

Estimation of three-phase induction motor parameters

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Abstract

Determination of a three-phase induction motor's characteristics from its equivalent circuit is very common and widely used. Computation of equivalent circuit parameters from test data is expensive. Alternative methods reported for parameter estimation, which do not require test data, give the parameter values of the approximate equivalent circuit of an induction motor. Characteristics obtained from the approximate equivalent circuit are, in general, not accurate enough for practical applications. This paper proposes a new method of estimating parameter values of the exact equivalent circuit of a three-phase induction motor. **The method requires the name plate data, ratio of starting to full load torque and the efficiency and power factor values at half and full load.** The proposed method was tested on several motors. The results/characteristics obtained by using the parameter values found by the proposed method and by Natarajan–Misra's method were compared and also checked with some of the actual values supplied by the manufacturer.

Keywords: induction motor parameters.

1. Introduction

The induction motor is the most commonly used electric motor. It has lower capital cost, simple and rugged construction and requires less maintenance than other types of electric motor. In fact, more than 60% of the connected load in the United States is comprised of induction motors of different sizes. It is of great interest for the user to know the performance characteristics (current, efficiency, power factor, torque, slip, etc.) of the motor for various load conditions (output horsepower). Based on these characteristics the user may determine the relative efficiency and economics of the motors.

Unfortunately, performance characteristics are, in general, not supplied by the manufacturer. However, these characteristics of an induction motor can be obtained from its equivalent circuit. The parameters of the equivalent circuit are usually determined from test data. **The test data required for this purpose are as follows:**

- (1) DC resistance measurement between the terminals;
- (2) no-load test data for different applied voltages;
- (3) blocked rotor test data (the test should be done at 15–25 Hz rather than at 60 Hz to compen-

sate for any effects of slot depth or winding configuration on the rotor [1]).

Procedures for determining parameter values of the equivalent circuit from the above test data are discussed in several textbooks on electric machinery [2–4] and in *IEEE Standard 112* [1]. Several methods are also reported in the literature to determine only the efficiency characteristic (efficiency vs. horsepower) of an induction motor from field test data [5–7].

The industrial user may have a large number of induction motors of different sizes. Determination of parameters of each motor from the above test data may be very laborious and prohibitive. Also, the industry may not have adequate laboratory facilities for testing motors of different sizes. In such cases, a simple and effective method of estimating parameter values with acceptable accuracy, without knowing the test data, is highly desirable.

Pereira [8] estimated the parameter values of an induction motor/generator by a simple method using the name plate information, starting torque and the efficiency and power factor values at three different operating points (50%, 75% and 100% of full load). These data can be obtained from the manufacturer at a nominal cost. Based on Pereira's method, Natarajan and Misra [9]

estimated the parameters of several induction motors using spreadsheet software on a personal computer. Both of the methods are suitable for determining the parameter values of the approximate equivalent circuit of an induction motor. Characteristics obtained from an approximate equivalent circuit are, in general, not accurate enough and comparison of relative efficiency and economics of the motors based on these characteristics may not be useful.

This paper presents a new method of estimating the parameter values of the exact equivalent circuit of a three-phase induction motor. The method requires the name plate data, ratio of starting to full load torque and the efficiency and power factor values at 50% and 100% of full load of the motor. The parameters and, hence, characteristics obtained by the proposed method were compared with some other reported methods and also with the data supplied by the manufacturer. The results of the proposed method were found to be in very good agreement with the actual values.

2. Background

The most commonly used equivalent circuit of a three-phase induction motor is shown in Fig. 1. Let us define the following in the equivalent circuit:

stator impedance

$$Z_1 = R_1 + jX_1 \quad \Omega$$

rotor impedance referred to stator

$$Z_2 = (R_2 + R_L) + jX_2 = \frac{R_2}{S} + jX_2 \quad \Omega \quad (1)$$

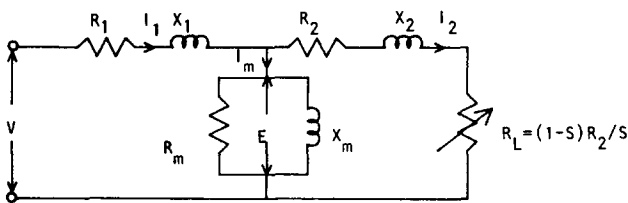


Fig. 1. Equivalent circuit of a three-phase induction motor.

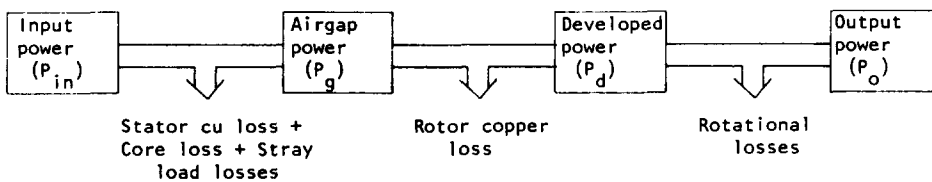


Fig. 2. Power flow diagram of a three-phase induction motor.

magnetizing impedance

$$Z_m = R_m // jX_m \quad \Omega$$

The stator current I_1 , rotor current I_2 referred to stator and the magnetizing current I_m for a given slip S are given by

$$I_1 = I_1/\theta_1 = \frac{V}{Z_1 + Z_{eq}}$$

$$I_2 = I_2/\theta_2 = \frac{E}{Z_2}$$

$$I_m = I_m/\theta_m = \frac{E}{Z_m}$$

where

$$Z_{eq} = Z_m // Z_2 \quad \text{and} \quad E = V - I_1 Z_1$$

The power flow diagram of an induction motor is shown in Fig. 2. From Figs. 1 and 2 the power at different stages can be written as follows.

Input power

$$P_{in} = 3VI_1 \cos \theta_1 \quad W$$

Stator copper loss

$$P_{Cu1} = 3I_1^2 R_1 \quad W$$

Core loss

$$P_{core} = \frac{3E^2}{R_m} \quad W \quad (2)$$

Stray load loss is an additional core loss caused by the increase of airgap leakage fluxes with load and by the higher harmonics of fluxes. It may be considered as 0.9% of the output for the motors of 2500 horsepower and larger [7]. However, this component of the losses is not considered in this study.

Airgap power

$$P_g = P_{in} - P_{Cu1} - P_{core} = 3I_2^2 \frac{R_2}{S} \quad W$$

Rotor copper loss

$$P_{Cu2} = 3I_2^2 R_2 = SP_g \quad W$$

Developed power

$$P_d = P_g - P_{Cu2} = (1 - S)P_g \quad \text{W}$$

Output power

$$P_o = (P_d - \text{rotational losses}) \quad \text{W}$$

Rotational losses, P_{rot} , are the mechanical losses due to bearing friction and windage. These may be considered as constant.

Efficiency

$$\eta = \frac{P_o}{P_{in}}$$

Power factor

$$\text{pf} = \cos \theta_1$$

Developed torque

$$T_d = \frac{P_d}{\omega_m} = \frac{P_g}{\omega_s} \quad \text{N m} \quad (3)$$

Output torque

$$T_o = \frac{P_o}{\omega_m} \quad \text{N m}$$

The sum of the core and rotational losses is considered as constant losses P_{const} and is independent of load:

$$P_{const} = P_{core} + P_{rot}$$

The developed torque at starting, T_{st} , can be obtained from eqn. (3) by using the value of P_g at $S = 1$. The approximate starting torque can also be written from the equivalent circuit, by neglecting the magnetizing impedance Z_m (because $Z_m \gg Z_2$ for $S = 1$):

$$T_{st} \approx \frac{1}{\omega_s} \frac{3V^2 R_2}{(R_1 + R_2)^2 + (X_1 + X_2)^2} \quad (4)$$

3. Proposed method

Estimation of induction motor parameters by the proposed method requires the following information/data about the motor:

(a) name plate data: horsepower, voltage, full load speed, NEMA design and connection (delta or wye);

(b) ratio of starting to full load torque;

(c) efficiency and power factor (or current) at 50% and 100% of full load.

The method first determines the rotor resistance from full load output and slip (or speed). The stator resistance and constant losses are determined from the real power balance equa-

tions at half and full load. The stator and rotor leakage reactances are determined to satisfy the starting torque criterion and NEMA design. Finally, the reactive power balance equation at full load condition is used to determine the magnetizing reactance. To get the parameter values more accurately, a simple iterative scheme is used. The procedure for determining the parameters is given in the following.

3.1. Determination of induction motor parameters

The electromechanical developed power P_d of an induction motor is the power dissipated in the dynamic load resistance R_L (Fig. 1). P_d is the sum of the output power and rotational losses. At full load, the developed power is

$$P_d = 3I_{2F}^2(1 - S_F) \frac{R_2}{S_F} = P_o + P_{rot}$$

or

$$R_2 = \frac{(P_o + P_{rot})S_F}{3I_{2F}^2(1 - S_F)} \quad (5)$$

Here, I_{2F} and S_F are the full load rotor current and slip, respectively.

From Fig. 2, the input power P_{in} ($= P_o/\eta$) comprises the output power, stator copper loss, rotor copper loss and the constant losses. Thus, the real power balance equation can be written as

$$P_{in} = \frac{P_o}{\eta} = P_o + 3I_1^2 R_1 + 3I_2^2 R_2 + P_{const} \quad (6)$$

Equation (6) at half and full load, respectively, becomes

$$3I_{1H}^2 R_1 + 3I_{2H}^2 R_2 + P_{const} = 746 \frac{\text{hp}}{2} \left(\frac{1}{\eta_H} - 1 \right) \quad (6a)$$

$$3I_{1F}^2 R_1 + 3I_{2F}^2 R_2 + P_{const} = 746 \text{ hp} \left(\frac{1}{\eta_F} - 1 \right) \quad (6b)$$

where hp is the rated output in horsepower (1 hp \cong 746 W) and subscripts H and F stand for half and full load, respectively.

By knowing the stator current, rotor current and efficiency at half and full load, eqns. (6a) and (6b) can be solved for the stator resistance R_1 and the constant losses P_{const} . It may be assumed with fair approximation that the core loss is 50% of the constant losses [8, 9]:

$$P_{core} = P_{rot} = 0.5P_{const} \quad (7)$$

From eqn. (2) the resistance R_m can be written as

$$R_m = \frac{3E_F^2}{0.5P_{const}} \quad (8)$$

The full load torque of a motor is

$$T_{FL} = \frac{746 \text{ hp}}{\omega_m} \text{ N m} \quad (9)$$

From eqns. (4) and (9) the ratio of starting to full load torque becomes

$$\frac{T_{st}}{T_{FL}} = \frac{1}{746 \text{ hp}} \frac{\omega_m}{\omega_s} \frac{3V^2 R_2}{R_T^2 + X_T^2}$$

or

$$X_T = \left[\frac{T_{FL}}{T_{st}} \frac{3V^2 R_2}{746 \text{ hp}} (1 - S_F) - R_T^2 \right]^{1/2} \quad (10)$$

where

$$R_T = R_1 + R_2, \quad X_T = X_1 + X_2$$

and

$$\omega_m = (1 - S)\omega_s$$

Once X_T is determined from eqn. (10), the stator and rotor leakage reactances can be obtained from the empirical proportions of NEMA design given in Table 1 [1].

From Fig. 1, the input reactive power to the motor is the sum of the reactive power absorbed by X_1 , X_2 and X_m . The reactive power balance equation at full load is

$$3VI_F \sin \theta_{1F} = 3I_{1F}^2 X_1 + 3I_{2F}^2 X_2 + 3E_F^2 / X_m$$

or

$$X_m = \frac{E_F^2}{VI_F \sin \theta_{1F} - I_{1F}^2 X_1 - I_{2F}^2 X_2} \quad (11)$$

Estimation of induction motor parameters by the proposed method involves the solution of eqns. (5), (6a), (6b), (8), (10) and (11). Solution of these equations requires the values of rotor induced voltage, stator current and rotor current at half and full load. The half and full load stator currents are obtained from

$$I_{1H} = \frac{746 \text{ hp}/2}{3V \text{ pf}_H \eta_H} \quad (12)$$

$$I_{1F} = \frac{746 \text{ hp}}{3V \text{ pf}_F \eta_F}$$

TABLE 1. Empirical proportion of leakage reactances

NEMA design	X_1	X_2
Class A	$0.5 X_T$	$0.5 X_T$
Class B	$0.4 X_T$	$0.6 X_T$
Class C	$0.3 X_T$	$0.7 X_T$
Class D	$0.5 X_T$	$0.5 X_T$
Wound rotor	$0.5 X_T$	$0.5 X_T$

where pf_H and pf_F are the half and full load power factors, respectively, and η_H and η_F are the half and full load efficiencies, respectively. The rotor induced voltage E and the rotor current I_2 at half and full load are given by

$$E_H = V - I_{1H} Z_1$$

$$E_F = V - I_{1F} Z_1 \quad (13)$$

and

$$I_{2H} = I_{1H} - E_H / Z_m$$

$$I_{2F} = I_{1F} - E_F / Z_m \quad (14)$$

Since the values of E , I_2 and P_{rot} are not known initially, the method estimates the parameter values iteratively (from eqns. (5), (6a), (6b), (8), (10) and (11)). At the end of each iteration the values of E , I_2 and P_{rot} are updated using eqns. (13), (14) and (7), respectively. The initial guesses of E and I_2 for the iterative procedure are called **Initial estimations** follows:

$$E_H = E_F = V, \quad I_{2H} = I_{1H} \text{pf}_H, \quad I_{2F} = I_{1F} \text{pf}_F \quad (15)$$

Under normal operating conditions, $R_2/S \gg X_2$. The rotor circuit becomes almost resistive and most of the active component of the stator current ($I_1 \text{pf}$) flows through the rotor. Also, the stator voltage drop is small. Thus eqn. (15) gives very good initial guesses to start the iterative scheme. The computational sequence of the proposed method is summarized in the following.

3.2. Summary of proposed method

The computational steps involved in estimating the parameter values of a three-phase induction motor by the proposed method are as follows.

Step 1. Obtain I_{1H} and I_{1F} from eqn. (12).

Step 2. Assume the initial values of E and I_2 at half and full load from eqn. (15). Also assume $P_{\text{rot}} = 0$.

Step 3. Determine R_2 from eqn. (5). Solve eqns. (6a) and (6b) for R_1 and P_{const} . Determine R_m from eqn. (8). Obtain X_1 and X_2 using eqn. (10) and Table 1. Finally, determine X_m from eqn. (11).

Step 4. Update the values of E , I_2 and P_{rot} using eqns. (13), (14) and (7), respectively.

Step 5. Repeat steps 3 and 4 until the algorithm converges with an acceptable tolerance.

It has been found in estimating the parameter values of several induction motors (about 30) that the proposed algorithm converged quickly in each case, and in fact only two or four iterations were required in all the cases studied.

3.3. Limitations of the proposed method

The following assumptions have been made in the proposed method for estimating the parameter values of a three-phase induction motor:

- (1) stray load loss is negligible;
- (2) frictional losses are assumed to be constant;
- (3) core loss is assumed to be 50% of constant losses [8, 9];
- (4) magnetizing impedance is neglected in determining the starting torque because, at $S = 1$, $Z_m \gg Z_2$;
- (5) stator and rotor leakage reactances are divided according to the NEMA standard [1].

4. Simulation results

The proposed method of estimating the parameter values of induction motors was tested on all of the 29 motors whose data are given in ref. 9. Results of the following two motors are given in detail in this paper:

- (a) 30 hp, 460 V, 1180 rpm, 60 Hz, Y-connected and NEMA design B;
- (b) 50 hp, 460 V, 3525 rpm, 60 Hz, Y-connected and NEMA design B.

The above information is usually available from the name plate. Other information required by the proposed method is also obtainable from ref. 9 and is listed in Table 2. These are data supplied by the manufacturer.

TABLE 2. Manufacturer-supplied data

hp	Efficiency (%)		Power factor (%)		T_{st}/T_{FL}
	Full load	Half load	Full load	Half load	
30	88.5	87.2	86.3	71.2	1.35
50	90.2	88.7	90.5	82.6	1.20

TABLE 3. Variation of parameters with number of iterations

hp	Iter. no.	R_1	X_1	R_2	X_2	R_m	X_m
30	1	0.3900	0.3095	0.1255	0.4642	447.29	17.423
	2	0.3899	0.2988	0.1237	0.4481	384.90	15.002
	3	0.3901	0.3016	0.1262	0.4523	385.74	15.022
	4	0.3901	0.3014	0.1261	0.4521	385.55	15.014
50	1	0.1599	0.2492	0.0982	0.3737	251.75	15.156
	2	0.1600	0.2406	0.0953	0.3608	223.26	13.416
	3	0.1602	0.2418	0.0966	0.3628	223.79	13.424
	4	0.1602	0.2418	0.0966	0.3626	223.72	13.420

In general, the convergence speed of any iterative method depends upon the initial guesses. By using the initial guesses given by eqn. (15), the proposed method of estimating the parameter values of the motors requires only three iterations to converge. Thus the method requires very little computation time and is comparable with the non-iterative approaches discussed in refs. 8 and 9. Table 3 shows the variation of parameter values for the first four iterations. It may be mentioned here again that, with the proposed initial guesses, the method requires only 2 to 4 iterations to converge for all the motors studied. Parameter values obtained by the proposed method and by the Natarajan–Misra (N–M) method [9] are compared in Table 4. This Table clearly indicates that the estimated parameters obtained by the two methods are not close. The accuracy of the estimated parameters was then tested by comparing different characteristics of the motors obtained from the equivalent circuit using the parameter values given in Table 4 with the manufacturer-supplied data shown in Table 2. Note that the values of efficiency and power factor of the 30 hp motor at 75% of full load are 88.6% and 81.9%, respectively, and the corresponding values for the 50 hp motor are 90.0% and 88.5% [9].

Figures 3 and 4 show the variation of efficiency and power factor with output power for the 30 and 50 hp motors, respectively. The curves were generated through the equivalent circuit (Fig. 1) using the parameter values obtained by the proposed method and by the N–M method. The actual values of efficiency and power factor at three different points (50%, 75% and 100% of full load) are also marked on the Figures by the symbols + and ×, respectively. Figure 5 shows the variation of stator current with output power for the 30 hp motor. The actual values of current (obtained from eqn. (12)) at three different points

TABLE 4. Comparison of induction motor parameters

hp	Method	R_1	X_1	R_2	X_2	R_m	X_m	P_{fric}
30	Proposed	0.3901	0.3016	0.1262	0.4523	385.7	15.02	477.0
	N-M	0.308	0.209	0.122	0.314	514.4	24.49	412.5
50	Proposed	0.1602	0.2418	0.0966	0.3628	223.8	13.42	848.3
	N-M	0.118	0.186	0.093	0.279	271.4	30.08	781.5

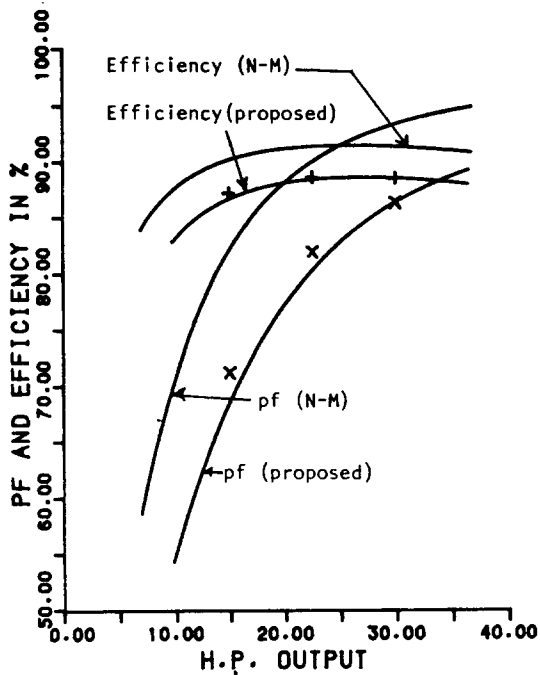


Fig. 3. Variation of efficiency and power factor with output power for the 30 hp motor.

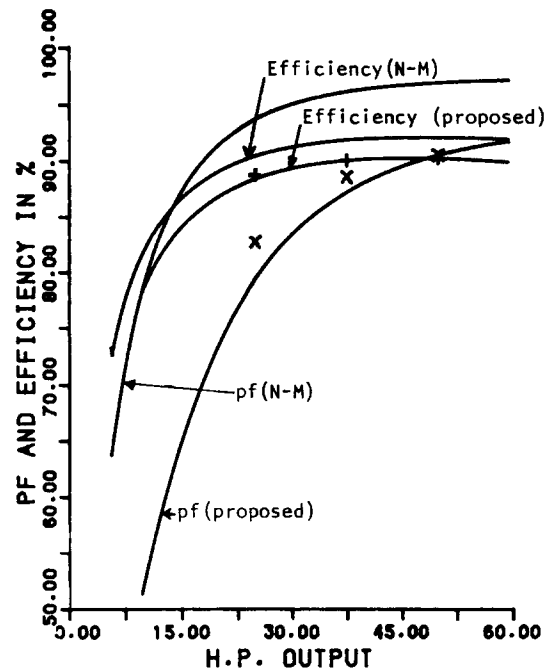


Fig. 4. Variation of efficiency and power factor with output power for the 50 hp motor.

are also shown on the Figure by the symbol +. Comparison of the characteristics (Figs. 3, 4 and 5) indicates that the results obtained through the proposed method of parameter estimation agree very well with the actual values. On the other hand, the corresponding characteristics obtained through the N-M method deviate from the actual values.

The starting torque of the motors obtained from the equivalent circuit (for $S = 1$ in Fig. 1) and its approximate value found from eqn. (4) are given in Table 5. The Table indicates that the torques obtained by using the parameter values of the proposed method are very close to the actual values, whereas the corresponding values obtained through the N-M method are significantly high. In fact, the latter are unrealistic numbers for a NEMA class B motor. Also, it can be seen from Table 5 that the approximate torques obtained from the simplified equation (4)

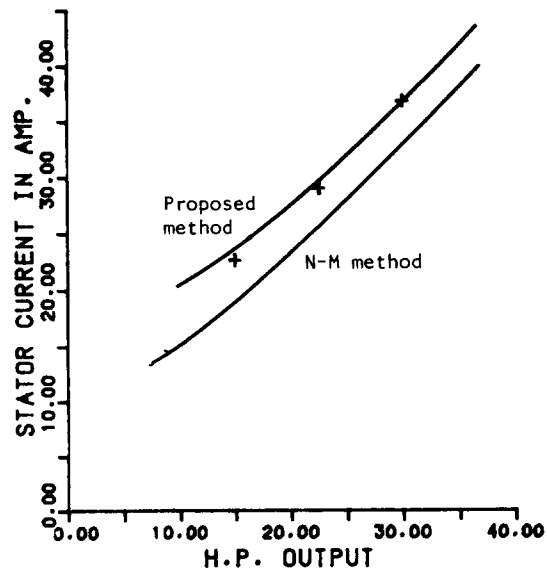


Fig. 5. Variation of stator current with output power for the 30 hp motor.

TABLE 5. Starting torque (in N m)

hp	T_{FL}	T_{st}	Method	T_{st} from equiv. circ.	T_{st} from eqn. (4)
30	181.11	$1.35T_{FL}$	Proposed	247.41	254.5
		= 244.5	N-M	442.6	448.21
50	101.05	$1.20T_{FL}$	Proposed	122.91	125.66
		= 121.26	N-M	198.61	200.19

are very close to the corresponding values found through the exact equivalent circuit (Fig. 1).

5. Conclusion

A simple and effective method of estimating the parameter values of the exact equivalent circuit of a three-phase induction motor using the name plate data, ratio of starting to full load torque and values of efficiency and power factor at half and full load has been presented. The parameters are obtained iteratively to meet the full load speed, starting torque and the active and reactive power balance criteria. The method requires only a few iterations to converge. From a large number of case studies, it has been found that the characteristics obtained by using the parameter values estimated through the proposed method agree very well with the actual values.

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Nomenclature

E	rotor induced phase voltage referred to stator
I_1	$= I_1/\theta_1$, stator phase current
I_2	$= I_2/\theta_1$, rotor phase current referred to stator

I_m	$= I_m/\theta_1$, magnetizing current per phase
P_{const}	constant losses
P_{core}	core loss
P_{rot}	rotational losses
pf	power factor
$R_1 (X_1)$	stator resistance (reactance) per phase
$R_2 (X_2)$	rotor resistance (reactance) per phase referred to stator
R_L	dynamic load resistance per phase referred to stator
R_m	resistance representing core loss per phase
S	slip
T_{FL}	full load torque
T_{st}	starting torque
V	stator phase voltage
X_m	magnetizing reactance per phase
η	efficiency
$\omega_s (\omega_m)$	synchronous (rotor) angular velocity, rad/s

Complex numbers are represented by bold letters. Subscripts H and F are used for half and full load, respectively.

References

- 1 *Standard Test Procedure for Polyphase Induction Motors and Generators*, ANSI/IEEE Standard 112, 1984.
- 2 A. E. Fitzgerald, C. Kingsley, Jr. and A. Kusko, *Electric Machinery*, McGraw-Hill Kogakusha, Tokyo, 3rd edn., 1971.
- 3 L. W. Matsch and J. D. Morgan, *Electromagnetic and Electromechanical Machines*, Harper Row, New York, 3rd edn., 1986.
- 4 S. A. Naser, *Electric Energy Conversion and Transmission*, Macmillan, New York, 1985.
- 5 A. Dell'Aquila, L. Salvatore and M. Savino, A new test method for determination of induction motor efficiency, *IEEE Trans.*, PAS-103 (1984) 2961–2973.
- 6 P. G. Cummings, W. D. Bowers and W. J. Martiny, Induction motor efficiency test methods, *IEEE Trans.*, IA-17 (1981) 253–272.
- 7 C. L. Becnel, J. W. Kilgore and E. F. Merrill, Determining motor efficiency by field testing, *IEEE Trans.*, IA-23 (1987) 440–443.
- 8 L. Pereira, Induction generators for small hydro plants, *Water Power Dam Construction*, (Nov.) (1981) 30–34.
- 9 R. Natarajan and V. K. Misra, Parameter estimation of induction motors using a spreadsheet program on a personal computer, *Electr. Power Syst. Res.*, 16 (1989) 157–164.