# Nanophotonic Computational Design

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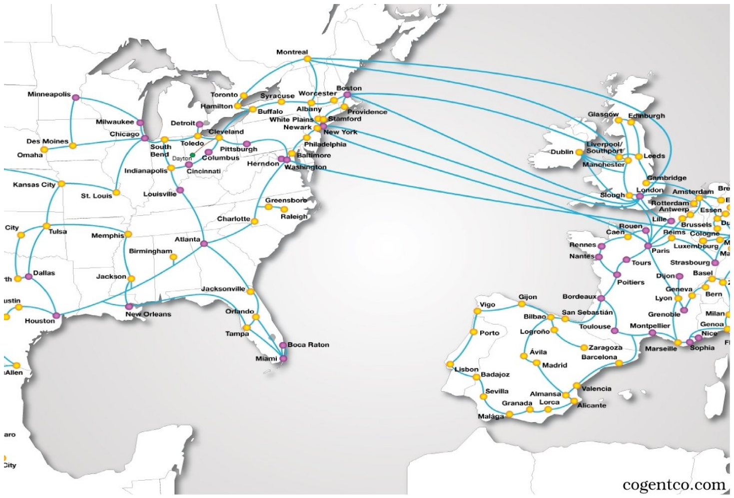
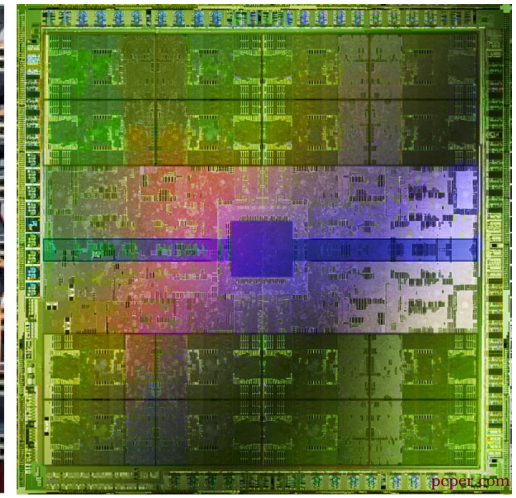
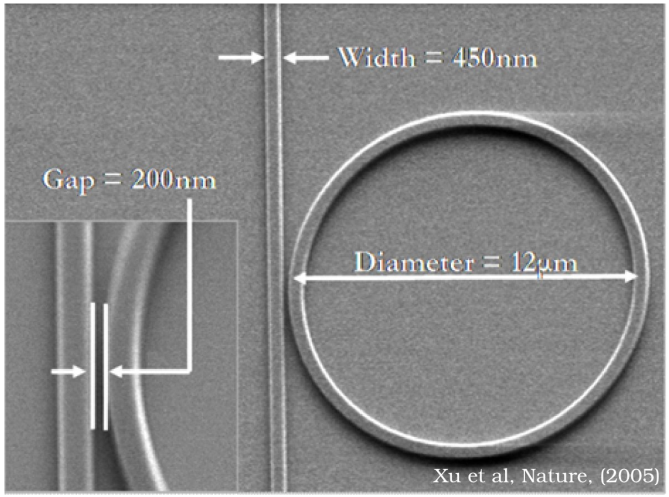
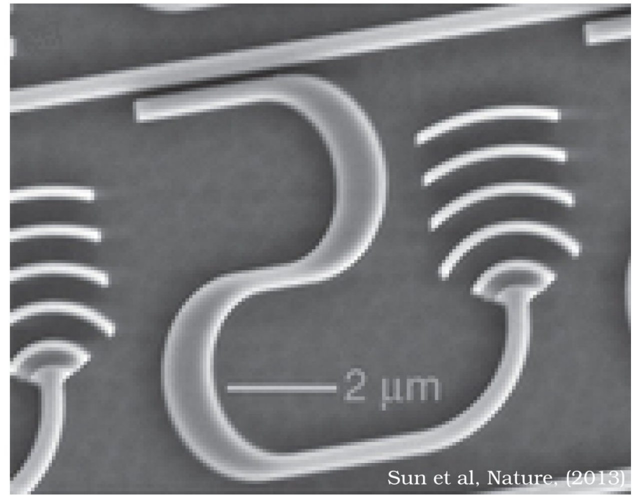
# Introduction






- Bring the optical network onto a computer chip



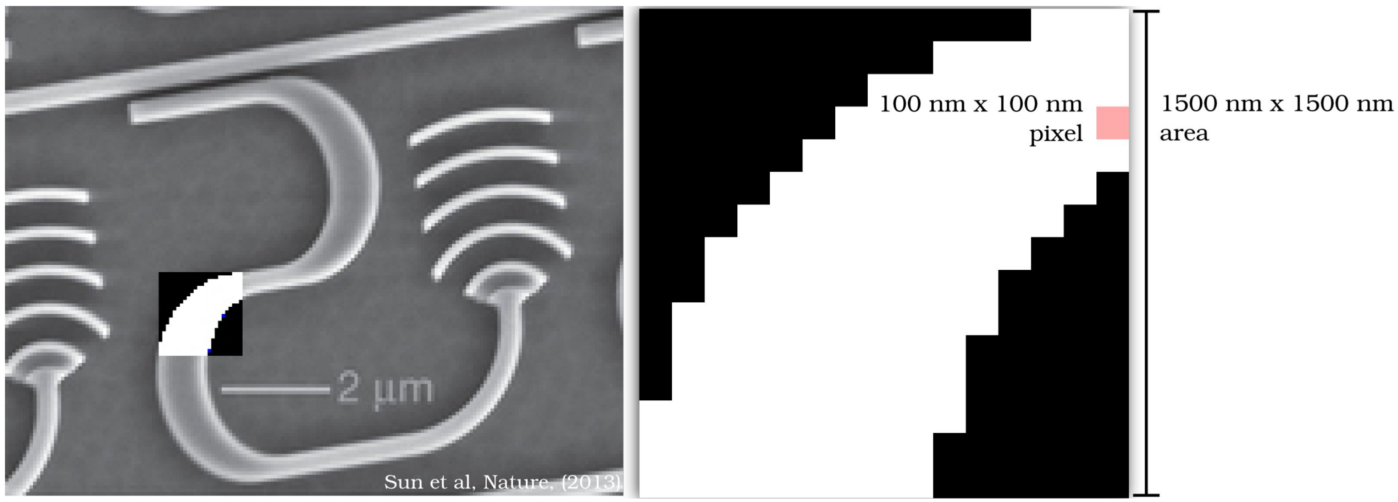




- Currently designed by hand, handful of parameters

- Design by computer, using the full design parameters

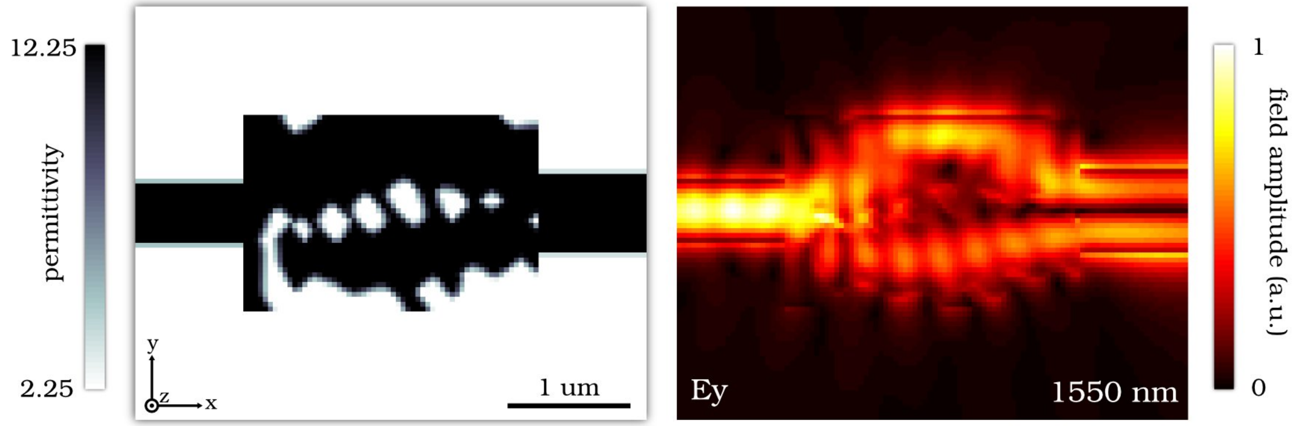
- Increasing design complexity requires additional degrees of freedom

- Fortunately, we have a virtually unlimited amount




- Include/exclude per pixel gives us $2^{(15^2)} = 2^{225}$ possibilities, uncountable

Goal: Show you how to design *any* linear nanophotonic device



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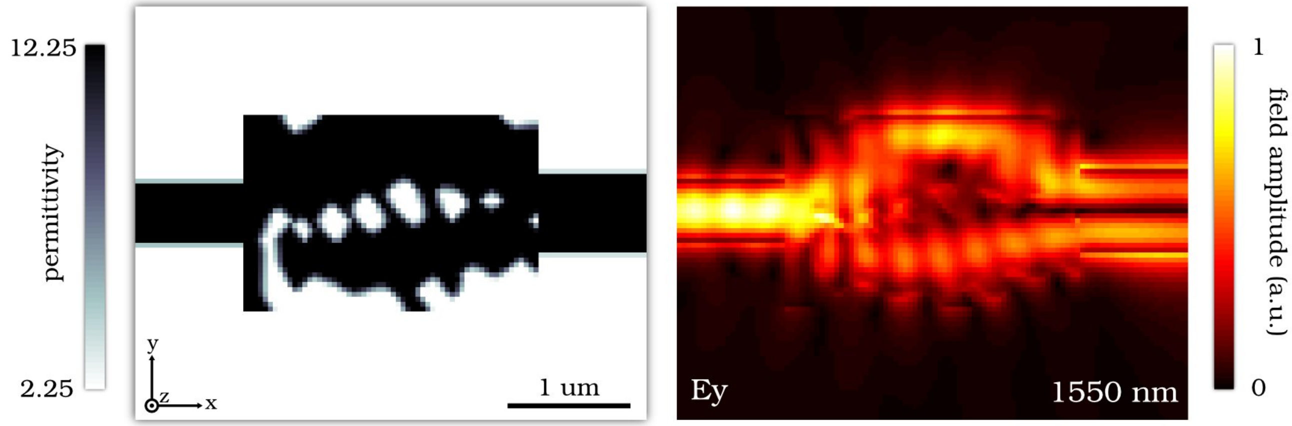


- Device properties:
  - Full 3D
  - Compact
  - Efficient
  - Multi-mode

– Multi-functional

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  - to the area of nanophotonics (physics)
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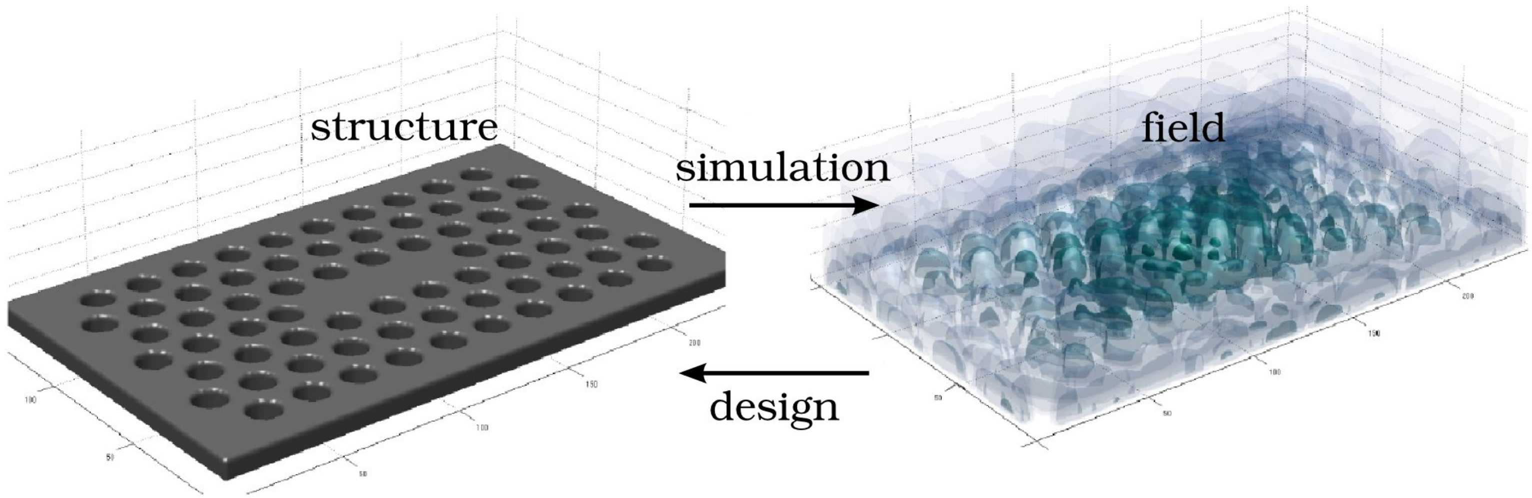
# Given a field, can we find its structure?



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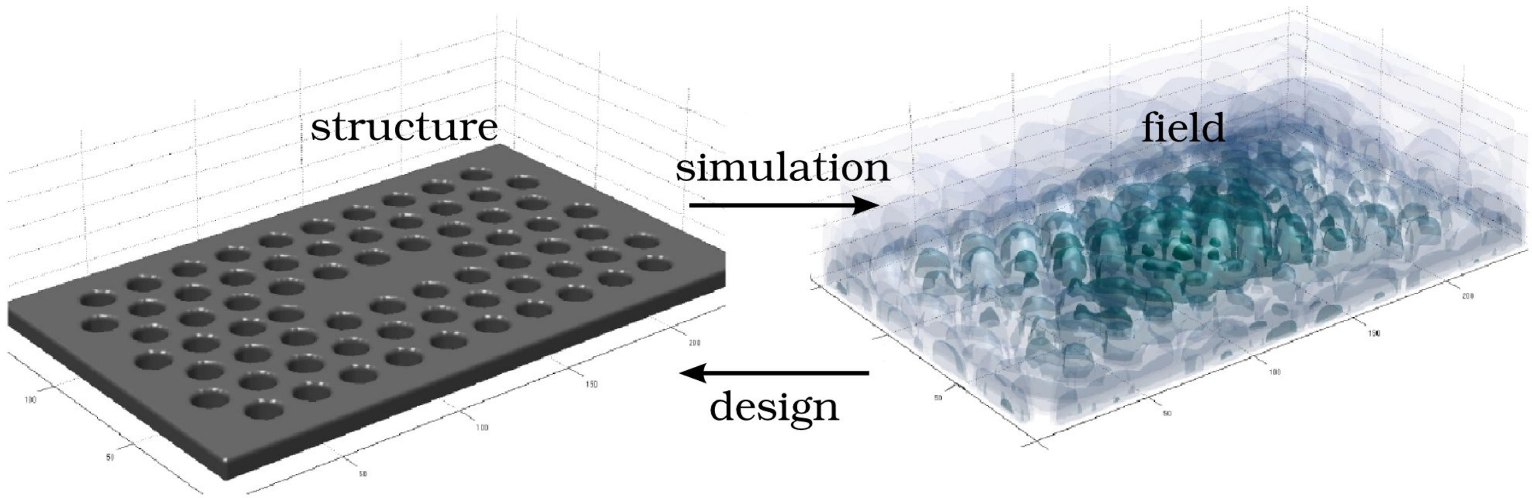


- Equivalently, find $\epsilon$ (structure) given $E$ (field)

$$\nabla \times \mu_0^{-1} \nabla \times E - \omega^2 \epsilon E = -i\omega J$$

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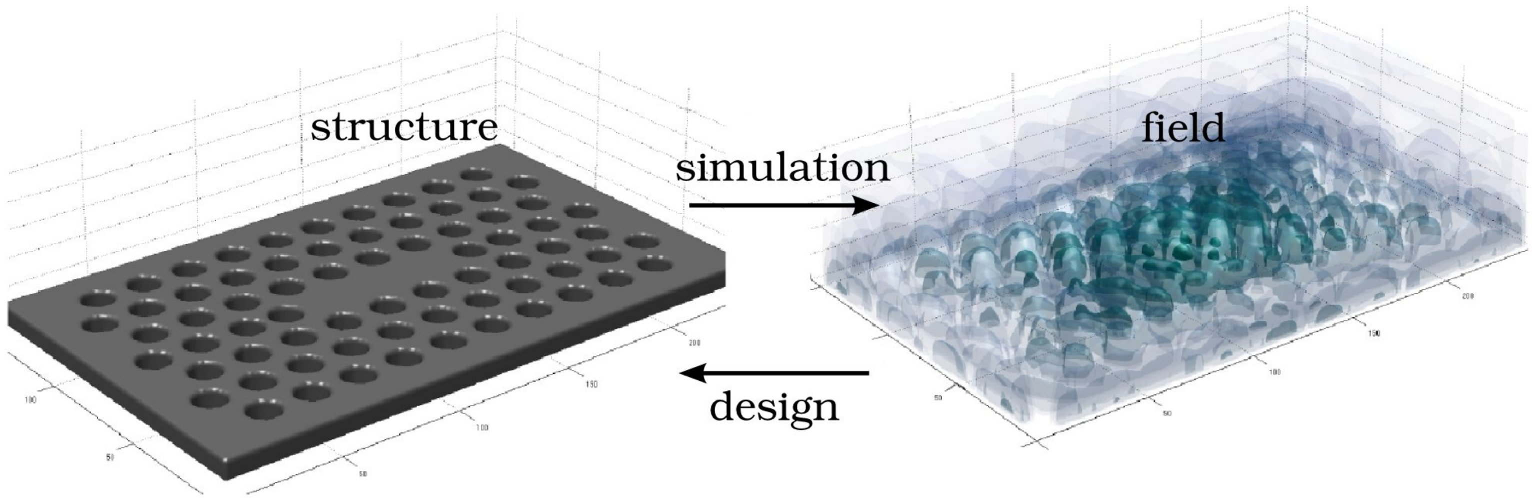


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- If possible, we can design *any* nanophotonic/optical component!

- Answer: Yes, given $E$ we *can* solve for $\epsilon$ (trivial!)

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$$\begin{aligned} \nabla \times \mu_0^{-1} \nabla \times E - \omega^2 \epsilon E &= -i\omega J \\ \omega^2 \epsilon E &= \nabla \times \mu_0^{-1} \nabla \times E + i\omega J \\ \omega^2 E \epsilon &= \nabla \times \mu_0^{-1} \nabla \times E + i\omega J \end{aligned}$$

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\omega^2 E \epsilon &= \nabla \times \mu_0^{-1} \nabla \times E + i\omega J \\
\epsilon &= (\nabla \times \mu_0^{-1} \nabla \times E + i\omega J)/\omega^2 E
\end{aligned}
$$

- Solving for $\epsilon$ actually way faster than simulation (solving for $E$)!

- Obvious and well-known from a mathematical perspective
  - Pre-requisite (200-level) class in optimization curriculum
  - Not yet taught (I think) in optics/photonics at Stanford

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$$\begin{aligned} E &\rightarrow x \\ \epsilon &\rightarrow z \\ \nabla \times \mu_0^{-1} \nabla \times -\omega^2 \epsilon &\rightarrow A(z) \\ -i\omega J &\rightarrow b \end{aligned}$$

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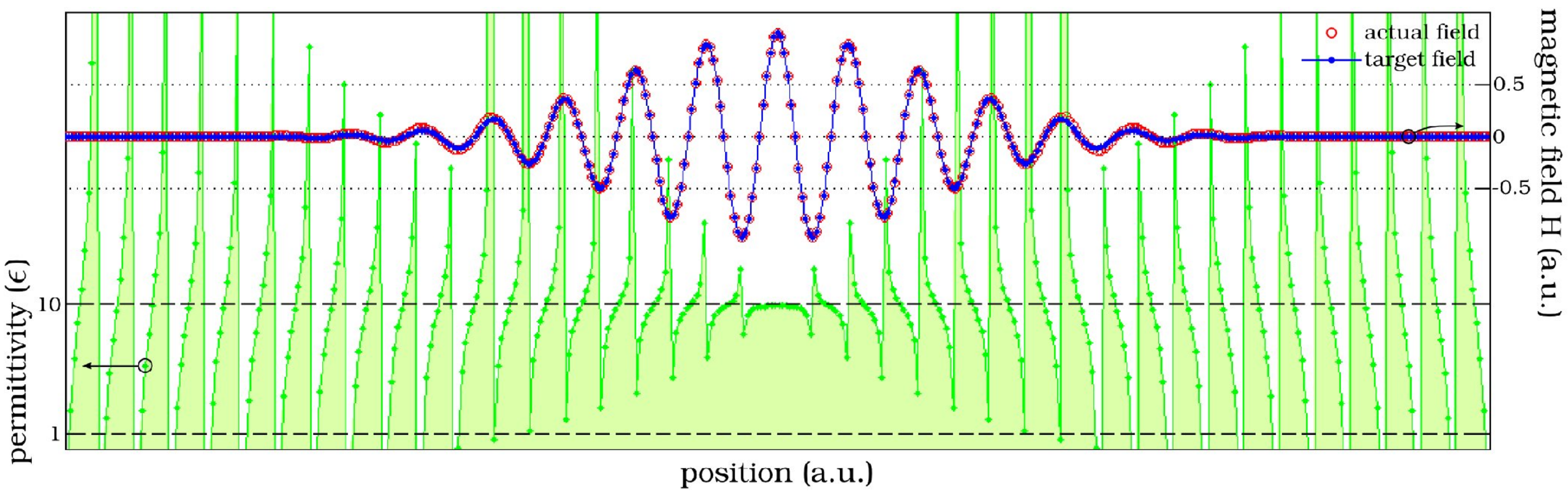
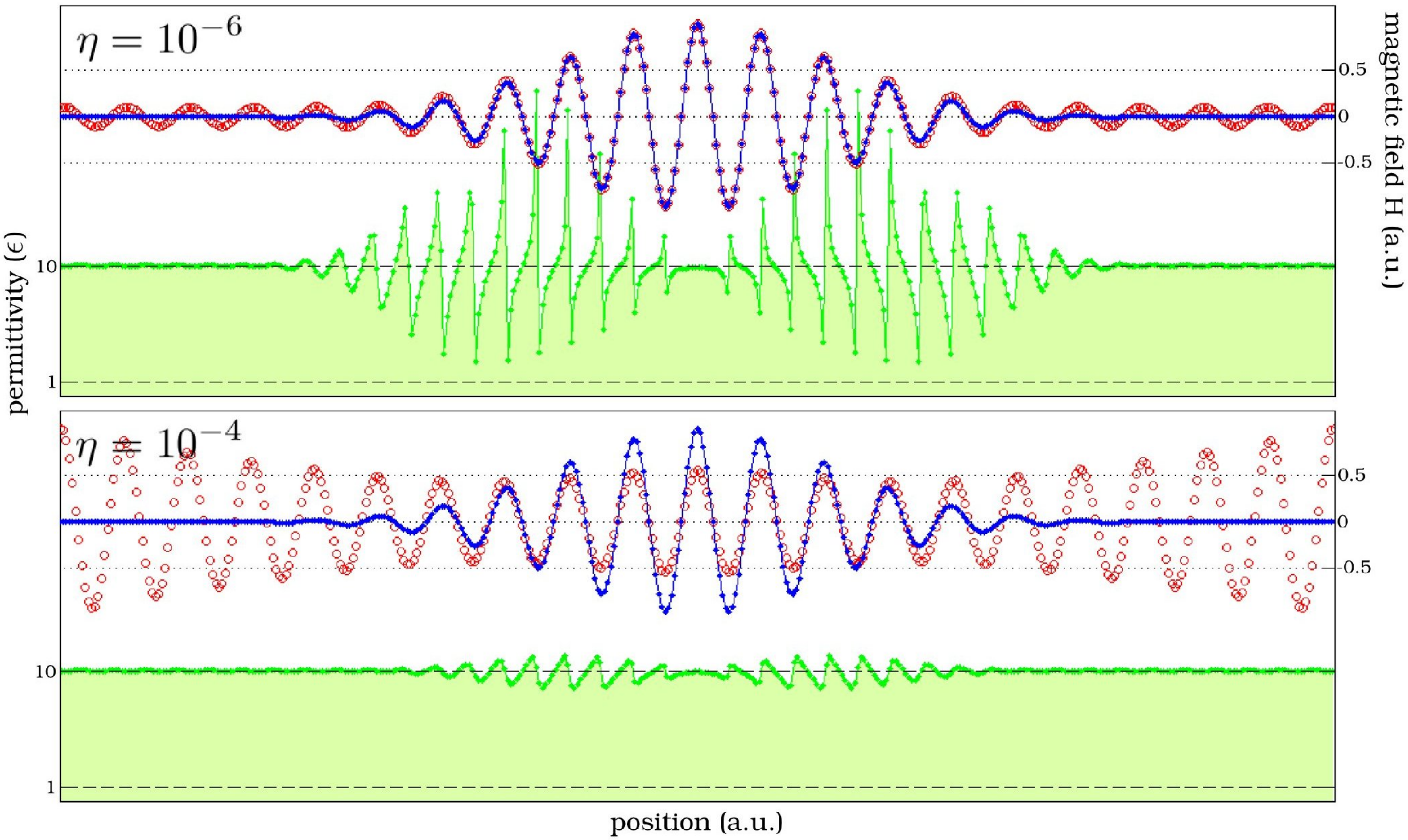
- Key: If $A(z)$ is linear in $z$ then $A(z)x = b$ is as well!

# Direct design of nanophotonic devices

- Choose $x$ (field) and solve for $z$ (structure)




- Perfect performance!. . . But unmanufacturable structure




- Regularization on $z$ decreases performance

- Conclusion:
  - Directly solving for $z$ (structure) is easy
  - Solving for manufacturable $z$ is hard

- However, we know that there exist some $x$ (fields) which produce well-behaved $z$ (structure)
  - But, cannot ask the user to choose such $x$

# Iterative design of nanophotonic devices

- Key lesson: Nanophotonic design involves optimizing *both* $x$ (field) and $z$ (structure)

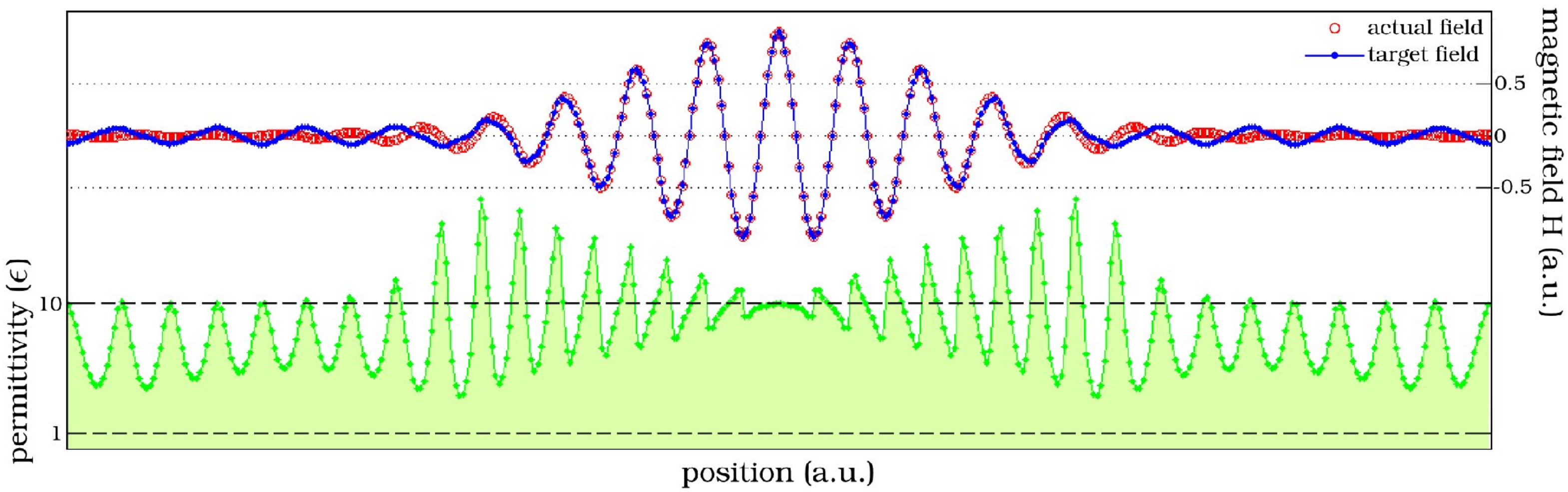
- New algorithm: Iteratively solve for $x$ and $z$

$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta_0 \|z - z_0\|^2$$

$$\underset{x}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta_1 \|x - x_0\|^2$$

- Here, $\eta_{0,1}$ are *regularization* parameters

- Iterative strategy produces $z$ (structure) that
  - is better behaved
  - more accurately produces $x$

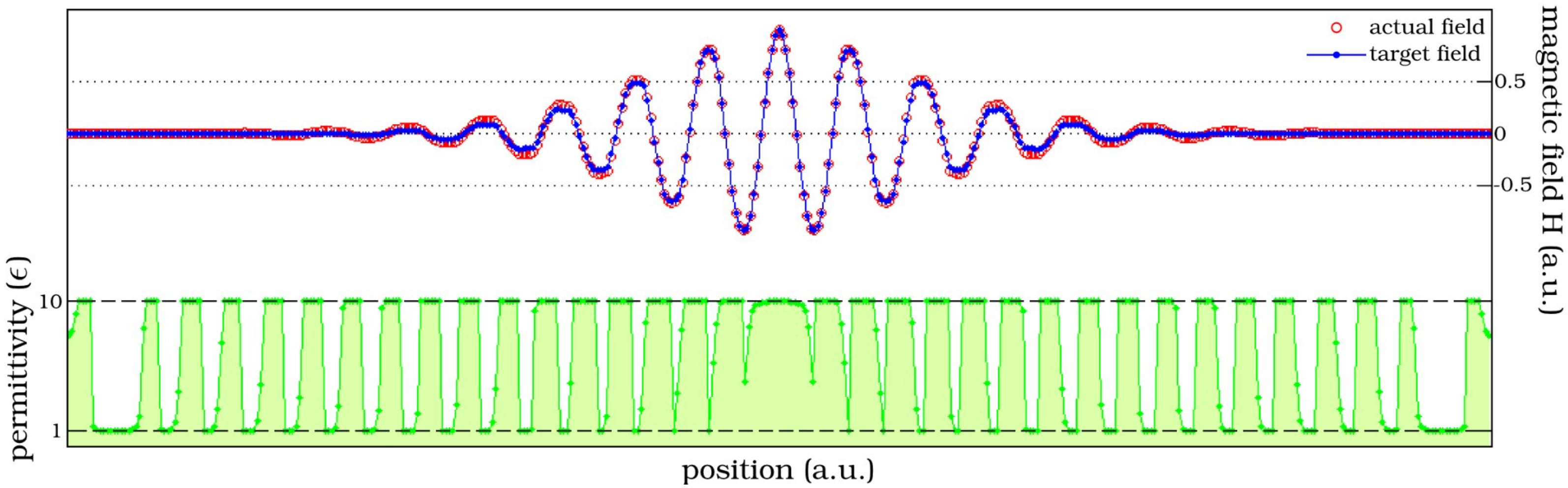


- We can also put hard limits on $z$

$$\begin{aligned} \underset{z}{\text{minimize}} \quad & \|A(z)x - b\|^2 \\ \text{subject to} \quad & z_0 \leq z \leq z_1 \end{aligned}$$

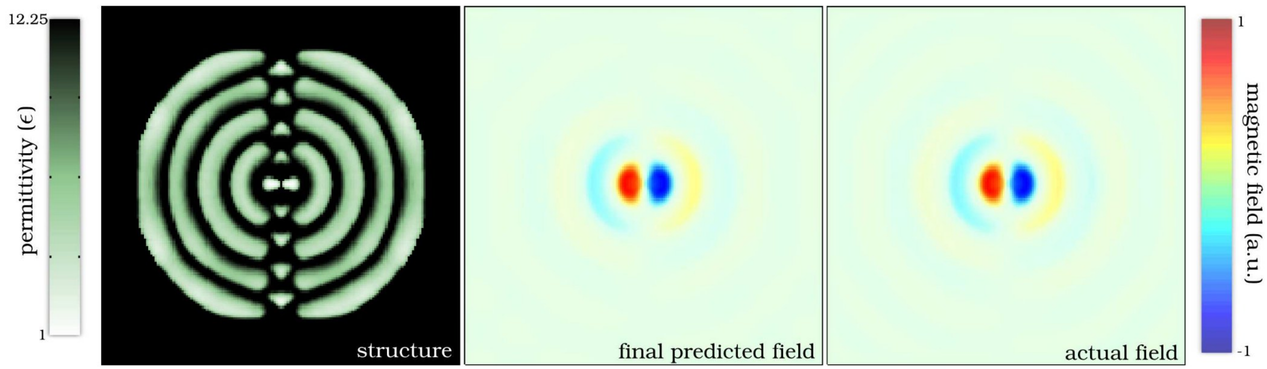
$$\underset{x}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta_1 \|x - x_0\|^2$$

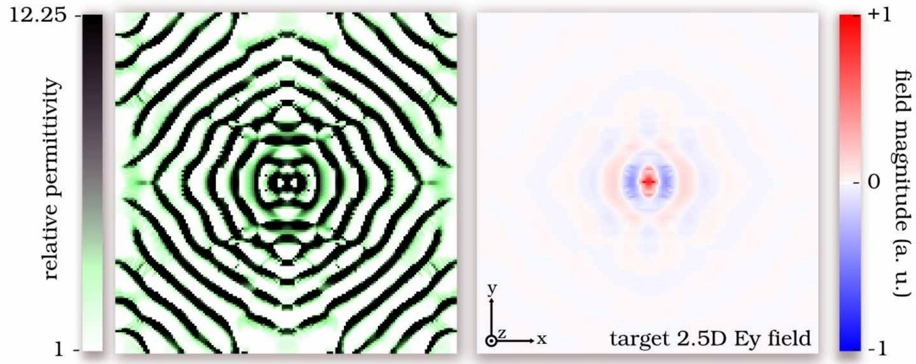
- $z_0 \leq z \leq z_1$ constraint better represents manufacturability constraint
  - Corresponds to a minimum and maximum allowable permittivity ($\epsilon$)




- Getting close!

- higher dimensions

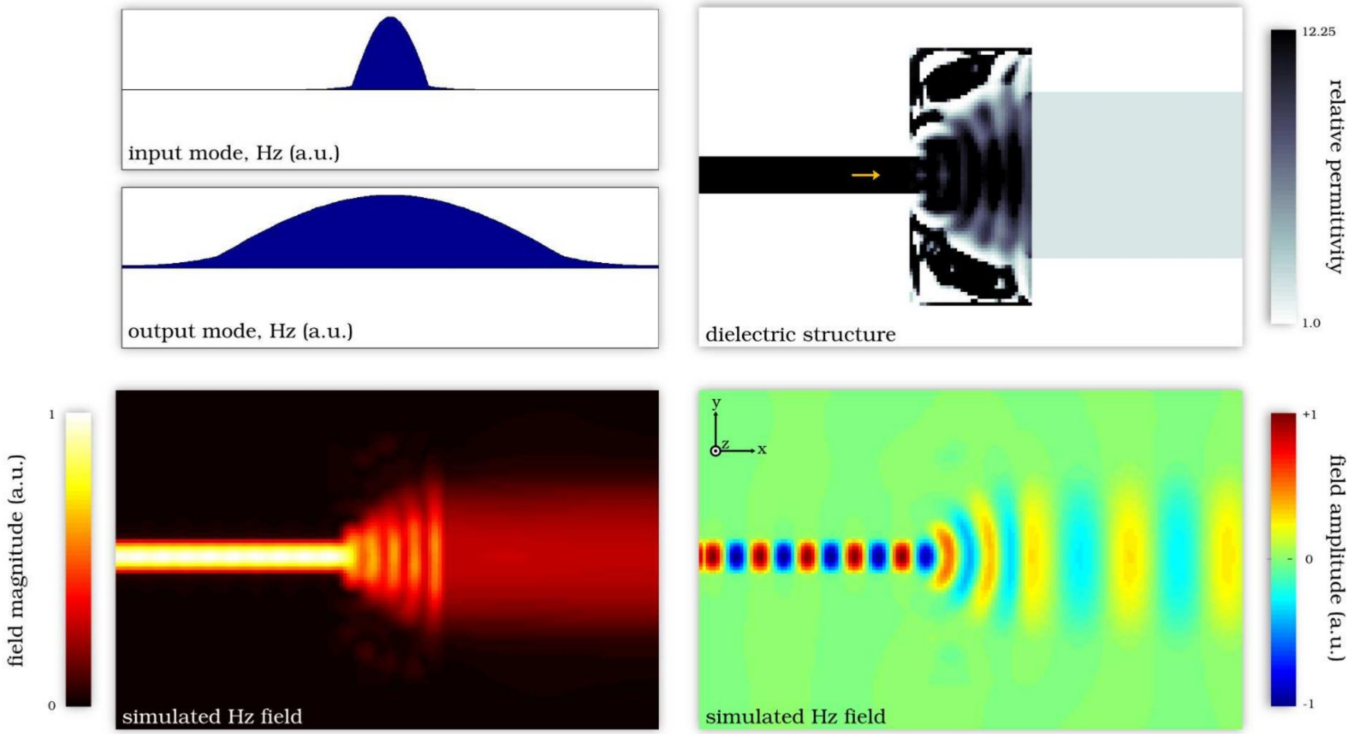


# Objective-first design of nanophotonic devices

- Next step in our thinking is that for most devices only some parts of the field matter

- waveguiding devices


input mode, Hz (a.u.)
output mode, Hz (a.u.)
dielectric structure
12.25
relative permittivity
1.0
1
field magnitude (a.u.)
0
simulated Hz field
y
z
x
simulated Hz field
+1
field amplitude (a.u.)
0
-1

# Design of 3D nanophotonic devices