

I. SUPPLEMENTAL MATERIAL

This material only contains the formulations. If you are interested in the detailed derivations, we strongly recommend You read through [1] and [2].

A. Kinematics Function $\mathbf{f}()$

Suppose the IMU sample interval is Δt , the kinematics function is:

$$\mathbf{f}(\mathbf{x}_i, \mathbf{u}_i, \mathbf{w}_i) = \Delta t \begin{bmatrix} \boldsymbol{\omega}_{m_i} - \mathbf{b}_{\omega_i} - \mathbf{n}_{\omega_i} \\ G \mathbf{v}_{I_i} \\ \mathbf{0}_{3 \times 1} \\ {}^G \mathbf{R}_{I_i} (\mathbf{a}_{m_i} - \mathbf{b}_{\mathbf{a}_i} - \mathbf{n}_{\mathbf{a}_i}) + {}^G \mathbf{g}_i \\ \mathbf{n}_{\mathbf{b}\omega_i} \\ \mathbf{n}_{\mathbf{b}\mathbf{a}_i} \\ \mathbf{0}_{3 \times 1} \end{bmatrix}. \quad (1)$$

The system input and noise are:

$$\mathbf{u} \doteq [\boldsymbol{\omega}_m^T \quad \mathbf{a}_m^T]^T, \mathbf{w} \doteq [\mathbf{n}_\omega^T \quad \mathbf{n}_a^T \quad \mathbf{n}_{\mathbf{b}\omega}^T \quad \mathbf{n}_{\mathbf{b}a}^T] \quad (2)$$

B. Formulation of \mathbf{J}^n

Recalling the definition of \mathbf{J}^n :

$$\mathbf{J}^n = \left(\frac{\partial (\hat{\mathbf{x}}_k^n \boxplus \delta \mathbf{x}_k^n) \boxminus \hat{\mathbf{x}}_k}{\partial \mathbf{x}_k^n} \bigg|_{\delta \mathbf{x}_k^n = \mathbf{0}} \right)^{-1}. \quad (3)$$

Combining the definition of $\delta \mathbf{x}$ and \mathbf{x} , we have:

$$\mathbf{J}^n = \begin{bmatrix} \mathbf{A}({}^G \mathbf{R}_{I_k}^n)^T & \mathbf{0}_{3 \times 15} \\ \mathbf{0}_{15 \times 3} & \mathbf{I}_{15 \times 15} \end{bmatrix} \quad (4)$$

where $\mathbf{A}()$ is defined as:

$$\mathbf{A}(\mathbf{u}) = \mathbf{I} + \left(\frac{1 - \cos \|\mathbf{u}\|}{\|\mathbf{u}\|} \right) \frac{[\mathbf{u}]_{\times}}{\|\mathbf{u}\|} + \left(1 - \frac{\sin(\|\mathbf{u}\|)}{\|\mathbf{u}\|} \right) \frac{[\mathbf{u}]_{\times}^2}{\|\mathbf{u}\|^2} \quad (5)$$

C. Kalman Update

Suppose we have m correspondences in total, the complete Jacobian matrix is $\mathbf{H}_k^n = [\mathbf{H}_{k,0}^n \quad \mathbf{H}_{k,1}^n, \dots, \mathbf{H}_{k,m}^n]$. In our implementation, we assume that the covariance matrix \mathbf{V}_k of the measurement noise is a constant denoted by \mathbf{V} . The optimal solution to the MAP problem can be solved by the Kalman update [2]:

$$\begin{aligned} \mathbf{P}^n &= \mathbf{J}^n \hat{\mathbf{P}}_k (\mathbf{J}^n)^T \\ \mathbf{K}^n &= ((\mathbf{H}^n)^T \mathbf{V}^{-1} \mathbf{H}^n + (\mathbf{P}^n)^{-1})^{-1} (\mathbf{H}^n)^T \mathbf{V}^{-1} \\ \delta \mathbf{x}_k^{n+1} &= \mathbf{K}^n (\mathbf{z}_k^n - \mathbf{h}(\hat{\mathbf{x}}_k^n)) + (\mathbf{K}^n \mathbf{H}^n - \mathbf{I}) \mathbf{J}^n (\hat{\mathbf{x}}_k^n \boxminus \hat{\mathbf{x}}_k) \\ \hat{\mathbf{x}}_k^{n+1} &= \hat{\mathbf{x}}_k^n \boxplus \delta \mathbf{x}_k^{n+1}. \end{aligned} \quad (6)$$

The iteration will stop if the error state $\delta \mathbf{x}_k^{n+1}$ satisfies the predetermined criterion or the count of iterations surpasses a specified threshold. Due to the fact that the error state is usually very small, and based on the experiment results, we didn't implement the error state reset procedure introduced in [1] and [2].

REFERENCES

- [1] J. Sola, "Quaternion kinematics for the error-state kalman filter," *arXiv preprint arXiv:1711.02508*, 2017.
- [2] D. He, W. Xu, and F. Zhang, "Kalman filters on differentiable manifolds," *arXiv preprint arXiv:2102.03804*, 2021.