# I. Supplemental Material

We only provide the formulation here. If you are interested in the detailed derivations, we recommend You can find all the relevant formula derivations in [1] and [2].

## A. Kinematics Function $\mathbf{f}()$

Suppose the IMU sample interval is $\Delta t$, the kinematics function is:

$$\mathbf{f}(\mathbf{x}_i, \mathbf{u}_i, \mathbf{w}_i) = \Delta t \begin{bmatrix} \boldsymbol{\omega}_{m_i} - \mathbf{b}_{\boldsymbol{\omega}_i} - \mathbf{n}_{\boldsymbol{\omega}_i} \\ {}^G\mathbf{v}_{I_i} \\ \mathbf{0}_{3\times 1} \\ {}^G\mathbf{R}_{I_i}(\mathbf{a}_{m_i} - \mathbf{b}_{\mathbf{a}_i - \mathbf{n}_{\mathbf{a}_i}}) + {}^G\mathbf{g}_i \\ \mathbf{n}_{\mathbf{b}\boldsymbol{\omega}_i} \\ \mathbf{n}_{\mathbf{b}\mathbf{a}_i} \\ \mathbf{0}_{3\times 1} \end{bmatrix}. \quad (1)$$

The system input and noise are:

$$\mathbf{u} \doteq \begin{bmatrix} \boldsymbol{\omega}_m^T & \mathbf{a}_m^T \end{bmatrix}^T, \mathbf{w} \doteq \begin{bmatrix} \mathbf{n}_{\boldsymbol{\omega}}^T & \mathbf{n}_{\mathbf{a}}^T & \mathbf{n}_{\mathbf{b}\boldsymbol{\omega}}^T & \mathbf{n}_{\mathbf{b}\mathbf{a}}^T \end{bmatrix} \quad (2)$$

## B. Formulation of $\mathbf{J}^n$

Recalling the definition of $\mathbf{J}^n$:

$$\mathbf{J}^n = \left( \frac{\partial \left( \hat{\mathbf{x}}_k^n \boxplus \delta\mathbf{x}_k^n \right) \boxminus \hat{\mathbf{x}}_k}{\partial \mathbf{x}_k^n} \bigg|_{\delta\mathbf{x}_k^n = \mathbf{0}} \right)^{-1}. \quad (3)$$

Combining the definition of $\delta\mathbf{x}$ and $\mathbf{x}$, we have:

$$\mathbf{J}^n = \begin{bmatrix} \mathbf{A}({}^G\mathbf{R}_{I_k}^n)^T & \mathbf{0}_{3\times 15} \\ \mathbf{0}_{15\times 3} & \mathbf{I}_{15\times 15} \end{bmatrix} \quad (4)$$

where $\mathbf{A}()$ is defined as:

$$\mathbf{A}(\mathbf{u}) = \mathbf{I} + \left( \frac{1 - \cos\|\mathbf{u}\|}{\|\mathbf{u}\|} \right) \frac{[\mathbf{u}]_\times}{\|\mathbf{u}\|} + \left( 1 - \frac{\sin(\|\mathbf{u}\|)}{\mathbf{u}} \right) \frac{[\mathbf{u}]_\times^2}{\|\mathbf{u}\|^2} \quad (5)$$

## C. Kalman Update

Suppose we have $m$ correspondences in total, the complete Jacobian matrix is $\mathbf{H}_k^n = \begin{bmatrix} \mathbf{H}_{k,0}^n & \mathbf{H}_{k,1}^n, ..., \mathbf{H}_{k,m}^n \end{bmatrix}$. In our implementation, we assume that the covariance matrix $\mathbf{V}_k$ of the measurement noise is a constant denoted by $\mathbf{V}$. The optimal solution to the MAP problem can be solved by the Kalman update [2]:

$$\begin{aligned} \mathbf{P}^n &= \mathbf{J}^n \hat{\mathbf{P}}_k (\mathbf{J}^n)^T \\ \mathbf{K}^n &= \left( (\mathbf{H}^n)^T \mathbf{V}^{-1} \mathbf{H}^n + (\mathbf{P}^n)^{-1} \right)^{-1} (\mathbf{H}^n)^T \mathbf{V}^{-1} \\ \delta\mathbf{x}_k^{n+1} &= \mathbf{K}^n \left( \mathbf{z}_k^n - \mathbf{h}\left( \hat{\mathbf{x}}_k^n \right) \right) + \left( \mathbf{K}^n \mathbf{H}^n - \mathbf{I} \right) \mathbf{J}^n (\hat{\mathbf{x}}_k^n \boxminus \hat{\mathbf{x}}_k) \\ \hat{\mathbf{x}}_k^{n+1} &= \hat{\mathbf{x}}_k^n \boxplus \delta\mathbf{x}_k^{n+1}. \end{aligned} \quad (6)$$

The iteration will stop if the error state $\delta\mathbf{x}_k^{n+1}$ satisfies the predetermined criterion or the count of iterations surpasses a specified threshold. Due to the fact that the error state is usually very small, and based on the experiment results, we didn't implement the error state reset procedure introduced in [1] and [2].

# References


[1] J. Sola, "Quaternion kinematics for the error-state kalman filter," *arXiv preprint arXiv:1711.02508*, 2017.

[2] D. He, W. Xu, and F. Zhang, "Kalman filters on differentiable manifolds," *arXiv preprint arXiv:2102.03804*, 2021.