Nanophotonic Computational Design

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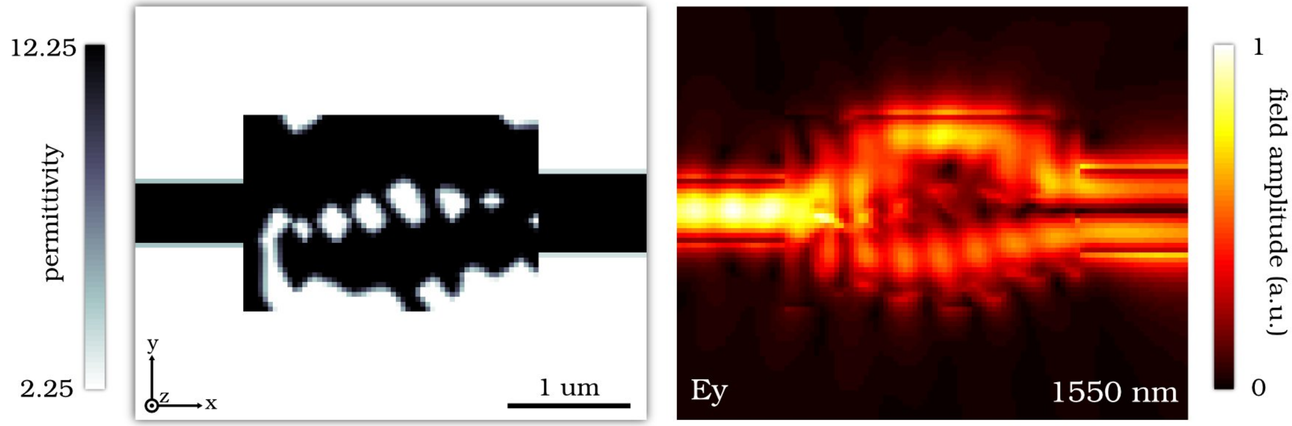
Goal: Show you how to design *any* linear nanophotonic device



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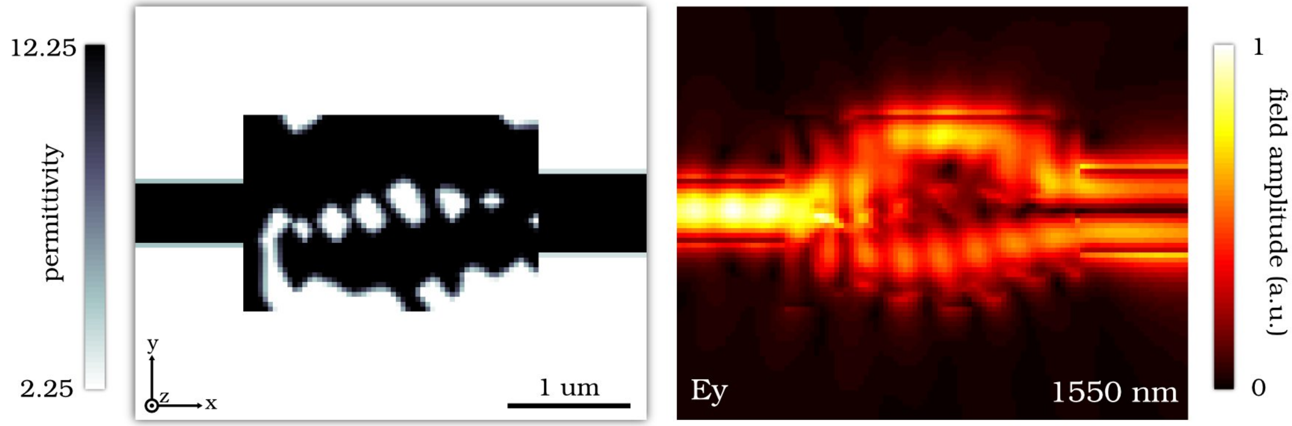

- Device properties:
  - Full 3D
  - Compact
  - Efficient
  - Multi-mode
  - Multi-functional

- Developed by
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  - to the area of nanophotonics (physics)
  - and implementing in software (programming)

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- Math Advisory:

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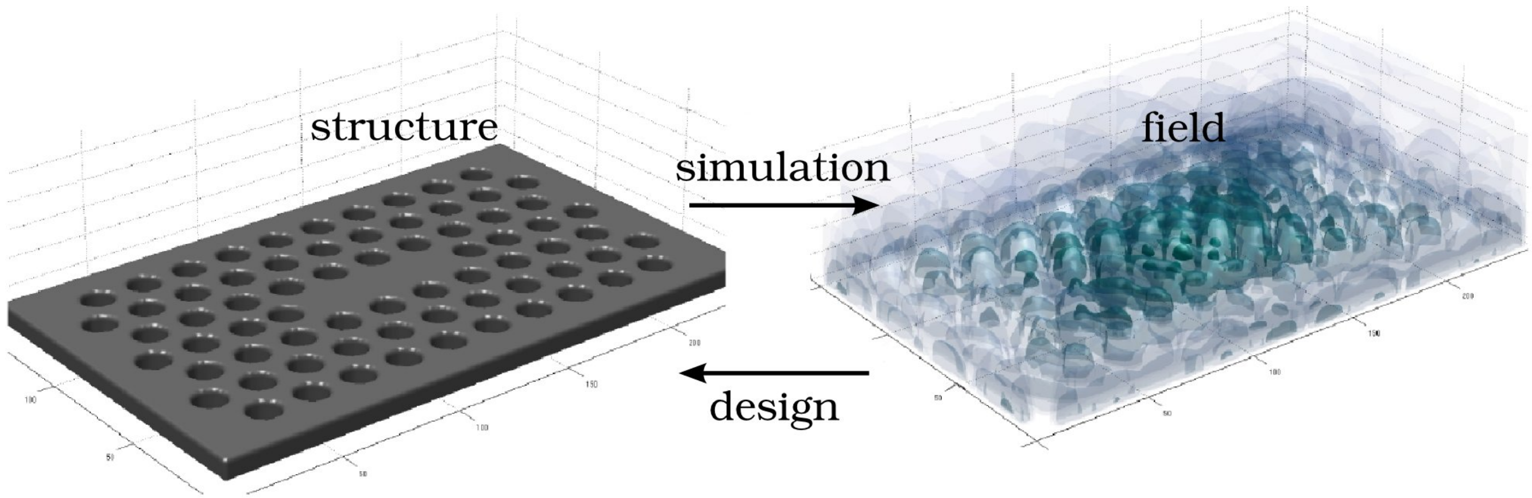
Given a field, can we find its structure?



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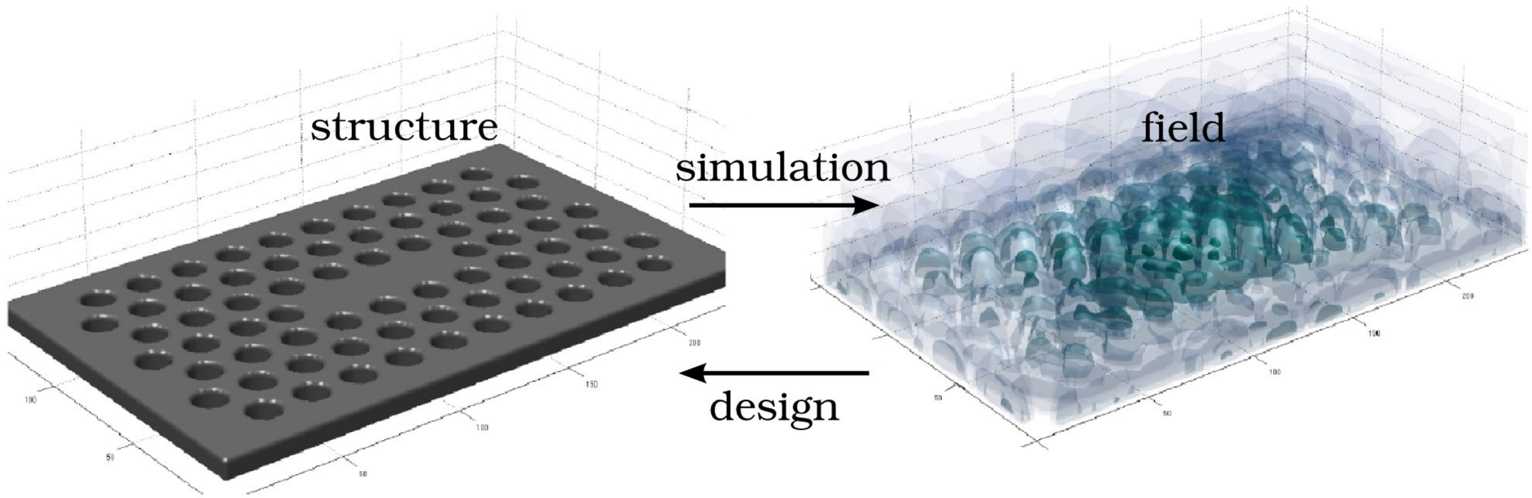

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$$\nabla \times \mu_0^{-1} \nabla \times E - \omega^2 \epsilon E = -i\omega J$$

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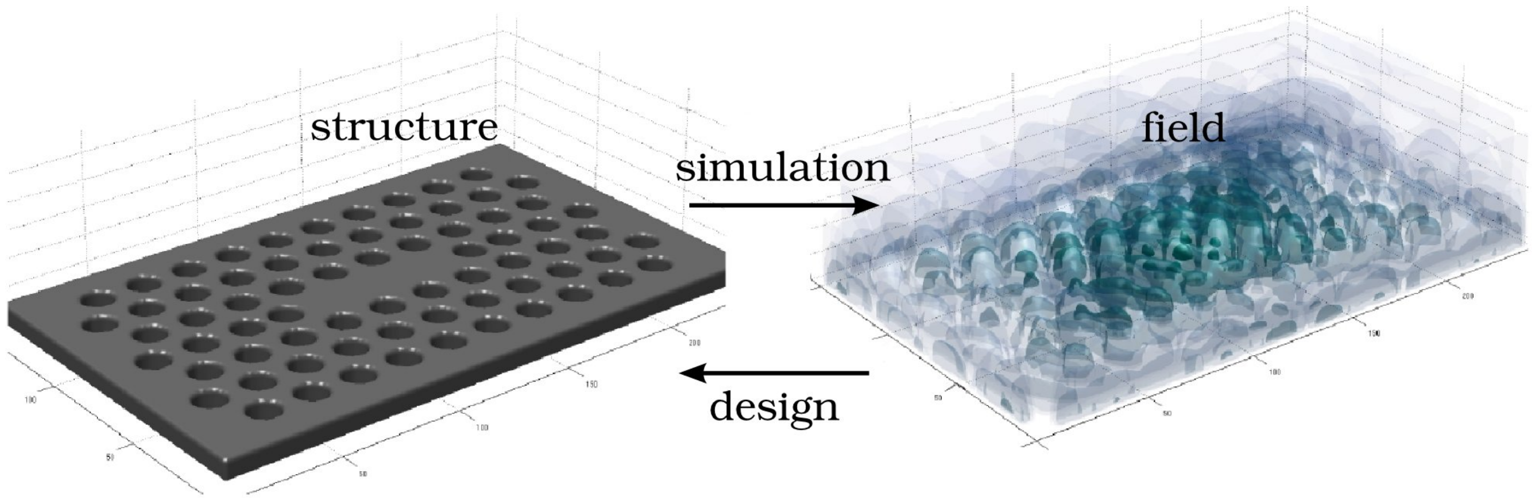

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- If possible, we can design *any* nanophotonic/optical component!

- Answer: Yes, given E we *can* solve for ϵ (trivial!)

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\omega^2 E \epsilon &= \nabla \times \mu_0^{-1} \nabla \times E + i\omega J \\
\epsilon &= (\nabla \times \mu_0^{-1} \nabla \times E + i\omega J) / \omega^2 E
\end{aligned}$$

- Solving for ϵ actually way faster than simulation (solving for E)!

- Obvious and well-known from a mathematical perspective
  - Pre-requisite (200-level) class in optimization curriculum
  - Not yet taught (I think) in optics/photonics at Stanford

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$$E \to x$$

$$\epsilon \to z$$

$$\nabla \times \mu_0^{-1} \nabla \times -\omega^2 \epsilon \to A(z)$$

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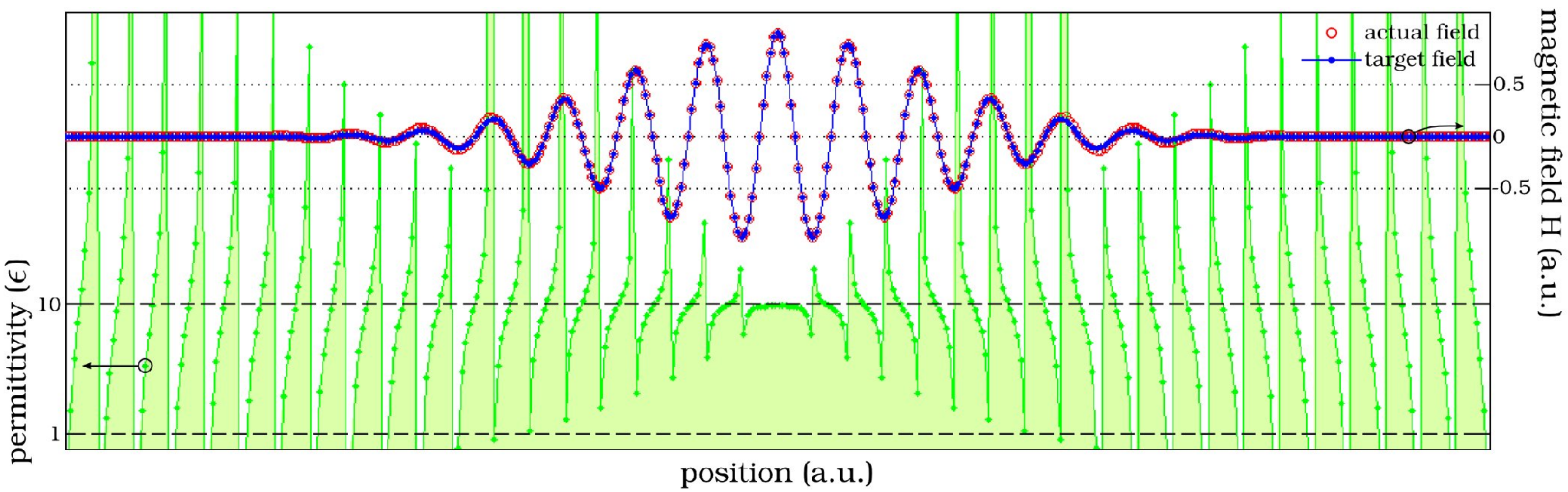
- Key: If $A(z)$ is linear in z then $A(z)x = b$ is as well!

Direct design of structure in 1D

- Chose x (field) and solved for z (structure)



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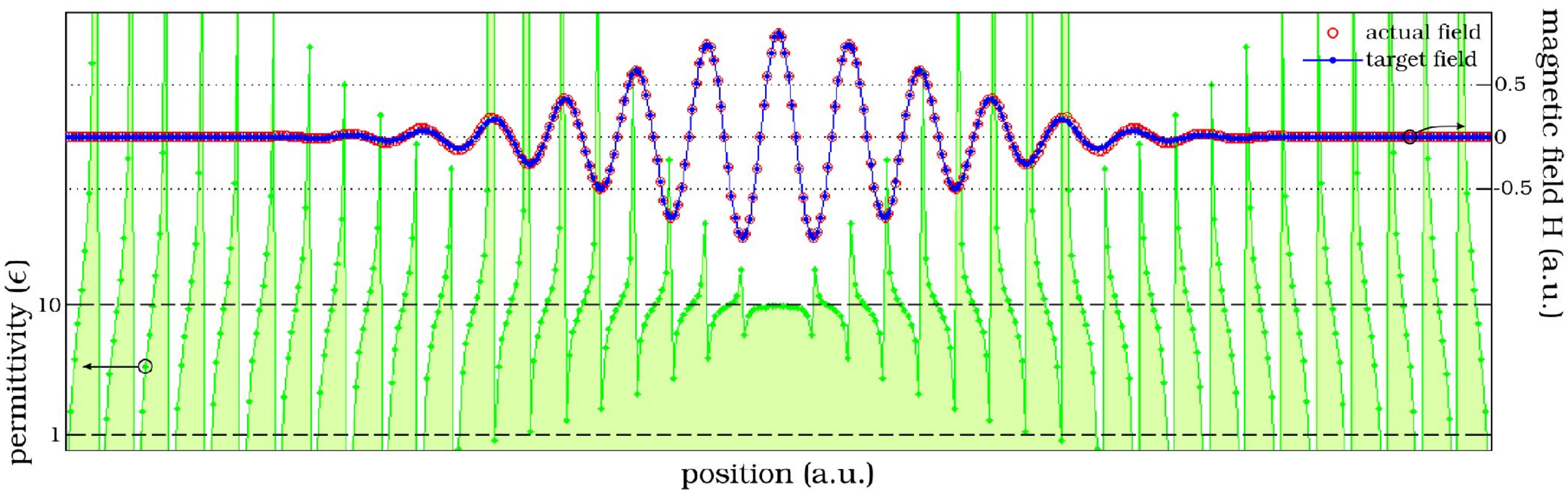
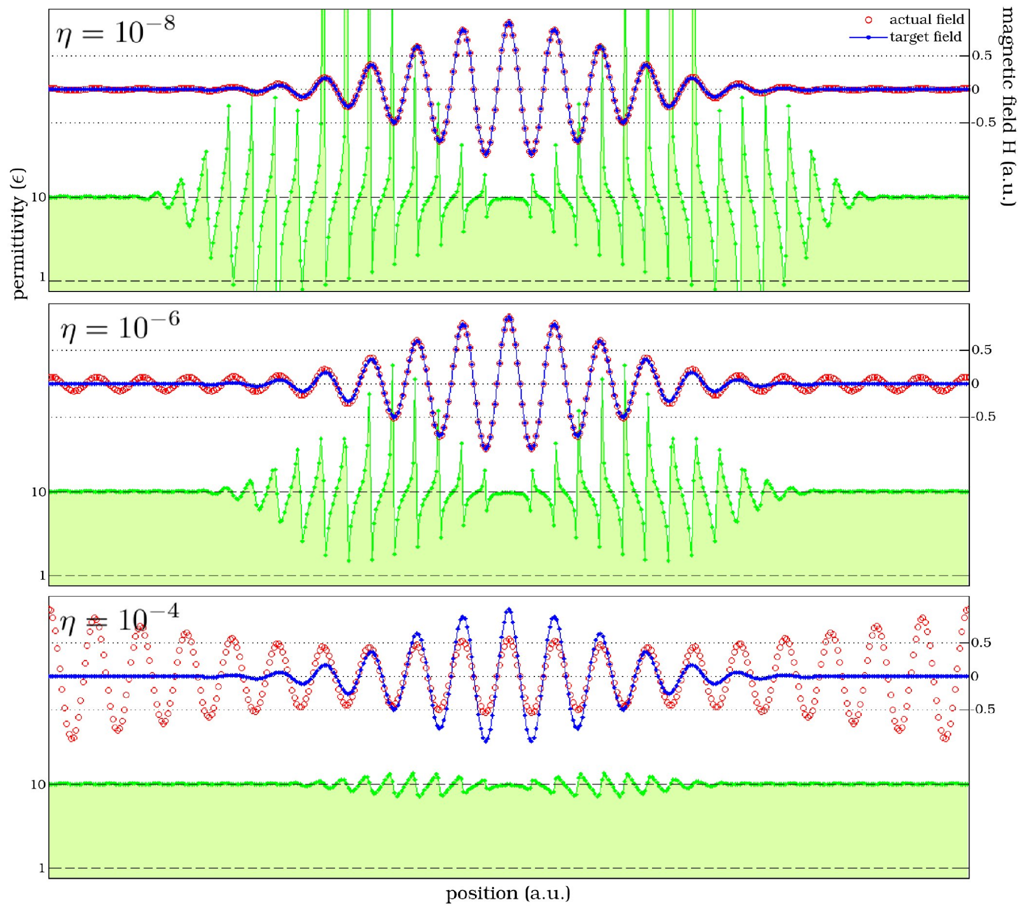


- Perfect performance but unmanufacturable structure



- Regularization on z decreases performance

- We know that there exist x which produce well-behaved z
- However, choosing such x is next to impossible
- Therefore, a practical design tool must include x-search

Iterative design of structure in 1D

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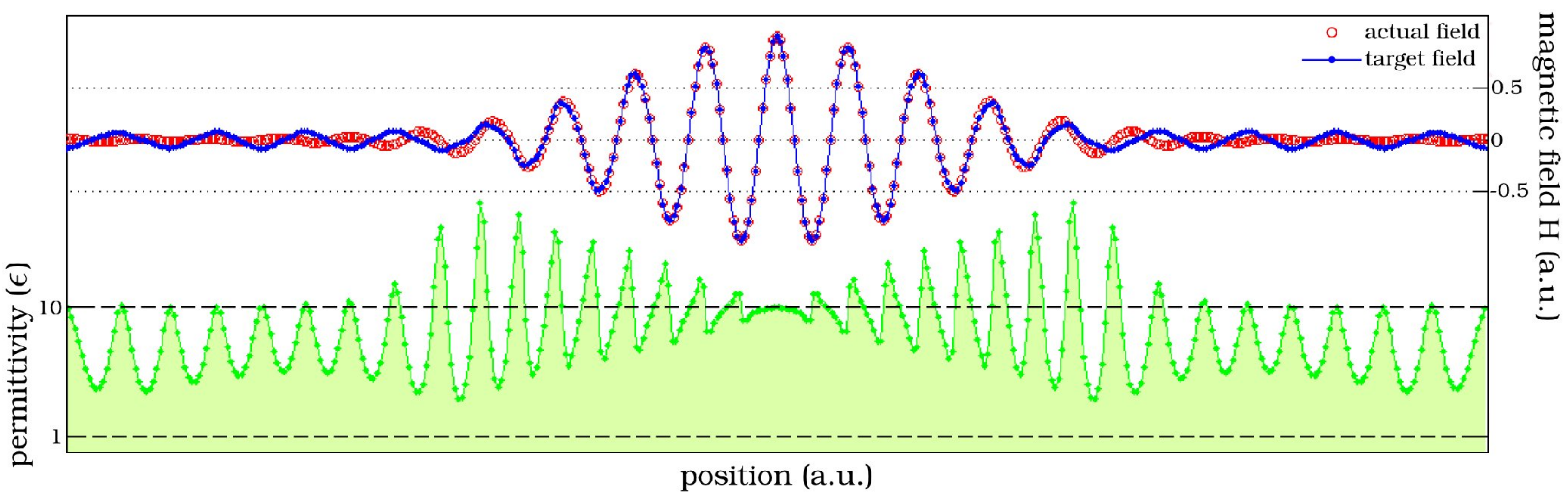
- Insight: Solving for well-behaved z requires fortuitous selection of x

- Therefore, vary x in order to allow for well-behaved z



- Variation in x allows z to be better behaved

- Alternately solve for x and z:

$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta_1 \|z - z_0\|^2$$

$$\underset{x}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta_0 \|x - x_0\|^2$$