously form clustered and un-clustered regions due only to repulsive collision forces and active Brownian motion [2]. This is one example of the surprising and rich physics of *active* particles compared to *passive* particles.

Listing 1: Minimal ABP Julia Implementation

```julia
using LinearAlgebra

function Fcol(X,radius=1.0)
    F = zeros(size(X,1),2)
    for i in 1:size(X,1)
        for j in i+1:size(X,1) # take advantage of symmetry
            if j != i
                rij = X[j,1:2] .- X[i,1:2] # displacement vector between i and j
                r = norm(rij) # distance between i and j
                if r < 2*radius # if overlapping
                    F[i,:] += (r-2*radius)*rij/r # apply a ‘‘spring force’’
                    F[j,:] -= (r-2*radius)*rij/r # take advantage of symmetry
                end
            end
        end
    end
    return F
end

function Step(X;radius=1.0,v0=1.0,dt=0.001,Dr=0.1,Dt=0.0,L=10,k=1.)
    Y = copy(X)
    F = Fcol(X,radius) # get collision forces
    for i in 1:size(X,1)
        d = [cos(X[i,3]),sin(X[i,3])] # current heading direction of i
        # rotational diffusion/ Brownian motion in 1D
        Y[i,3] += sqrt(2.0*Dr/dt)*randn()*dt
        Y[i,1:2] += dt*(v0*d + k*F[i,:] + sqrt(2.0*Dt/dt)*randn(2))
    end
    Y[:,1:2] = mod.(Y[:,1:2],L) #periodic bc's
    return Y
end
```

Real Examples

The simplest ABP models can be used to describe the motion of Janus particles suspended in a liquid, e.g Buttinoni *et al.* Janus particles are constructed with two different materials on each hemisphere (Janus is a Roman god with two faces), in this case Buttinoni *et al* use Graphite on one hemisphere which becomes heated due to absorbing laser light giving self propulsion [3].

[1] P. Pearle, B. Collett, K. Bart, D. Bilderback, D. Newman, and S. Samuels, "What brown saw and you can too," *American Journal of Physics*, vol. 78, no. 12, pp. 1278–1289, 2010. DOI: 10.1119/1.3475685. [Online]. Available: https://doi.org/10.1119/1.3475685.

[2] Y. Fily and M. C. Marchetti, "Athermal phase separation of self-propelled particles with no alignment," *Phys. Rev. Lett.*, vol. 108, p. 235 702, 23 Jun. 2012. DOI: 10.1103/PhysRevLett.108.235702. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevLett.108.235702.

[3] I. Buttinoni, J. Bialké, F. Kümmel, H. Löwen, C. Bechinger, and T. Speck, "Dynamical clustering and phase separation in suspensions of self-propelled colloidal particles," *Phys. Rev. Lett.*, vol. 110, p. 238 301, 23 Jun. 2013. DOI: 10.1103/PhysRevLett.110.238301.