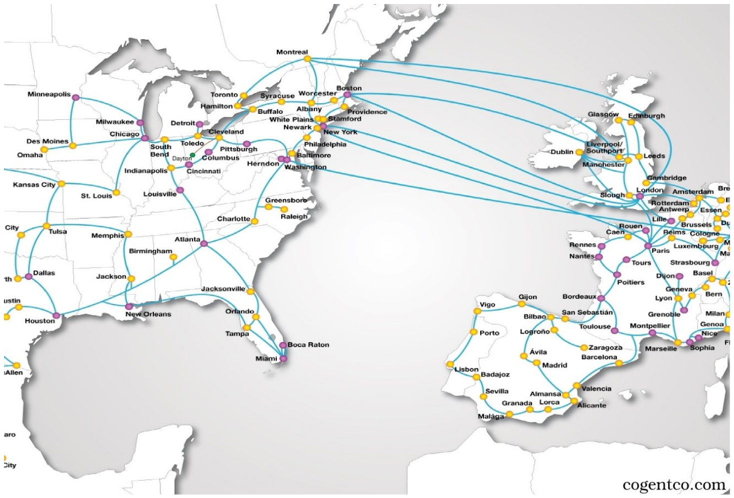
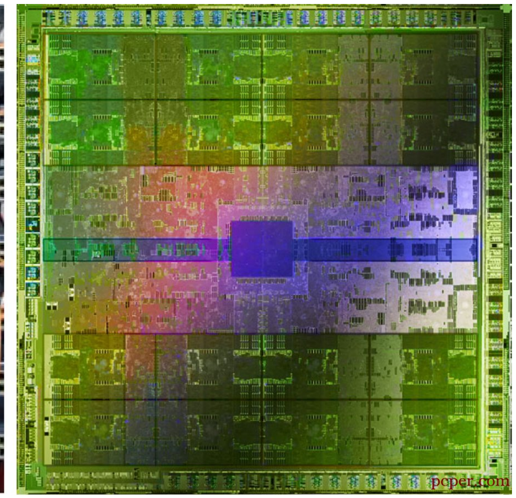
Nanophotonic Computational Design

Jesse Lu

February 25, 2013

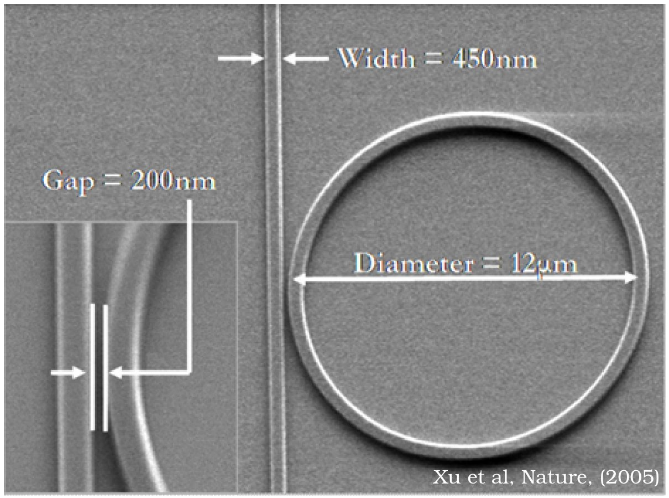
Introduction

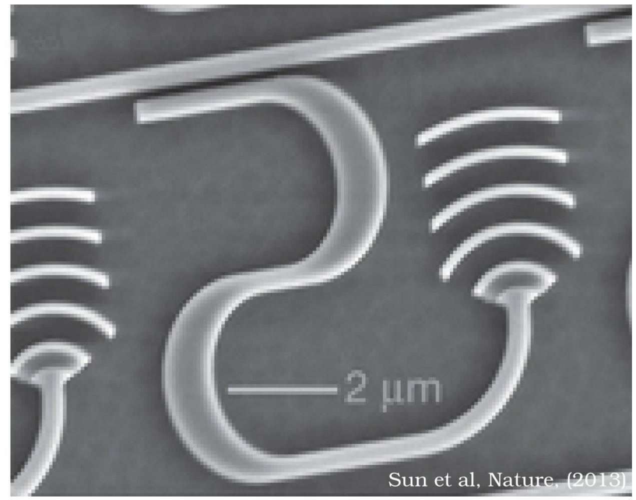




- As information grows, optical networks needed
  - across continents
  - within a datacenter
  - between chips and on-chip

- On-chip optical components are currently designed by tuning a small number of design parameters





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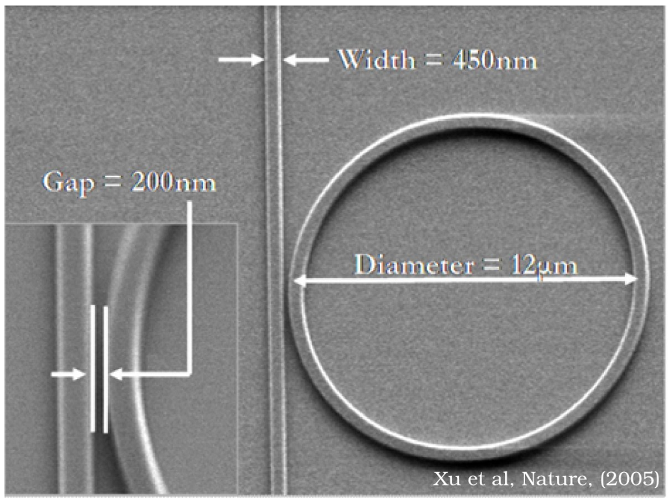
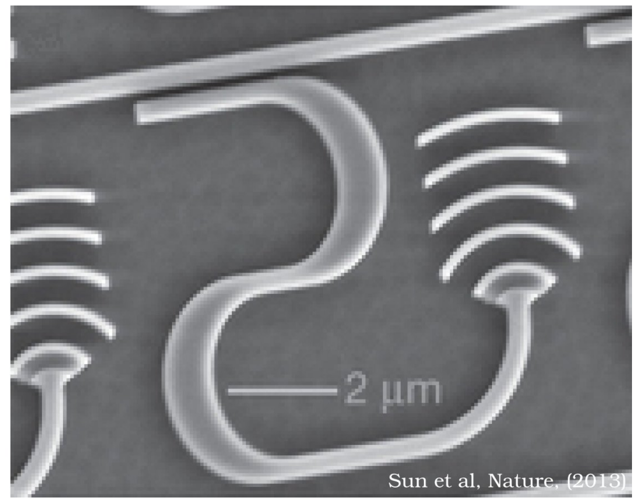
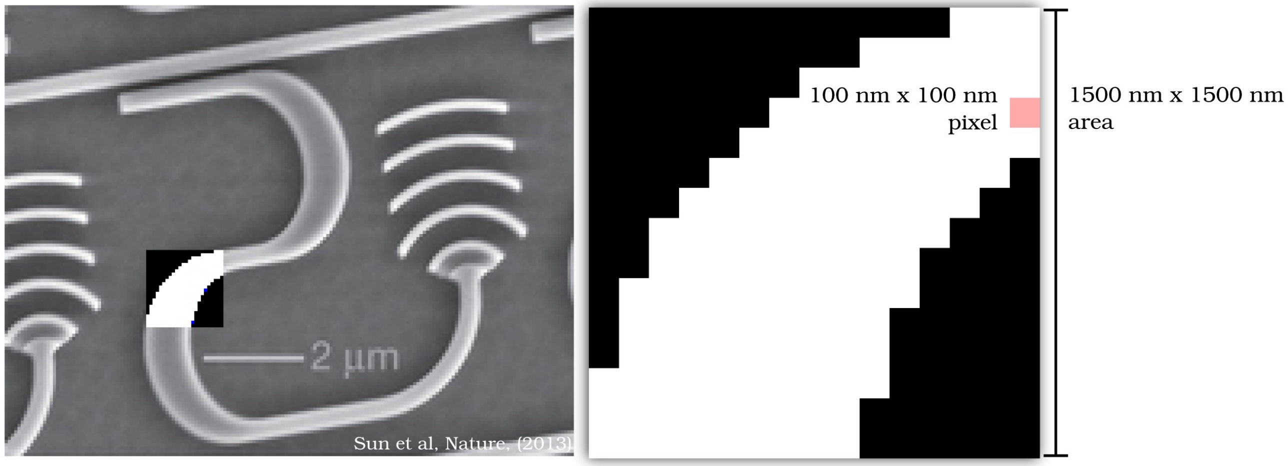


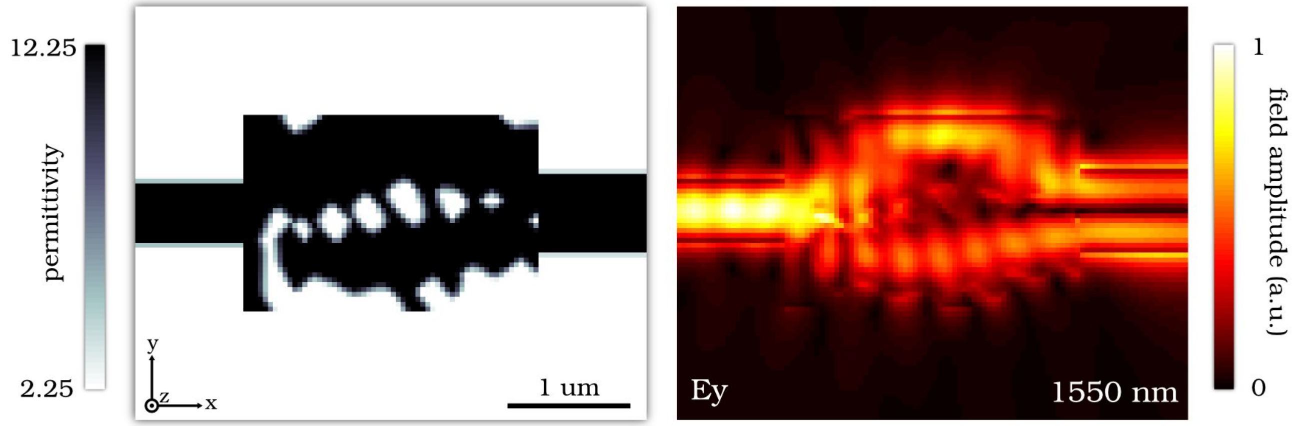


- What happens when we use the *full* parameter space for nanophotonic design?

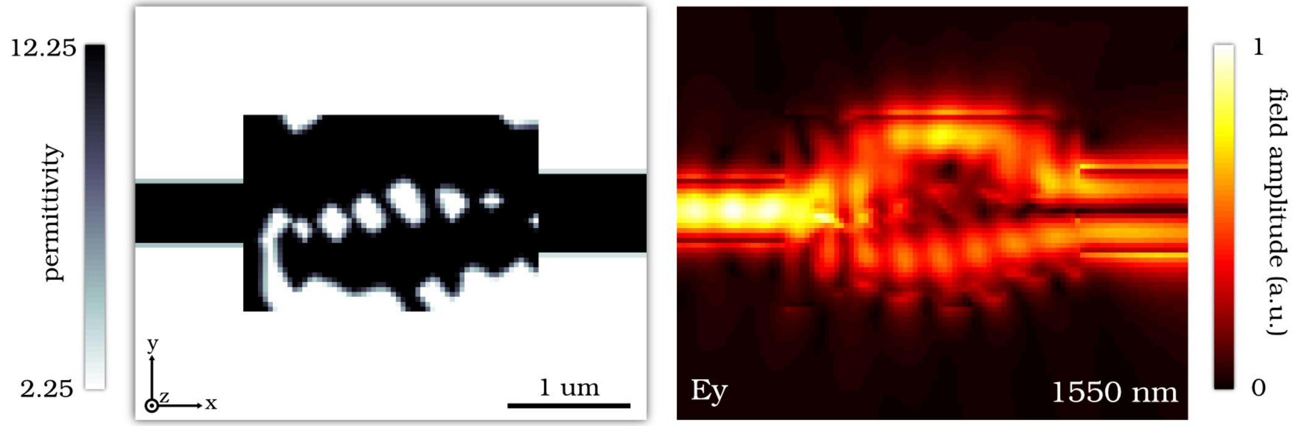
- The full parameter space is *vast*



- Include/exclude per pixel gives us $2^{(15^2)} = 2^{225}$ possibilities
  - A virtually uncountable number
  - Can only be leveraged computationally



- Our work: Software to design 3D linear nanophotonic devices using the full available parameter space



- Our work: Software to design 3D linear nanophotonic devices using the full available parameter space

- Many of these devices are
  - Completely novel (no previously known designs)
  - Extremely compact (footprints of a few vacuum wavelengths)
  - High efficiency ($> 80\%$ transmission)

- Developed by
  - applying (convex) optimization techniques (math)
  - to the area of nanophotonics (physics)
  - and implementing in software (programming)

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- Math Advisory:

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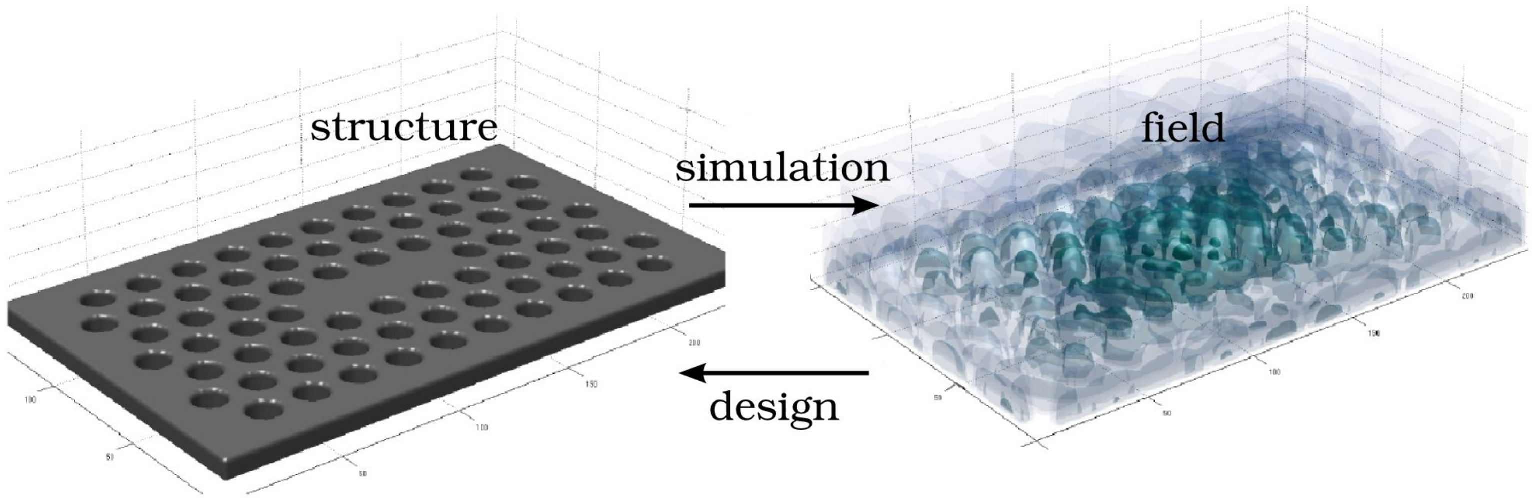
Given a field, can we find its structure?



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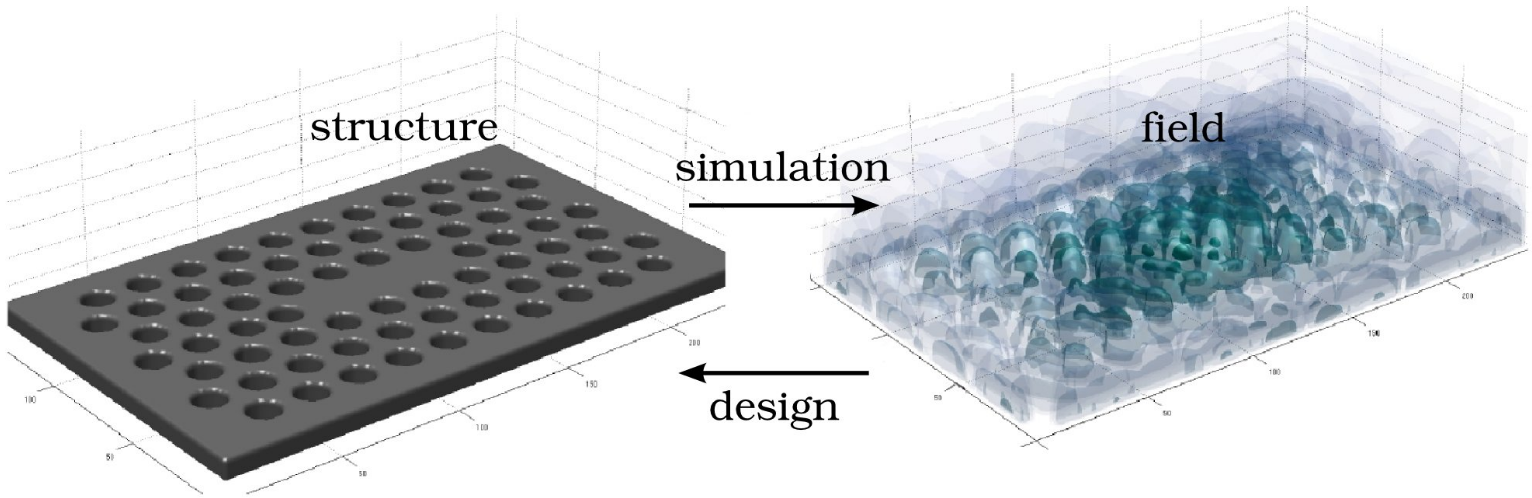

- Equivalently, find ϵ (structure) given E (field)

$$\nabla \times \mu_0^{-1} \nabla \times E - \omega^2 \epsilon E = -i\omega J$$

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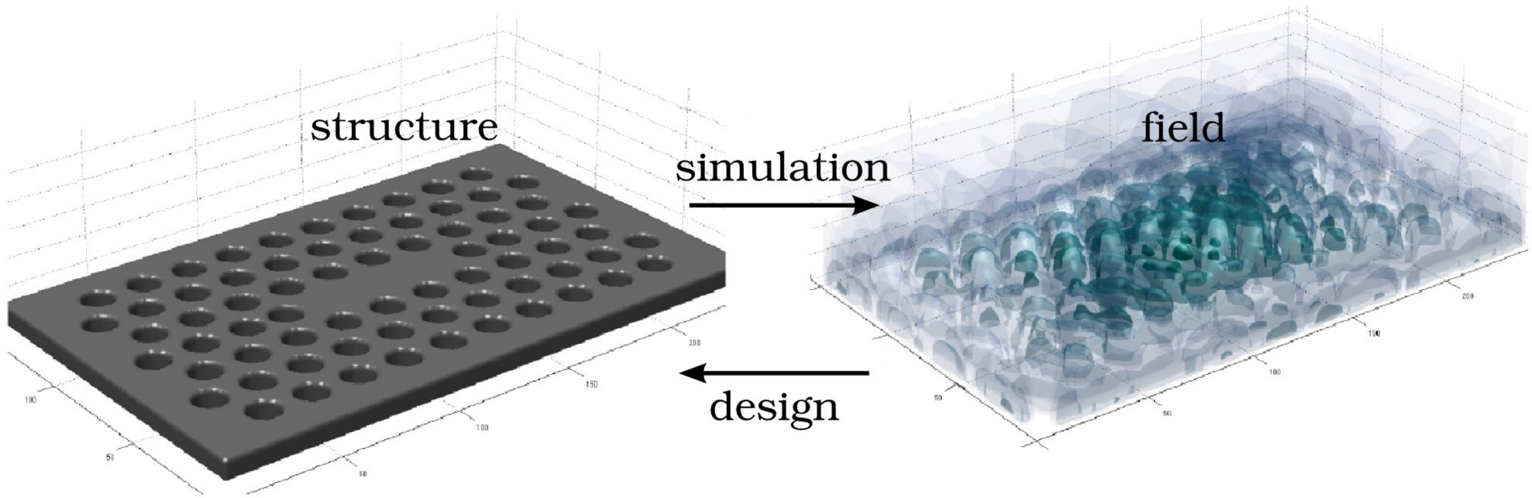

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- If possible, we can design *any* nanophotonic/optical component!

- Answer: Yes, given E we *can* solve for ϵ (trivial!)

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\epsilon &= (\nabla \times \mu_0^{-1} \nabla \times E + i\omega J)/\omega^2 E
\end{aligned}
$$

- Solving for ϵ actually way faster than simulation (solving for E)!

- Obvious and well-known from a mathematical perspective
  - Pre-requisite (200-level) class in optimization curriculum
  - Not yet taught (I think) in optics/photonics at Stanford

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$$\begin{aligned} E &\rightarrow x \\ \epsilon &\rightarrow z \\ \nabla \times \mu_0^{-1} \nabla \times -\omega^2 \epsilon &\rightarrow A(z) \\ -i\omega J &\rightarrow b \end{aligned}$$

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- Key: If $A(z)$ is linear in z then $A(z)x = b$ is as well!

Direct design of nanophotonic devices

- Let's try it already!
  - Choose x (field)
  - Solve for z (structure) by minimizing the *physics residual*, $A(z)x - b$

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$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2$$

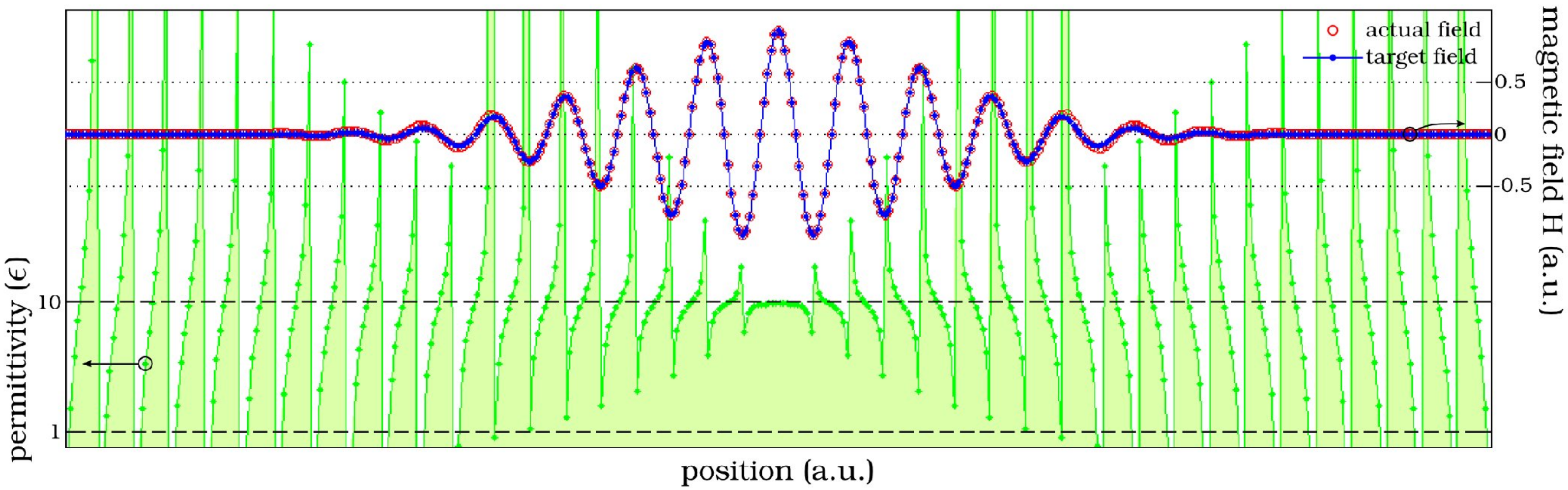
- Global minimum where $A(z)x - b = 0$ can be computed in one step

$$\epsilon = (\nabla \times \mu_0^{-1} \nabla \times E + i\omega J)/\omega^2 E$$

where $\epsilon \rightarrow z$

- Choose canonical 1D cavity field for x

- Solve for z (structure) and check design fidelity with simulation



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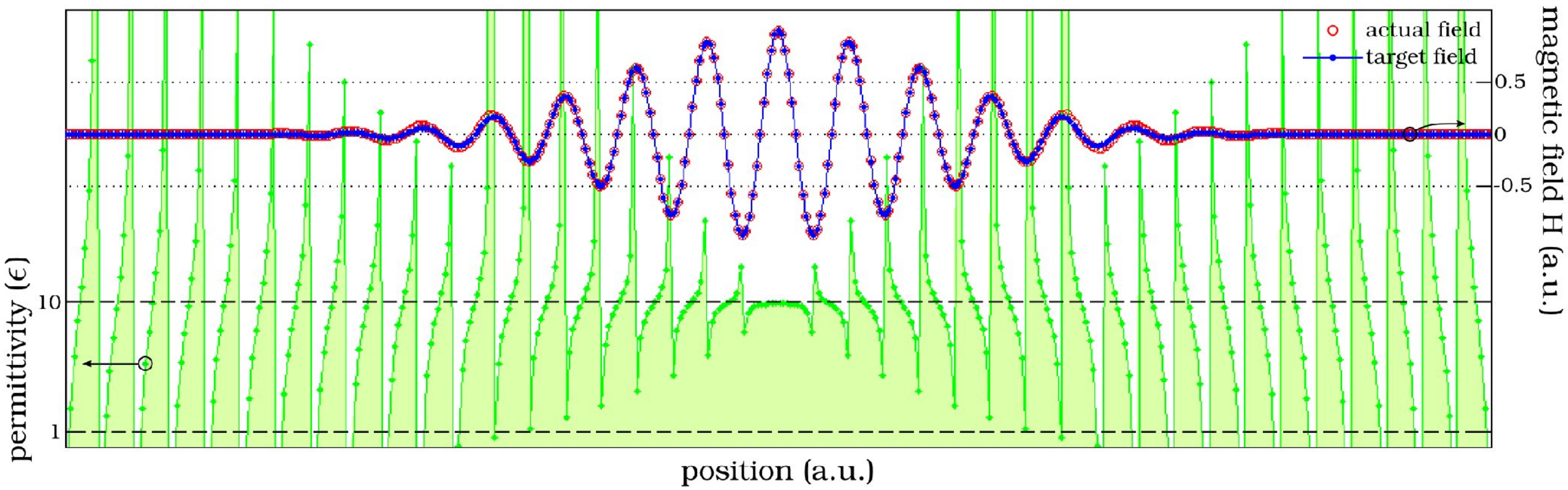


- Result
  - Perfect performance
  - But unmanufacturable structure (z not well-behaved)

Direct design with regularization

- Direct design "works" but not practical

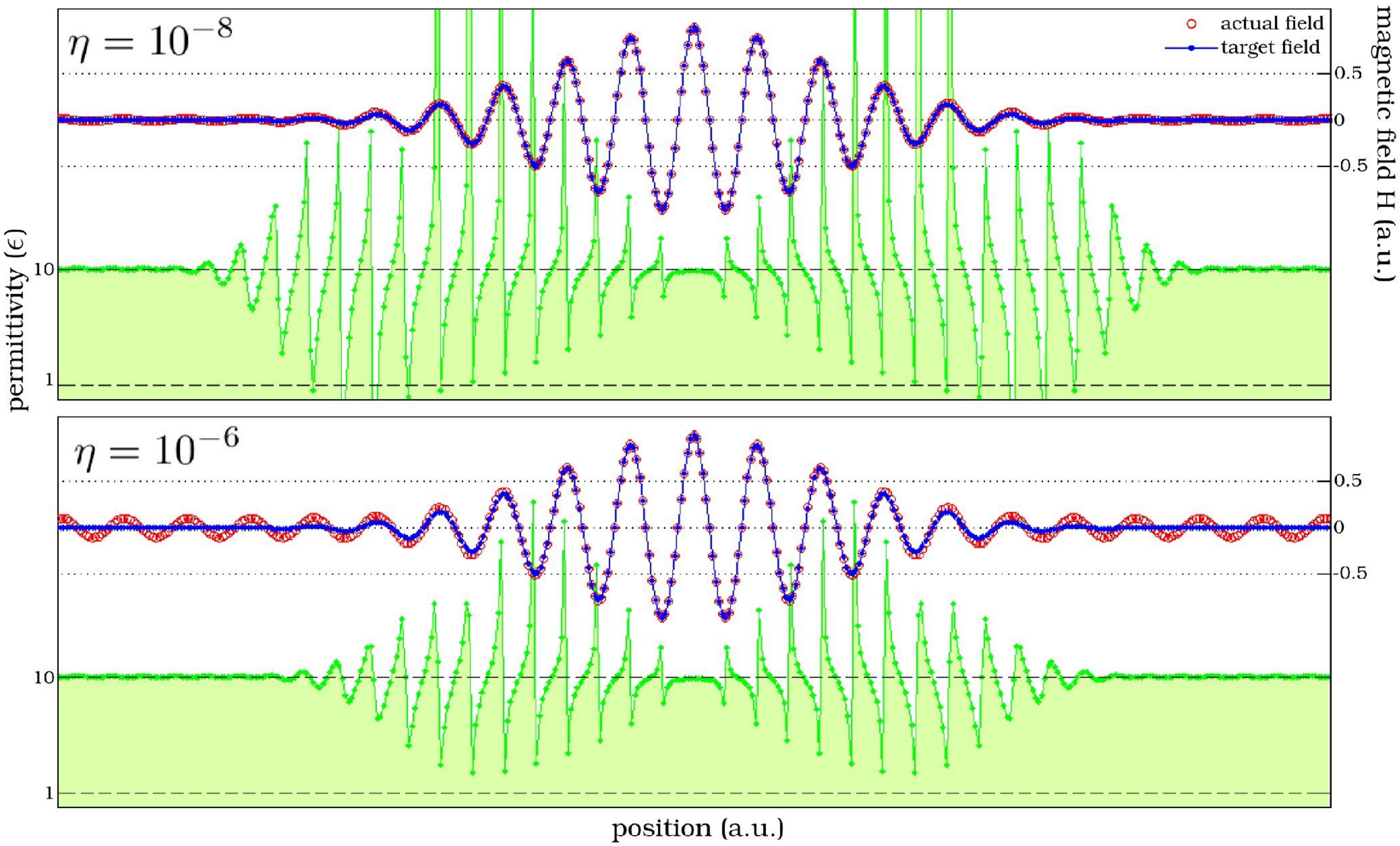
$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2$$

- So, let's add a *regularization* term to z and solve the following instead

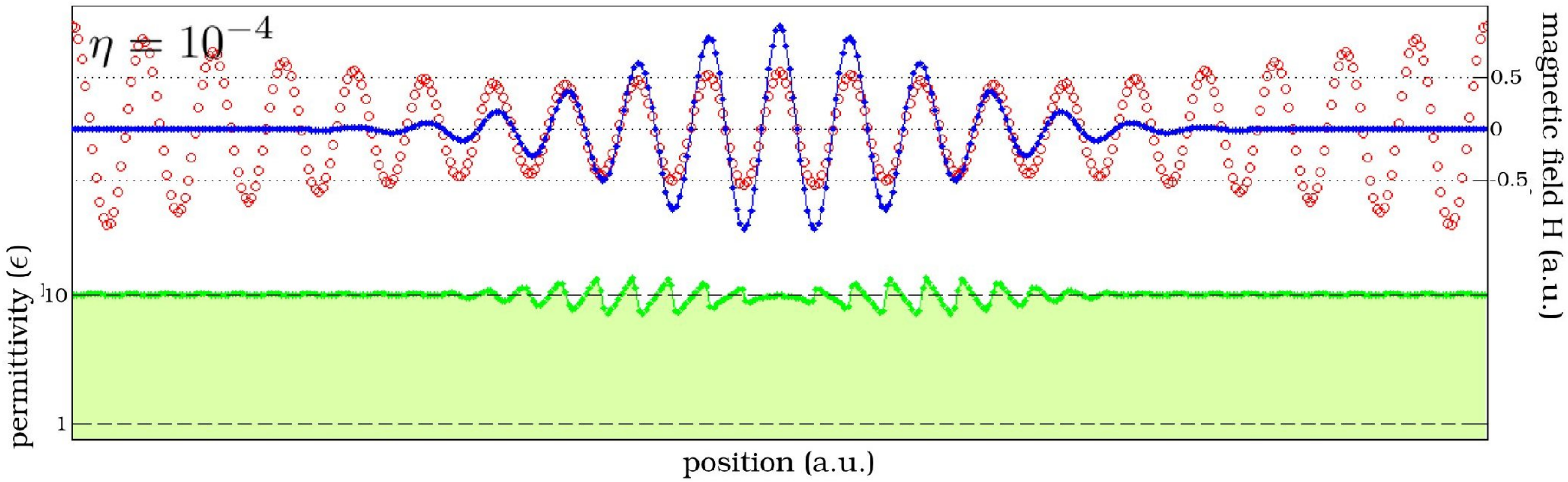
$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta\|z - z_0\|^2$$

  - $\|z - z_0\|^2$ term keeps z close to z_0
  - η controls the strength of the regularization

- Solution can still be computed in one step



- Unfortunately, regularization on z decreases performance



$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta \|z - z_0\|^2$$

- Decreased performance a result of non-zero physics residual at optimum

- How to overcome the apparent trade-off between manufacturability (z) and performance (x)?

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- Realized that there do exist some x (fields) which result from manufacturable z (structures)
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  - However, it's not useful to ask the user to choose such x

- Therefore, a *useful* tool would optimize for *both* x and z

Iterative design of nanophotonic devices

- New algorithm: Iteratively solve for x (field) and z (structure)

$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta_0 \|z - z_{\text{prev}}\|^2$$

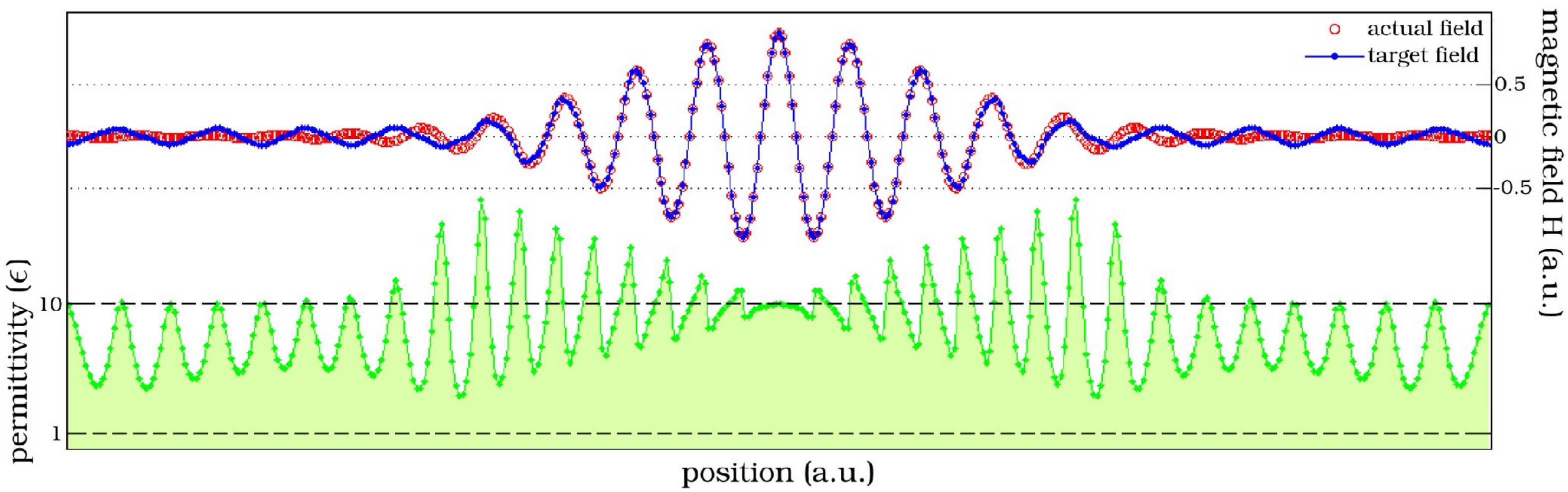
$$\underset{x}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta_1 \|x - x_{\text{prev}}\|^2$$

- Takes advantage of the *bi-linearity* of the physics residual
  - Jointly solving for x and z is a non-convex problem

- More concisely, we iteratively solve the following

$$\underset{\text{alternately } x \text{ then } z}{\text{minimize}} \ \|A(z)x - b\|^2 + \eta_0 \|z - z_{\text{prev}}\|^2 + \eta_1 \|x - x_{\text{prev}}\|^2$$

- Design process now consists of *multiple* computational steps
  - η_0, η_1 gradually decreased to bring physics residual toward 0



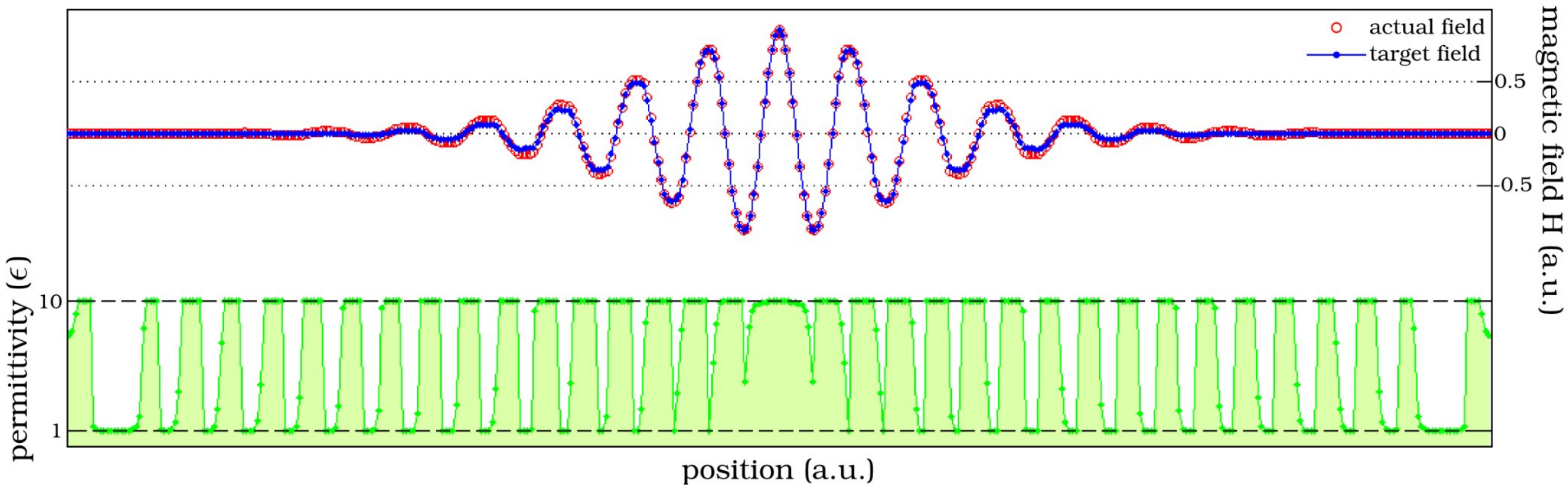
- Iterative strategy produces z (structure) that
  - is better behaved
  - more accurately produces x

Iterative design with hard constraints on z

- We can also put hard limits on z (structure)

$$\begin{array}{rl} \text{minimize} & \|A(z)x - b\|^2 + \eta_1 \|x - x_{\text{prev}}\|^2 \\ \text{subject to} & z_{\text{min}} \le z \le z_{\text{max}} \end{array}$$

- $z_{\text{min}} \le z \le z_{\text{max}}$ constraint better represents manufacturability constraint
  - Corresponds to a minimum and maximum allowable permittivity (ϵ)

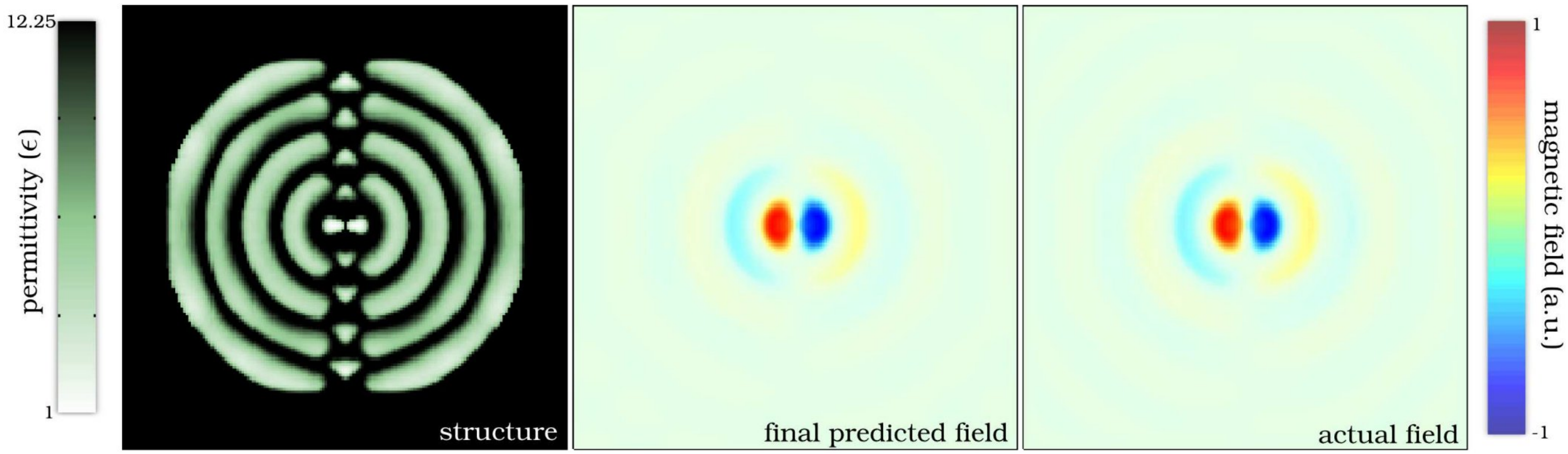


- Well-behaved, manufacturable z (structure)

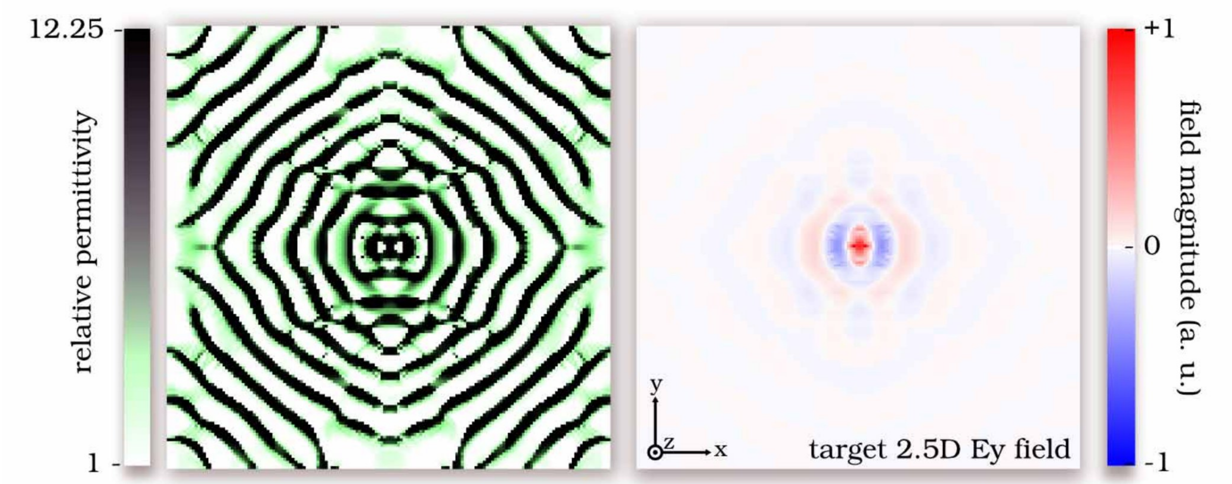
- Final x (field) accurately reproduced

- Majority of elements of z are fortuitously at one limit or the other!

- Can be used to create 2D resonators



- 3D resonators can be designed using a "2.5D" approximation



Objective-first design of linear devices

- Next realization: for linear components, only certain elements of x (field) matter
  - Specifically, the elements of x at the input/output ports

$$\begin{aligned} \text{minimize} \quad & \|A(z)x - b\|^2 \\ \text{subject to} \quad & x_{\text{boundary}} - \hat{x}_{\text{boundary}} = 0 \\ & z_{\text{min}} \leq z \leq z_{\text{max}} \end{aligned}$$

- Instead of regularization term, we force the elements of x at the boundary to be equal to the ideal case ($\hat{x}$)

- Called *objective-first* design because the design objective is prioritized above the physics residual

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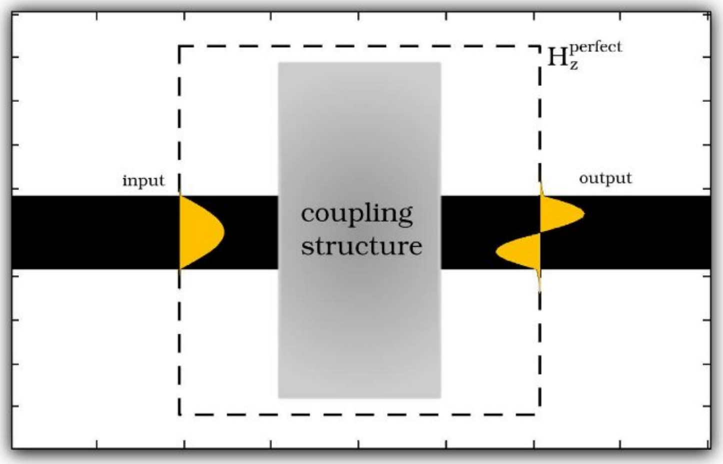
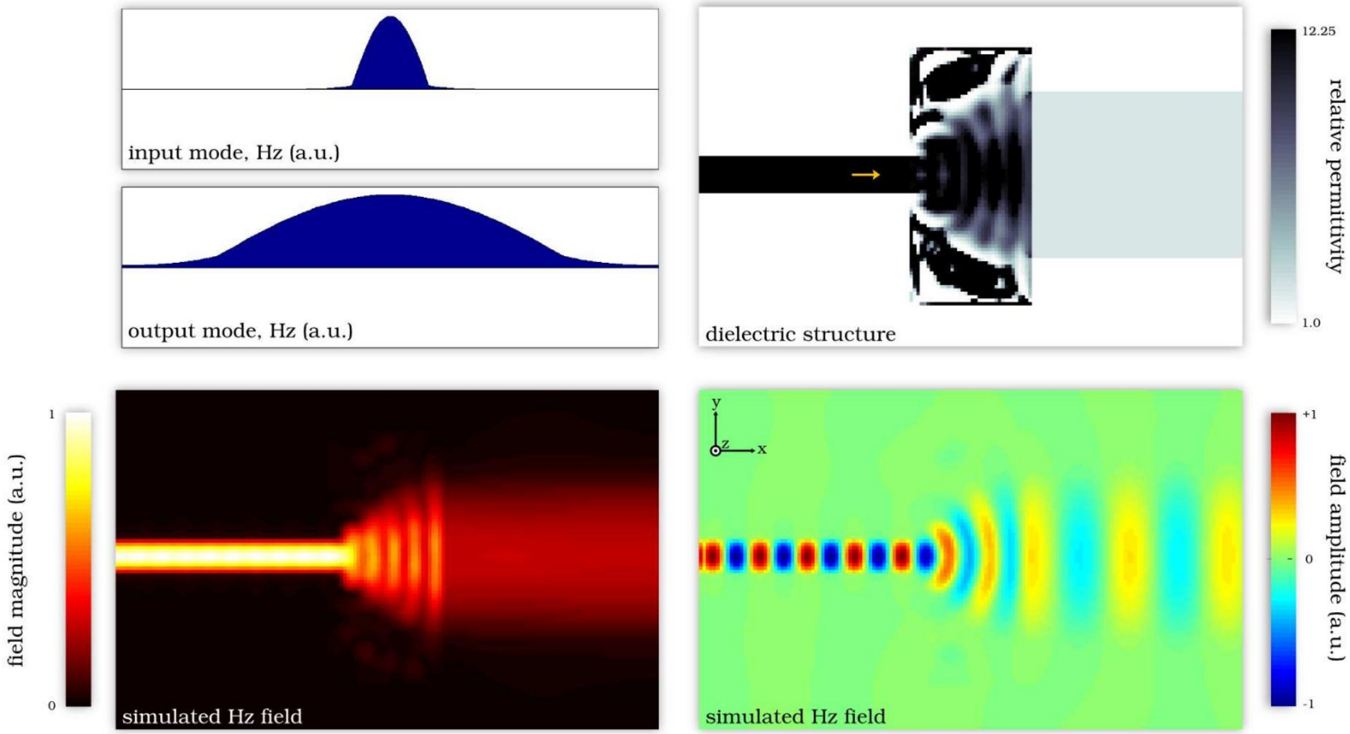
- Applied to the design of 2D waveguide mode couplers

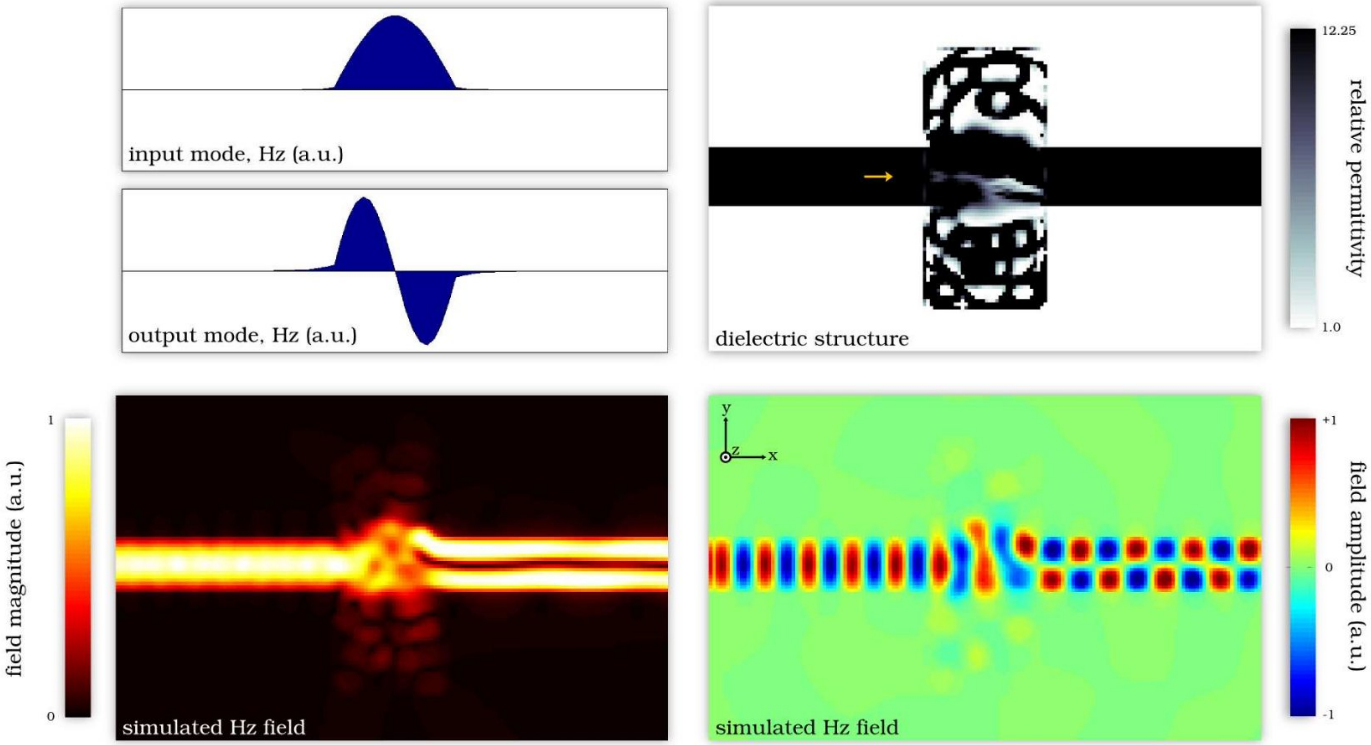


- Produced designs which exhibited
  - High efficiency ($\sim 98\%$)
  - Small device footprints ($1.5 - 4$ square vacuum wavelengths)

- Coupler to wide, low-index waveguide



- Coupler from fundamental to second-order waveguide mode



Design of 3D linear nanophotonic devices

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- Finally, enough understanding to tackle the *real* design problem
- Goal: Software to design *all* linear nanophotonic devices
  - Fully three-dimensional (no approximations)
  - Multi-mode
  - Discrete, planar, manufacturable structure

- Problem: Did not know how to solve $A(z)x - b = 0$ (simulation) in 3D
  - Millions of variables
  - Famously ill-conditioned
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- Fortunately, Wonseok had already solved this problem

Maxwell: Light-simulation supercomputer

- Partnered with Wonseok to develop a cloud-based electromagnetic solver using Amazon Web Services
  - GPU-accelerated implementation of Wonseok's algorithm
  - Cluster scales automatically to tens of nodes

> # Maxwell:
> **Your light-simulation supercomputer.**
>
> Maxwell supercharges your Matlab installation with the power of Amazon's Elastic Compute Cloud to enable it to solve full 3D electromagnetic simulations.

- Scalable
  - Far outstrips computing clusters such as Teragrid
  - Can perform multiple solves in parallel, on a single Matlab instance
  - All computation is performed externally (in the cloud)
- Easy to use
  - Installs with a single Matlab command
  - Solves completed with a single Matlab command: `maxwell(...);`
- The key technological enabler in achieving 3D design

3D design: problem statement

$$
\begin{aligned}
\text{minimize} \quad & \sum_i^M \|A_i(z)x_i - b_i\|^2 \\
\text{subject to} \quad & \alpha_{ij} \le |c_{ij}^\dagger x_i| \le \beta_{ij}, \quad \text{for } i = 1, \ldots, M \text{ and } j = 1, \ldots, N_i \\
& z_{\text{min}} \le z \le z_{\text{max}}
\end{aligned}
$$

- M modes with N_i monitored output modes each
  - b_i is the input excitation for each mode
  - $|c_{ij}^\dagger x_i|$ is the field overlap with output mode c_{ij}
  - α_{ij} and β_{ij} is the design range for the overlap