Nanophotonic Computational Design

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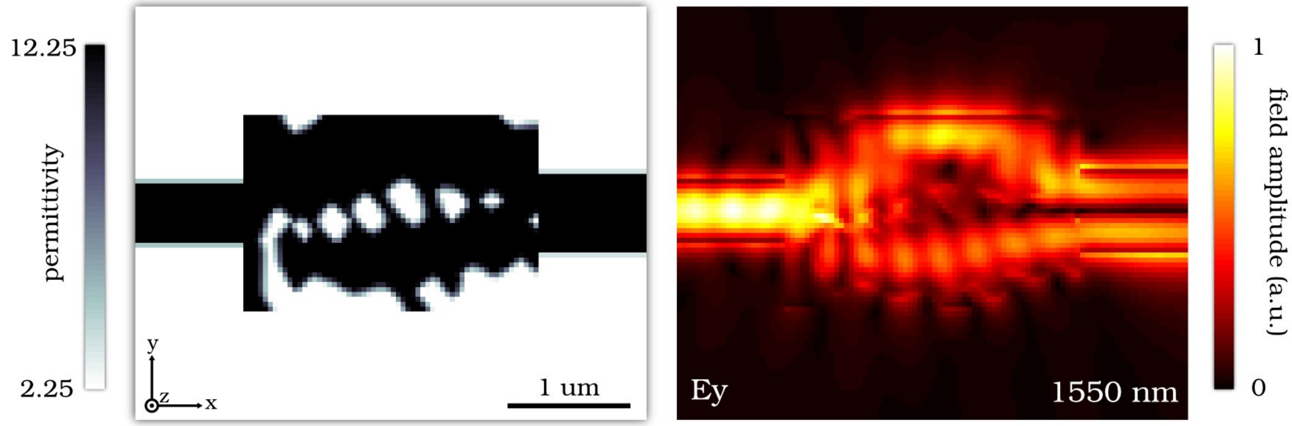
February 25, 2013

Goal: Show you how to design *any* linear nanophotonic device



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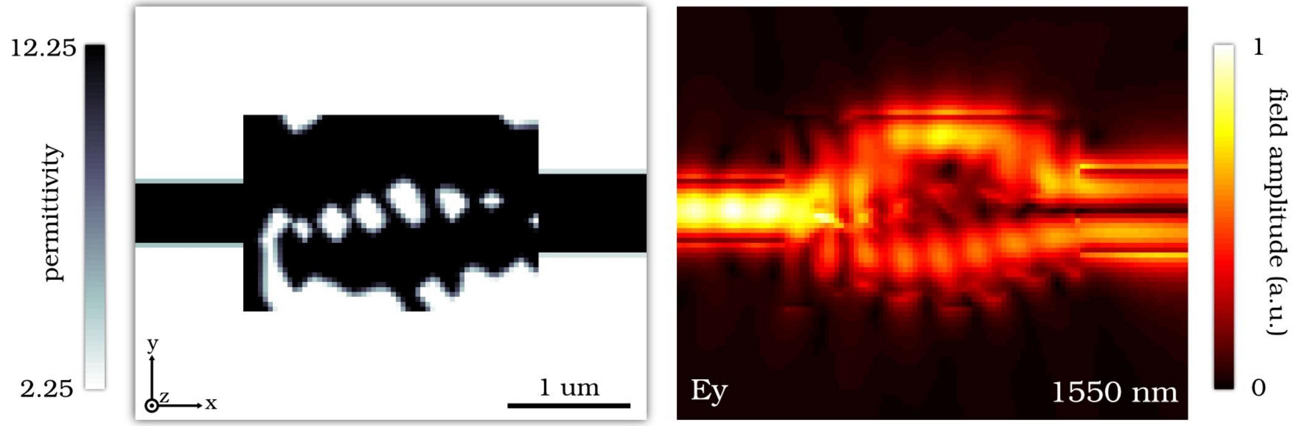

- Device properties:
  - Full 3D
  - Compact
  - Efficient
  - Multi-mode
  - Multi-functional

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  - to the area of nanophotonics (physics)
  - and implementing in software (programming)

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- Math Advisory:

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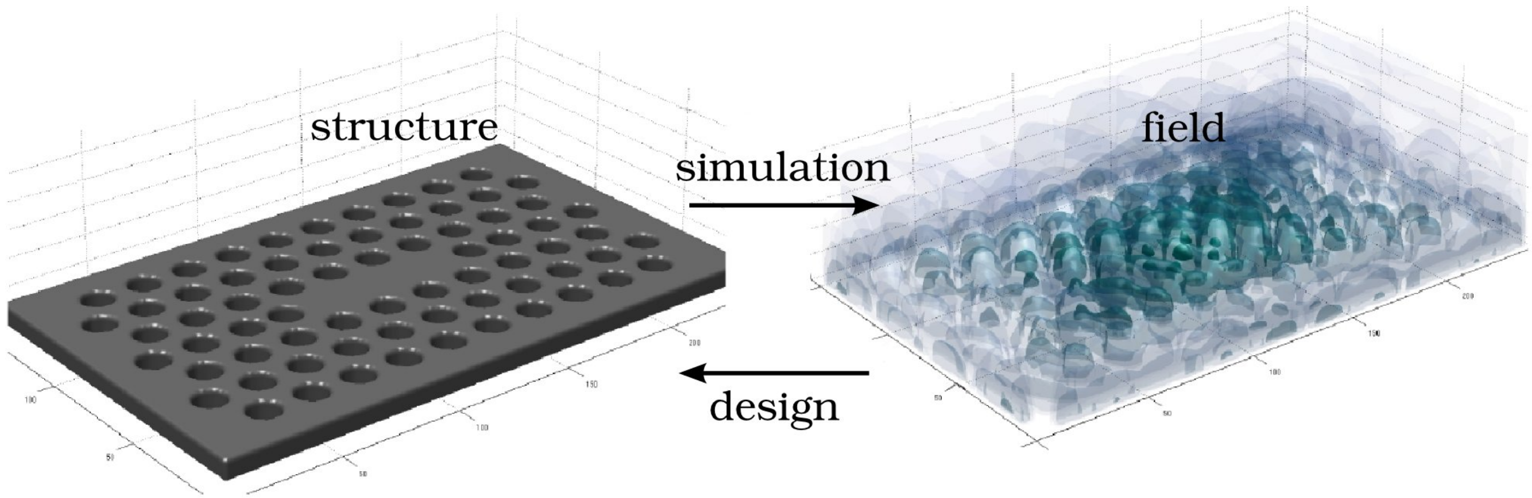
Given a field, can we find its structure?



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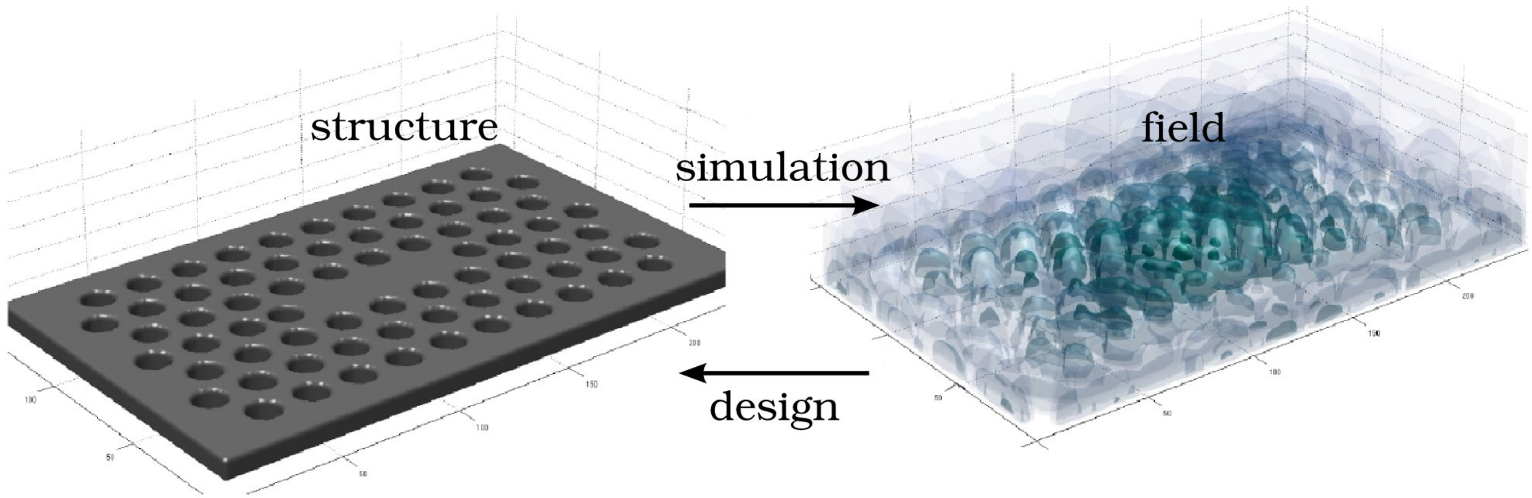

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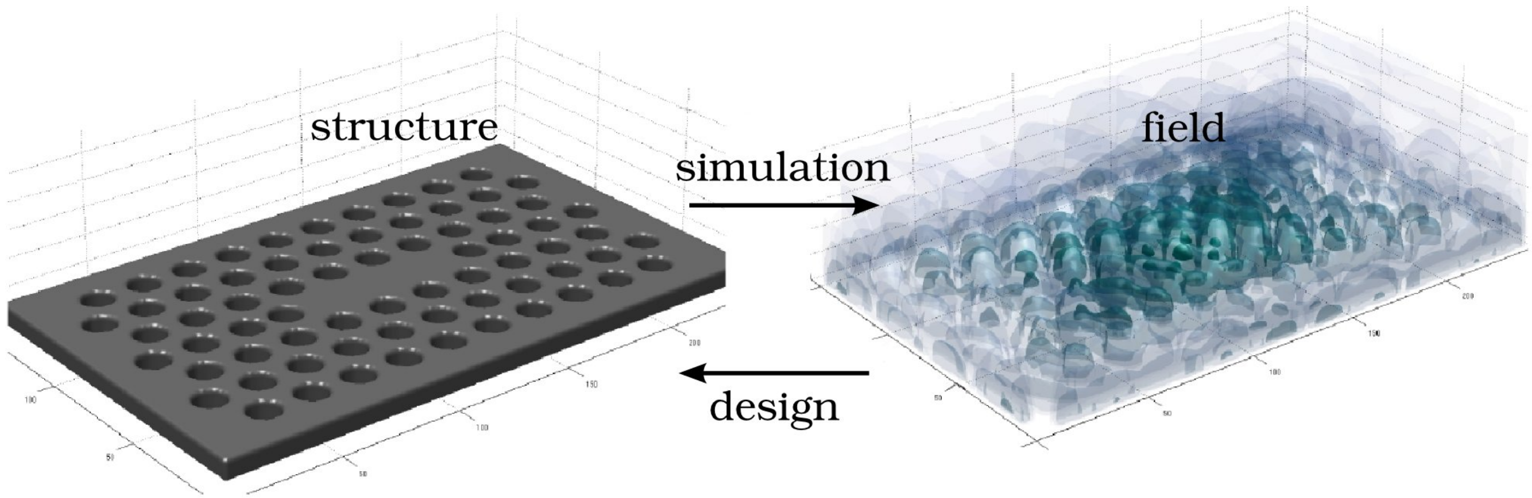

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- If possible, we can design *any* nanophotonic/optical component!

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- Solving for ϵ actually way faster than simulation (solving for E)!

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  - Pre-requisite (200-level) class in optimization curriculum
  - Not yet taught (I think) in optics/photonics at Stanford

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- Key: If $A(z)$ is linear in z then $A(z)x = b$ is as well!