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Wage inequality, segregation by skill and the price of capital in an assignment model

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ABSTRACT

Some pieces of empirical evidence suggest that in the U.S., from the 1970s to the 1990s, (i) wage inequality *between-plants* rose much more than wage inequality *within-plants* and (ii) there was an increase in the segregation of workers by skill into separate plants. This paper presents a frictionless assignment model in which these two features can be explained simultaneously as the result of the decline in the relative price of capital. Additional implications of the model regarding the skill premium and the dispersion in labor productivity across plants are also consistent with the empirical evidence. The model permits to consider changes in the skill distribution too. Combining these changes with falling capital prices provides a more comprehensive view of the overall trend of wage inequality and of workers' segregation by skill in the data, and it helps explaining some episodes of decreasing wage inequality.

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1. Introduction

It is a well-documented fact that wage inequality in the U.S. labor market increased substantially from the 1970s to the 1990s.[1] In the manufacturing sector, one important feature of this increase is that it came almost exclusively from an increase in the wage inequality *between-plants*. In this sense, Dunne et al. (2004) find that "virtually the entire increase in overall dispersion in hourly wages for U.S. manufacturing workers from 1975 to 1992 is accounted for by the between-plants components". More specifically, their decomposition of the overall wage inequality into the *within-plants* and *between-plants* wage inequality shows that the former only increased slightly during this period, while the latter increased in a similar manner as the overall wage inequality. As a consequence, the *between-plants* wage inequality increased its share in the overall wage inequality from 53% in 1977 to 64% in 1992. Covering the manufacturing and non-manufacturing sectors, Faggio et al. (2007) and Barth et al. (2011) confirm this finding for the U.K. (period 1984–2001) and the U.S. (period 1977–2002), respectively. For instance, Barth et al. (2011) report that, with some sectoral heterogeneity, more than 70% of the increase in earnings inequality in the U.S. over the period occurred across establishments.

Assuming that wages are closely related to skill, one could interpret this evidence as an indication that the composition of U.S. plants changed over this period in a way that increased the segregation of high- and low-skilled workers into separate plants. In this sense, greater segregation would reduce the skill heterogeneity within the workers of a plant and would make more likely to have plants composed of *either* high- or low-skilled workers rather than of *both* high- and low-skilled workers.

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[1] For instance, Acemoglu (2002) reports that, while in 1971 a worker at the 90th percentile of the wage distribution earned 266% more than a worker at the 10th percentile, in 1995 this number was 366%.

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In other words, it would make more likely to have high-skilled (low-skilled) individuals together with other high-skilled (low-skilled) individuals in the same plants. Thus intuitively, if wages are closely related to skill, an increase in the segregation of workers by skill into separate plants would increase the wage inequality *between-plants* and would reduce the wage inequality *within-plants* (or, when the overall wage inequality is increasing, would increase more the former than the latter).

Some empirical evidence supports this idea of increased segregation. For instance, using worker classification (production/non-production) as a proxy for skill, Kremer and Maskin (1996) find that, in the U.S. manufacturing sector, production workers are more likely to be together in the same plants. In particular, they consider a segregation index that lies between 0 (no segregation) and 1 (maximal segregation) and find that the segregation of these workers rose from 0.195 to 0.228 between 1976 and 1987.[2] They also provide similar evidence for Britain and France using this and other proxies for skill (seniority, experience, and other forms of worker classification) and this leads them to conclude that "wages, experience and worker classification are all imperfect indicators of skill, but together they paint a consistent picture of increasing segregation". In a similar vein, Acemoglu (1999) shows that in the Current Population Survey the sorting of workers across occupations increased in the U.S. between 1983 and 1993, with "middling" jobs being replaced by separate jobs for skilled and unskilled workers. Analogously, Kramarz et al. (1996) show that in France, between 1986 and 1992, "workers' specialization between firms has increased drastically during the period (i.e. workers with identical observed characteristics are employed more and more at the same firms)".

In light of this evidence this paper proposes that both (i) the larger increase in the wage inequality *between-plants* than in the wage inequality *within-plants*, and (ii) the increase in the segregation of workers by skill observed in the U.S. from the 1970s to the 1990s can be explained simultaneously by the decline in the relative price of capital.[3,4] In this sense, Krusell et al. (2000) report that the relative price of capital equipment (relative to consumption of nondurables and services) fell in the U.S. at an average rate of about 4.5% per year between 1954 and 1992.[5]

To illustrate this connection this paper presents a frictionless assignment model in which the relative price of capital decreases exogenously. In the model, individuals with different skill are imperfect substitutes in production and they must assign themselves to plants and to occupations within those plants. Specifically, the model assumes that plants are composed of one manager, one worker and a stock of capital. In production, the skill of the manager and the skill of the worker are complementary, but they play a non-symmetric role, and there is a form of capital–skill complementarity. In this setup, for a given price of capital, the shape of the equilibrium assignment (the equilibrium composition of the plants) depends on how strong is the complementary in production between skills relative to their asymmetry. In particular, stronger complementarity pushes the equilibrium towards greater segregation of individuals by skill into different plants.

Then, intuitively, the connection in the model between the decline in the relative price of capital and (i) and (ii) works in the following way. When the relative price of capital declines the overall wage inequality increases (due to the capital–skill complementarity in the economy) but also the complementarity in production between the skill of the manager and the skill of the worker becomes stronger relative to their asymmetry. This modifies the equilibrium composition of the plants in a way that the segregation of individuals by skill increases what, in turn, produces a larger increase in the wage inequality *between-plants* than in the wage inequality *within-plants*.[6]

The model has other interesting predictions. In particular, the model also predicts that, when the relative price of capital declines, the skill premium and the labor productivity dispersion across plants increase. These predictions are broadly consistent with the empirical evidence. A large empirical literature documents an overall increase in the skill premium from the 1970s to the 1990s in the U.S. For instance, Autor et al. (1998) report that the log relative wage of college and post-college workers to high-school workers went from 0.465 in 1970 to 0.557 in 1996.[7,8] Moreover, Dunne et al. (2004) find that the 90–10 differential of the log of labor productivity across U.S. manufacturing plants increased from around 1.7 to around 1.9 during the period 1975–1992.[9]

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[2] One could argue that this evidence could be the result of firms reorganizing their tasks across their plants in such a way that non-production workers and production workers are located in different plants. This paper does not consider multi-plant firms. However, as long as the purpose of this intra-firm reallocation of tasks is to improve productivity, this idea would be consistent with the essence of this paper.

[3] Certainly, many other factors, besides the decline in the relative price of capital, may have contributed to (i) and (ii) too. Section 6 explores the relevance of changes in the skill distribution over time to explain this evidence. Future work should integrate into the analysis some other factors concerning, for instance, with changes in the level of plant outsourcing, in the role of unions or in the competitive pressures from international trade.

[4] Contrary to the findings discussed above on increased segregation by skill between the 1970s and 1990s, for more recent years the empirical evidence is more mixed. For instance, Barth et al. (2011) do not find evidence of increased sorting of workers across plants based on observable skill in the U.S. between 1992 and 2002. Instead, Burgess et al. (2009) and Schlitte (2010) provide empirical support for the idea of increased workers' segregation in Maryland (between 1986 and 1998) and Germany (between 1993 and 2005), respectively. In light of this empirical controversy, the model presented in this paper could be most helpful to understand the evolution of wage inequality and of workers' segregation from the 1970s to the 1990s.

[5] Gordon (1990) and Cummins and Violante (2002) provide similar evidence. For the reasons behind such decline see, for instance, Krusell (1998).

[6] This model simultaneously obtains (i) and (ii). Of course, it is possible to obtain (i) without (ii), and many models in the literature could potentially do so, but that would miss part of the evidence mentioned above.

[7] The evidence in Autor et al. (1998) is for all workers in the economy and not only for manufacturing workers. However, both Davis and Haltiwanger (1991) and Dunne et al. (2004) find that the behavior of the overall wage inequality and of the skill premium for manufacturing workers closely tracks that for all the workers in the economy.

[8] Similar evidence is found in many other studies. In particular, Beaudry and Green (2005) find that the skill premium also increased from the mid-1990s through 2000.

[9] Faggio et al. (2007) find similar evidence for the U.K. over the period 1984–2001.

This paper is related to the literature in several ways. To begin with, there is a large literature that, as in this paper, connects the decline in the relative price of capital with increases in the overall wage inequality and in the skill premium through the existence of a form of capital–skill complementarity in production.[10] See, for instance, Krusell et al. (2000) that try to account for the recent evolution of the skill premium in the U.S. There is also a large literature that, as in this paper, links the decline in the relative price of capital (or, in many instances, the decline in the price of the Information Technology) to changes in the internal organization of plants and/or in their demand of skills. Some examples of this literature are Bresnahan et al. (2002), Autor et al. (2003), Cremer et al. (2004), and Garicano and Rossi-Hansberg (2006). This paper contributes to these two strands of the literature by showing how the decline in the relative price of capital, not only increases the overall wage inequality and modifies the equilibrium composition of the plants, but also affects both the *between*- and the *within-plants* wage inequality and the extent of individuals' segregation by skill.

Moreover, this paper is closely related to Kremer and Maskin (1996). They do not have capital in their model and explain the observed increases in workers' segregation by skill and wage inequality as the result of an increase in the mean and dispersion in the economy's skill distribution. Section 6 shows that the model proposed here can deliver their main results. In this sense, this model encompasses the one in Kremer and Maskin (1996) and, by considering *both* changes in the relative price of capital and in the skill distribution, it provides a more comprehensive view of the evolution of wage inequality and of workers' segregation by skill over time.

Finally, this paper obviously relates to (and benefits from) the assignment literature, especially to that one studying one-to-one matchings.[11] In this paper, individuals must endogenously divide themselves between managers and workers and (simultaneously) pair themselves creating plants. Then, in a sense, the assignment problem here could be considered a *two-step assignment*.[12] Two well-known papers in the literature with this kind of *two-step assignment* problem are Kremer (1993) and Lucas (1978).[13] In Kremer (1993) the equilibrium assignment involves complete segregation of individuals by skill into different plants (the best are paired with best and the worst with the worst). Instead, in Lucas (1978) in equilibrium there is a cutoff level of skill such that everybody above that level is a manager and everybody below it is a worker. A nice feature of the general framework presented in this paper is that, as illustrated in Section 5.4, it contains these two kinds of equilibrium as extreme cases (i.e., when the price of capital is free or infinitely expensive, respectively). An additional contribution of the paper is to show the implications for the economy of the transition from one type of equilibrium to the other. In the model, the equilibrium sets of managers and workers (and thus firms' composition) change endogenously in response to a fall in the relative price of capital, what affects the extent of wage inequality and of workers' segregation by skill.

The rest of the paper is organized as follows. Section 2 presents the production technology in the economy. Then, Section 3 describes the assignment problem in the paper and defines the equilibrium. Some basic properties of this equilibrium are presented in Section 4. Section 5 fully characterizes this equilibrium for a particular version of the model, and shows how it changes with the relative price of capital and the implications of those changes in terms of wage inequality and of segregation by skill. Then, Section 6 explores how the model responds to changes in the skill distribution and connects it with Kremer and Maskin (1996). Finally, Section 7 concludes the paper.

2. The production technology

There is only one good in the economy and this is produced by plants. A plant is composed of one manager, one worker and a stock of capital, and its output is given by the following production function[14]:

$$f(x,z,k) = x^{\mu}[\theta k^{\beta} + (1-\theta)z^{\beta}]^{(1-\mu)/\beta} \tag{1}$$

where x denotes the skill of the manager, z is the skill of the worker, k is the amount of capital in the plant, and μ, θ and β are parameters. In particular, consider that $\mu \in [\frac{1}{2},1)$, $\theta \in (0,1)$ and $1-\mu-\beta > 0$.

This characterization of the production technology has three crucial features that, as discussed in Kremer and Maskin (1996), are strictly required for the purposes of this paper:

- *Imperfect substitutability*. In the description above individuals with different skills are imperfect substitutes in production. One and only one person can be in charge of a given occupation within a plant, so it is impossible to substitute quality (skill) for quantity (number of persons) in that occupation. Imperfect substitutability is required in this paper in order to obtain

[10] There is a broad consensus in the empirical literature about the existence of capital–skill complementarity in the economy. See, for instance, Griliches (1969), Hamermesh (1993) or Goldin and Katz (1998).

[11] Sattinger (1993) and Legros and Newman (2002) are some examples of this literature.

[12] This feature differentiates this paper from many in the assignment literature where the assignment problem consists on pairing individuals/things that belong to two *ex ante defined groups*. For instance, Koopmans and Beckmann (1957) pair plants with locations, Becker (1973) pairs men with women and Caselli (1999) pairs workers with machines.

[13] The assignment problems in these two papers are not exactly the same as the one considered here. In particular, in Kremer (1993) plants have more than two occupations, and in Lucas (1978) a manager can hire more than one worker and skill is a two-dimensional variable (there is skill as a manager and skill as a worker although, in fact, he considers that everyone has the same skill as a worker).

[14] This convention calls "manager" to one of the members of the plant and "worker" to the other. This is done just for simplicity. What is really important is that a plant is composed of two types of workers. In this sense, what the model means by workers' segregation by skill below is simply individuals' segregation by skill.

implications about the composition of the plants. In particular, these implications could not be obtained if individuals with different skills were perfect substitutes in production, as they are in the classical efficiency units model. In that case, the output of a plant could be expressed as a function of an aggregate measure of skill in the plant. Then, plants with the same aggregate measure of skill would be observationally equivalent, even though they could have very different workforces.

- *Complementarity between skills*. In (1) the skill of the manager and the skill of the worker are complementary in production. This feature of the production technology is relevant in order to obtain sensible implications about the equilibrium composition of the plants. The empirical evidence broadly supports the idea that there is positive sorting among managers and workers in the economy (the best managers hire the best workers).[15] The fact that $\partial^2 f(x,z,k)/\partial x\, \partial z > 0$ delivers this result. Instead, if $\partial^2 f(x,z,k)/\partial x\, \partial z < 0$ the equilibrium composition of the plants would involve negative sorting, and if $\partial^2 f(x,z,k)/\partial x\, \partial z = 0$ one could not establish any kind of relationship between the skills of the two individuals paired together in a plant.
- *Asymmetry between skills*. In (1) there is an asymmetry in production between the skill of the manager and the skill of the worker. Basically, they affect the output of the plant in different ways. Again, this asymmetry is needed to avoid compositional implications that are not interesting for the purposes of this paper. In this sense, with a symmetric production technology, in which the skill of the manager and the skill of the worker affect the output of the plant in the same way, the equilibrium would always imply, as in Kremer (1993), no skill heterogeneity within-plants, perfect segregation of individuals by skill into different plants and zero wage inequality *within-plants*. Moreover, this asymmetry needs to be introduced in a sensible way. In particular, one could intuitively expect that, within a plant in equilibrium, the manager is more skilled than his worker. As it will be clear below, imposing that $\mu \in [\frac{1}{2},1)$ delivers this result. For now, just note that when $\mu \in [\frac{1}{2},1)$, the output produced by any plant composed of two individuals with different skills is always larger when the most skilled individual is the manager. Specifically, $\forall a > b$ and $\forall k > 0$,[16]

$$f(a,b,k) > f(b,a,k) \tag{2}$$

In addition to these features, note that (1) also shows complementarity in production (i) between the skill of the manager and the amount of capital and (ii) between the skill of the worker and the amount of capital (since, by assumption, $1-\mu-\beta > 0$). Thus, even though these conditions are not the conventional way in the literature of introducing capital–skill complementarity in production, one could say that this model exhibits a form capital–skill complementarity.[17] This is relevant for the results of this paper. In particular, given this complementarity, the decline in the relative price of capital will constitute a skill-biased technological change that will push wage inequality upwards.

Two additional comments about the particular functional form considered in (1). First, since there is not a standard production function involving capital in the assignment literature, this paper considers a functional form, the combination of a Cobb–Douglas and a CES, that has been used extensively in the literature to explain a wide variety of issues. Second, note that there are three possible ways of distributing x, z and k in a functional form that combines a Cobb–Douglas and a CES: one could place x outside of the CES, z or k. The option adopted in (1) is not arbitrary. Leaving k outside of the CES does not work for the purposes of this paper because, in that case, capital would affect x and z in the same way and, then, the equilibrium composition of the plants would not be affected by a change in the relative price of capital.[18] Leaving z outside of the CES is not a good choice either, as it could produce some unappealing assignment implications. In particular, under a production function of this type, it could happen that, for some prices of capital, the least skilled individuals within some plants choose to be the managers. One would expect, however, that within a plant the most skilled individual plays the role of the manager. Several pieces of empirical evidence provide support to this intuition, which is standard in the assignment literature. For instance, the empirical literature analyzing the wage structure and promotions within organizations most commonly find that there is sorting within the firms so that the higher ability workers promote and become managers and the workers with more job responsibilities are abler (see, for example, Baker et al., 1994; Dohmen et al., 2004; Lazear and Shaw, 2007).[19] Leaving x outside of the CES does not present any of these problems and that is why it is the option adopted in (1).

A final comment about this production technology. Considering that there are only two occupations within a plant and that skill is a one-dimensional variable is enough for the purposes of this paper. Assignment models are very demanding analytically and this strongly pushes for simplicity in the characterization of the production technology. This is why these two features are

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[15] See, for example, Doms et al. (1997) and Bartelsman and Doms (2000).

[16] Eq. (2) is satisfied if, $\forall a > b$ and $\forall k > 0$, $a^{\mu}[\theta k^{\beta}+(1-\theta)b^{\beta}]^{(1-\mu)/\beta} > b^{\mu}[\theta k^{\beta}+(1-\theta)a^{\beta}]^{(1-\mu)/\beta}$. This happens if $(\partial/\partial s)(s^{\mu}/[\theta k^{\beta}+(1-\theta)s^{\beta}]^{(1-\mu)/\beta}) > 0$, and this is always the case when $\mu \in [\frac{1}{2},1)$.

[17] The most usual approach in the literature to introduce capital–skill complementarity is through a production function with three inputs, skilled labor (S), unskilled labor (U), and capital (K), in which the direct elasticity of substitution (or, in other cases, the Allen–Uzawa partial elasticity of substitution) between skilled labor and capital (σ_{SK}) is lower than between unskilled labor and capital (σ_{UK}). This interpretation of capital–skill complementarity cannot be considered here. In particular, note that in the production function (1) one cannot associate x, the skill level of the manager (or the labor input in the managerial occupation), with the*skilled* labor input, nor z, the skill level of the worker (or the labor input in the workers' occupation), with the *unskilled* labor input (and, by extension, one cannot associate σ_{xk} and σ_{zk} with σ_{SK} and σ_{UK}, respectively). The reason why this association is not possible is that, as it will be clear below, in many instances it happens that in the equilibrium of the economy some plants have both the managerial and the workers' occupation filled by *unskilled* individuals while other plants have both of them filled by *skilled* individuals.

[18] For this same reason, it is not convenient for the purposes of this paper to consider a Cobb–Douglas production function with inputs x, z and k.

[19] Nevertheless, in line with the literature on the Dilbert's principle (see, for example, Faria, 2000), future work should explore the predictions of this model when workers are sometimes more skilled than their managers.

very common in the assignment literature. Obviously, more realism in these dimensions would be better but this would come at a great analytical cost.

2.1. The production function net of capital costs

The previous section characterized the plants' production function. However, in order to define and to characterize the equilibrium in the following sections, it is more useful to consider the plants' production function *net of the optimal capital costs*. In this sense, consider that plants do not have capital when they are created, but that they can buy any amount of it at an exogenously given price p. Then, the plants' production function net of the optimal capital costs can be defined as

$$h(x,z,p) \equiv f(x,z,k^*) - pk^*$$

where k^* is the solution to the following maximization problem:

$$\max_k \ f(x,z,k) - pk$$

This function $h(x,z,p)$ replicates the most relevant properties of the function $f(x,z,k)$ in Eq. (1). Specifically, $h(x,z,p)$ also (i) increases with the skill of the manager and with the skill of the worker, (ii) exhibits complementarity in production between the skill of the manager and the skill of the worker, and (iii) has an asymmetry in production between the skill of the manager and the skill of the worker. Similar to (2), the asymmetry in $h(x,z,p)$ makes that the *net* output produced by any plant composed of two individuals with different skill levels is always larger when the most skilled individual is the manager. Formally, $\forall a > b$ and $\forall p > 0$,

$$h(a,b,p) > h(b,a,p)$$

Results (i) and (ii) can be easily obtained using the envelope theorem. As for the last result, it comes directly from Eq. (2). To see this simply note that (2) implies that $f(a,b,k^*_{ba}) - pk^*_{ba} > f(b,a,k^*_{ba}) - pk^*_{ba}$, where k^*_{ba} is the amount of capital that a plant composed of a manager with skill b and a worker with skill a would optimally buy. But since, by definition, $h(a,b,p) = f(a,b,k^*_{ab}) - pk^*_{ab} \geq f(a,b,k^*_{ba}) - pk^*_{ba}$ and $f(b,a,k^*_{ba}) - pk^*_{ba} = h(b,a,p)$, this implies (iii). As it will be clear below, these properties of $h(x,z,p)$ will be crucial in determining the features of the economy's equilibrium assignment.

3. The assignment problem and the equilibrium

Consider that the economy is populated by a continuum of individuals with different skill, s, and that this skill is distributed across the population according to a continuous density function $\phi(s)$ defined over the interval $[s_{min}, s_{max}]$. Furthermore, consider that the assignment of individuals to plants and to occupations is frictionless. Specifically, everybody's skill is public information and the movement of individuals across plants and occupations is costless and it does not require time. Then, for a given price of capital p, the assignment problem in this paper is to allocate individuals to plants and to occupations within those plants and to allocate net output (payoff) to individuals in a way that is feasible given the production technology and the skill distribution and that is stable. Formally, the equilibrium (solution) of this problem is the combination of[20]:

- An occupational correspondence, $\Omega : [s_{min}, s_{max}] \rightrightarrows \{manager, worker\}$, that specifies, for each skill level, the occupational choice of the individuals with that skill. This in turn defines the sets:

$$M = \{s \in [s_{min}, s_{max}] : \Omega(s) = manager\}$$

$$WO = \{s \in [s_{min}, s_{max}] : \Omega(s) = worker\}$$

- A matching function, $\psi : M \longrightarrow WO$, that specifies the way managers are paired with workers to create plants.[21]
- A payoff function, $W : [s_{min}, s_{max}] \rightarrow \mathbb{R}$, that determines everybody's payoff.[22]

such that:

- The payoff structure is feasible. That is, in any plant, the combined payoff of its members is not greater than the net output they produce[23]:

$$W(s) + W(\psi(s)) \leq h(s, \psi(s), p) \quad \forall s \in M \tag{3}$$

[20] The equilibrium defined in this way is equivalent to the competitive market equilibrium considered in Kremer and Maskin (1996).

[21] To be precise, ψ could be a correspondence instead of a function. However, in the analysis that follows this is never the case and it is less intuitive to define the equilibrium when ψ is a correspondence. That is why ψ is considered to be a function here. See Legros and Newman (2002) for a definition of the equilibrium when ψ is a correspondence.

[22] This definition already incorporates one equilibrium result. In particular, the equilibrium in this model requires that individuals with identical skill obtain the same payoff. In other words, W is a function and not a correspondence.

[23] Due to condition (5) below, in equilibrium this equation is always satisfied with equality $\forall s \in M$.

- The assignments are feasible. That is, for any type of plant, the mass of managers is equal to the mass of workers[24]:

$$\int_{s\in A}\phi(s)\,ds=\int_{s\in\psi(A)}\phi(s)\,ds \text{ for every measurable set of managers } A\in M \tag{4}$$

- None has an incentive to deviate. That is:

$$\nexists a,b\in[s_{min},s_{max}]:\max\{h(a,b,p),h(b,a,p)\}>W(a)+W(b) \tag{5}$$

4. Basic properties of the equilibrium

The equilibrium assignment defined above has several interesting properties that do not depend on the particular skill distribution, price of capital or values of the parameters of the production function considered. These are formally stated in Lemma 1[25]:

Lemma 1. *Regardless of the economy's skill distribution, the price of capital and the values of the parameters of the production function, the equilibrium assignment*:

(i) *always exists*,
(ii) *maximizes the economy's aggregate net output among all the feasible assignments*,
(iii) *requires that the most skilled individual within any plant is the manager*,
(iv) *involves positive sorting between managers and workers, and*
(v) *requires a payoff function that is strictly increasing with respect to skill.*

The existence and efficiency of the equilibrium assignment should come at no surprise given the fact that the model does not contain any friction or imperfection. Instead, (iii) and (iv) come, respectively, from the asymmetry and the complementarity in production between the skill of the manager and the skill of the worker imposed in Section 2. Finally, the equilibrium payoff function needs to be increasing with respect to skill because the net output of a plant strictly increases both with the skill of its manager and with the skill of its worker.

Given the results presented in Lemma 1, only one additional piece of information is needed to fully characterize the equilibrium assignment: who are managers and who are workers in equilibrium. Now note that there are two forces in this model that play a role in determining these sets in equilibrium: the *complementarity* and the *asymmetry force*.[26] The former is due to the complementarity in production between the skill of the manager and the skill of the worker and the latter is due to the different roles that these skills play in production, as imposed in Section 2. These two forces push in different directions in determining who must be managers and who must be workers in equilibrium. To understand this, consider the case where there is complementarity in production between the skill of the manager and the skill of the worker but not asymmetry. In this case, the equilibrium assignment would be such that the best individuals would be paired with the best and the worst with the worst. Then, both low- and high-skilled individuals would be managers (and workers) in equilibrium and there would be maximal segregation of individuals by skill into different plants. This is the kind of equilibrium, for instance, in Kremer (1993). Instead, consider the case where there is asymmetry in production between the skill of the manager and the skill of the worker (as stated in Section 2) but not complementarity. In this case, the equilibrium assignment would be such that everybody with skill above the median skill in the population would be a manager while everybody else would be a worker. This equilibrium partition of individuals between of occupations is similar to one in Lucas (1978). Then, intuitively, the *complementarity force* pushes towards individuals' segregation by skill into different plants while the *asymmetry force* pushes high-skilled individuals into the managerial occupation inducing lower levels of skill segregation.

These two forces operate simultaneously in this model and, since they push in different directions in determining who must be managers and who must be workers in equilibrium, the exact shape of the equilibrium assignment depends on their relative strengths. These strengths, in turn, depend on the economy's skill distribution, on the price of capital and on the values of the parameters of the production function. Therefore, more structure is needed in order to know who are managers and who are workers in the equilibrium assignment and to completely characterize that assignment. In this sense, the two equilibrium assignments described above will only appear here as extreme cases while, in general, the equilibrium assignment will be a combination of both. That is, in general, in the equilibrium assignment in this model neither there will be complete segregation of individuals by skill nor everybody with skill below (above) the median skill in the population will be a worker (manager).

[24] Since plants are composed of one manager and one worker, it is unfeasible to pair, for instance, a mass of managers of measure $\frac{1}{3}$ to a mass of workers of measure $\frac{2}{3}$.
[25] See Appendix A for the proofs of all the lemmas in this paper.
[26] These two forces also appear simultaneously, for example, in Kremer and Maskin (1996) and in Davis (1997).

As stated in the Introduction, this paper claims that the decline in the relative price of capital, by changing the economy's equilibrium assignment, can produce simultaneously both (i) a larger increase in the wage inequality *between-plants* than in the wage inequality *within-plants* and (ii) an increase in the segregation of individuals by skill. Despite that, as mentioned above, more structure needs to be introduced in the model to be able to completely characterize the economy's equilibrium assignment and its evolution, one can intuitively see already how the model presented so far can deliver this connection. Basically, this happens whenever the decline in the relative price of capital, that increases the overall wage inequality due to the presence of capital–skill complementarity in production, also increases the strength of the *complementarity force* relative to the *asymmetry force*. As discussed above, in that case the economy moves towards an equilibrium assignment with higher segregation of individuals by skill what, in turn, induces larger increases in the wage inequality *between-plants* than in the wage inequality *within-plants*.

Again, whether a decline in the relative price of capital increases relatively more the strength of the *complementarity force* or the *asymmetry force* depends on the specific parameter values considered. As p falls, the strength of the *complementarity force* always increases. To see this note that, since $1-\mu-\beta>0$, $\partial^2 f(x,z,k)/\partial x\,\partial z$ increases with k $\forall(x,z)$. Instead, the strength of the *asymmetry force* may increase or decrease as the price of capital declines. In fact, it decreases if $\beta<0$ and it increases if $\beta>0$.[27] Thus, the condition that $\beta<0$ is sufficient to guarantee that the strength of the *complementarity force* increases relative to the *asymmetry force* as p falls. The following section illustrates the working of the model under this condition, using a particular version of the model described above for which it is possible to completely characterize the equilibrium assignment of the economy for any price of capital. Have in mind, however, that in principle the model could behave in a similar fashion even if that condition is not satisfied since it is a sufficient but not necessary condition.[28] Finally, considering that the decline in the price of capital decreases the strength of the *asymmetry force* is not unrealistic. For instance, Autor et al. (2003) suggest that the tasks within a firm can be divided in two types: routine tasks and non-routine tasks. They also consider that capital is used more intensively in the routine tasks that in the non-routine tasks.[29,30] Now, by assuming that managers do relatively more non-routine tasks than workers, one could conclude that workers use capital more intensively than managers. Then, since more capital increases the value of skill in the two positions (due to capital–skill complementarity), and capital is used relatively more intensively by workers than by managers, one could argue that more capital increases the value of the worker's skill relatively more than that of the managerial skill, thus reducing the extent of the asymmetry in production.

5. Full characterization of the equilibrium

Consider that:

(A1) $\mu=\frac{1}{2}$.
(A2) Skill is distributed across the population according to a uniform distribution between $[0,s_{max}]$.

These two assumptions help characterizing the equilibrium assignment of the economy to a larger extent than in Section 4. To begin with, assumption (A1) implies that the *complete assortative assignment*, the one in which individuals are perfectly segregated by skill into different plants (that is, all individuals with the same skill are paired among themselves), is the equilibrium assignment only when capital is free. Lemma 2 presents this result[31]:

Lemma 2. *When $\mu=\frac{1}{2}$, the complete assortative assignment is the equilibrium assignment of the economy if and only if $p=0$.*

The intuition behind this result is the following. When capital is free, plants buy an infinite amount of it.[32] Then, if $\mu=\frac{1}{2}$, the asymmetry in production between the skill of the manager and the skill of the worker disappears. More specifically, in that case, $h(x,z,0)=(1-\theta)^{1/2\beta}(xz)^{1/2}$. Therefore, only the *complementarity force* operates in the economy and the *complete assortative assignment*, where there is maximal segregation by skill, becomes the equilibrium. However, when $p\neq 0$, even if $\mu=\frac{1}{2}$, the *asymmetry force* operates in the economy and it keeps the equilibrium away from perfect segregation. In other words, one could argue that there are two sources of asymmetry in the model. On the one hand, there

[27] Note that the elasticity of output with respect to the managerial skill is equal to μ, which is higher than the elasticity of output with respect to the worker's skill that equals $(1-\mu)(1-\theta)z^\beta/[\theta k^\beta+(1-\theta)z^\beta]$. If $\beta<0$, the latter gets closer to the former as the price of capital declines and firms buy more of it. Thus, intuitively, the extent of the asymmetry in production between the skill of the manager and the worker gets reduced and so the strength of the *asymmetry force*.

[28] For continuity, at least for values of β slightly greater than 0, one should also expect that the strength of the *complementarity force* increases relatively more than the strength of the *asymmetry force* as p falls. Thus, the model should behave in a similar fashion for those values.

[29] The reasoning could be the following. The difference between a routine task and a non-routine task is that the former can be "proceduralized" while the latter cannot be (or it can be but at a very high cost). Now, think of capital as computers. Computers can only do tasks that can be codified in a fully specified sequence of logical programming commands. That is why they are used more intensively in routine tasks than in non-routine tasks.

[30] Holmes and Mitchell (2008) also consider that a firm is composed by a number of tasks with a different degree of complexity, and that capital tends to do the relatively easy-to master or routine tasks.

[31] Obviously, when the *complete assortative assignment* is the equilibrium of the economy, the equilibrium payoff function is $W(s)=\frac{1}{2}h(s,s,p)$.

[32] To see this, just note that $\lim_{k\to\infty} f_3(x,z,k)=0$ $\forall(x,z)$. Recall also that, as stated above, in this particular case of the model $\beta<0$.

is an *exogenous* source that has to do with μ being equal to or greater than $\frac{1}{2}$. On the other hand, there is an *endogenous* source that has to do with the different relationships between capital and workers' and managers' skills. By making $\mu=\frac{1}{2}$ in this section we concentrate in the latter more interesting source.[33] In this sense, when $\mu=\frac{1}{2}$ and $p=0$ both sources disappear, but when $p\neq 0$ the *endogenous* one still operates. Assumption (A2) helps characterizing the equilibrium assignment in the economy further in the latter case.

5.1. Equilibrium assignment

To find the equilibrium assignment in this economy, the following reasoning is useful. Assume that the solution to the assignment problem consisting of pairing individuals with skills in the interval $[0,s_{max}]$ requires that the individuals in the interval $[\lambda s_{max},s_{max}]$ are paired among themselves, with the top 50% more skilled individuals within this interval being managers and the rest being workers. Now consider how the equilibrium assignment of individuals in the interval $[0,\lambda s_{max}]$ should be.

It turns out that the assignment problem in which skill is distributed uniformly between $[0,s_{max}]$ and the assignment problem in which skill is distributed uniformly between $[0,\lambda s_{max}]$ are isomorphic. More specifically, the latter problem is just a redefinition of the former one in which $\tilde{s}=\lambda s$, and this redefinition does not affect the basic structure of the problem for two reasons:

- A uniform distribution between $[0,s_{max}]$ is identical to a uniform distribution between $[0,\lambda s_{max}]$ except for a multiplicative term.[34]
- The plants production function of net output is homogeneous of degree one in the skill of the manager and the skill of the worker. That is, $h(\lambda x,\lambda z,p)=\lambda h(x,z,p)$.

Therefore, since the only difference between the original and the redefined problem is a multiplicative term, the shape of the solution to both problems must be the same. To be more specific, if the solution to the original assignment problem implies that the individuals in the interval $[\lambda s_{max},s_{max}]$ are paired among themselves, then the solution to the redefined assignment problem must require that the individuals in the interval $[\lambda\tilde{s}_{max},\tilde{s}_{max}]$ must be paired among themselves according to the same rules. Taking this reasoning repetitively, it is possible to deduce the exact shape of the equilibrium assignment for an arbitrary price of capital p given $\lambda(p)$ and, in particular, to realize that the equilibrium assignment must have an infinite number of intervals. Lemma 3 presents this equilibrium assignment formally:

Lemma 3. *For each price of capital $p\neq 0$ the equilibrium assignment in this economy exhibits an infinite number of intervals of the form $[\lambda^i s_{max},\lambda^{i-1}s_{max}]$, $i=1,2,3,\ldots$, such that*:

- *in each one of these intervals,*
  - *the individuals with skill $s\in[\lambda^i s_{max},m_i]$ are workers, where $m_i=\lambda^{i-1}((1+\lambda)/2)s_{max}$.*
  - *the individuals with skill $s\in[m_i,\lambda^{i-1}s_{max}]$ are managers.*[35]
  - *each manager with skill s is paired with a worker with skill $\psi_i(s)$, where $\psi_i(s)=s-t_i$ and $t_i=\lambda^{i-1}((1-\lambda)/2)s_{max}$.*
- $\lambda=\lambda(p)\in(0,1)$ *is the solution to the following maximization problem*:

$$\max_{\lambda}\sum_{i=1}^{\infty}Y_i(\lambda) \tag{6}$$

where $Y_i(\lambda)$ denotes the net output produced within interval i.[36]

Fig. 1 provides a graphical representation of this assignment, where each arrow indicates that the individuals in the interval of origin hire the individuals in the interval of destination according to the matching function $\psi(s)$.

Note that the equilibrium assignment described above:

- Involves positive sorting between managers and workers. That is, $\partial\psi_i(s)/\partial s>0\ \forall i$.
- Is feasible given the uniform skill distribution assumed in (A2). In each interval i, m_i is the median skill level within the interval. Thus, the mass of managers in the subinterval $[m_i,\lambda^{i-1}s_{max}]$ is equal to the mass of workers in the

[33] The evolution of wage inequality and of segregation by skill as the price of capital declines is qualitatively the same in the model regardless of μ being equal to or greater than $\frac{1}{2}$. Considering that $\mu=\frac{1}{2}$ simply helps illustrating that behavior. The only qualitative difference between the two cases is that when $\mu>\frac{1}{2}$ the *complete assortative assignment* is never the equilibrium assignment in the economy (even when capital is free) and, therefore, when $p=0$ wage inequality *within-plants* is greater than zero.

[34] Here is where the uniformity of the skill distribution and the fact that $s_{min}=0$ (assumption (A2)) plays its role.

[35] In other words, $\Omega(s)=manager\ \forall s\in[m_i,\lambda^{i-1}s_{max}]$, $i=1,2,3,\ldots$ and $\Omega(s)=worker\ \forall s\in[\lambda^i s_{max},m_i]$, $i=1,2,3,\ldots$

[36] Although not shown explicitly, it must be clear that $Y_i(\lambda)$ depends on p.

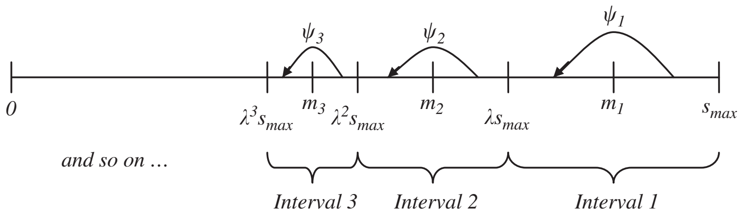

Fig. 1. Equilibrium assignment.

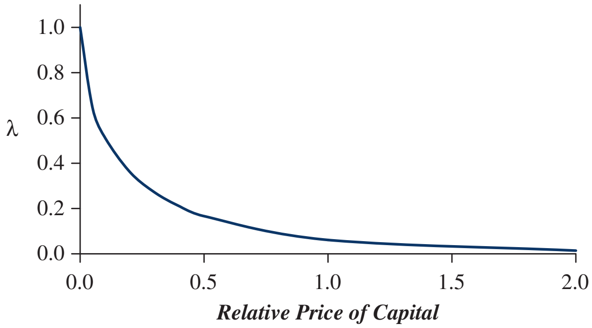

Fig. 2. Function $\lambda(p)$.

subinterval $[\lambda^i s_{max}, m_i]$. Furthermore, $\psi_i(s)$ is such that Eq. (7) is satisfied. It turns out that this implies that condition (4) is also satisfied.

$$\int_{\lambda^i s_{max}}^{m_i} \phi(s)ds = \int_{m_i}^{\lambda^{i-1} s_{max}} \phi(s)ds \quad \forall s \in [m_i, \lambda^{i-1} s_{max}]\ \forall i \tag{7}$$

As for the value of $\lambda = \lambda(p)$, that characterizes the equilibrium assignment for each price of capital, it is implicitly defined by Eq. (8), which is the first order condition of (6).

$$Y_1'(\lambda)[1-\lambda^2] + Y_1(\lambda)2\lambda = 0 \tag{8}$$

Very intuitively, the function $\lambda(p)$ that solves (8) is such that (i) it decreases continuously as p increases, (ii) it approaches to 1 as p goes to zero and (iii) it approaches to 0 as p goes to infinite. The behavior of this function is illustrated in Fig. 2 using, arbitrarily, $\beta = -1.5$, $\theta = 0.5$ and $s_{max} = 100$. The three panels of Fig. C1 show that this behavior is robust to different values of s_{max}, θ and β.

5.2. *Evolution of the equilibrium as p decreases*

From Fig. 2 it is clear that the smaller the price of capital the higher the value of $\lambda = \lambda(p)$. Therefore, as p decreases, the equilibrium assignment in the economy changes qualitatively as in Fig. 3.[37]

It is now clear that this model integrates the equilibrium assignments in Kremer (1993) and in Lucas (1978) within the same framework as two extreme cases. When $p=0$ ($\lambda = 1$) the equilibrium exhibits complete segregation of individuals by skill as in Kremer (1993). Instead, as p goes to infinite (λ goes to 0) the equilibrium assignment moves towards the one in Lucas (1978) in the sense that there is a cutoff level of skill such that everybody above that level is a manager and everybody below it is a worker. The essence of this paper lies precisely in understanding the implications, in terms of individuals' segregation by skill and of wage inequality, of moving from one kind of equilibrium to the other (driven by a fall in p). Regarding the former, Lemma 4 shows that individuals' segregation by skill increases in the economy as the price of capital decreases.

Lemma 4. *As p decreases, the difference between the skill of the manager and the skill of the worker within-plants in equilibrium decreases in average.*

[37] In each one of the equilibrium assignments depicted in Fig. 3 there is an infinite number of intervals but only the first ones are shown.

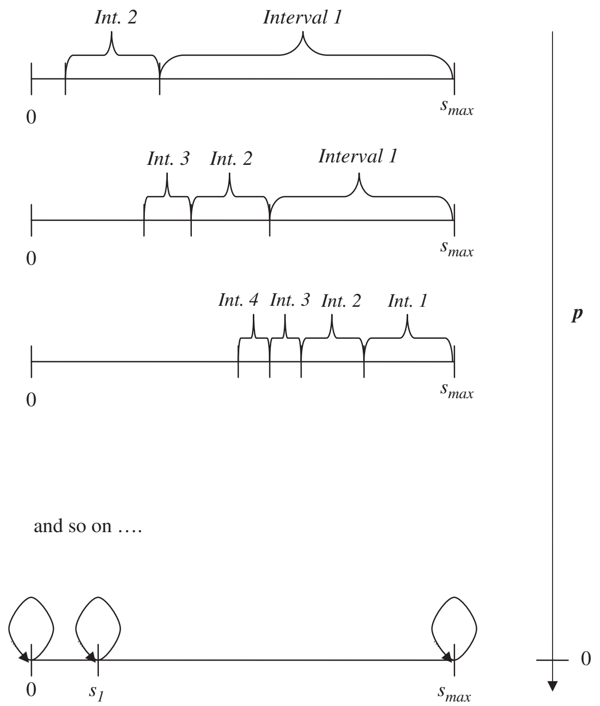

Fig. 3. Evolution of the equilibrium assignment as p decreases.

As for the predictions of the model regarding the evolution of wage inequality, consider the following decomposition of the overall wage inequality (σ_T^2) into the *between-plants* (σ_{BP}^2) and the *within-plants* (σ_{WP}^2) wage inequality[38]:

$$\sigma_T^2 = \frac{\sum_{l=1}^{N}[W(s_l)-\overline{W}]^2}{N} = \frac{\sum_{j=1}^{J} 2[\overline{W}^j-\overline{W}]^2}{N} + \frac{\sum_{j=1}^{J}([W(s_1^j)-\overline{W}^j]^2+[W(s_2^j)-\overline{W}^j]^2)}{N} = \sigma_{BP}^2+\sigma_{WP}^2 \tag{9}$$

where $\overline{W}$ is the average wage in the economy's equilibrium assignment and $\overline{W}^j$ is the average wage in the jth plant, $j=1,2,\ldots,J=N/2$, that in the equilibrium assignment is composed of two individuals, one with skill s_1^j and another one with skill s_2^j. In the second line of Eq. (9), the first term is the variance in average wage across plants and the second term averages the wage inequality within each type of plant in equilibrium. Thus, one could consider the former a measure of wage inequality *between-plants*, σ_{BP}^2, and the latter a measure of wage inequality *within-plants*, σ_{WP}^2.

Figs. 4 and 5 show that both σ_{BP}^2, σ_T^2 and the ratio σ_{BP}^2/σ_T^2 increase continuously as p decreases for a particular set of parameter values.[39,40] As a robustness check, Fig. C2 shows that σ_{BP}^2/σ_T^2 behaves in the same fashion as p decreases when different parameter values are considered. As for σ_{WP}^2, it first increases slightly when p decreases but eventually decreases towards 0 as p goes to 0.[41]

Another measure of wage inequality frequently used in the literature is the skill premium. One could define the skill premium in this model as the average wage in equilibrium for individuals with skill above s_{median} over the same measure for individuals with skill equal or lower than s_{median}. As Fig. 6 shows, the model clearly predicts an increase in this measure of the skill premium when p decreases. This behavior is robust to set of parameters considered, as illustrated in Fig. C3, and to the cutoff level of skill used to compute the premium.

Finally, the model has another interesting prediction. If one defines a plant's labor productivity as half the net output it produces, then the variance of labor productivity across plants coincides in this model with the variance of average wage across plants, that is, with σ_{BP}^2. Thus, according to Fig. 4, the model also predicts an increase in the dispersion in labor productivity across plants when p decreases.[42] The same conclusion can be reached measuring dispersion as the 90–10

[38] This decomposition is similar to the one in Davis and Haltiwanger (1991).

[39] See Appendix B for a description of the algorithm employed to find the optimal wages in the equilibrium assignment.

[40] All figures in this section are constructed using arbitrarily $\beta=-1.5$, $\theta=0.5$, $s_{max}=100$ and $I=50$. However, all results are *qualitatively* robust to the specific parameter values considered. For the more relevant cases, this robustness is illustrated with some additional figures in Appendix C.

[41] Recall that when $p=0$ the economy's equilibrium assignment is the *complete assortative assignment*. Therefore, when $p=0$ there is not skill heterogeneity within-plants, $\sigma_{WP}^2=0$ and $\sigma_{BP}^2=\sigma_T^2$.

[42] This model considers that skill is perfectly observable. Alternatively, if someone who does not observe skill perfectly analyzes this economy, he would conclude that there are TFP differences across plants, and that the dispersion in TFP across plants increases when p decreases.

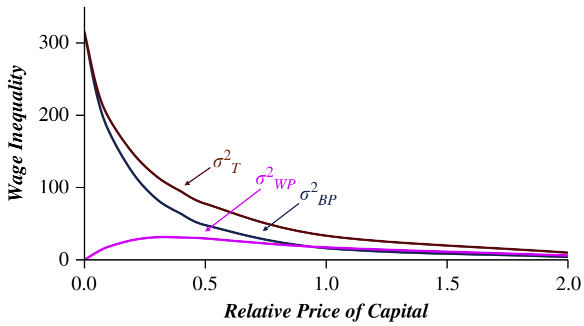

Fig. 4. Evolution of the overall, *between-plants* and *within-plants* wage inequality.

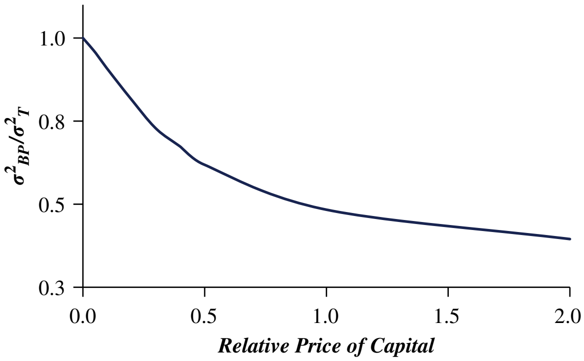

Fig. 5. Evolution of σ^2_{BP}/σ^2_T.

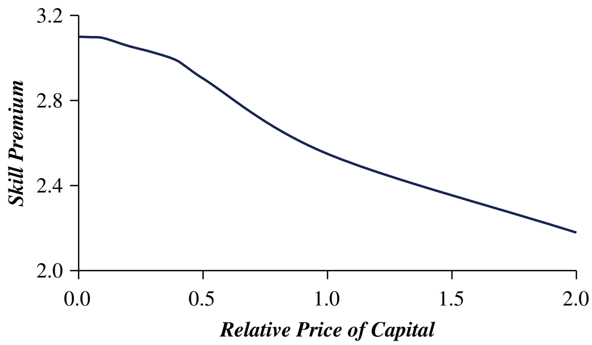

Fig. 6. Evolution of the skill premium.

differential of the log of labor productivity across plants. In the same vein, the model also predicts that dispersion in capital holdings across plants increases smoothly as the price of capital declines.

All these predictions of the model, when the economy faces a declining relative price of capital (as observed empirically in the U.S.), are broadly consistent with the empirical evidence. As mentioned in the Introduction, in the U.S. from the 1970s to the 1990s there was, *overall*,[43]

- *an increase in the segregation of workers by skill into separate plants.*
- *a large increase in the overall and the between-plants wage inequality and a slight increase in the within-plants wage inequality, and therefore an increase in the ratio* σ^2_{BP}/σ^2_T.
- *an increase in the skill premium.*
- *an increase in the dispersion of labor productivity across plants.*

[43] As already discussed, these facts are not exclusive of the U.S. For instance, in the U.K., Faggio et al. (2007) also find that, over the period 1984–2001, most of the increase in wage inequality was a between-firm phenomenon and that dispersion in labor productivity across firms increased.

These are *overall* patterns and the model, admittedly stylized, succeeds in capturing them. Of course, one would need to introduce additional elements into the model to be able to account for more detailed aspects of the evolution of these variables. For instance, to account for the decline in the skill premium observed in the U.S. during the second half of the 1970s (although it is clear that, *overall*, the skill premium increased between the 1970s and the 1990s). In this sense, the framework proposed in this paper is rich enough to consider *both* changes in the relative price of capital and in the economy's skill distribution over time. The following section shows that, by adding the latter, the model may provide a more comprehensive explanation of the evolution of the relevant variables in the data, and it may potentially account for episodes of decreasing wage inequality.

6. Changes in the skill distribution

Krusell et al. (2000) provide an "[…] explicit economic mechanism for understanding skill-biased technological change in terms of observable variables" and show that, in the presence of capital–skill complementarity in production, the development of better and cheaper capital equipment is the main driving force behind the increase in the U.S. skill premium in the last decades of last century. The assignment model presented in this paper extends this reasoning to show how this decline in the relative price of capital may also be responsible for the observed behavior of the *between*- and *within-plants* wage inequality and of workers' segregation by skill.

Kremer and Maskin (1996) propose an alternative explanation for these patterns. Namely, they provide some evidence that both the mean and the dispersion of the skill distribution have increased in the U.S. and the U.K. over time, and argue that these changes in the skill distribution may be liable for the observed increases in workers' segregation by skill and in wage inequality.[44] They note, however, that when the skill distribution is "tight", an increase in the mean skill level may reduce wage inequality. This section shows that the assignment model presented above is able to deliver these results. In this sense, this model encompasses the one in Kremer and Maskin (1996), and extends it by incorporating capital as an input in production which allows to obtain implications concerning the fall in its price. To the extent that the evolution of wage inequality and of segregation by skill in the data have been shaped *both* by changes in the skill distribution and in the relative price of capital, this feature of the model makes it more suitable for future quantitative analysis.

To show how the model reacts to changes in the skill distribution, this section adopts the following strategy:

- *Skill distribution.* Most typically, in the empirical and/or theoretical literature related to this paper, skill is assumed to be distributed according to a lognormal distribution, $s \sim \log N(\mu_s, \sigma_s)$.[45] To match some relevant features of the PSID wage distribution, Chang (2000) calibrates the parameters of this skill distribution such that $\mu_s = 2.11$ and $\sigma_s = 0.58$. This is the benchmark distribution considered in this section.
- *Model simulation.* For each pair price of capital–skill distribution, the model is solved numerically for a discrete version of the economy. This consists of 300 individuals, whose skill levels are randomly drawn from the corresponding theoretical skill distribution. The optimal assignment of these individuals and the wages sustaining that assignment are found by means of a linear programming problem and a wage algorithm like the ones described in the proofs of Lemmas 3 and 4, respectively. Considering a discrete economy with 300 individuals provides a reasonable description of the continuum economy. In particular, it was found that raising the number of individuals beyond 300 did not change significantly the results of the model and it only increased the computational burden. Nevertheless, to guarantee the robustness of the results, the model is solved for 1000 different random draws of 300 individuals each.

Fig. 7 shows how workers' segregation by skill (proxied by the ratio σ^2_{BP}/σ^2_T) and the wage inequality (proxied by the skill premium) respond to increases in σ_s, which imply a higher mean and dispersion of the skill distribution.[46] In the figures, the solid line shows the median estimate of the corresponding variable across the 1000 different simulations undertaken, while the two dashed lines show the 10th and 90th percentiles. Consistent with the main finding in Kremer and Maskin (1996), the model in this paper is able to deliver higher workers' segregation by skill and wage inequality when the mean and the dispersion of the skill distribution increases.

When a fall in the relative price of capital is combined with the aforementioned changes in the skill distribution, the increase in workers' segregation by skill and wage inequality is larger (Fig. 8).[47] To a very large extent, this scenario provides a good representation of the *overall* trend for these variables observed during the last decades of the 20th century, as this would have been driven both by falling capital prices (in a context of capital–skill complementarity) and by other forms of skill-biased technological change that increase the mean and dispersion of workers' *effective* skill levels.

[44] In their reasoning, it is indifferent whether these changes in the skill distribution happen through a change in *observable* skill levels or through a skill-biased technological change that raises the *effective* skill level of high-skilled workers.

[45] Some classical references are Roy (1951) and Mirrlees (1971). Other examples are Kanbur and Tuomala (1994) and Chang (2000).

[46] This exercise considers $\mu = 0.5$, $\beta = 1.5$, $\theta = 0.5$, and $p = 0.5$. Shifting σ_s from 0.48 to 0.68 implies a 12% increase in the mean and a 186% increase in the variance of the skill distribution. Similar qualitative results were obtained for different parameterizations of the model and for the uniform and the Pareto distributions.

[47] For the sake of clarity, only the median estimate across all simulations is reported for each variable and exercise. Note that, consistent with the findings in Section 5 for the uniform distribution, a fall in the price of capital (without skill change) also triggers higher workers' segregation by skill and wage inequality when skill is lognormally distributed.

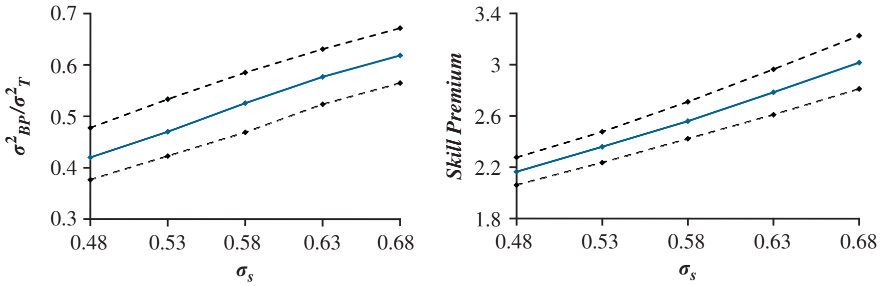

Fig. 7. Effects of a change in the skill distribution.

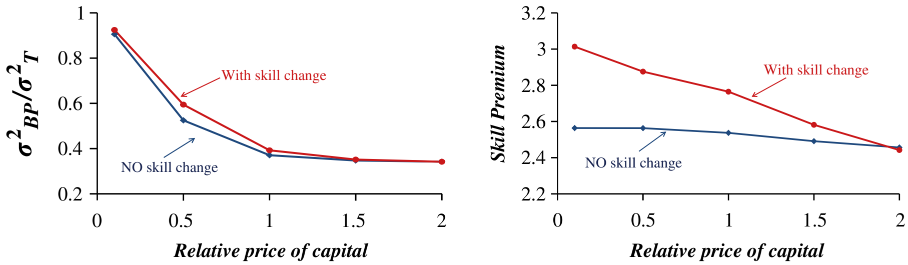

Fig. 8. Effects of changes in the skill distribution *and* in the price of capital.

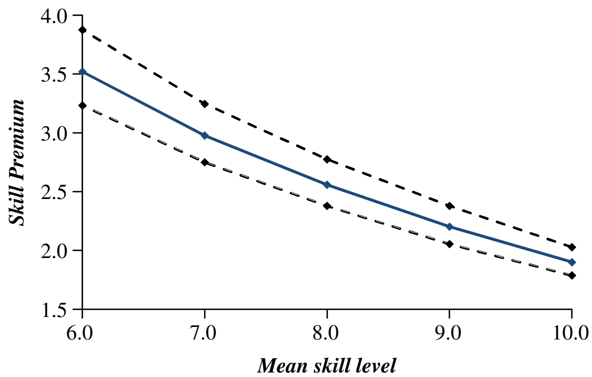

Fig. 9. Skill changes that reduce the skill premium.

For the particular example shown in Fig. 8, the fall in the relative price of capital is the main driver of higher workers' segregation by skill, while changes in the skill distribution towards higher mean and dispersion have a larger impact on the skill premium. Under the framework proposed in this paper, future quantitative work should try to evaluate the extent to which each of the two driving factors considered above is responsible, in different time periods, for the evolution of the *between-* and *within-plants* wage inequality and of the skill premium. This quantitative exercise should be able to explain, not only the *overall* trend in these variables, but also some particular episodes of decreasing wage inequality, like the fall in the skill premium observed in the U.S. in the second half of the 1970s. Again, the model in this paper is well suited for this purpose and it can rationalize a decreasing skill premium in response to certain changes in the skill distribution. This goes in line with another result in Kremer and Maskin (1996). In particular, they argue that, in some circumstances, when the skill distribution is not very dispersed, an increase in the mean skill level may reduce wage inequality in the economy. The model presented above is also able to deliver this result. For instance, for the same parameter values of Figs. 7 and 9 show that the skill premium decreases when the parameters of the lognormal skill distribution are rearranged such that the mean skill level in the economy increases while skill dispersion remains at a rather low level (e.g., 60% of the dispersion in the benchmark skill distribution).

The fact that the model is able to rationalize a fall in the skill premium in response to certain changes in the skill distribution is very important.[48] It shows that, by considering both changes in the price of capital and in the skill

[48] Naturally, if the size of the shock to the skill distribution is large relative to the shock to the price of capital, the model may also deliver a fall in the skill premium, like in Fig. 9, even when the price of capital is falling.

distribution, the model in this paper is very convenient to address, not only the *overall* trend in wage inequality and workers' segregation by skill, but also particular episodes of decreasing wage inequality. This goes in line with the findings in Krusell et al. (2000) that argue that the *relative quantity effect*, which is related to the growth rate of skilled labor relative to the growth rate of unskilled labor, ''contributed significantly to the decline of the skill premium during the 1970's'' while the capital–skill complementarity effect (and the fall in the price of capital) was the driving force behind the *overall* increase in the skill premium over the sample.

7. Conclusion

Empirical evidence for the U.S. suggests that, from the 1970s to the 1990s, (i) wage inequality *between-plants* rose much more than wage inequality *within-plants* and (ii) there was an increase in the segregation of workers by skill into separate plants. This paper presents a frictionless assignment model in which a decline in the relative price of capital, by affecting the equilibrium sets of managers and workers (and, thus, firms' composition) in the economy, is able to deliver these two empirical facts simultaneously. Additional predictions of the model regarding the skill premium and the dispersion in labor productivity across plants are also consistent with the empirical evidence.

It is important to mention that the connection shown in this paper between wage inequality, individuals' segregation by skill and the relative price of capital is not an exclusive feature of the particular functional form in (1) or of the particular version of the model considered in Section 5. With these elements one can illustrate the connection more clearly but the same connection can be found numerically using different functional forms or skill distributions.[49]

In the model presented above a decline in the relative price of capital always increases wage inequality. However, empirically, while in the U.S. the relative price of capital has fallen at least since the 1950s, only after the 1970s wage inequality increased substantially. In defense of the model one could argue two things. First, although the relative price of capital has fallen at least since the 1950s, Krusell et al. (2000) report that its rate of decline accelerated considerably in the period 1975–1992 relative to the period 1954–1975. This is precisely when wage inequality increased the most. And second, as mentioned in the Introduction, other factors besides the relative price of capital may have affected the extent of wage inequality in the economy too. Considering some of these factors (and their evolution over time) would certainly increase the empirical fit of the model. In this sense, Section 6 shows that the model proposed in this paper is rich enough to simultaneously consider changes in the relative price of capital and in the skill distribution. Consistent with other findings in the literature, the inclusion of the latter changes is shown to be helpful in addressing, not only the *overall* trend in wage inequality and workers' segregation by skill, but also particular episodes of decreasing wage inequality. In light of these results, future work that pursue a careful calibration of this extension of the model could be very informative about the relative importance of changes in the economy's skill distribution and in the price of capital in explaining the recent behavior of wage inequality and workers' segregation by skill.

Several extensions of the model are worth-pursuing by future work. First, one could endogenize the behavior of the relative price of capital and of the skill distribution in the model. Regarding the former, Krusell (1998) argues that the decline in the price of capital ''does reflect investment-specific technological change'' and that ''this technological change is a result of explicit R&D decisions on the level of the private firm''. Thus, one possible extension of the model would be to consider R&D decisions at the plant level which could affect the evolution of the relative price of capital in the economy. One preliminary thought on this extension would be that the fall in the relative price of capital and the increase in workers' segregation by skill could reinforce each other. For instance, to the extent that there are complementarities between managers and workers in R&D activities, the aggregate amount of R&D expenditures in the economy could become an increasing function of the level of workers' segregation by skill. In that case, a fall in the price of capital would trigger more segregation (as in this paper), which, in turn, could lead, via R&D expenditures, to an additional decrease in the price of capital. Making the skill distribution endogenous could also produce interesting implications. This could be done in multiple ways. For instance, in a dynamic framework, one could consider that individuals accumulate skill in their workplace and that more skill is accumulated by an individual the more skilled his colleagues are. One preliminary though on this extension would be that higher workers' segregation by skill could eventually lead to a bipolar skill distribution, where the initially high-skilled individuals, by being able to match with other high-skilled individuals, learn a lot and differentiate even more from the initially low-skilled individuals, who learn very little from their also low-skilled colleagues.

Second, it would be nice to drop the restriction that plants need to have one manager and one worker and to introduce endogenous plant size. This could be done, for example, within a hierarchical framework like the one in Garicano and Rossi-Hansberg (2006). Such an extended model would be more suitable for a quantitative examination and it could produce interesting implications about the evolution of firms' sizes and hierarchical structures. For instance, one could study how the decline in the relative price of capital has a differential impact across various firms in terms of their hierarchical structure (number of layers) and of managers' spans of control and wages at different layers of the organization. Extending the model to allow for more than one worker in each plant would also be interesting in another dimension. In particular, one may think that more capital (or technology) in a plant affects workers in two ways. On the

[49] In the case of alternative functional forms, they must still satisfy the crucial properties established in Section 2. Results under some alternative specifications are available from the author upon request.

one hand, capital may reduce the number of workers needed in production as a given machine could do the job of a certain number of them. On the other hand, in a more technological environment the skill requirements of the still needed workers will increase. The model in this paper focuses on the latter channel. Nevertheless, it would be nice that future work incorporates the former channel too by allowing plants to endogenously decide their number of workers. Again, this could produce interesting results regarding the evolution of firms' sizes and their relation with the price of capital.

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Appendix A. Proofs

Proof of Lemma 1. (i) *Existence*. The results in Gretsky et al. (1992) and in Kaneko and Wooders (1986), for a more general assignment problem than the one considered here, guarantee the existence of the equilibrium in this paper.

(ii) *Efficiency*. Consider that the equilibrium assignment for an arbitrary price of capital p is given by $\{\Omega,\psi,W\}$ and produces an aggregate net output equal to Y. Now, by contradiction with statement (ii) of this lemma, assume that there is another feasible assignment, $\{\hat{\Omega},\hat{\psi},\hat{W}\}$, that produces an aggregate net output $\hat{Y} > Y$.

We know that:

$$Y = \int_{s\in M} h(s,\psi(s),p)\phi(s)\,ds = \int_{s\in M} [W(s)+W(\psi(s))]\phi(s)\,ds = \int_{s\in \hat{M}} [W(s)+W(\hat{\psi}(s))]\phi(s)\,ds \tag{10}$$

In Eq. (10), the equality in the first line comes by definition. Instead, the equality in the second line comes from the equilibrium conditions (3) and (5). Finally, the equality in the third line holds since the assignments in $\{\hat{\Omega},\hat{\psi},\hat{W}\}$ are also feasible given the skill distribution. Basically, the second and third lines in this equation are simply two different ways of aggregating the equilibrium wages under $\{\Omega,\psi,W\}$.

Analogously, we know that:

$$\hat{Y} = \int_{s\in \hat{M}} h(s,\hat{\psi}(s),p)\phi(s)\,ds \tag{11}$$

Now, combining Eqs. (10) and (11), and using the fact that $\hat{Y} > Y$, one can easily see that there must exist at least one skill value $s \in \hat{M}$ for which $h(s,\hat{\psi}(s),p) > W(s)+W(\hat{\psi}(s))$. But this contradicts condition (5) of the equilibrium assignment. Therefore, the assumption above that there exists a feasible assignment different from the equilibrium one that produces more aggregate net output is incorrect. This proves statement (ii) of this lemma.

(iii) *Within-plant assignment*. Consider that two individuals with skills a and b, $a \neq b$, are together in the same plant in the equilibrium assignment for an arbitrary price of capital p. Without loss of generality, consider that $a > b$. Now, by contradiction with statement (iii) of this lemma, assume that, in that plant, the individual with skill a is the worker and the individual with skill b is the manager. But this contradicts efficiency of the equilibrium assignment. This is so because, in that plant, the output could be larger by changing the assignment of occupations within the plant. That is,

$$h(a,b,p) > h(b,a,p)$$

Therefore, if this plant exists in the equilibrium assignment, the individual with skill a must always be the manager.

(iv) *Positive sorting*. Consider that in the equilibrium assignment for an arbitrary price of capital p two individuals with skills a and b, such that $a > b$, are managers. Now, by contradiction with statement (iv) of this lemma, assume that they are paired with workers of skills c and d, respectively, such that $c < d$.

By statement (iii) in this lemma, we know that $a \geq c$ and $b \geq d$. Combining these inequalities with the previous ones, we know that $a > b \geq d > c$. Under these circumstances, the fact that $\partial^2 h(x,z,p)/\partial x\,\partial z > 0$ implies that:

$$h(a,d,p)+h(b,c,p) > h(a,c,p)+h(b,d,p)$$

This implies that the economy's aggregate net output is not maximized when individuals with skills a and b (such that $a > b$) are paired with individuals of skills c and d (such that $d > c$), respectively. Therefore, by statement (ii) in this lemma, that cannot constitute an equilibrium assignment. This proves statement (iv) of this lemma.

(v) *Payoff increasing with skill.* Consider, against statement (v) of this lemma, that in the equilibrium assignment for an arbitrary price of capital p it happens that $a>b$ but $W(a)\le W(b)$. In this equilibrium, one of the following two cases must happen:

- Case 1: The individual with skill b is paired with an individual with skill a.
- Case 2: The individual with skill b is paired with an individual with skill $c\neq a$.

In both cases, someone has an incentive to deviate from the equilibrium assignment. This is a contradiction and, therefore, it must always happen that, under the equilibrium assignment, $W(a)>W(b)$ whenever $a>b$.

In Case 1, the individual with skill a is better off by leaving the individual with skill b and matching with another individual with skill a. This is so because:

$$W(a)+W(a)\le W(a)+W(b)=h(a,b,p)<h(a,a,p)$$

In Case 2, the individual with skill c is better off by leaving the individual with skill b and matching with one individual with skill a. This is so because:

$$W(a)+W(c)\le W(b)+W(c)=\max\{h(b,c,p),h(c,b,p)\}<\max\{h(a,c,p),h(c,a,p)\} \qquad \square$$

Proof of Lemma 2. (i) When $\mu=\frac{1}{2}$, the *complete assortative assignment* is the equilibrium assignment of the economy *if* $p=0$.

Assume that in the equilibrium assignment when $p=0$ individuals with an arbitrary skill a are paired with individuals with skill $b\neq a$. In this case, since $h(x,z,0)=(1-\theta)^{1/2\beta}(xz)^{1/2}$, it is very easy to show that:

$$h(a,a,0)+h(b,b,0)>2h(a,b,0)$$

But this implies that the economy's aggregate net output is not maximized when individuals with skill a are paired with individuals with skill $b\neq a$. Therefore, by Lemma 1, that cannot constitute an equilibrium assignment, and the equilibrium assignment when $p=0$ requires that all the individuals of a given skill level are paired among themselves. That is, $\Omega(s)=\{worker,manager\}$ and $\psi(s)=s\ \forall s\in[s_{min},s_{max}]$ in the equilibrium.

(ii) When $\mu=\frac{1}{2}$, the *complete assortative assignment* is the equilibrium assignment of the economy *only if* $p=0$.

Assume, by contradiction, that $p\neq 0$ and the *complete assortative assignment* is the equilibrium assignment of the economy. Because in equilibrium none has incentive to move, it must happen (in particular) that, $\forall a\in[s_{min},s_{max}]$ and $\forall\lambda\in[s_{min}/a,1]$:

$$F(a,\lambda a,p)\equiv h(a,\lambda a,p)-\tfrac{1}{2}[h(a,a,p)+h(\lambda a,\lambda a,p)]\le 0 \tag{12}$$

Now note that $F(a,\lambda a,p)|_{\lambda=1}=0$ and that:

$$F_\lambda(a,\lambda a,p)=a[h_2(a,\lambda a,p)-\tfrac{1}{2}[h_1(\lambda a,\lambda a,p)+h_2(\lambda a,\lambda a,p)]] \tag{13}$$

Evaluating (13) at $\lambda=1$ one gets that:

$$F_\lambda(a,\lambda a,p)|_{\lambda=1}=\frac{a}{2}[h_2(a,a,p)-h_1(a,a,p)]=\frac{a}{2}[f_2(a,a,k^*_{aa})-f_1(a,a,k^*_{aa})]$$

where k^*_{aa} is the amount of capital that a plant composed of two individuals with skill a optimally buys and the last equality comes from applying the envelope theorem.

Now note that, for the production function in (1), since $\mu=\frac{1}{2}$ it happens that:

$$f_2(a,a,k^*_{aa})-f_1(a,a,k^*_{aa})<0 \quad \forall a$$

Therefore, $F_\lambda(a,\lambda a,p)|_{\lambda=1}<0$ which contradicts (12), as there must exist a λ sufficiently close to 1 for which $F(a,\lambda a,p)>0$. This proves this part of the lemma. $\square$

Proof of Lemma 3. Lemma 3 is proved numerically according to the following computational strategy. Consider arbitrary values for p, β, θ and s_{max}. Then, consider I different *skill types* evenly distributed over the interval $(0,s_{max})$ and the corresponding I different *skill types* that are paired with them according to the assignment proposed in Lemma 3 for those arbitrary values of p, β, θ and s_{max}. This produces a discrete economy with $N=2I$ different *skill types*.[50] For notational convenience, denote by $S=\{s_1,s_2,\ldots,s_N\}$ the set of sorted skill types.

The assignment described in Lemma 3 is the equilibrium assignment in the economy only if it maximizes the aggregate net output produced among all the feasible assignments. This implies, among other things, that the aggregate net output produced under such an assignment by the N skill types defined above must be equal to the maximum they could produce in isolation. Therefore, it must not exist as an alternative feasible way of pairing those N skill types that produces more

[50] Since (A2) imposes that skill is distributed uniformly across the population, this description implicitly considers that all skill types have the same mass of individuals (mass 1, without loss of generality).

aggregate net output. Checking if this is the case is a way of proving Lemma 3. This can be done easily with the following linear programming problem.

Linear programming problem. The aggregate net output produced in the discrete economy defined above when the price of capital is p can be expressed as[51]:

$$Y = \sum_{l,j} a_{lj}^{p} e_{lj} \tag{14}$$

where

- a_{lj}^{p} denotes the net output optimally produced by a plant composed of one individual with skill s_l and one individual with skill s_j when the price of capital is p. That is, $a_{lj}^{p} \equiv \max\{h(s_l,s_j,p),h(s_j,s_l,p)\}$.
- e_{lj} denotes the fraction of individuals with skill s_l that are paired with individuals with skill s_j. For instance, $e_{lj}=1$ when all the individuals with skill s_l are paired with individuals with skill s_j, and $e_{lj}=0$ when no individual with skill s_l is paired with an individual with skill s_j.

Obviously, the assignments described by the e_{lj}'s must be feasible. More specifically, they must satisfy the following easy-to-interpret conditions:

$$e_{lj} \in [0,1] \quad \forall l,j = 1,\ldots,N \tag{15}$$

$$\sum_{l} e_{lj} = 1 \quad \forall l = 1,\ldots,N \tag{16}$$

$$\sum_{j} e_{lj} = 1 \quad \forall j = 1,\ldots,N \tag{17}$$

$$e_{lj} = e_{jl} \quad \forall l,j = 1,\ldots,N \tag{18}$$

Then, for any price of capital p, the assignment among the N skill types defined above that maximizes the aggregate net output that they can produce in isolation is simply the solution to the linear programming problem that looks for the e_{lj}'s that maximize (14) subject to conditions (15)–(18). Now, it turns out that the optimal assignment of those N skill types obtained solving this maximization problem always coincides with the one established in Lemma 3.[52,53] This result is robust to the specific values of the parameters of the model and of I considered.[54] Thus, one could argue that the assignment described in Lemma 3 is really the equilibrium assignment in the economy. In addition, this numerical strategy also suggests that the proposed equilibrium assignment is the *unique* equilibrium assignment in the economy. □

Proof of Lemma 4. The difference between the skill of the manager and the skill of the worker for all plants in interval i in the equilibrium assignment for an arbitrary price of capital p is equal to:

$$s-\psi_i(s) = t_i = \lambda^{i-1} s_{max}\left(\frac{1-\lambda}{2}\right)$$

Moreover, given the economy's skill distribution, the relative weight of interval i in the whole economy is equal to:

$$\int_{\lambda^{i} s_{max}}^{\lambda^{i-1} s_{max}} \phi(s)\, ds = \lambda^{i-1}(1-\lambda)$$

Therefore, the average difference between the skill of the manager and the skill of the worker within the plants that exist in the equilibrium assignment for an arbitrary price of capital p, $dif(p)$, is equal to:

$$dif(p) = \sum_{i=1}^{\infty}\left(\frac{s_{max}}{2}\right)(1-\lambda)^2\lambda^{2i-2} = \left(\frac{s_{max}}{2}\right)\frac{(1-\lambda)^2}{1-\lambda^2} = \left(\frac{s_{max}}{2}\right)\frac{1-\lambda}{1+\lambda}$$

Now, since $\partial\lambda(p)/\partial p<0$ (see Fig. 2), it is immediate to show that $\partial dif(p)/\partial p>0$. □

51 See Koopmans and Beckmann (1957) for a somehow similar problem.

52 It is important to mention that, although condition (15) allows *fractional assignment* (the e_{lj}'s are allowed to take any value in the interval [0,1], and not only 0 or 1), the solution to this maximization problem always involves corner solutions (that is, the optimal e_{lj}'s are always either 0 or 1) as in the equilibrium assignment proposed in Lemma 3.

53 This solution can be found numerically using the linear programming tools in Matlab. The exact Matlab program employed to obtain it is available from the author upon request. Basically, it solves the problem using a Matlab's large scale algorithm based on interior point methods.

54 Obviously, the larger the I the more guarantees this procedure provides.

Appendix B. Algorithm to find equilibrium wages

To determine everybody's payoff in the equilibrium assignment presented in Lemma 3, we apply the following computational strategy over the discrete version of the economy described in the proof of Lemma 3. Namely, consider I different *skill types* evenly distributed over the interval $(0,s_{max})$ and the corresponding I different *skill types* that are paired with them according to the assignment proposed in Lemma 3 for some arbitrary values of p, β, θ and s_{max}. In this discrete economy, denote by s_l^* the skill type that is optimally paired with the skill type $s_l \in S$. Then conditions (3) and (5) together impose the following restrictions on the equilibrium payoffs:

$$W(s_l)+W(s_l^*)=\max\{h(s_l,s_l^*,p),h(s_l^*,s_l,p)\} \quad \forall l=1,2,\ldots,N \tag{19}$$

$$W(s_l)=\begin{cases}\max\limits_{s_j}[\max\{h(s_l,s_j,p),h(s_j,s_l,p)\}-W(s_j)],\\ \text{s.t. } s_j\in S\end{cases} \quad \forall l=1,2,\ldots,N \tag{20}$$

Together, conditions (19) and (20) impose some bounds on each $W(s)$ but do not completely define it. In other words, it is possible to find different equilibrium payoff functions associated to the same equilibrium assignment. This non-uniqueness is due to the discreteness of the economy (this does not happen in a continuous economy) but, fortunately, it is not a big problem for the purposes of this paper. On the one hand, the bounds imposed by conditions (19) and (20) on each $W(s)$ for a given equilibrium assignment are tighter the greater the number of skill types in the economy. Thus, by considering sufficiently large values of I one can make the range of variation (across all the equilibrium payoff functions associated to the same equilibrium assignment) of any $W(s)$ very small. On the other hand, even if a given $W(s)$ varies a little bit across the different equilibrium payoff functions consistent with a given equilibrium assignment, this has very little effect on the variables that are relevant for this paper since they are aggregate variables (skill premium, wage inequality *between-* and *within-plants*, etc.). These two claims are confirmed by the following iterative procedure that, departing from an initial guess about everybody's equilibrium payoff, can deliver a set of equilibrium payoffs for everybody consistent with a given equilibrium assignment:

1. Make a guess about everybody's equilibrium payoff associated with a given equilibrium assignment.
2. Define a new set of equilibrium payoffs, $\tilde{W}(s)$, as

$$\tilde{W}(s_l)=\begin{cases}\max\limits_{s_j}[\max\{h(s_l,s_j,p),h(s_j,s_l,p)\}-W(s_j)],\\ \text{s.t. } s_j\in S\end{cases} \quad \forall l=1,2,\ldots,N$$

3. Check if $\tilde{W}(s)$ satisfies conditions (19) and (20).
4. If $\tilde{W}(s)$ satisfies condition (19) but not condition (20), then begin the process again using $\hat{W}(s)$ as the initial guess, where $\hat{W}(s)$ is defined as

$$\hat{W}(s_l)=\begin{cases}\max\limits_{s_j}[\max\{h(s_l,s_j,p),h(s_j,s_l,p)\}-\tilde{W}(s_j)],\\ \text{s.t. } s_j\in S\end{cases} \quad \forall l=1,2,\ldots,N$$

5. If $\tilde{W}(s)$ satisfies condition (20) but not condition (19), then begin the process again using $\hat{W}(s)$ as the initial guess, where $\hat{W}(s)$ is defined as:

$$\hat{W}(s_l)=\tilde{W}(s_l)+\tfrac{1}{2}[\max\{h(s_l,s_l^*,p),h(s_l^*,s_l,p)\}-\tilde{W}(s_l)-\tilde{W}(s_l^*)] \quad \forall l=1,2,\ldots,N \tag{21}$$

6. If $\tilde{W}(s)$ fails to satisfy both conditions (19) and (20), then begin the process again using $\hat{W}(s)$ as the initial guess, where $\hat{W}(s)$ is defined as in Eq. (21).

As mentioned above, it turns out that:

- Departing from an initial guess, this iterative procedure always converge to a set of payoffs satisfying conditions (19) and (20) and, therefore, consistent with the equilibrium assignment being considered.
- By changing the initial guess, this iterative procedure produces different sets of equilibrium payoffs associated to the same equilibrium assignment.
- When I is large, there is very little difference between the different sets of equilibrium payoffs systems associated to a given equilibrium assignment. In fact, they all behave almost identically in terms of the skill premium, and the overall, *between-plants* and *within-plants* wage inequality, that are the most relevant variables for this paper. In other words, when I is a large number, this strategy provides a good approximation to the actual equilibrium payoff function.

Appendix C. Robustness

See Figs. C1–C3.

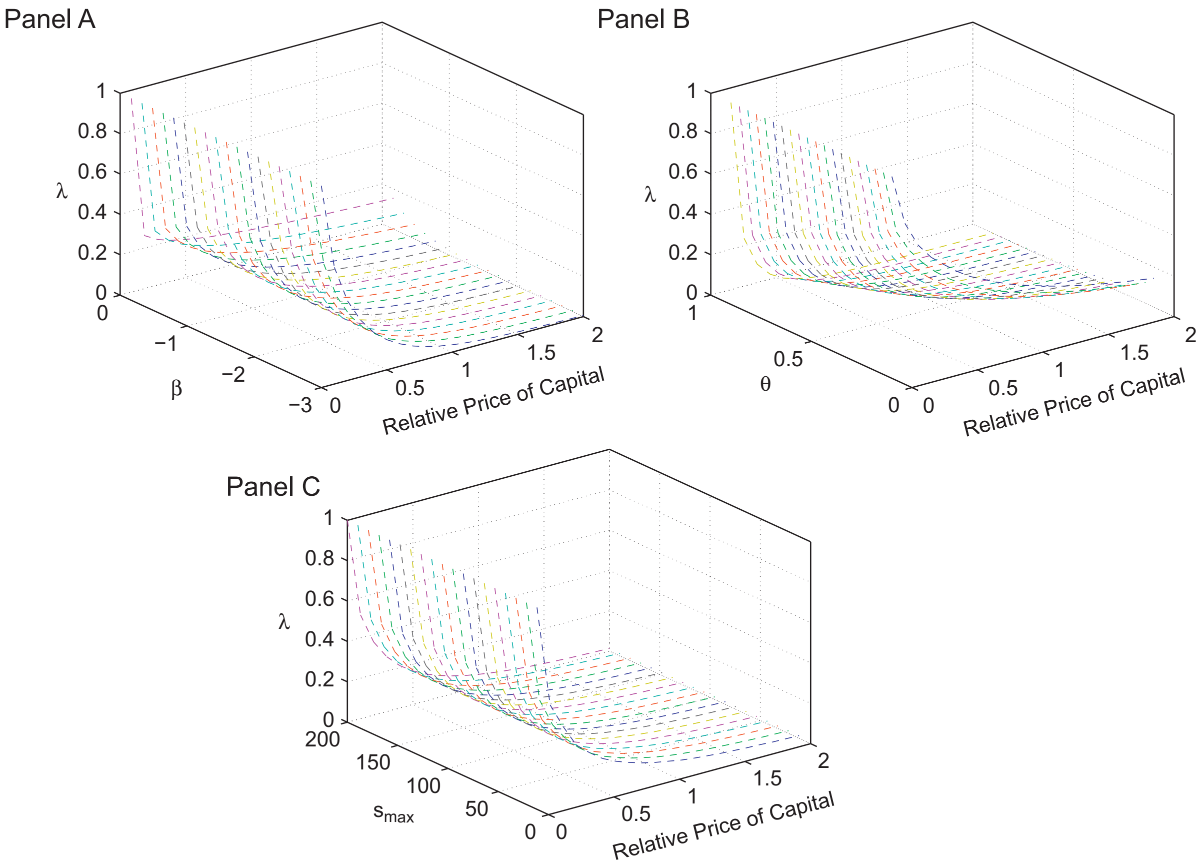

Fig. C1. Function $\lambda(p)$. Robustness.
Panel A: $\lambda(p)$ for different values of $\beta \in (-3,0)$, when $\theta = 0.5$ and $s_{max} = 100$.
Panel B: $\lambda(p)$ for different values of $\theta \in (0,1)$, when $\beta = -1.5$ and $s_{max} = 100$.
Panel C: $\lambda(p)$ for different values of $s_{max} \in (1,200)$, when $\beta = -1.5$ and $\theta = 0.5$.

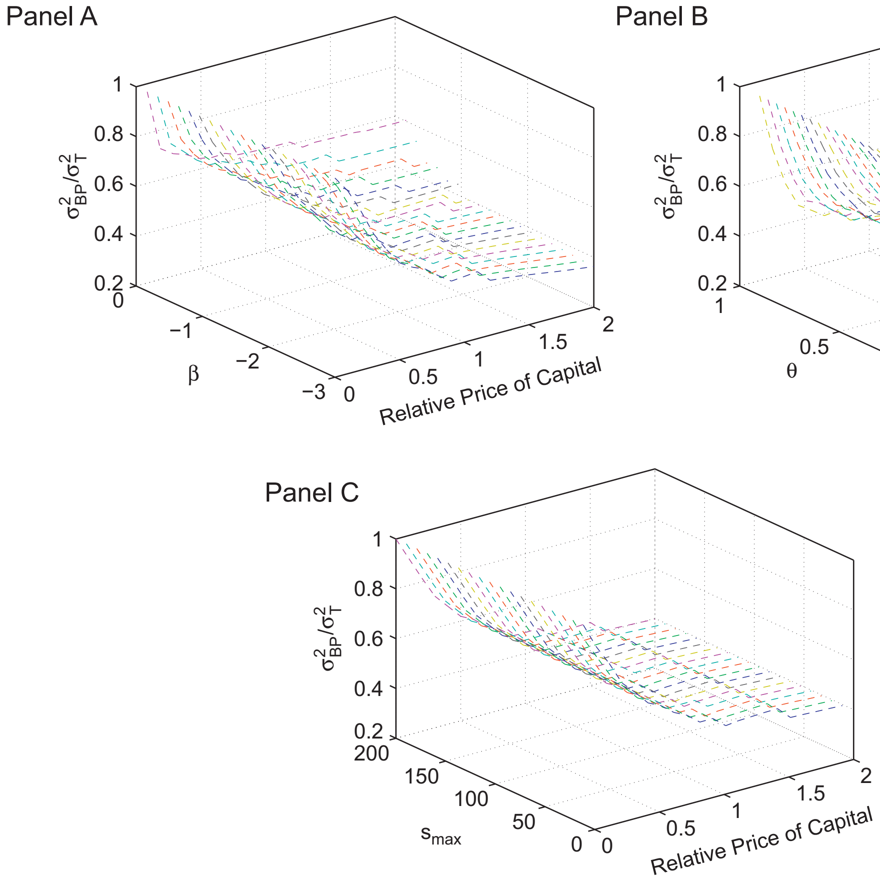

Fig. C2. Evolution of σ^2_{BP}/σ^2_T. Robustness.
Panel A: $\sigma^2_{BP}/\sigma^2_T(p)$ for different values of $\beta \in (-3,0)$, when $\theta = 0.5$ and $s_{max} = 100$.
Panel B: $\sigma^2_{BP}/\sigma^2_T(p)$ for different values of $\theta \in (0,1)$, when $\beta = -1.5$ and $s_{max} = 100$.
Panel C: $\sigma^2_{BP}/\sigma^2_T(p)$ for different values of $s_{max} \in (1,200)$, when $\beta = -1.5$ and $\theta = 0.5$.

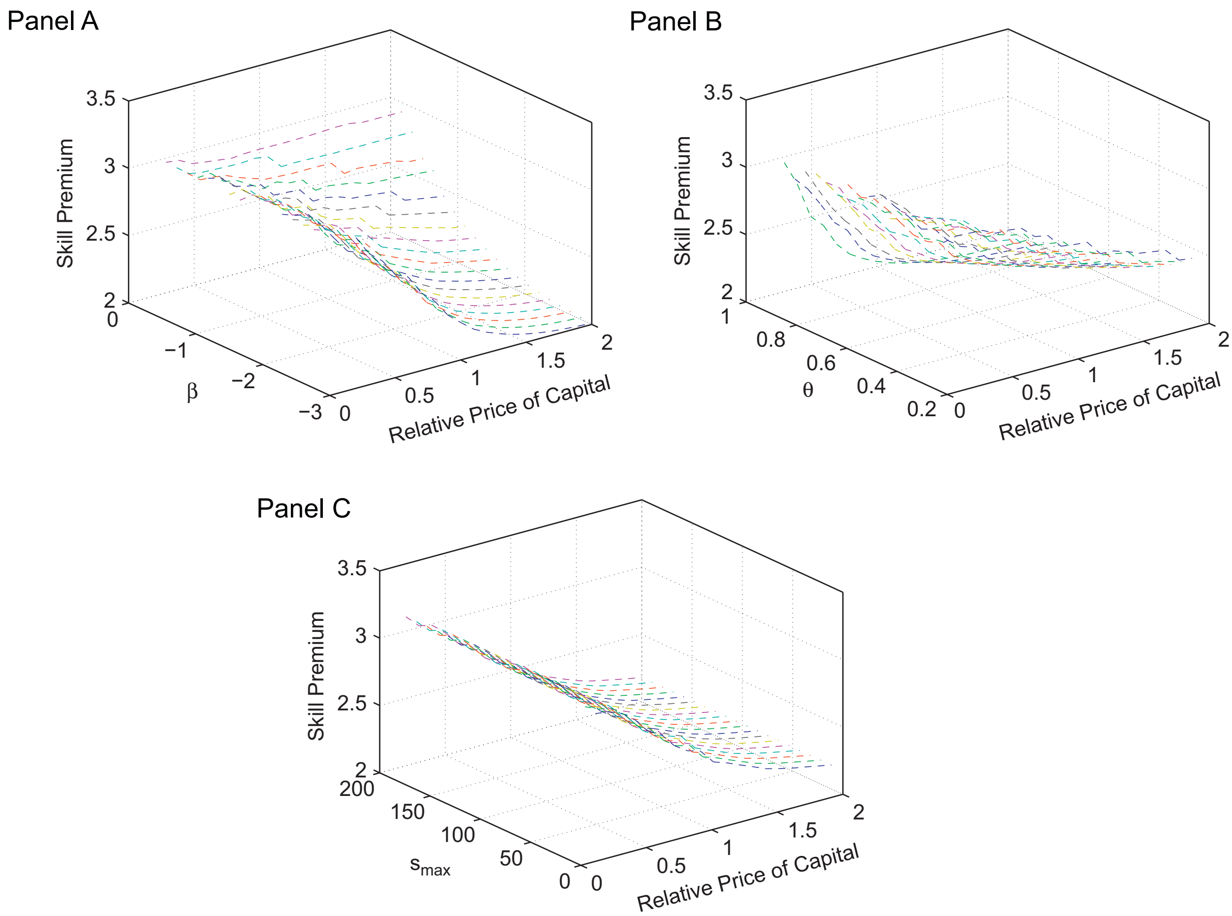

Fig. C3. Evolution of the skill premium. Robustness.
Panel A: $SP(p)$ for different values of $\beta \in (-3,0)$, when $\theta = 0.5$ and $s_{max} = 100$.
Panel B: $SP(p)$ for different values of $\theta \in (0,1)$, when $\beta = -1.5$ and $s_{max} = 100$.
Panel C: $SP(p)$ for different values of $s_{max} \in (1,200)$, when $\beta = -1.5$ and $\theta = 0.5$.

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