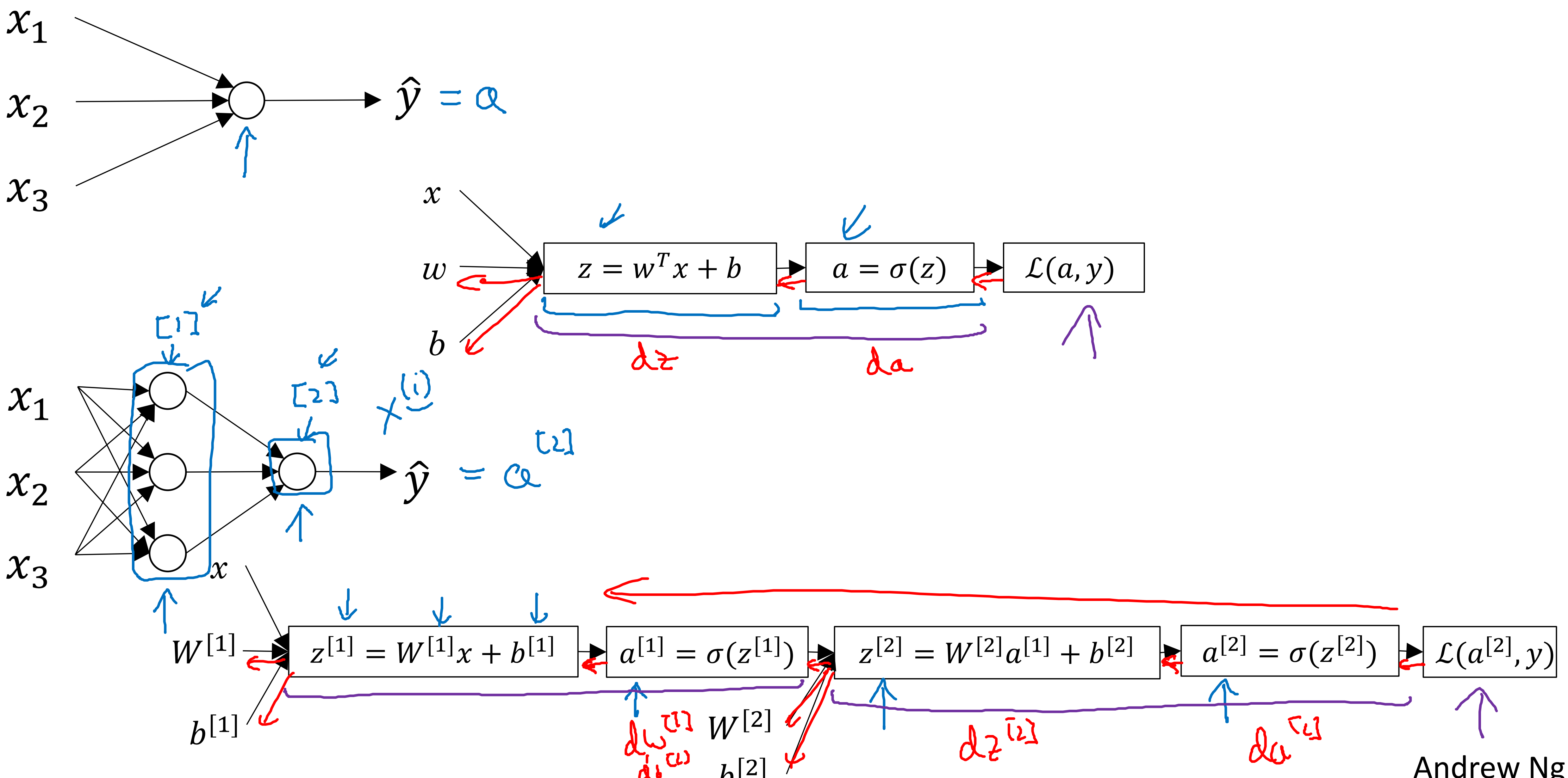
deeplearning.ai

One hidden layer Neural Network

Neural Networks Overview

What is a Neural Network?



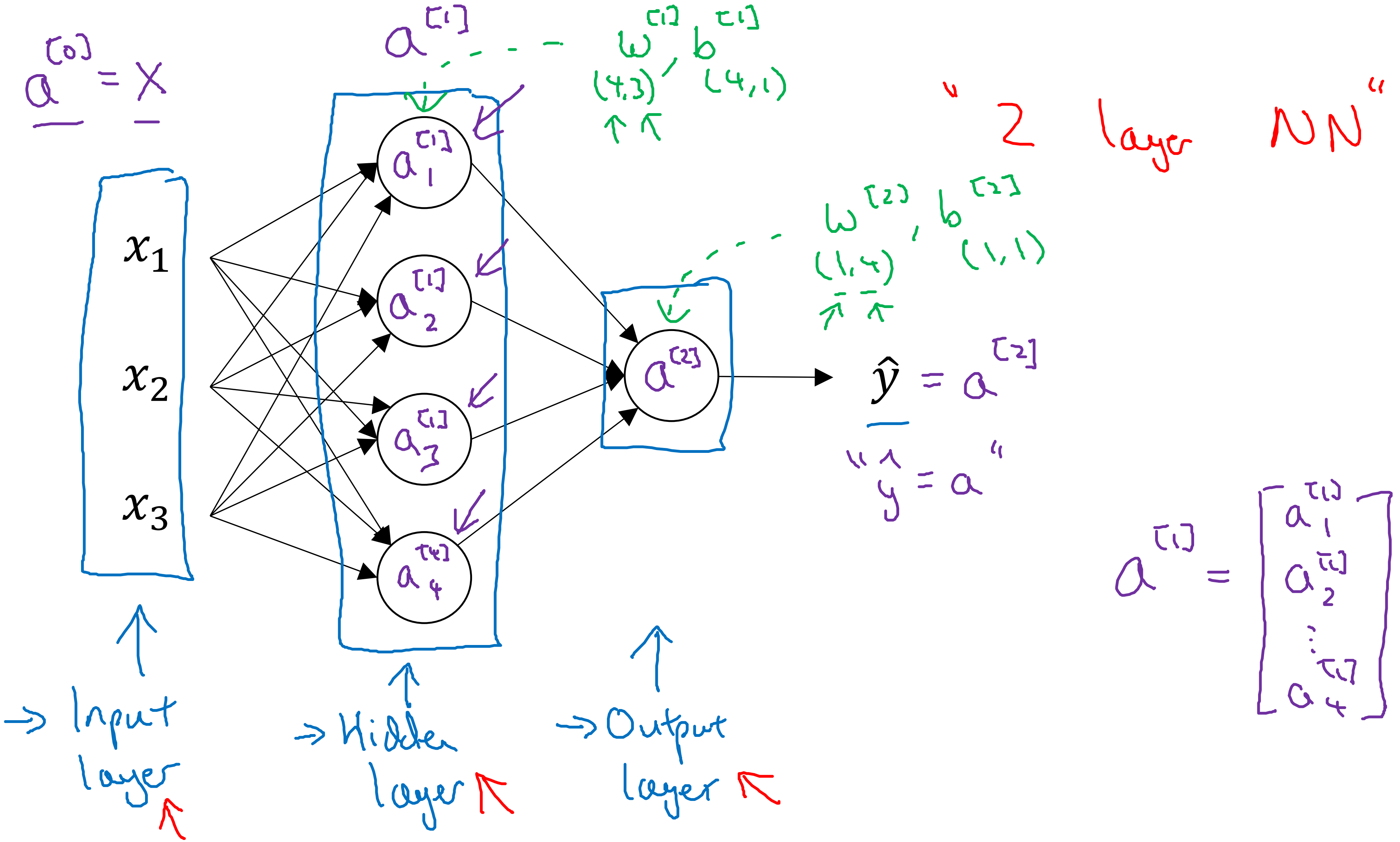
Andrew Ng

deeplearning.ai

One hidden layer Neural Network

Neural Network Representation

Neural Network Representation

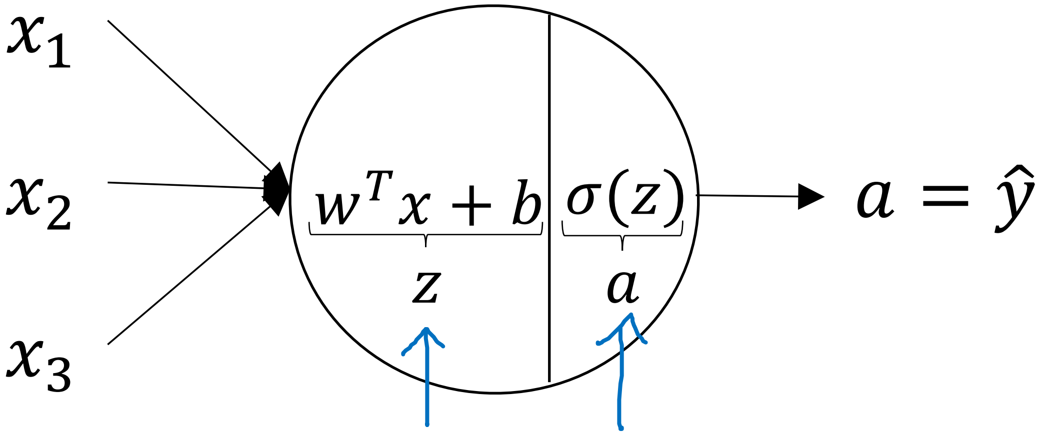


deeplearning.ai

One hidden layer Neural Network

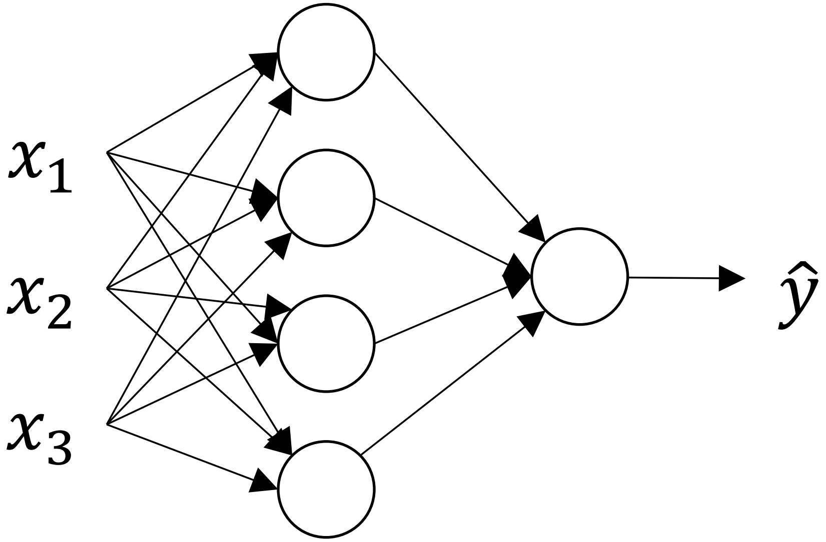
Computing a Neural Network's Output

Neural Network Representation

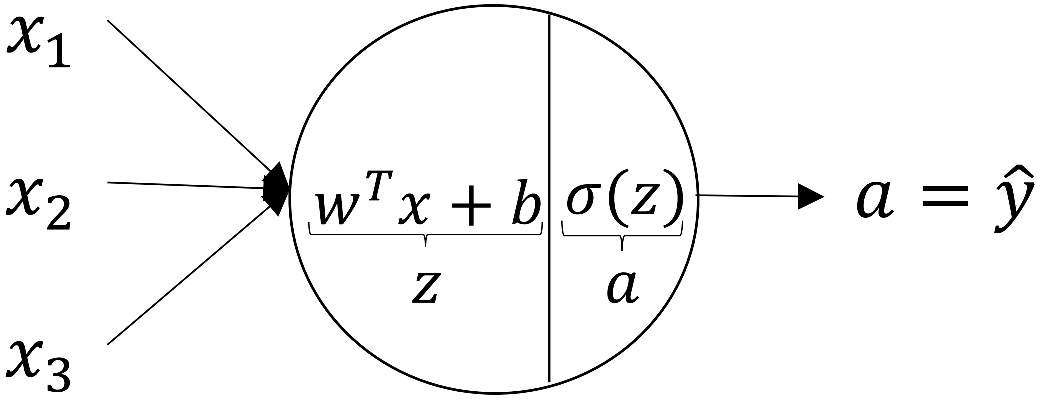


$$z = w^T x + b$$

$$a = \sigma(z)$$

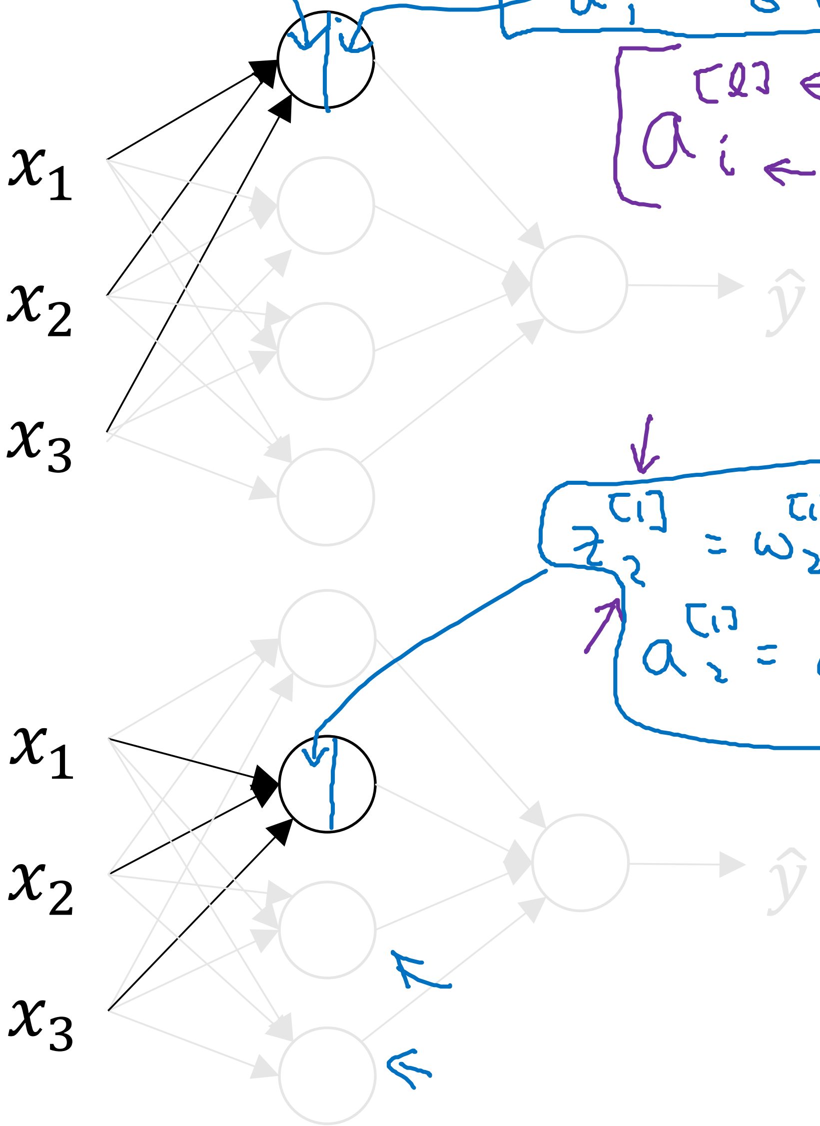


Neural Network Representation



$$z = w^T x + b$$

$$a = \sigma(z)$$



$$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]}$$

$$a_1^{[1]} = \sigma(z_1^{[1]})$$

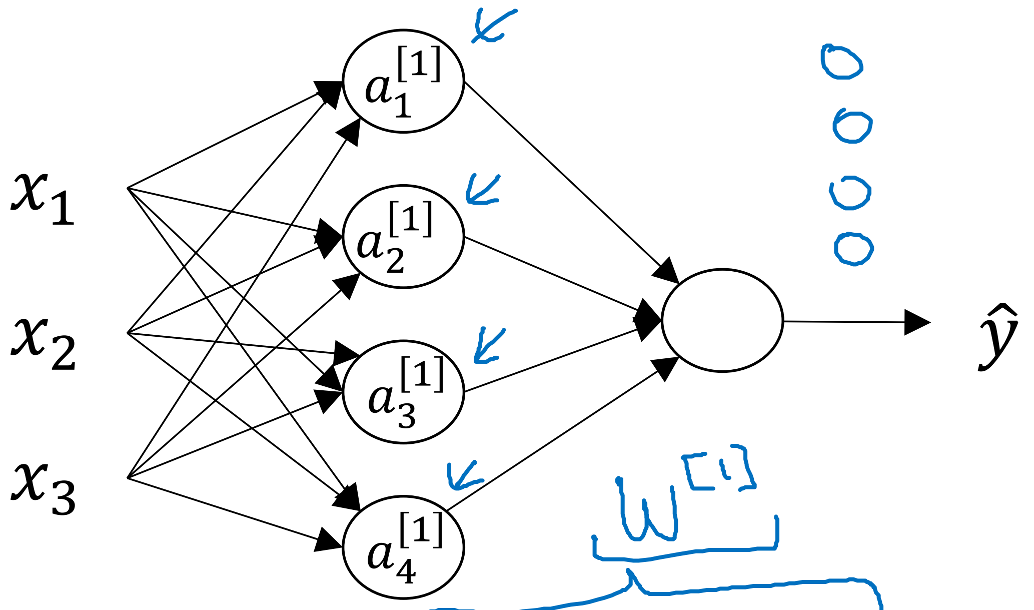
$a_i^{[l]}$ ← layer; i ← node in layer.

$$z_2^{[1]} = w_2^{[1]T} x + b_2^{[1]}$$

$$a_2^{[1]} = \sigma(z_2^{[1]})$$

Neural Network Representation

$(w_1^{[1]})^T x \qquad a^{[1]}$



$$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]}, \quad a_1^{[1]} = \sigma(z_1^{[1]})$$

$$z_2^{[1]} = w_2^{[1]T} x + b_2^{[1]}, \quad a_2^{[1]} = \sigma(z_2^{[1]})$$

$$z_3^{[1]} = w_3^{[1]T} x + b_3^{[1]}, \quad a_3^{[1]} = \sigma(z_3^{[1]})$$

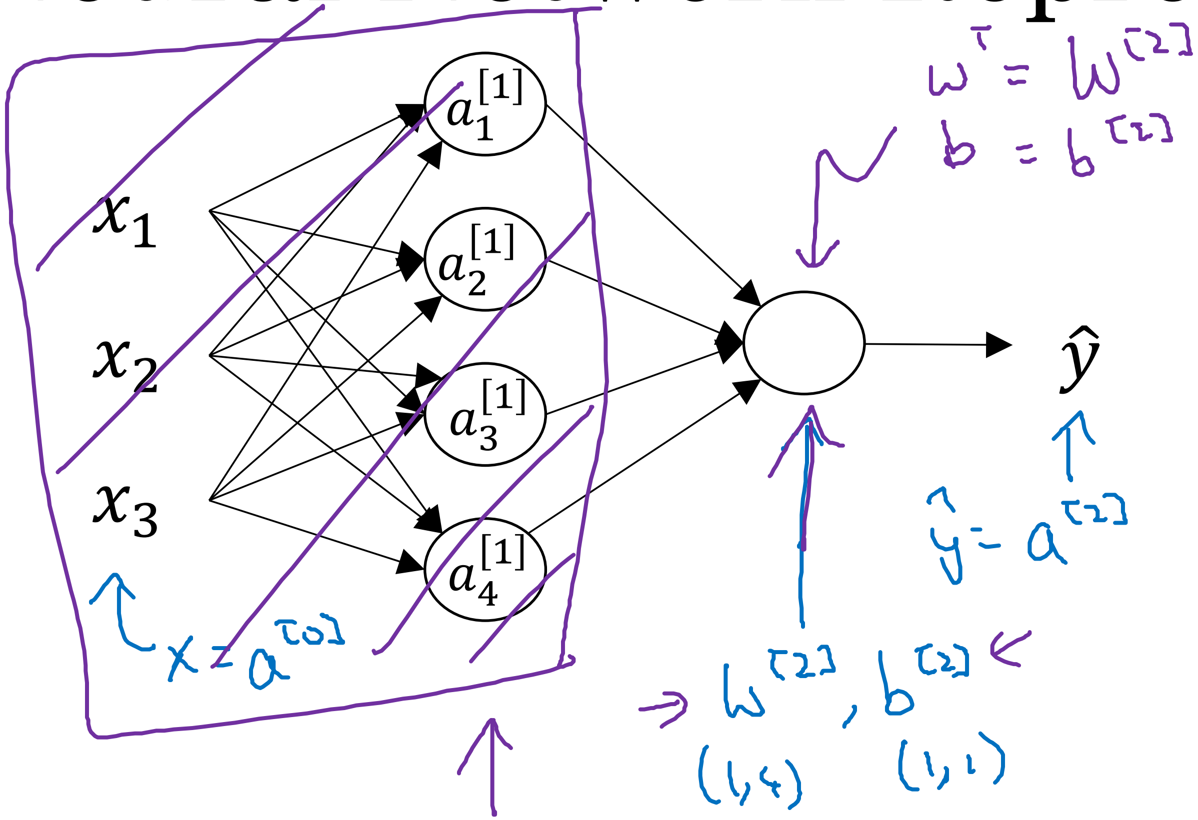
$$z_4^{[1]} = w_4^{[1]T} x + b_4^{[1]}, \quad a_4^{[1]} = \sigma(z_4^{[1]})$$

$$z^{[1]} = \underbrace{\begin{bmatrix} — w_1^{[1]T} — \\ — w_2^{[1]T} — \\ — w_3^{[1]T} — \\ — w_4^{[1]T} — \end{bmatrix}}_{W^{[1]}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix}}_{b^{[1]}} = \begin{bmatrix} w_1^{[1]T} x + b_1^{[1]} \\ w_2^{[1]T} x + b_2^{[1]} \\ w_3^{[1]T} x + b_3^{[1]} \\ w_4^{[1]T} x + b_4^{[1]} \end{bmatrix} = \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix}$$

$(4,3) \qquad (4,1)$

$$a^{[1]} = \begin{bmatrix} a_1^{[1]} \\ \vdots \\ a_4^{[1]} \end{bmatrix} = \sigma(z^{[1]})$$

Neural Network Representation learning



$z = w^T x + b$

$\hat{y} = a = \sigma(z)$

Given input x:

$$\underset{(4,1)}{z^{[1]}} = \underset{(4,3)}{W^{[1]}} \underset{(3,1)}{\overset{a^{[0]}}{\cancel{x}}} + \underset{(4,1)}{b^{[1]}}$$

$$\underset{(4,1)}{a^{[1]}} = \sigma(\underset{(4,1)}{z^{[1]}})$$

$$\underset{(1,1)}{z^{[2]}} = \underset{(1,4)}{W^{[2]}} \underset{(4,1)}{a^{[1]}} + \underset{(1,1)}{b^{[2]}}$$

$$\underset{(1,1)}{a^{[2]}} = \sigma(\underset{(1,1)}{z^{[2]}})$$

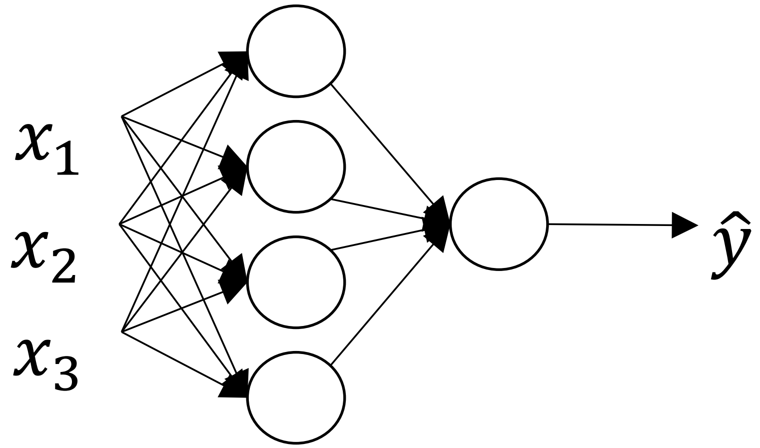
Andrew Ng

One hidden layer Neural Network

deeplearning.ai

Vectorizing across multiple examples

Vectorizing across multiple examples



$$z^{[1]} = W^{[1]}x + b^{[1]}$$
$$a^{[1]} = \sigma(z^{[1]})$$
$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$
$$a^{[2]} = \sigma(z^{[2]})$$

$x \longrightarrow a^{[2]} = \hat{y}$

$x^{(1)} \longrightarrow a^{[2](1)} = \hat{y}^{(1)}$

$x^{(2)} \longrightarrow a^{[2](2)} \quad \hat{y}^{(2)}$

$\vdots$

$x^{(m)} \longrightarrow a^{[2](m)} \quad \hat{y}^{(m)}$

$a^{[2](i)}$ — (i): example i; $[2]$: layer 2

for $i = 1$ to m,

$z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$

$a^{[1](i)} = \sigma(z^{[1](i)})$

$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$

$a^{[2](i)} = \sigma(z^{[2](i)})$

Andrew Ng

Vectorizing across multiple examples

for i = 1 to m:

$$\boxed{z^{[1](i)}} = W^{[1]}x^{(i)} + b^{[1]}$$

$$\boxed{a^{[1](i)}} = \sigma(z^{[1](i)})$$

$$\boxed{z^{[2](i)}} = W^{[2]}a^{[1](i)} + b^{[2]}$$

$$\boxed{a^{[2](i)}} = \sigma(z^{[2](i)})$$

$$\boxed{\begin{aligned} Z^{[1]} &= W^{[1]}X + b^{[1]} \\ A^{[1]} &= \sigma(Z^{[1]}) \\ Z^{[2]} &= W^{[2]}A^{[1]} + b^{[2]} \\ A^{[2]} &= \sigma(Z^{[2]}) \end{aligned}}$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

(n_x, m)

training examples

hidden units.

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

hidden units

Andrew Ng

One hidden layer Neural Network

deeplearning.ai

Explanation for vectorized implementation

Justification for vectorized implementation

$z^{[1](1)} = W^{[1]}x^{(1)} + \cancel{b^{[1]}} \rightarrow 0$, $\quad z^{[1](2)} = W^{[1]}x^{(2)} + \cancel{b^{[1]}} \rightarrow 0$, $\quad z^{[1](3)} = W^{[1]}x^{(3)} + \cancel{b^{[1]}} \rightarrow 0$

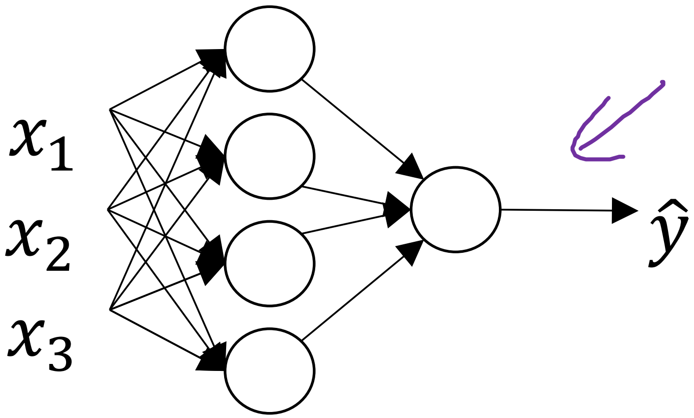
$W^{[1]} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}$ $\qquad W^{[1]}x^{(1)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$ $\qquad W^{[1]}x^{(2)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$ $\qquad W^{[1]}x^{(3)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$

$$W^{[1]} \underbrace{\begin{bmatrix} | & | & | & \\ x^{(1)} & x^{(2)} & x^{(3)} & \cdots \\ | & | & | & \end{bmatrix}}_{X} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} | & | & | & \\ z^{[1](1)} & z^{[1](2)} & z^{[1](3)} & \cdots \\ | & | & | & \\ +b^{[1]} & +b^{[1]} & +b^{[1]} & \end{bmatrix} = Z^{[1]}$$

$Z^{[1]} = W^{[1]}X + b^{[1]}$

$W^{[1]}x^{(i)} = z^{[1](i)}$

Recap of vectorizing across multiple examples




```
for i = 1 to m
```

$$\boxed{z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}}$$

$$a^{[1](i)} = \sigma(z^{[1](i)})$$

$$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$

$$a^{[2](i)} = \sigma(z^{[2](i)})$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$A^{[0]}$ $\quad X = a^{[0]}$ $\quad x^{(i)} = a^{[0](i)}$

$$Z^{[1]} = W^{[1]}X + b^{[1]} \leftarrow W^{[1]}A^{[0]} + b^{[1]}$$

$$A^{[1]} = \sigma(Z^{[1]})$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = \sigma(Z^{[2]})$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

Andrew Ng

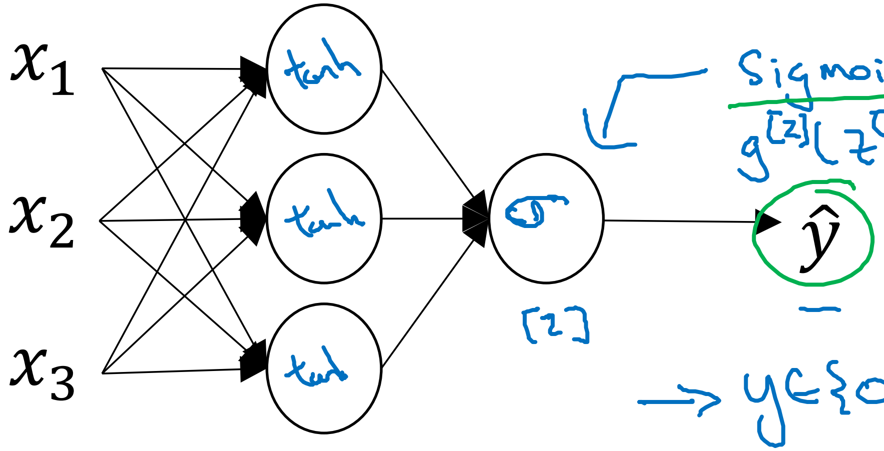
One hidden layer Neural Network

Activation functions

deeplearning.ai

Activation functions

$g^{[1]}(z^{[1]}) = \tanh(z^{[1]})$



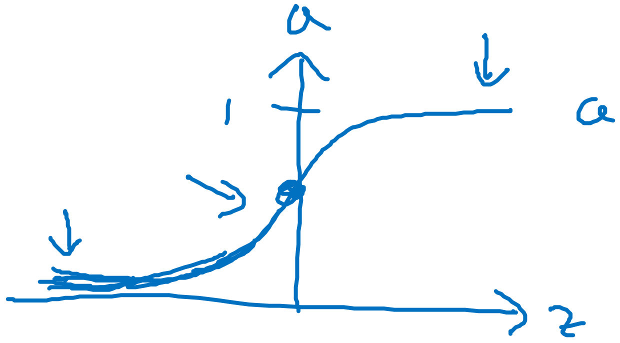
Sigmoid

$g^{[2]}(z^{[2]}) = \sigma(z^{[2]})$

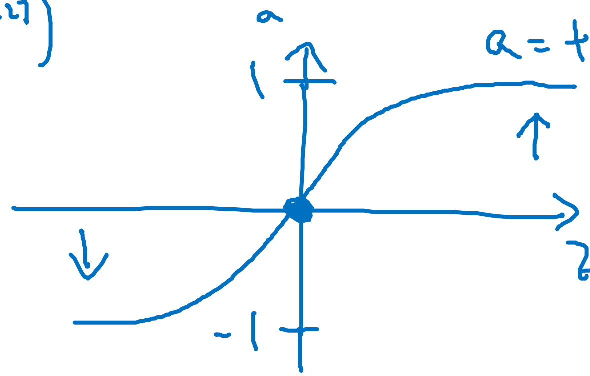
$\rightarrow y \in \{0, 1\}$

$0 \le \hat{y} \le 1$

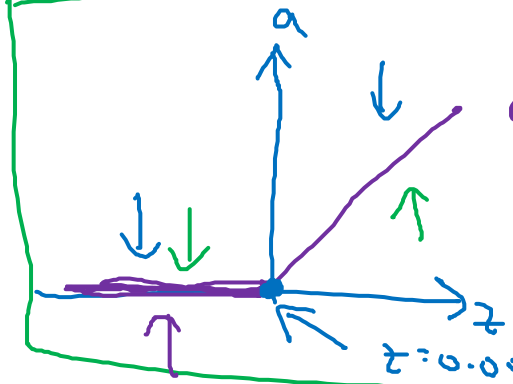
-1 1



$a = \frac{1}{1+e^{-z}}$



$a = \tanh(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$

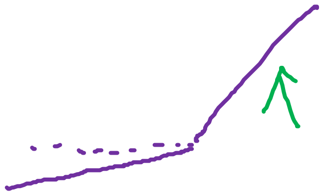


$z > 0$

$a = \max(0, z)$

ReLU

Rectified Linear Unit



Leaky ReLU

Given x:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

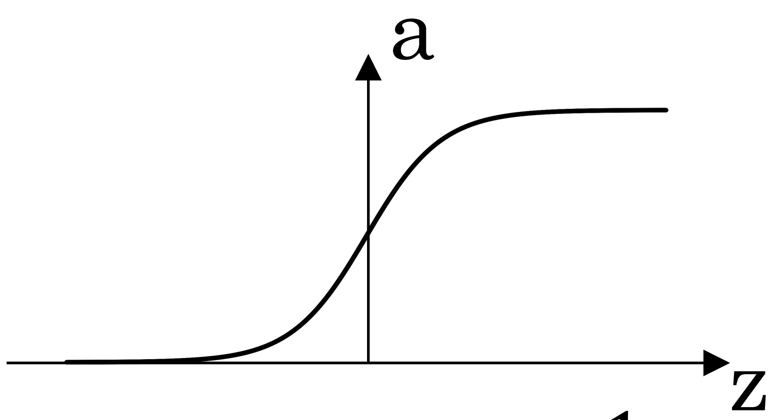
$$\rightarrow a^{[1]} = \cancel{\sigma(z^{[1]})}\ g^{[1]}(z^{[1]})$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

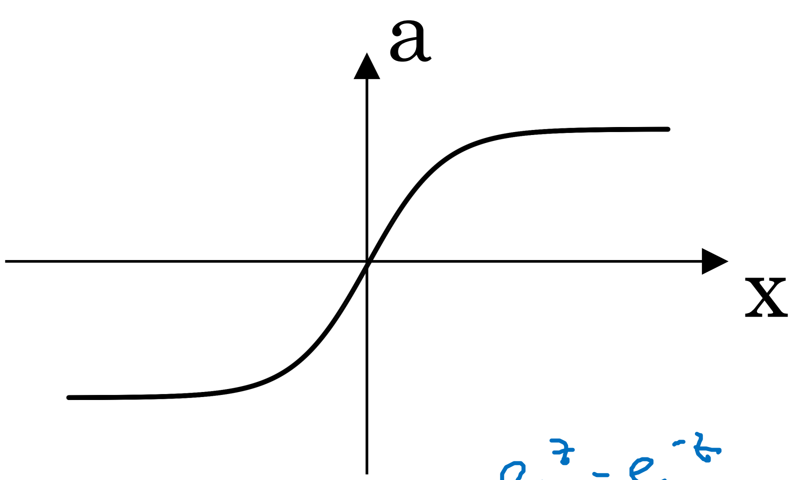
$$\rightarrow a^{[2]} = \cancel{\sigma(z^{[2]})}\ g^{[2]}(z^{[2]})$$

Andrew Ng

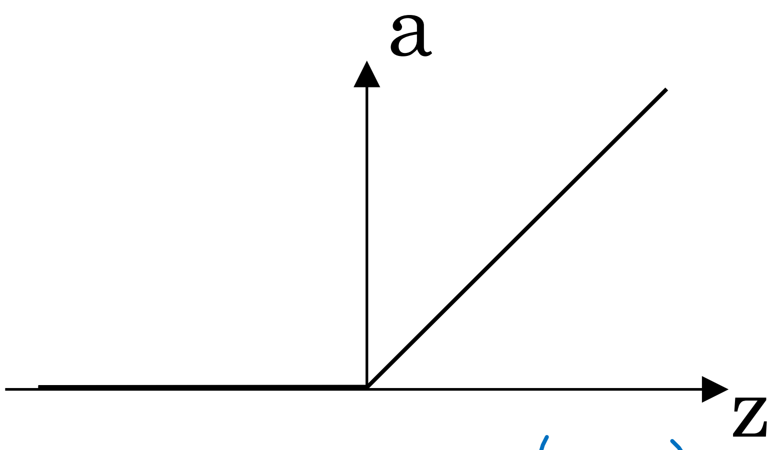
Pros and cons of activation functions



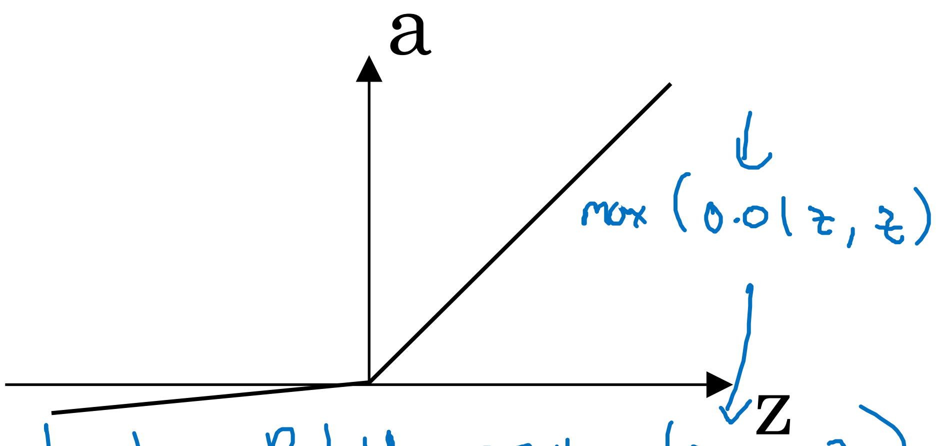
sigmoid: $a = \dfrac{1}{1 + e^{-z}}$



tanh : $a = \dfrac{e^{z} - e^{-z}}{e^{z} + e^{z}}$



ReLU $\quad a = \max(0, z)$



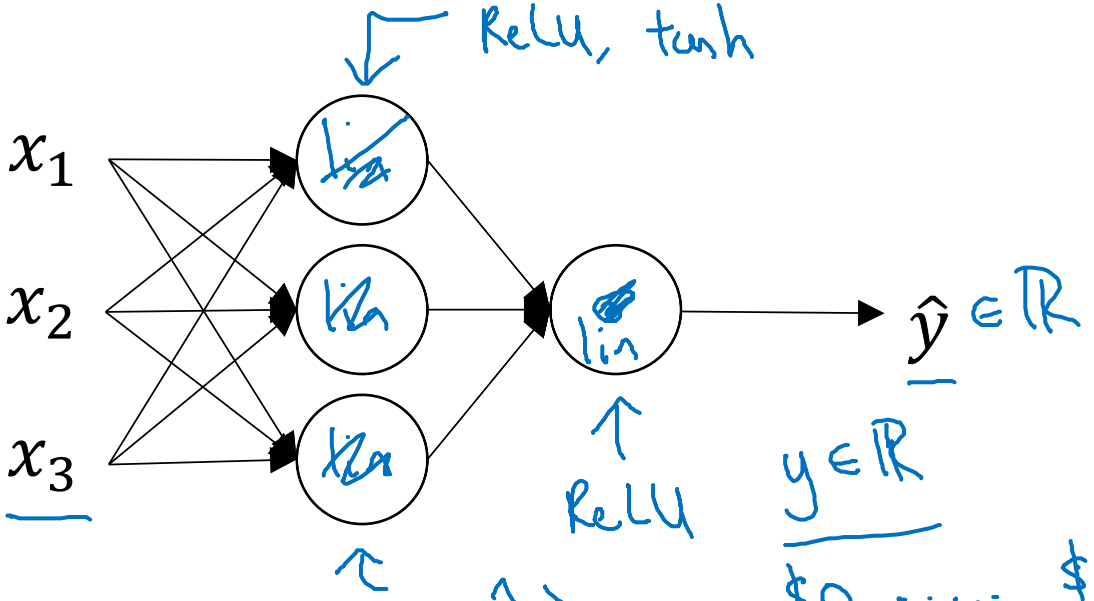
Leaky ReLU $\quad a = \max(0.01z, z)$

deeplearning.ai

One hidden layer Neural Network

Why do you need non-linear activation functions?

Activation function



$y \in \mathbb{R}$

\$0 ..... \$1,000,000s

$\hat{y} \geq 0.$

Given x:

$\rightarrow z^{[1]} = W^{[1]}x + b^{[1]}$

$\rightarrow a^{[1]} = \cancel{g^{[1]}(z^{[1]})}\ z^{[1]}$

$\rightarrow z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$

$\rightarrow a^{[2]} = \cancel{g^{[2]}(z^{[2]})}\ z^{[2]}$

$g(z) = z$

"linear activation function"

$a^{[1]} = z^{[1]} = W^{[1]}x + b^{[1]}$

$a^{[2]} = z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$

$a^{[2]} = W^{[2]}(\underbrace{W^{[1]}x + b^{[1]}}_{a^{[1]}}) + b^{[2]}$

$= \underbrace{(W^{[2]}W^{[1]})}_{W'}x + \underbrace{(W^{[2]}b^{[1]} + b^{[2]})}_{b'}$

$= \underline{W'x + b'}$

$g(z) = z$

One hidden layer Neural Network

Gradient descent for neural networks

deeplearning.ai

Gradient descent for neural networks

Parameters: $\underset{(n^{[1]}, n^{[0]})}{W^{[1]}}, \underset{(n^{[1]}, 1)}{b^{[1]}}, \underset{(n^{[2]}, n^{[1]})}{W^{[2]}}, \underset{(n^{[2]}, 1)}{b^{[2]}}$ $\quad n_x = n^{[0]}, n^{[1]}, n^{[2]} = 1$

Cost function: $J(W^{[1]}, b^{[1]}, W^{[2]}, b^{[2]}) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$ $\quad \hat{y} \leftarrow a^{[2]}$

Gradient descent:

→ Repeat {

→ Compute predicts $(\hat{y}^{(i)}, i = 1 \ldots, m)$

$dW^{[1]} = \frac{\partial J}{\partial W^{[1]}}, \quad db^{[1]} = \frac{\partial J}{\partial b^{[1]}}, \ldots$

$W^{[1]} := W^{[1]} - \alpha \, dW^{[1]}$

$b^{[1]} := b^{[1]} - \alpha \, db^{[1]}$

$W^{[2]} := \ldots \quad b^{[2]} := \ldots$

}

Andrew Ng

Formulas for computing derivatives

Forward propagation:

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]}) \leftarrow$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = g^{[2]}(Z^{[2]}) = \sigma(Z^{[2]})$$

Back propagation:

$$dZ^{[2]} = A^{[2]} - Y \qquad Y = [y^{(1)}\ y^{(2)} \ldots, y^{(m)}]$$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T}$$

$(n^{[2]},) \leftarrow$

$(n^{[2]}, 1) \leftarrow$

$$db^{[2]} = \frac{1}{m} \underline{\text{np.sum}}(dZ^{[2]}, \underline{\text{axis}=1}, \underline{\text{keepdims}=\text{True}})$$

$$dZ^{[1]} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]}, m)} * \underbrace{g^{[1]\prime}(Z^{[1]})}_{(n^{[1]}, m)}$$

element-wise product

$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$$

$$db^{[1]} = \frac{1}{m} \text{np.sum}(dZ^{[1]}, \text{axis}=1, \underline{\text{keepdims}=\text{True}})$$

$\underline{(n^{[1]}, 1)}$ $\qquad (n^{[1]},)$

reshape

Andrew Ng

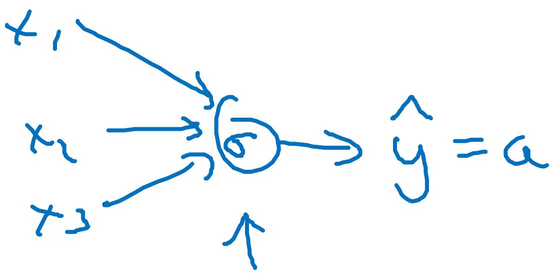
deeplearning.ai

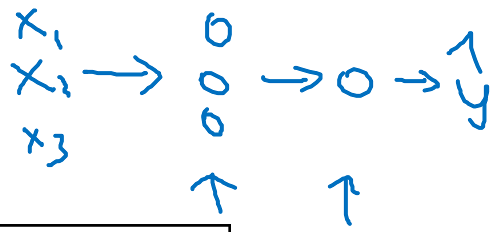
One hidden layer Neural Network

Backpropagation intuition (Optional)

Computing gradients

Logistic regression





$$x,\ w,\ b \rightarrow \boxed{z = w^T x + b} \rightarrow \boxed{a = \sigma(z)} \rightarrow \boxed{\mathcal{L}(a, y)}$$

$dz = a - y$

$da = \frac{d}{da}\mathcal{L}(a,y) = -y\log a - (1-y)\log(1-a)$

$= -\frac{y}{a} + \frac{1-y}{1-a}$

$dw = dz \cdot x$

$db = dz$

$\boxed{dz = da \cdot g'(z)}$

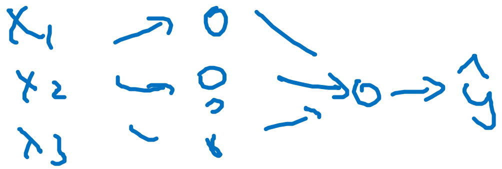
$g(z) = \sigma(z)$

$\boxed{\frac{\partial \mathcal{L}}{\partial z} = \frac{\partial \mathcal{L}}{\partial a} \cdot \boxed{\frac{da}{dz}}}$

"dz" = "da"

$\frac{d}{dz} g(z) = g'(z)$

Neural network gradients



$n_x = n^{[0]}$ $\quad n^{[1]}$ $\quad n^{[2]} = 1$

x, $W^{[1]}$, $b^{[1]}$ → $z^{[1]} = W^{[1]}x + b^{[1]}$ → $a^{[1]} = \sigma(z^{[1]})$ → $z^{[2]} = W^{[2]}x + b^{[2]}$ → $a^{[2]} = \sigma(z^{[2]})$ → $\mathcal{L}(a^{[2]}, y)$

$W^{[2]}$ $\quad dW^{[2]}$

$b^{[2]}$ $\quad db^{[2]}$

$dW^{[1]}$ $\quad db^{[1]}$

$dz^{[1]} = W^{[2]T} dz^{[2]}$ $\quad da^{[1]} = \dots$ $\quad dz^{[2]} = a^{[2]} - y$ $\quad da^{[2]}$

$* \, g^{[1]'}(z^{[1]})$

element-wise product

$dW^{[2]} = dz^{[2]} a^{[1]T}$

$db^{[2]} = dz^{[2]}$

"$dw = dz \cdot x$"

$W^{[1]}$: [ ]

$W^{[2]}$ $\quad (n^{[2]}, n^{[1]})$

$z^{[2]}, dz^{[2]}$ $\quad (n^{[2]}, 1)$ - (1,1)

$z^{[1]}, dz^{[1]}$ $\quad (n^{[1]}, 1)$

$dz^{[1]} = \boxed{W^{[2]T} dz^{[2]}} * \boxed{g^{[1]'}(z^{[1]})}$

$(n^{[1]}, 1) = (n^{[1]}, n^{[2]}) \; (n^{[2]}, 1) * (n^{[1]}, 1)$

foo $\quad W^{[1]}$

dfoo $\quad dW^{[2]}$

$dW^{[1]} = dz^{[1]} \cdot x^T$

$db^{[1]} = dz^{[1]}$

$a^{[0]T}$

Andrew Ng

Summary of gradient descent

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]} {a^{[1]}}^T$$

$$db^{[2]} = dz^{[2]}$$

$$dz^{[1]} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(\mathrm{z}^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T$$

$$db^{[1]} = dz^{[1]}$$

Vectorized Implementation:

$$z^{[1]} = W^{[1]} x + b^{[1]}$$

$$a^{[1]} = g^{[1]}(z^{[1]})$$

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$Z^{[1]} = W^{[1]} X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]})$$

Summary of gradient descent

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]} a^{[1]^T}$$

$$db^{[2]} = dz^{[2]}$$

$$\underset{(n^{[1]},1)}{dz^{[1]}} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(z^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T$$

$$db^{[1]} = dz^{[1]}$$

$$dZ^{[2]} = A^{[2]} - Y$$

$$J(\ldots) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]^T}$$

$$db^{[2]} = \frac{1}{m} np.sum(dZ^{[2]}, axis = 1, keepdims = True)$$

elementwise product

$$\underset{(n^{[1]},m)}{dZ^{[1]}} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]},m)} * \underbrace{g^{[1]\prime}(Z^{[1]})}_{(n^{[1]},m)}$$

$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$$

$$db^{[1]} = \frac{1}{m} np.sum(dZ^{[1]}, axis = 1, keepdims = True)$$

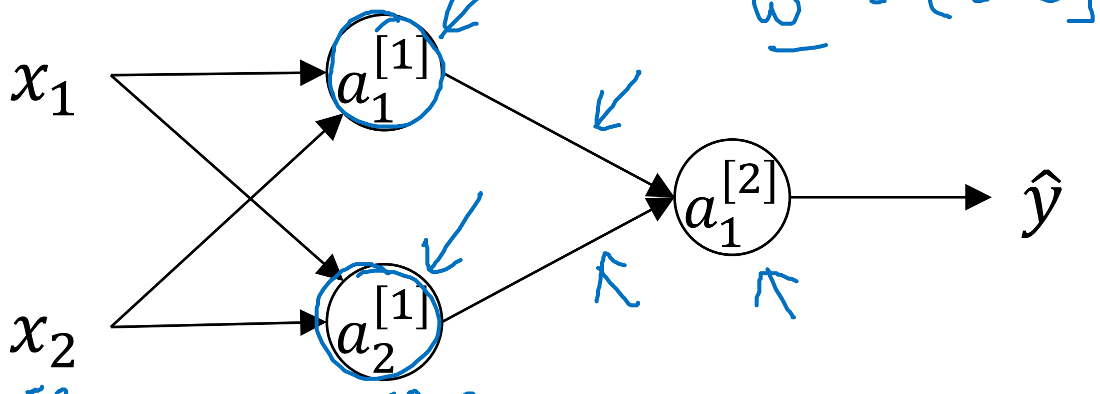
One hidden layer Neural Network

Random Initialization

deeplearning.ai

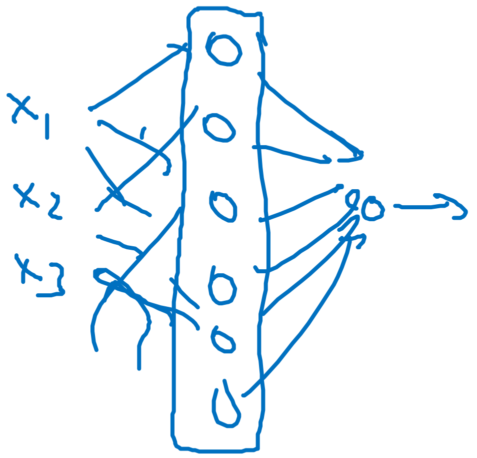
What happens if you initialize weights to zero?

$W^{[2]} = \begin{bmatrix} 0 & 0 \end{bmatrix}$



$n^{[0]} = 2$ $\quad$ $n^{[1]} = 2$

Symmetric



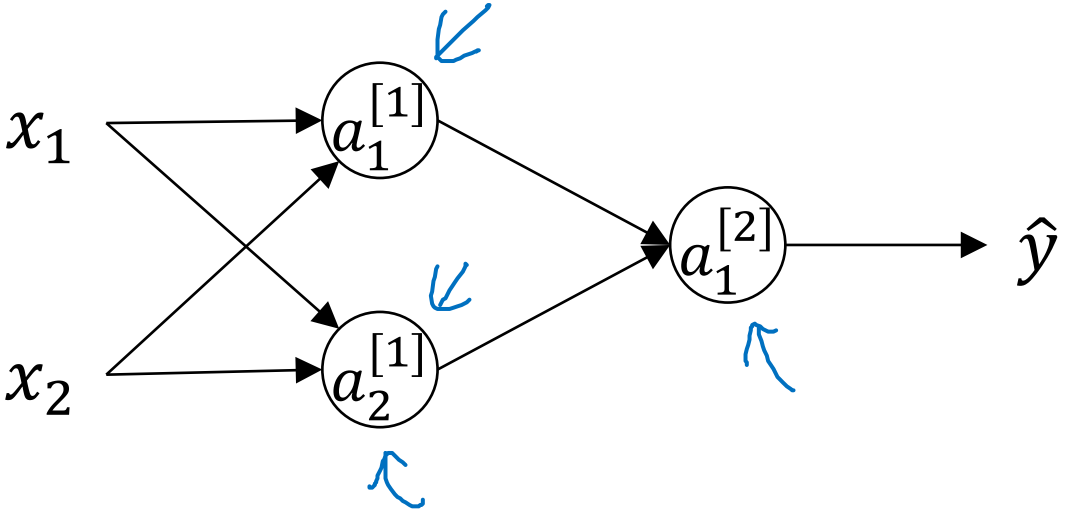
$W^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\qquad$ $b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$W^{[1]} = \begin{bmatrix} \cdots\cdots \\ \cdots\cdots \end{bmatrix}$

$a_1^{[1]} = a_2^{[1]}$ $\qquad$ $dz_1^{[1]} = dz_2^{[1]}$

$dW = \begin{bmatrix} u & v \\ u & v \end{bmatrix}$ $\qquad$ $W^{[1]} = W^{[1]} - \alpha \, dW$

Random initialization



$\rightarrow W^{[1]} =$ np.random.randn((2,2)) $* \; 0.01$

100?

$b^{[1]} =$ np.zeros((2,1))

$W^{[2]} =$ np.random.randn((1,2)) $* \; 0.01$

$b^{[2]} = 0$

$\rightarrow z^{[1]} = W^{[1]} x + b^{[1]}$

$a^{[1]} = g^{[1]}(z^{[1]})$