As taken from section 19.3 of [1], the interior point method obtains step direction p by solving

$$\begin{bmatrix} \nabla^2_{xx}L & 0 & A_E^T(x) & A_I^T(x) \\ 0 & \Sigma & 0 & -I \\ A_E(x) & 0 & 0 & 0 \\ A_I(x) & -I & 0 & 0 \end{bmatrix} \begin{bmatrix} p_x \\ p_s \\ -p_y \\ -p_z \end{bmatrix} = - \begin{bmatrix} \nabla f(x) - A_E^T(x)y - A_I^T(x)z \\ z - \mu S^{-1}e \\ c_E(x) \\ c_I(x) - s \end{bmatrix}. \tag{1}$$

This equation can be simplified by removing p_s and then p_z. The reduced system is then

$$\begin{bmatrix} \nabla^2_{xx}L + A_I^T(x)\Sigma A_I^T(x) & A_E^T(x) \\ A_E(x) & 0 \end{bmatrix} \begin{bmatrix} p_x \\ -p_y \end{bmatrix} = - \begin{bmatrix} \nabla f(x) - A_E^T(x)y - A_I(x)(z - \Sigma c_I(x) + \mu S^{-1}e) \\ c_E(x) \end{bmatrix}, \tag{2}$$

where

$$p_z = -\Sigma A_I(x)p_x - \Sigma c_I(x) + \mu S^{-1}e \tag{3}$$

$$p_s = A_I(x)p_x + c_I(x) - s \tag{4}$$

References

[1] Nocedal and Wright, Numerical Optimization, Second Edition (Cambridge 2004)