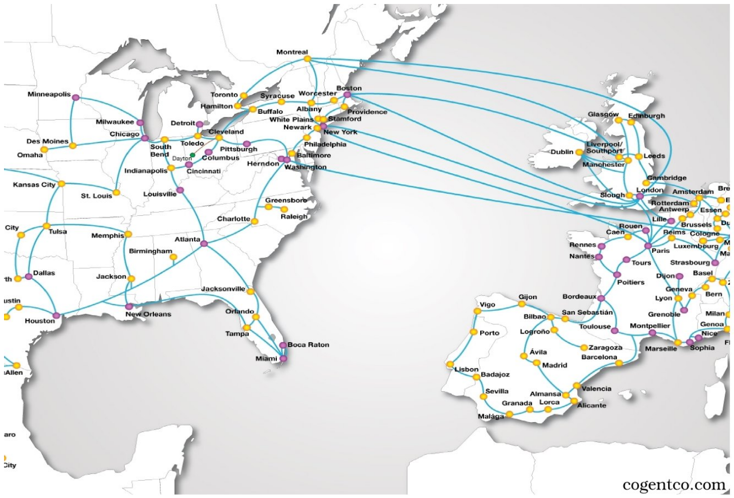
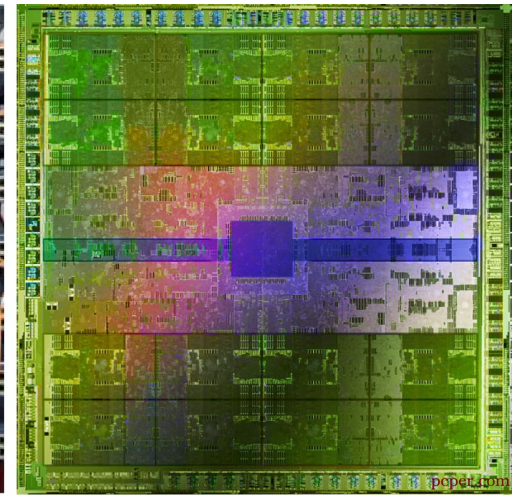
# Nanophotonic Computational Design

Jesse Lu

February 25, 2013

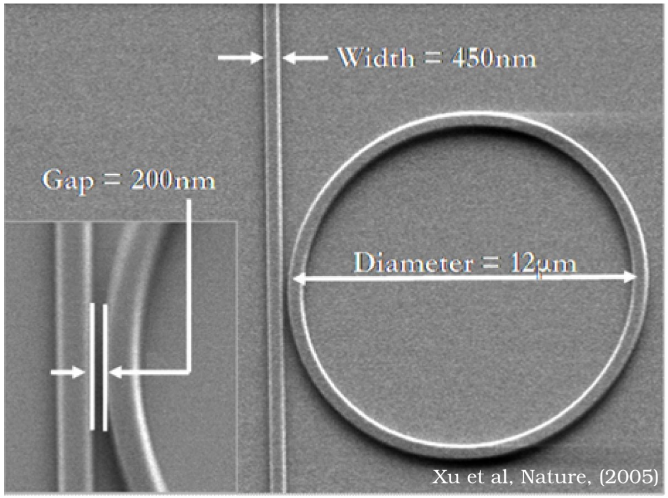
# Introduction

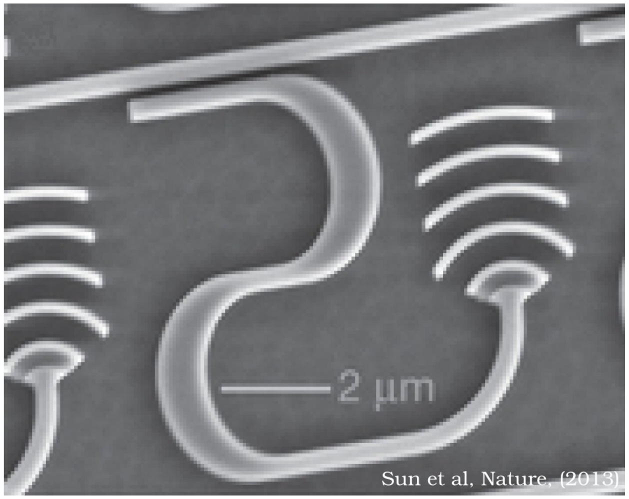




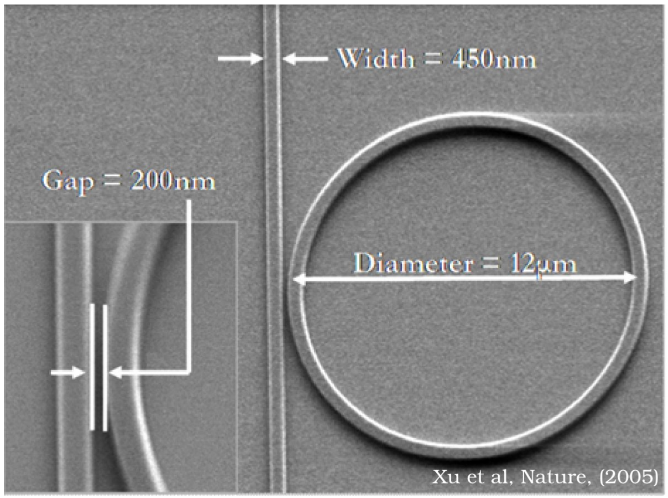

- As information grows, optical networks needed
  - across continents
  - within a datacenter
  - between chips and on-chip

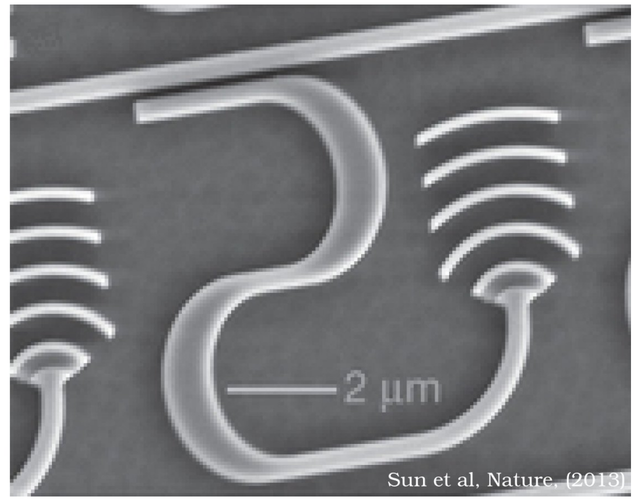
- On-chip optical components are currently designed by tuning a small number of design parameters



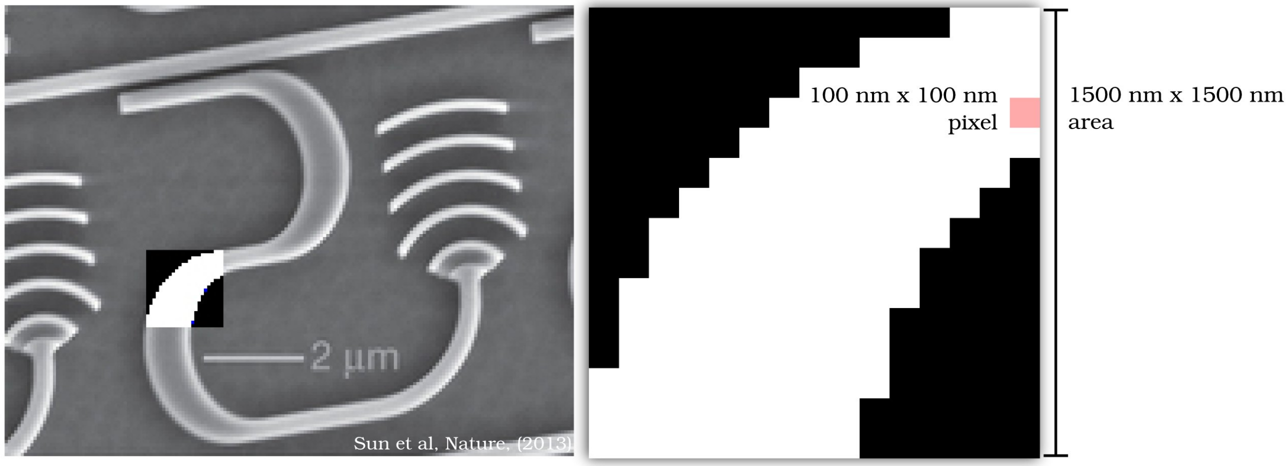
- On-chip optical components are currently designed by tuning a small number of design parameters



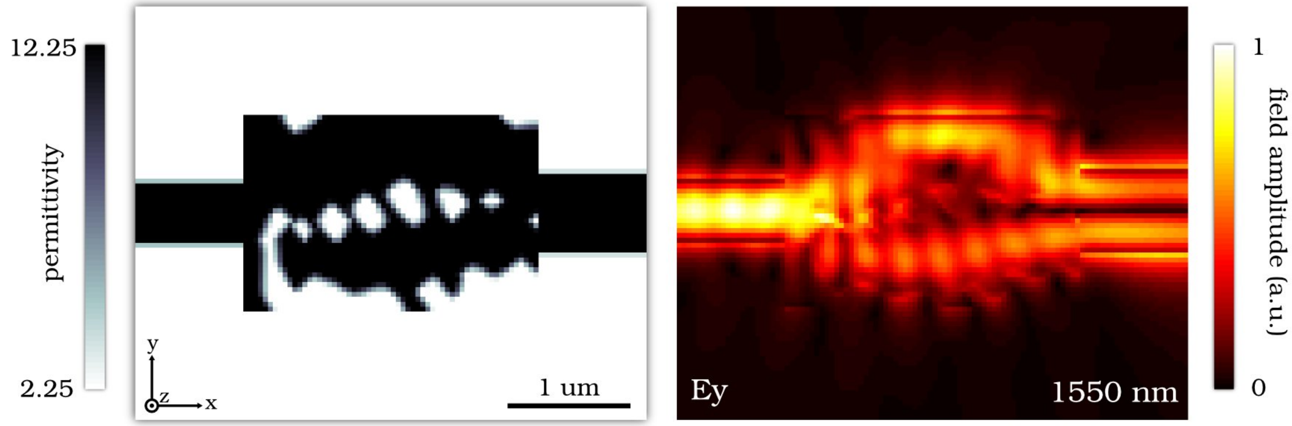




- What happens when we use the *full* parameter space for nanophotonic design?

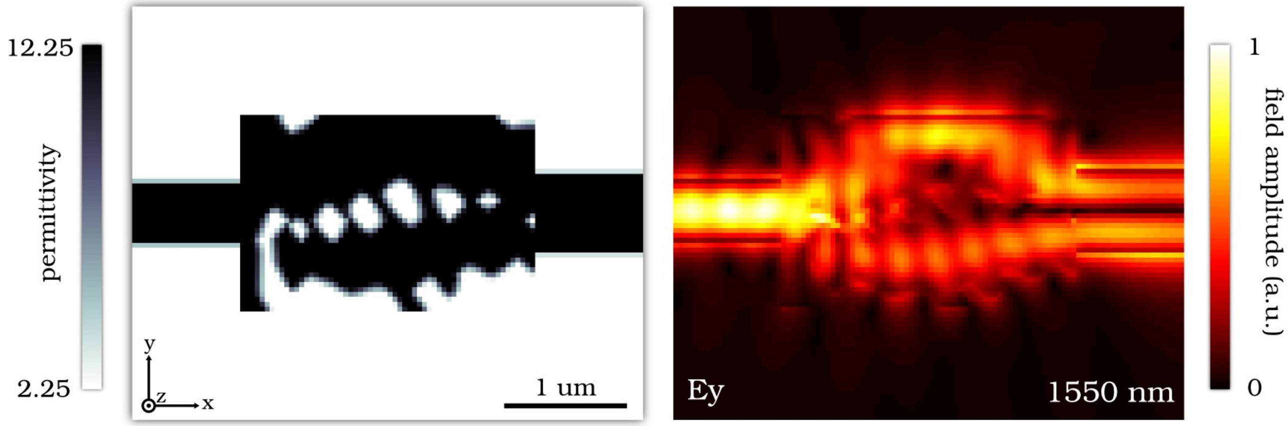
- The full parameter space is *vast*




- Include/exclude per pixel gives us $2^{(15^2)} = 2^{225}$ possibilities
  - A virtually uncountable number
  - Can only be design by a computer




- Our work: Software to design full 3D, multi-mode, and multi-functional linear nanophotonic devices—using the fully available parameter space




- Our work: Software to design full 3D, multi-mode, and multi-functional linear nanophotonic devices—using the fully available parameter space

- Many of these devices are
  - Completely novel (no previously known designs)
  - Extremely compact (footprints of a few vacuum wavelengths)
  - High efficiency ($> 80\%$ transmission)

- Developed by
  - applying (convex) optimization techniques (math)
  - to the area of nanophotonics (physics)
  - and implementing in software (programming)

- Developed by
  - applying (convex) optimization techniques (math)
  - to the area of nanophotonics (physics)
  - and implementing in software (programming)

- Physics Advisory:

  CONTAINS INVOLVED MATHEMATICAL CONTENT

- Developed by
  - applying (convex) optimization techniques (math)
  - to the area of nanophotonics (physics)
  - and implementing in software (programming)

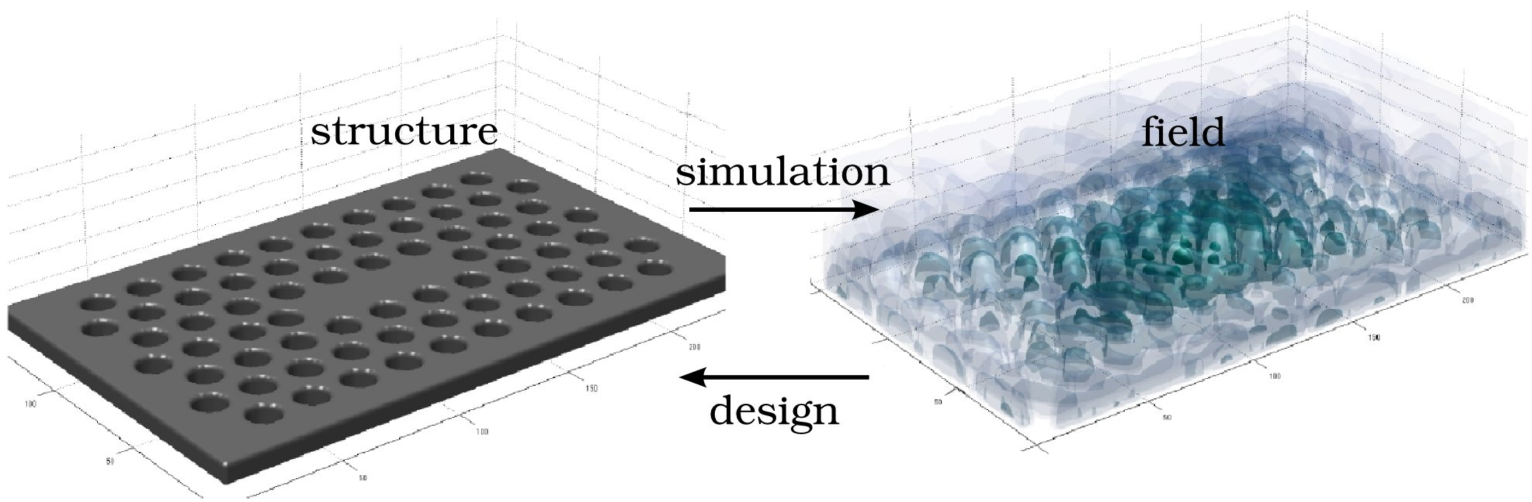
- Physics Advisory:

  CONTAINS INVOLVED MATHEMATICAL CONTENT

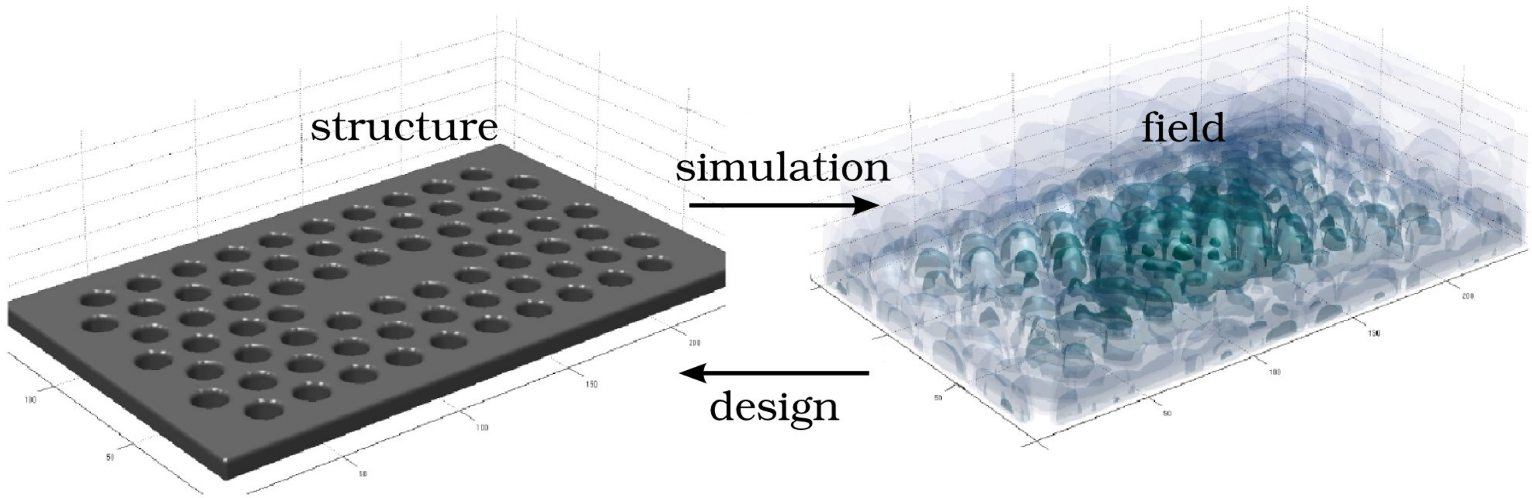
- Math Advisory:

  CONTAINS INVOLVED NANOPHOTONIC CONTENT

# Given a field, can we find its structure?



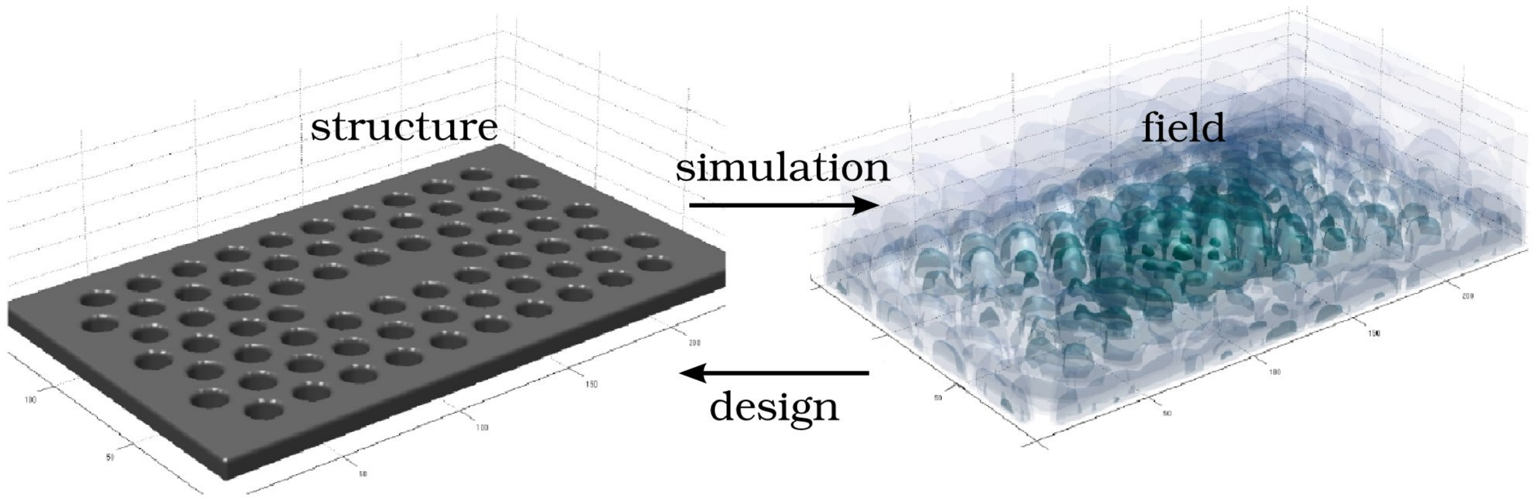
# Given a field, can we find its structure?




- Equivalently, find $\epsilon$ (structure) given $E$ (field)

$$\nabla \times \mu_0^{-1} \nabla \times E - \omega^2 \epsilon E = -i\omega J$$

## Given a field, can we find its structure?




- Equivalently, find $\epsilon$ (structure) given $E$ (field)

$$\nabla \times \mu_0^{-1} \nabla \times E - \omega^2 \epsilon E = -i\omega J$$

- If possible, we can design *any* nanophotonic/optical component!

- Answer: Yes, given $E$ we *can* solve for $\epsilon$ (trivial!)

$$\nabla \times \mu_0^{-1} \nabla \times E - \omega^2 \epsilon E = -i\omega J$$

- Answer: Yes, given $E$ we *can* solve for $\epsilon$ (trivial!)

$$\nabla \times \mu_0^{-1} \nabla \times E - \omega^2 \epsilon E = -i\omega J$$
$$\omega^2 \epsilon E = \nabla \times \mu_0^{-1} \nabla \times E + i\omega J$$

- Answer: Yes, given $E$ we *can* solve for $\epsilon$ (trivial!)

$$\nabla \times \mu_0^{-1} \nabla \times E - \omega^2 \epsilon E = -i\omega J$$

$$\omega^2 \epsilon E = \nabla \times \mu_0^{-1} \nabla \times E + i\omega J$$

$$\omega^2 E \epsilon = \nabla \times \mu_0^{-1} \nabla \times E + i\omega J$$

- Answer: Yes, given $E$ we *can* solve for $\epsilon$ (trivial!)

$$
\begin{aligned}
\nabla \times \mu_0^{-1} \nabla \times E - \omega^2 \epsilon E &= -i\omega J \\
\omega^2 \epsilon E &= \nabla \times \mu_0^{-1} \nabla \times E + i\omega J \\
\omega^2 E \epsilon &= \nabla \times \mu_0^{-1} \nabla \times E + i\omega J \\
\epsilon &= (\nabla \times \mu_0^{-1} \nabla \times E + i\omega J)/\omega^2 E
\end{aligned}
$$

- Solving for $\epsilon$ actually way faster than simulation (solving for $E$)!

- Obvious and well-known from a mathematical perspective
  - Pre-requisite (200-level) class in optimization curriculum
  - Not yet taught (I think) in optics/photonics at Stanford

- Obvious and well-known from a mathematical perspective
  - Pre-requisite (200-level) class in optimization curriculum
  - Not yet taught (I think) in optics/photonics at Stanford

$$\underbrace{\nabla \times \mu_0^{-1} \nabla \times E - \omega^2 \epsilon E = -i\omega J}_{\text{physics}} \quad \longrightarrow \quad \underbrace{A(z)x = b}_{\text{linear algebra}}$$

- Obvious and well-known from a mathematical perspective
  - Pre-requisite (200-level) class in optimization curriculum
  - Not yet taught (I think) in optics/photonics at Stanford

$$\underbrace{\nabla \times \mu_0^{-1} \nabla \times E - \omega^2 \epsilon E = -i\omega J}_{\text{physics}} \quad \longrightarrow \quad \underbrace{A(z)x = b}_{\text{linear algebra}}$$

$$\begin{aligned} E &\rightarrow x \\ \epsilon &\rightarrow z \\ \nabla \times \mu_0^{-1} \nabla \times -\omega^2 \epsilon &\rightarrow A(z) \\ -i\omega J &\rightarrow b \end{aligned}$$

- Obvious and well-known from a mathematical perspective
  - Pre-requisite (200-level) class in optimization curriculum
  - Not yet taught (I think) in optics/photonics at Stanford

$$\underbrace{\nabla \times \mu_0^{-1} \nabla \times E - \omega^2 \epsilon E = -i\omega J}_{\text{physics}} \quad \longrightarrow \quad \underbrace{A(z)x = b}_{\text{linear algebra}}$$

$$\begin{aligned} E &\rightarrow x \\ \epsilon &\rightarrow z \\ \nabla \times \mu_0^{-1} \nabla \times -\omega^2 \epsilon &\rightarrow A(z) \\ -i\omega J &\rightarrow b \end{aligned}$$

- Key: If $A(z)$ is linear in $z$ then $A(z)x = b$ is as well!

# Direct design of nanophotonic devices

- Let's try it already!
  - Choose $x$ (field)
  - Solve for $z$ (structure) by minimizing the *physics residual*, $A(z)x - b$

## Direct design of nanophotonic devices

- Let's try it already!
  - Choose $x$ (field)
  - Solve for $z$ (structure) by minimizing the *physics residual*, $A(z)x - b$

$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2$$

# Direct design of nanophotonic devices

- Let's try it already!
  - Choose $x$ (field)
  - Solve for $z$ (structure) by minimizing the *physics residual*, $A(z)x - b$

$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2$$

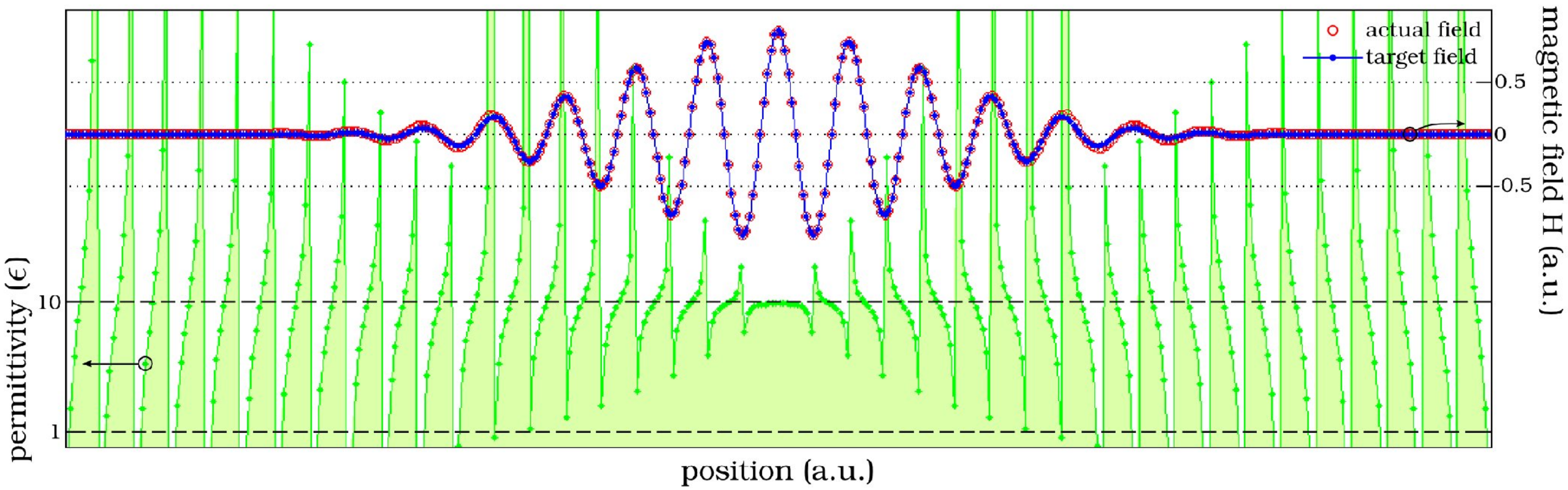
- Global minimum where $A(z)x - b = 0$ can be computed in one step

$$\epsilon = (\nabla \times \mu_0^{-1} \nabla \times E + i\omega J)/\omega^2 E$$

where $\epsilon \rightarrow z$

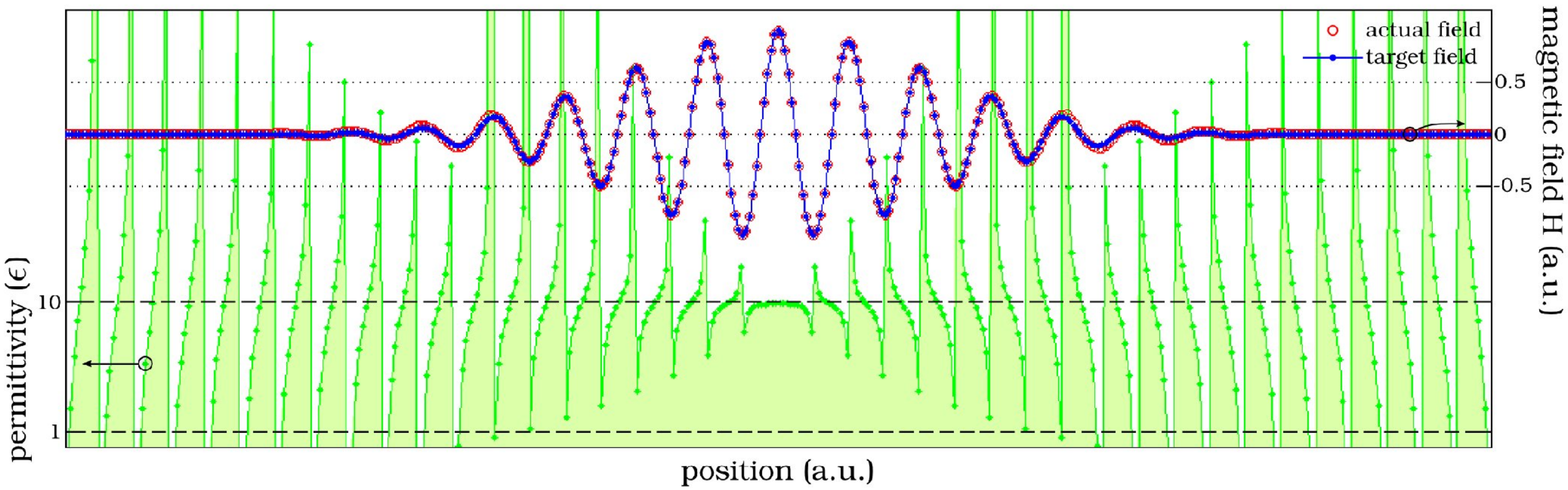
- Choose canonical 1D cavity field for $x$

- Solve for $z$ (structure) and check design fidelity with simulation



- Choose canonical 1D cavity field for $x$

- Solve for $z$ (structure) and check design fidelity with simulation




- Result
  - Perfect performance
  - But unmanufacturable structure ($z$ not well-behaved)

# Direct design with regularization

- Direct design "works" but not practical

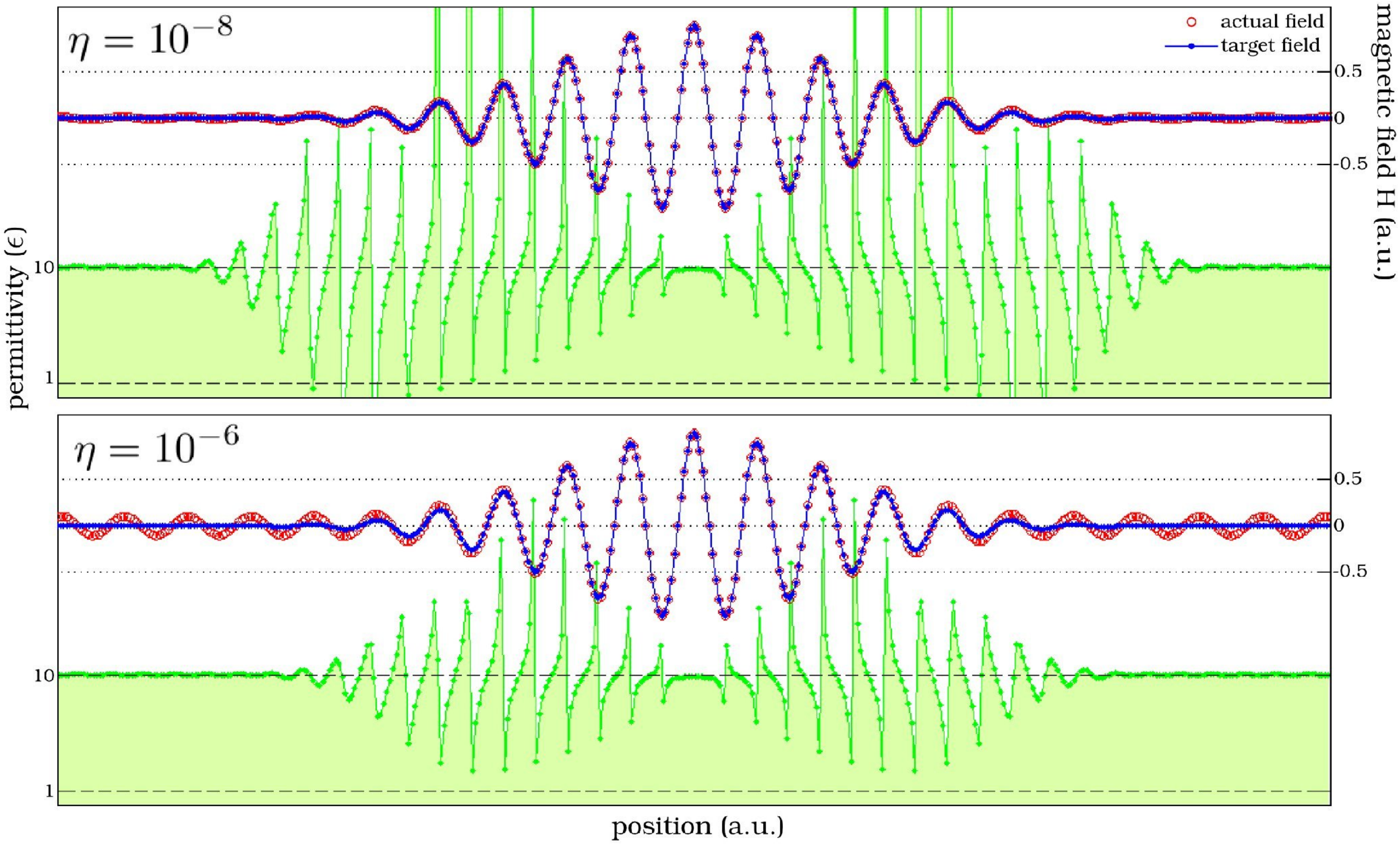
$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2$$

- So, let's add a *regularization* term to $z$ and solve the following instead

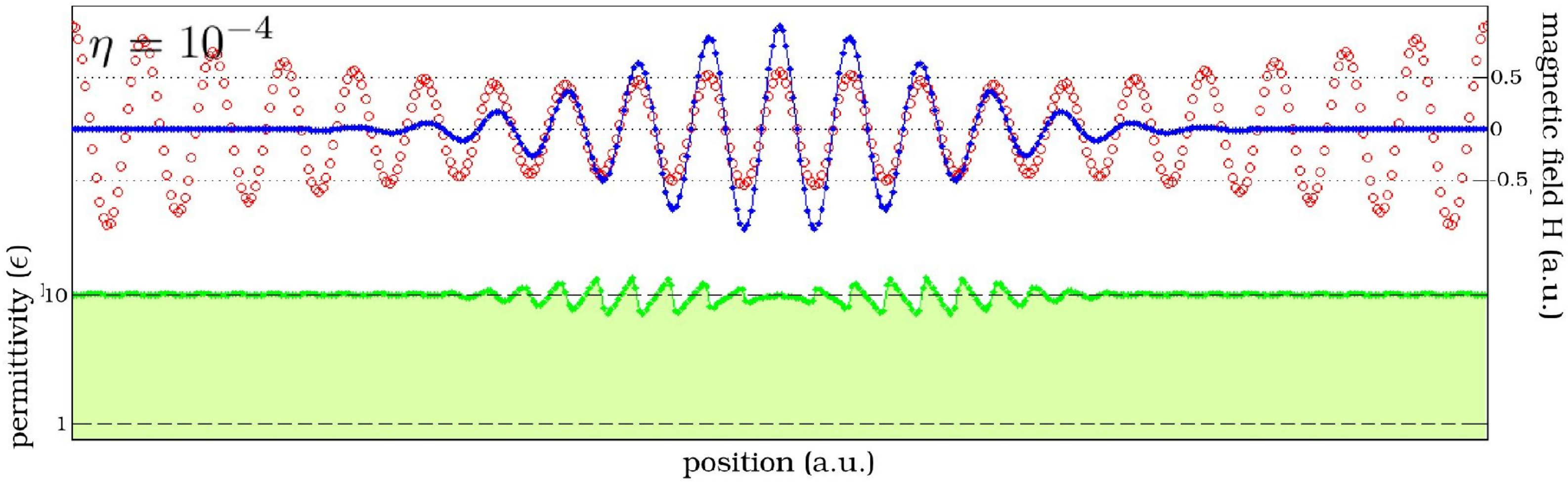
$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta\|z - z_0\|^2$$

  - $\|z - z_0\|^2$ term keeps $z$ close to $z_0$
  - $\eta$ controls the strength of the regularization

- Solution can still be computed in one step




- Unfortunately, regularization on $z$ decreases performance




$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta\|z - z_0\|^2$$

- Decreased performance a result of non-zero physics residual at optimum

- How to overcome the apparent trade-off between manufacturability ($z$) and performance ($x$)?

- How to overcome the apparent trade-off between manufacturability ($z$) and performance ($x$)?

- Realized that there do exist some $x$ (fields) which result from manufacturable $z$ (structures)

  – These can be produced via simulation
  – However, it's not useful to ask the user to choose such $x$

- How to overcome the apparent trade-off between manufacturability ($z$) and performance ($x$)?

- Realized that there do exist some $x$ (fields) which result from manufacturable $z$ (structures)
  - These can be produced via simulation
  - However, it's not useful to ask the user to choose such $x$

- Therefore, a *useful* tool would optimize for *both* $x$ and $z$

## Iterative design of nanophotonic devices

- New algorithm: Iteratively solve for $x$ (field) and $z$ (structure)

$$\underset{z}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta_0 \|z - z_{\text{prev}}\|^2$$

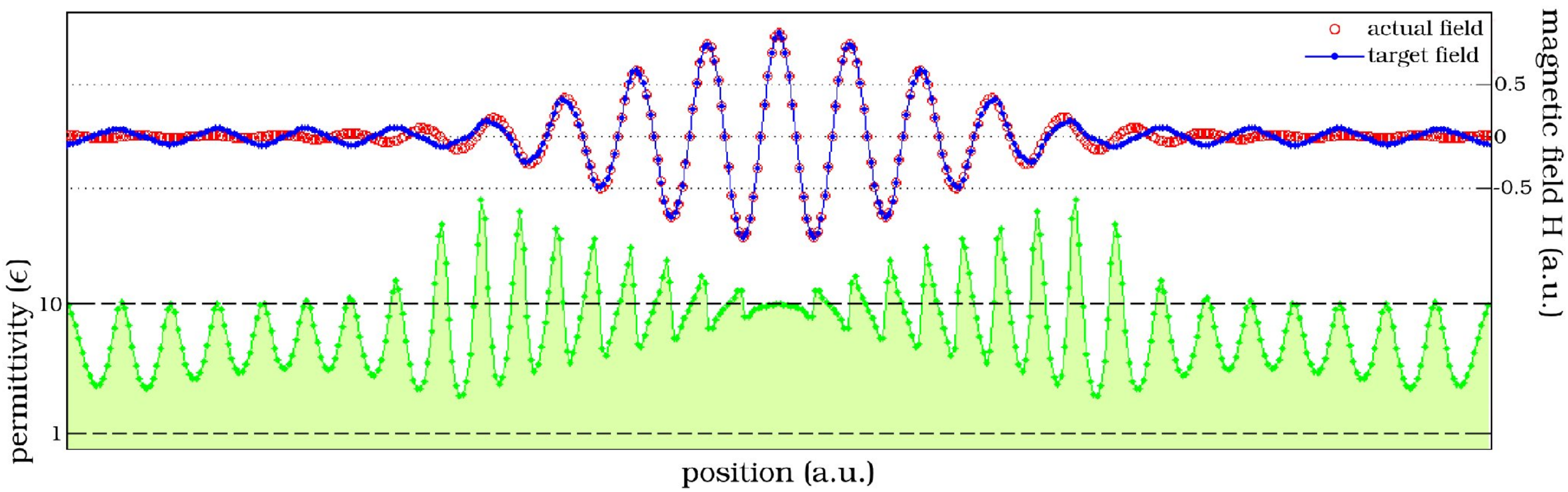
$$\underset{x}{\text{minimize}} \quad \|A(z)x - b\|^2 + \eta_1 \|x - x_{\text{prev}}\|^2$$

- Takes advantage of the *bi-linearity* of the physics residual
  - Jointly solving for $x$ and $z$ is a non-convex problem

- More concisely, we iteratively solve the following

$$\underset{\text{alternately } x \text{ then } z}{\text{minimize}} \; \|A(z)x - b\|^2 + \eta_0\|z - z_{\text{prev}}\|^2 + \eta_1\|x - x_{\text{prev}}\|^2$$

- Design process now consists of *multiple* computational steps
  - $\eta_0, \eta_1$ gradually decreased to bring physics residual toward 0



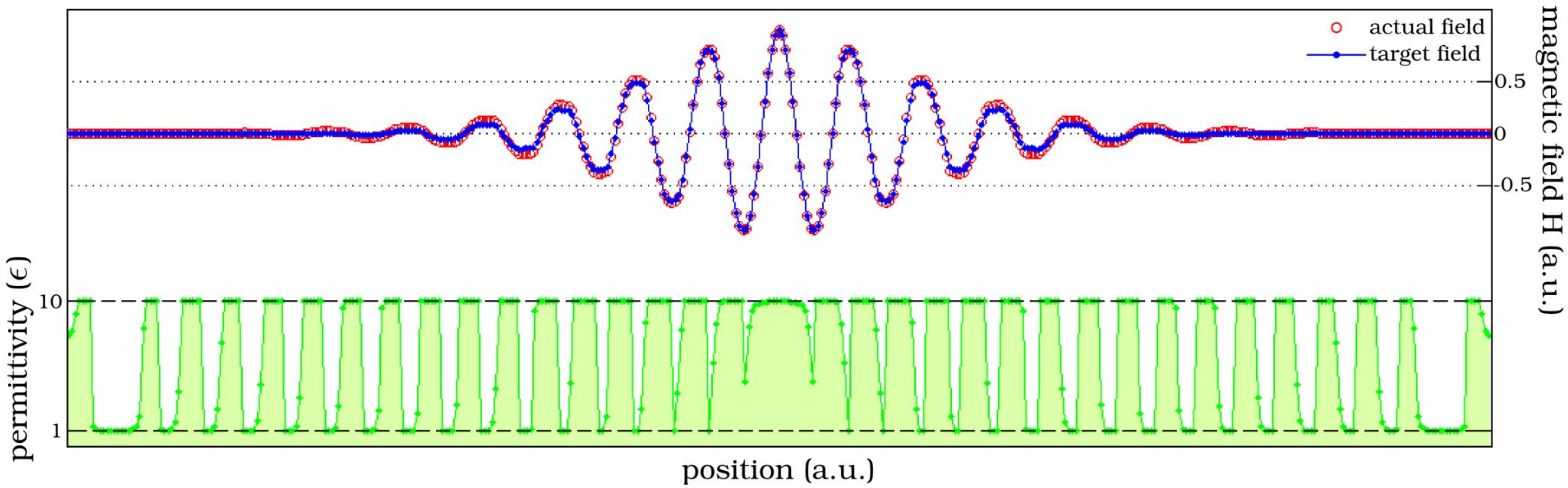

- Iterative strategy produces $z$ (structure) that
  - is better behaved
  - more accurately produces $x$

## Iterative design with hard constraints on $z$

- We can also put hard limits on $z$ (structure)

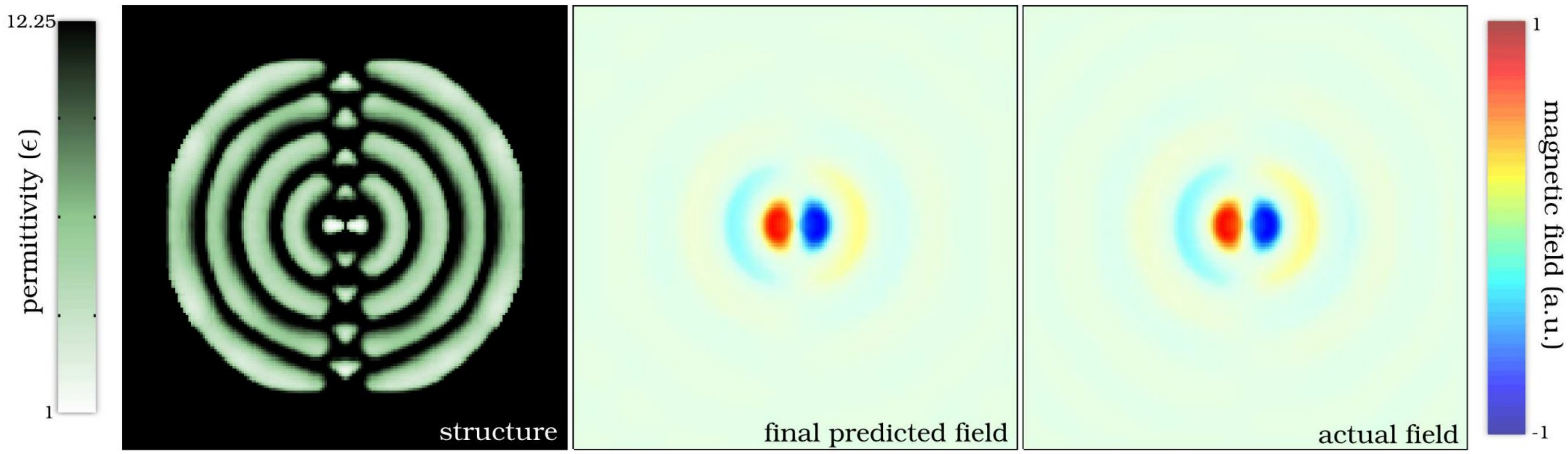
$$\begin{array}{rl} \text{minimize} & \|A(z)x - b\|^2 + \eta_1 \|x - x_{\text{prev}}\|^2 \\ \text{subject to} & z_{\text{min}} \leq z \leq z_{\text{max}} \end{array}$$

- $z_{\text{min}} \leq z \leq z_{\text{max}}$ constraint better represents manufacturability constraint
  - Corresponds to a minimum and maximum allowable permittivity ($\epsilon$)

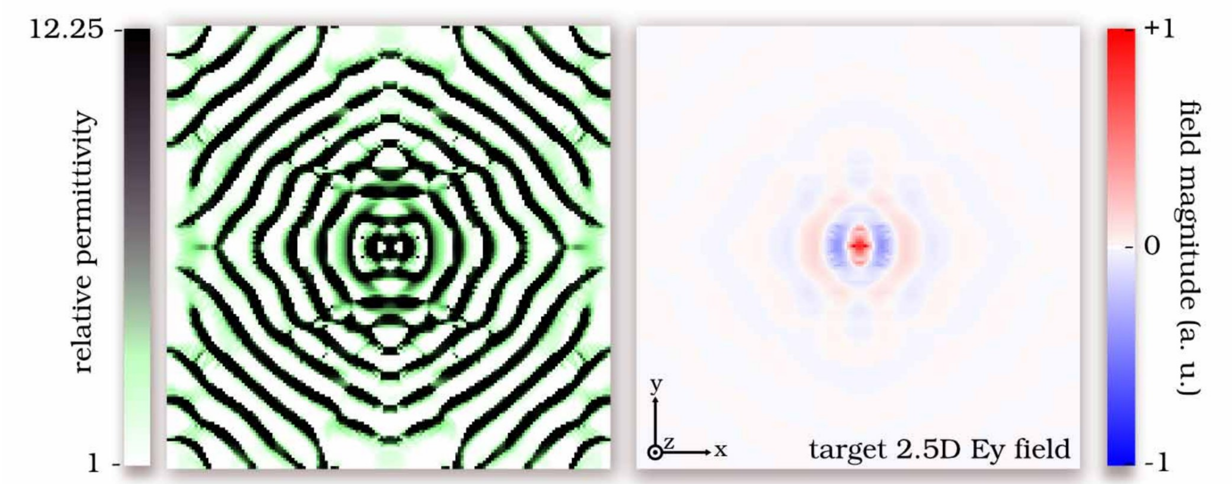



- Well-behaved, manufacturable $z$ (structure)
- Final $x$ (field) accurately reproduced
- Majority of elements of $z$ are fortuitously at one limit or the other!

- Can be used to create 2D resonators



- 3D resonators can be designed using a "2.5D" approximation



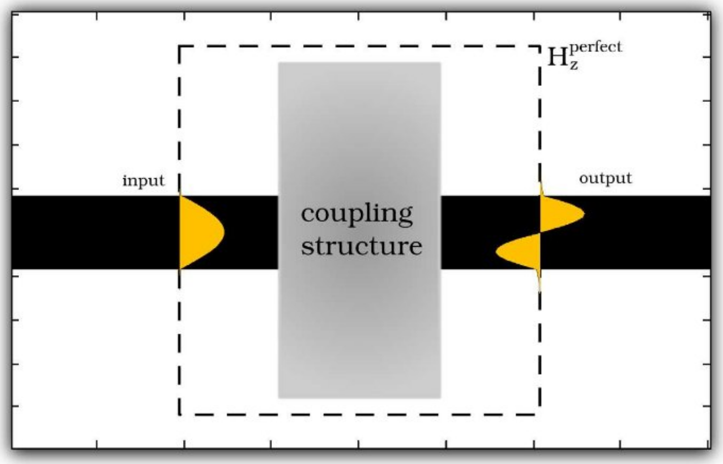
# Objective-first design of linear devices

- Next realization: for linear components, only certain elements of $x$ (field) matter
  - Specifically, the elements of $x$ at the input/output ports

$$\begin{array}{rl} \text{minimize} & \|A(z)x - b\|^2 \\ \text{subject to} & x_{\text{boundary}} - \hat{x}_{\text{boundary}} = 0 \\ & z_{\text{min}} \leq z \leq z_{\text{max}} \end{array}$$

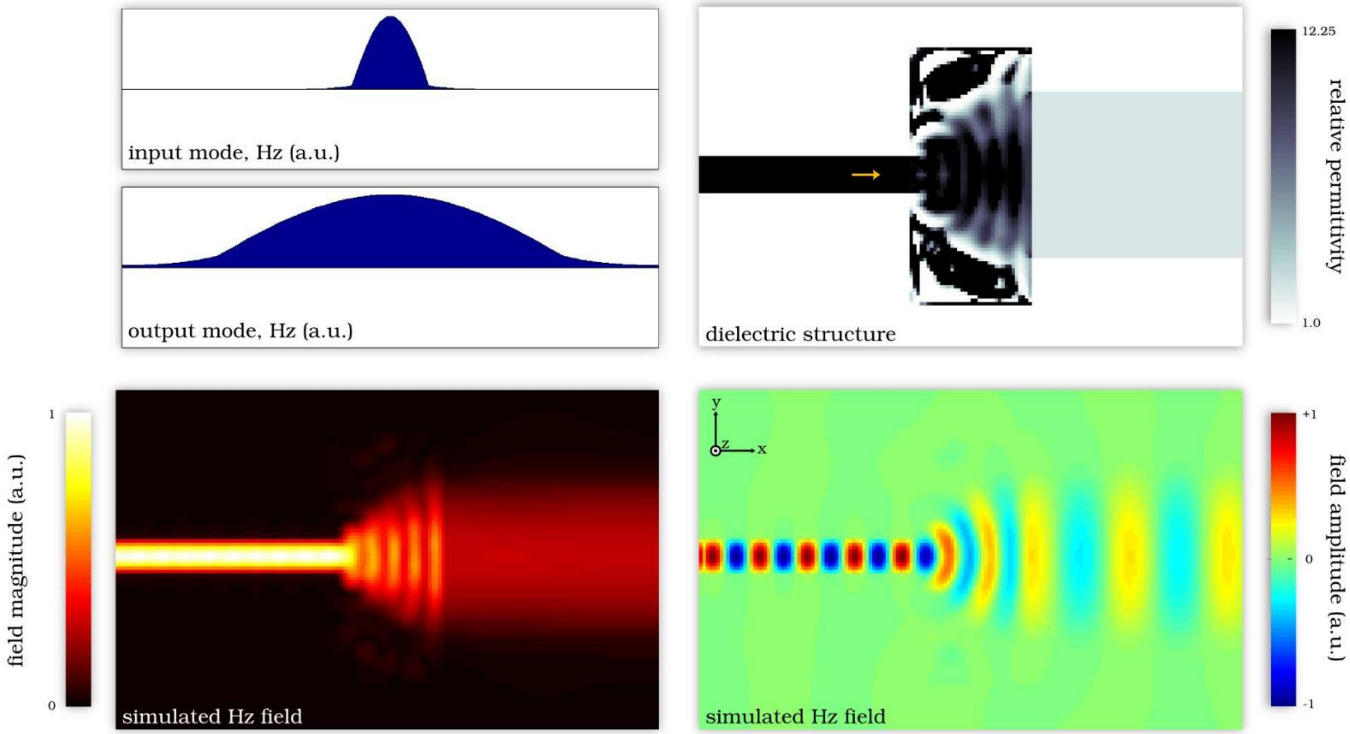
- Instead of regularization term, we force the elements of $x$ at the boundary to be equal to the ideal case ($\hat{x}$)

- Applied to the design of 2D waveguide mode couplers

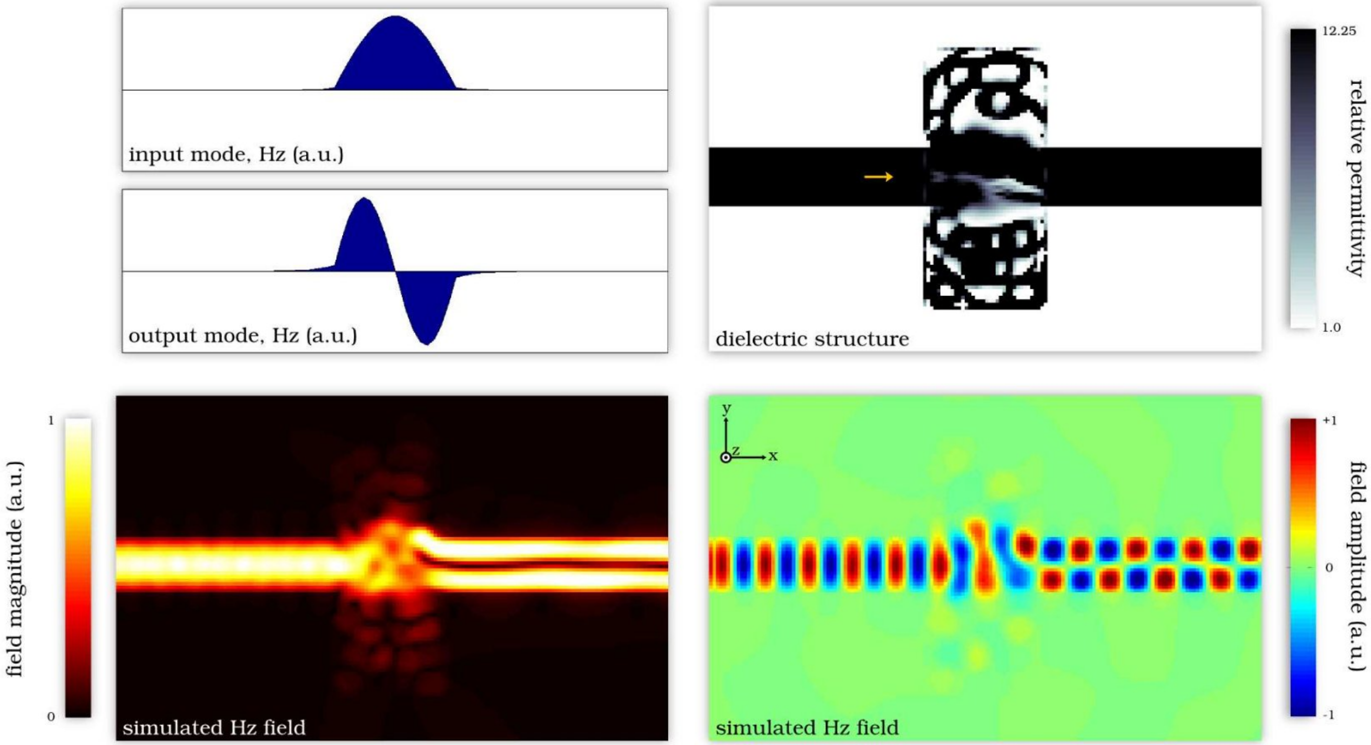



- Produced designs which exhibited
  - High efficiency ($\sim 98\%$)
  - Small device footprints ($1.5 - 4$ square vacuum wavelengths)

- Coupler to wide, low-index waveguide



- Coupler from fundamental to second-order waveguide mode



# Design of 3D linear nanophotonic devices

- Finally, enough understanding to tackle the *real* design problem

## Design of 3D linear nanophotonic devices

- Finally, enough understanding to tackle the *real* design problem
- Goal: Software to design *all* linear nanophotonic devices

# Design of 3D linear nanophotonic devices

- Finally, enough understanding to tackle the *real* design problem
- Goal: Software to design *all* linear nanophotonic devices
  - Fully three-dimensional (no approximations)
  - Multi-mode
  - Discrete, manufacturable structure

- Problem: Did not know how to solve $A(z)x - b = 0$ (simulation) in 3D
  - Millions of variables
  - Famously ill-conditioned
  - No known commercial *solvers* that can handle arbitrary structures

- Problem: Did not know how to solve $A(z)x - b = 0$ (simulation) in 3D
  - Millions of variables
  - Famously ill-conditioned
  - No known commercial *solvers* that can handle arbitrary structures

- Fortunately, Wonseok had already solved this problem

## Maxwell: Light-simulation supercomputer

- Partnered with Wonseok to develop a cloud-based electromagnetic solver using Amazon Web Services
  - GPU-accelerated implementation of Wonseok's algorithm
  - Cluster scales automatically to tens of nodes

- Scalable
  - Far outstrips computing clusters such as Teragrid
  - Can perform multiple solves in parallel, on a single Matlab instance
  - All computation is performed externally (in the cloud)
- Easy to use
  - Installs with a single Matlab command
  - Solves completed with a single Matlab command: `maxwell(...);`
- The key technological enabler ...