Interior cells

Cell geometry

In[∘]:=

```
$Assumptions = {xᵢ ∈ Reals && hᶜᵢ₊₁ ∈ Reals && hᶜᵢ ∈ Reals && hᶜᵢ₋₁ ∈ Reals && xᵢ > 0 && hᶜᵢ₊₁ > 0 && hᶜᵢ > 0 &&
    hᶜᵢ₋₁ > 0 && xᵢ - hᶜᵢ / 2 - hᶜᵢ₋₁ > 0 && z ∈ Reals};
```

$$X_{i+\frac{1}{2}} = \text{Interval}\left[\left\{x_i,\ x_i + \frac{h^c{}_i}{2} + \frac{h^c{}_{i+1}}{2}\right\}\right];$$

$$X_{i-\frac{1}{2}} = \text{Interval}\left[\left\{x_i - \frac{h^c{}_i}{2} - \frac{h^c{}_{i-1}}{2},\ x_i\right\}\right];$$

$$X_i = \text{Interval}\left[\left\{x_i - \frac{h^c{}_i}{2},\ x_i + \frac{h^c{}_i}{2}\right\}\right];$$

$$X_{i-1} = \text{Interval}\left[\left\{x_i - \frac{h^c{}_i}{2} - h^c{}_{i-1},\ x_i - \frac{h^c{}_i}{2}\right\}\right];$$

$$X_{i+1} = \text{Interval}\left[\left\{x_i + \frac{h^c{}_i}{2},\ x_i + \frac{h^c{}_i}{2} + h^c{}_{i+1}\right\}\right];$$

Piecewise constant variables

In[∘]:=

```
Df[z_, i_] = Piecewise[{{D_{i-1} * ϕ_{i-1}, Between[z, X_{i-1}]}, {D_i * ϕ_i, Between[z, X_i]}, {D_{i+1} * ϕ_{i+1}, Between[z, X_{i+1}]}}, 0];
u[z_, i_] = Piecewise[{{u_{i-1} * ϕ_{i-1}, Between[z, X_{i-1}]}, {u_i * ϕ_i, Between[z, X_i]}, {u_{i+1} * ϕ_{i+1}, Between[z, X_{i+1}]}}, 0];
S[z_, i_] = Piecewise[{{S_{i-1} * ϕ_{i-1}, Between[z, X_{i-1}]}, {S_i * ϕ_i, Between[z, X_i]}, {S_{i+1} * ϕ_{i+1}, Between[z, X_{i+1}]}}, 0];
```

Auxiliary functions

In[∘]:=

$$\text{Rules} = \left\{\frac{h^c{}_i\, u_i}{D_i} \to \text{Pe}_i,\ \frac{h^c{}_{i+1}\, u_{i+1}}{D_{i+1}} \to \text{Pe}_{i+1},\ \frac{h^c{}_{i-1}\, u_{i-1}}{D_{i-1}} \to \text{Pe}_{i-1},\ \frac{h^c{}_1\, u_1}{D_1} \to \text{Pe}_1,\ \frac{h^c{}_N\, u_N}{D_N} \to \text{Pe}_N\right\};$$

$$\text{Rules2} = \left\{\text{Pe}_{1+j} \to \frac{h^c{}_{j+1}\, u_{j+1}}{D_{j+1}},\ \text{Pe}_j \to \frac{h^c{}_j\, u_j}{D_j},\ \text{Pe}_{j-1} \to \frac{h^c{}_{j-1}\, u_{j-1}}{D_{j-1}},\ \text{Pe}_1 \to \frac{h^c{}_1\, u_1}{D_1},\ \text{Pe}_2 \to \frac{h^c{}_2\, u_2}{D_2},\ \text{Pe}_N \to \frac{h^c{}_N\, u_N}{D_N},\ \text{Pe}_{N-1} \to \frac{h^c{}_{N-1}\, u_{N-1}}{D_{N-1}}\right\};$$

Λ in equation (3), Llorente 2020 AMC

In[∘]:=

$$p^{intgr}{}_{j+1/2}[z\_, i\_] = \text{Assuming}\left[\text{Between}\left[z, X_{i+\frac{1}{2}}\right], \text{FullSimplify}\left[\int_{x_i+\frac{1}{2} h^c{}_i}^{z} \frac{u[t, i]}{\text{Df}[t, i]}\, dt\right]\right]$$

$$p^{intgr}{}_{j-1/2}[z\_, i\_] = \text{Assuming}\left[\text{Between}\left[z, X_{i-\frac{1}{2}}\right], \text{FullSimplify}\left[\int_{x_i-\frac{1}{2} h^c{}_i}^{z} \frac{u[t, i]}{\text{Df}[t, i]}\, dt\right]\right]$$

Out[∘]=

$$\begin{cases} -\frac{u_i\,(-2 z + h^c{}_i + 2 x_i)}{2 D_i} & 2 z < h^c{}_i + 2 x_i \\ -\frac{u_{1+i}\,(-2 z + h^c{}_i + 2 x_i)}{2 D_{1+i}} & 2 z > h^c{}_i + 2 x_i \\ 0 & \text{True} \end{cases}$$

Out[∘]=

$$\begin{cases} \frac{u_{-1+i}\,(2 z + h^c{}_i - 2 x_i)}{2 D_{-1+i}} & 2 z + h^c{}_i < 2 x_i \\ \frac{u_i\,(2 z + h^c{}_i - 2 x_i)}{2 D_i} & 2 z + h^c{}_i > 2 x_i \\ 0 & \text{True} \end{cases}$$

S in equation (3), Llorente 2020 AMC

In[∘]:=

```
s^intgr_{j+1/2}[z_, i_] = Assuming[Between[z, X_{i+1/2}], FullSimplify[∫_{x_i + 1/2 h^c_i}^{z} s[t, i] ⅆt]]
s^intgr_{j-1/2}[z_, i_] = Assuming[Between[z, X_{i-1/2}], FullSimplify[∫_{x_i - 1/2 h^c_i}^{z} s[t, i] ⅆt]]
```

Out[∘]=

$$\begin{cases} \frac{1}{2} S_i \left(2 z - h^c{}_i - 2 x_i\right) \phi_i & 2 z < h^c{}_i + 2 x_i \\ \frac{1}{2} S_{1+i} \left(2 z - h^c{}_i - 2 x_i\right) \phi_{1+i} & 2 z > h^c{}_i + 2 x_i \\ 0 & \text{True} \end{cases}$$

Out[∘]=

$$\begin{cases} \frac{1}{2} S_{-1+i} \left(2 z + h^c{}_i - 2 x_i\right) \phi_{-1+i} & 2 z + h^c{}_i < 2 x_i \\ \frac{1}{2} S_i \left(2 z + h^c{}_i - 2 x_i\right) \phi_i & 2 z + h^c{}_i > 2 x_i \\ 0 & \text{True} \end{cases}$$

<λ, 1> in equation (6), Llorente 2020 AMC

In[∘]:=

```
Pe_{j+1/2}[i_] = FullSimplify[∫_{x_i}^{x_i + 1/2 h^c_i + 1/2 h^c_{i+1}} u[z, i]/Df[z, i] ⅆz]
Pe_{j-1/2}[i_] = FullSimplify[∫_{x_i - 1/2 h^c_i - 1/2 h^c_{i-1}}^{x_i} u[z, i]/Df[z, i] ⅆz]
```

Out[∘]=

$$\frac{1}{2}\left(\frac{h^c{}_i\, u_i}{D_i} + \frac{h^c{}_{1+i}\, u_{1+i}}{D_{1+i}}\right)$$

Out[∘]=

$$\frac{1}{2}\left(\frac{h^c{}_{-1+i}\, u_{-1+i}}{D_{-1+i}} + \frac{h^c{}_i\, u_i}{D_i}\right)$$

<λ, $e^{-\Lambda}$> in equation (6), Llorente 2020 AMC

In[∘]:=

```
IntgrPe_{j+1/2}[i_] = FullSimplify[∫_{x_i}^{x_i + 1/2 hC_i + 1/2 hC_{i+1}} u[z, i]/Df[z, i] Exp[-pintgr_{j+1/2}[z, i]] dz]
IntgrPe_{j-1/2}[i_] = FullSimplify[∫_{x_i - 1/2 hC_i - 1/2 hC_{i-1}}^{x_i} u[z, i]/Df[z, i] Exp[-pintgr_{j-1/2}[z, i]] dz]
```

$$\mathrm{Intgr}^{Pe}{}_{j+1/2}[i\_] = \mathrm{FullSimplify}\left[\int_{x_i}^{x_i+\frac{1}{2}h^C{}_i+\frac{1}{2}h^C{}_{i+1}} \frac{u[z,i]}{Df[z,i]}\,\mathrm{Exp}\left[-p^{intgr}{}_{j+1/2}[z,i]\right] dz\right]$$

$$\mathrm{Intgr}^{Pe}{}_{j-1/2}[i\_] = \mathrm{FullSimplify}\left[\int_{x_i-\frac{1}{2}h^C{}_i-\frac{1}{2}h^C{}_{i-1}}^{x_i} \frac{u[z,i]}{Df[z,i]}\,\mathrm{Exp}\left[-p^{intgr}{}_{j-1/2}[z,i]\right] dz\right]$$

Out[∘]=

$$e^{\frac{h^C{}_i u_i}{2 D_i}} - e^{-\frac{h^C{}_{1+i} u_{1+i}}{2 D_{1+i}}}$$

Out[∘]=

$$e^{\frac{h^C{}_{-1+i} u_{-1+i}}{2 D_{-1+i}}} - e^{-\frac{h^C{}_i u_i}{2 D_i}}$$

$<\Gamma^{-1}, e^{-\Lambda}>$ in equation (6), Llorente 2020 AMC

In[∘]:=

$$\mathrm{Intgr}^{D}{}_{j+1/2}[i\_] = \mathrm{FullSimplify}\left[\int_{x_i}^{x_i+\frac{1}{2}h^C{}_i+\frac{1}{2}h^C{}_{i+1}} \frac{1}{Df[z,i]}\,\mathrm{Exp}\left[-p^{intgr}{}_{j+1/2}[z,i]\right] dz\right]$$

$$\mathrm{Intgr}^{D}{}_{j-1/2}[i\_] = \mathrm{FullSimplify}\left[\int_{x_i-\frac{1}{2}h^C{}_i-\frac{1}{2}h^C{}_{i-1}}^{x_i} \frac{1}{Df[z,i]}\,\mathrm{Exp}\left[-p^{intgr}{}_{j-1/2}[z,i]\right] dz\right]$$

Out[∘]=

$$\frac{-1 + e^{\frac{h^C{}_i u_i}{2 D_i}}}{u_i\,\phi_i} + \frac{1 - e^{-\frac{h^C{}_{1+i} u_{1+i}}{2 D_{1+i}}}}{u_{1+i}\,\phi_{1+i}}$$

Out[∘]=

$$\frac{-1 + e^{\frac{h^C{}_{-1+i} u_{-1+i}}{2 D_{-1+i}}}}{u_{-1+i}\,\phi_{-1+i}} + \frac{1 - e^{-\frac{h^C{}_i u_i}{2 D_i}}}{u_i\,\phi_i}$$

$<S\Gamma^{-1}, e^{-\Lambda}>$ in equation (6), Llorente 2020 AMC

In[∘]:=

$$\text{Intgr}^{S}{}_{j+1/2}[i\_] = \text{FullSimplify}\left[\int_{x_i}^{x_i+\frac{1}{2}h^{C}{}_{i}+\frac{1}{2}h^{C}{}_{i+1}} \frac{S^{\text{intgr}}{}_{j+1/2}[z,i]}{\text{Df}[z,i]} \text{Exp}\left[-P^{\text{intgr}}{}_{j+1/2}[z,i]\right] dz\right]$$

$$\text{Intgr}^{S}{}_{j-1/2}[i\_] = \text{FullSimplify}\left[\int_{x_i-\frac{1}{2}h^{C}{}_{i}-\frac{1}{2}h^{C}{}_{i-1}}^{x_i} \frac{S^{\text{intgr}}{}_{j-1/2}[z,i]}{\text{Df}[z,i]} \text{Exp}\left[-P^{\text{intgr}}{}_{j-1/2}[z,i]\right] dz\right]$$

Out[∘]=

$$\frac{\left(-1+e^{\frac{h^{C}{}_{i}\,u_i}{2\,D_i}}\right) D_i\,S_i}{u_i^2} + \frac{1}{2}\left(-\frac{e^{\frac{h^{C}{}_{i}\,u_i}{2\,D_i}}\,h^{C}{}_{i}\,S_i}{u_i} + \frac{S_{1+i}\left(2\,D_{1+i} - e^{-\frac{h^{C}{}_{1+i}\,u_{1+i}}{2\,D_{1+i}}}\left(2\,D_{1+i} + h^{C}{}_{1+i}\,u_{1+i}\right)\right)}{u_{1+i}^2}\right)$$

Out[∘]=

$$\frac{e^{-\frac{h^{C}{}_{i}\,u_i}{2\,D_i}}\left(2\left(-1+e^{\frac{h^{C}{}_{i}\,u_i}{2\,D_i}}\right) D_i\,S_i\,u_{-1+i}^2 + u_i\left(-h^{C}{}_{i}\,S_i\,u_{-1+i}^2 + e^{\frac{h^{C}{}_{i}\,u_i}{2\,D_i}}\,S_{-1+i}\left(-2\,D_{-1+i} + e^{\frac{h^{C}{}_{-1+i}\,u_{-1+i}}{2\,D_{-1+i}}}\left(2\,D_{-1+i} - h^{C}{}_{-1+i}\,u_{-1+i}\right)\right) u_i\right)\right)}{2\,u_{-1+i}^2\,u_i^2}$$

In[∘]:=

```
B[z_] = z / (Exp[z] - 1);
```

Fluxes

Homogeneous flux

Equation (6), Llorente 2020 AMC

$$F^H{}_{i+\frac{1}{2}} = \frac{<\lambda,e^{-\Lambda}>/<\lambda,1>}{<\Gamma^{-1},e^{-\Lambda}>}[B(-<\lambda,1>)\,C_i - B(<\lambda,1>)\,C_{i+1}]$$

$$F^I{}_{i+\frac{1}{2}} = -\frac{<S\Gamma^{-1},e^{-\Lambda}>}{<\Gamma^{-1},e^{-\Lambda}>}$$

In[ ]:=
```
FH_{j+1/2}[i_] = Collect[IntgrPe_{j+1/2}[i] / Pe_{j+1/2}[i] / IntgrD_{j+1/2}[i] * (B[-Pe_{j+1/2}[i]] * C_i - B[Pe_{j+1/2}[i]] * C_{i+1}),
    {C_i, C_{i+1}}, Simplify] /. Rules;
FH_{j-1/2}[i_] = Collect[IntgrPe_{j-1/2}[i] / Pe_{j-1/2}[i] / IntgrD_{j-1/2}[i] * (B[-Pe_{j-1/2}[i]] * C_{i-1} - B[Pe_{j-1/2}[i]] * C_i),
    {C_{i-1}, C_i}, Simplify] /. Rules;
ΔFH_j[i_] = Collect[FH_{j-1/2}[i] - FH_{j+1/2}[i], {C_{i-1}, C_i, C_{i+1}}] /. Rules;
```

Inhomogeneous flux

In[ ]:=
```
FI_{j+1/2}[i_] = Collect[-IntgrS_{j+1/2}[i] / IntgrD_{j+1/2}[i], {S_i, S_{i+1}}, Simplify] /. Rules;
FI_{j-1/2}[i_] = Collect[-IntgrS_{j-1/2}[i] / IntgrD_{j-1/2}[i], {S_{i-1}, S_i}, Simplify] /. Rules;
ΔFI_j[i_] = Collect[FI_{j-1/2}[i] - FI_{j+1/2}[i], {S_{i-1}, S_i, S_{i+1}}] /. Rules;
```

Total flux

In[ ]:=
```
ΔF_j[i_] = Collect[FH_{j-1/2}[i] - FH_{j+1/2}[i] + FI_{j-1/2}[i] - FI_{j+1/2}[i], {C_{i-1}, C_i, C_{i+1}, S_{i-1}, S_i, S_{i+1}}, Simplify];
```

Boundary conditions

x @ 0

In[∘]:=
```
Y[x_] =
  y[x] /. First@ DSolve[{-D₁ ϕ₁ * y''[x] + u₁ ϕ₁ * y'[x] == S₁ ϕ₁, α₀ * y[0] + β₀ * y'[0] == γ₀, y[hᶜ₁ / 2] == C₁}, y[x], x];
F1/2 = Collect[-D₁ ϕ₁ * Y'[0] + u₁ ϕ₁ * Y[0], {C₁, S₁, r₀}, Simplify] /. Rules;
```

In[∘]:=
```
F1/2 == Aml[1] * C₁ + Bml[1] * S₁ + b[1] /. Rules2 // Simplify
```

Out[∘]= True

In[∘]:=
```
FH1/2 = Coefficient[F1/2, {C₁}] * C₁
FI1/2 = FullSimplify[Coefficient[F1/2, {S₁}]] * S₁
ΔFH₁ = FH1/2 - FHj+1/2[1] /. Rules;
ΔFI₁ = FI1/2 - FIj+1/2[1] /. Rules;
```

Out[∘]=
$$\left\{\frac{C_1\, u_1\, (D_1\, \alpha_0 + u_1\, \beta_0)\, \phi_1}{-\left(-1 + e^{\frac{Pe_1}{2}}\right) D_1\, \alpha_0 + u_1\, \beta_0}\right\}$$

Out[∘]=
$$\left\{\frac{S_1 \left(2 \left(-1 + e^{\frac{Pe_1}{2}}\right) D_1 - h^c{}_1\, u_1\right) (D_1\, \alpha_0 + u_1\, \beta_0)\, \phi_1}{2\, u_1 \left(-\left(-1 + e^{\frac{Pe_1}{2}}\right) D_1\, \alpha_0 + u_1\, \beta_0\right)}\right\}$$

x @ L

In[∘]:=

```
Z[x_] =
  z[x] /. First @ DSolve[{-D_N ϕ_N * z''[x] + u_N ϕ_N * z'[x] == S_N ϕ_N, α_L * z[L] + β_L * z'[L] == γ_L, z[L - h^c_N / 2] == C_N},
     z[x], x];
F_(N+1/2) = Collect[-D_N ϕ_N * Z'[L] + u_N ϕ_N * Z[L], {C_N, S_N, r_L}, Simplify] /. Rules;
```

In[∘]:=

```
F^H_(N+1/2) = Coefficient[F_(N+1/2), {C_N}] * C_N;
F^I_(N+1/2) = FullSimplify[Coefficient[F_(N+1/2), {S_N}]] * S_N;
ΔF^H_N = F^H_(j+1/2)[N - 1] - F^H_(N+1/2) /. Rules;
ΔF^I_N = F^I_(j+1/2)[N - 1] - F^I_(N+1/2) /. Rules;
```

Matrix form

$\Delta F = A{*}C + B{*}S + b$

Boundary terms

$[b_1, 0, \ldots, 0, b_N]$

In[∘]:=
```
b[1] = D_1 u_1 γ_0 ϕ_1 / (-D_1 α_0 e^(-Pe_1/2) + D_1 α_0 - u_1 β_0 e^(-Pe_1/2));
b[N] = e^(-Pe_N/2) D_N u_N γ_L ϕ_N / (-D_N α_L e^(-Pe_N/2) + D_N α_L + u_N β_L);
```

$$b[1] = \frac{D_1 u_1 \gamma_0 \phi_1}{-D_1 \alpha_0 e^{-\frac{Pe_1}{2}} + D_1 \alpha_0 - u_1 \beta_0 e^{-\frac{Pe_1}{2}}};$$

$$b[N] = \frac{e^{-\frac{Pe_N}{2}} D_N u_N \gamma_L \phi_N}{-D_N \alpha_L e^{-\frac{Pe_N}{2}} + D_N \alpha_L + u_N \beta_L};$$

In[∘]:=
```
b[1] - Coefficient[F_(1/2), γ_0] * γ_0 // Simplify
b[N] - (-Coefficient[F_(N+1/2), γ_L] * γ_L) // Simplify
```

Out[∘]= 0

Out[∘]= 0

Homogeneous terms

$A \times C,\ A = [\ldots,\ A_{lower},\ A_{mL} + A_{mu},\ A_{upper},\ \ldots]$

In[•]:=

$$\text{Afun}[i\_] = \frac{u_i\,\phi_i\,u_{i+1}\,\phi_{i+1}}{\left(-e^{-\frac{Pe_i+Pe_{i+1}}{2}} + e^{-\frac{Pe_i}{2}}\right) u_i\,\phi_i + \left(1 - e^{-\frac{Pe_i}{2}}\right) u_{i+1}\,\phi_{i+1}};$$

$$A_{lower}[j\_] = \text{Afun}[j-1];$$

$$A_{ml}[j\_] = -e^{-\frac{Pe_{-1+j}+Pe_j}{2}}\,\text{Afun}[j-1];$$

$$A_{mu}[j\_] = -\text{Afun}[j];$$

$$A_{upper}[j\_] = e^{-\frac{Pe_j+Pe_{1+j}}{2}}\,\text{Afun}[j];$$

$$A_{ml}[1] = \frac{u_1\,\phi_1\,(D_1\,\alpha_0 + u_1\,\beta_0)\,e^{-\frac{Pe_1}{2}}}{D_1\,\alpha_0\,e^{-\frac{Pe_1}{2}} - D_1\,\alpha_0 + u_1\,\beta_0\,e^{-\frac{Pe_1}{2}}};$$

$$A_{mu}[N] = \frac{u_N\,\phi_N\,(D_N\,\alpha_L + u_N\,\beta_L)}{D_N\,\alpha_L\,e^{-\frac{Pe_N}{2}} - D_N\,\alpha_L - u_N\,\beta_L};$$

Interior cells

In[•]:=

$$\left(\text{Coefficient}\left[\Delta F^H{}_j[j],\ C_{j-1}\right] - A_{lower}[j]\right)\ /.\ \text{Rules2}\ //\ \text{Simplify}$$

$$\left(\text{Coefficient}\left[\Delta F^H{}_j[j],\ C_j\right] - (A_{ml}[j] + A_{mu}[j])\right)\ /.\ \text{Rules2}\ //\ \text{Simplify}$$

$$\left(\text{Coefficient}\left[\Delta F^H{}_j[j],\ C_{j+1}\right] - A_{upper}[j]\right)\ /.\ \text{Rules2}\ //\ \text{Simplify}$$

Out[•]= 0

Out[•]= 0

Out[•]= 0

Boundary cells

Boundary conditions only go into $A_{ml}[1]$ and $A_{mu}[N]$

In[•]:=
```
(Coefficient[ΔF^H_1[[1]], C_1] - (A_ml[1] + A_mu[1])) /. Rules2 // Simplify
(Coefficient[F^H_(1/2)[[1]], C_1] - A_ml[1]) /. Rules2 // Simplify
(Coefficient[-F^H_(j+1/2)[1], C_1] - A_mu[1]) /. Rules2 // Simplify

(Coefficient[ΔF^H_N[[1]], C_N] - (A_ml[N] + A_mu[N])) /. Rules2 // Simplify
(Coefficient[F^H_(j+1/2)[N - 1], C_N] - A_ml[N]) /. Rules2 // Simplify
(Coefficient[-F^H_(N+1/2)[[1]], C_N] - A_mu[N]) /. Rules2 // Simplify
```

Out[•]= 0

Out[•]= 0

Out[•]= 0

Out[•]= 0

Out[•]= 0

Out[•]= 0

Inhomogeneous terms

$B \times S$, $B = [\ldots, B_{lower}, B_{ml} + B_{mu}, B_{upper}, \ldots]$

In[∘]:=

$$\mathrm{Bfun}_l[i\_] = h^c{}_i \frac{e^{-\frac{Pe_i}{2}} + \frac{Pe_i}{2} - 1}{u_i\, Pe_i \left(\frac{1-e^{-\frac{Pe_i}{2}}}{u_i\, \phi_i} + \frac{e^{-\frac{Pe_i}{2}} - e^{-\frac{Pe_i + Pe_{1+i}}{2}}}{u_{1+i}\, \phi_{1+i}}\right)};$$

$$\mathrm{Bfun}_u[i\_] = h^c{}_{i+1} \frac{e^{-\frac{Pe_i}{2}} - e^{-\frac{Pe_i + Pe_{1+i}}{2}} - \frac{Pe_{1+i}}{2} e^{-\frac{Pe_i + Pe_{1+i}}{2}}}{u_{1+i}\, Pe_{1+i} \left(\frac{1-e^{-\frac{Pe_i}{2}}}{u_i\, \phi_i} + \frac{e^{-\frac{Pe_i}{2}} - e^{-\frac{Pe_i + Pe_{1+i}}{2}}}{u_{1+i}\, \phi_{1+i}}\right)};$$

$$B_{lower}[j\_] = \mathrm{Bfun}_l[j-1];$$
$$B_{ml}[j\_] = -\mathrm{Bfun}_u[j-1];$$
$$B_{mu}[j\_] = -\mathrm{Bfun}_l[j];$$
$$B_{upper}[j\_] = \mathrm{Bfun}_u[j];$$

$$B_{ml}[1] = \frac{h^c{}_1\, \phi_1\, (D_1\, \alpha_0 + u_1\, \beta_0) \left(1 - e^{-\frac{Pe_1}{2}} - \frac{Pe_1}{2} e^{-\frac{Pe_1}{2}}\right)}{Pe_1 \left(D_1\, \alpha_0\, e^{-\frac{Pe_1}{2}} - D_1\, \alpha_0 + u_1\, \beta_0\, e^{-\frac{Pe_1}{2}}\right)};$$

$$B_{mu}[N] = \frac{h^c{}_N\, \phi_N\, (D_N\, \alpha_L + u_N\, \beta_L) \left(1 - e^{-\frac{Pe_N}{2}} - \frac{Pe_N}{2}\right)}{Pe_N \left(-D_N\, \alpha_L\, e^{-\frac{Pe_N}{2}} + D_N\, \alpha_L + u_N\, \beta_L\right)};$$

In[•]:=

```
(Coefficient[ΔF^I_j[j], S_(j-1)] - B_lower[j]) /. Rules2 // Simplify
(Coefficient[ΔF^I_j[j], S_j] - (B_ml[j] + B_mu[j])) /. Rules2 // Simplify
(Coefficient[ΔF^I_j[j], S_(j+1)] - B_upper[j]) /. Rules2 // Simplify

(Coefficient[ΔF^I_1[[1]], S_1] - (B_ml[1] + B_mu[1])) /. Rules2 // Simplify
(Coefficient[ΔF^I_N[[1]], S_N] - (B_ml[N] + B_mu[N])) /. Rules2 // Simplify
```

Out[•]=

```
0
```

Out[•]=

```
0
```

Out[•]=

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0
```

Out[•]=

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0
```

Out[•]=

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0
```