MATHS 2P : Graphs & Networks
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Contents

1 Fundamentals 2
1.1 Graphs 2
1.2 Graph Properties 3

These lecture notes were collated by me from a mixture of sources , the two main sources being the lecture notes provided by the lecturer and the content presented in-lecture. All other referenced material (if used) can be found in the *Bibliography* and *References* sections.

The primary goal of these notes is to function as a succinct but comprehensive revision aid, hence if you came by them via a search engine , please note that they're not intended to be a reflection of the quality of the materials referenced or the content lectured.

Lastly, with regards to formatting, the pdf doc was typeset in LaTeX, using a modified version of Stefano Maggiolo's class

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1 Fundamentals

Lecture 1
September 25^{th}, 2019

1.1 *Graphs*

A graph G is a pair (V, E), where V is any finite set, and E is a set whose elements are pairs of elements of V. We call the elements of V the *vertices** of G and those of E its *edges*. e.g. $G = \{\{a, b, c\}, \{ab, ac\}\}$

** often also referred as nodes*

1.1 definition. Adjacent Vertices are vertices connected directly through an edge. Formally, if $e = \{u, v\} \in E$, then u, v are adjacent

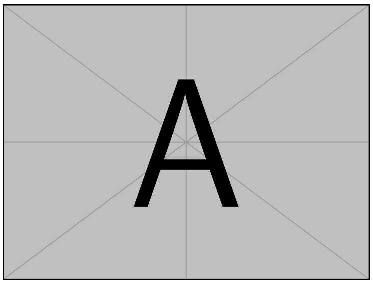
1.2 definition. Incident Edges are edges which share a vertex. We say that they are *"incident to v"*

Lecture 2
September 27^{th}, 19

Representing Graphs

Note that the representation need not be unique

Pictorially



Note that this definition only holds for simple graphs, i.e without loops. But , it is easily generalised if the binary requirement is dropped

Adjacency Matrix

1.3 definition. Adjacency Matrix is the $n \times n$ binary matrix , where $n = |V|$ and $a_{ij} = 1 \iff e = \{u, v\} \in E$; i.e iff u, v are adjacent

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

1.4 remark. In this course, we'll only deal with *simple*, *undirected* graphs. Note that AMs of this type have the nice property of being symmetric (see 2B notes for properties)

Subgraphs

1.5 definition. Subgraphs are graphs obtained by deleting edges and/or edges of another graph

1.6 definition. Induced Subgraph is a graph formed by deleting only nodes and its incident edges. Formally: Let $W \subset V$, then the induced subgraph of G is given by $G[W] = \{W, \{\{xy\} | xy \in G\}\}$. We say that *"G is induced by W"*

1.7 example.

$G(V) = \{\{a, b, c\}, \{ab, ac\}\}$ and $U = \{c\}$ then $G[U] = \{\{a, b, \cancel{c}\}, \{ab, \cancel{ac}\}\}$

1.8 definition. Spanning Subgraph similar to the induced, but edges are deleted instead

Lecture 3
October 2^{nd}, 19

1.2 *Graph Properties*