COURSE CODE
LECTURER

*Joao Almeida-Domingues**

University of Glasgow

July 7^{th}, 1993 – July 7^{th}, 1994

Contents

These lecture notes were collated by me from a mixture of sources , the two main sources being the lecture notes provided by the lecturer and the content presented in-lecture. All other referenced material (if used) can be found in the *Bibliography* and *References* sections.

The primary goal of these notes is to function as a succinct but comprehensive revision aid, hence if you came by them via a search engine , please note that they're not intended to be a reflection of the quality of the materials referenced or the content lectured.

*2334590D@student.gla.ac.uk

Lastly, with regards to formatting, the pdf doc was typeset in LaTeX, using a modified version of Stefano Maggiolo's class

1 Propositional Logic

1.1 definition. Propositional Logic the logic of compound statements built from simpler statements using Boolean connectives

1.2 definition. Propostion declarative sentence which is either true or false

To do (1)

To do (2) **To do** (3) **To do** (4)

2

3

4

5 Proofs

5.1 *Rules of Inference*

5.1 definition. Valid Argument the truth of the premises logically guarantees the truth of the conclusion , *i.e.* $(p \land q \land r \land \dots) \to q$ is a tautology

Memorising not required

Modus Ponens

$$\frac{p \quad (p \to q)}{q}$$

p	q	$($	p	$\land$	$($	p	$\to$	q	$))$	$\to$	q
1	1		1	1		1	1	1		1	1
1	0		1	0		1	0	0		1	0
0	1		0	0		0	1	1		1	1
0	0		0	0		0	1	0		1	0

Modus Tolens

$$\frac{\neg q \quad (p \to q)}{\neg q}$$

p	q	$($	$\neg$	q	$\land$	$($	p	$\to$	q	$))$	$\to$	$\neg$	p
1	1		0	1	0		1	1	1		1	0	1
1	0		1	0	0		1	0	0		1	0	1
0	1		0	1	0		0	1	1		1	1	0
0	0		1	0	1		0	1	0		1	1	0

To do (5)

Hypothetical Syllogism

$$\frac{(p \to q) \quad (q \to r)}{p \to r}$$

Disjunctive Syllogism

$$\frac{p \lor q \quad \neg p}{q}$$

Resolution

$$\frac{p \lor q \quad \neg p \lor r}{q \lor r}$$

Addition

$$\frac{p}{p \lor q}$$

Simplification

$$\frac{p \land q}{p}$$

Conjunction

$$\frac{p \quad q}{p \land q}$$

To do (6)

5.2 *Constructing a Proof Tree*

Continuing with the example above, we can construct a *proof tree*, where each *leaf* is composed of axioms - the premises, and the conclusion sits at the *root*
To do (7)

5.3 *Quantifiers*

6 Induction and Recursive Definitions

6.1 definition. Kleene Star is a unary operation on strings or , more generally , sets of symbols

6.2 notation. V^* , where V is the set being operated on

7 Graphs

See 2P notes `https://github.com/Joe-a-d/LectureNotesUniversityOfGlasgow/tree/master/Year2/Graphs&Networks/2P.pdf`

- Fundamentals
- Isomorphism
- Eulerian
- Hamiltonian

7.1 example. How many simple undirected graphs are there with 20 vertices and 60 edges?

For every pairs of vertices one has a possible edge, hence K_{20} has $\binom{20}{2} = 190$ edges For a graph to have 60 edges we need to choose 60 out of 190 possible edges. The number of graphs therefore equals $\binom{190}{60} = \frac{190!}{60!30!}$

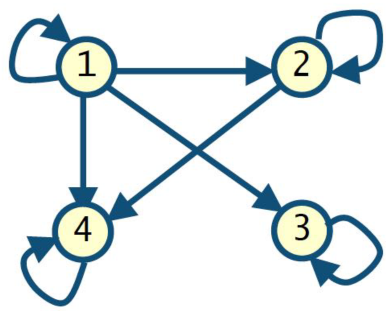
8 Relations

8.1 definition. Binary Relation for two sets A, B , $R \subseteq A \times B$, where each element of R is the set of related ordered pairs (a, b) for $a \in A, b \in B$. We say that *"a is related to b by R"*

We can also represent relations as graphs, where the nodes are the individual elements of A, B present in R, and its edges are the ordered pairs

8.2 example. Take $A = 1, 2, 3, 4$ and $R = a|b$ for $a, b \in A$. In this case R operates over the same set, hence $R \subseteq A \times A$. Now let $G(V, E)$ be the graph representing this relation then :

$$V = \{1, 2, 3, 4\} \text{ and } E = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)$$



8.3 remark. Relations are not exclusively binary , for a relation between n sets we have a set R of $n-$tupples

8.1 *Properties*

Reflexive : $a \in A \implies (a, a) \in R$

Symmetric $a, b \in A, a \neq b$ and $(a, b) \in R \implies (b, a) \in R$

Anti-Symmetric $a, b \in A, a \neq b$ and $(a, b) \in R \implies (b, a) \notin R$

Transitive $a, b, c \in A, a \neq b \neq c$ $(a, b), (b, c) \in R \implies (a, c) \in R$

8.4 definition. Equivalence Relation R is an equivalence relation *iff* it is reflexive, symmetric and transitive

8.5 example. Take $(a, b) \in R \iff$ "a is strictly taller than b" , i.e $height(a) > height(b)$. Then, (1) R is not reflexive, since $height(a) = height(a)$ (2) R is not symmetric given that b is strictly smaller (not taller) than a , $height(b) < height(a)$ (3) R is anti-symmetric since, $height(b) < height(a)$. Finally (4) R is transitive since if a is taller than b and b is taller than c then a is taller than c

To do (8)

To do...

- ☐ 1 (p. 3): See slide annotations
- ☐ 2 (p. 3): Truth table for all connectives , use the succinct style, where for large statements the result is written under the connective
- ☐ 3 (p. 3): Write a remark explaining the use of the succinct form of the tables for large compound propositions (e.g. (p ¿ q) & (p v q) one would not make a separate column for the conjunction, but instead just write the result under the &)
- ☐ 4 (p. 3): Write a remark explaining that for any n propositions, rows = 2^n, since for every p 1,0
- ☐ 5 (p. 3): Tautology truth tables
- ☐ 6 (p. 4): Example slides 26,27
- ☐ 7 (p. 4): slide 33
- ☐ 8 (p. 7): add bib: Truth Tables : Michael Rieppel "mrieppel " @https://github.com/mrieppel/TruthTableGenerator