2B : Linear Algebra
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Contents

These lecture notes were collated by me from a mixture of sources , the two main sources being the lecture notes provided by the lecturer and the content presented in-lecture. All other referenced material (if used) can be found in the *Bibliography* and *References* sections.

The primary goal of these notes is to function as a succinct but comprehensive revision aid, hence if you came by them via a search engine , please note that they're not intended to be a reflection of the quality of the materials referenced or the content lectured.

Lastly, with regards to formatting, the pdf doc was typeset in LaTeX, using a modified version of Stefano Maggiolo's class

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1 Vectors

1.1 *Basics*

Most of this has been covered already, if too succinct check 1R/1S notes

1.1 definition. Vector displacement from one point to another in space ; geometrically represented by a directed line segment

1.2 remark. In general we use the origin of the cartesian coordinate system as the displacement origin

1.3 definition. $\mathbb{R}^n$ the set of all n-tuples of real number , for $n \in \mathbb{R}$

$$\mathbb{R}^n = \{(x_1, \ldots, x_n) : x_1, \ldots, x_n \in R\}$$

1.4 remark. remember that () are used for ordered objects

1.5 notation. $\mathbf{v}$; $\underline{v}$; $\mathbf{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$; $\mathbf{v} = \begin{bmatrix} v_1 \ldots v_n \end{bmatrix}$

1.6 definition. Vector Addition $\mathbf{u} + \mathbf{v} = (u_1 + v_1, \ldots, u_n + v_n)$

1.7 definition. Scalar Multiplication $\lambda\mathbf{v} = (\lambda v_1, \ldots, \lambda v_n)$

Algebraic Properties of Vector Addition

$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ (commutativity of vector addition)
$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$ (commutativity of vector addition)
$\mathbf{u} + \mathbf{0} = \mathbf{u}$
$\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
$c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$ (distributivity of vector addition)
$(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$ (distributivity of scalar addition)
$c(d\mathbf{u}) = (cd)\mathbf{u}$
$\mathbf{1u} = \mathbf{u}$

1.2 *Linear Combination and Independence*

1.8 definition. Linear Combination Sum of the members of a set, where each member is multiplied by a constant

It follows from **??** that for a set composed of vectors , we can say that an arbitary vector $\mathbf{v}$ is a linear combination of $\mathbf{u}, \mathbf{w}$ *iff*

$$\mathbf{v} = j\mathbf{u} + k\mathbf{w} \quad , \quad \text{for } j, k \in \mathbb{R}$$

Systems of Linear Equations and Matrices

Note that for a vector in $\mathbb{R}^n$ we have a linear equation in n variables whose solution is a vector of size n. In other words, the linear equation $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$ has solution $a_1s_1 + a_2s_2 + \cdots + a_ns_n = b$

Hence, we can find the values which satisfy **??** by finding the solution vector which satisfies all linear equations simultaneously, i.e by constructing and solving the appropriate system of equations. This becomes clearer if we rewrite **??** in vector form

$$\begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} = j \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} + k \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$$

Recall also that we can transform the above in an augment matrix, where each column represents a vector and its unknown coefficient, and by performing successive EROs we can simplify the system enough so as to hopefully be able to glean the solution vector from the matrix

1.9 definition. Pivot leading non-zero entry

1.10 definition. Row Echelon Form Matrix which satisfies the following:

1. All pivots in lower rows are strictly to the right of those of the rows above it
2. All non-zero rows are above zero rows

$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 5 \end{bmatrix}$$

1.11 definition. Reduced Row Echelon Form Matrix which is in REF and

1. All pivots in lower rows are equal to 1
2. Every element above the pivots is equal to 0

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{cc} j & k \\ \end{array}$$
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$