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Spatial Decomposition of Segregation Indices: A Framework Toward Measuring Segregation at Multiple Levels

Segregation measures based upon data gathered at different geographical levels cannot provide consistent results because of the scale effect under the Modifiable Areal Unit Problem (MAUP) umbrella. This paper proposes a framework, which decomposes segregation attributable to different geographical levels, to conceptually link segregation values obtained from multiple geographical levels together such that differences in segregation values among levels are accounted for. Using two different indices, the dissimilarity index D and the diversity index H, this paper illustrates the decomposition methods specific to these indices. When these indices are decomposed, local measures of segregation pertaining to multiple geographical levels are computed. These local segregation measures indicate the levels of segregation contributed by the local units and the regional units to the entire study area.

The spatial scale effect, or the large umbrella of the modifiable areal unit problem (MAUP), has been a persistent issue in spatial analysis and geographical research (Tate and Atkinson 2001). The inconsistency of analytical results derived from data gathered at different scale levels and/or gathered from different spatial partitioning systems are found in almost all types of analysis involving spatial data. Measuring levels of segregation is no exception. In general, using data of higher spatial resolution or smaller enumeration units will yield a higher level of segregation reflected by measures such as the dissimilarity index D (Wong 1997).

Different approaches have been suggested to deal with the scale effect, including a call for multiscale analyses to obtain a more comprehensive understanding of the geographical issues involved (Fotheringham 1989). Using Geographic Information Systems (GIS), spatial analysis can be performed repeatedly with multiple scale data to assess the scale effect. Another appealing approach is to develop scale-insensitive spatial analytical techniques such that analyses conducted at different scale levels are

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relatively consistent. Unfortunately, few successes have emerged so far (e.g., Tobler 1991; Wong 2001).

This paper takes the approach between merely recognizing different results due to the scale effect and the quest for scale insensitive techniques to address measurement issues in segregation. Because segregation values based upon data gathered from different scales are different, the purpose of this paper is to develop a framework to relate the segregation values obtained from different scales such that these values can provide a consistent but more comprehensive depiction of the phenomenon or situation. In general, the levels of segregation found at the lower or local level (using smaller enumeration units) are higher than that at the higher or regional level (larger enumeration units). Similar to the approach used by Moellering and Tobler (1972) to decompose the total variance into different components attributable to various geographical levels, the approach adopted here decomposes segregation values obtained from the dissimilarity index at the local level (with higher observed segregation) to the pure local segregation and regional segregation, and decomposes the segregation values obtained from the diversity index at the regional level (with lower observed segregation) to the local segregation and the pure regional segregation. The purpose of this decomposition exercise is to account for the sources of segregation at different scale levels so that segregation values from different scale levels can be linked conceptually. This approach can provide a consistent accounting of segregation across multiple scales as long as the areal units at different scale levels are nested in a hierarchical manner. In addition, local measures can be derived in the process to reflect how much segregation from local units at a given geographical level is attributable to the overall segregation level.

The next section will provide a brief overview of the scale effect issue with reference to measuring segregation. Section 2 will introduce the framework to decompose the dissimilarity index D and the diversity index H. A simulation experiment and two empirical studies follow.

1. SCALE EFFECT AND MEASURING SEGREGATION

The scale effect is one of the two sub-problems (the other is the zoning effect) under the MAUP, which has been documented extensively. Because several comprehensive overview articles of the MAUP already exist (e.g., Fotheringham and Wong 1991; Wong and Amrheim 1996; Sui 2000), this paper will not duplicate these efforts. Nevertheless, it is important to point out that the impact of the MAUP is widespread and significant partly because the correlation of variables, which is an important component in statistical analysis, will change when data gathered at different scale levels are used (Openshaw and Taylor 1979). In general, data are spatially "smoothed" when they are aggregated to adjacent values, and thus less variation is preserved at the aggregated level (Fotheringham and Wong 1991). But if data have a strong positive spatial autocorrelation, the aggregation process will not remove much information as compared to negatively spatially autocorrelated data. Amrhein and Reynolds (1997) and Reynolds and Amrhein (1997) evaluated the relationships between the changes in spatial autocorrelation and the changes in variance through the aggregation process, and provided a detailed explanation of the smoothing process.

Several research efforts have tried to correct or adjust for correlation among variables (e.g., Holt, Steel, and Tranmer 1996) in order to obtain relatively consistent statistical results across scale levels, but most of these procedures are either too computationally intensive or impractical. Focusing on the regression framework, Fotheringham, Brunsdon, and Charlton (2001) speculated that the spatially weighted regression could be a potential solution to the scale effect, yet it has to be demonstrated. Using a different approach to tackle the problem, King (1995) suggested an error-bound method to handle the MAUP. Some geographers are skeptical if this approach can provide a reasonable solution to the MAUP (e.g., Anselin 2000), but some see promise (Withers 2001). Besides these rather general "solutions" to the MAUP, more application-specific solutions have been proposed. Tobler (1989) suggested a "frame-independent" model specifically for modeling interregional migration. In analyzing count variables, Wong (2001) proposed a spatial correlation approach, which can yield results relatively consistent across scale levels.

In the area of segregation analysis, several papers have documented the scale effect on measuring segregation level. Wong (1997) provided a conceptual discussion on how segregation levels, as reflected by the index of dissimilarity D, changed with scale. Because the D index is purely a function of the homogeneity within an areal unit, therefore, in general, the smaller the areal unit, the more homogeneous the population mix and thus the higher the value of D. Theoretically, if the unit of analysis is reduced to individuals, D will be one, indicating a perfectly segregated situation. To investigate both scale and zoning effects on the dissimilarity index D, Wong, Lasus, and Falk (1999) computed D values for thirty metropolitan areas with a wide range of scale levels. All these studies pointed to the direction that measuring segregation, as with other spatial analysis methods, is not immune to the scale effect. In addition, these studies demonstrated that the scale effect, which generates multiple results, could be a major impediment to formulate consistent public policy to address segregation issues. When regions were ranked according to their segregation levels, their rankings were not consistent when data at different scales were used. Counties were ranked differently according to their segregation levels when census tract and block group data were used (Wong 1997). When thirty metropolitan areas were ranked by their segregation levels based upon tract and block group data, their ranks were not consistent, even though the differences in D values between the tract and block group levels were relatively small for most of the thirty metropolitan areas (an average difference of 0.0378) (Wong 2003).

It would be unlikely to derive a scale-invariant segregation index based upon the dissimilarity concept first introduced by Duncan and Duncan (1955). Many scholars, including geographers, have challenged the effectiveness of the D index in capturing spatial segregation, especially in its ability in handling the checkerboard situation (Morrill 1991). Nevertheless, the index is still strongly recommended for segregation studies, and many sociologists uphold its status as the most effective index (Massey and Denton 1988). On the other hand, some researchers endorse the entropy-based diversity index H because it possesses several desirable mathematical properties (White 1986; Massey and Denton 1988, Reardon and Firebaugh 2002). A major advantage of using the H index is that it can evaluate situations with multiethnic groups, while the D index is limited to two-group comparisons. This paper will use these two segregation indices to illustrate the proposed spatial decomposition framework.

2. SPATIAL DECOMPOSITION OF SEGREGATION MEASURES

2.1. *The Dissimilarity Index D*

Using the traditional two-group setting, the D index is generally defined as

$$D = \frac{1}{2}\sum_{i} \left|\frac{w_i}{W} - \frac{b_i}{B}\right|, \tag{1}$$

where w_i and b_i are the white and black population counts in areal unit i, and W and B are the total whites and blacks in the entire study region. The D value is a function of how the population in each of the two groups is distributed among areal units. If all units have the same proportion among the two groups, then there will be no segregation, and D will be zero. If one group exclusively occupies each areal unit, then D will be one, indicating a perfectly segregated situation.

When D is calculated for a given region but with data at different aggregation levels, the results are usually not consistent over scale levels. When the areal units are smaller, more variation in the population distribution is captured by the detailed data and thus D tends to be higher. At the more aggregated level, less variation is found in the data and therefore D is lower (Wong 1997). This pattern is very similar to the changes in variance in data at different scale levels (Moellering and Tobler 1972).

Let W and B be the total white and black counts in the entire area as defined in equation (1). Assume that there are only two geographical levels, regions, and local units, in the area. These regions and local units could be analogous to census tracts and census block groups of the census geography in the United States. Then let W_j and B_j be the white and black population counts in region j, which is denoted as Ω_j. Within region Ω_j, w_{ij} and b_{ij} are the white and black counts in local unit i. Formally,

$$W_j = \sum_{i \in \Omega_j} w_{ij}, \qquad B_j = \sum_{i \in \Omega_j} b_{ij}, \tag{2}$$

and

$$W = \sum_j W_j, B = \sum_j B_j. \tag{3}$$

Then at the less aggregated or local unit level, the computation of the index of dissimilarity D is defined as

$$D_l = \frac{1}{2}\left[\sum_j \sum_{i \in \Omega_j} \left|\frac{w_{ij}}{W} - \frac{b_{ij}}{B}\right|\right]. \tag{4}$$

And at the aggregated or regional level, the computation of the index is defined as

$$D_r = \frac{1}{2}\left[\sum_j \left|\frac{W_j}{W} - \frac{B_j}{B}\right|\right]. \tag{5}$$

Using the relationships in equation (2), equation (5) can be rewritten into

$$D_r = \frac{1}{2}\left[\sum_j \left|\frac{\sum_{i \in \Omega_j} w_{ij}}{W} - \frac{\sum_{i \in \Omega_j} b_{ij}}{B}\right|\right]. \tag{6}$$

D_r will be lower than D_l because the aggregated data from which D_r is computed are smoothed or averaged. Based on equations (4) and (6), two observations can be derived. Segregation at the region level and at the local unit level will be the same for a given region j only if

$$\sum_{i \in \Omega_j} \left| \frac{w_{ij}}{W} - \frac{b_{ij}}{B} \right| = \left| \frac{\sum_{i \in \Omega_j} w_{ij}}{W} - \frac{\sum_{i \in \Omega_j} b_{ij}}{B} \right| . \tag{7}$$

If both sides of equation (7) are expanded for all i values, the left hand side (LHS) will equal the right hand side (RHS) for a given j if

$$w_{ij} \geq b_{ij} \ \forall\, i , \tag{8}$$

or

$$b_{ij} \geq w_{ij} \ \forall\, i . \tag{9}$$

Each of these inequalities is a necessary, but not sufficient condition for equation (7) to hold for a given j. If either inequality is true, and the difference in the distribution of the two population groups in region j captured by the RHS of equation (7) has passed along to all the local units in the same manner, then equation (7) will hold. In other words, the local units in region j have a uniform difference in the two-group distribution, or a perfect positive spatial autocorrelation in the difference. Then the local unit level does not contribute additional segregation to the magnitude as constrained by the regional level.

If neither inequality (8) nor (9) is true, then LHS of equation (7) will be larger than the RHS. That is the local unit level or the disaggregated level will have a higher level of segregation than the region level or the more aggregated level. When LHS of equation (7) is larger than the RHS, a situation much more likely than the equality situation, it implies that there is more variability in the distributions of the two population groups at the disaggregated level than that found at the more aggregated level. Therefore, the difference in segregation level between the local unit and the regional levels can be regarded as the additional segregation introduced by the local level.

Before computing the segregation introduced purely by the local level, we have to compute segregation at the regional level. Based upon Wong (1996), segregation for each region at the regional level can be obtained by spatially disaggregating D_r for each j such that for each region j, the segregation level in the region is

$$RD_j = \left| \frac{\sum_{i \in \Omega_j} w_{ij}}{W} - \frac{\sum_{i \in \Omega_j} b_{ij}}{B} \right| . \tag{10}$$

In all areal units inside region j, levels of segregation are unlikely to be the same as RD_j. Some units maybe higher than RD_j and some may be lower, but their aggregated level of segregation is conditioned by the population distribution at the regional level. Given the population distribution of the regions, relatively uneven distribution of population among local units within each region generates segregation additional to the regional level. Thus, the local segregation introduced by the local unit level for region j, but separated from the contribution at the regional level, can be defined as

$$LD_j = \sum_{i\in\Omega_j} \left| \frac{w_{ij}}{W} - \frac{b_{ij}}{B} \right| - \left| \frac{\sum_{i\in\Omega_j} w_{ij}}{W} - \frac{\sum_{i\in\Omega_j} b_{ij}}{B} \right| . \tag{11}$$

Or using equation (10),

$$LD_j = \sum_{i\in\Omega_j} \left| \frac{w_{ij}}{W} - \frac{b_{ij}}{B} \right| - RD_j \, . \tag{12}$$

Therefore, D_l (4), the segregation level for the entire study area based upon the more disaggregated or local level data, can be decomposed into the segregation level at the regional scale and the local segregation attributable to the local units under each region. Formally,

$$D_l = D_r + \sum_j LD_j \, . \tag{13}$$

If the differences in the distribution of the two groups among all local units in j are small or have a strong positive spatial autocorrelation, then LD_j will be rather small. LD_j in equations (11) and (12) can then be regarded as a local measure of segregation, but it is different from other local segregation measures in that it reflects the contribution of segregation only from the local units, or the segregation at the local level conditioned by the regional level (Wong 2002). Therefore, LD_j can also be labeled as a conditional segregation measure. It will be relatively low if all local units within the same region have similar differences in the distribution of the two groups (i.e., local units have strong positive spatial autocorrelation). When it is mapped, it can indicate the spatial variation of segregation contributed purely by the local units. On the other hand, RD_j is also a local measure, which is the level of segregation of each region contributed to the segregation of the entire study area measured at the regional scale. Using both local measures allows us to explore the spatial variations of segregation at different scales and from different sources.

2.2. The Diversity Index H

The entropy-based diversity index is widely adopted in diversity or segregation studies (e.g., White 1986). It is also widely used in ecological studies, especially in measuring species diversity, and is known as the Shannon-Wiener index (Krebs 1989). The index is purely a function of the ethnic or racial mix within the areal unit. The more evenly distributed in the population among population groups, the higher the index. After Plane and Rogerson (1994), the entropy-based index is defined as

$$H = -\sum_k p_k \log p_k \ \ and \ \ 0 \le p_k \le 1 \, , \tag{14}$$

where p_k is the proportion of population in group k in the areal unit. The maximum value of the index is log m, where m is the number of population groups. The index will reach its maximum when population is distributed evenly across all population groups. The minimum will approach zero.

The use of the diversity index is strongly supported by some sociologists. Besides its advantage in handling multiple group comparisons, it has several desirable mathematical properties (White 1986; Krebs 1989; Reardon and Firebaugh 2002). Nevertheless, the diversity index is a local aspatial index in the sense that a diversity value, which is inversely related to the level of segregation, can be computed for each areal unit, but the value is not affected by the population composition of the surrounding units (Wong 2002). In the following discussion of the decomposition framework, the index is applied to a two-group situation (white and black) to parallel the previous discussion on the two-group dissimilarity index. Using the same notation as in equation (1), the diversity of areal unit i is

$$H_i = -\left[\frac{w_i}{N_i}\log\left(\frac{w_i}{N_i}\right) + \frac{b_i}{N_i}\log\left(\frac{b_i}{N_i}\right)\right], \tag{15}$$

where N_i is the total population of the two groups in areal unit i

In a multi-level geographical system, a diversity index can be computed for the entire area based upon the individual local areal unit data or the aggregated regional data. Due to the same reason discussed in section 1, the individual local unit data retain more variation in the population mixes and thus will indicate a lower level of diversity. On the other hand, local differences are smoothed when data of smaller areal units are spatially aggregated to larger units, and the aggregated data can only report the more evenly distributed population mixes at the regional level. Therefore, the level of diversity is higher. Specifically, the diversity index for region j based upon the aggregated data is computed by

$$H_j = -\left[\frac{W_j}{N_j}\log\left(\frac{W_j}{N_j}\right) + \frac{B_j}{N_j}\log\left(\frac{B_j}{N_j}\right)\right], \tag{16}$$

where the notation follows that of equations (2) and (3), and N_j is the total population of the two groups in region j. When local areal unit data are used to compute the diversity for the region j, and areal unit i values are inside j, the regional diversity value can just be the average of the diversity values of all local areal units, H_i values. But averaging the local diversity values will not take into account the differences in population sizes among local areal units, as each local areal unit will be weighted equally. A more appropriate diversity value for the entire region is based upon population weighted local diversity values. That is

$$\overline{H}_j = \left(\sum_{i\in\Omega_j} N_i * H_i\right) / \left(\sum_{i\in\Omega_j} N_i\right), \tag{17}$$

where N_i is the total population of whites and blacks in local areal unit i, and

$$H_j \geq \overline{H}_j. \tag{18}$$

Expression (18) will become an equality, which rarely happens when all H_i values in j have the same value. Please note that lower values in the diversity index H means higher level of segregation. Population mix dictates the value of the diversity index. It is likely that the population mixes in local areal units are different from their regional aggregates. Some of local areal units may even have diversity values higher than their corresponding regional values if they have more balanced population mix than their regional population mix. However, given that equation (17) is weighted by the population of each local unit, and population counts of local units are constrained by the regional population counts, therefore, the diversity value computed from the region aggregates (16) will be larger or equal to the diversity value computed from the weighted average of local areal unit diversity values (17). Therefore, expression (18) depicts a situation consistent with the previous discussion that more detailed data (RHS of the expression) gives a higher or equal level of segregation than that based upon the more aggregated data (LHS of the expression).

Because the diversity value of region j is based upon the aggregated population counts of all i values within j, the variations in population mix exhibited in local areal units are removed in the aggregated values. Therefore, the increased diversity value of j as compared to that obtained from local areal units can be regarded as the increased diversity created by the spatial aggregation process. Formally, the increased diversity due to spatial aggregation for region j is defined as

$$\Delta H_j = H_j - \overline{H}_j . \tag{19}$$

Then the higher level of diversity of a region based upon the aggregated regional level data can be decomposed into the weighted average level of diversity for all local units within the region and the additional level of diversity attributable to the region when the between-units differences are removed. Formally,

$$H_j = \overline{H}_j + \Delta H_j . \tag{20}$$

The first term on the RHS can be regarded as the local diversity (LH_j), and the second term can be treated as the additional regional diversity (RH_j) incurred onto the local units through the spatial aggregation or smoothing process.

3. A SIMULATED LANDSCAPE AND EMPIRICAL EXAMPLES

3.1. A Simulated Landscape

To demonstrate the usefulness of the spatial decomposition framework, a hypothetical landscape with two population groups (white and black) at two nested hierarchical levels is used first. It is followed by two empirical studies. Figures 1a and 1b show the 4-unit and 16-unit configurations of the same hypothetical area. While the population sizes of different units are not uniform, the two figures indicate the proportions of white in all areal units. In the 4-unit configuration, the two regions on the right are 0.75 white, one region on the left is completely white, and the other region is 0.6 white. Within each of these regions, all local units have the same proportions of white as their corresponding regions (or aggregated units), except the region on the lower left. Within this region, two local units are all white and two units are 0.429 white. The D for the 4-unit configuration is 0.25 (D_r), and the D for the 16-unit configuration is 0.375 (D_l), a difference of 0.125. The question is how to account for the difference of 0.125 between the two levels.

First, we compute the regional segregation level (RD_j) for each region based upon the regional data, and the results are shown in Figure 1c. The two regions on the right have proportions of white (0.75) equal to the proportion of white for the entire study area. Therefore, their RD_j values are zero. But for the two regions on the left, both of them deviate from the global proportion of 0.75, with one above and one below. Each

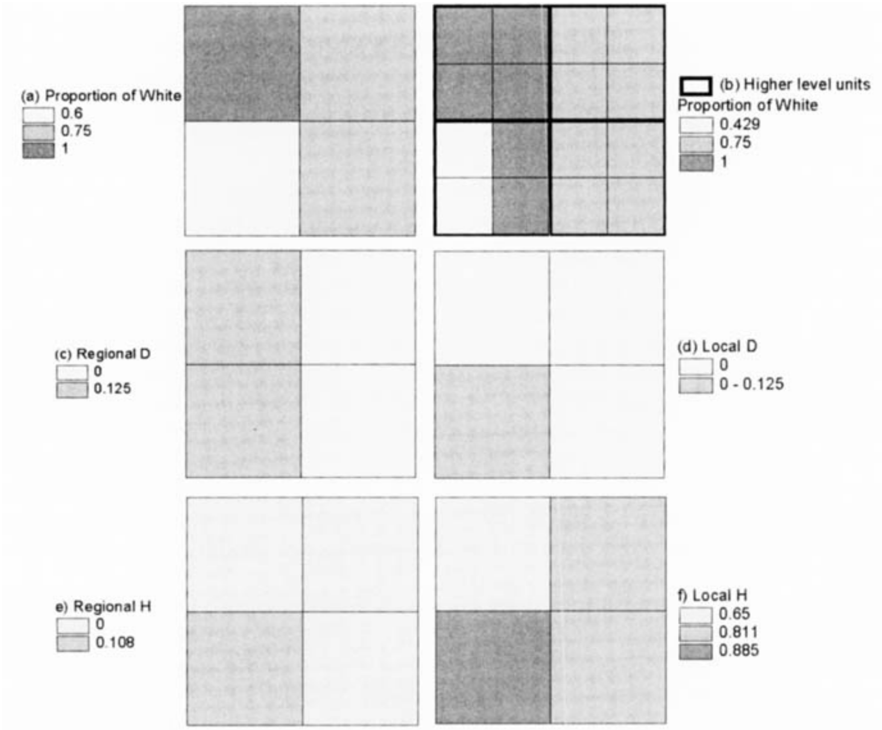

FIG. 1. The 4-unit and 16-unit Hypothetical Configurations, and the Decomposed Segregation Values

of them has a regional segregation level of 0.125. At the 16-unit local level, we expect that the lower left region should have a higher level of segregation at the local level than the other three regions, and other regions should not have contributed to the overall segregation at the regional level because all their local units have the same proportions of white as their corresponding regions. These speculations are confirmed by Figure 1d, which shows that the local segregation (LD_j) is the highest in the lower left region. That is, the local units of that region contribute the extra 0.125 of segregation to the local level computation. Also note that even though the local units in the upper left region have higher proportions of white than the entire study area proportion, these local units do not contribute extra amounts of local segregation to the overall level because the local units are constrained by the proportion of white at the regional level.

Figures 1e and 1f show the results of the decomposed diversity indices. Note that in this two-group case, diversity will be the highest when the two groups have equal shares of the population. Therefore, if the most disaggregated data (local areal unit data) are used based upon Figure 1b, the upper left region should have the lowest diversity level. While it may not be apparent, the lower left region probably should have a relatively high level of diversity because of the highly varied local proportions of white. The local H (LH_j) in Figure 1f confirms these expectations. But when the population counts data were aggregated to the regional level, the aggregation process "removes" or smoothes the differences of population mixes among local areal units and introduces additional diversity to the regional level analysis. Because of the great internal or within-region variations in the lower left region, we expect that the aggregation process will add the most diversity to that region. But for other regions, each of them does not have any internal variation. Therefore, the aggregation process does not increase their diversity levels. These results are shown in Figure 1e, which depicts the increased diversity introduced at the regional level (RH_j).

3.2. *Two Empirical Examples*

In this paper, Washington, D.C., and Birmingham, Alabama, are chosen as the empirical examples to illustrate how segregation measures can be decomposed into different geographical levels, and segregation values at different levels can be linked together. These two areas are chosen partly because of the results reported in a prior study on thirty metropolitan areas (Wong 2003). Among the thirty metro areas, the two areas chosen for this paper have a moderate degree of segregation based upon D at the tract level (D ranges from 0.4923 to 0.8845). When block group level data were used, the rank of Washington, D.C., according to the D value, shifted slightly to become less segregated, but Birmingham shifted to the opposite direction to become more segregated. Another reason that these two regions were chosen is that they have relatively high proportions of black, and the significant presence of minority population can demonstrate the decomposition framework more effectively. Census tract and block group levels data for the 1990 census are used in these empirical examples.

Figure 2 shows some of the results. Figure 2a is the proportion of white in Washington, D.C. The northwest quadrant of the city has the highest concentration of whites, while the southeast and northeast have a black majority. Using tract and block group level data, the dissimilarity index D values are 0.7669 and 0.7886, respectively. Adopting the framework described above, and regarding tracts as regions, the regional D values, or RD_j values in equation (10), which are the D values for census tracts, are shown in Figure 2b. Given the citywide racial mix, several tracts in northwest have the highest regional D values, indicating that they deviate from the citywide racial mix the most due to the high concentration levels of whites. On the other hand, moderate levels of regional D are found in certain parts of northeast and southeast where blacks are disproportionally represented. Using equations (11) or (12), the local D values (LD_j values) are computed for all census tracts using block group level data, and the results are shown in Figure 2c.

Most census tracts have low levels of local D values, indicating that the block group level, especially those in the northwest section of the city, does not add much segregation to the regional segregation pattern. As Figure 2a indicates that whites are highly clustered in northwest, while the rest of the city has a black majority, the levels of spatial autocorrelation, both at the regional and local scales, are quite high. Given the high level of spatial autocorrelation at the block group level, the spatial aggregation process does not remove much local variation and thus enables the same spatial pattern that existed at the block group level to appear at the tract level. Therefore, the Local D (LD_j) values are relatively low throughout the city. The most outstanding local pattern is the census tract found slightly northeast of the city center. This tract has the highest local D value because it consists of four very different block groups in terms of their racial compositions, and the populations of the two groups are reported in Table 1. It is clear that the segregation level is relatively high among these four block groups. The first one (# 0095011), though not completely, is very much dominated by blacks, and the last one on the list (# 0095014) is completely dominated by blacks. For the other two block groups, one has a black majority, but the remaining one is just the opposite—white dominant. Because of the high degree of variation or negative spatial autocorrelation in the racial mixes among these four block groups, the local segregation level turns out to be relatively high. The total of all local D values, which are derived from block group data, is 0.02155. Adding this total local

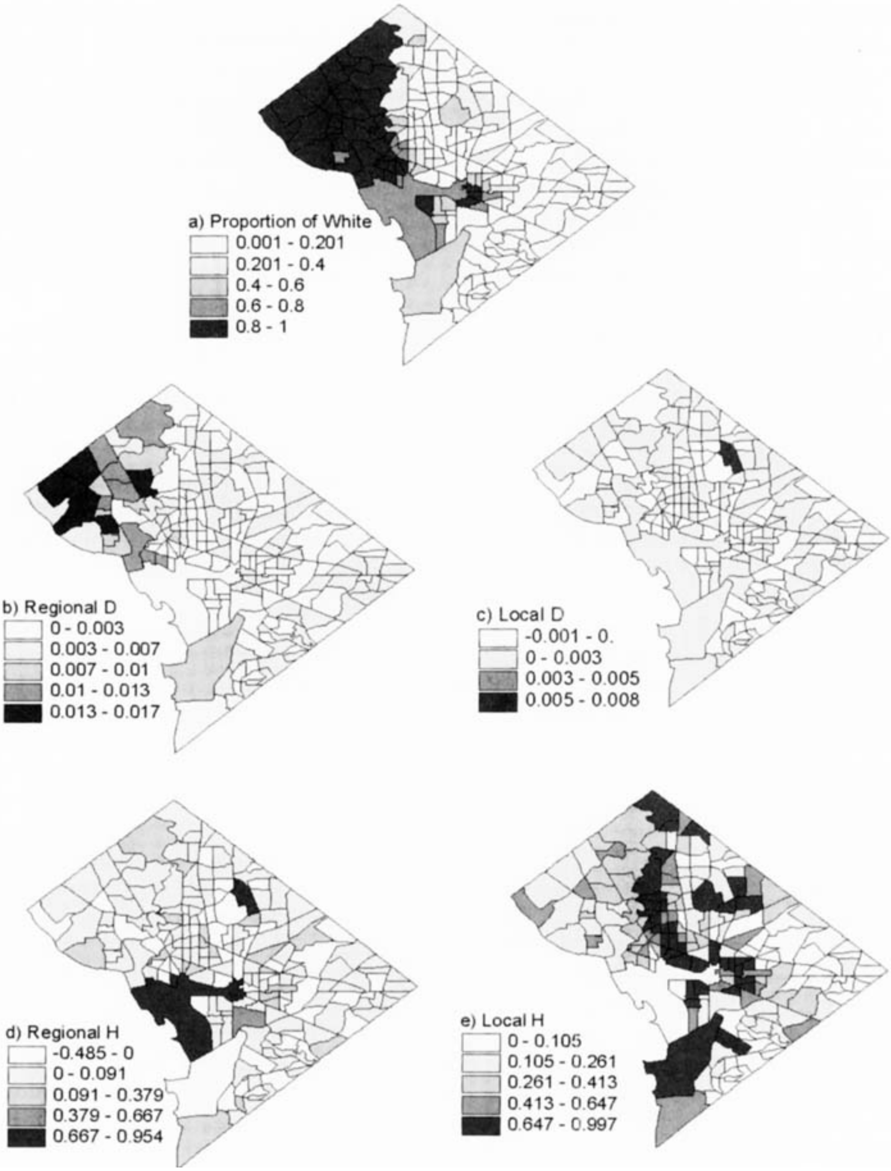

FIG. 2. The Proportion of White and All Decomposed Segregation Measures for Washington, D.C.

segregation to the tract level segregation (0.7669) gives a total segregation of 0.78845 in equation (13). This is not exactly the level of segregation computed at the block group level (0.7886). Besides rounding errors, another reason that they do not match exactly is because the census data between census levels are not perfectly consistent. In many tracts, the population counts do not equal to the sum of the population counts at the block group level. Some of these inconsistencies are also due to the censoring of population data at the block group levels when the counts are relatively low. As a result, some regional D values (RD_j) are slightly larger than the first term in equations (11) and (12), and thus some of the local D values are slightly negative. Note that those census tracts with local D values equal to zeros were also grouped into the first class in Figure 2c.

The diversity value for Washington, D.C., based upon the citywide white and black population counts is 0.6697. A diversity value was also computed for each census tract and block group using equation (15). The weighted diversity values for the entire city based upon equation (17) were computed at the tract level and block group level. Their values are 0.1124 and 0.1024, respectively. Figure 2e shows the local H values (LH_j values) for census tracts. These local H values were derived from the weighted averages of H values computed for all block groups within each census tract. The tracts with the highest levels of local diversity, as expected, are found in the middle section of the city where the proportions of white are similar to the proportions of black. The areas with lower local diversity are on the east and west where areal units are highly skewed toward one group or the other in their racial compositions.

In terms of the increased diversity at the regional level introduced by the aggregation process (i.e., the regional H in Figure 2d), tracts with the greatest increases are limited to the south and northeast of the city center, while the lowest increases are scattered throughout the entire city. The same data inconsistency problem across census levels found in the decomposition of *D* discussed earlier partly generates the extreme regional H (RH_j) values in some of the tracts. Some of the tracts with relatively high regional H values, or large increases in diversity value at the regional level include block groups with zero population counts in either group or both. When these block group data are aggregated with other non-zero count block groups within the same tract, the diversity at the tract level becomes much higher than the weighted average based upon the block group data, and thus a relatively large RH_j. This situation is caused by either having block groups without population in the same tract, or due to the censoring of population data at the block group level when the population counts are very small. In Washington, D.C., the high $\mathrm{RH_j}$ value in the tract south of the city center falls into this situation but, the tract slightly northeast from the city center, which is the same tract with the highest local D value as identified in Figure 1c, has the highest $\mathrm{RH_j}$ value, reflecting the real increase in diversity introduced at the census tract level. Because of the large variations in racial mixes among the block groups within this tract (Table 1), aggregation at the tract level increases the level of diversity tremendously. On the other extreme, because of the same data issue as discussed in the decomposition of *D*, the $\mathrm{RH_j}$ values for certain

TABLE 1.
Population Counts for the Four Block Groups with Highest Local D values in Washington, D.C.

Block group #	White	Black
0095011	38	1937
0095012	1944	45
0095013	192	777
0095014	0	809

tracts are negatives, indicating the inconsistencies of population count data between the block group and the tract levels provided by the census.

The difference in the D values, 0.02155, between the census tract and block group levels in Washington, D.C., is relatively small given that the theoretically range of *D* is between 0 and 1. But this small difference in *D* between the two levels implies that the segregation at the block group level is very much captured by the segregation computed at the census tract level. But in Birmingham, the D value at the census tract level is 0.7204, and the D value at the block group level is 0.7789, a difference of 0.0585. As mentioned earlier, these small differences were significant enough to change the rankings of these cities in opposite directions when they were compared with other metro areas (Wong 2003). Figure 3 shows some of the results from the Birmingham example. Figure 3a shows the proportion of whites at the census tract level. In Birmingham, most whites reside around the city center and in the suburbs while blacks are highly confined to the center of the city. Even with this large spatial disparity pattern of the two population groups in the city, its level of segregation according to *D* was just slightly higher than that in Washington, D.C.

Figure 3b shows the regional D value computed using census tract data. High values indicate that those tracts deviate greatly from the citywide racial mix. In the map, many smaller (in area) census tracts in or near the city center have high regional D values, which are likely the result of high concentrations of blacks in those tracts. Detailed examinations of some of those tracts confirmed that blacks were the majority there. Several larger tracts in the north and south, and a few on the east of the city center have moderate levels of regional D values. These tracts, opposite to those in the center, have the white majority or are almost exclusively whites.

In terms of local D values, Figure 3c does not show any specific spatial pattern. Moderate levels of local D values are found in several tracts in the outskirt of the city and in a few tracts south and west of the city center. The areas with the highest local D values scatter around, with one tract in the north, one in the west of the city center, and another one in the northwestern portion of the city. The tract with the high local D value north of the city center is examined in more detailed. Table 2 shows the population counts of all seven block groups in that tract. One of the block groups has no population. For the remaining six, three are exclusively white and one is exclusively black. The remaining two block groups are dominated by whites. Therefore, the segregation level in this tract between the two groups is relatively high based upon the block group data. High levels of within-tract variation are also found in other tracts with high levels of local D values. Similar to the situation in Washington, D.C., several tracts have slightly negative local D values due to the inconsistency of the population count data between the tract and block group levels.

The decomposition procedure for the H index was also performed for the Birmingham area. At the citywide level, the H value was 0.2724, much lower than that in Washington, D.C., because of the relatively large black population size in Birming-

TABLE 2
Population Counts for the Seven Block Groups with Highest Local D values in Birmingham, Alabama.

Block Group #	White	Black
0120021	799	0
0120022	1034	73
0120023	334	0
0120024	300	0
0120025	0	1203
0120028	0	0
0120029	1738	46

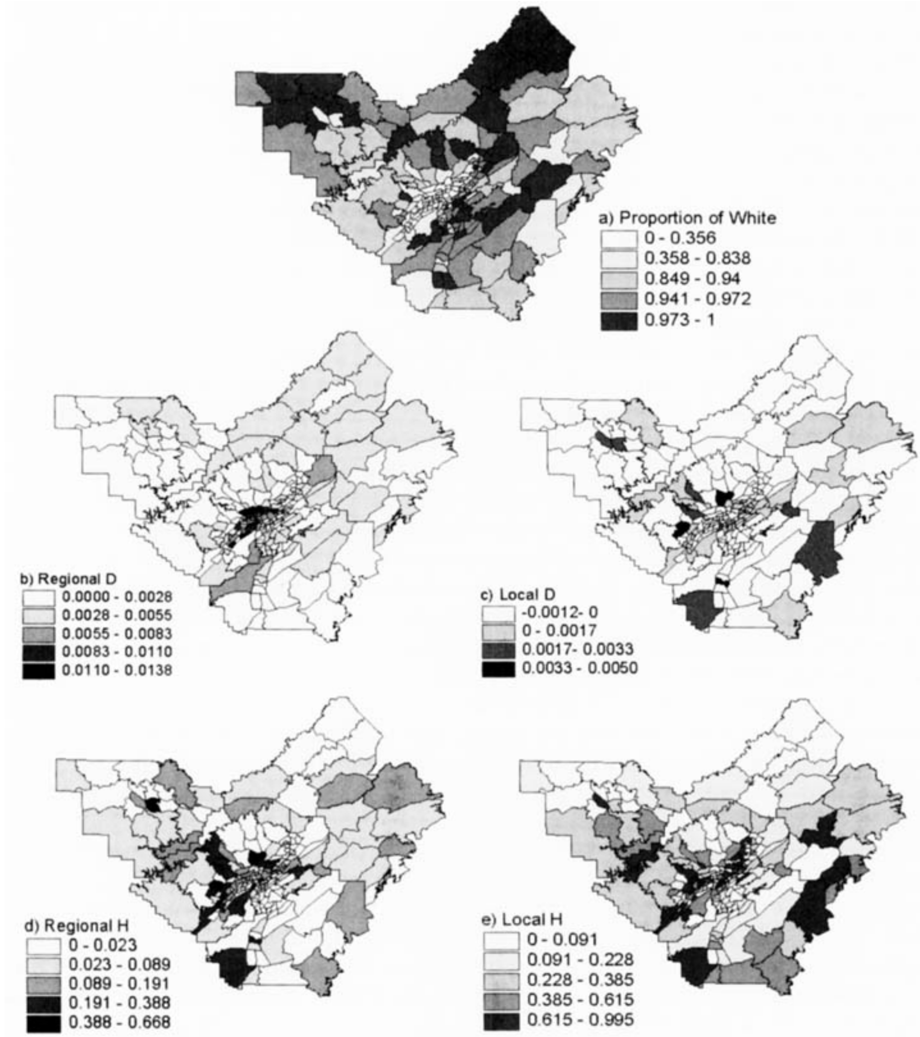

FIG. 3. The Proportion of White and All Decomposed Segregation Measures for Birmingham, Alabama.

ham. The weighted diversity at the tract level was 0.1181, and the weighted diversity at the block group level was 0.0938. The second category in Figure 3a (0.358–0.838) includes the census tracts in which the two population groups have similar shares. These tracts are scattered around the periphery of the city and a few tracts near the city center. Because these tracts have relatively similar proportions of the two groups, they likely have high local diversity levels (Figure 3e). Tracts with the highest regional H values or the greatest increase in diversity at the tract level through aggregation should be those tracts that include block groups dominated by different groups. These tracts are likely found in areas where the neighboring block groups are dominated by different population groups and belong to the same tract. Given the population distribution pattern in Birmingham, these tracts are mostly found around the inner city centers, and a few suburban tracts with relatively high proportions for both groups and high degrees of heterogeneity among the block groups. As a result, these tracts have relatively high levels of RH_j (Figure 3d). Please note that all RH_j values in Birmingham are positive, while some RH_j values in Washington, D.C., are negative. This difference between the two areas indicates that the inconsistency of population counts between the two census levels is more of a problem in Washington, D.C., than that in Birmingham, Alabama.

4. CONCLUSIONS

This paper proposes a framework to identify the segregation levels attributable to different geographical levels. Different levels of segregation measured at multiple geographical levels can then be linked together such that a consistent understanding of the segregation values based upon results from different geographical levels become possible. In a two-level setting and using the dissimilarity index D, this paper suggests a method to decompose segregation values into the regional level and the local level. Because the distribution of population among the regions or larger areal units imposes constraints on the distribution of population at the local level, therefore, the segregation measured at the local scale refers to the contribution from the local units only, conditioned upon the distribution of population at the higher geographical levels or among the larger units. In decomposing the entropy-based diversity measure H, the suggested method is to decompose diversity values based upon the more aggregated data (higher level of diversity) into the weighted diversity values computed from smaller areal units data (lower level of diversity) and the additional diversity contributed by merging smaller areal units into more diverse regions. Even though these two methods are specific to the two indices and they approach the decomposition process from opposite directions (in terms of geographical levels), the general framework is applicable to other segregation measures. This paper addresses only the two-level situation, and situations with more than two levels can be accommodated easily with slight modifications.

The resulting regional and local measures can be mapped to show the spatial patterns of segregation or diversity. The map from the regional D measure shows how each regional areal unit deviates from the area-wide racial mix. The map from the local D measure shows the within-area or within-tract racial mix variation. A high value implies large variation of racial mix or high level of segregation within the areal unit, and thus segregation among the sub-units contributes to the overall segregation level of the entire study area. In terms of the diversity index, the map of local H indicates the weighted average diversity within regions, while the map of regional H shows the increased diversity at the regional level due to the aggregation process. Therefore, these local measures together with their maps can serve as tools for exploratory spatial data analysis (ESDA) (Fotheringham and Rogerson 1993). Specifically, they are useful in exploring the spatial variations of ethnic mix at multiple geographical levels. Results can help identify smaller regions for more detailed analyses on their ethnic mixes and their geographical variations.

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