Theory for Boundary-Value Objective-First Optimization

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1 Setting up the boundary-value problem

We begin by writing down a generic (sourceless) physics problem,

$$A(p)x = 0. \tag{1}$$

Here, x is the field variable, p is the structure variable, and $A(p)$ represents the physics of the problem.

Now, suppose that we want to fix the value of the field at certain grid-points. We denote these fixed points as *boundary values* of the problem. Specifically, we break up x into

$$x = S_1 x_1 + S_0 x_0, \tag{2}$$

where S_1 and S_0 are selection matrices for the varying and fixed elements of x respectively.

We can now attempt to solve eq. 1 by finding x_1 in the following manner. Let

$$A(p)S_1 x_1 = -S_0 x_0, \tag{3}$$

or

$$\hat{A}(p)x_1 = b, \tag{4}$$

where $\hat{A}(p) = A(p)S_1$ and $b = -S_0 x_0$.

Note that there often will not be a valid x_1 to satisfy eq. [**?**] since $\hat{A}(p)$ will in general be skinny and full-rank. Instead, we can minimize the *physics residual*, defined as

$$\|A(p)x\|^2 = \|\hat{A}(p)x_1 - b\|^2. \tag{5}$$

We call solving the problem

$$\underset{x_1}{\text{minimize}} \quad \|\hat{A}(p)x_1 - b\|^2 \tag{6}$$

improving the field. Note that an equivalent problem definition is

$$\underset{x}{\text{minimize}} \quad \|A(p)x\|^2 \tag{7}$$

$$\text{subject to} \quad S_0^T x = x_0 \tag{8}$$

References