2T

Dr Christian Korff

Joao Almeida-Domingues[*]

University of Glasgow

January 13th, 2020 – March 25th, 2020

Contents

These lecture notes were collated by me from a mixture of sources , the two main sources being the lecture notes provided by the lecturer and the content presented in-lecture. All other referenced material (if used) can be found in the *Bibliography* and *References* sections.

The primary goal of these notes is to function as a succinct but comprehensive revision aid, hence if you came by them via a search engine , please note that they're not intended to be a reflection of the quality of the materials referenced or the content lectured.

Lastly, with regards to formatting, the pdf doc was typeset in LaTeX, using a modified version of Stefano Maggiolo's class

[*]2334590D@student.gla.ac.uk

1 Sets

1.1 remark. This topic has been covered in length on other courses. See 2F, 2R notes

set , cardinality , subset , union , intersection , complement , cartesian product , domain , codomain , image , injective , surjective , bijective , set operations

2 Basic Counting Principles

2.1 remark. Most of this has also been covered in 2R. It is partially presented here with a greater emphasis on the cardinalities of sets. See 2R for more

sample space , event , σ-algebra , Borel σ-algebra , distribution , Kolmogrov axioms , probability rules , power set , conditional probability , Bayes' theorem

2.2 definition. **σ-algebra** on a set L is a collection Σ of subsets of L which includes L itself and is *closed under complement* and *under countable unions*

- $L \in \Sigma$
- $E \in L \implies E^{\complement} \in \Sigma$
- $E_1, \ldots, E_n \in \Sigma \implies \bigcup_{i=1}^{n} E_i \in \Sigma$

An event of the sample space L must therefore form a σ-algebra on L. In light of this view, a probability distribution can be defined as the following mapping

2.3 definition. Probability Distribution $P : \Sigma \to [0, 1]$ s.t. $P(L) = 1$ and $P(\bigcup_{i=1}^{n} E_i) = \sum_{i=1}^{n} P(E_i)$, for disjoint events

In words, a probability distribution is just a function which maps events in L to a valid probability

2.4 definition. Probability Space is represented by the triple (L, Σ, P)

2.1 *Discrete Uniform Distribution*

The distribution where all outcomes $S \in L$ are equally likely to occur

Let $\Sigma = \{E | E \subset L\}$ the the classical definition of probability can be written in terms of the cardinalities of sets E, L such that $P(E) = \frac{|E|}{|L|}$

Probability Rules

The following probability rules follow from the set rules

2.5 definition. Addition $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2.6 definition. Subtraction $P(A^{\complement}) = 1 - P(A)$

2.7 definition. Multiplication For a sequence of independent experiments with sample spaces L_k , then $L = L_1, \ldots, L_n$

2.8 remark. Using cardinality as a map from the category of finite sets to the natural set $\{\mathbb{N} \cup 0\}$ we see that the set operations *induce* arithmetic operations