Electromagnetic Theory Handbook for Objective-First Optimization

Jesse Lu, jesselu@stanford.edu

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Contents

1 Maxwell's equations **1**

2 Perfectly matched layers **1**

1 Maxwell's equations

According to Eqs. 3.7 and 3.8 in [1], Maxwell's time-harmonic equations are

$$-i\omega H = -\frac{1}{\mu}\nabla \times E - \frac{1}{\mu}M \tag{1}$$

$$-i\omega E = \frac{1}{\epsilon}\nabla \times H - \frac{1}{\epsilon}J \tag{2}$$

where M and J are the magnetic and electric current densities, respectively.

The wave equations are then,

$$\nabla \times \frac{1}{\mu}\nabla \times E - \omega^2 \epsilon E = i\omega J - \nabla \times \frac{1}{\mu}M \tag{3}$$

$$\nabla \times \frac{1}{\epsilon}\nabla \times H - \omega^2 \mu H = i\omega M + \nabla \times \frac{1}{\epsilon}J \tag{4}$$

2 Perfectly matched layers

The upshot of ref. [2] is that a PML can be implemented by simply substituting partial derivatives in the following manner,

$$\frac{\delta}{\delta x} \rightarrow \frac{1}{1 + i\frac{\sigma_x(x)}{\omega}}\frac{\delta}{\delta x}, \tag{5}$$

where $\sigma_x(x) > 0$ in the PML and $\sigma_x = 0$ outside of it.

Further considerations include complex σ, Im $\sigma < 0$, to attenuate evanescent waves. Quadratic or cubic growth of σ to reduce numerical reflections arising from discretization error.

Generally, a half-wavelength thick PML layer is sufficient for acceptable attentuation.

References

[1] Allen Taflove, Susan C. Hagness, *Computational Electrodynamics, Third Edition* (Artech House, 2005).

[2] Steven G. Johnson, *Notes on Perfectly Matched Layers (PMLs)* (2007).