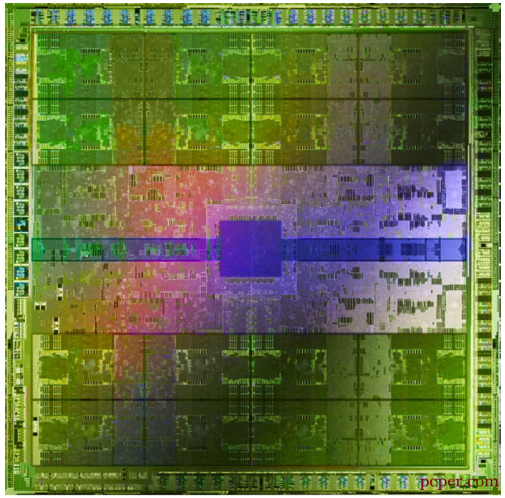
Nanophotonic Computational Design

Jesse Lu

February 25, 2013

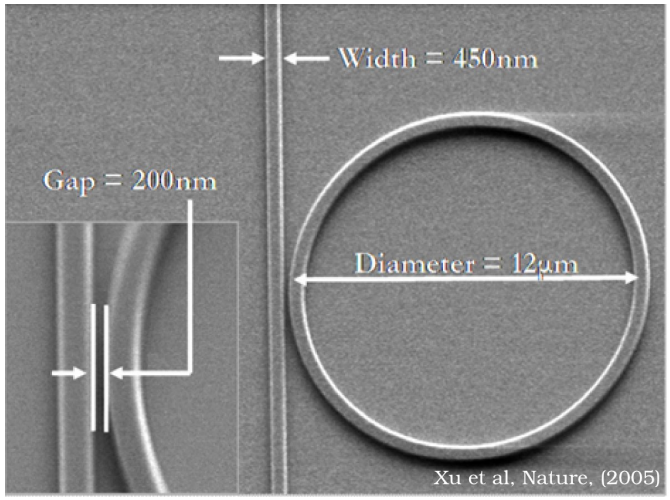
- Introduction
- Summary
- Incremental results
- Final result

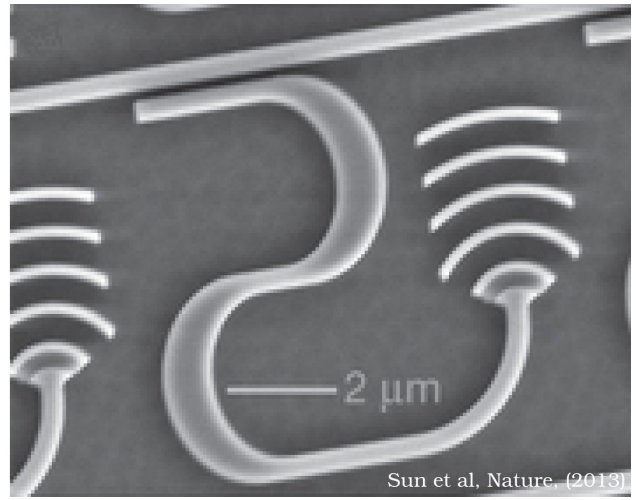
Introduction



- As information grows, optical networks needed
  - across continents
  - within a datacenter
  - between chips and on-chip

- On-chip optical components are currently designed by tuning a small number of design parameters





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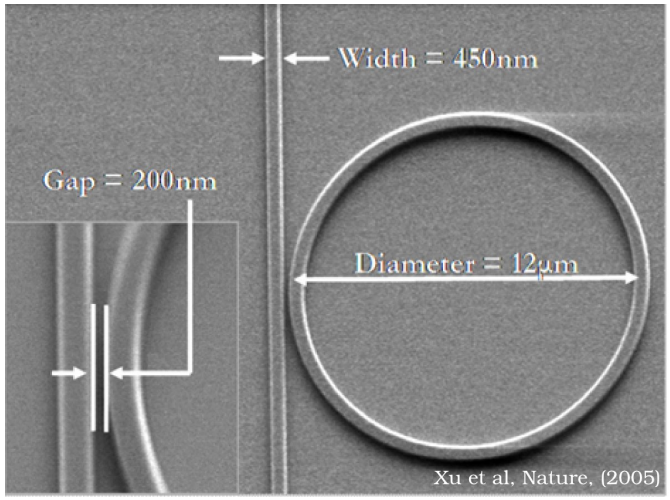
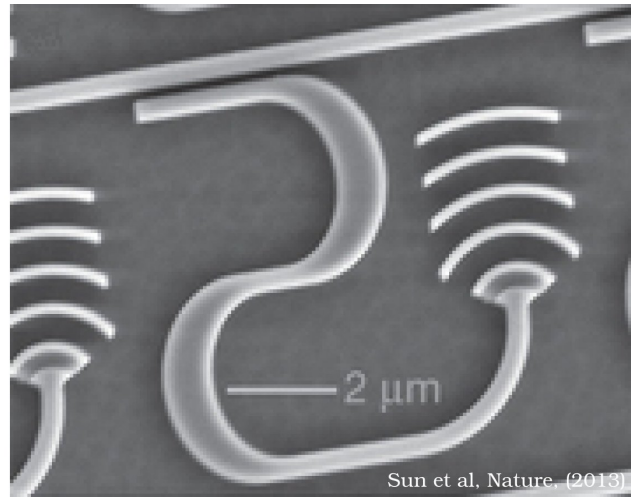
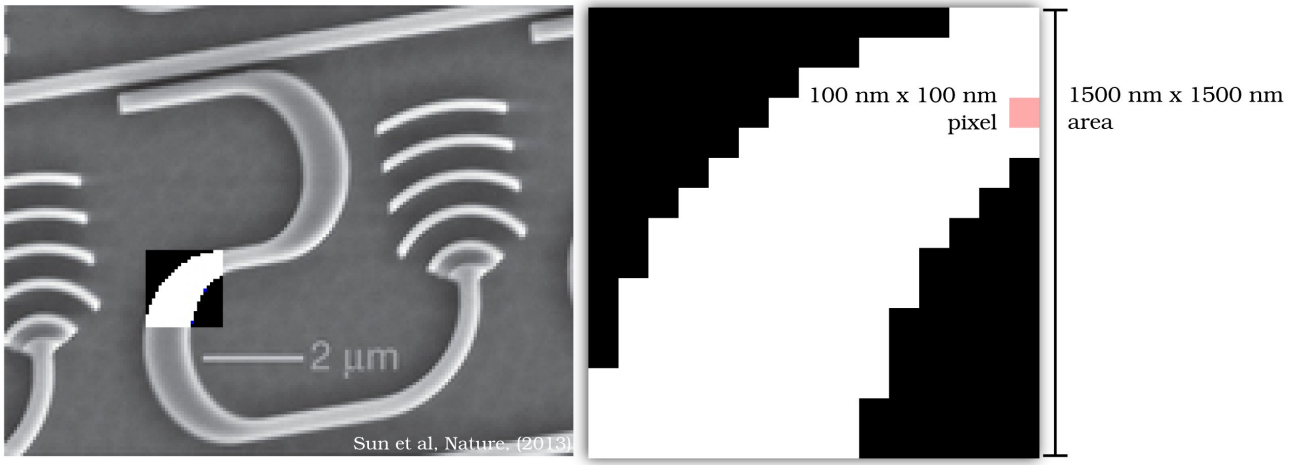




- Our idea: Design using *all* available parameters
  - If successful, full-parameter design *guarantees* better devices
  - i.e. We can do better by using *all* the design parameters

- The number of available design parameters for nanophotonic devices is *enormous*



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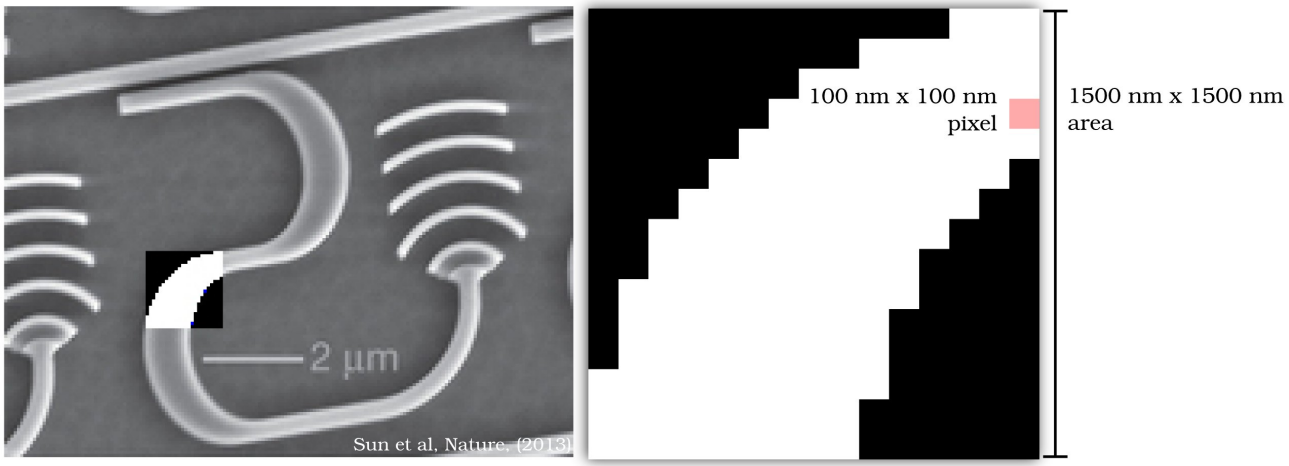

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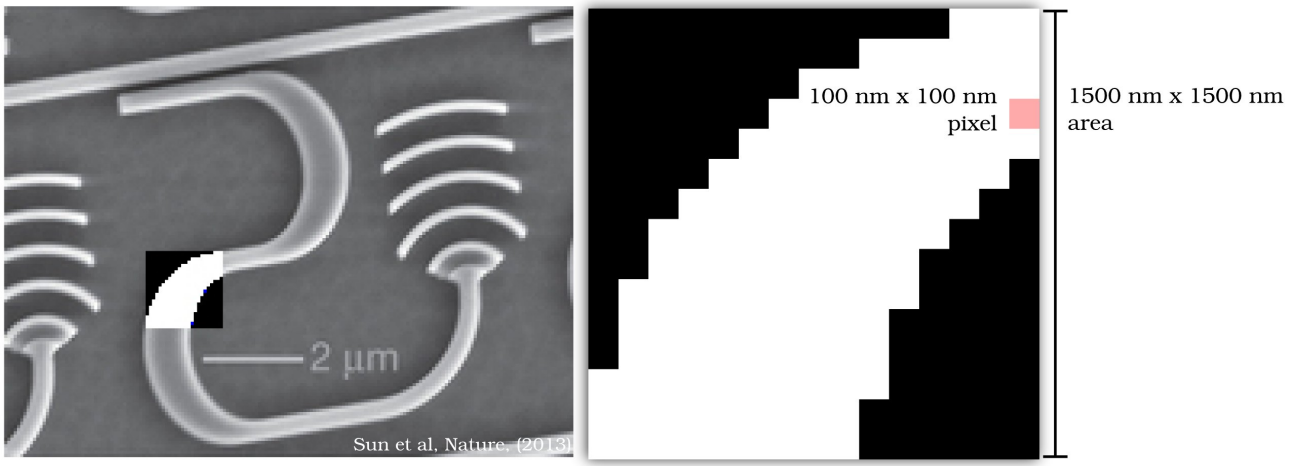

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- Realization: Hand-tuning won't work for full-parameter design

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    in: {freq1, freq2}
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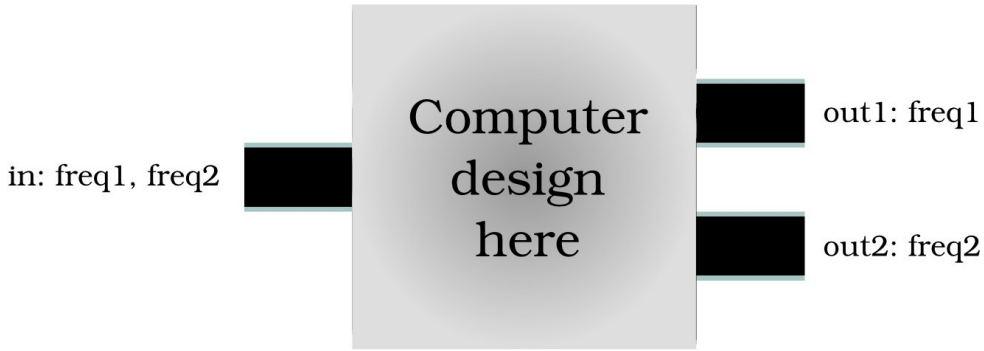

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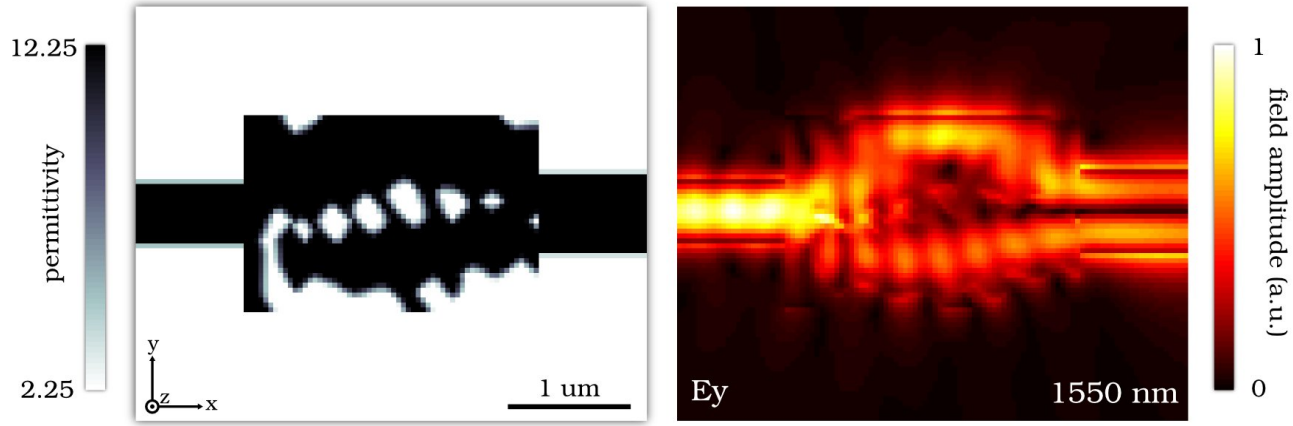
- Our goal: Make it work

Summary



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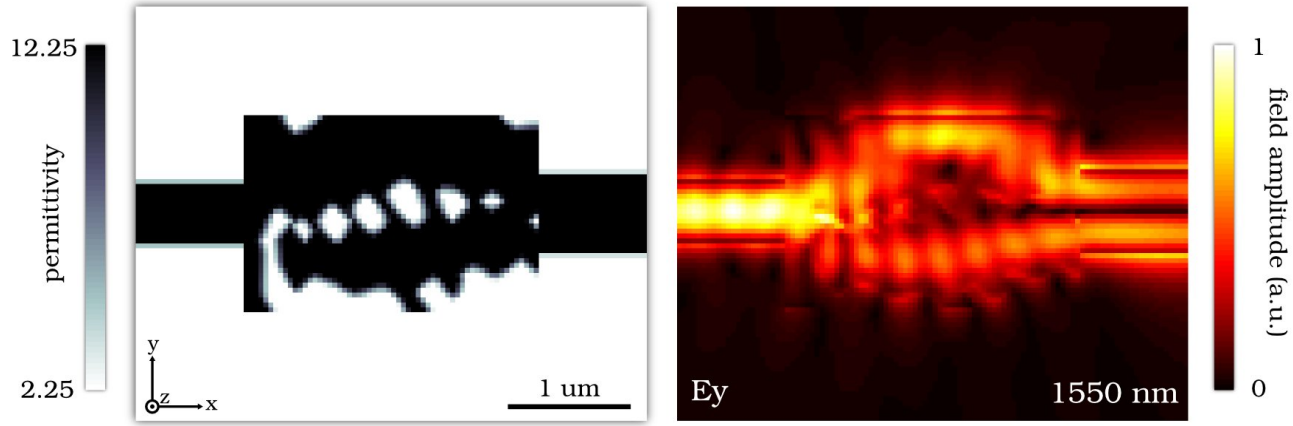

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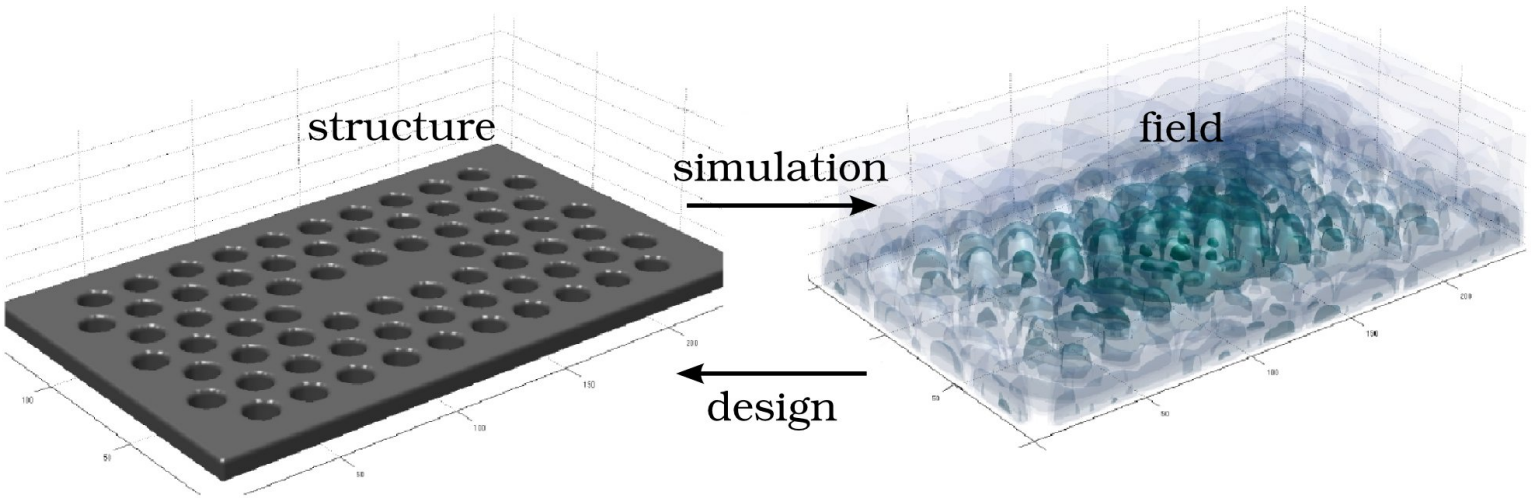
- Physics Advisory:

  CONTAINS INVOLVED MATHEMATICAL CONTENT

Given a field, can we find its structure?



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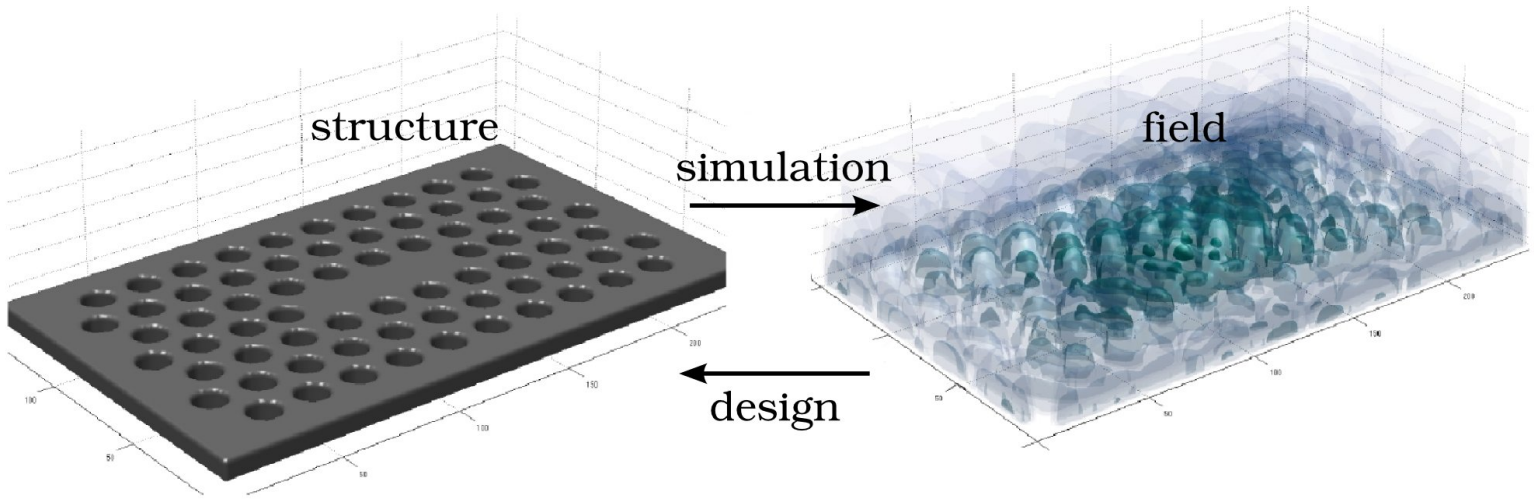

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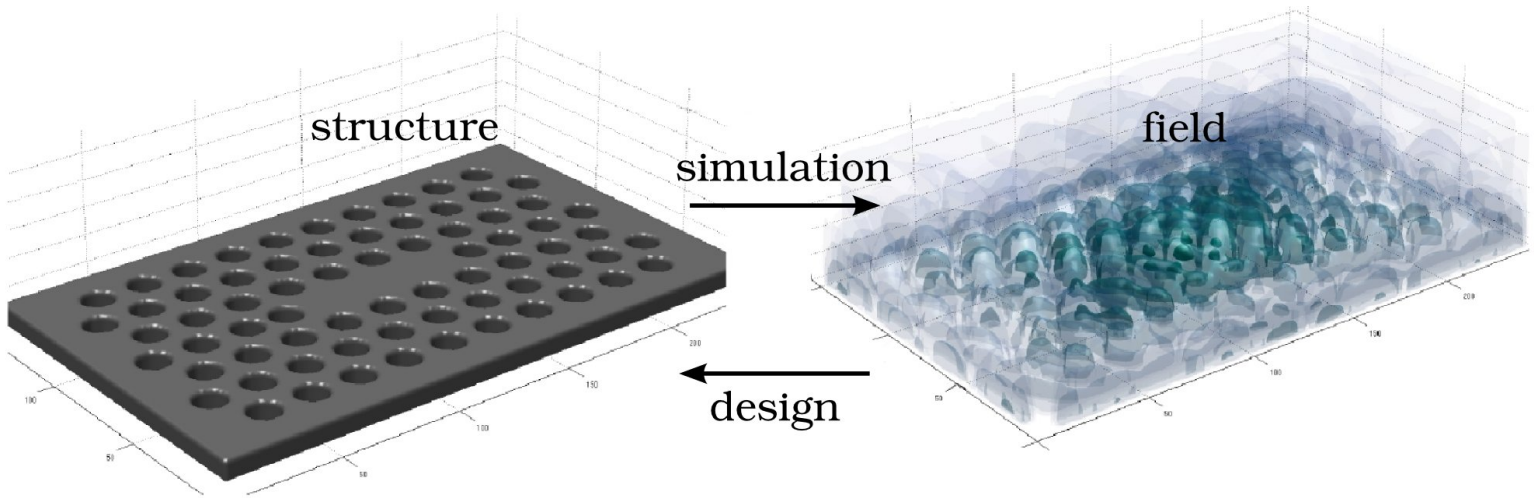

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- If possible, we can design *any* nanophotonic/optical component!

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\epsilon &= (\nabla \times \mu_0^{-1} \nabla \times E + i\omega J)/\omega^2 E
\end{aligned}$$

- Solving for ϵ actually way faster than simulation (solving for E)!

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  - Pre-requisite (200-level) class in optimization curriculum
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- Key: If $A(z)$ is linear in z then $A(z)x = b$ is as well!

Direct design of nanophotonic devices

- Let's try it already!
  - Choose x (field)
  - Solve for z (structure) by minimizing the *physics residual*, $A(z)x - b$

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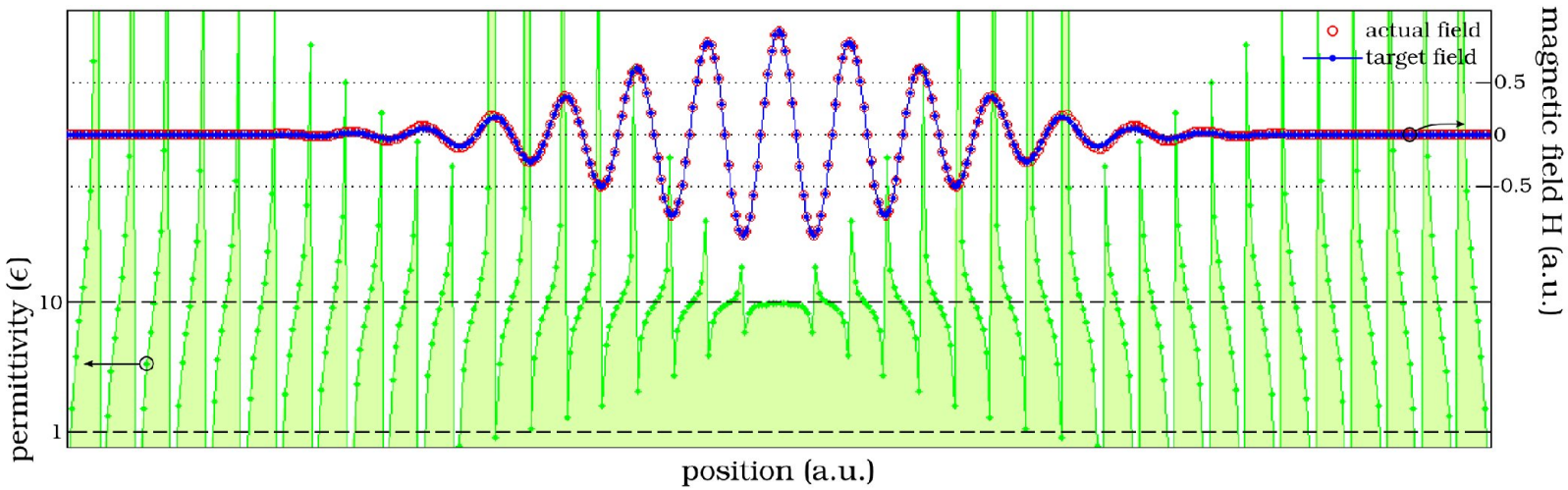
- Global minimum where $A(z)x - b = 0$ can be computed in one step

$$\epsilon = (\nabla \times \mu_0^{-1} \nabla \times E + i\omega J)/\omega^2 E$$

where $\epsilon \rightarrow z$

- Choose canonical 1D cavity field for x
- Solve for z (structure) and check design fidelity with simulation



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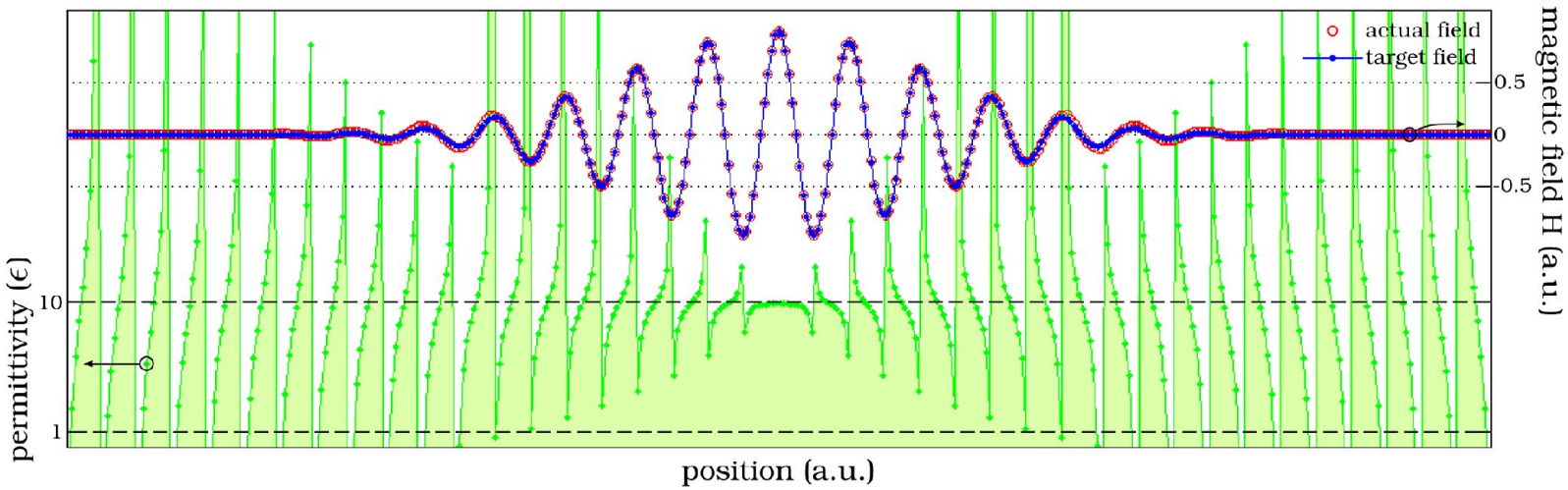

- Result
  - Perfect performance
  - But unmanufacturable structure (z not well-behaved)

Direct design with regularization

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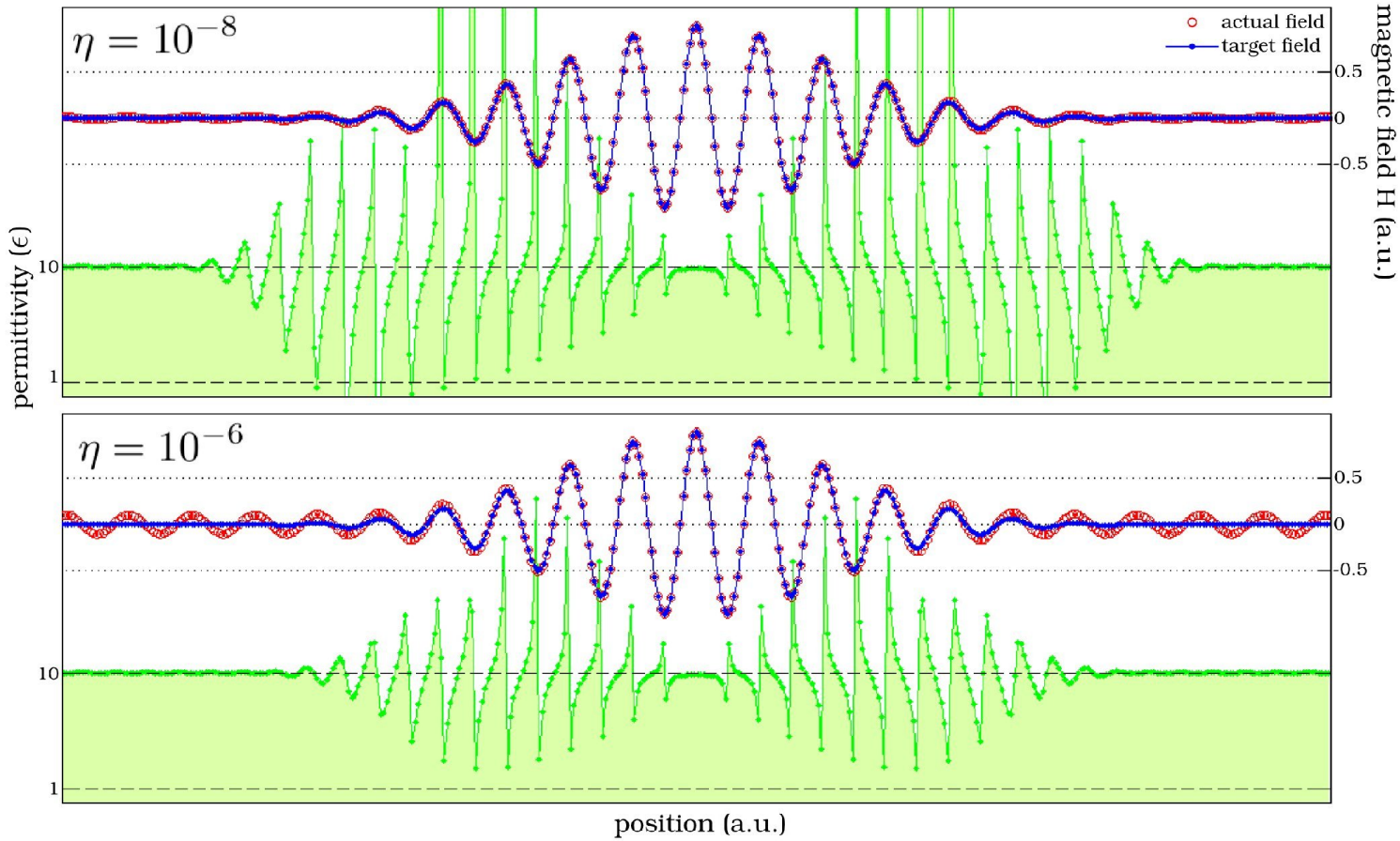
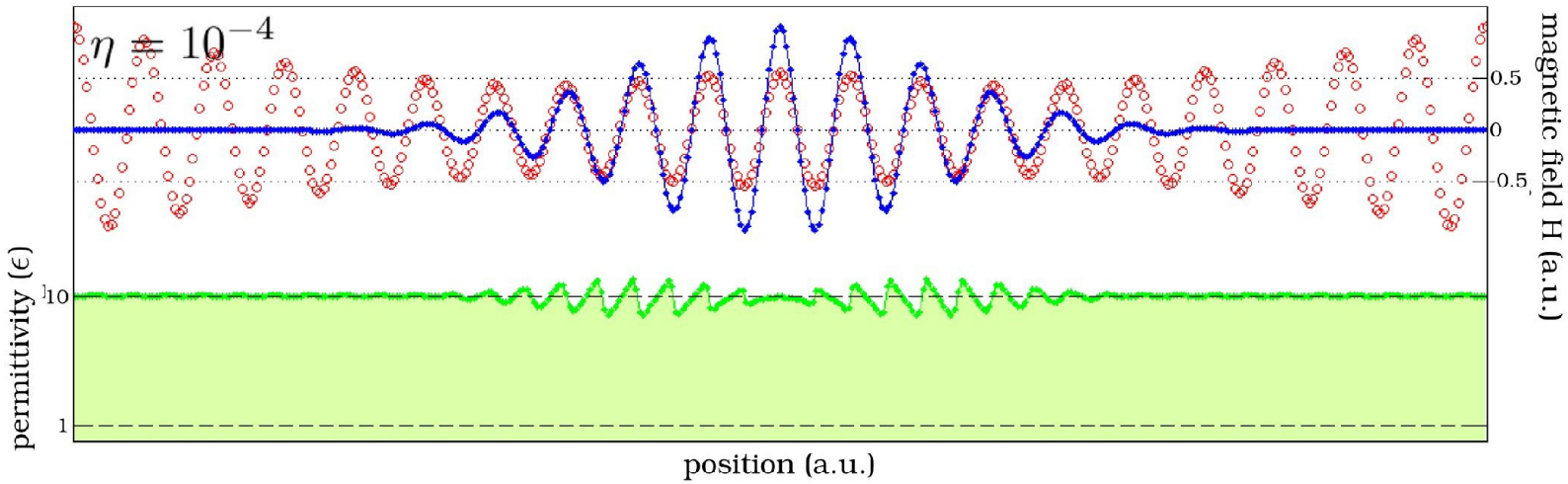
  - $\|z - z_0\|^2$ term keeps z close to z_0
  - η controls the strength of the regularization

- Solution can still be computed in one step



- Unfortunately, regularization on z decreases performance



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- Decreased performance a result of non-zero physics residual at optimum

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- Therefore, a *useful* tool would optimize for *both* x and z

Iterative design of nanophotonic devices

- New algorithm: Iteratively solve for x (field) and z (structure)

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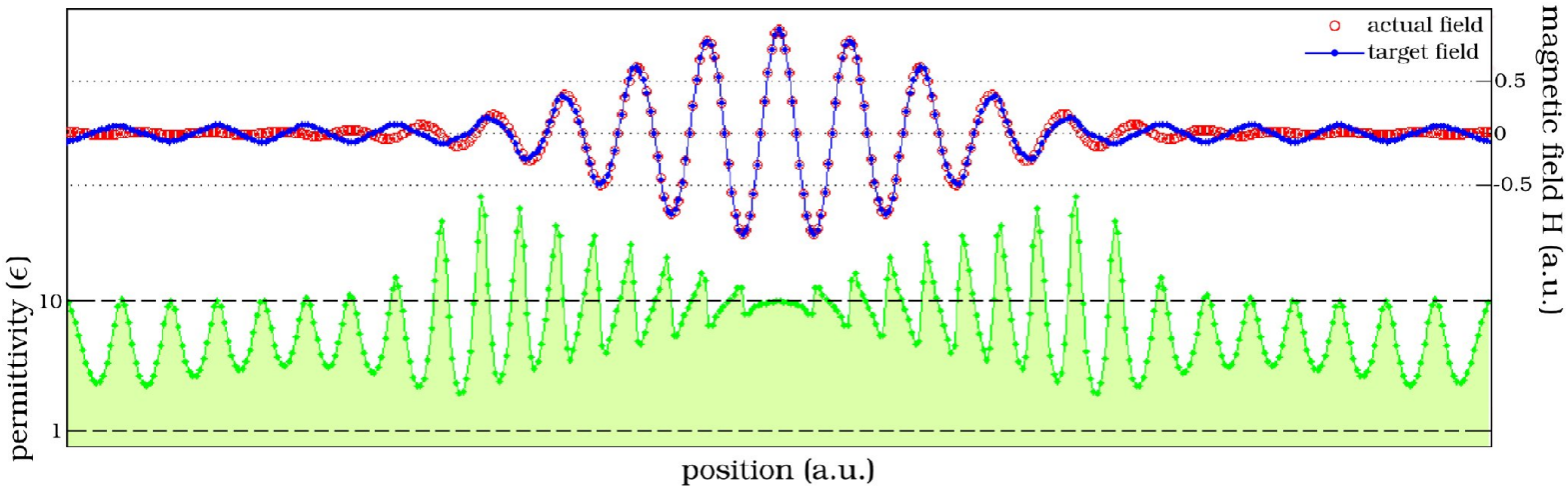
- Takes advantage of the *bi-linearity* of the physics residual
  - Jointly solving for x and z is a non-convex problem

- More concisely, we iteratively solve the following

$$\underset{\text{alternately } x \text{ then } z}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta_0 \|z - z_{\text{prev}}\|^2 + \eta_1 \|x - x_{\text{prev}}\|^2$$

- Design process now consists of *multiple* computational steps
  - η_0, η_1 gradually decreased to bring physics residual toward 0



- Iterative strategy produces z (structure) that
  - is better behaved
  - more accurately produces x

Iterative design with hard constraints on z

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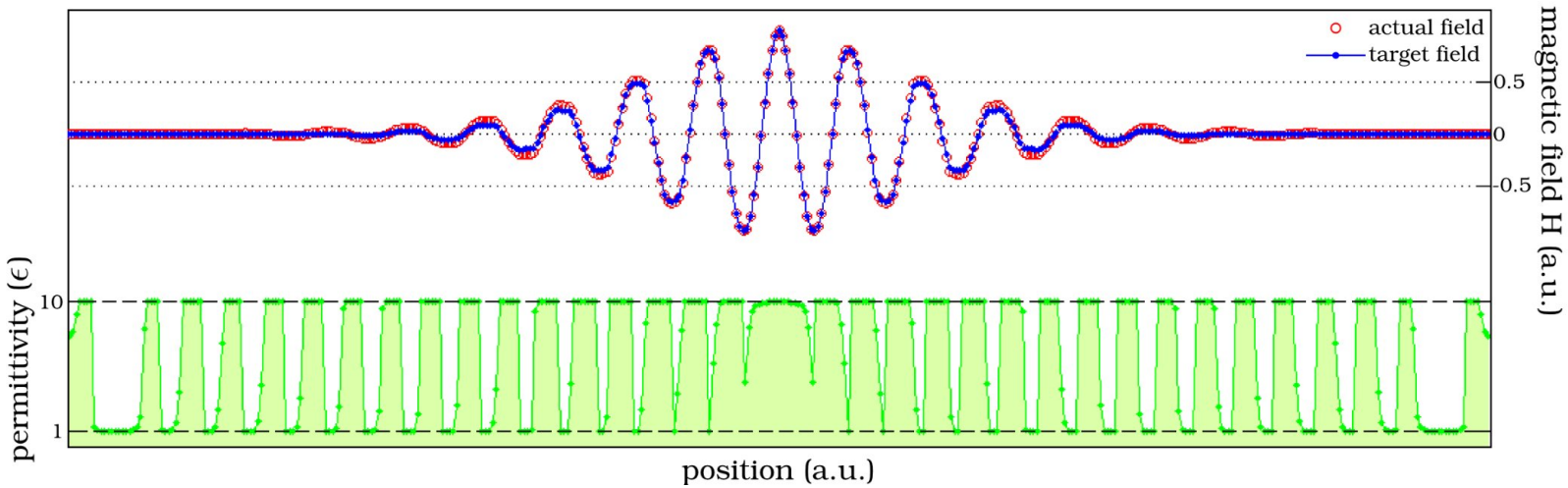
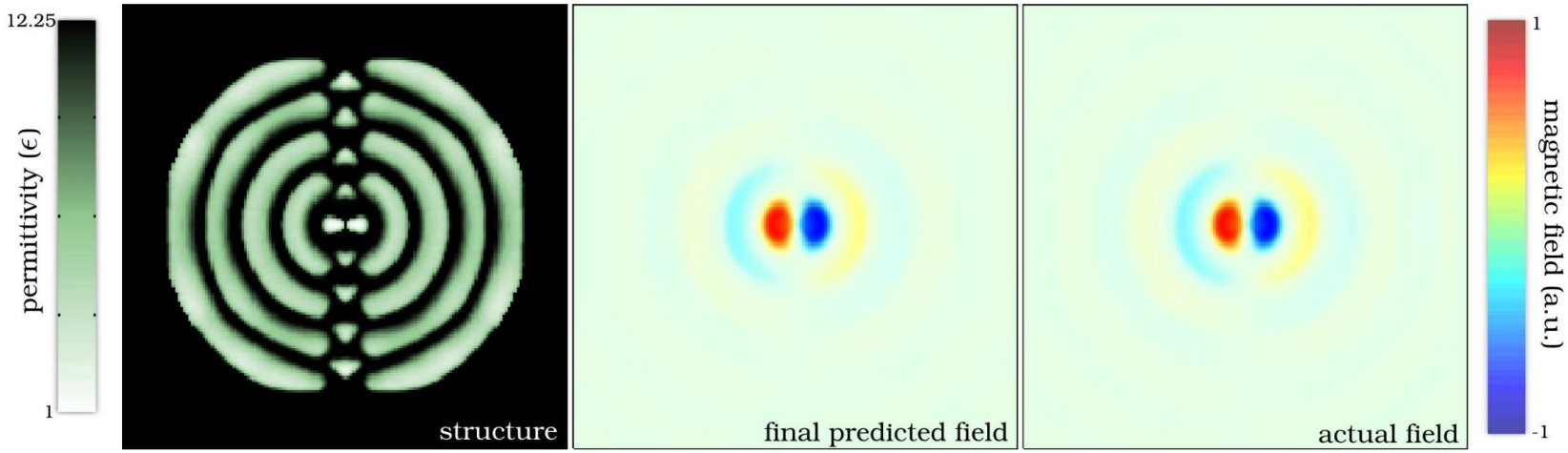
- $z_{\text{min}} \leq z \leq z_{\text{max}}$ constraint better represents manufacturability constraint
  - Corresponds to a minimum and maximum allowable permittivity (ϵ)

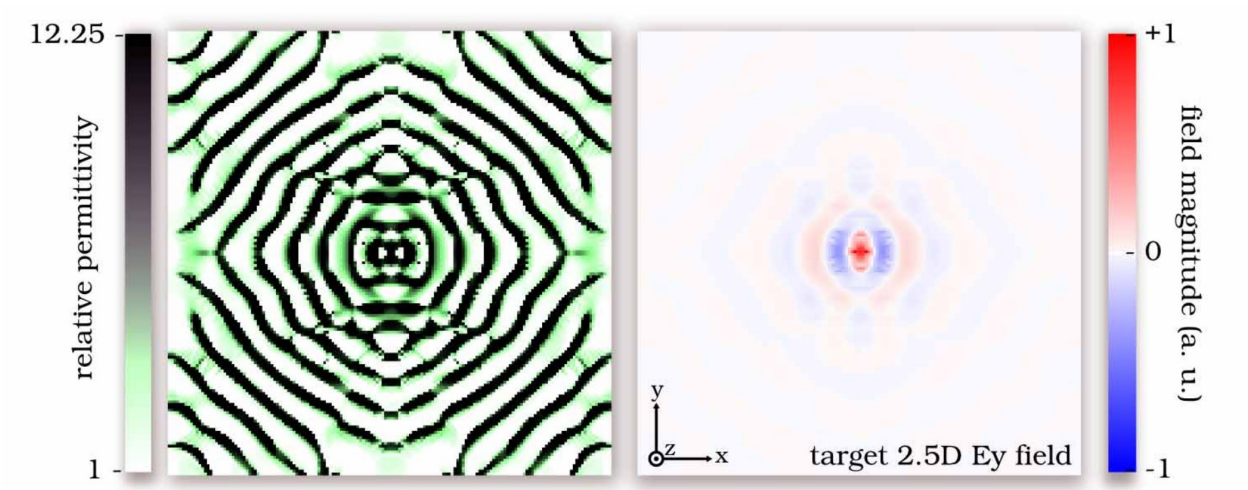


- Well-behaved, manufacturable z (structure)
- Final x (field) accurately reproduced
- Majority of elements of z are fortuitously at one limit or the other!

- Can be used to create 2D resonators



- 3D resonators can be designed using a "2.5D" approximation



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- Instead of regularization term, we force the elements of x at the boundary to be equal to the ideal case ($\hat{x}$)

- Called *objective-first* design because the design objective is prioritized above the physics residual

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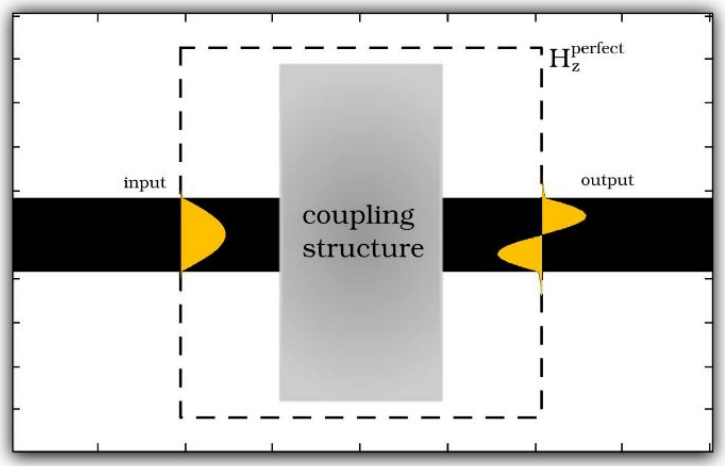
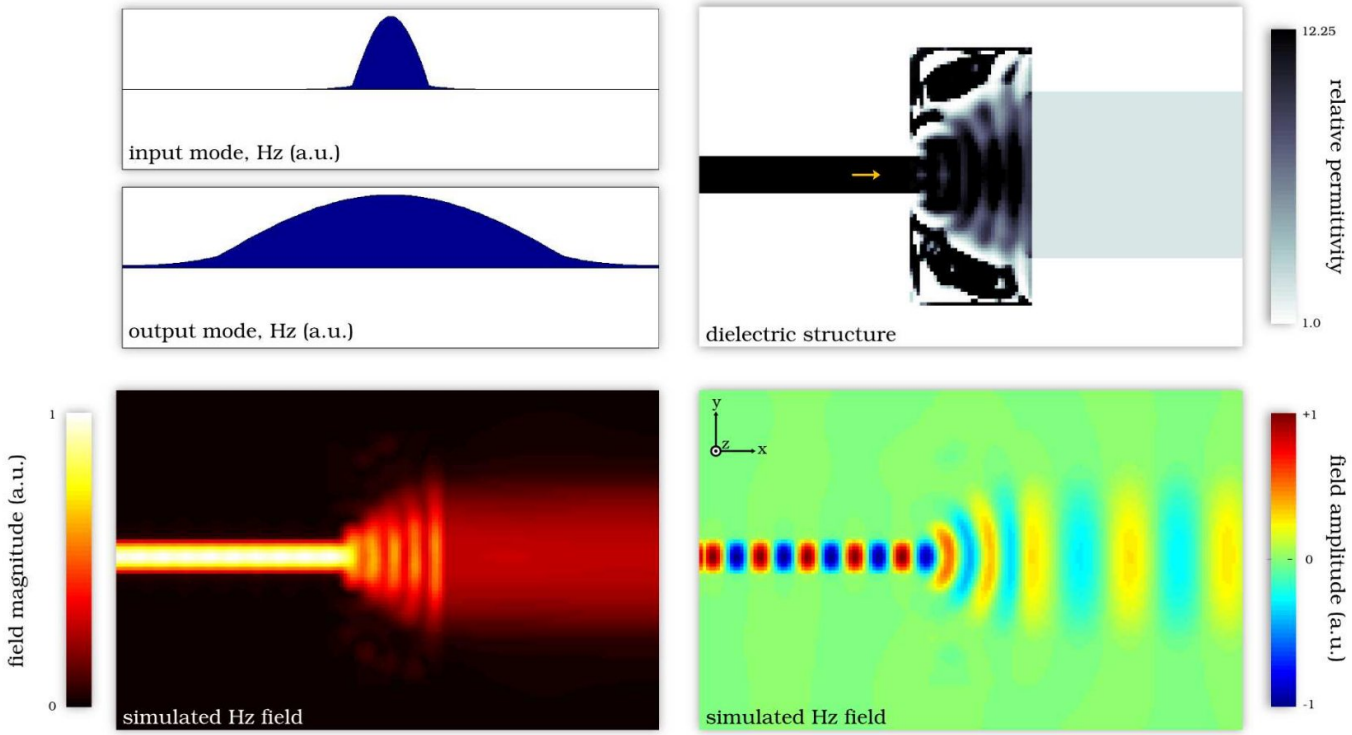
- Applied to the design of 2D waveguide mode couplers

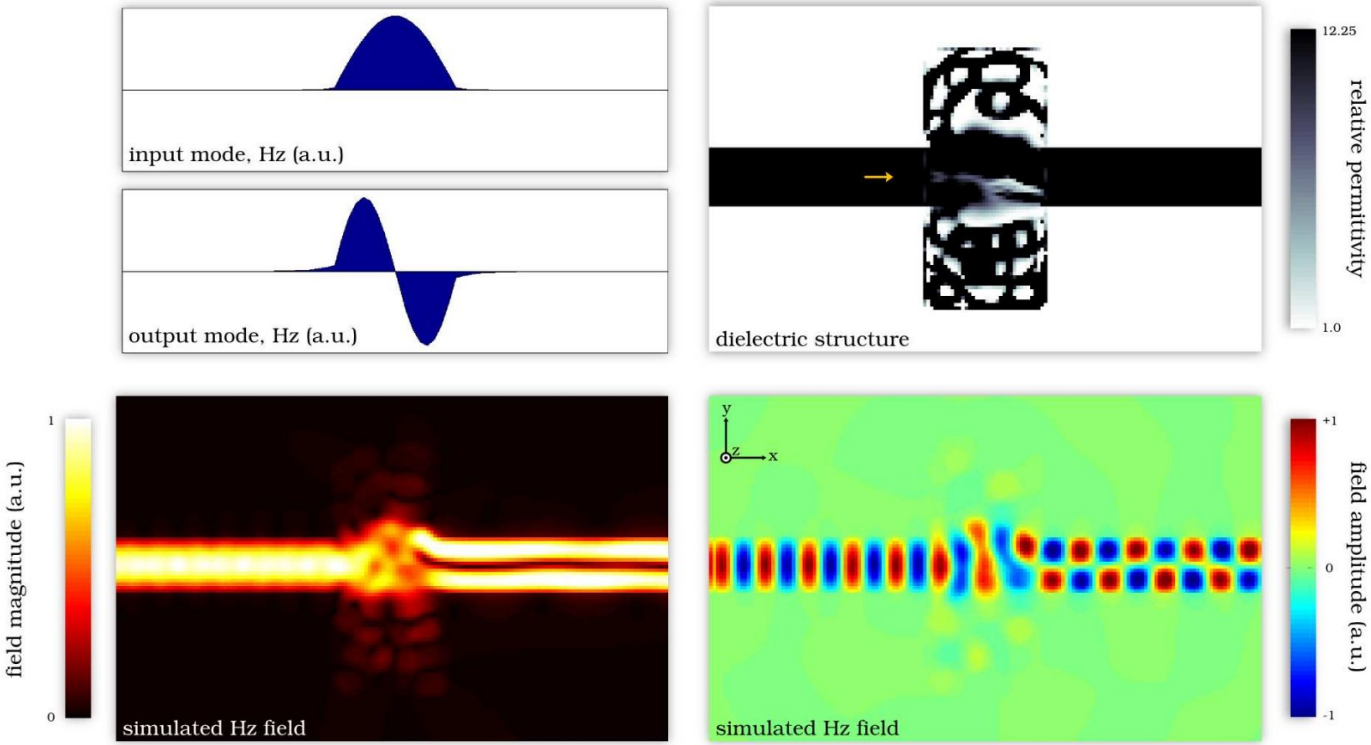


- Produced designs which exhibited
  - High efficiency ($\sim 98\%$)
  - Small device footprints ($1.5 - 4$ square vacuum wavelengths)

- Coupler to wide, low-index waveguide



- Coupler from fundamental to second-order waveguide mode



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- Goal: Software to design *all* linear nanophotonic devices
  - Fully three-dimensional (no approximations)
  - Multi-mode
  - Discrete, planar, manufacturable structure

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  - Famously ill-conditioned
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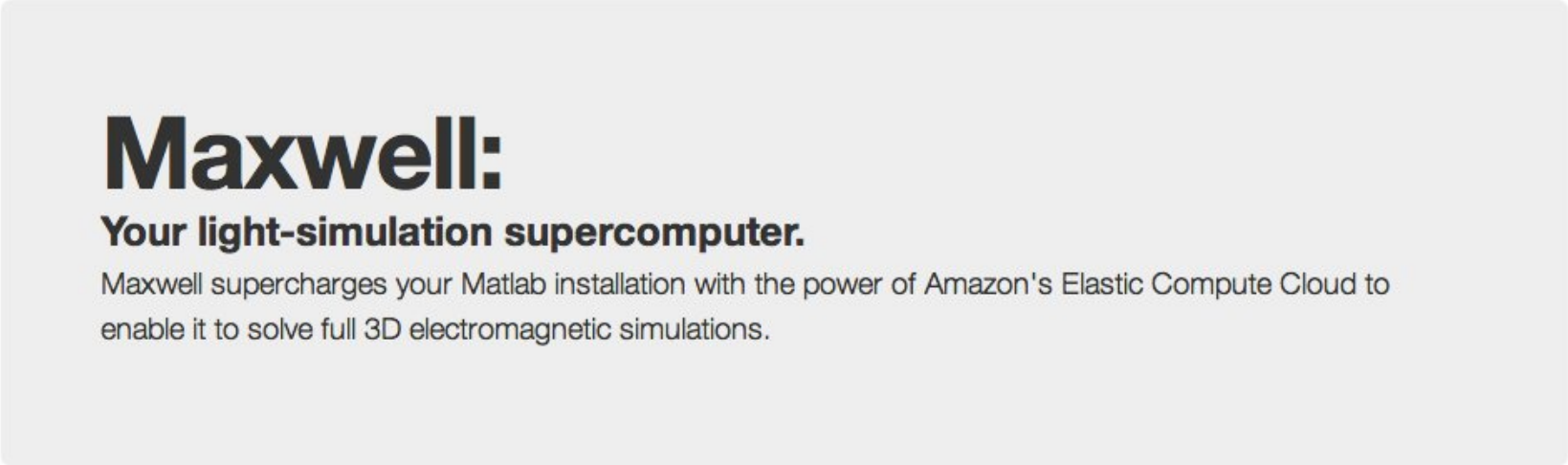
- Fortunately, Wonseok had already solved this problem

Maxwell: Light-simulation supercomputer

- Partnered with Wonseok to develop a cloud-based electromagnetic solver using Amazon Web Services
  - GPU-accelerated implementation of Wonseok's algorithm
  - Cluster scales automatically to tens of nodes



- Scalable
  - Far outstrips computing clusters such as Teragrid
  - Can perform multiple solves in parallel, on a single Matlab instance
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- The key technological enabler in achieving 3D design

3D design: problem statement

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$$
\begin{array}{ll}
\text{minimize} & \displaystyle\sum_{i}^{M} \|A_i(z)x_i - b_i\|^2 \\
\text{subject to} & \alpha_{ij} \leq |c_{ij}^{\dagger} x_i| \leq \beta_{ij}, \quad \text{for } i = 1, \ldots, M \text{ and } j = 1, \ldots, N_i \\
& z_{\text{min}} \leq z \leq z_{\text{max}}
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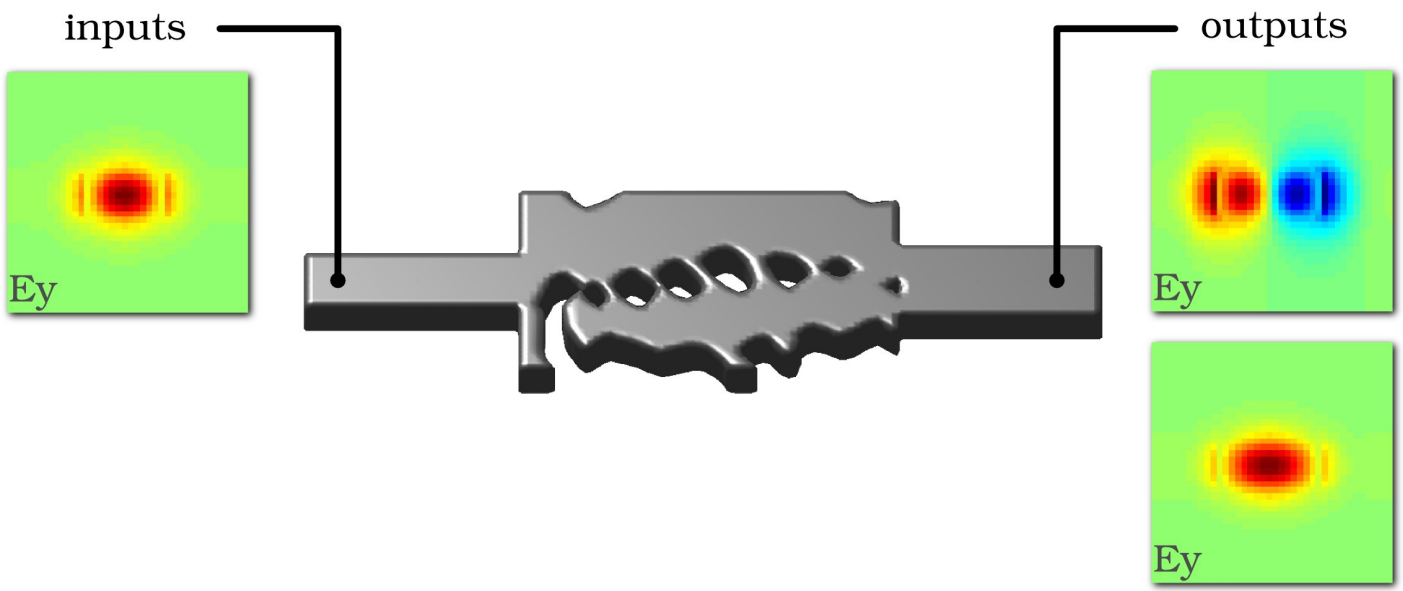
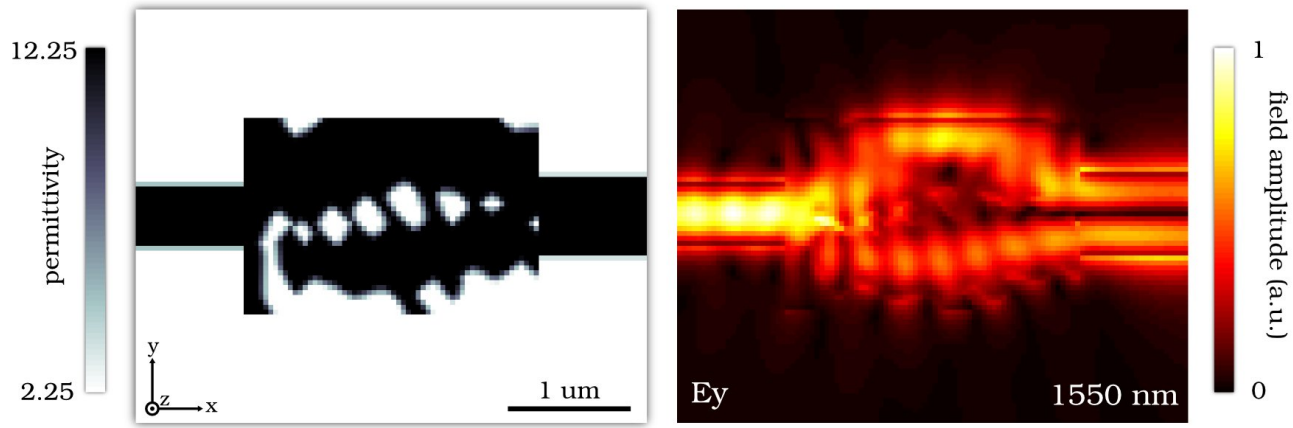
- M modes with N_i monitored output modes each
  - b_i is the input excitation for each mode
  - $|c_{ij}^{\dagger} x_i|$ is the field overlap with output mode c_{ij}
  - α_{ij} and β_{ij} is the design range for the overlap



- Input mode is excited using current source (b_i or $-i\omega J$)
- Output mode is enforced using overlap integrals ($c_{ij}^{\dagger} x_i$ or $\int \hat{E}_{ij} E$)

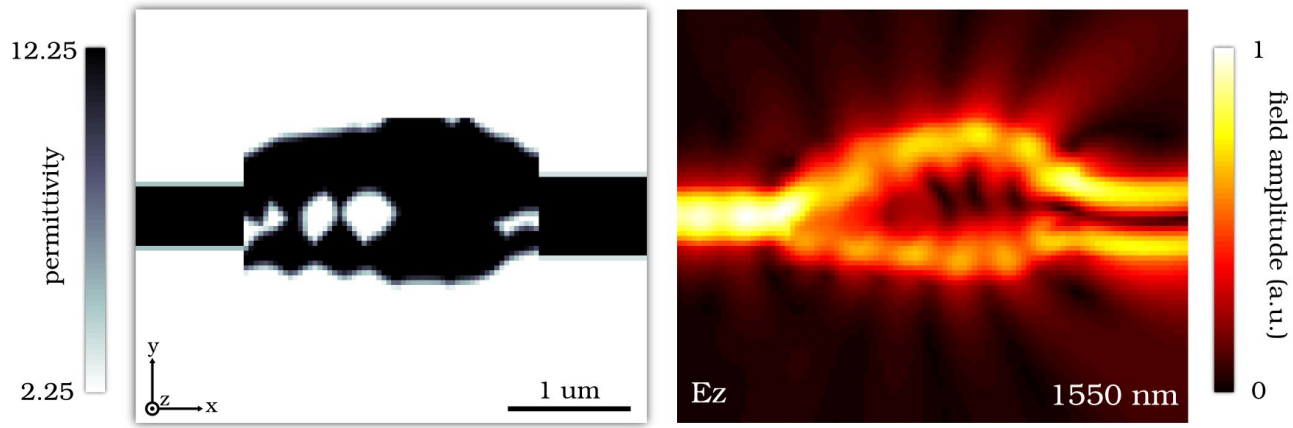
- TE mode converter



- Efficiency: 86.4%
- Power in rejection mode (transmitted fundamental): 0.7%
- 250 nm silicon slab surrounded by silica

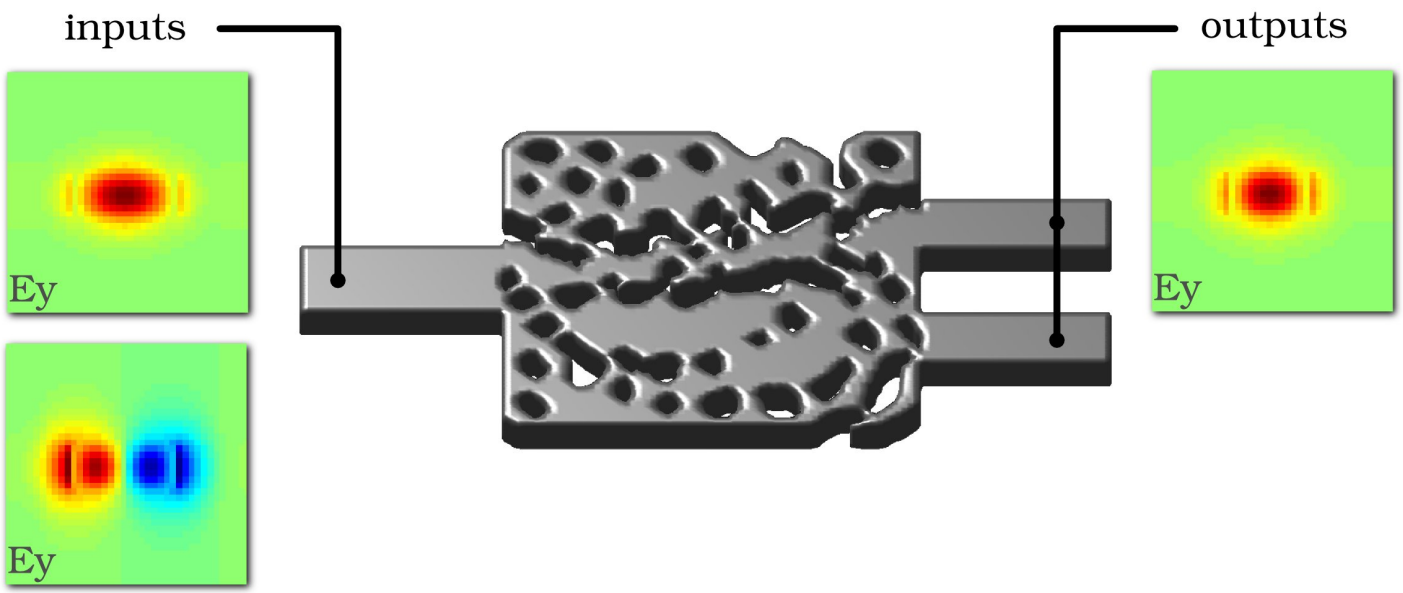
inputs
outputs
Ez
Ez
Ez

- TM mode converter

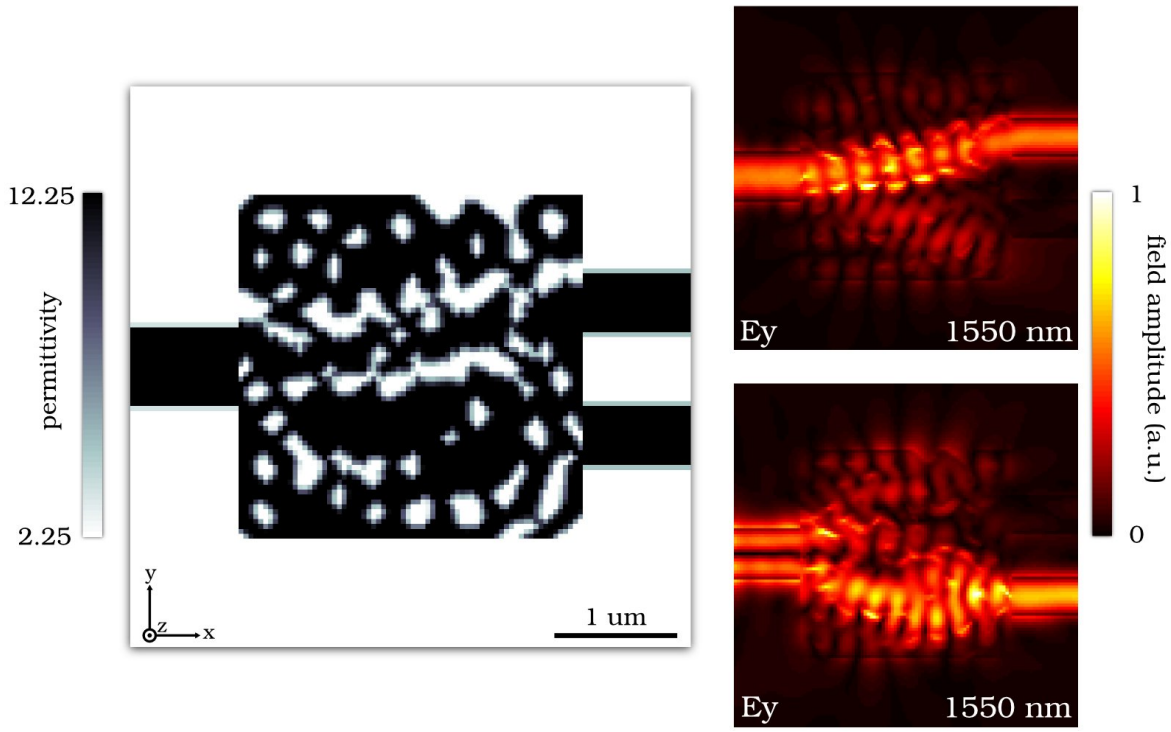


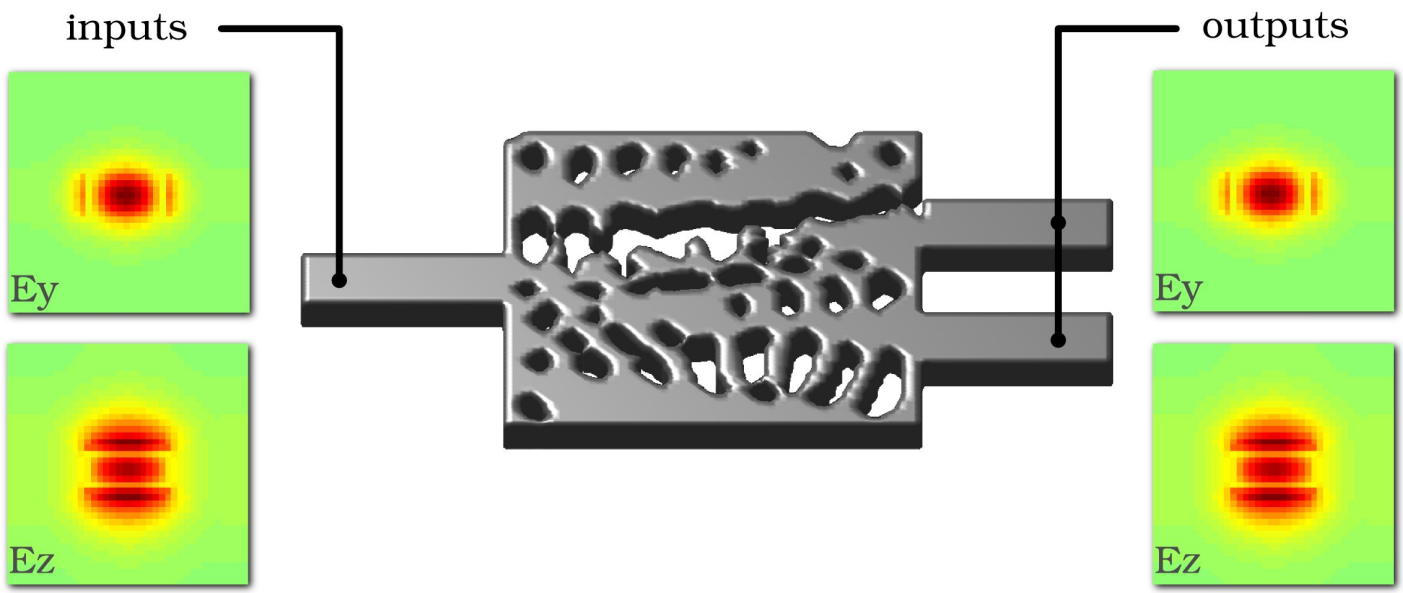
- Efficiency: 76.9%

- Power in rejection mode (transmitted fundamental): 1.0%

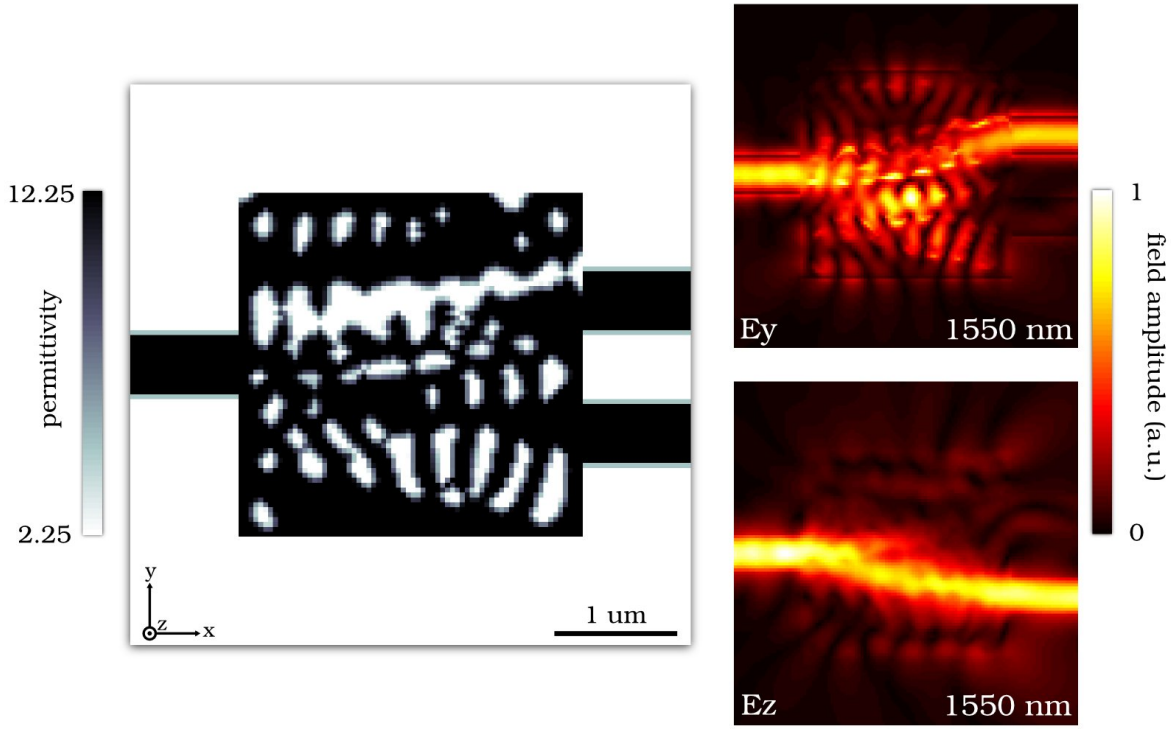

inputs
outputs
Ey
Ey
Ey

- Spatial mode splitter
  - Efficiency: 88.7%, 77.4% (0.27%, 0.20%)



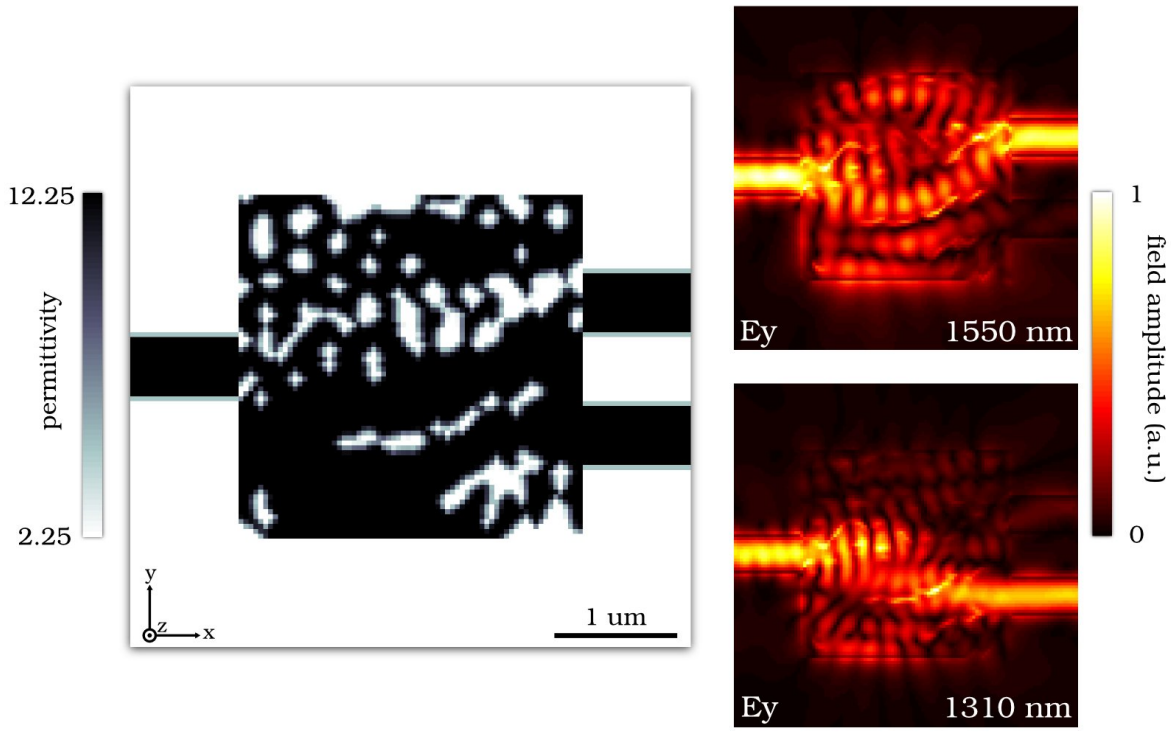

inputs
outputs
Ey
Ez
Ey
Ez

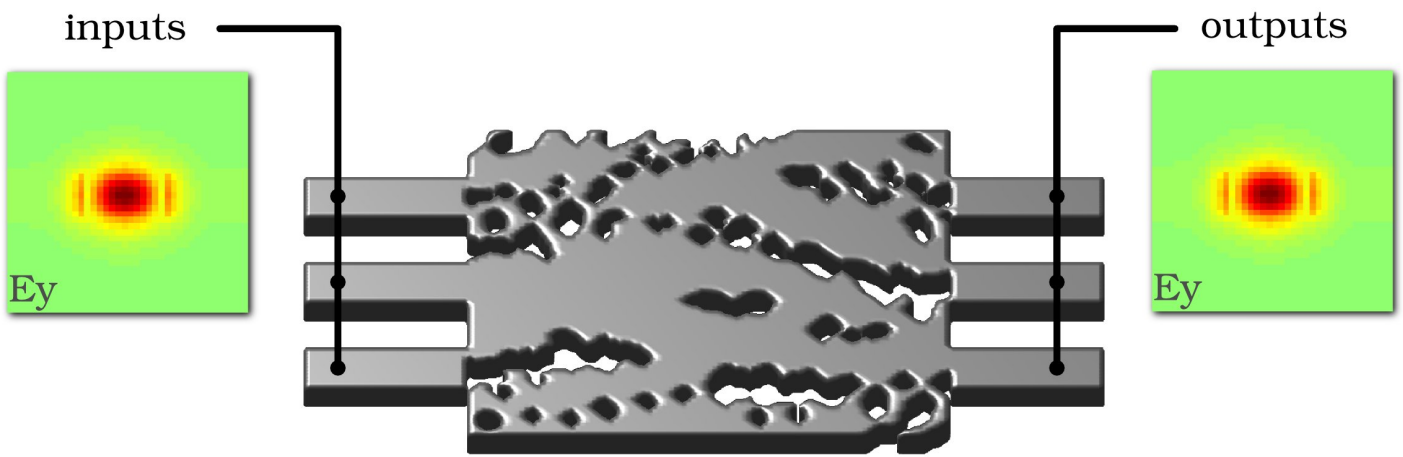
- TE/TM splitter
  - Efficiency: 87.6%, 88.8% (1.06%, 0.58%)



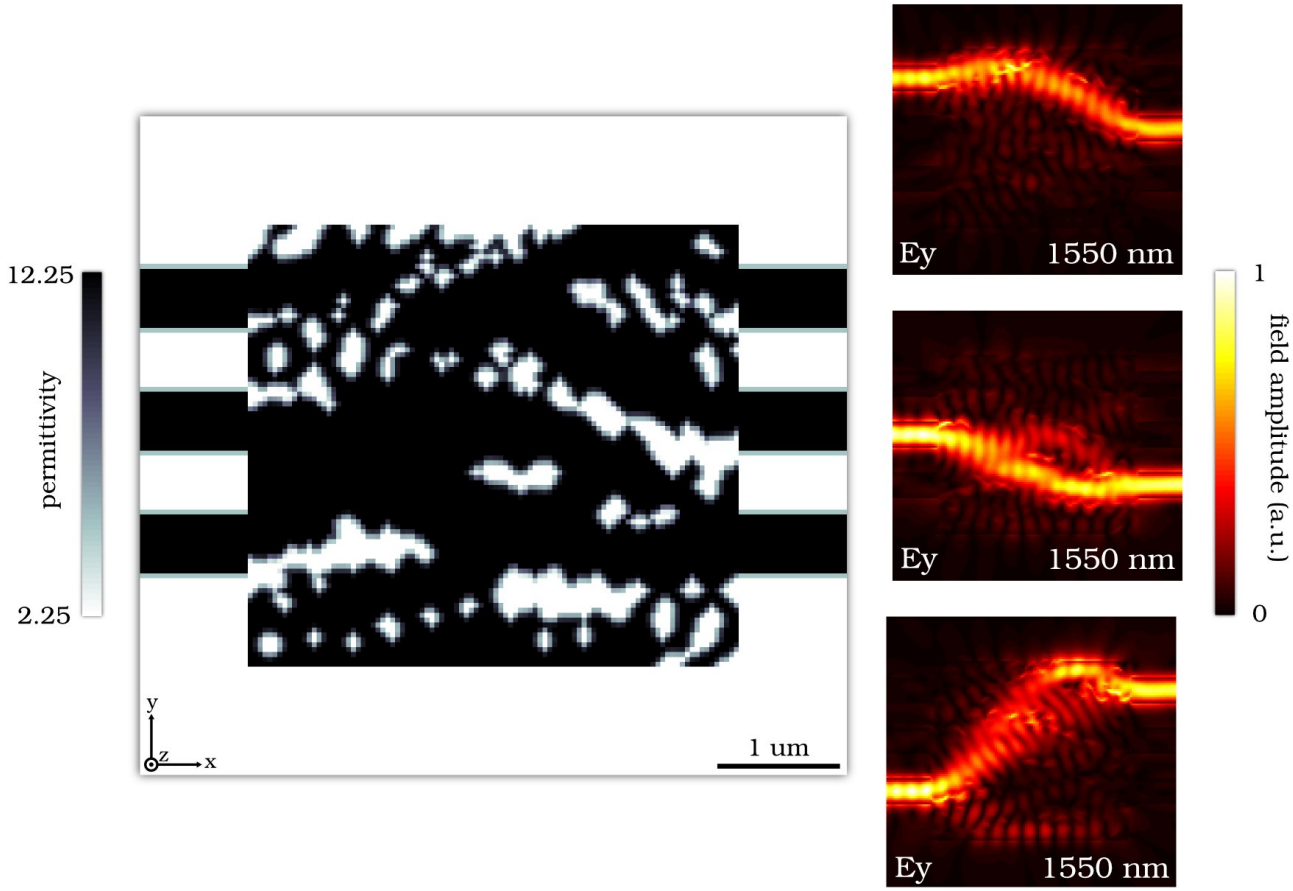
inputs
outputs
Ey
Ey

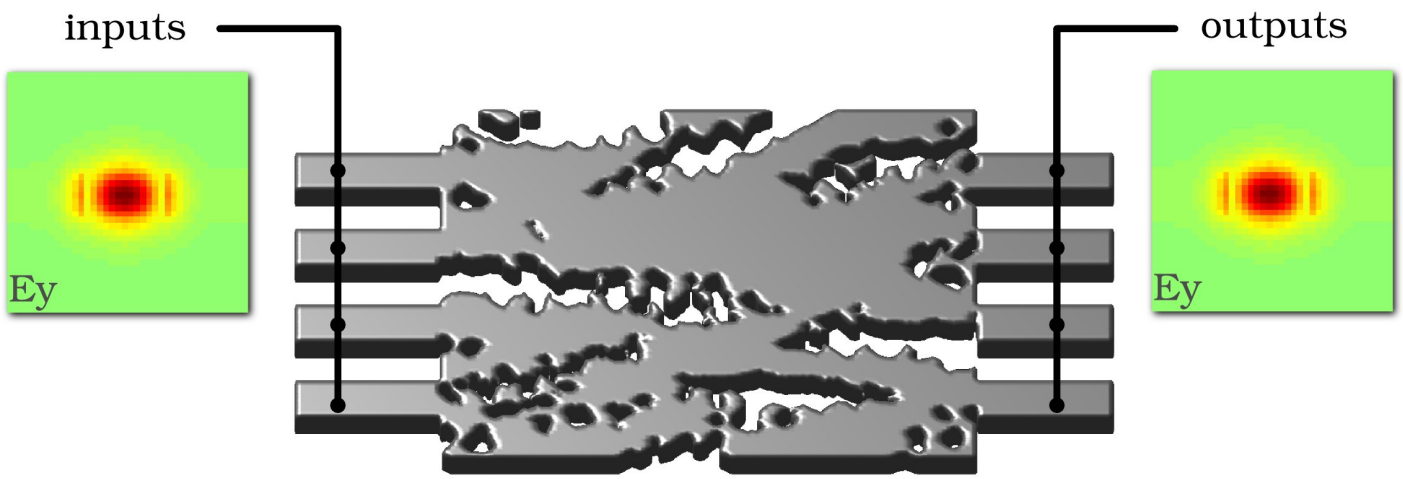
- Wavelength splitter
  - Efficiency: 83.2%, 78.7% (0.49%, 1.66%)



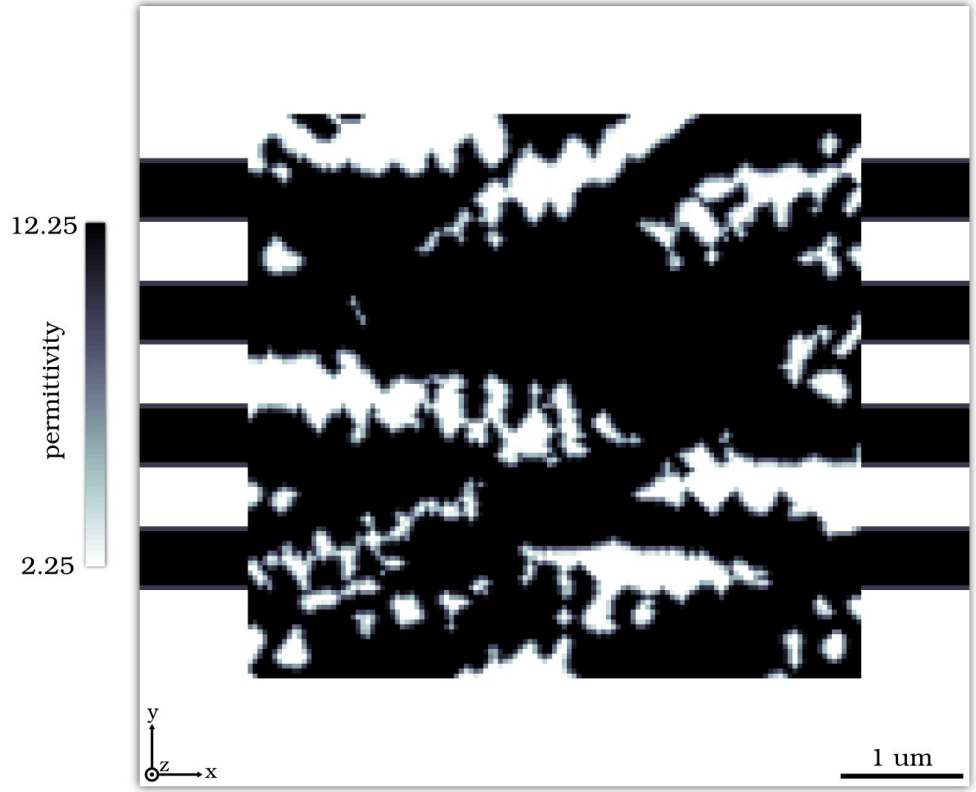

inputs
outputs
Ey
Ey

- 3x3 hub (88.6%, 90.6%, 87.3% efficiency)

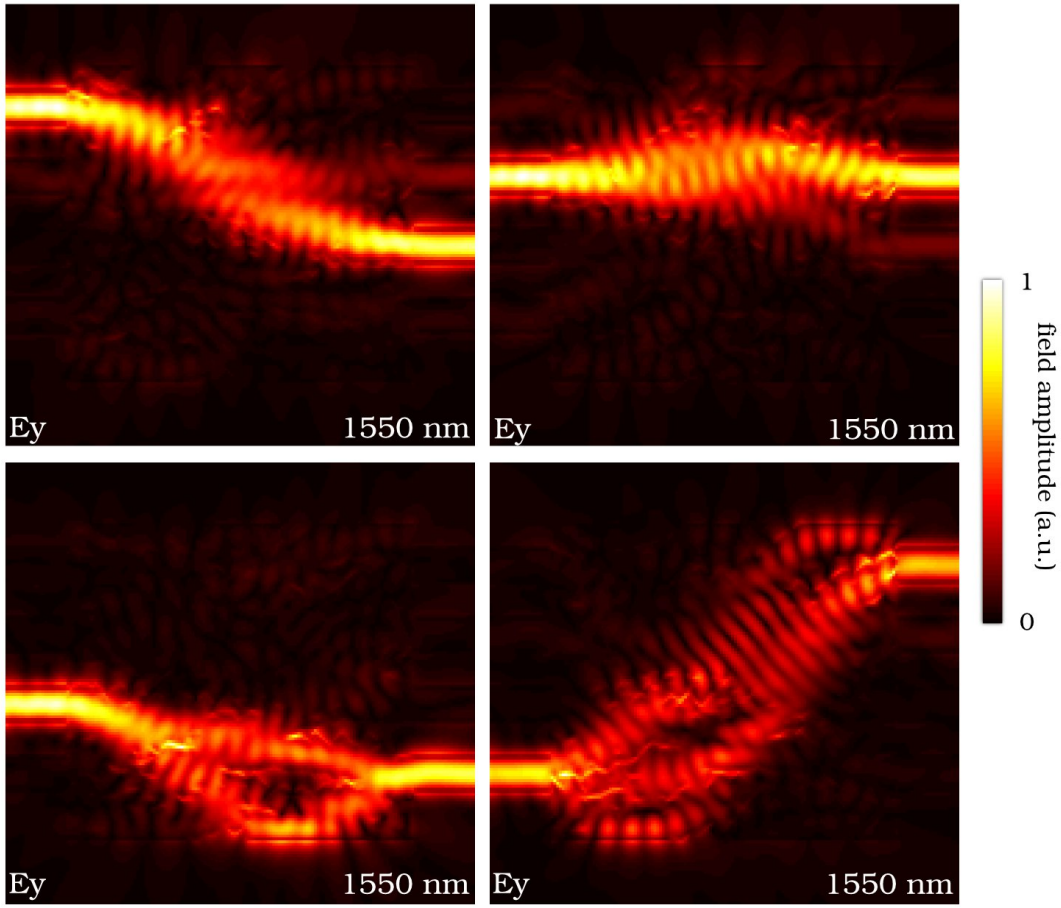


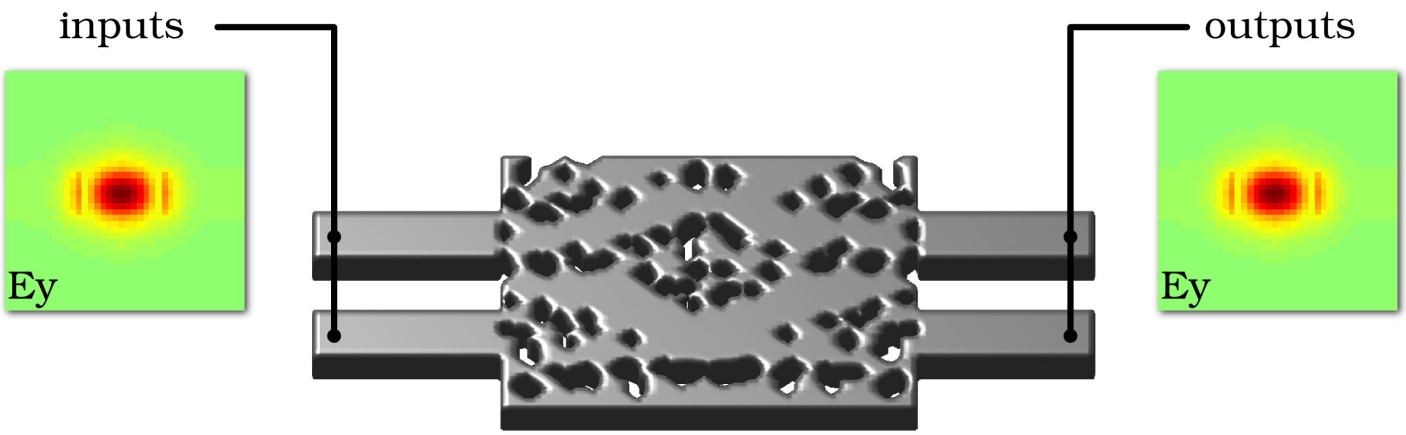

inputs
outputs
Ey
Ey

- 4x4 hub

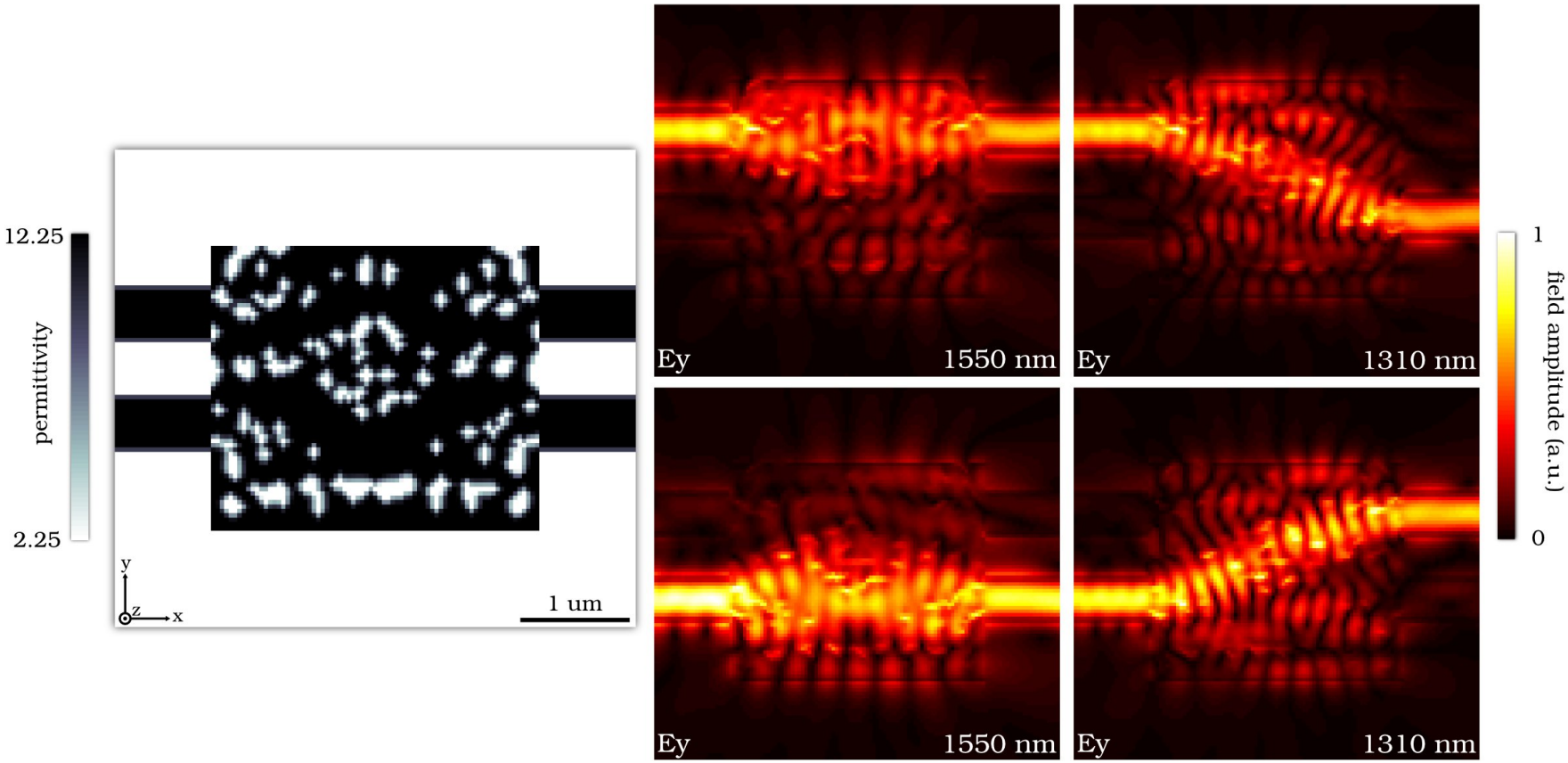


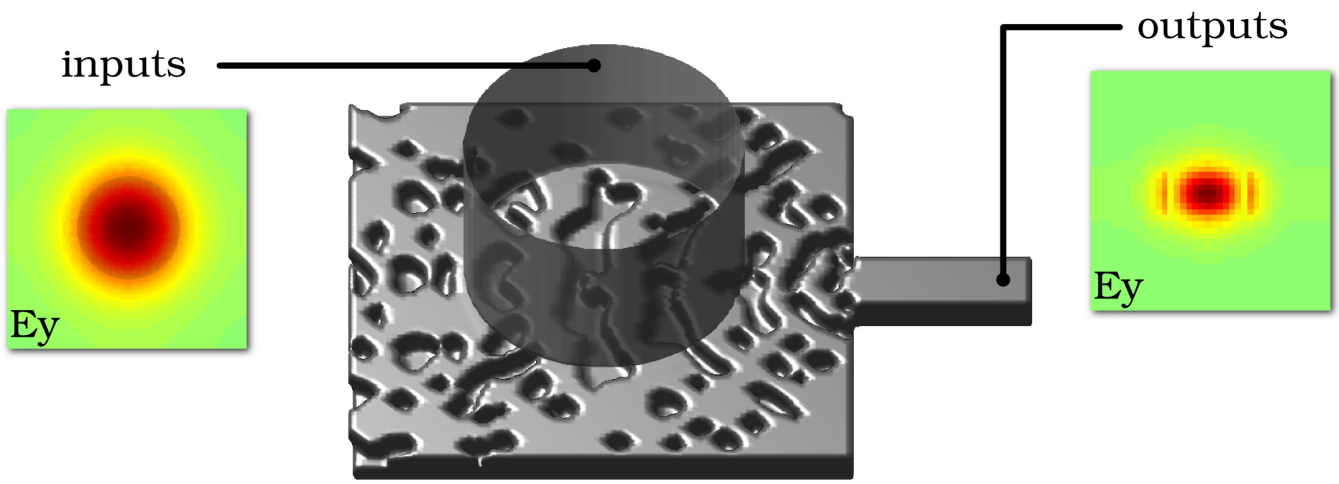
- Efficiency: 85.9%, 88.1%, 85.4%, 84.3%



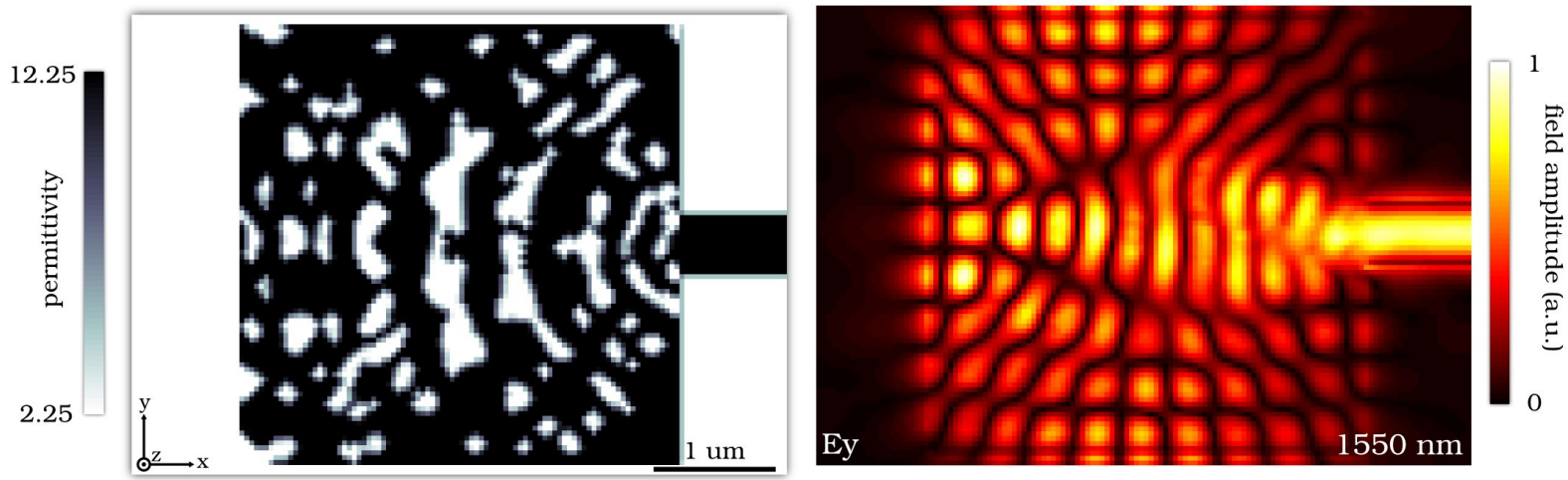

inputs
outputs
Ey
Ey

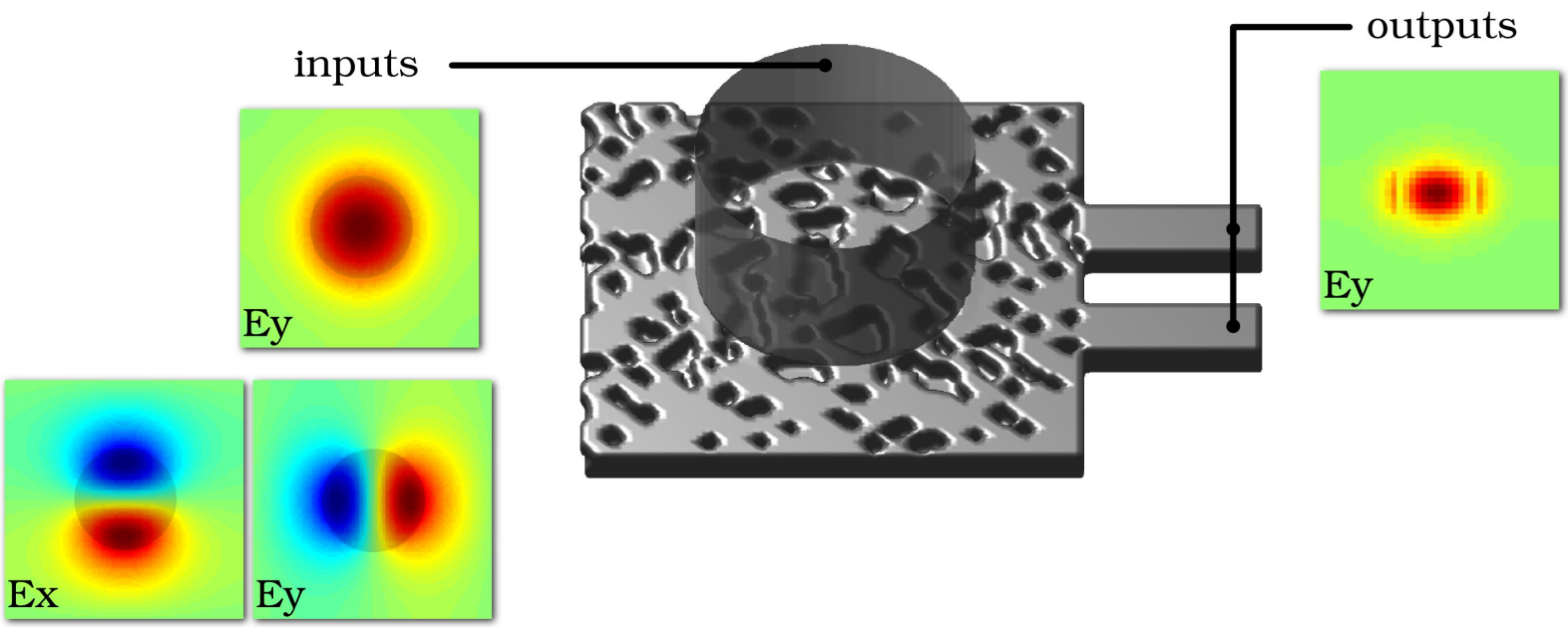
- 2x2x2 hub (77.6%, 73.7%, 75.7%, 75.2% efficiency)



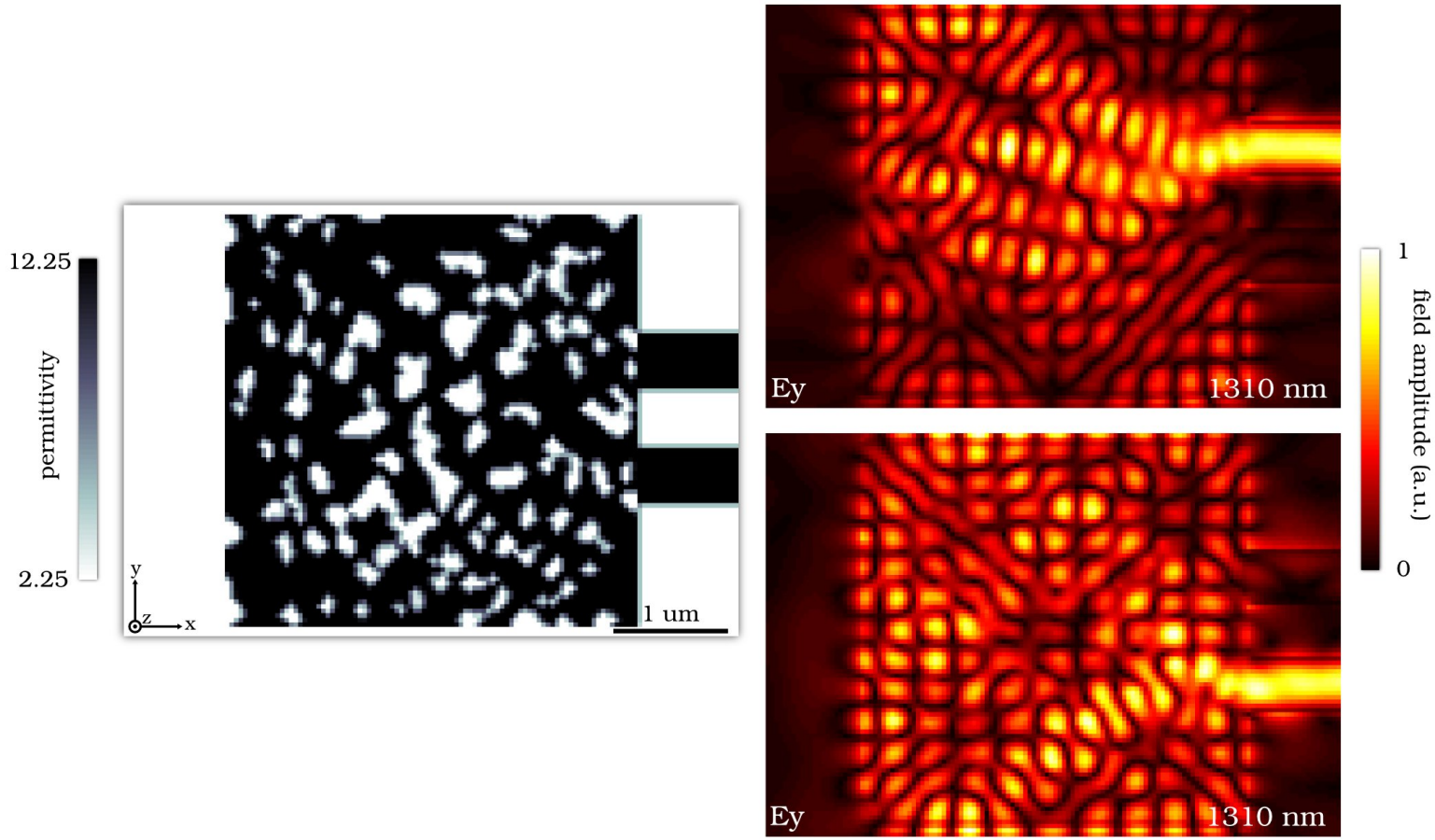

inputs
outputs
Ey
Ey

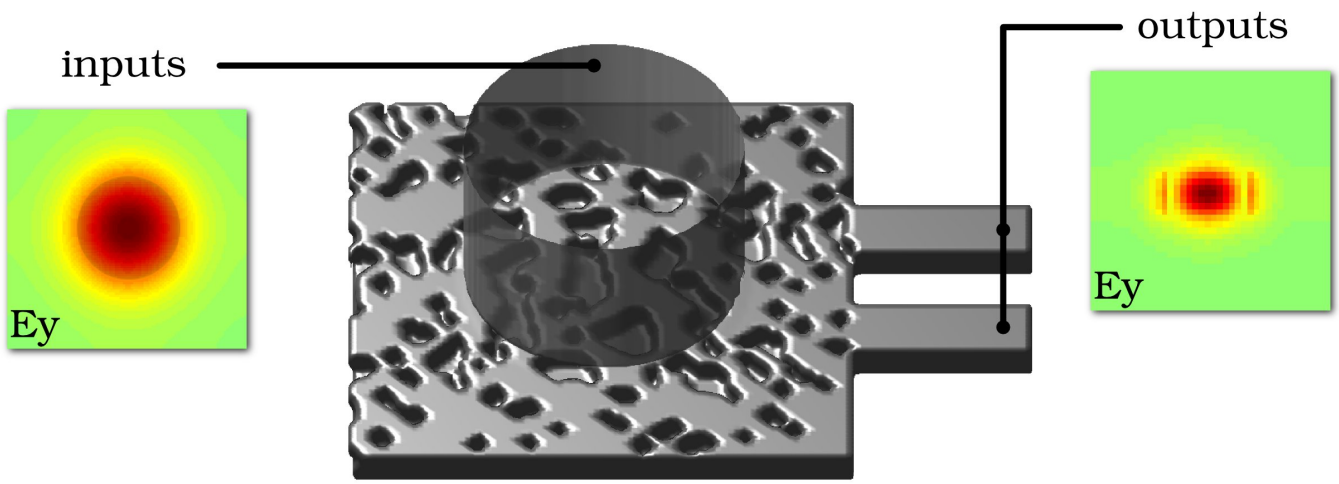
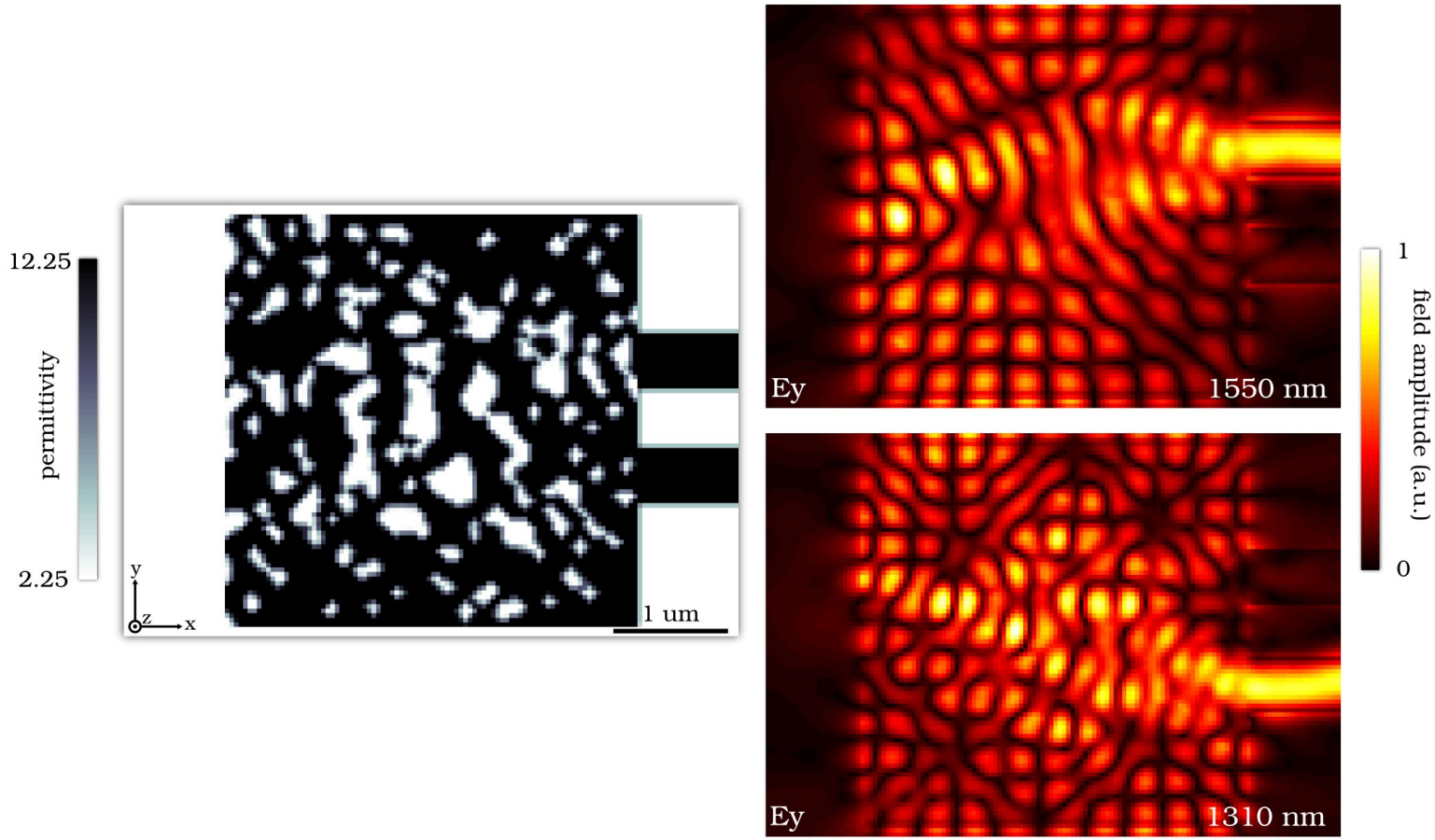
- fiber coupler (51.5%)




inputs
outputs
Ey
Ex
Ey
Ey

- fiber coupler mode splitter (32.6%, 22.7%)




inputs
outputs
Ey
Ey

- fiber coupler wdm (31.6%, 28.6%)