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Formally Modeling a Gender-Segregated Economy: A Reply to Campbell and Warner

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Summary. — Campbell and Warner contend that there are two major problems with my presentation of a formal structure for analysis of a gender-segregated low-income economy in an earlier issue of this journal. Their first argument that there is no explicit treatment of women negotiating their wage is unimportant. Their second argument is correct; my construction of the male's decision-making process is underspecified, precluding an internal maximization solution. Two options for formalization are advanced that make an internal maximization solution feasible. © 1997 Elsevier Science Ltd. All rights reserved.

Campbell and Warner (1997) have presented a thoughtful critique of my paper originally published in a special issue of this journal devoted to the implications for macro-analysis of engendering economics (Darity, 1995). They identify two major problems with my paper. First, Campbell and Warner complain that my model "offers no explicit treatment of the woman negotiating her wages and places direct determination under the control of the man who is maximizing his income." They even argue that my model implies that maximization of male incomes means setting female compensation at zero. Second, Campbell and Warner complain that my model is "underspecified" in the formal sense that there is no internal maximum possible from the exercise developed in my paper.

Their first complaint is not substantive; their second complaint is correct. With respect to the first complaint, Equation 3 in my original paper (Darity, 1995, p. 1964) constitutes a female labor supply equation in Cobb-Douglas form. The real compensation (w/p_v) for women enters the equation with exponent ρ. The exponent ρ could be the outcome of custom, tradition, or even non-symmetric inter-gender bargaining, given relative male dominance, the latter as Campbell and Warner prefer. But an explicit exploration of the game-theoretic/bargaining foundations for the economic outcomes in societies of this type is the subject matter for another paper. Moreover, if men were to set w=0, female labor supply would vanish and, given the gender division of labor, no cash crops production or male income could be generated.

With respect to the second complaint, Campbell and Warner are right. When I set up the exercise I sought the solution for optimal values of the female wage, w, and the amount of labor supplied by men for cash crops production, M_c. But since I tried to solve by direct substitution with manic inversion of exponents rather than driving to set up the system in matrix notation, I never detected the impossibility of deriving a nontrivial solution. Because the system was nonlinear and because I assumed the system had a solution via direct substitution, I never contemplated displaying the first-order conditions in matrix notation. Therefore, I evidently "forced" a solution, despite the insolubility of the notationally simplified version of my system Campbell and Warren present as:

$$JX = D \tag{1}$$

where

$$J = \begin{bmatrix} (\alpha+\sigma\beta)A-\sigma B \\ \rho\beta A-(\rho+I)B \end{bmatrix}, X = \begin{bmatrix} M_c^{\alpha+\sigma(\beta-I)} \\ w^{I+\rho(I-\beta)} \end{bmatrix}, \text{ and } D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A \equiv P_c C^{\beta} K_c^{\gamma} (1/P_v)^{\rho\beta}$$

$$B \equiv C(1/P_v)\rho \tag{2}$$

The central technical problem is the fact that D is the zero vector, so that neither matrix inversion nor Cramer's rule yield a nontrivial solution. If the system were respecified so that a constant term appeared in either the upper or lower cell of the D vector then, in principle, the system would have non-

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zero solutions for M_C and w. But it is unlikely that a respecified system could be linearized and written in matrix notation. Direct substitution still would have to be used as the method of solution; at least an internal maximum would be feasible.

One route out of the impasse is to follow Campbell and Warner's inclination to treat the male optimization problem as inclusive of an old-fashioned labor-leisure choice. Suppose men in this community had a utility function in three arguments:

$$U = U(Y_M, M_C, w/P_v) \quad U_1 > 0, U_2 < 0, U_3 < 0 \qquad (3)$$

In Equation 3 men gain utility from additional income, disutility from increased labor (or decreased leisure), and disutility from paying women higher real compensation. The latter effect arises, apart from any reduction in male incomes that might be associated with a higher female wage, because higher real incomes for women can mean their exercise of greater independence.

The Lagrange equation now can be formed, again using Campbell and Warner's notational simplification:

$$L = U(Y_M, M_C, w/P_v) + \lambda[Y_M - AM_C^{\alpha+\sigma\beta}w^{\rho\beta}BM_C^{\alpha}w^{\rho+1}] \qquad (4)$$

First-order conditions for a maximum are:

$$\frac{\partial \mathcal{L}}{\partial Y_M} = U_1 + \lambda = 0 \qquad (5)$$

$$\frac{\partial \mathcal{L}}{\partial M_C} = U_2 - \lambda[(\alpha + \sigma\beta)AM^{(\alpha+\sigma\beta-1)}w^{\rho\beta} - \sigma BM_C^{\sigma-1}w^{\rho+1}] = 0 \qquad (6)$$

$$\frac{\partial \mathcal{L}}{\partial w} = U_3 - \lambda[\rho\beta AM_C^{\alpha+\sigma\beta}w^{\rho\beta-1} - (\rho+1)BM_C^{\sigma}w^{\rho}] = 0 \qquad (7)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = Y_M - AM_C^{\alpha+\sigma\beta}w^{\rho\beta} + BM_C^{\sigma}w^{\rho+1} = 0 \qquad (8)$$

By using expressions (5), (6), and (7) and by eliminating λ we arrive at the following two equation system in two unknowns, M_C and w:

$$-U_1/U_2 = [(\alpha + \sigma\beta)AM_C^{(\alpha+\sigma\beta-1)}w^{\rho\beta} - \sigma BM_C^{(\sigma-1)}w^{(\rho+1)}]^{-1} \qquad (9)$$

$$-U_1/U_3 = [\rho\beta AM_C^{\alpha+\sigma\beta}w^{\rho\beta-1} - (p+1)BM_C^{\sigma}w^{\rho}]^{-1} \qquad (10)$$

Equation (9) captures the labor-leisure tradeoff and equation (10) captures the male income-female wage tradeoff at the optimum for men. Capacity to solve the system for optimal values of M_C and w depends upon the specific form given to the male utility function.

Aesthetically I prefer an alternative route that involves a modest extension of my original model. This idea evolved in conversations with Korkut Ertürk. Simply deduct from men's income a sum proportionate to the amount of labor they supply. This could be interpreted as the male "psychic" cost of leisure foregone, if one so desires, or it could capture some inefficiency in generating income as men devote more hours to cash crops production. Net male income now is defined as follows:

$$Y_M = P_C X_C = wF_C - gM_C \qquad g > 0 \qquad (11)$$

The term gM_C represents the male income lost due to the "expense" men incur in devoting more labor to cash crops production. Note that males do not consider any costs associated with increased female labor devoted to cash crops production, ignoring, for example, the adverse impact on household or subsistence production. Thus, what is optimal for males plainly need not be optimal for the community as a whole.

Once again using Campbell and Warner's notational simplification (11) can be rewritten as:

$$Y_M = AM_C^{\alpha+\sigma\beta}w^{\rho\beta} - BM_C^{\sigma}w^{\rho+1} - gM_C \qquad (11')$$

First-order conditions for a male maximum now become:

$$\frac{\partial Y_M}{\partial M_C} = (\alpha + \sigma\beta)AM_C^{\alpha+\sigma\beta-1}w^{\rho\beta} - \sigma BM_C^{\sigma-1}w^{\rho+1} - g = 0 \qquad (12)$$

$$\frac{\partial Y_M}{\partial w_C} = \rho\beta AM_C^{\alpha+\sigma\beta}w^{\rho\beta-1} - (\rho+1)BM_C^{\sigma}w^{\rho} = 0 \qquad (13)$$

Dividing (12) by $M_C^{\sigma-1} w^{\rho\beta}$ and (13) by $M_C^{\sigma} w^{\rho\beta-1}$ yields:

$$(\alpha + \sigma\beta)AM_C^{\alpha+\sigma(B-1)} - \sigma B w^{1+\rho(\beta-1)} = gM_C^{1-\sigma} w^{-\rho\beta} \quad (12')$$

$$\rho\beta AM_C^{\alpha+\sigma(\beta-1)} - (\rho + 1) - Bw^{1+\rho(1-\beta)} = 0 \quad (13')$$

Equations (12′) and (13′) are used to solve for w and M_c.

Barring another error the solutions are the following:

$$w^\star = \left[\frac{H}{gR^{1/U(1-\sigma)}}\right]^{Q/T} \quad (14)$$

$$M_C^\star = R^{1/U} w^{\star Q/U}$$

$$R \equiv \frac{(\rho + 1)B}{\rho\beta A}; Q \equiv 1 + \rho(1 - \beta)$$

$$U \equiv \alpha + \sigma(\beta - 1); H \equiv [(\alpha + \beta)AR - \sigma B];$$

$$T \equiv [Q/U(1 - \sigma) - \rho\beta] \quad (15)$$

obviously, these closed form solutions are messy but they do provide a consistent basis for the findings I presented in sections 5 and 6 of my earlier paper. For example, the price of cash crops, P_C, and the price of village goods, P_v, do indeed enter 14 and 15 in an inverse fashion and an increase in each will lead to opposite effects on $W^\star$ and $M_C^\star$. Just as I suggested in my earlier paper a rise in P_C will tend to promote a rise in both $w^\star$ and $M_C^\star$; the reverse effect will follow from an increase in P_v. But the effect of either price change on $w^\star$ is stronger and less ambiguous than the effect on $M_C^\star$.

Note also the effect of the new parameter g, that captures any losses in net income for men due to increased male labor devoted to cash crops production. A larger value of g will necessarily depress both the equilibrium values of male labor, $M_C{}^\star$, and women's real compensation, $w^\star$, as long as the exponential term T/Q is positive.

At the core of the model is the characterization of the female labor supply process as decomposable into three effects: coercion, cooperation, and compensation. Of particular importance in determining outcomes in the model are the relative magnitudes of the parameters α,β,σ and ρ. α and β are the elasticities of cash crops output with respect to inputs of male and female labor respectively, while σ and ρ are the elasticities of female labor supply with respect to male employment and the real wage for women. At least now the analysis can be performed with equilibrium values of $w^\star$ and $M_C^\star$ truly "in hand."

Campbell and Warner's useful criticisms have led to two avenues toward solution of the system. Having two routes out of perdition *is* preferable to having one.

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