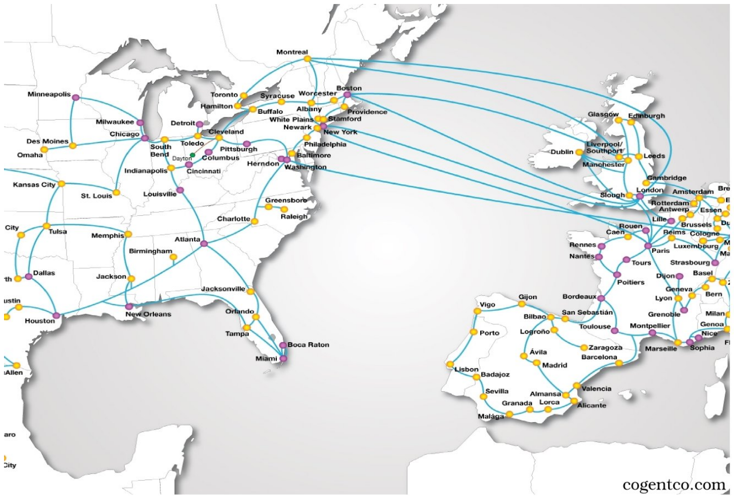
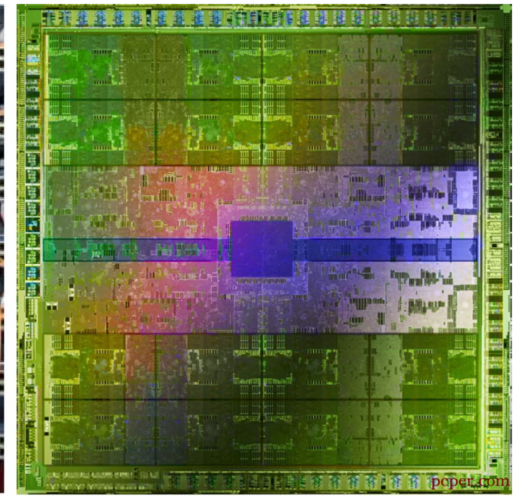
Nanophotonic Computational Design

Jesse Lu

February 25, 2013

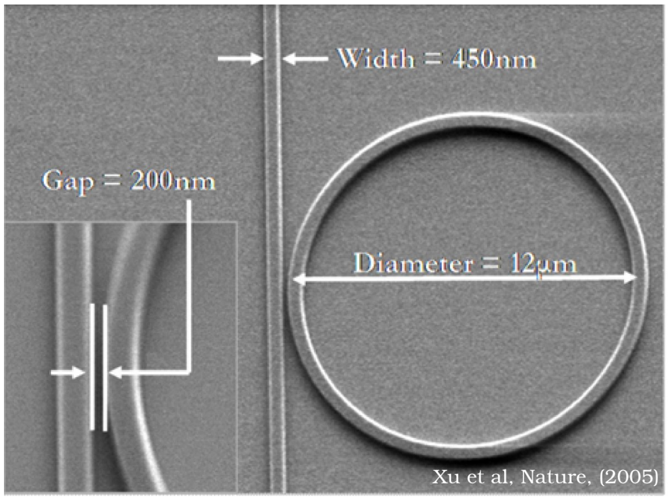
Introduction

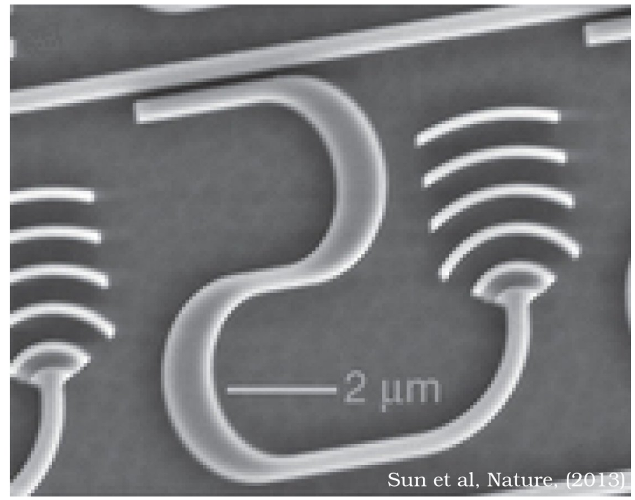




- As information grows, optical networks needed
  - across continents
  - within a datacenter
  - between chips and on-chip

- On-chip optical components are currently designed by tuning a small number of design parameters





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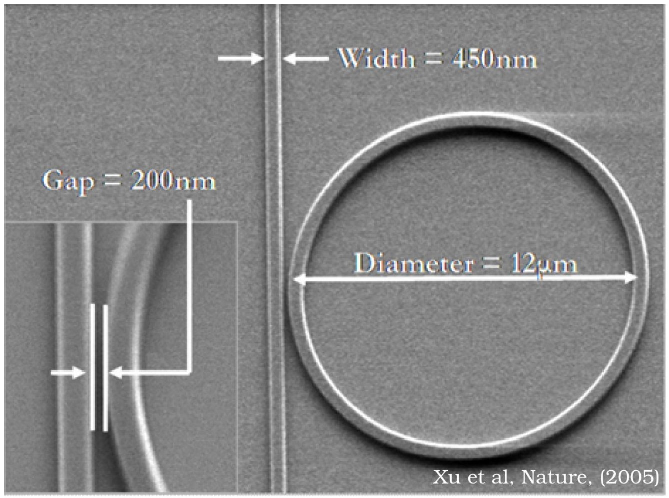
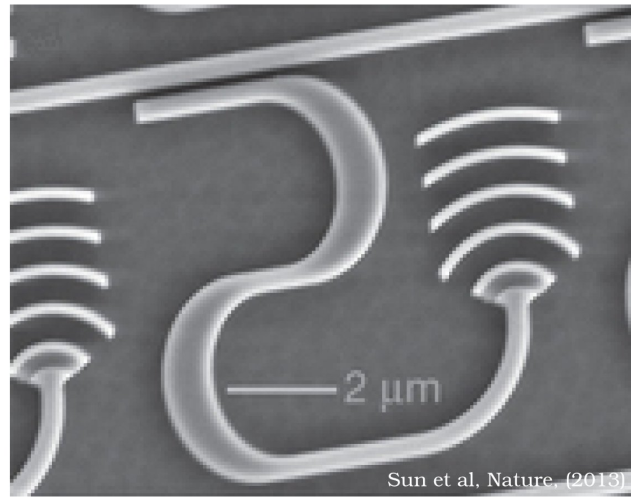
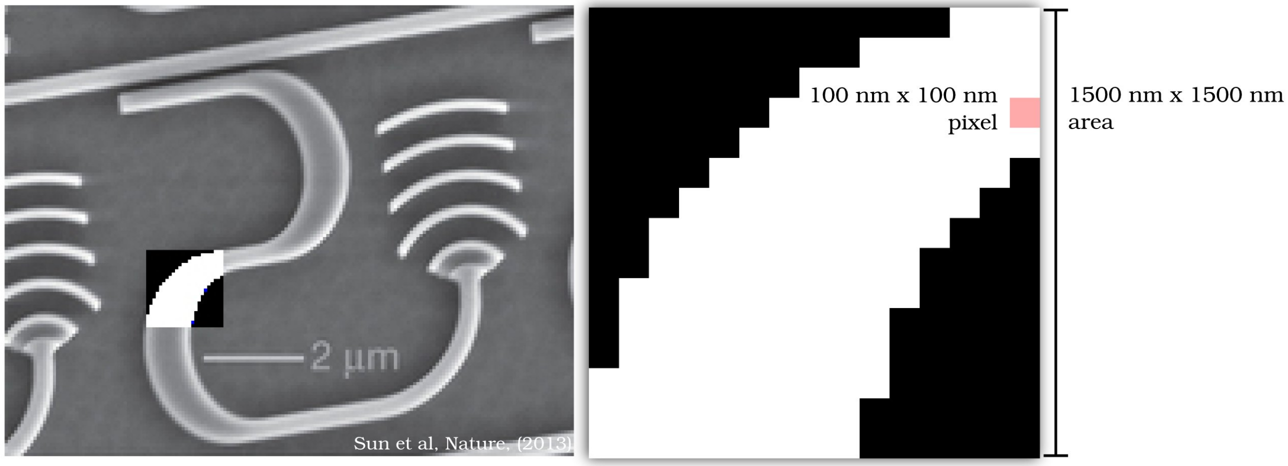


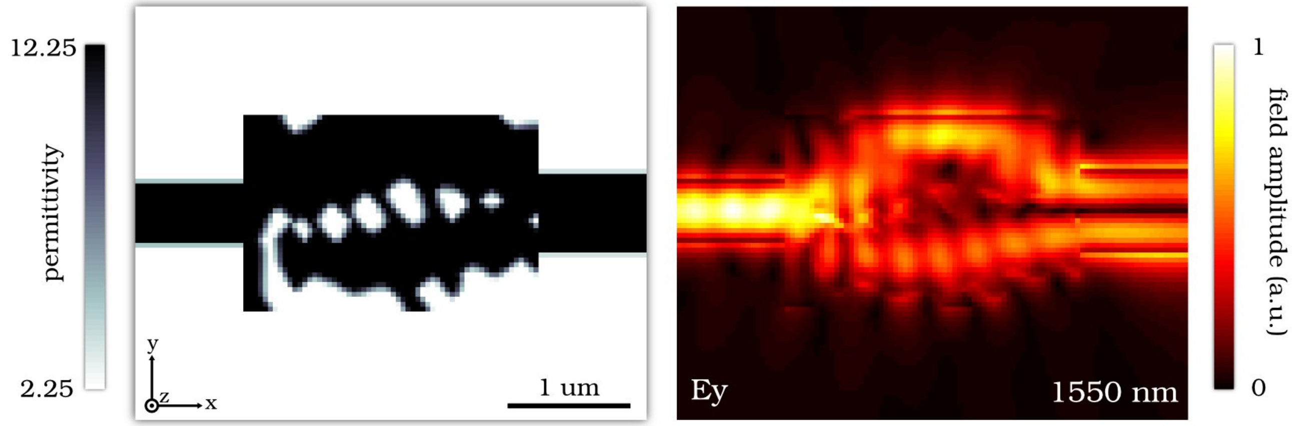


- What happens when we use the *full* parameter space for nanophotonic design?

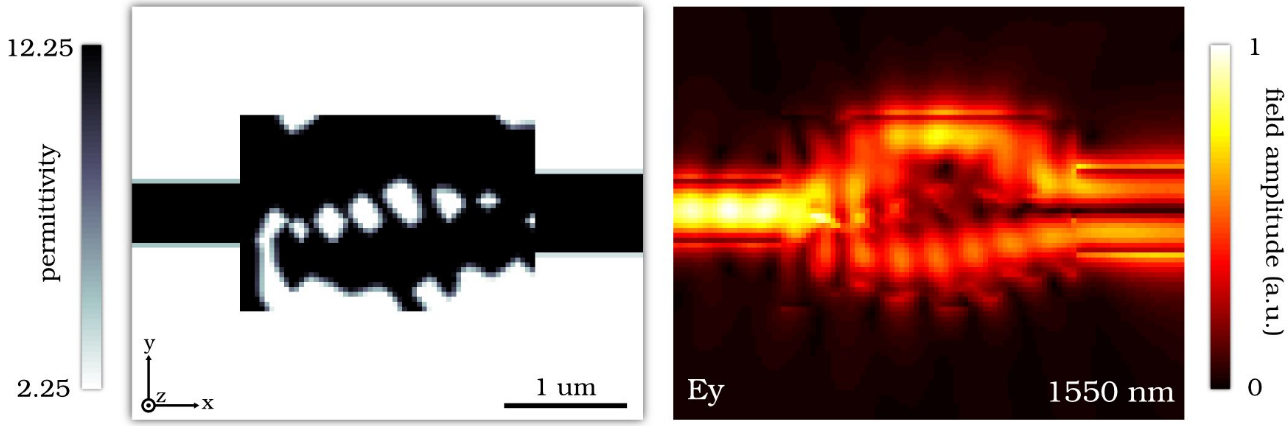
- The full parameter space is *vast*



- Include/exclude per pixel gives us $2^{(15^2)} = 2^{225}$ possibilities
  - A virtually uncountable number
  - Can only be design by a computer



- Our work: Software to design full 3D, multi-mode, and multi-functional linear nanophotonic devices—using the fully available parameter space



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- Many of these devices are
  - Completely novel (no previously known designs)
  - Extremely compact (footprints of a few vacuum wavelengths)
  - High efficiency ($> 80\%$ transmission)

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  - to the area of nanophotonics (physics)
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- Math Advisory:

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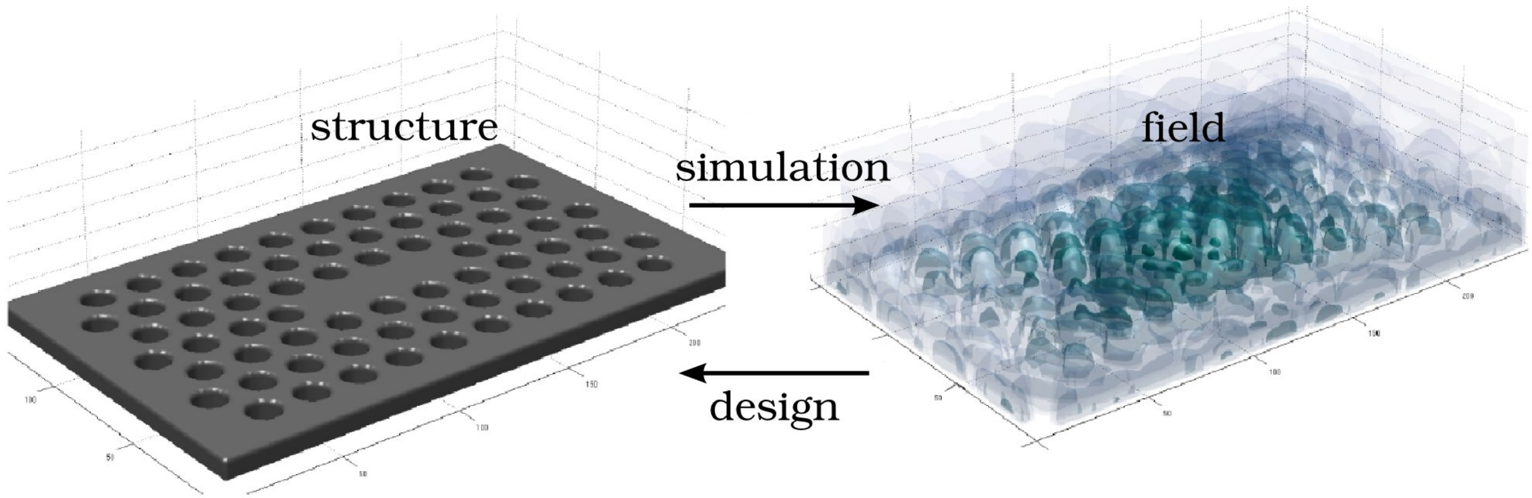
Given a field, can we find its structure?



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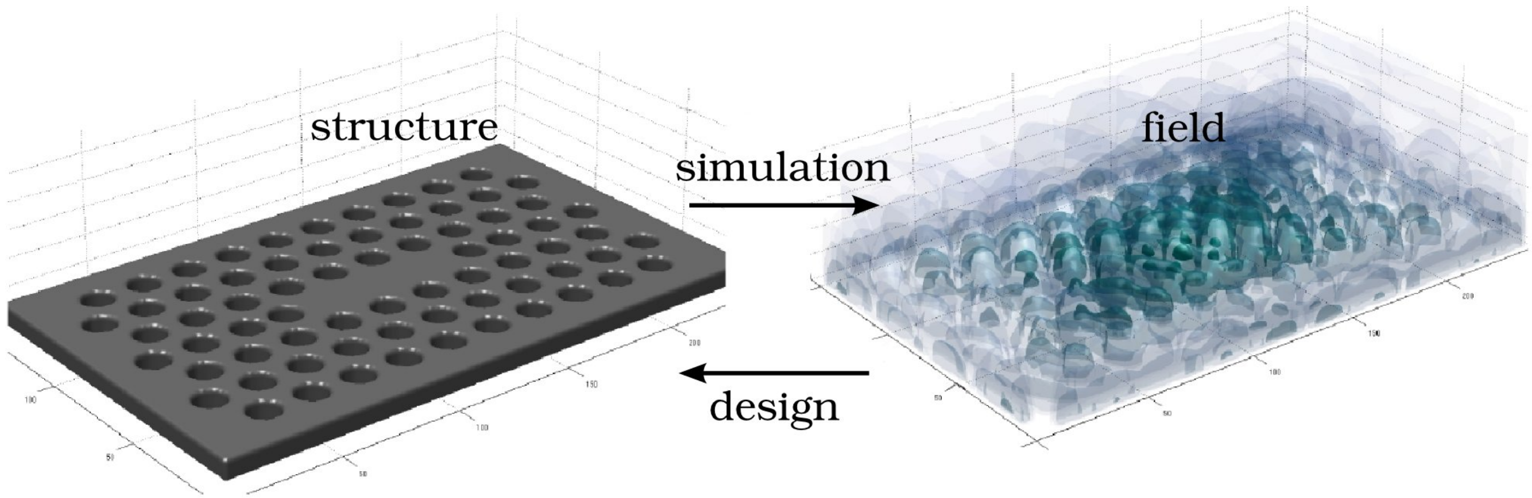

- Equivalently, find ϵ (structure) given E (field)

$$\nabla \times \mu_0^{-1} \nabla \times E - \omega^2 \epsilon E = -i\omega J$$

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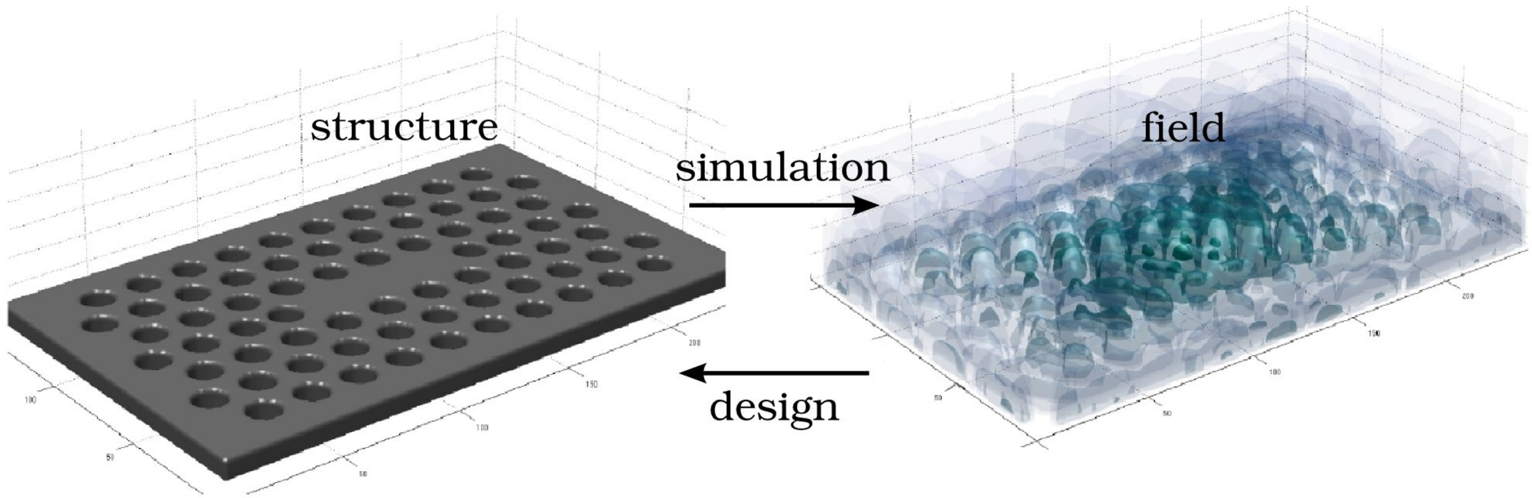

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- If possible, we can design *any* nanophotonic/optical component!

- Answer: Yes, given E we *can* solve for ϵ (trivial!)

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$$\nabla \times \mu_0^{-1} \nabla \times E - \omega^2 \epsilon E = -i\omega J$$
$$\omega^2 \epsilon E = \nabla \times \mu_0^{-1} \nabla \times E + i\omega J$$

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$$\begin{aligned} \nabla \times \mu_0^{-1} \nabla \times E - \omega^2 \epsilon E &= -i\omega J \\ \omega^2 \epsilon E &= \nabla \times \mu_0^{-1} \nabla \times E + i\omega J \\ \omega^2 E \epsilon &= \nabla \times \mu_0^{-1} \nabla \times E + i\omega J \end{aligned}$$

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$$
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\epsilon &= (\nabla \times \mu_0^{-1} \nabla \times E + i\omega J)/\omega^2 E
\end{aligned}
$$

- Solving for ϵ actually way faster than simulation (solving for E)!

- Obvious and well-known from a mathematical perspective
  - Pre-requisite (200-level) class in optimization curriculum
  - Not yet taught (I think) in optics/photonics at Stanford

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$$\begin{aligned} E &\rightarrow x \\ \epsilon &\rightarrow z \\ \nabla \times \mu_0^{-1} \nabla \times -\omega^2 \epsilon &\rightarrow A(z) \\ -i\omega J &\rightarrow b \end{aligned}$$

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$$E \rightarrow x$$

$$\epsilon \rightarrow z$$

$$\nabla \times \mu_0^{-1} \nabla \times -\omega^2 \epsilon \rightarrow A(z)$$

$$-i\omega J \rightarrow b$$

- Key: If $A(z)$ is linear in z then $A(z)x = b$ is as well!

Direct design of nanophotonic devices

- Let's try it already!
  - Choose x (field)
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$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2$$

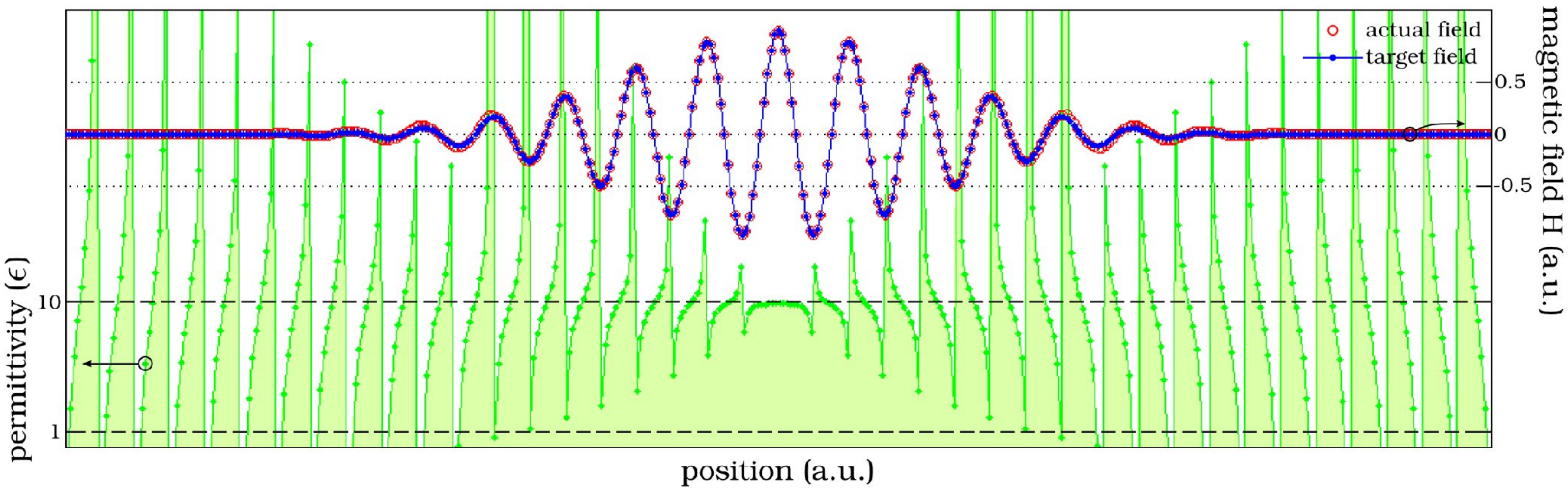
- Global minimum where $A(z)x - b = 0$ can be computed in one step

$$\epsilon = (\nabla \times \mu_0^{-1} \nabla \times E + i\omega J)/\omega^2 E$$

where $\epsilon \rightarrow z$

- Choose canonical 1D cavity field for x

- Solve for z (structure) and check design fidelity with simulation



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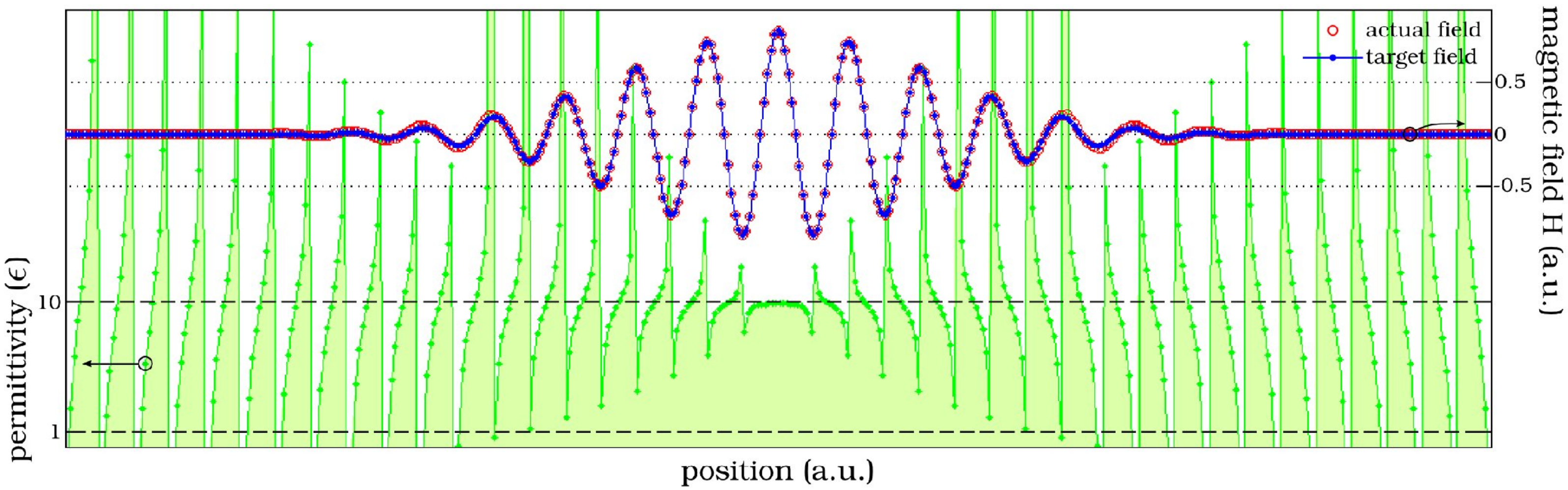


- Result
  - Perfect performance
  - But unmanufacturable structure (z not well-behaved)

Direct design with regularization

- Direct design "works" but not practical

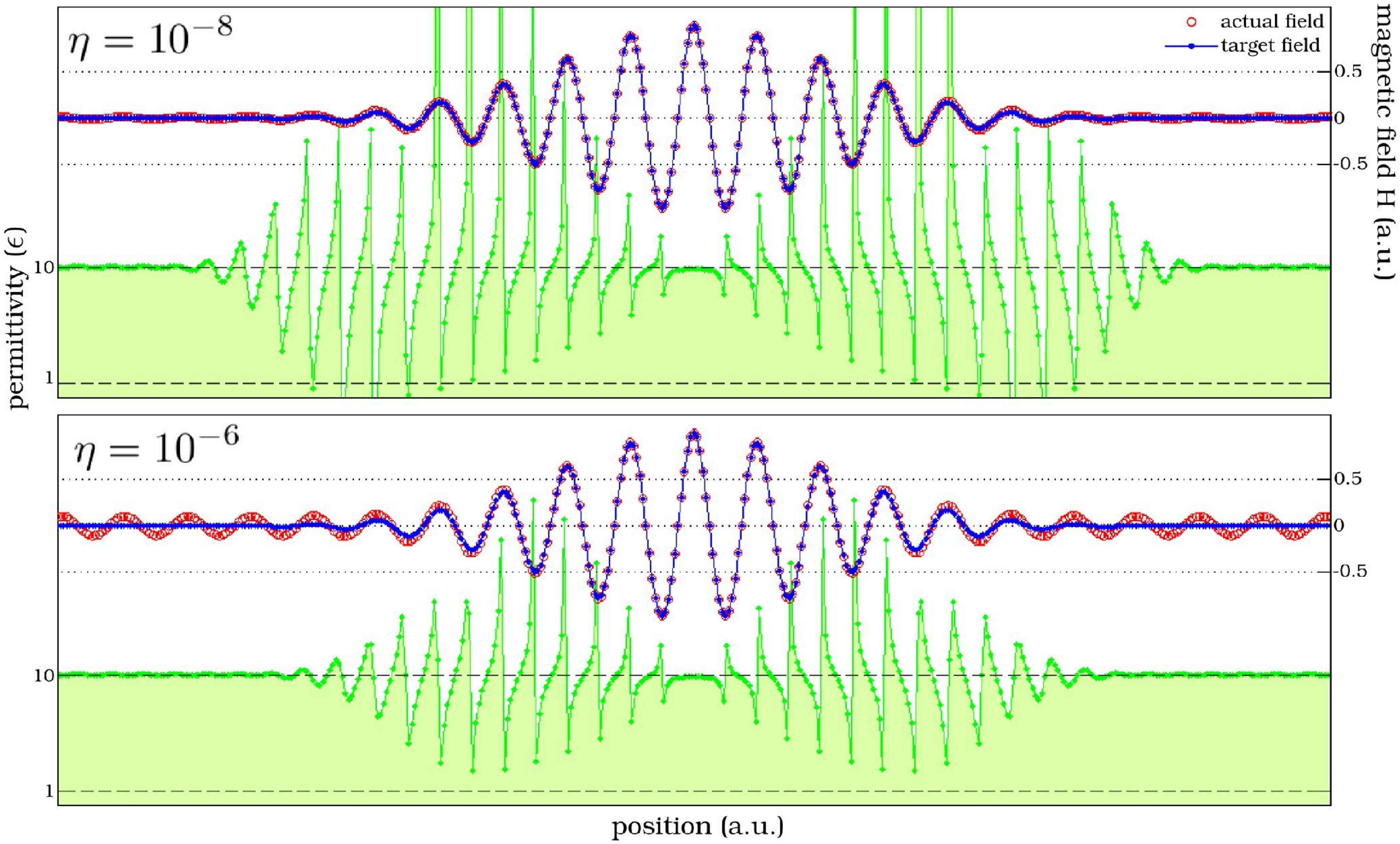
$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2$$

- So, let's add a *regularization* term to z and solve the following instead

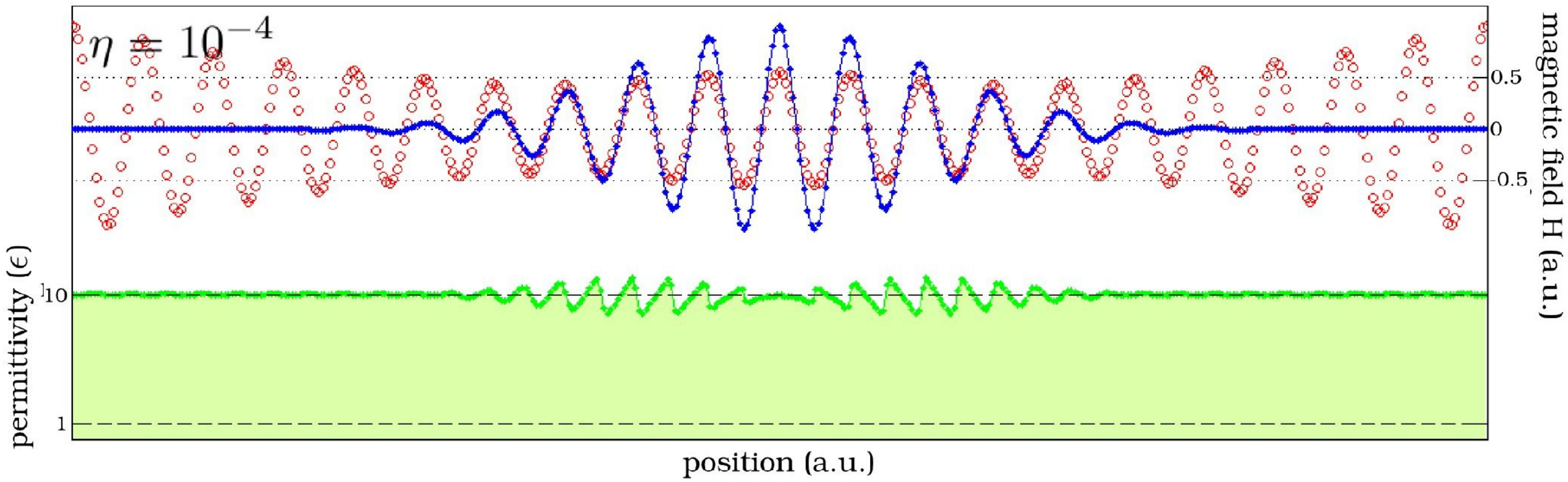
$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta\|z - z_0\|^2$$

  - $\|z - z_0\|^2$ term keeps z close to z_0
  - η controls the strength of the regularization

- Solution can still be computed in one step



- Unfortunately, regularization on z decreases performance



$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta\|z - z_0\|^2$$

- Decreased performance a result of non-zero physics residual at optimum

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- Realized that there do exist some x (fields) which result from manufacturable z (structures)

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- How to overcome the apparent trade-off between manufacturability (z) and performance (x)?

- Realized that there do exist some x (fields) which result from manufacturable z (structures)
  - These can be produced via simulation
  - However, it's not useful to ask the user to choose such x

- Therefore, a *useful* tool would optimize for *both* x and z

Iterative design of nanophotonic devices

- New algorithm: Iteratively solve for x (field) and z (structure)

$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta_0 \|z - z_0\|^2$$

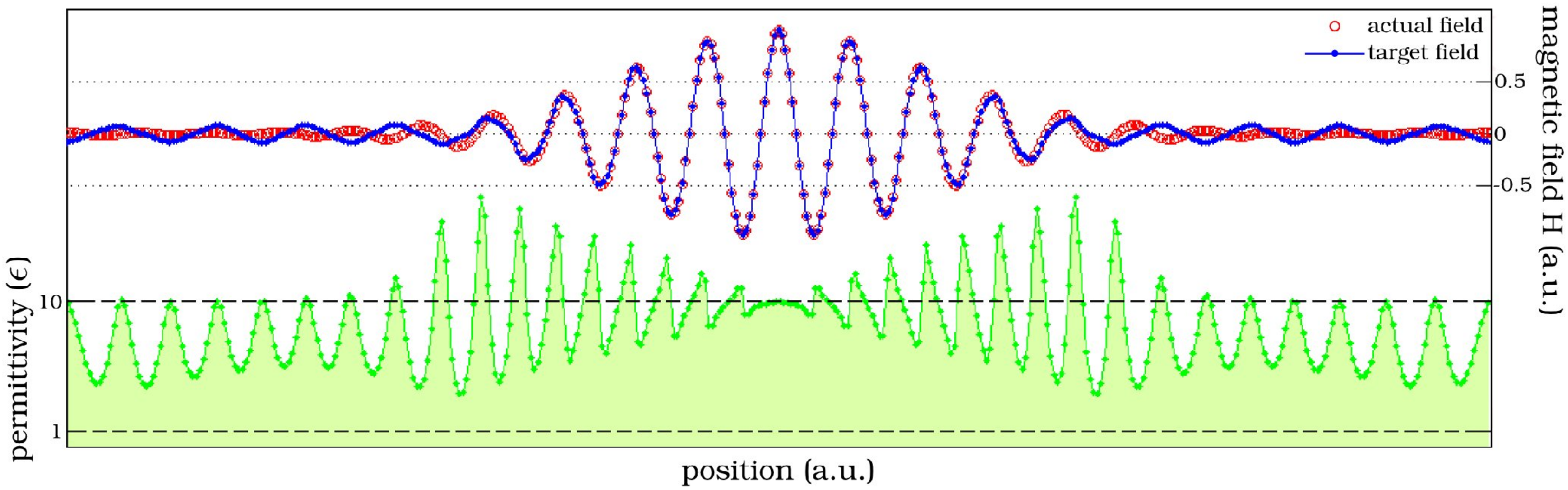
$$\underset{x}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta_1 \|x - x_0\|^2$$

- Takes advantage of the *bi-linearity* of the physics residual
  - Jointly solving for x and z is a non-convex problem

- More concisely, we iteratively solve the following

$$\underset{\text{alternately } x \text{ then } z}{\text{minimize}} \; \|A(z)x - b\|^2 + \eta_0\|z - z_0\|^2 + \eta_1\|x - x_0\|^2$$

- Design process now consists of *multiple* computational steps
  - η_0, η_1 gradually decreased to bring physics residual toward 0



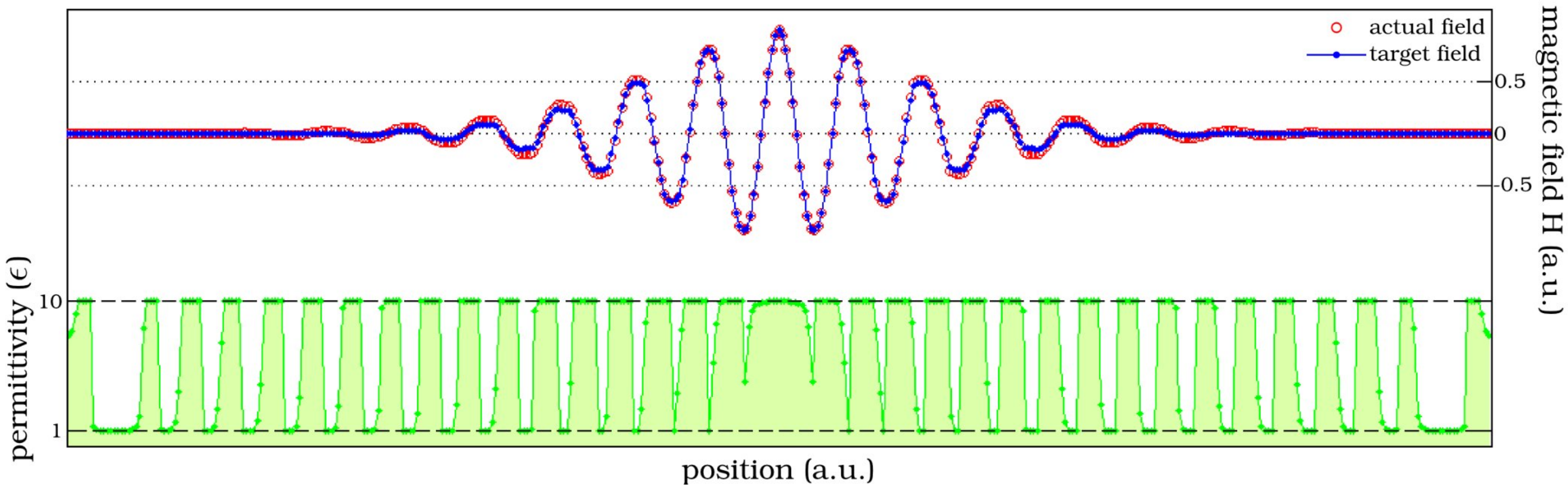
- Iterative strategy produces z (structure) that
  - is better behaved
  - more accurately produces x

Iterative design with hard constraints on z

- We can also put hard limits on z (structure)

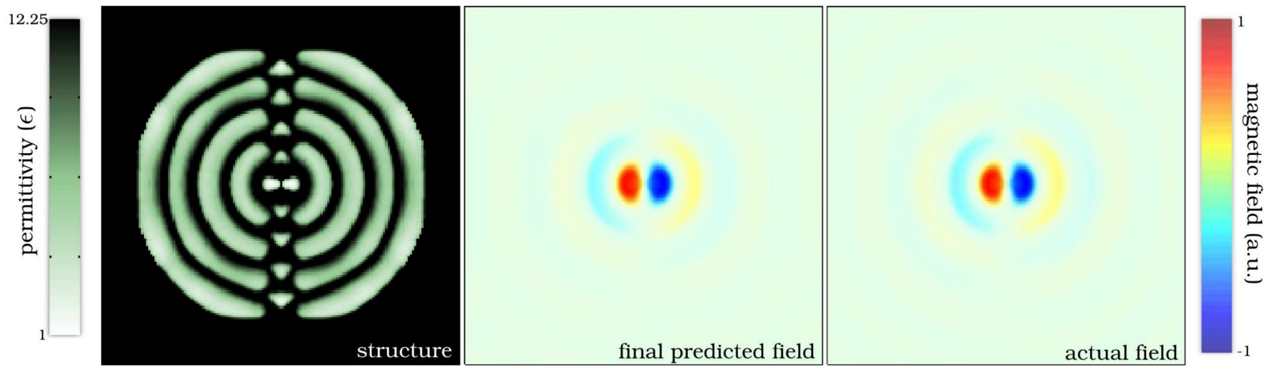
$$\begin{array}{ll} \text{minimize} & \|A(z)x - b\|^2 + \eta_1 \|x - x_0\|^2 \\ \text{subject to} & z_{\text{min}} \leq z \leq z_{\text{max}} \end{array}$$

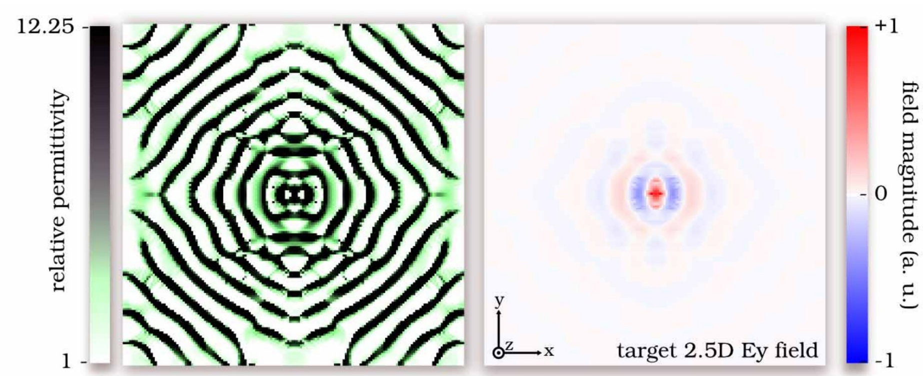
- $z_{\text{min}} \leq z \leq z_{\text{max}}$ constraint better represents manufacturability constraint
  - Corresponds to a minimum and maximum allowable permittivity (ϵ)



- Getting close!

- higher dimensions

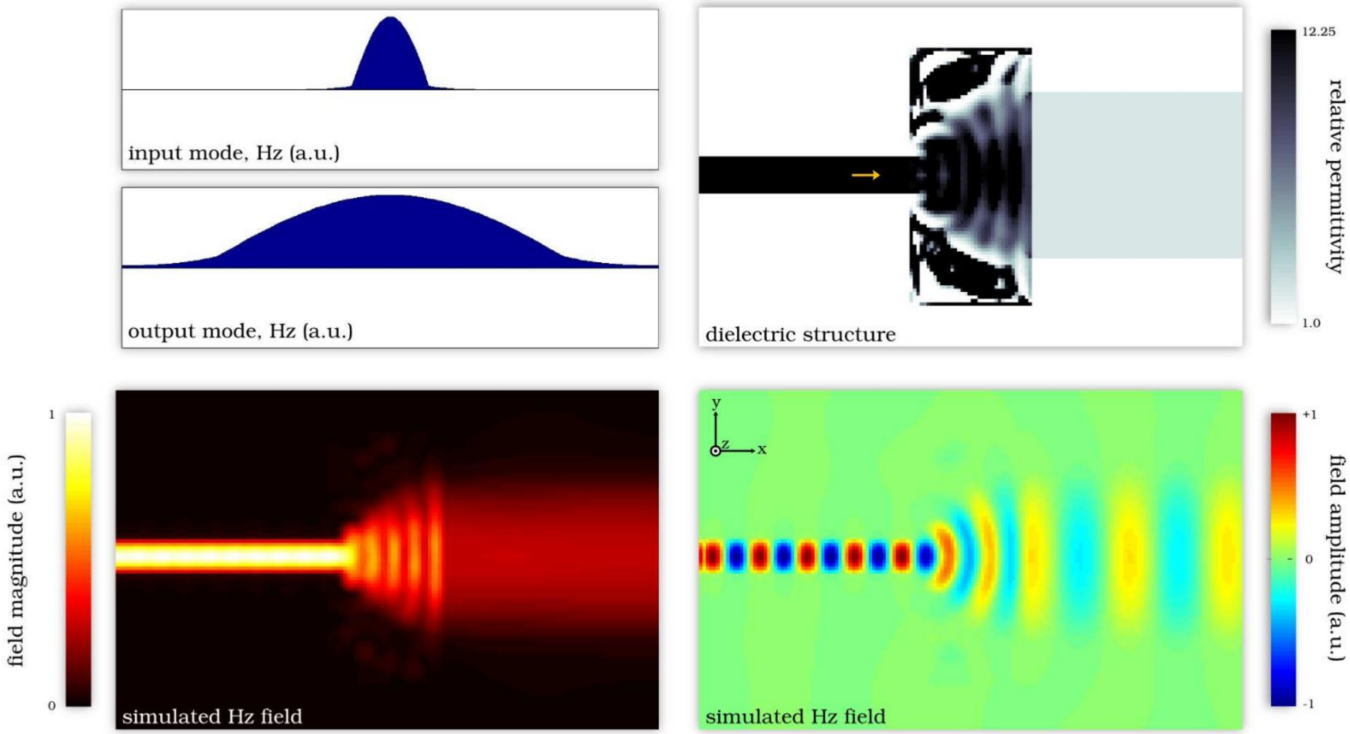




Objective-first design of nanophotonic devices

- Next step in our thinking is that for most devices only some parts of the field matter

- waveguiding devices


input mode, Hz (a.u.)
output mode, Hz (a.u.)
dielectric structure
12.25
relative permittivity
1.0
1
field magnitude (a.u.)
0
simulated Hz field
y
z
x
simulated Hz field
+1
field amplitude (a.u.)
0
-1

Design of 3D nanophotonic devices