2T
Dr Christian Korff

*Joao Almeida-Domingues**

University of Glasgow

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These lecture notes were collated by me from a mixture of sources , the two main sources being the lecture notes provided by the lecturer and the content presented in-lecture. All other referenced material (if used) can be found in the *Bibliography* and *References* sections.

The primary goal of these notes is to function as a succinct but comprehensive revision aid, hence if you came by them via a search engine , please note that they're not intended to be a reflection of the quality of the materials referenced or the content lectured.

Lastly, with regards to formatting, the pdf doc was typeset in LaTeX, using a modified version of Stefano Maggiolo's class

*2334590D@student.gla.ac.uk

1 Permutations & Combinations

1.1 definition. Arrangement selection of objects in a particular order

1.2 definition. k-Permutation The number of arrangements of k elements from a set of size n wihout repetition. ${}^{n}P_k = \frac{{}^{n}P_n}{{}^{n-k}P_{n-k}}$

1.3 notation. ${}^{n}P_k$

1.4 remark. When $k = n$, then we have the total number of permutations of the set

1.5 remark. with repetition, we have the total number of arrangements equal to n^r, i.e. the number of maps $A \to B$, with $|A| = r$ $|B| = n$

1.6 definition. k-Combination the number of unordered selections of k-objects of a set A of size n without repetition, is equal to the number of subsets of size k

$$nC_r = \left|\left\{B \in 2^A||B| = r\right\}\right| = \frac{n(n-1)\cdots(n+r)}{r(r-1)\cdots 1} = \left(\begin{array}{c} n \\ r \end{array}\right)$$

1.7 notation. $\binom{n}{k}$

1.8 definition. k-subsets of A $\binom{A}{k}$

1.9 definition. Power set of A 2^A

Hence, for a finite set A, we can also represent the k-subsets of A as $\binom{A}{k} \subset 2^A$

1.10 proposition. $\left|\binom{A}{k}\right| = \left(\binom{|A|}{k}\right)$

1.1 *Binomial Theorem : Combinational Proof*

1.11 definition. Binomial Theorem $(x+y)^n = \sum_{r=0}^{n} \left(\begin{array}{c} n \\ r \end{array}\right) x^r y^{n-r}$

Proof.

For $(x+y)^n$ we have $(x+y)(x+y)\cdots(x+y)$, n times. Hence for the r^{th} term, we want the $r - subset$, i.e. given r factors we need all possible combinations of x, y. We count how many times we obtain $x^r yn - r$, and sum the individual $r - subsets$ of n, to get the desired result

1.12 remark. Note that the symmetric nature of combinations, follows from the above. Since, the order does not matter, and given the commutative nature of multiplication; for example , for the x^2y^3 term in a $(x+y)^5$ expansion we can see it as either choosing x 2 times, or y 3 times, i.e $\binom{5}{2} = \binom{5}{3}$

1.13 definition. Combinatorial Proof : double counting or identity, is when one counts the same set in different ways

1.14 example.

$$2^n = \sum_{r=0}^{n} \binom{n}{r}$$

For an $n - set, A$, we have $2^A = \bigcup_{r=0}^{n} \binom{n}{r}$. Hence,

$$\left|2^A\right| = 2^{|A|} = \left|\bigcup_{r=0}^{n} \binom{A}{r}\right| = \sum_{r=0}^{n} \left|\binom{A}{r}\right| = \sum_{r=0}^{n} \binom{|A|}{r}$$

1.15 definition. Combinatorial Proof : bijective where two sets are shown to have the same cardinality by proving the existence of a bijection between them