Environment and Planning B: Planning and Design 2009, volume 36, pages 989 – 1007

doi:10.1068/b33029

The interaction of segregation and suburbanization in an agent-based model of residential location

Ciriyam Jayaprakash
Department of Physics, The Ohio State University, 191 West Woodruff Avenue, Columbus, OH 43210-1117, USA; e-mail: .jay@mps.ohio-state.edu

Keith Warren ¶
College of Social Work, The Ohio State University, Stillman Hall, 1947 N College Road, Columbus, OH 43210, USA; e-mail: warren.193@osu.edu

Elena Irwin
Department of Agricultural, Environmental and Development Economics, The Ohio State University, 2120 Fyffe Road, Columbus, OH 43210, USA; e-mail: irwin.78@osu.edu

Kan Chen
Department of Physics, National University of Singapore, 2 Science Drive, Singapore 117542; e-mail: phykchen@yahoo.com
Received 23 February 2006; in revised form 7 July 2008; published online 10 September 2009

Abstract. We present a model of the interaction of segregation and suburbanization in determining residential location. The model incorporates differential income between two classes of agents, a simplified market mechanism for the purchase of housing, and a simple geographic structure of one central city and four symmetrically arranged suburbs. Agents derive utility from neighborhood racial composition, the size of their lot, private amenities that are specific to neighborhoods, and public amenities that stretch across municipalities. We find that the public-amenities term leads to a positive-feedback loop in which migration to suburbs increases the public amenities in those municipalities while lowering amenities in the central city, thus sparking further migration. When the minority agents are uniformly less affluent than the majority agents, this dynamic produces discontinuity in segregation as measured by centralization. Such discontinuities are typical of first-order phase transitions. When minority and majority incomes overlap, significant regions appear over which there are multistable equilibria at high and low levels of segregation, along with considerable sensitivity to the initial distribution of minority agents. We discuss the implications of these findings.

1 Introduction

Ethnic, religious, and economic segregation is an international problem (Bowles et al, 2007; Broccolichi and van Zanten, 2000; Lim et al, 2007; McLoughlin, 2005; Maloutas and Karadimitriou, 2001). Theory and empirical evidence support the position that segregation concentrates poverty and crime (Charles, 2003; Massey and Denton, 1993) and maintains economic inequality, even when the segregation is not primarily economic in nature (Bowles et al, 2007; Fischer and Massey, 2000; Neckerman and Torche, 2007).

Suburbanization is also an issue of international scope, which can draw residents from and weaken the financial bases of central cities (European Environment Agency, 2006; Mieszkowski and Mills, 1993; Orfield, 2003; Rusk, 1996). Authors frequently regard segregation and suburbanization as linked, but the nature of the link is unclear. It is possible that suburbanization increases segregation by giving rise to neighborhoods and municipalities that are differentiated by desirability and affordability, while concentrating members of minority groups and the poor in less desirable neighborhoods (Anderson, 2003; European Environment Agency, 2006;

¶ Corresponding author.

Nechyba and Walsh, 2004; Orfield, 2003; Squires and Kubrin, 2005; Yang and Jargowsky, 2006). For instance, in the United States economically disadvantaged individuals are often concentrated in aging central cities and inner suburbs, a particular problem since entry-level jobs are increasingly concentrated in newer, more distant suburbs (Briggs, 2005; Squires and Kubrin, 2005). Others point out that, particularly in the United States, ethnic segregation appears to have influenced patterns of suburbanization (Frey, 1979; Galster, 1990; Morton, 2002; Rusk, 2006).

Feedback between segregation and suburbanization is therefore an obvious possibility (Anderson, 2003; European Economic Agency, 2006; Orfield, 2003). Such feedback poses challenges to analysis, since it operates across systemic levels—individuals, the neighborhoods that they construct, and urban or suburban municipalities that encompass multiple neighborhoods.

Agent-based models offer a methodology for thinking about such multilevel feedback and its possible consequences (Epstein, 2007; Miller and Page, 2006). However, no multilevel model focused on the interaction of segregation and suburbanization exists at this time. In his seminal agent-based work on segregation, Schelling (1969; 1978) did not include any geographic structure beyond neighborhood size. Later work has begun to incorporate neighborhood boundaries. O'Sullivan et al (2003) found that a model in which interactions occur within bounded neighborhoods leads to more rapid segregation than the standard Schelling model. In a particularly detailed treatment of geography, Benenson et al (2002) integrate architectural preference and a variety of household characteristics into a detailed model of Arab–Israeli segregation in Yaffo, Israel. Their model reproduces important aspects of residential settlement and residential change in Yaffo. Fossett (2006) varies the housing quality in different neighborhoods and gives different ethnic groups more, or less, ability to afford different status neighborhoods. These conditions do not lead to high levels of segregation.

In all of these models, racial composition of neighborhoods is determined endogenously by the choices of the agents. However, other factors are determined exogenously. In O'Sullivan et al (2003) the boundaries of the neighborhoods are a given and the status of the different neighborhoods is assumed to be equal. In Fossett (2006) the housing quality in the different neighborhoods is varied consciously, but determined before the outset of the modeling run. In Benenson et al (2002) the architecture of the houses is also fixed before the outset of the modeling run.

Two recent articles have sought to endogenize housing prices within an agent-based context. Torrens (2007) has proposed a very complex model of residential mobility in which such factors as household life cycle, family size, ethnic composition of neighborhoods, and price range are taken into account. The value of houses in a given neighborhood is determined endogenously through a market simulation. Torrens touches on the issue of segregation, and finds that poor families are often priced out of affluent neighborhoods, as has been documented in empirical studies (Farley and Frey, 1994), and that the purchase of homes by affluent families of one ethnic group can alter the value of homes occupied by another group. Benard and Willer (2007) found that endogenous housing prices are a precondition for economic segregation, which also rises as income and status become more highly correlated.

While these models represent important steps beyond Schelling's initial formulation, there still has been little work directed at the ways in which segregation and suburbanization influence each other. Any such model must incorporate the fact that neighborhood geography, and particularly neighborhood quality, is determined endogenously through the interaction of residents (Florida, 2008). A neighborhood has high status because high-status people have chosen to live there, perhaps to be near to their own kind, perhaps to be separate from those of lower status, or perhaps because of amenities that are available in that neighborhood but not in others. A neighborhood has amenities, whether public or private, because those who live there are able to pay for them (Oates, 1969). Spacious neighborhoods emerge, in part, because individuals wish to settle at some distance from each other, and choose to move to municipalities where zoning accommodates their preferences (European Environmental Agency, 2006).

In the remainder of this paper we describe and then analyze a series of models in which racial segregation, economic segregation, and neighborhood quality are all determined endogenously. All of the models include black agents and white agents and in all of them the white agents are, on average, more affluent. In two of the models all agents seek to maximize the space available to them, while in two others white agents seek to isolate themselves from black agents. All four models play out in a geographic realm which consists of one central city and four suburbs arrayed around its edges, and in which both neighborhood and municipality-wide amenities play a role. In the first pair of models black agents are uniformly less wealthy than white agents, while in the second pair there is income overlap between white agents and black agents. We systematically investigate the way in which variability in preference for space and neighborhood racial composition interact with the endogenously determined level of amenities in municipalities and neighborhoods to produce metropolitan settlement patterns.

2 The model

We start with a brief overview of the model and then describe the details in subsequent sections. The model consists of a city divided into five municipalities (a central city and four suburbs) inhabited by 100 000 households (agents), 20% of which are black. We consider different income distributions for white agents and black agents, with and without overlap between income distribution of the two groups. The relocation of agents depends on their utility function and the total costs at the new location, which must not exceed the household's income. The utility function for both groups has a private-amenity component that is determined 'locally', a public-amenity component that is determined at the level of the municipality, and a spatial-preference component (more space is available in the suburbs than in the central city). In addition, the white agents have a disutility arising from black agents in their local neighborhood. The costs include housing expenditure, living costs, and transportation costs, with the remaining disposable income spent on private amenities. In discrete time steps corresponding to three months, a fraction of the population evaluates its utility at the current location and at all affordable locations and moves to the location with the highest utility that exceeds the current utility. We begin the simulation with different initial conditions, run the simulation for eighty to one hundred time steps, and examine the spatial patterns of population distribution that emerge as we vary the spatial preferences of all occupants and the racial preference of the white occupants. The details of the model are presented below.

2.1 Model geography

The model describes a square metropolitan region with a total of $n \times n$ cells that contains a smaller central area of $m \times m$ cells. The smaller $m \times m$ area is referred to as the central city while the surrounding region is partitioned into four equally sized areas referred to as suburbs. For the simulation of the model reported here, n is set to 100 and m to 40. Each cell within each of these areas can be interpreted as a block. The spatial capacity of a block is the maximum number of units that a block can physically hold: thirteen households are allowed in the suburbs and twenty five in the central city. The neighborhood around each cell is assumed to be a 5×5 cell area, thus consisting of twenty-five cells, the cell under consideration and the twenty-four cells that surround it, except at the edge of the region where the neighborhood consists of fewer cells, ranging from twenty to nine (depending on their precise location).[1]

2.2 Model agents

Agents, which are meant to represent households, are divided between 80 000 white agents and 20 000 black agents. This distribution roughly reflects the average proportion of white households to black households within US metropolitan regions in 2000.[2] Since the geography of the model will allow a maximum of 149 500 agents, there is a substantial amount of open space. White households are assumed to be more affluent than black households on average, but we allow for variation in income within the groups. We normalize the mean income ratio of black households to 1.65, while white households are assumed to have a mean income ratio of 2.15.[3] With regards to income, we examine two sets of models. In the first, total variation within each group ranges between 0.1 and 0.2. Thus, in these models there is income variability within the group but no actual income overlap; the poorest white household is more affluent than the wealthiest black household. In the second set of models, the mean incomes of the two groups remain the same, but the variation within each group is increased to 0.30 so that there is some overlap. We increase the overlap by allowing 10% of the black household to have incomes ratios between 1.85 and 2.15 corresponding to the bottom half of the white households. This is a simple way of mimicking the positive skew of the income distribution. Thus, while white households are still substantially wealthier on average, there is considerable overlap. All households are assumed to occupy 1.6 units of space in the central city and 2.6 units in the suburbs, reflecting more stringent zoning constraints to development often found in the suburbs. This difference allows us to explore spatial preferences.

2.3 Agent utility

Agent utility, which reflects the extent to which an agent values a given location, motivates all agent-location decisions and is influenced by four factors. Two of these factors, racial-composition preference and spatial preference, are varied between model runs. Two others, preference for private amenities (such as coffee shops or retail services available in a given neighborhood) and preference for public amenities (such as schools that are available throughout a suburb or central city), are held constant throughout the model. The combination of the factors is linear, as are the factors themselves. We assume a simplified, linear utility specification. This allows us to isolate the potential nonlinear effects of interactions among agents. Agents choose their location to maximize the following utility:

$$U = U_{\text{race}} + U_{\text{space}} + U_{\text{private.amenity}} + U_{\text{public.amenity}} \,. \tag{1}$$

(1) We have also considered a smaller neighborhood of 3×3 cells and found no qualitative or even semiquantitative differences in the model results.

(2) African Americans comprise about 13% of the population overall. However, African Americans are somewhat more urbanized than other ethnic groups, and the actual percentage of African Americans varies widely between metropolitan areas. For instance, the New Orleans metropolitan area was 37.4% African American before the collapse of the city levees following hurricane Katrina, while San Diego, CA is about 5.4% African American (United States Census, 2003a). We chose 20% as a rough middle point between the two extremes.

(3) According to the United States Census (2003b), the median income for white households within metropolitan areas in 2003 was \$48 807, whereas the median income for corresponding black households was \$30 674. This yields a white household income to black household income ratio of 1.6. Over the course of model development we varied our ratio between 1.3 and 2.2; we found that the ratio makes little difference to the results as long as there is no overlap.

The constraint imposed by income is incorporated into the private-amenity utility term, which is further elaborated below and, thus, U can be interpreted as an expression of net utility. We allow white agents to be averse to living in black-agent neighborhoods, but we treat black agents as being indifferent to neighborhood racial composition. The indifference of black agents is a conscious simplification. The effect of neighborhood racial-composition preference varies with the percentage of the minority group and the different preferences of minority and majority members (Macy and Van de Rijt, 2006). By having only one group express preferences and holding group percentages constant in all runs, we avoid considerable complexity.

The contribution of racial preference to the utility of white agents is negative and is proportional to the number of black agents in the neighborhood:

$$U_{\text{race}} = -R_{\text{p}} \frac{N_{\text{b}}(x_i, y_i)}{N_{\text{b, max}}} , \tag{2}$$

where R_{p} is the racial preference coefficient and $N_{\text{b}}(x_i, y_i)$ is the number of black agents in the neighborhood of block i labeled by the coordinates (x_i, y_i), normalized by $N_{\text{b, max}}$, the normalization constant to be 125. This is clearly arbitrary since we are merely setting the scale for R_{p}. The factor R_{p} determines the degree of preference and we study the behavior of the model as R_{p} is varied. Since R_{p} is negative, white agent utility falls as the percentage of black agents increases. While the neighborhood is the twenty-four blocks surrounding any given block along with the block itself, multiple agents can live in any given block. We have used the fraction of sites that black agents occupy, rather than the percentage of black people in the local population which is the coefficient used in Schelling (1969).

Spatial utility is given by:

$$U_{\text{space}} = S_{\text{p}} \left[s(x_i, y_i) - C(x_i, y_i) \right] , \tag{3}$$

where $s(x_i, y_i)$ is the space available in block (x_i, y_i), and $C(x_i, y_i)$ is the congestion in the block. Space is seen as desirable, but if too many people move in the block becomes, at least, somewhat congested. The space available is set at 2.6 in arbitrary units in the suburbs and 1.6 in the central city. The congestion is proportional to the fractional occupancy of the neighborhood, with a maximum value of 0.4. The spatial preference factor S_{p} determines the magnitude of the contribution from spatial preferences, and the behavior of the model is studied as a function of S_{p}.

The spatial preference term described above ignores the potential amenities available in higher-density areas, including greater proximity to retail shops, commercial services, restaurants, and neighborhood churches. These amenities are important to consider in a model of segregation, since there is some evidence that suggests that such amenities contribute to the maintenance of integrated neighborhoods (Nyden et al, 1997). To capture these effects, we introduce a private-amenities component into the utility function that is meant to capture the potential benefits of a local shopping district. This term also reflects the agent's income constraint:

$$U_{\text{private.amenities}} = \alpha_{\text{private.amenities}} \Delta_k f_{\text{private.amenities}}(x_i, y_i) F_{\text{private.amenities}, j} , \tag{4}$$

where $f_{\text{private.amenities}}(x_i, y_i)$ is proportional to the fractional occupation of the neighborhood of the cell (x_i, y_i); this reflects the dependence of private amenities on the presence of customers. Δ_k is the disposable income of household k whose utility function is being calculated. Disposable income is determined by the total income of the household net of housing, transportation, and living expenses. We do not allow for savings and, thus, agents are assumed to spend their entire disposable income on private amenities. This implies that, all else being equal, wealthier households are more able than poorer ones to increase their utility by increasing their consumption of private amenities. The term $F_{\text{private.amenities},\, j}$ is a factor that depends on the municipality j, set at 1.0 in the suburbs and 1.8 in the city. This reflects a preexisting stock of private amenities in the city. The coefficient $\alpha_{\text{private.amenities}}$ is fixed to be 0.30 for all model runs and has the same value for all households. The private-amenities term thus combines input from individual agents, local neighborhoods, and municipalities.

Lastly, it is well established that the quality of local public goods within a municipality (for example, police protection and school quality) is an important determinant of household location (Bayoh et al, 2006; Broccolichi and van Zanten, 2000; Oates, 1969; Tiebout, 1956). Our model is comprised of five municipalities: a central city and four equally sized suburbs. The level of public amenities is assumed to vary across these municipalities, but to be constant within a municipality. The equation for public amenities is:

$$U_{\text{public.amenities}} = \alpha_{\text{public.amenities}} T_j \ , \tag{5}$$

where T_j is the taxes collected in municipality j. Thus, public amenities are proportional to the taxes collected in the municipality j. Taxes represent 3% of the total income of all residents in the municipality. Thus, the wealthier the residents of the municipality, the better the public amenities will be. Also the public-amenities utility term introduces interactions between agents on the scale of an entire municipality rather than a neighborhood. The coefficient $\alpha_{\text{public.amenities}}$ is set to 0.24.

2.4 Costs

We assume three costs: (i) housing, (ii) living expenses such as food and clothing, and (iii) transportation costs. Any income over and above costs is considered to be disposable income, spent on private amenities. It is assumed that living expenses are higher for white households, since they have a higher overall income, and they are fixed at 0.70 of household income across the model. This represents roughly a third of the mean white-household income ratio of 2.15 ($0.70/2.15 = 0.32$). For black households, living expenses are 0.50 of household incomes in the suburbs and 0.40 in the central city (Conley, 1999). Transportation costs include a fixed component at 0.20 and a variable component that is proportional to the distance from the city center using the so-called Manhattan metric: the distance of the block at (x_i, y_i) from the city center that is at $(n/2, n/2)$ is given by $|(n/2) - x_i| + |(n/2) - y_i|$, where the city is $n \times n$ in size. The coefficient of proportionality was chosen so that the maximum additional cost was 0.10.

The price of housing is dependent on demand. Simulating a full-bid auction model for housing prices, while attractive from the point of view of economic theory, is computationally prohibitive. Instead, housing cost depends primarily on the lot area and the demand for the neighborhood. Lot area is taken to be 1.6 units in the central city and 2.6 units in the suburbs, thus yielding extra spatial utility in the suburbs. The cost per unit area consists of a fixed part, set at 0.30 per unit across the city. In addition, we include terms that capture the relative supply and demand for housing in the neighborhood: a term that is the product of a coefficient c_1 and the fractional occupancy of the neighborhood and another that is a product of a coefficient c_2 and the fractional change in the occupancy of the neighborhood from the previous time step. The coefficients were chosen to be between 0.02 and 0.04 and had negligible effect on the results.

2.5 Updating

At each time step 10% of black households and 10% of white households are randomly chosen. If a household has not moved in the previous four time steps, the household attempts a move. We varied the waiting time in a few cases to higher (eight time steps) or lower (two time steps) values: while this changed the rate at which steady state was reached it typically did not lead to qualitative changes. We assume perfect information. All of the available choices are checked for available space and living cost—the total cost to live in the neighborhood had to be 1% less than the available income of the household. (Changing this to 0% had no effect on the results in the cases we have checked.) In addition, we impose the condition that the utility increases by 2%. (Changing this to 1% had very little effect in the cases that we have checked.) All available blocks were ordered by the utility that would be obtained by the household moving there. Once the highest utility location is identified, the household moves there if by doing so it increases its utility compared with the current location. This is repeated for eighty to one hundred time steps, which can be interpreted as twenty to twenty-five years if each time step is taken to be a quarter. Initially, agents are distributed randomly across the metropolitan region. Agents are never placed initially in a location that their income would not allow them to afford. Particularly for those runs in which there is no income overlap between white agents and black agents, this constrains the initial distribution of black agents to the center of the model.

2.6 Measures of segregation and suburbanization

We use three measures of segregation, the first two of which also serve as measures of suburbanization. The first is visual; we reproduce model output that indicates where black agents and white agents eventually settle. Such displays can reveal patterns that no statistic can assess adequately. We also use a simple measure of centralization (Massey and Denton, 1988), the number of white households in the central city. We emphasize this measure because the public-amenities term in our model leads to de facto competition between municipalities for affluent households, and would be expected to lead to considerable segregation along jurisdictional lines. The number of white agents in the central city is a measure of segregation that is jurisdictionally based, and therefore congruent with the structure of the model and the focus of the article on the interaction of segregation and suburbanization. Finally, on occasion we use the well-known dissimilarity index D (Massey and Denton, 1988), which measures the number of members of the minority population who would have to move to attain an even distribution within a geographic region. We further calculated the Gini coefficient and entropy measures of segregation. We do not report these since they do not provide additional information beyond the measures of centralization and dissimilarity.

3 Results

3.1 Race versus space with no income overlap

Our first line of questioning focuses on differences in suburbanization patterns when segregation is motivated by racial preference versus economic considerations. We begin by comparing the model in which agents exhibit racial preference with that in which agents exhibit a preference for space (larger lots). In this case we simplify the model further by allowing no income overlap. Figure 1 shows the change in the number of white agents in the central city as a function of increasing values of the R_{p} for white agents—as mentioned above, black agents are always defined as indifferent. As racial preference among white agents increases, the ratio decreases in a nonlinear manner. There is a discontinuous transition into a segregational regime at a value of about 0.08, and by 0.09 virtually all the white agents disappear from the central city. It is, of course, unsurprising that a change in racial preference will lead to an increase in segregation, but such a large discontinuous increase has not been a feature of

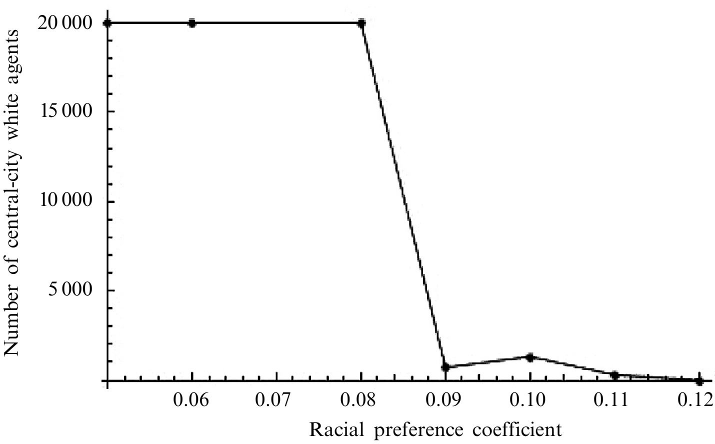

Figure 1. Change in the number of white agents in the central city as racial preference increases, in the case of no income overlap.

previous models. Such discontinuities are typical of first-order phase transitions, for instance when water freezes into ice (Kittel and Kroemer, 1980; Mazenko, 2006).

It is not apparent from this graph, but post hoc analysis indicates that, for a very narrow range of the parameter R_p, roughly from 0.084 – 0.086, there are two reachable equilibria in the model, one that is highly segregated and one that is highly integrated. This is a form of sensitivity to initial conditions that is known as bistability or coexistence, the existence of two locally stable states that occur for a given set of parameters. Bistability is known to occur in spatially extended systems with a large number of dynamical variables at a first-order phase transition, where there is a discontinuity or jump in the order parameter, as occurs in binary fluid mixtures and alloys (Kittel and Kroemer, 1980). In these systems, complex spatial patterns have been observed experimentally on short time scales before equilibrium is reached. In the case of our initial model, with no income overlap between black agents and white agents, bistability occurs over a sufficiently narrow range of the parameter R_p as to be of theoretical interest only. As we shall discuss below, once income overlap is included, the range of parameter values over which bistability occurs expands considerably.

Figure 2(a) shows the location of black agents and figure 2(b) the location of white agents in the case of $R_p = 0.10$, just higher than the transition value. In figure 2(a), darker shading means a higher concentration of the black agents, while in figure 2(b) lighter shading means a higher concentration of the white agents. Any region that is white in figure 2(a) and black in figure 2(b) is empty. Lines demarcate the central city and four suburbs. In this figure, white agents crowd entirely into three suburbs, while the black agents are overwhelmingly centralized. A slight change in racial preference sets off a positive-feedback cycle in which public amenities rise as more white agents move to a suburb, while utility associated with residence in the central city falls as those same agents leave. This tends to concentrate the white agents. Orfield (2003) has noted the tendency in American metropolitan regions for affluent individuals to concentrate in a subset of suburbs. This concentration leads to a robust tax base and a high level of amenities, particularly public amenities, which in turn draws further affluent agents. This model mimics such a positive-feedback loop. The black agents remain in the central city because prices in the three highly occupied suburbs were already or have been bid beyond their reach, while the unoccupied suburb has amenities that are lower than those in the central city. Several central-city neighborhoods are completely unoccupied.

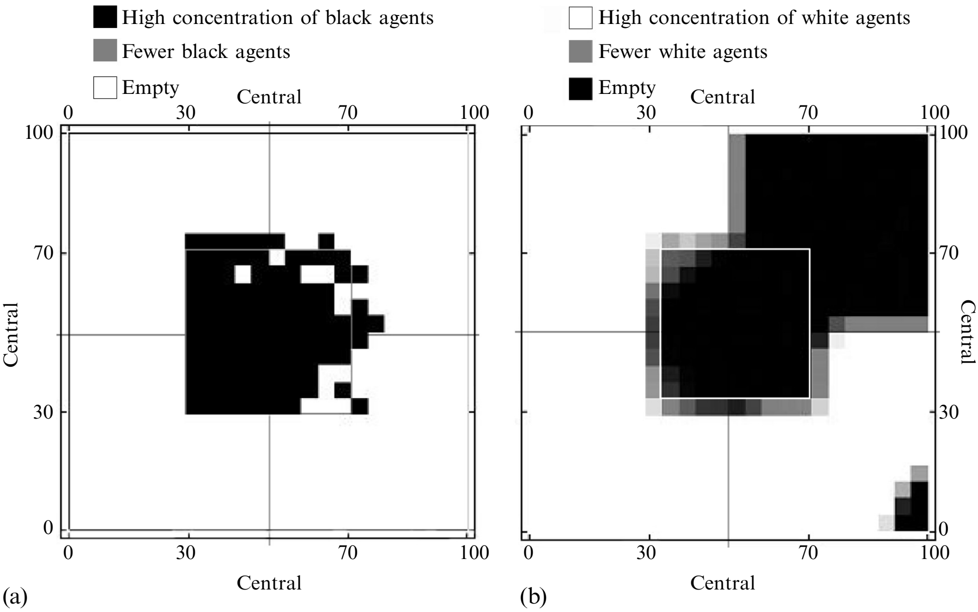

Figure 2. Distribution of (a) black agents, and (b) white agents, respectively, at racial preference coefficient 0.10. In (a) the black neighborhoods show the highest concentration of black agents, while in (b) the white neighborhoods show the highest concentration of white agents.

Figure 3 shows the change in the number of white agents in the central city as a function of change in spatial preference, which we assume to be identical for both groups. Because R_p is set to zero for white agents in this figure, segregation is economic in nature, on the basis of the ability to purchase property. Again, while it is unsurprising that there is eventual segregation by income, the discontinuous transition into a segregated regime is striking. A relatively small change in S_p around the value of 0.15 generates a discontinuous change in the ratio of white agents to black agents in the central city. While it is not visually obvious from the graph, there is a tiny region of bistability around a value of 0.155.

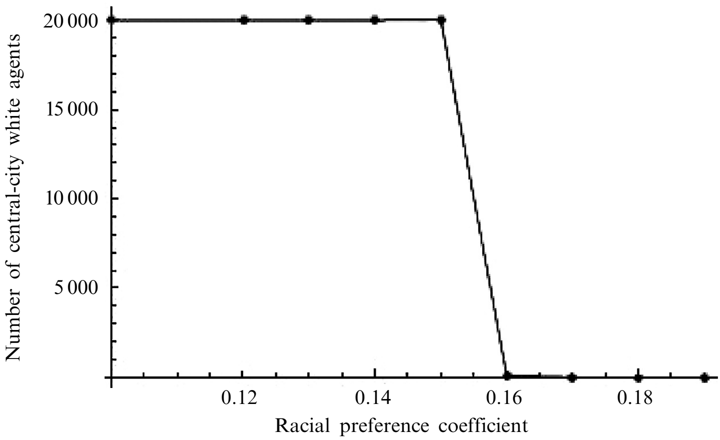

Figure 3. Change in the number of white agents in the central city as spatial preference increases, in the case of no income overlap.

Figure 4 shows the model when spatial preference is at 0.16, immediately above the transition value. It is similar to figure 3 in that white agents have again crowded into three suburbs. This cannot be a homophily effect, since this model includes no racial preference whatsoever. Concentration of white agents arises because of the positive-feedback loop arising from improvements in public amenities as affluent agents occupy a suburb. Figure 4 is different from figure 3 in that the highest income black agents have expanded into the fourth, previously unoccupied suburb. This occurs because the added space available in suburban lots now adds to their utility. However, their income is insufficient to begin the feedback cycle in which increases in public amenities draw wealthier agents. Thus, the suburb remains underoccupied and offers a low level of public amenities when compared with the suburbs that white agents occupy.

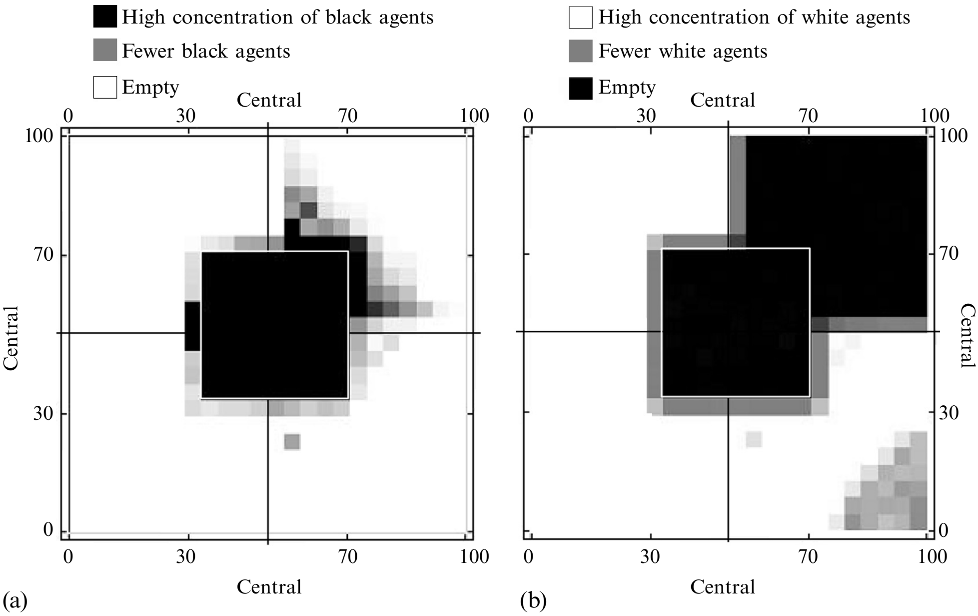

Figure 4. Distribution of (a) black agents, and (b) white agents, respectively, at spatial preference coefficient 0.16 with no income overlap.

3.2 The model with income overlap [4]

We expected overlap in income between black agents and white agents to simply raise the point at which segregation occurs as S_{p} and R_{p} increases. We were wrong. Instead, the model exhibits much greater sensitivity to the initial distribution of black agents, and mulitistability occurs over a much greater range of model parameters. We describe each of these phenomena in turn.

[4] Once we included income overlap, the model did not always settle into an equilibrium state in the number of time steps, 80 to 100, that we completed per run. There were often a small number of agents, two percent or less, who were dissatisfied with their location. We performed some experimental runs of up to 400 time steps aimed at finding the longest transient before reaching equilibrium. These runs typically reached an equilibrium state eventually, but there were times—even in the longest runs—when the model did not reach it. However, these transient states are steady states in the sense that 'macroscopic' spatial average values, for example, fraction of white households in a neighborhood, have small fluctuations. We believe that the model will eventually reach a final equilibrium, albeit with some very long transients, but it is possible that some configurations do not. We report results when less than 2% of all agents are dissatisfied with their location.

3.3 Sensitivity to the breadth of the initial geographic placement of agents

The model with overlap is sensitive to the breadth of the initial geographic area that agents occupy regardless of whether spatial preferences or racial preferences drive segregation. Segregation at equilibrium falls when black agents spread more broadly across the region at the beginning of the model run. Figure 5 demonstrates this sensitivity in the case of racial preferences with R_p set at 0.10 for all runs, using the dissimilarity index. As the initial spread of black agents grows wider, the dissimilarity index falls. When the initial spread of black agents is 40×40 blocks—that is, the black agents all start out in the inner city—the dissimilarity index is highest at the end of the run. This did not occur in the model without income overlap because in that case the black agents were so poor that they could not afford to be in the suburbs at all, so the initial placement did not put them there. Once there is income overlap, the more affluent black agents can afford to be in the suburbs, and once they are placed there the rising number of amenities keeps them there.

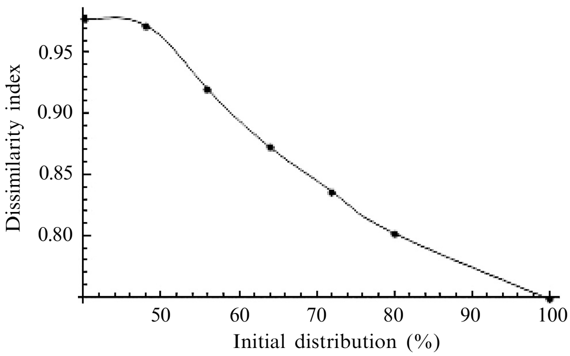

Figure 5. Sensitivity of the dissimilarity index to the initial distribution of black agents at a fixed value of racial preference coefficient of 0.10.

While we have not graphed this, the model also appears sensitive to the initial area that the white agents occupy. When those agents are more tightly clustered toward the center of the model we find that in the cases we have studied black suburbs are more likely to form when we vary R_p. This occurs because decentralization of white agents is not favored since the net utility from public and private amenities and racial preferences remains high in the central city.

3.4 Bistability in centralization of white agents and black agents

In the previous model, without income overlap between black agents and white agents, there are very narrow regions of the racial preference parameter and the spatial preference parameter in which two possible equilibrium numbers of white agents in the central city arise. With income overlap, these regions grow. Figure 6 illustrates this in the case of spatial preference and the case in which black agents and white agents are uniformly distributed across the city subject to income constraints. As spatial preference increases from zero, somewhat more than 25 000 white agents remain in the central city until S_p reaches 0.06. At that point, two equilibria appear, one in which roughly 25 000 white agents remain in the central city and another in which approximately 21 000 white agents remain. At S_p of 0.07 the equilibria are at 21 000 and 10 000 white agents in the central city. By S_p of 0.09 the system can not reach the upper equilibrium. At S_p of 0.10 the bistable equilibria lie at approximately 3000 and 100 white agents in the inner city. Because the affluent black agents also seek more space and better amenities, black agent settlement in the central city also evolves through a

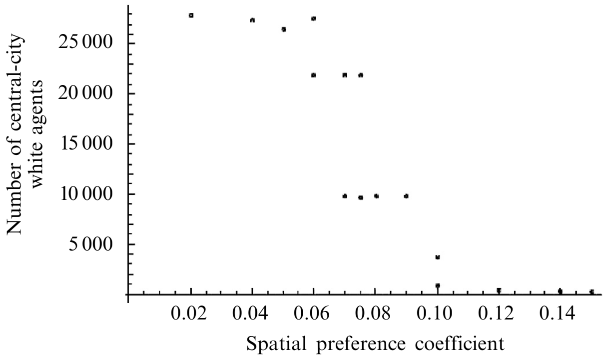

Figure 6. Change in the number of white agents in the central city as spatial preferences increases, in the case of income overlap. We have not included a connecting line in this case because of the combination of multiple plateaus and bistability.

series of bistable regimes. It never drops to zero, however, because a significant fraction of black agents cannot afford to move to the suburbs.

The transition into a bistable regime occurs at a lower level of S_p than does the transition into a highly segregated regime in the model without income overlap. There appear to be two reasons for this. First, the collapse in public amenities in the central city begins earlier since black agents who would once have started out in the central city can now afford to begin in the suburbs. Second, affluent black agents flee the central city along with white agents, again hastening the collapse of public amenities.

The transition in the racial preference case is even more complex. Figure 7 illustrates this. The first bistable regime begins at $R_p = 0.06$, and includes stable equilibria of roughly 27 000 and 21 000 white agents in the central city. By $R_p = 0.075$, the higher of those equilibria has collapsed, leaving two equilibria points at about 21 000 white agents and about 5000 white agents in the central city. We have not illustrated this, but as income overlap increases this bistable region increases as well, for modest increases in the overlap. Within the more highly suburbanized regime above $R_p = 0.09$, a second, somewhat less dramatic, bistable region emerges. Depending on whether the white agents form one or two predominantly white neighborhoods within the central city, roughly 3500 or 7000 remain. Even tristability can occur, as shown at $R_p = 0.13$ where there are stable points at roughly 7000, 3500, and 0 white agents in the central city. Eventually the bistability disappears and no white agents remain in the central city. As in the spatial-preference model, the presence of black agents in the central city shows

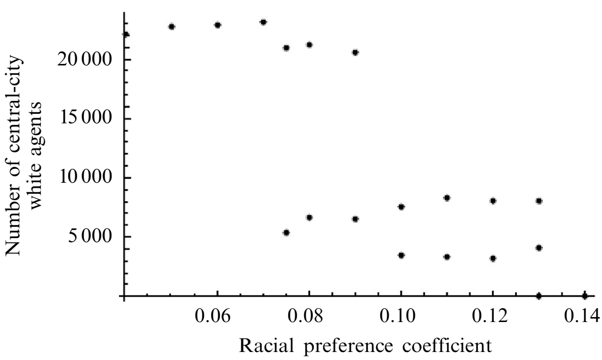

Figure 7. Change in the number of white agents in the central city as the racial preference coefficient increases, in the case of income overlap.

a similar evolution through a series of bistable regimes as the more affluent black agents become increasingly likely to flee the central city, in this case drawn by suburban amenities.

Bistability arises because in some cases affluent white agents happen to form neighborhoods within the central city that are sufficiently large to maintain both public and private amenities within the central city at a level that is equal to that of the suburbs. This can happen either because the initial distribution of black agents was larger, thus leaving additional room for the formation of white neighborhoods in the central city, or simply because the initial random distribution of agents made the formation of such neighborhoods more likely. Either case creates a gentrified equilibrium with a large number of white agents remaining in the inner city.

Figures 8 and 9 give a geographically based view of the two equilibria with $R_\mathrm{p} = 0.08$. Again, figures 8(a) and 9(a) show the concentration of black agents and figures 8(b) and 9(b) that of white agents. In figure 8, there are about 6700 white agents in the center city in two predominantly white neighborhoods. In the equilibrium in this figure, in which far fewer white agents remain in the central city, the flight of the white agents has once again collapsed both private and public amenities within the central city. Once again, those black agents who have the means to do so flee to the suburbs so as to take advantage of the increased suburban amenities. Suburbanizing black agents are noticeably concentrated on the edges of the central city, where black agents lower the utility of white agents. Thus, segregation occurs in the suburbs as well as in the central city, and black agents to some extent find themselves concentrated in those neighborhoods that are least desirable from the point of view of the white agents. This effect echoes empirical findings on the concentration of African Americans in segregated, and often less desirable, suburbs (Charles, 2003). Those black agents left behind in the central city are disproportionately impoverished and subject to a low level of both private and public amenities. A large stretch of the

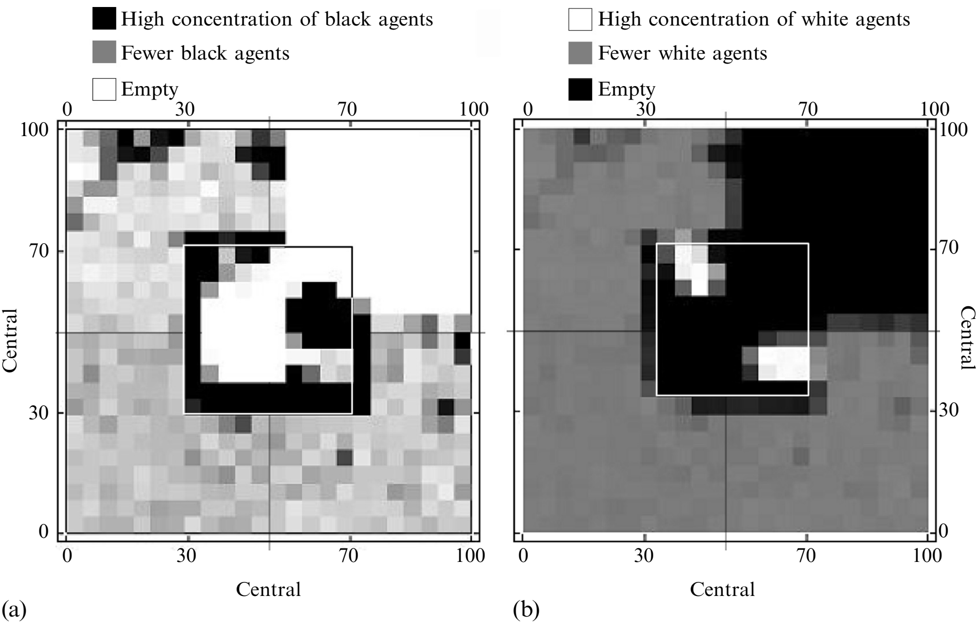

Figure 8. Concentrations of (a) black agents, and (b) white agents, at racial preference coefficient 0.08, in the case of income overlap and the lower equilibrium number of white agents remaining in the city.

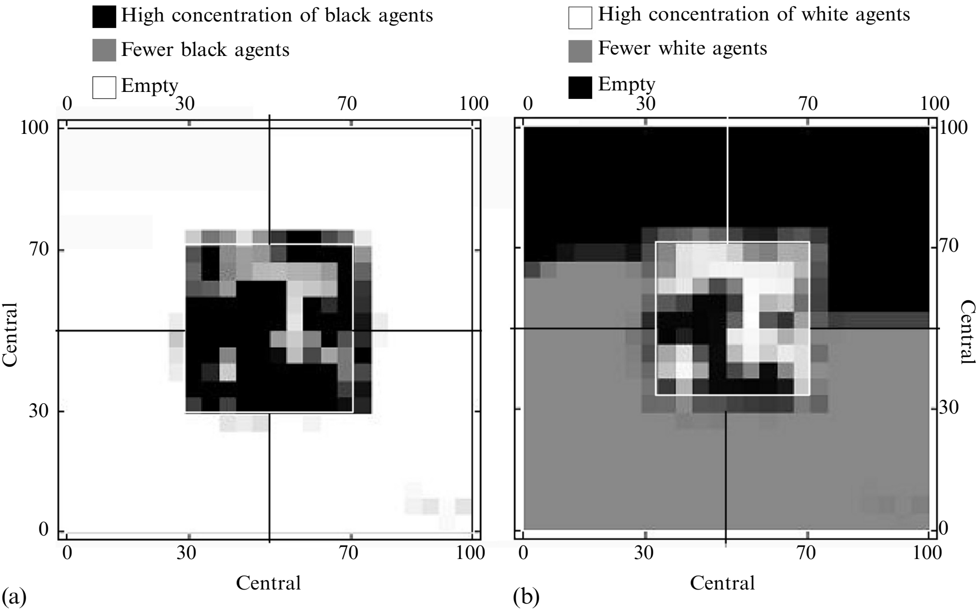

Figure 9. Concentrations of (a) black agents, and (b) white agents, at racial preference coefficient 0.08, in the case of income overlap and the higher equilibrium number of white agents remaining in the city.

central city is white in figure 8(a) and black in figure 8(b), and therefore empty, stricken with urban blight.

Figure 9 illustrates the results for the same parameter values, but different initial conditions. There are noticeably more white agents in the central city, about 21 000. The black agents are still highly segregated; the white agents simply form predominantly white central-city neighborhoods. Nevertheless, the presence of the white agents in the central city can be construed as being beneficial to black agents, since they help to maintain public and private amenities in the central city. This leads to two noticeable effects. First, black suburbanization is almost nonexistent; affluent black agents find higher amenities in the central city. Second, a suburb that was full in figure 8 is nearly empty in figure 9. The central city has been able to keep most of the agents that formerly filled that suburb.

One question that arises is whether these effects still occur if the incomes of black agents and of white agents are equal. We therefore equalized the income of the two groups by simultaneously decreasing white agents' income and increasing black agents' income, while giving them identical income distributions and identical living expenses. This model produced three results that were different from previous models. In the case of purely spatial preference, segregation does not occur at all. This is hardly surprising. In the case of racial preference, segregation occurs at high levels of R_p but centralization of black agents does not. Rather, the model produces mixed neighborhoods of black agents and white agents with no obvious relationship to municipal boundaries. The existence of both white agents and black agents with incomes insufficient to live in the affluent suburbs means that the poorest of both groups share the central city, albeit in different neighborhoods, as R_p increases. Thus, the centralization observed in earlier models appears to depend on the existence of different incomes between the two groups of agents, in particular on the income of white agents being sufficient to live in the suburbs. The lack of centralization meant that the number of white agents in the central city was no longer an appropriate measure of segregation. But, as measured by D, segregation increases smoothly with R_p and, while some bistability does occur as a result of neighborhood amenities drawing affluent agents, it is not nearly as dramatic as when incomes differ.

4 Model validity and limitations

A concern with any agent-based model is whether the results stem from idiosyncratic aspects of the model, such as the timing of the movement of the agents or the specific size of the neighborhoods. We must emphasize particularly that, as with most agent-based models, the specific parameter values that determine changes and outcomes are only valid within the model. For instance, if we increase the size of the suburbs without increasing the number of agents, wealthier agents tend to crowd into fewer of them. While this would mark a quantitative change in the settlement pattern of agents, we do not regard it as a qualitative change—the same pattern of concentration of affluent white agents into a subset of possible municipalities occurs. The results do not seem sensitive to the length of the movement delay—we did extensive modeling at delays of four and eight time steps and found no difference. If we do not give the central city an advantage in private amenities centralization and isolation of the less affluent black agents is even more likely to occur.

5 Discussion

This model is considerably more complex than that of Schelling (1969; 1978), and in particular incorporates feedback across multiple systemic levels. This complexity leads to four important model outcomes. The first is that this model produces favored suburbs in which affluent white agents concentrate, and centralization of less-affluent black agents; such centralization is an important and deleterious aspect of segregation, particularly in the United States (Orfield, 2003). The second is that the model produces segregation in the case of both racial and spatial preferences. The third is that the model produces two phenomena characteristic of a first-order phase transition, a tipping point in centralization in which the number of white agents in the central city falls by between fifty and one hundred percent, depending on the income overlap between black residents and white residents, and a sequence of multistable states. Finally, the model is sensitive to the initial spatial distribution of the agents. We will discuss each of these points in turn.

5.1 Favored suburbs and centralization

The utility term incorporates three different factors in any given run and thereby forces the agents to balance competing preferences. Especially important is the powerful role of endogenous amenities. Driven by racial preference or spatial attraction, a few affluent white agents migrate to one or another suburb. This increases the quality of amenities in the suburb while simultaneously reducing their quality in the central city. More white agents and, in some versions of the model, affluent black agents, are then drawn into the suburb. This positive-feedback loop leads to a form of path dependence (Arthur, 1994; Batty, 2007; Florida, 2008); municipalities which gain an initial edge in attracting affluent individuals have a strong tendency to keep them. Racial or spatial preference may prime this positive-feedback loop, but the amenity terms translate it into overwhelmingly white suburbs and the centralization of economically disadvantaged black agents.

The idea that suburban amenities play an important role in the pattern of suburban settlement is hardly news (Charles, 2003; European Environmental Agency, 2006; Orfield, 2003). It is usually assumed that this stems from a qualitative difference between public and private amenities, and family preferences for quality schools and public safety in suburbs over, say, quality restaurants downtown (European Environmental Agency, 2006). This explanation cannot be true for the current model, since agents are not comparing schools and public safety with restaurants and coffee shops; they are simply comparing two numbers. Why, then, do we see the concentration of affluent agents in the suburbs and the isolation of low-income black agents in the central city? This is particularly surprising since we assume that cities have an advantage in the private-amenity term.

The reason appears to be the geographic reach and stability of public amenities, those that are determined by the incomes of residents and extend over entire municipalities. Once a suburb has achieved a high level of public amenities, the departure of a few wealthy agents is unlikely to erode that level seriously, simply because there are so many others. Also, no departure ever shifts the boundaries of a municipality, and therefore no departure ever shifts the region over which public amenities are shared. Private amenities apply only to a neighborhood; neighborhoods contain fewer people and the boundaries of a neighborhood are somewhat fluid. The departure of relatively few wealthy agents from a neighborhood can erode private amenities, beginning a downward spiral. This occurs more readily in neighborhoods because the interactions are spatially local. Neighborhoods are therefore less stable than municipalities.

In recent years numerous cities have sought to revitalize themselves through neighborhood gentrification, a process that, in terms of this model, consists of a cycle of improvements in local amenities meant to attract affluent residents who, in turn, patronize the amenities which have attracted them and act to maintain the urban tax base (Florida, 2008; Orfield, 2003). This model raises the disquieting possibility that individual neighborhoods may simply be too unstable to assure the long-term vitality of a city. If this is correct, then the fundamental task of city renewal is the maintenance and improvement of public services that stretch across the entire municipality—a far more daunting task than gentrifying individual neighborhoods.

5.2 Income and segregation

Sociologists have demonstrated convincing empirical evidence that the segregation of African Americans in the United States is not based primarily on differences in income between African Americans and European Americans. For instance, the segregation of Asian Americans and Latino Americans declines more rapidly with increased income than that of African Americans, strongly suggesting that race must play a role in the segregation of the latter (Charles, 2003). While the current model does not (and cannot) overthrow empirical findings, it is worth noting that when the incomes of black agents and white agents were equal the model produced segregation, but not centralization, of black agents at increasing levels of R_{p}. This raises the possibility that differential income may play an important role in the geographic form that American segregation has taken, with poor African Americans concentrated in increasingly depopulated central cities and, in recent years, in aging first suburbs (Orfield, 2003; Rusk, 1996; Squires and Kubrin, 2005).

When spatial preference drives settlement decisions our model also produces high levels of segregation. Again, this cannot overthrow empirical findings, but it does demonstrate that, in theory, a positive-feedback loop between agent settlement and public amenities can produce high levels of racial segregation in the absence of any neighborhood racial-composition preference. This stands in contrast to Fossett's (2006) model, which, lacking multilevel feedback, does not produce segregation without such preference. This finding may be relevant to segregation outside the United States.

5.3 Tipping points and bistability

When the incomes of black agents and white agents do not overlap, this model creates a sudden collapse in the number of white agents in the central city in the case of either racial preference or spatial preference. Income overlap alters this sudden collapse to reveal a more complex and less predictable set of multistable outcomes. These patterns are typical of a first-order phase transition (Kittel and Kroemer, 1980) and carry important implications for our understanding of the processes of segregation and centralization of minorities.

The first implication is that identical preference functions can produce very different outcomes, depending on the initial distribution of agents in a metropolitan region. This erodes our ability to predict macrolevel patterns from the microlevel characteristics of agents. It also strongly implies that historical factors should be taken into consideration when studying the variation in segregation between different metropolitan areas. The importance of historical factors, even highly idiosyncratic ones, is typical of systems that include strong positive-feedback loops which can generate path dependence (Batty, 2007).

Further, bistable equilibria imply hysteresis. Once a particular level of individual racial preference or spatial preference has led to suburbanization, individual preferences will have to fall below that level before white agents return to the central city. This is most obvious from an examination of figure 7. The racial-preference factor leads to the collapse of the first bistable region and the certain flight of roughly three-quarters of those white agents who were previously in the central city to the suburbs at around an R_p value of 0.09. If racial preference declines following suburbanization, settlement of the white agents will remain at the lower-level equilibrium until R_p falls to roughly 0.06. This is actually a more hopeful result than one can derive from previous agent-based models of segregation; for instance, in Schelling's initial model segregation locks-in permanently once established (Schelling, 1969). This does not happen in our model because as racial preference decreases white agents can increase their utility by moving into a more integrated neighborhood with higher private or public amenities. Nevertheless, it is clear that racial preference will have to decrease past the point at which white agent flight began before white agents will return to the central city in large numbers.

The appearance of sudden transitions and multiple equilibria in centralization carries an important implication for our understanding of changes in city population. In the United States, these have often been rapid and have often failed to mirror the increasing population of the country as a whole. For instance, between 1980 and 2000, Cleveland, OH lost nearly one sixth of its residents (United States Census Bureau, 2003a). New York City lost nearly 900 000 residents during the 1970s, but gained over 800 000 during the next two decades. The population of Pittsburgh, PA has fallen by over 10% per decade since 1960 (see http://www.demographia.com). On a much longer time scale, Batty (2006) has documented high levels of volatility in world rankings of the populations of cities. Changes such as these are consistent with shifts between equilibria; a city can reach one equilibrium or another, but it cannot maintain a population between them for long. This model shows that feedback between neighborhoods, municipalities, and households is sufficient to establish multiple equilibria.

5.4 Initial spatial distribution

The model with income overlap is sensitive to the initial distribution of black agents. We believe that this has implications for understanding patterns of segregation in the Unites States. It has long been noted that metropolitan areas in the northeastern United States are more highly segregated than those in the southeast.

Massey and Denton (1993) argue that this stems from greater residential integration in the southeast in the first half of the twentieth century, as well as the preexistence of rural black neighborhoods in areas that later suburbanized, and the subsequent persistence of that pattern. While our model obviously does not correspond to any specific city, the sensitivity of final outcomes to the width of the initial distribution of black agents does indicate that such geographic patterns can persist over considerable periods of time.

6 Conclusion

Incorporation of feedback across multiple scales allows this model to reproduce geographic patterns that are common in metropolitan areas in the United States and in other parts of the developed world. These patterns include affluent suburbs in which the majority group concentrates in a subset of the possible suburbs, impoverished inner cities that sometimes experience gentrification, and the isolation of affluent minority members in the less-desirable suburban neighborhoods (Charles, 2003; European Environment Agency, 2006; Orfield, 2003). Since agent-utility functions consist of a set of linear equations, the settlement patterns in this model stem from interactions between the agents and should not depend on anything much more specific than a range of incomes, a tendency to avoid living next to members of other social groups, and a preference for space and amenities. All of these are widespread (Charles, 2003; Wilson, 1997). The similarity between the results in this model and actual patterns of metropolitan settlement suggests that, while segregation can result from purely local feedback, the interplay of feedback at multiple spatial scales offers a better explanation for the interaction of segregation and suburbanization.

Acknowledgements. This project was funded by NSF Grant DEB-0410336.

References

Anderson H S, 2003 *Urban Sores: On the Interaction Between Segregation, Urban Decay and Deprived Neighborhoods* (Ashgate, Aldershot, Hants)

Arthur W B, 1994 *Increasing Returns and Path Dependence in the Economy* (University of Michigan Press, Ann Arbor, MI)

Batty M, 2006, "Rank clocks" *Nature* **444** 592 – 596

Batty M, 2007 *Cities and Complexity: Understanding Cities With Cellular Automata, Agent-based Models and Fractals* (MIT Press, Cambridge, MA)

Bayoh I, Irwin E G, Haab T, 2006, "Determinants of residential location choice: how important are local public goods in attracting homeowners to central city locations?" *Journal of Regional Science* **46** 97 – 120

Benard S, Willer R, 2007, "A wealth and status based model of segregation" *The Journal of Mathematical Sociology* **31** 149 – 174

Benenson I, Omer I, Hatna E, 2002, "Entity-based modeling of urban residential dynamics: the case of Yaffo, Tel Aviv" *Environment and Planning B: Planning and Design* **29** 491 – 512

Bowles S, Loury G C, Sethi R, 2007, "Is equal opportunity enough? A theory of persistent group inequality", http://www.columbia.edu/~rs328/GroupInequality.pdf

Briggs X D S (Ed.), 2005 *The Geography of Opportunity: Race and Housing Choice in America* (Brookings Institution Press, Washington, DC)

Broccolichi S, van Zanten A, 2000, "School competition and pupil flight in the urban periphery" *Journal of Education Policy* **15** 51 – 60

Charles C Z, 2003, "The dynamics of racial segregation" *Annual Review of Sociology* **29** 167 – 207

Conley D, 1999 *Being Black, Living in the Red: Race, Wealth and Social Policy in America* (University of California Press, Berkeley, CA)

Epstein J M, 2007 *Generative Social Science: Studies in Agent-based Computational Modeling* (Princeton University Press, Princeton, NJ)

European Environmental Agency, 2006, "Urban sprawl in Europe: the ignored challenge", http://www.reports.eea.europa.eu/eea_report_2006_/10/en/eea.report_10_2006.pdf

Farley R, Frey W H, 1994, "Changes in the segregation of whites from blacks during the 1980s: small steps toward a more integrated society" *American Sociological Review* **59** 23 – 45

Fischer M J, Massey D S, 2000, "Residential segregation and ethnic enterprise in US metropolitan areas" *Social Problems* **47** 408 – 424

Florida R, 2008 *Who's Your City? How the Creative Economy is Making Where to Live the Most Important Decision of Your Life* (Basic Books, New York)
Fossett M, 2006, "Ethnic preferences, social distance dynamics and residential segregation: theoretical explorations using simulation analysis" *The Journal of Mathematical Sociology* **30**(3 – 4) 185 – 273
Frey W H, 1979, "Central city white flight: racial and non-racial causes" *American Sociological Review* **44** 425 – 448
Galster G, 1990, "White flight from racially integrated neighborhoods in the 1970s: the Cleveland experience" *Urban Studies* **27** 385 – 399
Kittel C, Kroemer H, 1980 *Thermal Physics* 2nd edition (W H Freeman, San Francisco, CA)
Lim M, Metzler R, Bar-Yam Y, 2007, "Global pattern formation and ethnic/cultural violence" *Science* **317** 1540 – 1544
McLoughlin S, 2005, "Mosques and the public space: conflict and cooperation in Bradford" *Journal of Ethnic and Migration Studies* **31** 1045 – 1066
Macy M W, Van de Rijt A, 2006, "Ethnic preferences and residential segregation: theoretical explorations beyond Detroit" *Journal of Mathematical Sociology* **30**(3 – 4) 275 – 288
Maloutas T, Karadimitriou N, 2001, "Vertical social differentiation in Athens: alternative or complement to community segregation?" *International Journal of Urban and Regional Research* **25** 699 – 716
Massey D S, Denton N A, 1988, "The dimensions of racial segregation" *Social Forces* **67** 281 – 315
Massey D S, Denton N A, 1993 *American Apartheid: Segregation and the Making of the Underclass* (Harvard University Press, Cambridge, MA)
Mazenko G F, 2006 *Non-equilibrium Statistical Mechanics* (Wiley-VCH, Weinheim, Germany)
Mieszkowski P, Mills E, 1993, "The causes of metropolitan suburbanization" *Journal of Economic Perspectives* **7** 135 – 147
Miller J H, Page S E, 2006 *Complex Adaptive Systems: An Introduction to Computational Models of Social Life* (Princeton University Press, Princeton, NJ)
Morton M J, 2002, "The suburban ideal and suburban realities: Cleveland Heights, Ohio, 1860 – 2001" *Journal of Urban History* **28** 671 – 698
Nechyba T J, Walsh R P, 2004, "Urban sprawl" *Journal of Economic Perspectives* **18**(4) 177 – 200
Neckerman K, Torche F, 2007, "Inequality: causes and consequences" *Annual Review of Sociology* **33** 335 – 357
Nyden P, Maly N, Lukehart J, 1997, "The emergence of stable racially and ethically diverse communities: a case study of neighborhoods in nine US cities" *Housing Policy Debate* **8** 491 – 534
Oates W E, 1969, "Effects of property taxes and local public spending on property values: empirical study of tax capitalization and Tiebout hypothesis" *Journal of Political Economy* **77** 957 – 971
Orfield M, 2003 *American Metropolitics: The New Suburban Reality* (Brookings Institution Press, Washington, DC)
O'Sullivan D, Macgill J, Yu C, 2003, "Agent-based residential segregation: a hierarchically structured spatial model", paper presented at Agent 2003, University of Chicago, Argonne National Laboratory, 2 – 4 October, http://www.geog.psu.edu/people/osullivan/publications/conf/agent2003/Agent2003_Paper_OSullivan_online.pdf
Rusk D, 1996 *Cities Without Suburbs* (Woodrow Wilson Center Press, Washington, DC)
Rusk D, 2006, "Social framework: sprawl, race, and concentrated poverty—changing the 'rules of the game' ", in *Urban Sprawl: A Comprehensive Resource Guide* Ed. D C Soule (Greenwood Press, Westport, CT) pp 90 – 102
Schelling T, 1969, "Model of segregation" *American Economic Review* **59** 488 – 493
Schelling T, 1978 *Micromotives and Macrobehavior* (Norton, New York)
Squires G D, Kubrin C E, 2005, "Privileged places: race, uneven development and the geography of opportunity in urban America" *Urban Studies* **42** 47 – 68
Tiebout C M, 1956, "A pure theory of local expenditures" *Journal of Political Economy* **64** 416 – 424
Torrens P M, 2007, "A geographic automata model of residential mobility" *Environment and Planning B: Planning and Design* **34** 200 – 222
United States Census, 2003a, "American Community Survey", http://www.census.gov/acs/
United States Census, 2003b, "Statistical Abstract of the United States", http://www.census.gov/prod/www/statistical-abstract04.html
Wilson W J, 1997 *When Work Disappears: The World of the New Urban Poor* (Vintage, New York)
Yang R, Jargowsky P, 2006, "Suburban development and economic segregation in the 1990s" *Journal of Urban Affairs* **28** 253 – 273