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Formally Modelling a Gender-Segregated Economy: A response to William Darity, Jr

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Summary. — William Darity, Jr.'s model, presented in the November 1995 issue of *World Development* (Darity, 1995), is an important attempt to provide an outline for gender-based analysis at the household level. The model falls short, however, because it focuses on income maximization without specific constraints on the man's time. This paper demonstrates that as formulated there is no internal maximization solution possible, and then offers additional economic information that would be meaningful and appropriate to make Darity's model well-specified. © 1997 Elsevier Science Ltd. All rights reserved.

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In recent years there has been growing awareness that gender relations are an important component for understanding agricultural response in sub-Saharan Africa. Unfortunately, there have been few attempts at formally modeling this gender component. William Darity, Jr.'s model, presented in the November 1995 issue of *World Development* (Darity, 1995), is an important attempt to provide an outline for this task. A particular strength of Darity's model is that, unlike a number of earlier attempts to integrate gender into a household model, it specifically incorporates the subordinate role of women in the decision making process. The model falls short, however, because it focuses on income maximization without specific constraints on the man's time. This paper demonstrates that as formulated there is no internal maximization solution possible, and then offers additional economic information that would be meaningful and appropriate to make Darity's model well-specified.

In Darity's model, the man's objective is to maximize his income subject to his own labor, the degree of coercion over the woman, and the wage he is willing to pay her. There are two problems concerning Darity's model and both involve the man maximizing his income.

The first problem is concerned with the man's ability to appropriate a portion of the proceeds of the woman's labor time or, alternatively, what portion of the proceeds of the woman's labor she retains.[1] This, within the context of Darity's model, means the determination of the women's wage (w). Darity's model offers no explicit treatment of the woman negotiating her wage and places direct determination under the control of the man who is maximizing his income. While Darity claims that "Men pull women out of social maintenance by... dint of wage payments..." (p. 1964), his formal model allows the man to freely set this choice variable as desired. Maximizing his income suggests that wages should be (locally[2]) set at $w = 0$. Though $w = 0$ yields the man's maximum income, this does not correspond to the reality for "wages" set for women in household-based cash crop cultivation.

An appeal to societal norms or customs for determining women's wages (i.e. wages exogenously decided) would both technically solve the problem and, at least in some sense, be a reasonable assumption. But in our view there is a deeper issue involved. Both the wage the woman gets, and the amount of time she works, would be better modeled using some form of bargaining process that shapes her choices. At the foundation of this bargaining analysis is the unequal power relations between men and women. This power, determined in part by

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patriarchal societal norms and customs, dictates the need for a non-symmetric bargaining model. Hence it is in the interest of the man to offer a wage such that the losses from offering a higher wage are equal to the gains from the increase in work effort the woman would expend. If the wage is too low, the woman would refuse to participate. Darity correctly captures the dominance of the man over the woman, but he goes too far in that he eliminates her choice and therefore the results of her ability to bargain altogether. While this might be appropriate for a slave relationship, or even a reasonable approximation to some gender relations, we do not think it captures the reality of African peasant households. The lack of price responsiveness that is prevalent throughout Africa is due, at least in part, to gender relations and women choosing not to participate in male-controlled agricultural production. A bargaining model, within the context of non-symmetric power relations, can capture this reality.

The second issue, the man's labor-leisure decision, is formally more problematic for appropriate inclusion in Darity's underspecified model. As the model is presented, there is no reason that the man should not always want to work more if his goal is to maximize his income. It can be shown that the marginal income for the man with respect to his work is always positive, and thus no maximization is possible. More formally, we begin with Equation 5 of Darity's paper.[3]

$$Y_M = P_c M_c^{\alpha}[CM_c^{\sigma}(w/p_v)^{\rho}]^{\beta}K_c^{\gamma} - wCM_c^{\sigma}(w/p_v)^{\rho} \tag{5}$$

In order to simplify notation the choice variables are isolated from the constants and parameters.

$$Y_M = AM_c^{\alpha+\sigma\beta}w^{\rho\beta} - BM_c^{\sigma}w^{\rho+1}$$
$$A = P_c C^{\beta}K_c^{\gamma}(1/p_v)^{\rho\beta}$$
$$B = C(1/p_v)^{\rho} \tag{5'}$$

Taking the derivatives of the modified Equation 5' with respect to the choice variables yields;[4]

$$\frac{\partial Y_M}{\partial M_c} = \frac{(\alpha+\sigma\beta)AM_c^{\alpha+\sigma\beta}w^{\rho\beta}}{M_c} - \frac{\sigma BM_c^{\sigma}w^{\rho+1}}{M_c} \tag{6'}$$

$$\frac{\partial Y_M}{\partial w} = \frac{\rho\beta AM_c^{\alpha+\sigma\beta}w^{\rho\beta}}{w} - \frac{(\rho+1)BM_c^{\sigma}w^{\rho+1}}{w} \tag{7'}$$

Setting Equation 6' and Equation 7' equal to zero and dividing Equation 6' by $M_c^{\sigma-1}w^{\rho\beta}$, and Equation 7' by $M_c^{\sigma}w^{\rho\beta-1}$ yields;

$$(\alpha+\sigma\beta)AM_c^{\alpha+\sigma(\beta-1)} - \sigma Bw^{1+\rho(1-\beta)} = 0 \tag{8}$$

$$\rho\beta AM_c^{\alpha+\sigma(\beta-1)} - (\rho+1)Bw^{1+\rho(1-\beta)} = 0 \tag{9}$$

Let $x=M_c^{\alpha+\sigma(\beta-1)}$ and $y=w^{1+\rho(1-\beta)}$ and take the determinant of the 2×2 matrix.

$$-(\alpha+\sigma\beta)(\rho+1)AB + \rho\beta\sigma AB \tag{10}$$

To have nontrivial solutions, the determinant must equal zero.[5] But dividing by AB and simplifying yields an expression that cannot equal zero.[6]

$$-\alpha(\rho+1) - \sigma\beta \neq 0 \tag{11}$$

Equation 5 does not attain an internal maximum.

Another method for proving the inability of attaining maximization is achieved using the first order conditions Equation 6' and Equation 7'. This solution provides further insight into the economic maximization problem. Assume that M_c and w were such that the first order condition concerning women's wages obtains (i.e. $\partial Y_M/\partial w = 0$). In this case Equation 7', set equal to zero and solved in terms of B, becomes;

$$B = \frac{\rho\beta AM_c^{\alpha+\sigma(\beta-1)}w^{-1-\rho(1-\beta)}}{(\rho+1)} \tag{12}$$

Substituting Equation 12 into Equation 6' results in the following equation.

$$\partial Y_M/\partial M_c = (\alpha+\sigma\beta)AM_c^{\alpha+\sigma\beta-1}w^{\rho\beta} - \sigma\left[\frac{\rho\beta AM_c^{\alpha+\sigma(\beta-1)}w^{-1-\rho(1-\beta)}}{(\rho+1)}\right] \times M_c^{\sigma-1}w^{\rho+1} \tag{13}$$

Equation 13 can be simplified to obtain;

$$\partial Y_M/\partial M_c = AM_c^{\alpha+\sigma\beta-1}w^{\rho\beta}\frac{1}{\rho+1} \times [(\alpha+\sigma\beta)(\rho+1) - \sigma\rho\beta)] \tag{14}$$

The first four factors are clearly positive. The fifth factor can be expanded.

$$\alpha\rho + \sigma\beta\rho + (\alpha+\sigma\beta) - \sigma\rho\beta = \alpha\rho + \alpha + \sigma\beta \tag{15}$$

Given the initial parameter assumptions, Equation 15 must be positive. This demonstrates that $\partial Y_M/\partial w>0$, and again, the first-order conditions (necessary for a local maximum) cannot both be satisfied. Further, this approach demonstrates that if the man chooses a wage payment for the woman (w) and self-effort (M_c) such that his income is maximized with respect to her wage, then the marginal return of his own labor must be positive, and the man will still want to increase his work time. Additional constraints and a broader objective function are needed.

There are several ways this model could be modified to generate a finite solution. A simple solution could be generated by imposing an absolute limit on the supply of men's labor or, equivalently, setting a socially determined limit on the woman's labor with a fixed production technology for the inputs of men's and women's labor time. But, this would cause men to work up to some prescribed level and remove any decision concerning labor and leisure. Such solutions do not focus enough attention on what we think is the primary issue of this type of model; the allocation of men's and women's labor time. It seems more realistic to model the man's labor choice within the context of obtaining more leisure to the extent that he is willing to trade off some income. Thus, the man should be represented in the standard way as an individual who chooses between items in his objective function (here, labor and leisure).

Maximizing utility, which depends upon both income and leisure, and subject to time and budget constraints, has been the standard approach in the few attempts at formally modeling the effects of gender relations on economic activity. McElroy and Horney (1981) used this approach in a broad attempt to look at how the household would behave differently if it rested upon Nash Bargaining between a man and a woman instead of an individual utility maximization. Jones (1983) discussed in a general way some considerations for applying this model to cash crop production in Africa, but she does not build a specific model. Recently, Smith (1994) used this approach to look at the effects of structural adjustment on welfare in rural Africa. Her formal model, unlike Darity's model which seems more pertinent for this type of analysis, treats the two agents as symmetric. Smith includes a verbal discussion of why women are likely to be subordinate but does not incorporate this discussion into her formal models.

While some form of the man's labor/leisure tradeoff is sufficient to close Darity's model, there is one extension that we suggest which would enrich his model. Darity's model does capture the dominance of the man as a decision maker, but we think it goes too far in that direction and thereby rules out certain responses that might occur. In Darity's model, women's labor effort is determined solely by men's labor effort, societal custom, and the wage she receives. The woman herself has no choice between labor and leisure. We believe that women do make these decisions, not always as their husbands would want them to, though certainly choosing from a menu that is shaped by their husband's choices. We think a non-symmetric bargaining model would be the most appropriate technique to capture the unequal power relations between men and women within the African household. Furthermore, the model must allow for choice of labor allocation within the context of these power relations and changing social and economic parameters. The result of this approach is a nexus between the reality of the inner dynamics of agriculture households and overall economic performance.

The importance of gender in agricultural response is critical for understanding the policy implications for developing countries. The goal should be to incorporate assumptions that adequately capture this economic reality.

NOTES

1. It is assumed that the man controls the receipts of cash crop marketing activities and therefore determines payments made to the woman who must, in essence, work for the man since she is unable to directly sell what she produces.

2. The exact configuration of this neighborhood depends on the particular parameters in the problem.

3. It should be noted that the typographic error w_c, in the second part of the right hand equation, has been amended to be the correct term, wC. This error was not incorporated into Darity's mathematics.

4. These equations conform to Equation 6' and Equation 7' in Darity's paper, with the important exception of w being in the denominator rather than the numerator in the first part of Equation 7'.

5. See Chiang (1984, pp. 110–111).

6. This is predicated upon the initial assumptions that α,ρ,σ, and β are all positive.

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