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A Surface-Based Approach to Measuring Spatial Segregation

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Quantitative indices of residential segregation have been with us for half a century, but suffer significant limitations. While useful for comparison among regions, summary indices fail to reveal spatial aspects of segregation. Such measures generally consider only the population mix within zones, not between them. Zone boundaries are treated as impenetrable barriers to interaction between population subgroups, so that measurement of segregation is constrained by the zoning system, which bears no necessary relation to interaction among population subgroups. A segregation measurement approach less constrained by the chosen zoning system, which enables visualization of segregation levels at the local scale and accounts for the spatial dimension of segregation, is required. We propose a kernel density estimation approach to model spatial aspects of segregation. This provides an explicitly geographical framework for modeling and visualizing local spatial segregation. The density estimation approach lends itself to development of an index of spatial segregation with the advantage of functional compatibility with the most widely used index of segregation (the dissimilarity index D). We provide a short review of the literature on measuring segregation, briefly describe the kernel density estimation method, and illustrate how the method can be used for measuring segregation. Examples using a simulated landscape and two empirical cases in Washington, DC and Philadelphia, PA are presented.

Introduction

In spite of the half century that has elapsed since Duncan and Duncan's (1955) use of the dissimilarity index D to measure segregation, formal analysis of residential segregation still suffers significant limitations in at least two aspects. First, summary index approaches continue to dominate the literature (Massey and Denton 1988). Using summary indices to depict the level of segregation of a region is useful for comparison among regions. However, adopting this approach may fail to reveal significant spatial aspects of segregation, such as the nonuniformity of local levels

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of segregation across a study area (see Fotheringham 1997 on the shift toward localized methods in spatial analysis). The summary index approach also fails to support an effective visualization framework for spatial aspects of segregation.

A second weakness of many segregation measures results from the use of census data. Calculation of segregation levels based on census data considers only the population mix within individual areal units, not the spatial relationships among them. The artificially delineated boundaries of statistical areas are effectively regarded as blockages to interaction between population subgroups in different units. Thus, evaluation of the segregation level is constrained by the boundary structure of a spatial partitioning system, which has no necessary relation to levels of segregation or interaction among population subgroups. This issue is illustrated by the "checkerboard problem," whereby a grid landscape with alternating exclusively white and black areas is not evaluated by traditional segregation measures as less segregated than the case where the same areas are rearranged into larger exclusively White and Black regions (White 1983).

There are other more subtle problems associated with using census data aggregated to zones. Many zoning schemes are initially designed with the aim of optimizing how they represent population characteristics of interest. In the case of census zones, homogeneity of population within zones may be considered desirable. As D is a function of the internal homogeneity of areal units, when zones are initially defined, segregation is exaggerated to the extent that racial characteristics are deemed an aspect of homogeneity. Conversely, over time, segregation appears to fall more rapidly than may actually be the case, as populations move but the boundaries of zones by which they are aggregated remain fixed (to support longitudinal studies). A further side effect of the impact of homogeneity on segregation measurement is that smaller statistical enumeration units produce higher measured segregation levels. This is the so-called scale effect in measuring segregation (Wong 1997).

Methodological developments in segregation studies have only partially addressed these issues. There is a need to develop a segregation measurement approach that lessens the effect of the zoning system defined a priori, that enables the visualization of local segregation levels, and that accounts for spatial aspects of segregation. In this article, we propose a kernel density estimation method to model spatial aspects of segregation. While the effect of enumeration unit boundaries cannot be completely removed, their impact on evaluating segregation becomes less significant. Adopting this approach to segregation provides a geographical framework for modeling and visualizing local variations in spatial segregation. Specifically, it can aid in identifying local contributions to the segregation of a study area based upon local imbalances of subgroup populations. In the next section, we provide a short review of the segregation measurement literature most relevant to the objectives of this article. In the subsequent section, we briefly describe the kernel density estimation method and illustrate how the method can be used to measure segregation. This is followed by an example using a simulated landscape, and two empirical examples using Washington, DC and Philadelphia, PA in the penultimate section. The article ends with conclusions and suggestions for further work.

Measuring segregation: Approaches and measures

The literature on measuring segregation is dominated by the dissimilarity approach introduced by Duncan and Duncan (1955). The dissimilarity index D for two subgroups is defined as

$$D = \frac{1}{2}\sum_i \left|\frac{a_i}{A} - \frac{b_i}{B}\right| \tag{1}$$

where a_i and b_i are the population counts of two population subgroups of concern in areal unit i, and A and B are the total population counts of the two subgroups in the entire study region. In the traditional segregation literature, two subgroups are usually referred to as White and Black, given the historical context. Although D does not meet all the mathematical criteria desirable in a measure of segregation (see Reardon and Firebaugh 2002 for a detailed discussion), its range from 0 to 1 makes for simple interpretation, and it has been widely applied in the literature.

However, as has been noted, this approach is problematic, as evaluation of D is constrained by the definition of the boundaries of the enumeration units used. The calculation of D effectively assumes that people in different areal units as defined by some zoning scheme do not mix or interact. The spatial separation of population subgroups that is one aspect of the conceptualization of spatial segregation (Newby 1982) is thus completely determined by the boundary system. This drawback of D is shared by most popular measures of segregation due to the aspatial formulation of those measures. Geographers have made several attempts to capture the spatial interaction aspect of segregation. Distance-based approaches were suggested by Jakubs (1981) and Morgan (1983), while more recently, Morrill (1991) and Wong (1993) adopted a neighborhood interaction approach to moderate the level of D. Additional terms were introduced into the calculation of D to account for the potential interaction among population subgroups in neighboring units. The effect of these modifications is to diminish the aspatial segregation level based upon D. Adapting concepts from the development of spatial versions of D, Dawkins (2004) has also proposed a spatial version of the Gini coefficient to measure residential segregation.

In an alternative approach, Wong (1999) suggested that segregation can be regarded as the level of spatial correspondence among population subgroups. Using the centrographic measure of standard deviational ellipses to capture the spatial distribution of a population, a new spatial measure S was introduced. In the study area, a deviational ellipse is computed or fitted for each population subgroup based upon the locations of the population. This is shown schematically in Fig. 1, where black filled diamonds are summarized by the smaller ellipse, and open

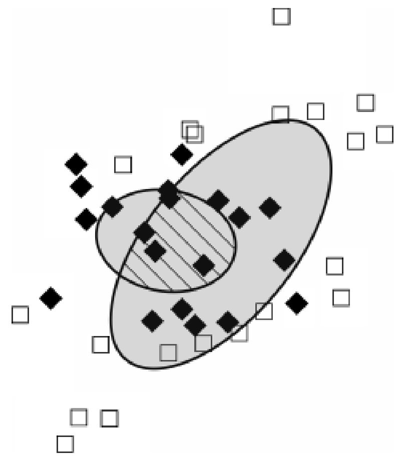

Figure 1. Illustration of calculation of the ellipse-based *S* segregation index. Each population group's location is summarized by a standard deviational ellipse and the intersection (hatched) and union (shaded gray) areas of the ellipses are combined to arrive at a summary index (see text for details).

squares by the larger one. Using tools in a geographic information system (GIS), ellipses from different subgroups can be overlaid on each other to identify their spatial intersection and union. The *S* index is then calculated as one minus the ratio of the intersection and union areas (see equation [8]). If two population subgroups have a relatively high degree of spatial correspondence, then they are less likely to be segregated. This method can lead to problems, however, when one subgroup is enveloped by the other, as often occurs with minority groups concentrated in inner urban neighborhoods. Nevertheless, the ellipse-based *S* index serves as a point of departure for our proposed surface-based index.

Either explicitly accounting for interaction among population subgroups across areal units or explicitly incorporating the location of populations provides spatial measures of segregation. These measures are also limited in the sense that they remain summary indices that describe segregation levels across an entire region, when segregation is unlikely to be uniform across any region. In order to depict the spatial variability of segregation across a region, one may use the entropy-based diversity index preferred by White (1983, 1986). Recently, Reardon and Firebaugh (2002) have also offered strong evidence that the diversity index is highly desirable. The diversity index has an additional desirable property from a geographical perspective because the index is computed for each areal unit, and therefore, a map can be generated for the study region to show spatial variation in diversity. However, the diversity index is also aspatial in the sense that the level of diversity of a given unit is not dependent upon the racial or ethnic mixes of neighboring units (Wong 2002). In other words, boundaries of areal units are still treated as borders that prevent interaction between different areal units.

Advocating a local approach to depicting the spatial variation of segregation, Wong (2002) reports earlier work involving disaggregation of the regional index D into D_i, a value for each areal unit in the region. The local D_i, unfortunately, is also aspatial. Subsequently, using the notion of composite population count to accommodate interaction across unit boundaries, Wong (2002) proposed a spatial version of the diversity index. In addition, adopting the exposure or interaction concept suggested by Lieberson (1981) to measure segregation, Wong (2002) proposed a local spatial segregation index. This index is local because each areal unit has a segregation value. It is spatial because the measured segregation level is dependent on the neighborhood racial compositions. Because it is a local measure, maps can be used to visualize the spatial variation of segregation.

In this article, we expand on the notion of spatial correspondence among population subgroups adopted in the formulation of the ellipse-based measure S. The ellipse-based index reflects the overall spatial correspondence among population subgroups across the entire region. However, as argued above, segregation is not uniform across a region, so we model the distribution of population subgroups using density surfaces, to enable a more localized picture of nonuniformity in segregation to be developed. Surface-based approaches to modeling segregation are not new. Spriggs (1984) used trend surface (i.e., regression) modeling of minority group populations and argued that locations with large negative residuals were more integrated. A comprehensive spatial measure of segregation has also been proposed recently by Reardon and O'Sullivan (2004). In their work, population surfaces represent the composition of the local spatial environment of each location as a first step in the development of spatial versions of standard aspatial segregation measures. This is similar to our approach, although the role of the population surface in the present work is simply as an estimate of the overall distribution of each population subgroup. Both approaches effectively tackle the problem of regarding zone boundaries in population data as barriers to interaction among subgroups.

The difficulties of measuring segregation based on counts aggregated to zones are not limited to the implicit assumption that populations do not interact across zone boundaries. The more general modifiable areal unit problem (MAUP) is also present (Openshaw and Taylor 1979; Openshaw 1983), arising from the arbitrary nature of zones relative to the phenomenon being measured. The MAUP may be considered as a combined scale and zoning effect. The scale effect describes the impact of using progressively larger zones to aggregate data. The zoning effect refers to the fact that the observed aggregate data may be altered by spatially shifting any given zoning scheme. While many of the methods discussed above partially address zoning effects, there has been only limited progress in addressing scale effects. A relatively scale-insensitive correlation measure based on the comparison of cumulative distribution functions over space was proposed by Wong (2001). This measure combines the characteristics of a Kolmogorov–Smirnov statistic and Ripley's K function analysis (Ripley 1976) and has the potential to provide relatively consistent results when population distributions described along transects are compared using data at different geographical scales. Although K functions are useful for detecting point clustering at a range of scales, they are primarily used to describe an entire study area and are therefore not useful for identifying localized effects, which is a major objective of the present article. While recent developments in point pattern analysis may lead to more localized measures (see Baddeley et al. 2005), in the present context, point pattern methods are inappropriate as the points we use (see Figs. 4 and 5) are centroids of census units, not the locations of actual point events.

Kernel density estimation and measurement of segregation

The standard deviational ellipses that form the basis of the S index of segregation are just one possible approach to summarizing the spatial distribution of a population (O'Sullivan and Unwin 2003, pp. 79–81, 85–88). An alternative approach is kernel density estimation, which was originally developed to allow estimation of an underlying population probability density function from a sample, as an alternative to histograms (Silverman 1986; Wand and Jones 1995). Histograms are prone to problems arising from the dependence of the histogram on the choice of bin-widths and origin. Kernel density estimation avoids some of these problems, by multiplying (i.e., convolving) the sample data by a probability density function continuously applied across the range of the sample data and summing the results to produce a single empirical estimate of the underlying probability density function. This may be written as

$$y(x) = \frac{1}{n}\sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right) \tag{2}$$

where y is the estimated probability density function, $\{X_i \ldots X_n\}$ are the sample data, and K is some kernel function, with bandwidth h.

The shape of y is not much dependent on the choice of kernel function (Silverman 1986; Wand and Jones 1995). Common kernel functions are the familiar Gaussian, and the less familiar quartic function

$$K(x) = \begin{cases} c(h)\left(1 - \left(\frac{x}{h}\right)^2\right)^2 & \text{where } x \leq h \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

and

$$c(h) = \frac{15}{16h} \tag{4}$$

where h is the kernel bandwidth and $c(h)$ is a scaling factor that ensures the function sums to unity (so that it is a true PDF). In general, kernel functions are more heavily weighted in the center of their range, although uniform functions are occasionally used, in which case the estimation is equivalent to calculation of a simple moving average. The density estimate y is strongly dependent on the kernel function bandwidth. Wide bandwidths tend to oversmooth the data by including many distant data points in each estimate, so that important features in the distribution may not be apparent. On the other hand, narrow bandwidths tend to overemphasize local variations. In practice, it is necessary to experiment with a range of bandwidths to determine an appropriate value in empirical cases.

The extension of kernel density estimation to the spatial case is straightforward (Diggle 1983, 1985; Brunsdon 1995). The kernel function becomes a two-dimensional function, usually radially symmetric, so that it can be expressed as a function of r, the distance from its central point. The quartic function becomes

$$K(r) = \begin{cases} c(h)\left(1 - \left(\frac{r}{h}\right)^2\right)^2 & \text{where } r \leq h \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

and

$$c(h) = \frac{3}{\pi h^2} \tag{6}$$

to ensure proper scaling. The sample data are now the spatial locations of observations. In the present example where data are only known at some level of aggregation, counts are treated as multiple data points at a single location (often a zone centroid). The values of the kernel density estimate from a set of point data are proportional to the density of point events in the sample and enclose a total volume equal to the total count of the data.

Previous geographic applications of kernel density estimation have included point pattern analysis examples in crime analysis (see, e.g., Chainey, Reid, and Stuart 2003), spatial epidemiology (Gatrell 1994), the visualization of population distribution (Wood et al. 1999), and as a method for interpolation between different zoning schemes (Bracken and Martin 1995). There has been significant interest in the utility of surface-based analysis of demographic data in recent years, arising in part from realization of the limitations of zoned data (see Coombes and Raybould 2000; Martin, Tate, and Langford 2000; Thurstain-Goodwin and Unwin 2000 in a special issue of *Transactions in GIS* devoted to this topic).

In the present context, for any given areal unit, subgroup populations originally can either be assigned to the centroid of the areal unit or spread uniformly over the areal unit. When a kernel function is applied, the population is spread or distributed around the areal unit according to the shape of the kernel. Provided that the kernel function crosses the boundary of the areal units, this process of redistributing the population to a large extent counteracts the implicit assumption that populations do not interact across boundaries. This is analogous to the role of the kernel density method in dealing with the bin-width problem of histograms in aspatial situations. In practice, when kernel density estimation is applied, a finite resolution grid must be imposed defining locations at which estimates are made. The choice of location of this grid marginally affects the density estimates made, although provided the grid resolution is substantially smaller than the bandwidth (say by a factor of 5 or more, and minimally by a factor of 2), the location effect is negligible.

Developing a segregation measure using kernel density estimation

The segregation measure we present in this article is derived from kernel density-estimated functions of different population subgroups in a manner similar to the *S* measure based on standard deviational ellipses (Wong 1999).

First, we calculate a two-dimensional spatial function that is a probability density function for each population subgroup of interest. We assume two subgroups with total populations *W* and *B*, and sets of counts $\{w_1 \ldots w_n\}$ and $\{b_1 \ldots b_n\}$ across *n* zones. These data are converted to the fraction of each population subgroup resident in each zone, which we denote p_{Wi} and p_{Bi}. Note that

$$\sum_i p_{Wi} = \sum_i \frac{w_i}{W} = 1 \text{ and } \sum_i p_{Bi} = \sum_i \frac{b_i}{B} = 1 \tag{7}$$

by definition. Now, applying kernel density estimation to these sets of numbers, we obtain two-dimensional probability density functions $p_W(x, y)$ and $p_B(x, y)$. Integration of either function across some part *A* of the study region *R* yields an estimate of the fraction of that population subgroup that is resident in *A*. Given the equalities in (7), these functions can be regarded as true probability density functions with a total enclosed volume of 1.

For illustrative purposes, we present the calculation of a segregation index between two subgroups based on the one-dimensional functions shown in Fig. 2 (although we actually work with two-dimensional spatial functions). Recall that, in the two-subgroup case, *S* is calculated from

$$S = 1 - \frac{A_\cap}{A_\cup} \tag{8}$$

where $A_\cap$is the area of intersection between two summary ellipses of population, and $A_\cup$ is the area of the union of the two summary ellipses. Applying the same logic to two probability density functions of population, we have

$$S = 1 - \frac{V_\cap}{V_\cup} \tag{9}$$

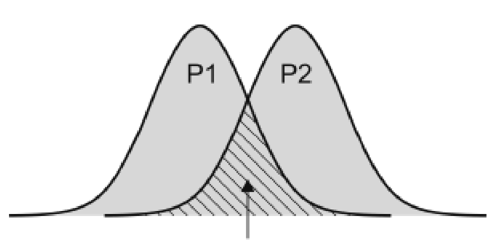

Figure 2. Illustration of calculation of the segregation index.

where $V_\cap$ is the volume of intersection between the two density functions, and $V_\cup$ is the volume of the union of the two functions. In Fig. 2, the intersection volume is shown as the hatched area, and the union volume by the sum of the hatched and shaded areas.

Fig. 2 makes it clear that calculation of the required volumes of intersection and union is straightforward. The intersection volume is given by integration over the study region R of the lesser of the two functions at each location:

$$V_\cap = \oiint_R \min(p_W, p_B)\, dR \tag{10}$$

and the union volume is given by integration of the greater of the two functions at each location:

$$V_\cup = \oiint_R \max(p_W, p_B)\, dR \tag{11}$$

In practice, the integration is approximate and is carried out by summation across a grid of cells of finite size, so that the intersection volume is given by

$$V_\cap = \sum_i \min(p_{Wi}, p_{Bi}) \tag{12}$$

and the union volume is given by

$$V_\cup = \sum_i \max(p_{Wi}, p_{Bi}) \tag{13}$$

It may not be immediately apparent from this derivation of the new index that it is closely related to the index of dissimilarity D. In fact, S can be shown to be a spatial form of D as follows: combining equations (9), (12), and (13), we have

$$S = 1 - \frac{\sum_i \min(p_{Wi}, p_{Bi})}{\sum_i \max(p_{Wi}, p_{Bi})} \tag{14}$$

which gives us

$$\begin{aligned} S &= \frac{\sum_i \max(p_{Wi}, p_{Bi}) - \sum_i \min(p_{Wi}, p_{Bi})}{\sum_i \max(p_{Wi}, p_{Bi})} \\ &= \frac{\sum_i |p_{Wi} - p_{Bi}|}{\sum_i \max(p_{Wi}, p_{Bi})} \end{aligned} \tag{15}$$

Comparing this expression with (1) and (13), we arrive at

$$S = \frac{2D'}{V_\cup} \tag{16}$$

where the prime reminds us that D' is not the index of dissimilarity, as it would be calculated from the original set of zonal counts, but calculated from a discrete set of values drawn from two estimated density functions. However, in the special case of density estimation with a small (but nonzero) bandwidth, the numerator in (16) will be very close to twice the traditional index of dissimilarity D. The denominator in (16) will have a value between 1 (for completely integrated populations, when $S \approx D = 0$) and 2 (for completely segregated populations, when $S \approx D = 1$). The direct relationship with the index of dissimilarity D is an advantage of the proposed approach, insofar as much previous quantitative work on segregation is based on the dissimilarity index.

Development of the proposed measure proceeded as described above from the premise of a measure analogous to the ellipse-based measure, but based on volumes under kernel density-estimated surfaces. The only nonobvious step is the choice of base data in the estimation procedure. The rationale for this choice was that it yields surfaces that enclose unity volume for all population subgroups and that are therefore representative of the spatial distribution of that subgroup, scaled so as to allow for the subgroup's total population. This avoids problems presented by using population counts, which would result in population surfaces enclosing widely divergent volumes. In extreme cases, with a small minority group, the population count approach would see the majority population volume completely enclosing the minority group, so that the intersection–union formulation of S would have little meaning.

Examples

In this section, we apply the segregation measure developed in the preceding discussion in two-subgroup settings to demonstrate the applicability of the measure, to better understand some of its properties, and to show the potential for useful visualizations inherent in the approach.

Synthetic data

First, we apply the method to synthetic data to demonstrate some properties of the measure. For these experiments, four synthetic landscapes, as shown in Fig. 3 below, were used. All four landscapes are developed on an idealized 40 km $\times$ 40 km grid of 1 km square cells. Each cell has a population of 1000 that is either exclusively White or exclusively Black. These landscapes are intended to show how the smoothing involved in kernel density estimation removes the checkerboard problem of traditional segregation methods, so the first two landscapes feature checkerboards at different scales. In the first landscape, 1 km cells are organized in a checkerboard pattern with a 1-km resolution, so that every cell has four orthogonal neighbors of the opposite type to itself. In the second landscape, the checkerboard pattern is on a larger 5-km scale with groups of 25 cells each with 1000 residents of the same type. The third landscape is a highly segregated one where all Whites live in the western half of the city and all Blacks live in the eastern half. The fourth landscape is similarly segregated, with a central circular ''ghetto'' of cells whose population is exclusively Black, surrounded by all White cells.

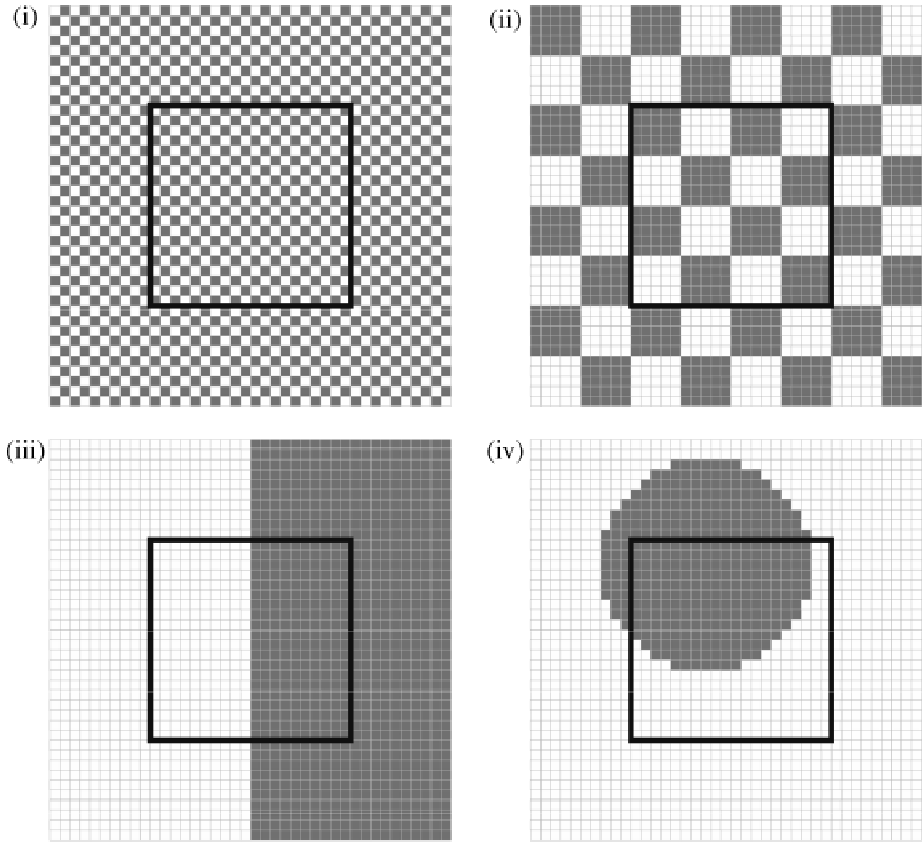

Figure 3. Synthetic grids used in preliminary analyses. (i) 1-km resolution checkerboard; (ii) 5-km resolution checkerboard; (iii) globally divided region; (iv) a "ghetto." In each case, dark squares and white squares contain a population of 1000 individuals of one type only. Squares are 1 km on a side. The central, bold outlined 20 km × 20 km square is used for summation of the segregation measure, whereas the 40 km × 40 km region is used for the density estimation step in the measurement (see text for details).

Measurement of the segregation of these landscapes was carried out as follows:

1. Convert population counts to population proportions for each 1-km square.
2. Assign each 1-km square's population proportion to a point at its centroid.
3. Apply a kernel density function to the resulting set of points for each subgroup's population proportion, to obtain probability density functions summarizing the spatial distribution of each subgroup. As previously noted, other functions could equally be used, and the choice is unlikely to affect the results by much.
4. Determine maximum and minimum values of the two subgroup surfaces at each location.

5. Calculate the final value of the segregation measure by determining the volume under the maximum and minimum value surfaces and combining them using equation (14).

The kernel density function used in this case was a quartic function as in equation (5) above, with bandwidth h set to 2.5 km. Given the characteristic scales of the synthetic data, this bandwidth allows us to observe the effect of a bandwidth much larger and also much smaller than the spatial scale of the data. Kernel density estimation was applied to the whole $40\,\text{km} \times 40\,\text{km}$ region in each case, while summation operations for calculation of the final segregation measure were only calculated over the $20\,\text{km} \times 20\,\text{km}$ central zone indicated by the heavily outlined squares in Fig. 3. This was done to avoid edge effects resulting from inclusion of regions with zero population outside the study region. The resulting segregation values are summarized in Table 1.

These results are consistent with our expectations of these data. In particular, it is noteworthy that the 1-km checkerboard pattern results in very low measured segregation, due to the 1-km resolution variation in population being substantially smoothed by a 2.5 km kernel bandwidth. On the other hand, the 5-km checkerboard pattern remains relatively highly segregated under this smoothing. Both the east–west segregated landscape and the ghetto landscape result in very high measured levels of segregation.

Further light is shed on these results by examination of maps of the intermediate calculation in each case. Specifically, we focus on the minimum and maximum values after comparing the volume functions. As can be seen in Fig. 4(i) and (ii), the smoothed 1-km checkerboard landscape is almost level, with little difference between the minimum and maximum values. In Fig. 4(iii) and (iv), the wide variation between minimum and maximum values for the 5-km checkerboard landscape is evident. Toward the center of each $5\,\text{km} \times 5\,\text{km}$ region, the value of the minimum surface approaches zero in each case, as the center of any White neighborhood has a very low smoothed density of Blacks, and the center of any Black neighborhood has a very low smoothed density of Whites (note that only the central $20\,\text{km} \times 20\,\text{km}$ region from Fig. 3 is shown in Fig. 4). On the other hand, the center of each neighborhood has a high maximum value for whichever population is in the majority there. On the boundaries between neighborhoods in this landscape,

Table 1 Values of Segregation Measure for the Synthetic Landscapes in Fig. 3

Landscape		Segregation, S
Number	Description	
(i)	1-km checkerboard	0.025
(ii)	5-km checkerboard	0.681
(iii)	East–west segregated	0.964
(iv)	"Ghetto"	0.963

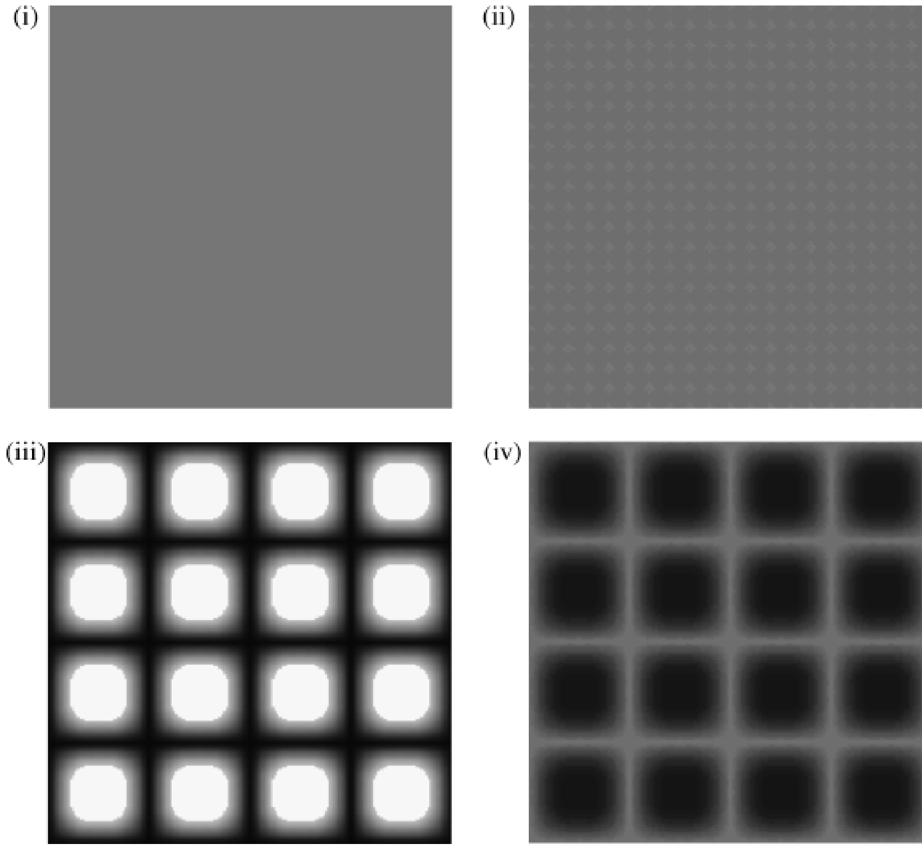

Figure 4. Results of smoothing the checkerboard landscapes. These figures show results for the central 20 km $\times$ 20 km zone. The minimum (i) and maximum (ii) values for the 1-km (top) and 5-km ([iii] and [iv] in the bottom) checkerboards after smoothing by a 2.5 km bandwidth quartic function. Note that the grayscale color scheme in all four figures is the same, with darker shading indicating higher values.

the maximum and minimum values are similar to one another, so that these regions contribute less to the segregation measure that results.

Census data

Next, we apply our method to calculation of segregation indices in two U.S. metropolitan areas: Washington, DC and Philadelphia, PA. Calculations are based on census block data from the 2000 U.S. Census using the White-only and Black-only counts for each block. Maps of the regions are shown in Figs. 5 and 6. Again, density smoothing was carried out across a wider area than for summation of the minimum and maximum surfaces to reduce edge effects. Density smoothing was applied across a region extending 10 km in all directions beyond the city boundary. In the Washington, DC study area, this area included some 19,698 census blocks,

Figure 5. The Washington, DC study area. Block centroids within 10 km of the District of Columbia are shown. These points were used to generate smoothed population surfaces for Whites and Blacks.

while the Philadelphia, PA study included 41,729 blocks. Using these input data, density smoothing of population proportion data for Whites and Blacks was carried out with bandwidths 1.0, 2.5, 5.0, 7.5, and 10 km. The resulting probability density surfaces for the two subgroups were combined as before by determining minimum and maximum value surfaces, summing the values of these within the city limits, and calculating a segregation index in each case. All calculations were carried out on a grid with a 250-m resolution. In this example, we use multiple bandwidths to explore the sensitivity of the results to the choice of this value.

The results of the calculations for the Washington, DC and Philadelphia, PA examples are summarized in Table 2. The most obvious characteristic of the measure for both cities is that segregation between Whites and Blacks is high. The result recorded for zero bandwidth is calculated from raw block population data as a

Figure 6. The Philadelphia, PA study area. Block centroids within 10 km of Philadelphia County are shown. These points were used to generate smoothed population surfaces for Whites and Blacks.

traditional aspatial *D* segregation index for White-only and Black-only populations inside the city limits. It is a strength of the proposed index that it can be related to the *D* index in this way. A second obvious feature of these results is that the measured segregation decreases as the bandwidth used increases, as we would expect. Increasing the bandwidth corresponds to assuming that interaction between subgroups occurs over longer distances, so that segregation necessarily decreases. Notably, segregation falls more rapidly with increasing bandwidth for Washington, DC compared with Philadelphia, PA. This is not simply an artifact of the relative geographical scale of the two metropolitan areas. It also reflects the greater presence of Blacks throughout the Washington metropolitan area, compared with

Table 2 Measured Segregation Values for Washington, DC and Philadelphia, PA

Kernel bandwidth (km)	Segregation, S	
	Washington, DC	Philadelphia, PA
0.0	0.895	0.907
1.0	0.870	0.862
2.5	0.813	0.815
5.0	0.720	0.770
7.5	0.629	0.736
10.0	0.553	0.702

Philadelphia, where almost the entire Black population is concentrated in inner urban neighborhoods within Philadelphia County. Thus, as bandwidth increases, in Washington, DC, proportionally more interaction between subgroups becomes likely, whereas in Philadelphia, PA this does not occur. Thus, by exploring different bandwidth sizes and the corresponding segregation levels, the spatial extents or scales of segregated neighborhoods may be revealed.

This variability of measured segregation with kernel bandwidth means that it is important both to choose a sensible bandwidth for comparisons between cases and that the variability of the measure with bandwidth should be examined. Clearly, the optimal bandwidth should not be so small that some census units include no neighbors. On the other hand, the bandwidth size should not be too large to include too many neighbors. One possible approach to determining an appropriate bandwidth would be to use the average nearest-neighbor distance among all polygon centroids. A variety of approaches might be used, based on ensuring that some minimum numbers of neighbors are included within the kernel bandwidth applied to each centroid. An alternative would be to use locally adaptive kernel bandwidths as suggested in the context of geographically weighted regression (see Fotheringham, Brunsdon, and Charlton 2002, pp. 57–58).

Intermediate map products

An important feature of the proposed spatial segregation measure is the potential it offers for visualization of stages in the calculation, which may assist in understanding the particular neighborhoods that are giving rise to the observed overall segregation value. As examples of these, we show displays of (i) the population probability density functions for Whites and Blacks and (ii) the resulting maximum minus minimum surface representing, respectively, the numerator surface in equation (15).

Fig. 7 shows population probability density surfaces for Whites and Blacks in Washington, DC. The strong east–west segregation of the city is evident. These maps were generated using a 2.5-km bandwidth. The only part of the city where strong concentrations of both population subgroups are in close proximity to one another is just to the northwest of the central city. These are the regenerating

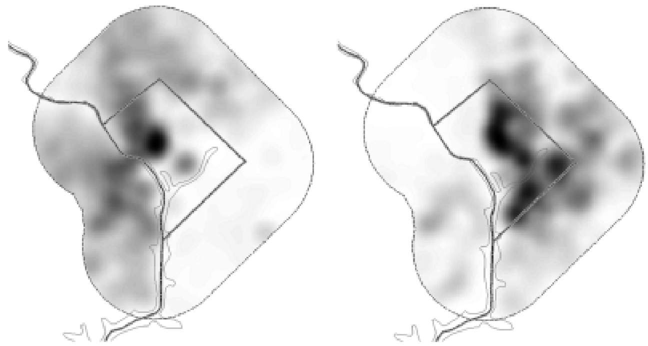

Figure 7. Population probability density surfaces for Whites (left) and Blacks (right) in Washington, DC. These were calculated using a 2.5-km bandwidth and census block data.

neighborhoods of Washington, NW, such as Dupont Circle, where more affluent young professionals—often White—have moved into previously more Black-dominated neighborhoods. Fig. 8 tells a similar story for Philadelphia, PA. Here, the most obvious aspect of the distributions is that the Black population is concentrated in the city itself, to the north and west of the central city. Only a small proportion of the Black population lives outside the city boundary. By contrast, the White population has some concentrations in neighborhoods inside the city limits in the central city and to the northeast, but a significant proportion of the White population lives around the city distributed fairly evenly in all directions.

To emphasize the usefulness of these intermediate stages of the calculation of S for visualizing local aspects of segregation, it is instructive to view maps of the maximum minus minimum population proportion surface for each city. These are shown in Fig. 9. These maps give a disaggregate view of the local contributions to overall segregation, by showing the numerator at each location in the summation presented in equation (15). There is a distinct difference between the two cities in this case. In both cases, the inner urban neighborhoods with dense concentrations

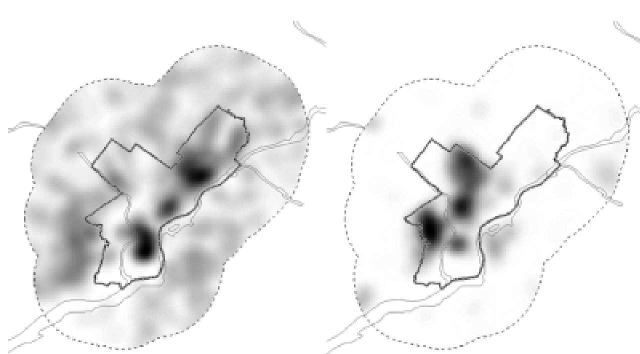

Figure 8. Population probability density surfaces for Whites (left) and Blacks (right) in Philadelphia, PA. These were calculated using a 2.5-km bandwidth and census block data.

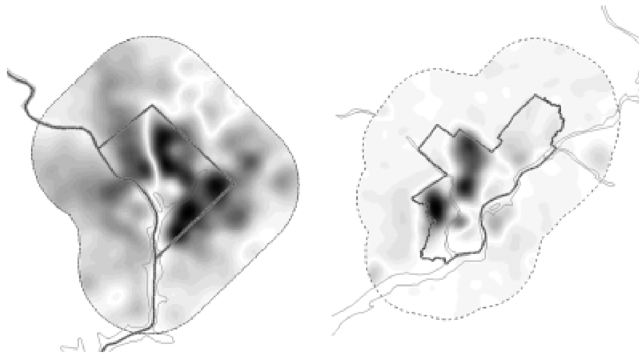

Figure 9. Maximum population proportion minus minimum population proportions for each city. These maps are derived from the data shown in Figs. 7 and 8. The local contribution to overall segregation can clearly be seen in each map. (Note that maps are not to the same scale.)

of Black populations contribute heavily to overall segregation (compare the right-hand maps in Figs. 7 and 8). However, in the case of Washington, DC, relatively White neighborhoods also contribute relatively strongly to segregation. By contrast, the relative contribution to overall segregation from the suburban regions of Philadelphia, PA is much less. In both maps, regions with some degree of mixing are highlighted as lighter-colored regions. In both cases, these tend to be around the edges of contiguous regions dominated by the Black population, so that the overall effect is that this surface picks out boundaries of areas of a relatively homogeneous population.

Much of the above discussion on the two cases will appear trivial to those who know the local population landscapes reasonably well. But when this approach is applied to places with less-rich local information, the final results and intermediate maps may offer a wealth of information to help explain overall segregation and the situations at the neighborhood level. Most popular segregation measures are global measures, which fail to depict the spatial variability of segregation. The current approach can depict segregation levels at the local scale and provide important information in understanding overall levels of segregation. The two cases of Washington, DC and Philadelphia, PA have readily discernible spatial patterns and thus can be recognized easily from simple population maps by race. For places with more subtle patterns, the potential utility of the proposed approach is apparent. Further, we have limited our discussions to the two-subgroup situation. Extending the current approach to multigroup situations would allow identification of local concentrations of segregation that may not be revealed by simple mapping exercises.

Further research and conclusions

Many aspects of the present approach call for further research. While the reported segregation for any region will generally fall as the kernel bandwidth is increased, it is likely that the decay characteristics of different regions may vary, and this behavior remains to be characterized. We have used density estimation applied to census block centroids as the basis for estimation of population proportion density distributions. In practice, we have to work with aggregated data in most cases, so that the proposed method remains limited by the smallest geographical scale at which sufficiently detailed counts of subgroups are available. This forces us to make simplifying assumptions about the distribution of populations within areal units. Both these issues draw attention to the need for research into the combined effects of bandwidth and population surface resolution or cell size on the method. As a practical rule of thumb, we suggest that bandwidth should be of the same order as typical distances between areal unit centroids. The choice of the estimated surface resolution is related to this and should be no more than half of the bandwidth, and preferably much less than this where practical (say 0.1–0.2 times the bandwidth).

As blocks in urban areas are generally of limited geographical extent in comparison with the kernel bandwidths that we have used, this is not a concern in the examples reported here. However, alternative methods of estimating population distribution based on zoned and aggregated data are available and may yield different results. A "prismatic" distribution that assumes an even distribution of population across spatial units might be used, or one based on pycnophylactic interpolation (Tobler 1979), or dasymetric methods (Mennis 2003). Each of these methods would yield a different starting population distribution and could result in different final segregation scores. The sensitivity of the final measure to these different approaches remains to be determined and might enable the satisfactory measurement of segregation based on less detailed aggregate data than that used here.

We have not offered significance tests for the proposed method. The proposed method is a local spatial method, in common with many recently introduced spatial analytical techniques focusing on local patterns (Fotheringham 1997). Analytic significance testing of local spatial is often not feasible, and Monte Carlo methods are likely to be the most useful in developing an inferential framework in this case. It is worth noting, however, that in many real-world settings, segregation patterns are so strong that the need for an inferential framework for significance testing is minimal.

We have not discussed extending the present method to the multigroup case, which is of increasing importance in empirical settings. The most obvious approach would simply extend the intersection-and-union approach based on minimum and maximum population proportion surfaces. This would probably result in high apparent segregation measures, however, as in the multigroup setting one or other of the population subgroups would be likely to be poorly represented at every location. An alternative would be to use the maximum population proportion and second highest population proportion value in all cases, which would result in low segregation scores, provided that at least two subgroups were reasonably well represented at every location. Note that the proposed approach will be useful for identifying clusters of highly segregated neighborhoods in a multiethnic setting and for visualizing local segregation levels. An alternative approach would be to report all two-subgroup segregation scores separately (see Morrill 1995). A further possibility would use entropy-based diversity indices (see Reardon and O'Sullivan 2004).

The proposed method shows considerable potential as a method for measuring residential spatial segregation. It has a significant advantage over many previous measures in reducing the artificial barriers to interaction imposed by enumeration unit boundaries provided that the kernel bandwidth, and the resolution of the grid across which density estimates are made, are chosen appropriately. It also has a significant advantage in being directly related to the most widely used nonspatial measure of segregation: the dissimilarity index D. With a small nonzero kernel bandwidth, the measure approaches the D index. The probability density functions and other intermediate surfaces used in the calculation of the proposed measure also lend themselves to visualization by standard GIS methods and may yield further insight into local particulars of population distributions that give rise to especially high or low overall segregation scores.

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