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Optimization Algorithms

Mini-batch gradient descent

Batch vs. mini-batch gradient descent

X, Y $\qquad$ $X^{\{t\}}, Y^{\{t\}}$.

Vectorization allows you to efficiently compute on m examples.

$$X = [\underbrace{x^{(1)}\ x^{(2)}\ x^{(3)} \ldots x^{(1000)}}_{X^{\{1\}}\ (n_x, 1000)} \mid \underbrace{x^{(1001)} \ldots x^{(2000)}}_{X^{\{2\}}\ (n_x, 1000)} \mid \ldots \mid \underbrace{\ldots x^{(m)}}_{X^{\{5,000\}}\ (n_x, 1000)}]$$

(n_x, m)

$$Y = [\underbrace{y^{(1)}\ y^{(2)}\ y^{(3)} \ldots y^{(1000)}}_{Y^{\{1\}}\ (1, 1000)} \mid \underbrace{y^{(1001)} \ldots y^{(2000)}}_{Y^{\{2\}}\ (1, 1000)} \mid \ldots \mid \underbrace{\ldots y^{(m)}}_{Y^{\{5,000\}}\ (1, 1000)}]$$

$(1, m)$

What if $m = 5,000,000$?

5,000 mini-batches of 1,000 each

Mini-batch t: $X^{\{t\}}, Y^{\{t\}}$.

$x^{(i)}$

$z^{[l]}$

$X^{\{t\}}, Y^{\{t\}}$.

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Mini-batch gradient descent

repeat {
for $t = 1, \ldots, 5000$ {

1 step of gradient descent using $X^{\{t\}}, Y^{\{t\}}$ (as if $m = 1000$)

X, Y

Forward prop on $X^{\{t\}}$.

$$\left.\begin{aligned} Z^{[1]} &= W^{[1]} X^{\{t\}} + b^{[1]} \\ A^{[1]} &= g^{[1]}(Z^{[1]}) \\ &\vdots \\ A^{[L]} &= g^{[L]}(Z^{[L]}) \end{aligned}\right\}$$

Vectorized implementation (1000 examples)

from $X^{\{t\}}, Y^{\{t\}}$.

Compute cost $J^{\{t\}} = \frac{1}{1000} \sum_{i=1}^{l} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) + \frac{\lambda}{2 \cdot 1000} \sum_{l} \|W^{[l]}\|_F^2$.

Backprop to compute gradients wrt $J^{\{t\}}$ (using $(X^{\{t\}}, Y^{\{t\}})$)

$W^{[l]} := W^{[l]} - \alpha dW^{[l]}, \quad b^{[l]} := b^{[l]} - \alpha db^{[l]}$

}

}

"1 epoch"

pass through training set.

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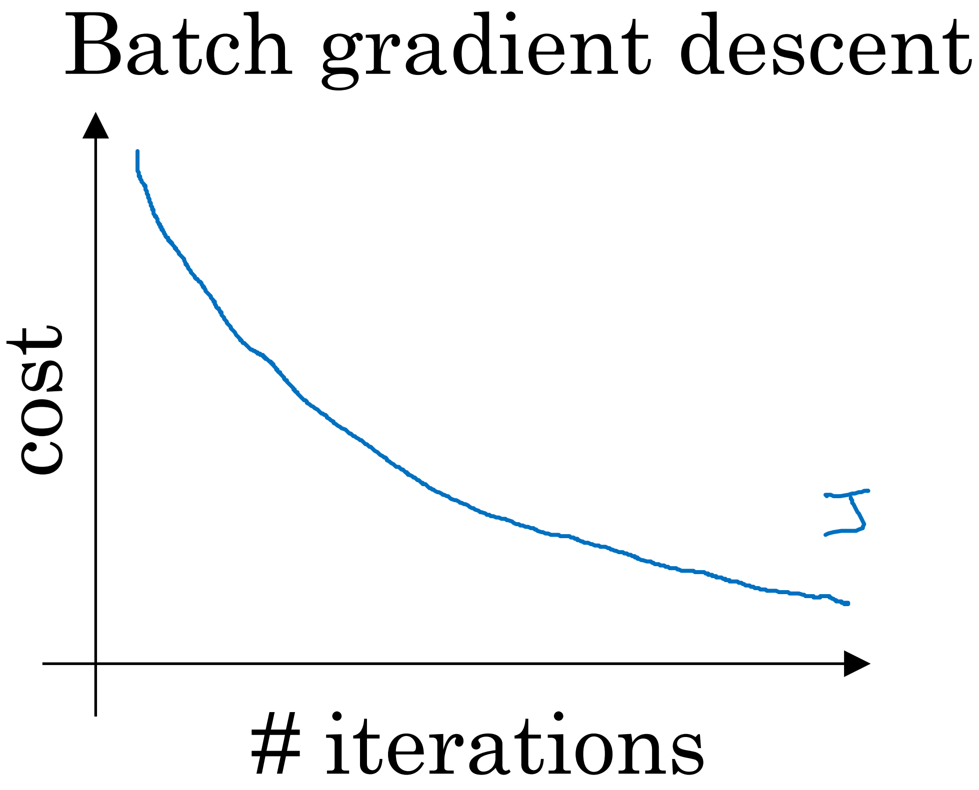
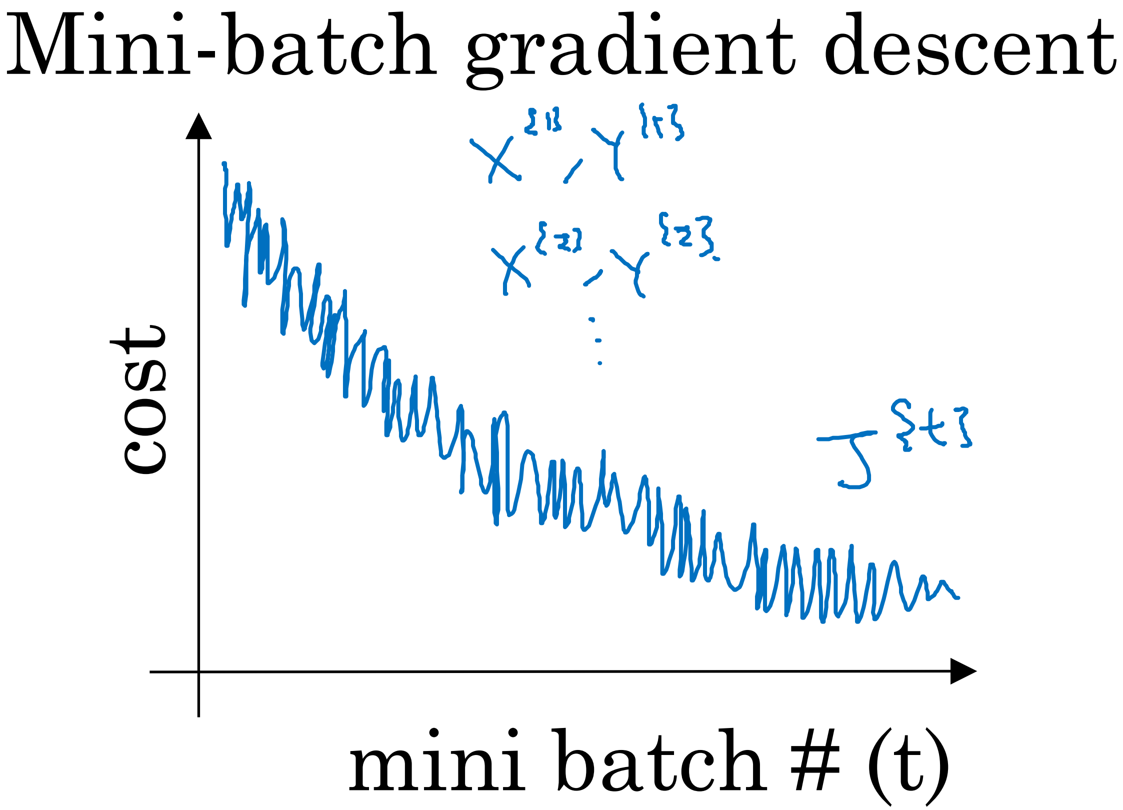
Optimization Algorithms

Understanding mini-batch gradient descent

Training with mini batch gradient descent





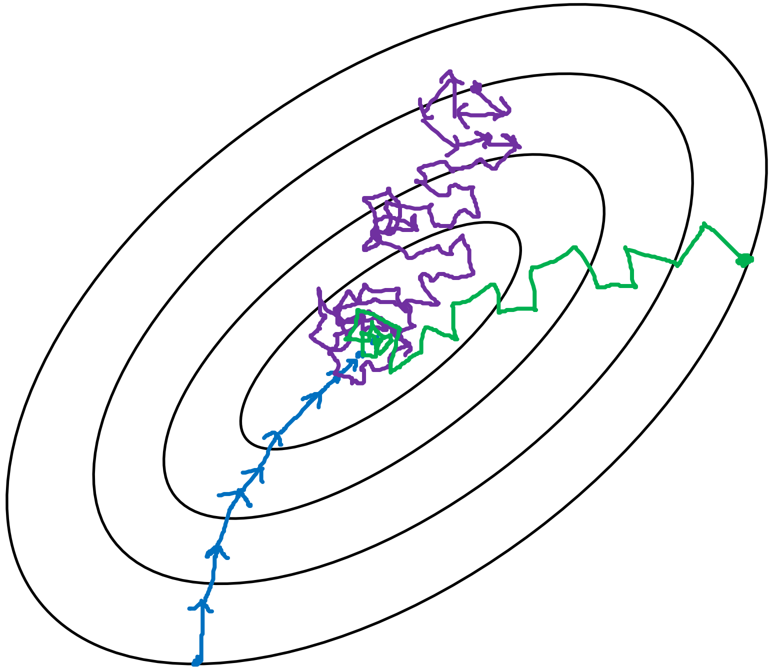
Plot $J^{\{t\}}$ computed using $\underline{X^{\{t\}}, Y^{\{t\}}}$

Choosing your mini-batch size

→ If mini-batch size = m : Batch gradient descent. $(X^{\{1\}}, Y^{\{1\}}) = (X, Y)$.

→ If mini-batch size = 1 : Stochastic gradient descent. Every example is it own
$(X^{\{1\}}, Y^{\{1\}}) = (x^{(1)}, y^{(1)})$ ... $(x^{(2)}, y^{(2)})$ mini-batch.

In practice: Somewhere in-between 1 and m



Stochastic gradient descent
↓
Lose speedup from vectorization

In-between (mini-batch size not too big/small)
↓
Fastest learning.
- Vectorization. (~1000)
- Make progress without processing entire training set.

Batch gradient descent (mini-batch size = m)
↓
Too long per iteration

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Choosing your mini-batch size

If small trainig set : Use batch gradient descent.
($m \leq 2000$)

Typical mini-batch sizes:

$\rightarrow$ 64, 128, 256, 512 $\qquad$ 1024

$\underbrace{2^6 \quad 2^7 \quad 2^8 \quad 2^9}$ $\qquad$ 2^{10}

Make sure minibatch fit in CPU/GPU memory.

$X^{\{t\}}, Y^{\{t\}}$

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Optimization Algorithms

Exponentially weighted averages

Temperature in London

$\theta_1 = 40°F$ 4°C ←

$\theta_2 = 49°F$ 9°C

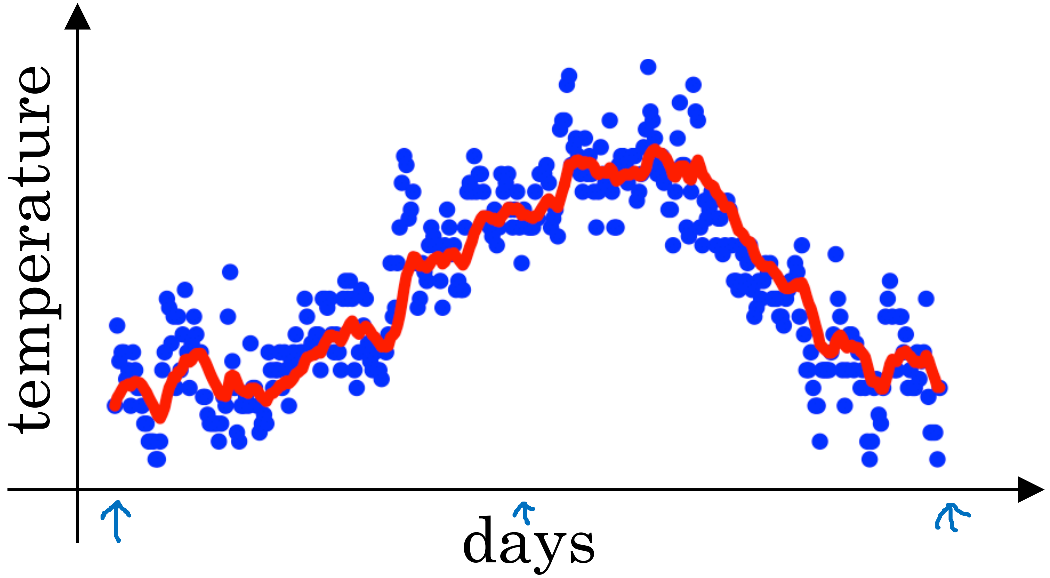
$\theta_3 = 45°F$ ⋮

⋮

$\theta_{180} = 60°F$ 15°C

$\theta_{181} = 56°F$ ⋮

⋮



$V_0 = 0$

$V_1 = 0.9\,V_0 + 0.1\,\theta_1$

$V_2 = 0.9\,V_1 + 0.1\,\theta_2$

$V_3 = 0.9\,V_2 + 0.1\,\theta_3$

⋮

$$\boxed{V_t = 0.9\,V_{t-1} + 0.1\,\theta_t}$$

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Exponentially weighted averages

moving

$$V_t = \beta V_{t-1} + (1-\beta)\theta_t \leftarrow$$

$\beta = 0.9$: ≈ 10 days' temperature.

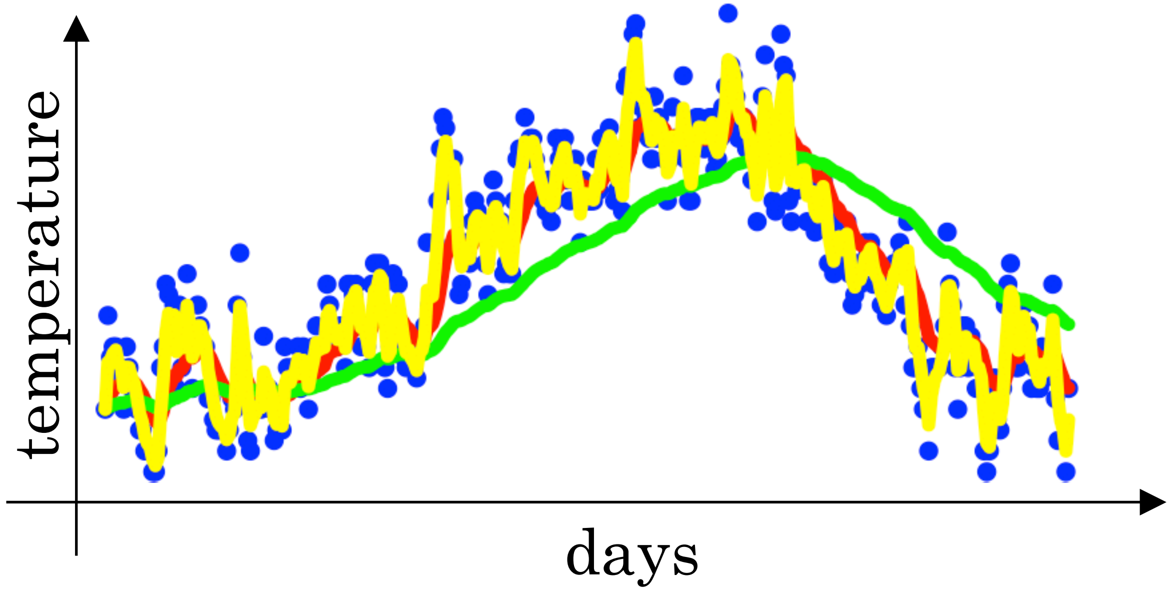
$\beta = 0.98$: ≈ 50 days

$\beta = 0.5$: ≈ 2 days

V_t as approximately average over

$\rightarrow \approx \frac{1}{1-\beta}$ days' temperature.

$$\frac{1}{1-0.98} = 50$$



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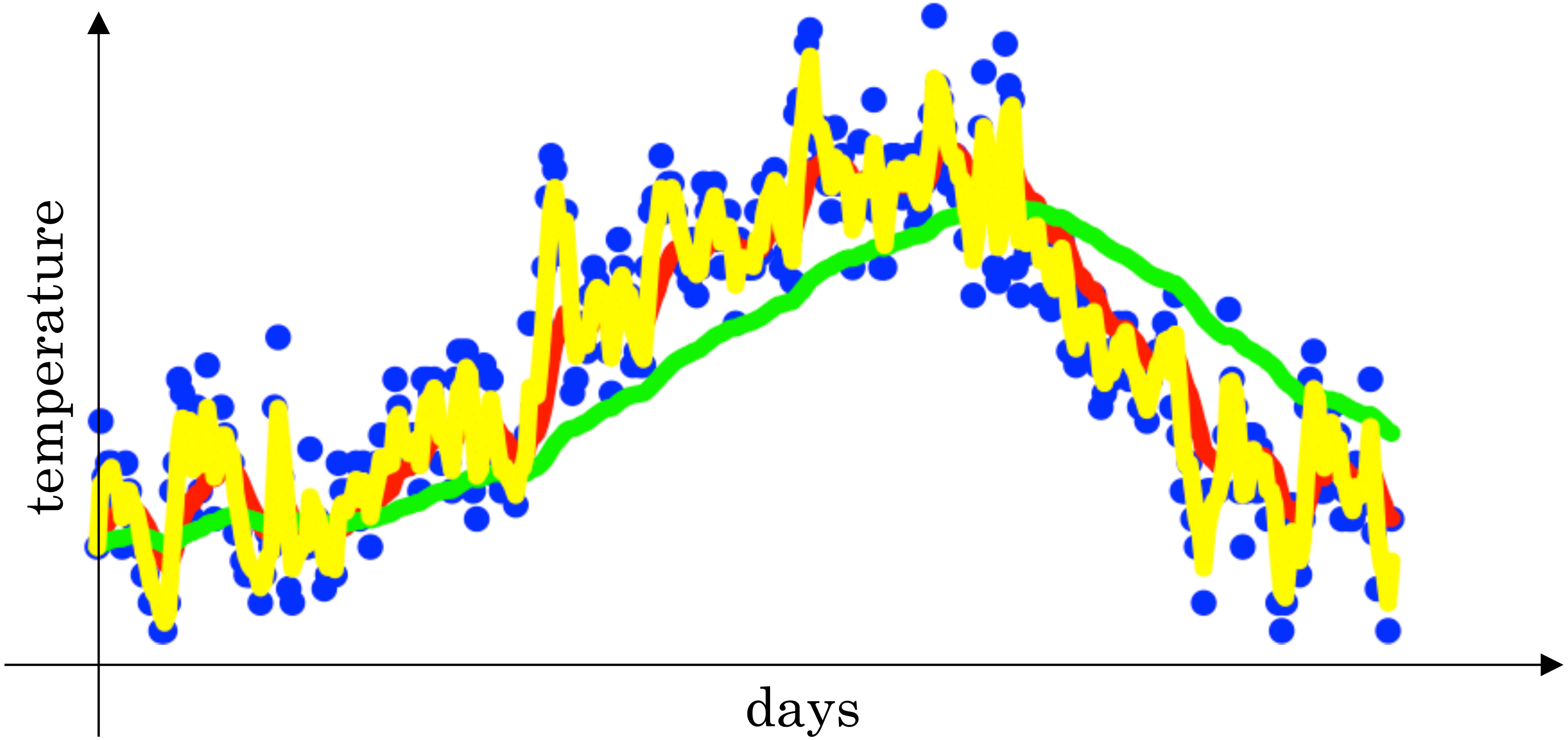
Optimization Algorithms

Understanding exponentially weighted averages

Exponentially weighted averages

$$\boxed{v_t = \beta v_{t-1} + (1-\beta)\theta_t}$$

$\beta = 0.9 \qquad 0.98 \qquad 0.5$



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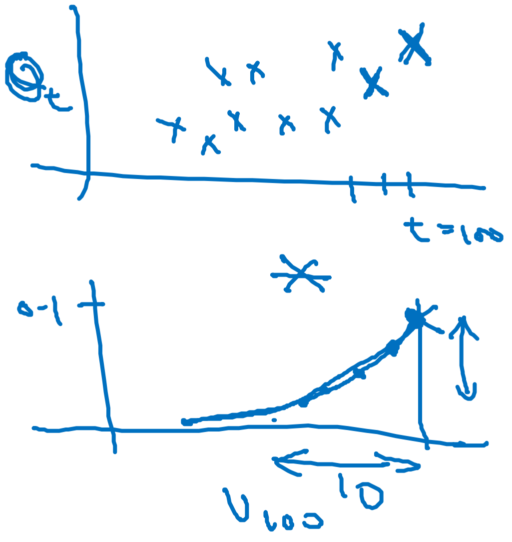
Exponentially weighted averages

$$v_t = \beta v_{t-1} + (1-\beta)\theta_t$$



$\approx \frac{1}{1-\beta}$

$\varepsilon = 1-\beta$

$$v_{100} = 0.9v_{99} + 0.1\theta_{100}$$

$$\boxed{v_{99} = 0.9v_{98} + 0.1\theta_{99}}$$

$$\boxed{v_{98} = 0.9v_{97} + 0.1\theta_{98}}$$

...

$0.1\,\theta_{98} + 0.9\,v_{97}$

$$V_{100} = 0.1\,\theta_{100} + 0.9\,\cancel{v_{99}}\left(0.1\,\theta_{99} + 0.9\,\cancel{v_{98}}\right)$$

$$= 0.1\,\theta_{100} + 0.1\times 0.9\cdot\theta_{99} + 0.1\,(0.9)^2\,\theta_{98} + 0.1\,(0.9)^3\,\theta_{97} + 0.1\,(0.9)^4\,\theta_{96} + \dots$$

$0.9^{10} \approx 0.35 \approx \frac{1}{e}$ $\qquad \underbrace{(1-\varepsilon)}_{0.9}{}^{1/\varepsilon} = \frac{1}{e}$ $\qquad 0.98$?

$\varepsilon = 0.02 \to 0.98^{50} \approx \frac{1}{e}$

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Implementing exponentially weighted averages

$$v_0 = 0$$
$$v_1 = \beta v_0 + (1-\beta)\,\theta_1$$
$$v_2 = \beta v_1 + (1-\beta)\,\theta_2$$
$$v_3 = \beta v_2 + (1-\beta)\,\theta_3$$
$$\dots$$

$$V_\theta := 0$$
$$V_\theta := \beta v + (1-\beta)\,\theta_1$$
$$V_\theta := \beta v + (1-\beta)\,\theta_2$$
$$\vdots$$

$\rightarrow V_\theta = 0$

Repeat {

Get next θ_t

$V_\theta := \beta V_\theta + (1-\beta)\,\theta_t \leftarrow$

}

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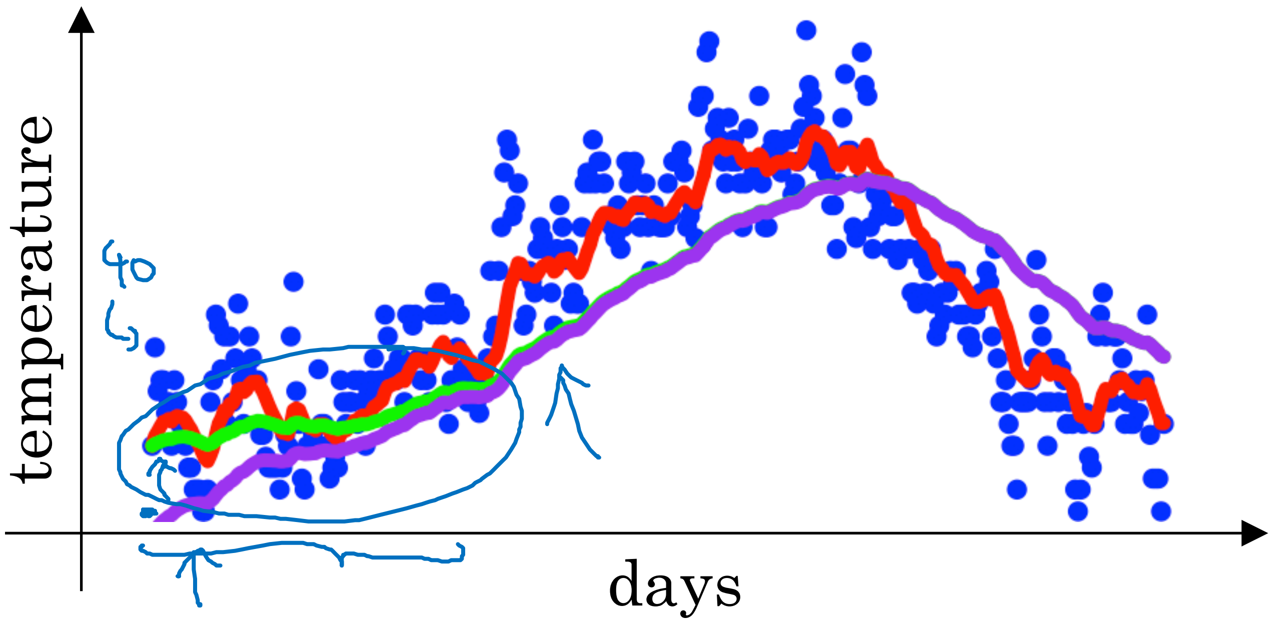
Optimization Algorithms

Bias correction in exponentially weighted average

Bias correction



$\beta = 0.98$

$\rightarrow v_t = \beta v_{t-1} + (1-\beta)\theta_t$

$v_0 = 0$

$v_1 = \cancel{0.98 v_0 +} \, 0.02\, \theta_1$

$v_2 = 0.98\, v_1 + 0.02\, \theta_2$

$= 0.98 \times 0.02 \times \theta_1 + 0.02\, \theta_2$

$= 0.0196\, \theta_1 + 0.02\, \theta_2$

$\dfrac{v_t}{1-\beta^t}$

$t=2: \quad 1-\beta^t = 1-(0.98)^2 = 0.0396$

$\dfrac{v_2}{0.0396} = \dfrac{0.0196\,\theta_1 + 0.02\,\theta_2}{0.0396}$

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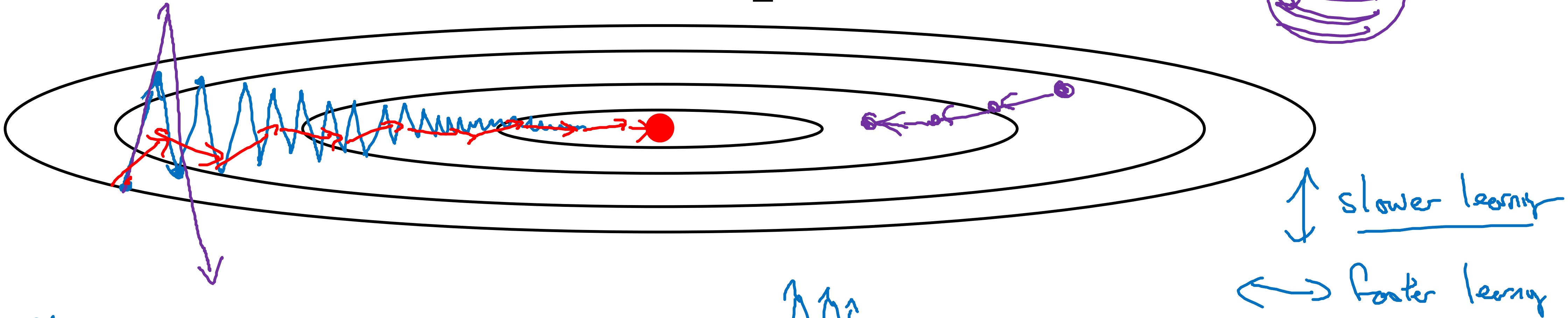
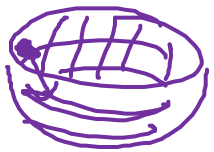
Optimization Algorithms

Gradient descent with momentum

Gradient descent example





Momentum:

On iteration t:

Compute dW, db on current mini-batch.

$$V_{dW} = \beta V_{dW} + (1-\beta)\,dW$$

$$V_{db} = \beta V_{db} + (1-\beta)\,db$$

friction ↑ velocity ↑ acceleration

"$V_\theta = \beta V_\theta + (1-\beta)\theta_t$"

$$W := W - \alpha V_{dW}, \quad b := b - \alpha V_{db}$$

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Implementation details

$V_{dW} = 0, \quad V_{db} = 0$

On iteration t:

Compute dW, db on the current mini-batch

$$\to v_{dW} = \beta v_{dW} + \cancel{(1-\beta)}\, dW$$

$$\to v_{db} = \beta v_{db} + \cancel{(1-\beta)}\, db$$

$$W = W - \alpha v_{dW}, \quad b = b - \alpha v_{db}$$

$V_{dW} = \beta V_{dW} + dW \leftarrow$

~~V_{dW} t $1-\beta$~~

Hyperparameters: α, β

$\beta = 0.9$

average over last ≈ 10 gradients

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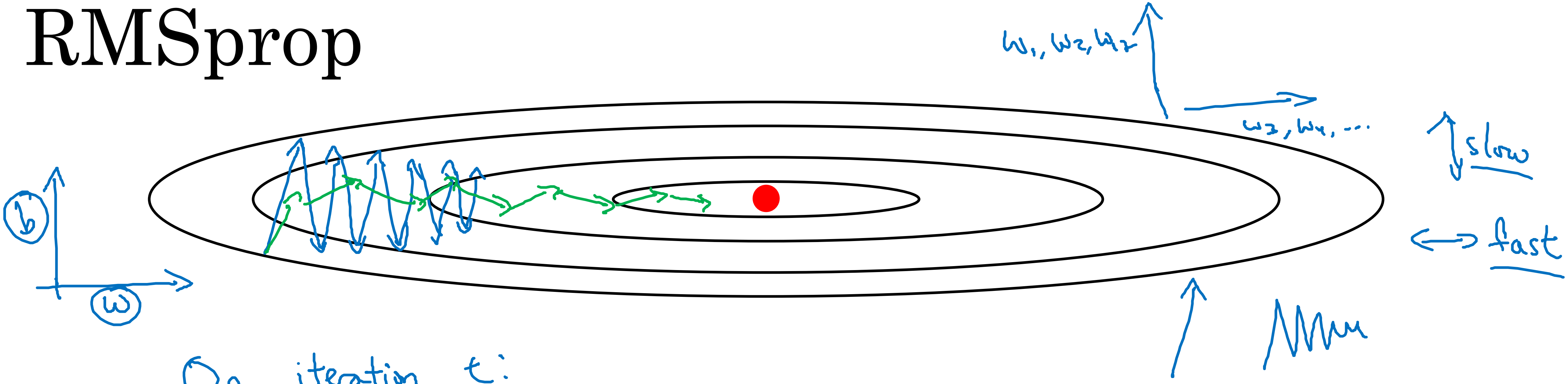
Optimization Algorithms

RMSprop

RMSprop



On iteration t:

Compute dW, db on current mini-batch

element-wise

$$S_{dW} = \beta_2 S_{dW} + (1-\beta_2)\, dW^2 \leftarrow \text{small}$$

$$\rightarrow S_{db} = \beta_2 S_{db} + (1-\beta_2)\, db^2 \leftarrow \text{large}$$

$$W := W - \alpha \frac{dW}{\sqrt{S_{dW}+\varepsilon}} \qquad b := b - \alpha \frac{db}{\sqrt{S_{db}+\varepsilon}}$$

$$\varepsilon = 10^{-8}$$

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Optimization Algorithms

Adam optimization algorithm

Adam optimization algorithm

$V_{dw} = 0,\ S_{dw} = 0.\quad V_{db} = 0,\ S_{db} = 0$

On iteration t:

Compute dW, db using current mini-batch

$V_{dw} = \beta_1 V_{dw} + (1-\beta_1) dW, \quad V_{db} = \beta_1 V_{db} + (1-\beta_1) db \quad \leftarrow$ "momentum" β_1

$S_{dw} = \beta_2 S_{dw} + (1-\beta_2) dW^2, \quad S_{db} = \beta_2 S_{db} + (1-\beta_2) db \quad \leftarrow$ "RMSprop" β_2

yhat = np.array([.9, 0.2, 0.1, .4, .9])

$V_{dw}^{corrected} = V_{dw} / (1-\beta_1^t), \quad V_{db}^{corrected} = V_{db} / (1-\beta_1^t)$

$S_{dw}^{corrected} = S_{dw} / (1-\beta_2^t), \quad S_{db}^{corrected} = S_{db} / (1-\beta_2^t)$

$$W := W - \alpha \frac{V_{dw}^{corrected}}{\sqrt{S_{dw}^{corrected}} + \varepsilon} \qquad b := b - \alpha \frac{V_{db}^{corrected}}{\sqrt{S_{db}^{corrected}} + \varepsilon}$$

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Hyperparameters choice:

→ α : needs to be tune

→ β_1 : 0.9 → $(\underline{dw})$

→ β_2 : 0.999 → $(\underline{dw^2})$

→ ε : 10^{-8}

Adam: Adaptive moment estimation

Adam Coates

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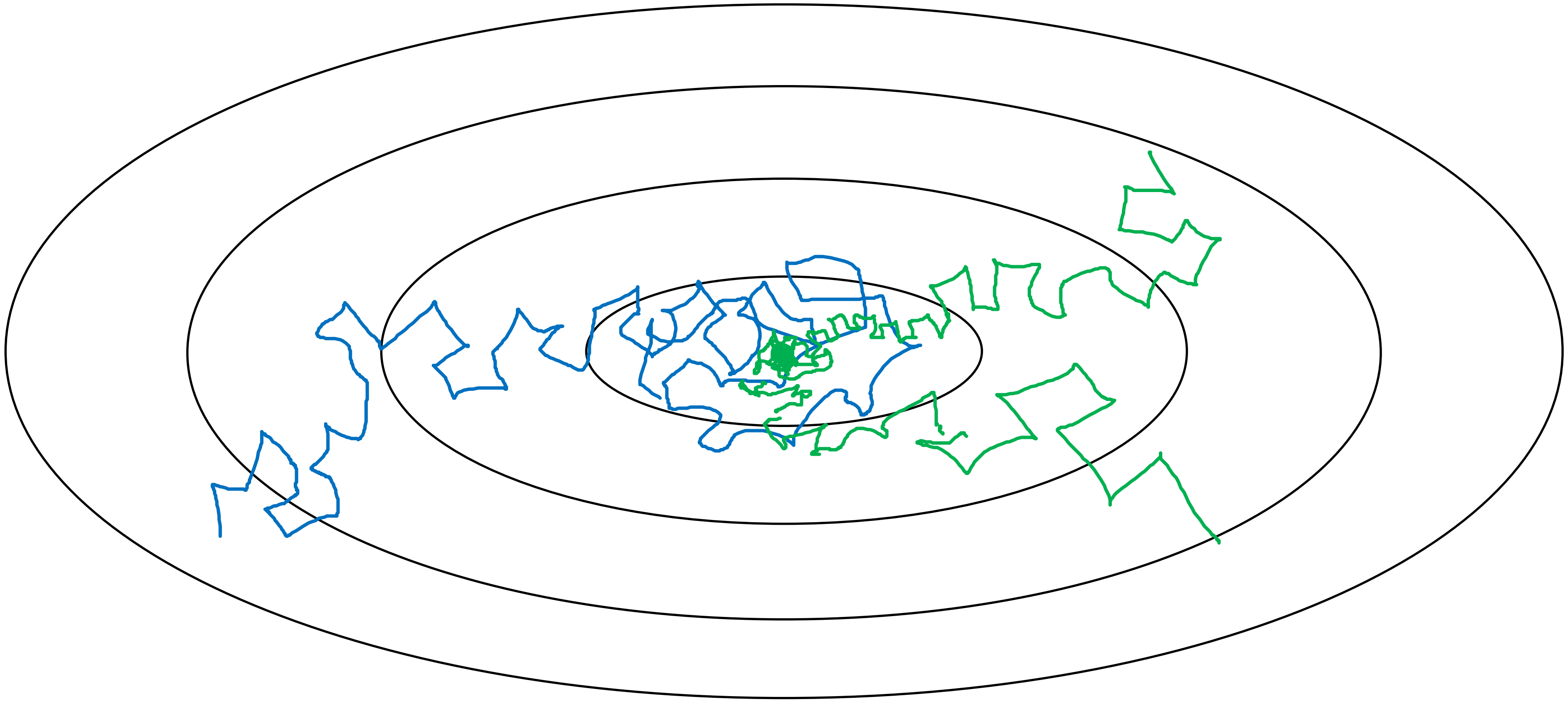
Optimization Algorithms

Learning rate decay

Learning rate decay

Slowly reduce α



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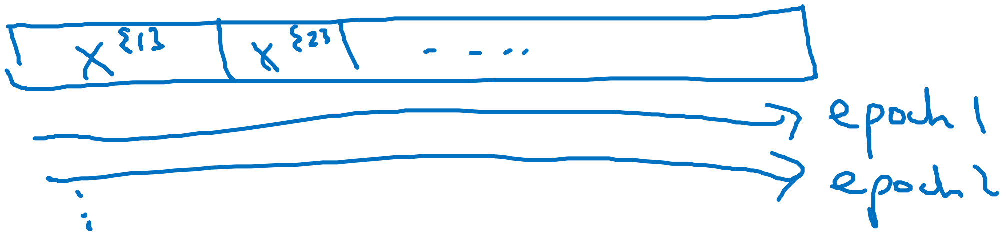
Learning rate decay

1 epoch = 1 pass through data.

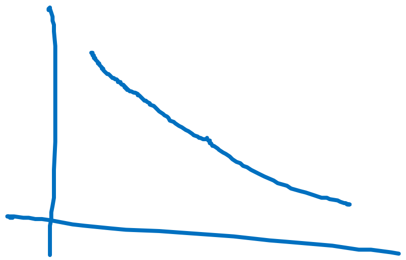


$$\underbrace{\alpha = \frac{1}{1 + \underline{\text{decay-rate}} * \text{epoch-num}}\ \underline{\alpha_0}}$$

$\alpha_0 = 0.2$

decay.rate = 1

Epoch	α
1	0.1
2	0.67
3	0.5
4	0.4
⋮	⋮



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Other learning rate decay methods

formula

$$\alpha = 0.95^{\text{epoch-num}} \cdot \alpha_0$$ — exponentially decay.

$$\alpha = \frac{k}{\sqrt{\text{epoch-num}}} \cdot \alpha_0 \quad \text{or} \quad \frac{k}{\sqrt{t}} \cdot \alpha_0$$

discrete staircase

Manual decay.

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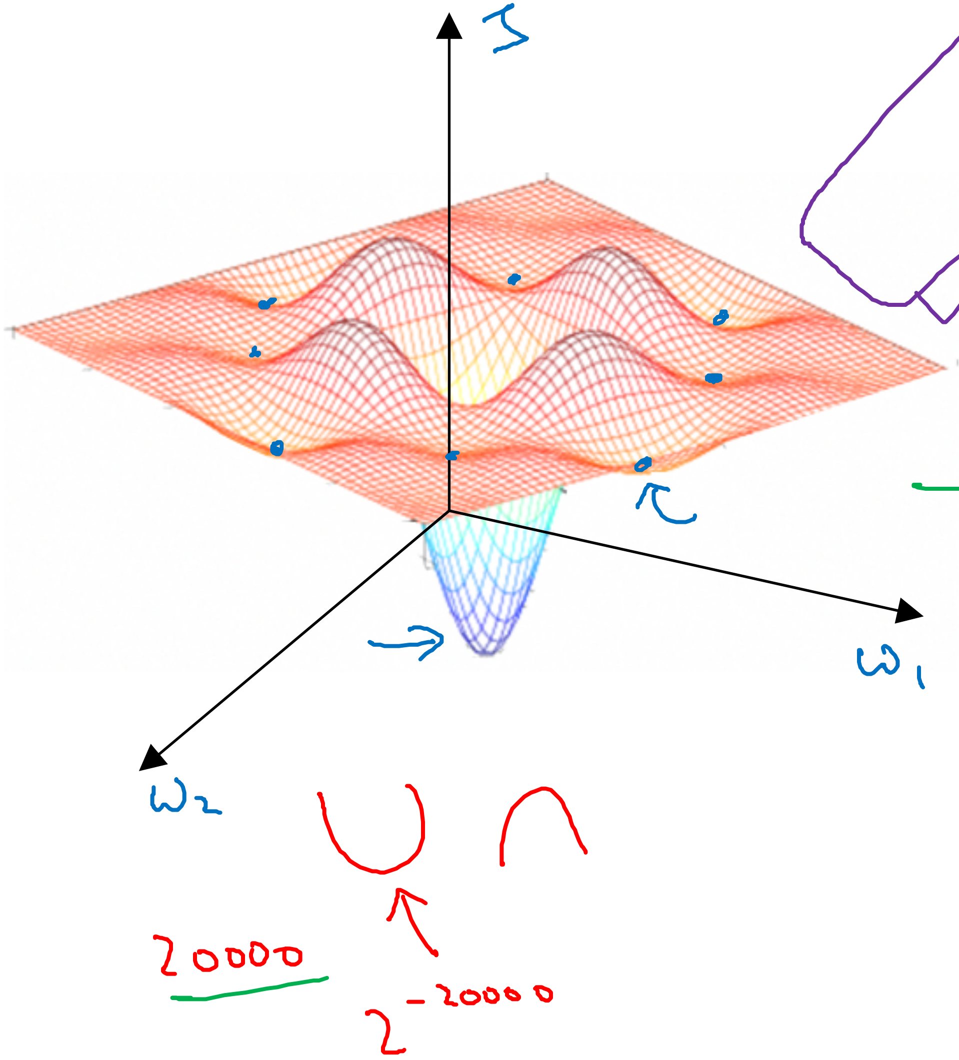
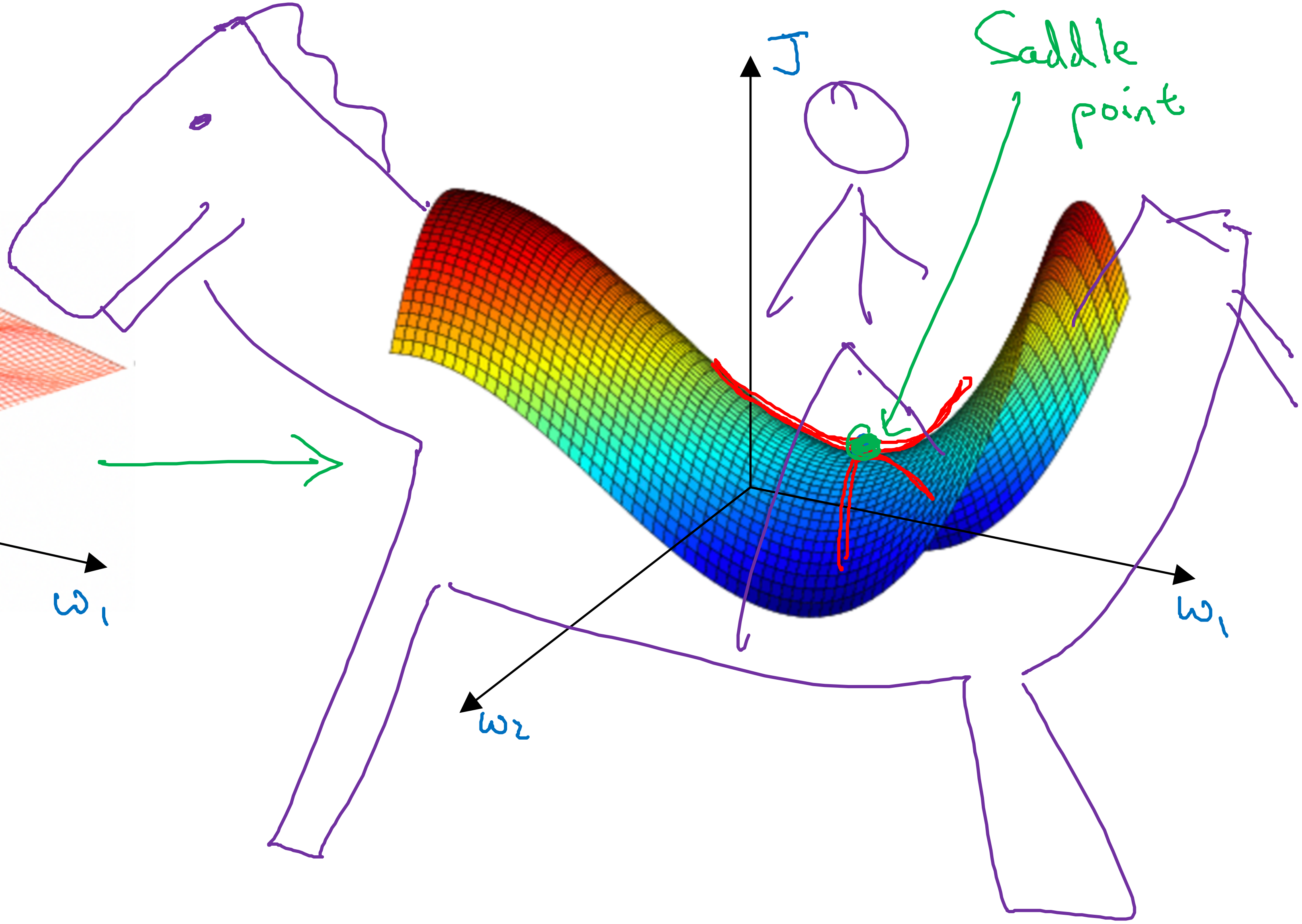
Optimization Algorithms

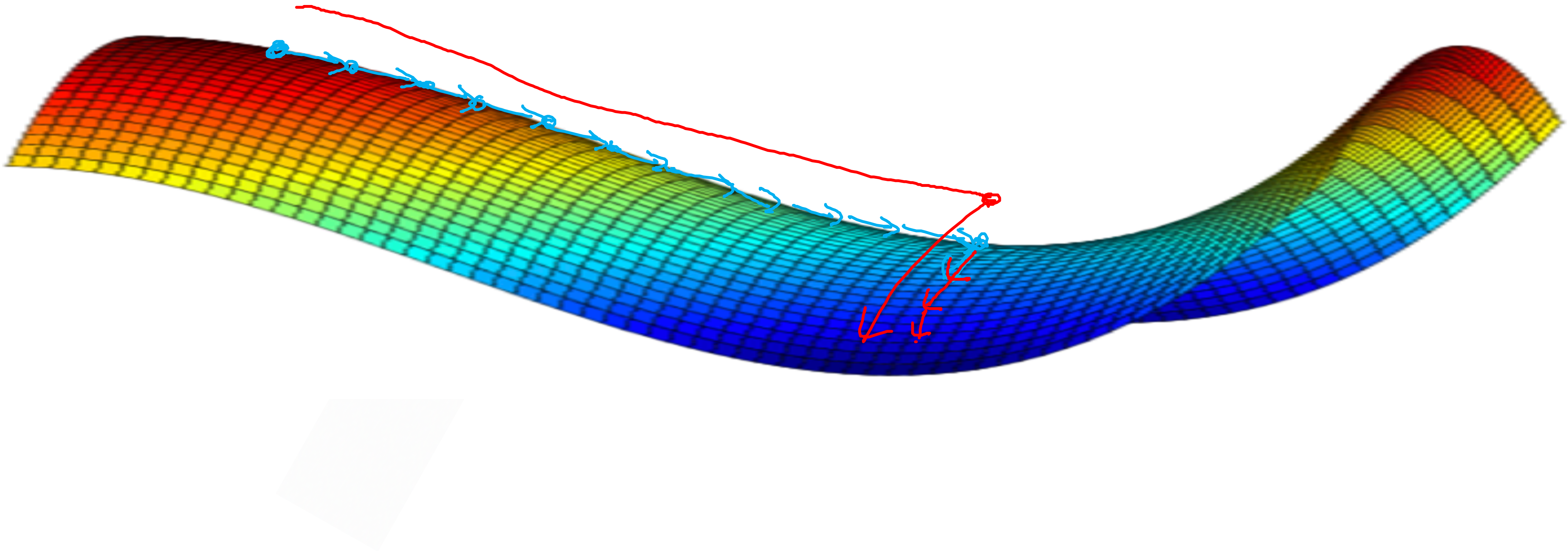
The problem of local optima

Local optima in neural networks





Problem of plateaus



- Unlikely to get stuck in a bad local optima
- Plateaus can make learning slow

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