A Markov Model of Referral Based Hiring and Workplace Segregation

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Abstract

I construct a Markov model of referral hiring to look at two issues: inequality and workplace segregation. The model differs from most models of referral hiring in that it explicitly considers a firm structure where employment opportunities arise. The model suggests that referral hiring does not directly produce inequality between groups at the population level. The result highlights a difference between outcomes of referral hiring at the population and individual levels. However, referral hiring does produce segregation of groups across firms in a given industry and the degree of segregation monotonically increases in the amount of referral hiring.

Keywords: Referral Hiring, Segregation, Inequality, Social Networks.

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1 Introduction

Approximately 50% of job placement in the US occurs through social contacts (Granovetter 1995). Recent research in sociology and economics suggests that the importance of social contacts in the labor market may be one cause of observed levels of persistent inequality for some minority groups.[1] Most of the work focuses on the effect of social structure on individual outcomes (such as income or employment rates) or group level inequality. Another potential effect of referral hiring has received much less attention: workplace segregation.[2] Since large amounts of job information travel through social networks, friends with overlap in social networks will tend to receive job offers from the same sources and thereby work in the same firms. Thus referral hiring may produce firm level segregation by race or gender if such segregation exists in social networks.

Most previous models of referral hiring follow the tradition of the labor search literature and assume random origination and destruction of jobs from an anonymous firm. Thus in order to consider issues of segregation one needs to extend the analysis to include explicit firms and firm structure. The introduction of explicit firms complicates the modeling process. In addition to accounting for the employment states of individuals, one must also account for specific jobs within firms.

In order to model these complex dynamics I construct a Markov model to analyze referral hiring. In a Markov process the transition probabilities are endogenously determined by the current employment states of workers and firms. This allows me to let hiring decisions depend in a clear and precise way on both the make-up of

[1] See the appendix to Granovetter (1995) for a comprehensive literature review through 1995. Some of the better known papers include Bridges and Villemez (1986), Campbell and Marsden (1990), Granovetter (1973), Holzer (1988), Montgomery (1991), Montgomery (1992), and Mortensen and Vishwanath (1994). More recent work includes: Arrow and Borzekowski (2000), Calvo-Armengol (2001), Calvo-Armengol and Jackson (2002) and Mouw (1999).

[2] One exception is Mouw (1999)

workers at specific firms and the employment success of individual groups. To my knowledge Montgomery (1994) is the only other researcher to use a Markov process in a model of referral hiring. In his model he concentrates on the macro-level effects of weak-tie versus strong-tie interactions on employment rates and inequality. He does not explicitly address issues of segregation. Other related models include the vacancy chain models of ?)white70) and ?). In these models the authors study the likelihood of a worker moving between different job classes (or strata) using a Markov model.

In my model two exogenously defined groups exist in the population. As an example the groups may be thought of as women and men, black and white, or two distinct ethnic groups such as Italian and Irish. Firms hire workers from an endogenously created unemployed population. When hiring occurs firms may either use traditional search methods or referral hiring. If hiring is done by referral the type of worker hired depends on the makeup of workers employed at the hiring firm. When not hiring by traditional means no such bias exists. The likelihood of non-referral hiring is given as a parameter of the model.

My results suggest that the influence of social contacts in hiring decisions will not produce inequality between population groups directly. In steady state there is a unique level of unemployment that is equal across population groups as long as some arbitrarily small amount of traditional hiring occurs. However, I find that referral hiring creates workplace segregation within industries. As more referral hiring occurs, firms develop biases toward hiring workers of a type the firm already employs. But the measure of the bias for a given type across all firms is proportional to the measure of agents of that type in the population. Thus both groups have equal opportunities in the labor market.

In section 2 I show analytical results for a number of special cases and the general model. Next I present a series of computer experiments showing how workplace segregation changes with the amount of referral hiring in the economy in section 3. As firms move toward hiring exclusively from referrals the distribution of workers across firms approaches complete segregation. But for relatively small decreases in the propensity to hire by referral, segregation quickly decreases. The result may be considered in the spirit of Schelling (1978) who showed that slightly biased preferences for having neighbors of one's own type can produce large amounts of residential segregation. Here, I get an opposite result, even small limits on referrals in the hiring process create large amounts of workplace integration.

Section 4 addresses two extension to the structure of firms: horizontal and vertical organization of industries. My results suggest that referral hiring will cause segregation across firms within industries but will not create segregation across industries.

My model suggests that referral hiring does not produce inequality directly. However I acknowledge that referral hiring mixed with other social interactions or education choices may produce inequality indirectly due to the creation of workplace segregation. Segregation may allow for statistical discrimination or diversity of social norms which may be a disadvantage for minority groups.[3] The investigation of these interactions is important for understanding the effects of public policy related to equal opportunity. I more fully discuss these issues in section 5.

2 The Model

Consider a finite population of N individuals where there are two groups. Label the groups A and B. Let the group to which an individual belongs be exogenously determined. Let γ define the likelihood an individual belongs to group A and the

[3] Mailath, Samuelson, and Shaked (2000) provide an excellent example of the influence of referral hiring on education choices . In their model they find an example of an equilibrium where search intensity of firms aimed at a particular group (a proxy for network connections) may result in low education choices for the group who is not the target of firm search efforts.

complement, $1-\gamma$, belong to group B. Without loss of generality let group A be a minority group, $\gamma < 1/2$. Let E be the number of employed workers in the population and U be the number of unemployed members of the population. Assume all individuals are either employed or unemployed, $N = E + U$. There are no firm specific productivities; all workers are equally productive at each job and each firm. There are a finite number of firms, M, each with a finite number of jobs, Θ. Let A_E and A_U be the number of employed and unemployed members of group A. And similarly define B_E and B_U. So, $A_U + B_U = U$ and $A_E + B_E = E$.

The dynamics of the model work as follows: Each period every firm fires one worker at the firm. The fired workers enter the unemployed pool in the next period. After the workers are fired each firm instantaneously hires a new worker from the unemployed population to replace the one just fired. Since all firms and workers are equivalent in the model there is no reason for a worker to leave a firm for another firm or for a firm to desire the worker of another firm. [4] The type of the new worker hired is determined as a function of the composition of workers at the hiring firm, a propensity to hire by referral, and the distribution of types in the unemployed population. Let q be the percentage the unemployed pool who belong to group A at the beginning of the period (before the workers are fired), $q = A_U/U$. Let p be the number of group A workers at employed at the hiring firm after the firing occurs divided by $\Theta - 1$. If a firm hires by referral, the type of worker hired (A or B) is determined by the fraction of type A and B workers employed at the hiring firm after the firings occur. A firm hiring by referral chooses a member of Group A with probability p and a member of the majority group with probability $1-p$, provided $A_U > 0$. If a firm does not hire by referral it uses traditional search and thus chooses

[4]Note that this assumption is relaxed in the extensions considered later in the paper. In the extension firms are arranged in a hierarchy. Workers prefer to work for firms higher in the hierarchy. Thus firms can hire workers from other firms below them in the hierarchy.

a worker randomly from the unemployed population. Thus a firm that does not hire by referral hires a member of group A with probability q and a member of group B with probability $1 - q$. Define s as the likelihood a firm does not hire by referral. Given s, q, and p one can see that the newly hired worker will be a member of the minority group with probability $sq + (1 - s)p$, $s \in (0, 1]$. Note that s is a parameter of the model but that q and p are endogenously determined.

The parameter s may be considered the intensity with which firms (workers) search for workers (jobs) where no current referral connection exists. As s decreases more referral hiring occurs in the model. Note this is a model of random social networks where there is segregation of types across the network. Thus one representation of the model is a random directed network with two unconnected components, a and b. Component a consists exclusively of members of Group A and component b consists exclusively of members of Group B. Such stark segregation may seem a strict assumption. But, in a study using MCSUI data Mouw found that white individuals who used a contact to find a job used a white contact 88% of the time; black individuals using contacts used a black contact 86% of the time (Mouw 1999). Thus assuming such stark segregation in terms of referrals approximates reality.

Note that the networks of agents in the model are exogenous. Others (such as **?**) and **?**)) have considered endogenously created referral networks where agents pay a cost in order to maintain specific connections within a network or to originate new connections. While this may be a more realistic construction of networks in some cases here I chose to model the networks more simply for three reasons: First, and most importantly, the addition of a firm structure to the model of referral hiring complicates the analysis. Thus future work could consider endogenously created agent networks within the existing model of firm structure. Because the primary innovation of the model in this paper is the firm structure I concentrate on isolating its effect here. Second, in the model I am concentrating on the effect of referral hiring on exogenously defined groups such as racial, ethnic groups and gender. Thus the groups most concerned in this paper are exogenously defined. Third, individuals choose friends for a variety of reasons (similar interests in activities, personal attraction, geography, job acquisition). And it is not clear that job acquisition is the most important feature for selecting friends. Thus an accurate model of endogenous network creation should consider a model of multi-dimensional choice for network creation not just a choice along the dimension of job acquisition. This point is especially prominent when one considers that many job referrals come through weak as opposed to strong ties (Granovetter 1973). For these reasons one would expect endogenously created networks to generate additional insights and results. But for the present I restrict attention to the more simple consideration of exogenous networks.

Now consider the distribution of workers across firms. Define the state of each firm as the number of minority employees at the firm. A firm can have from 0 to Θ group A workers employed at the firm. Let the fraction of firms in the state with 0 group A workers be given by d_0, the fraction with 1 by d_1,..., the fraction with Θ by d_θ. This defines a distribution $D = [d_0, d_1, ..., d_\theta]$ with $\Theta + 1$ discrete states. Define T as the transition matrix between firm states. T is endogenously determined as a function of D, q, and s. I will fully derive it below.

Note that for $s > 0$ and sufficiently large γN it is possible for any firm to transition to any of the $\Theta + 1$ states in a finite number of periods. Thus this model defines an ergodic Markov process. As such the model may be solved by finding the asymptotic steady states which define the long run distribution of the model. A steady state distribution of workers will be such that $DT = D$.

2.1 An Example

As a means of exposition first consider a specific example with firms of size two. Many of the properties of this special example carry forward to the general case where they are more cumbersome to attain. The general case with firms of arbitrary size Θ will be considered in the next sub-section.

In this example firms may be in one of three states: firms who currently employ two group B workers, those that employ one group A worker and one group B worker, and those that employ two group A workers. As defined above let the fraction of the firm population with two group B workers be d_0, the fraction with one worker from each group be d_1, and the fraction with two group A workers be d_2. (Note that $d_0 + d_1 + d_2 = 1$.)

For this example the transition matrix T is given by:

$$\begin{vmatrix} (1-sq) & sq & 0 \\ \frac{(1-sq)}{2} & \frac{s}{2} & \frac{1-s(1-q)}{2} \\ 0 & s(1-q) & [1-s(1-q)] \end{vmatrix}$$

The first row represents transition probabilities for firms in state 0; the second row firms in state 1; and the third row firms in state 2. Consider the second row, firms in state 1. They are firms that employ one group A and one group B worker. The first element of the row is the likelihood that a firm in state 1 transitions to state 0; a firm in state 1 fires the group A worker and hires a group B worker. Since there are two workers at a firm in this example the group A worker is fired with probability 1/2 given the worker is from a firm in state 1 (recall a firm in state 1 employs one worker from each group.) Given the firing of a Group A worker, a member of Group

B must be hired to move to state 0. Since the only employed worker at this firm after the firing is from Group B the only way a Group A worker can be hired is through traditional search. Thus a Group A worker is hired in this case with probability sq and a Group B worker is hired with probability $1 - sq$. So, the joint probability of a Group A worker currently employed at a firm in state 1 being fired and a Group B worker being hired to replace her occurs with probability $\frac{(1-sq)}{2}$. This is the first element of row 2. The third element of row 2 and the other non-zero off-diagonal elements of the other rows can be derived in a similar manner.

The second element of row two is the probability that firms in state 1 remain in state 1. This can happen in two ways: First, it could be that a Group A worker employed at a firm in state 1 is fired and another Group A worker is hired to replace her. Thus no change in the state of the firm occurs. This happens with probability $\frac{sq}{2}$. Or second, a Group B worker could be fired from a firm in state 1 and another Group B worker hired to replace him. Again, no change in the state of the firm occurs. This happens with probability $\frac{s(1-q)}{2}$. Adding these three possibilities together yields: $\frac{sq}{2} + \frac{s(1-q)}{2}$ which reduces to $\frac{s}{2}$. The other diagonal elements of T can be calculated similarly. Lastly note that the upper right and lower left elements are 0. This is because firms only fire one worker at a time. Thus a firm can only make a transition to one state above or one state below its current state. Moves from state 0 to state 2 or from state 2 to state 0 are not possible in one time period.

Once T is known, a fixed point is given by a fixed point of the distribution D such that $DT = D$. This is given as the solution to the following system of equations:

$$d_0(1 - sq) + \frac{d_1}{2}(1 - sq) = d_0 \tag{1}$$

$$d_0sq + \frac{d_1s}{2}) + d_2(1-q) = d_1 \tag{2}$$

$$[\frac{d_1}{2}[1 - s(1-q)] + d_2[1 - s(1-q)] = d_2 \tag{3}$$

Note that each diagonal element of T represents the case where either the fired worker is from a firm in a different state or a worker of group i was fired and a worker of the same group was hired. Thus there is no change in the measure of firms in that state. Elements to the right of the diagonal represent instances where a firm increases the number of Group A workers it employs by one. Thus a member of group A is hired following the firing of a group B member. Elements to the left of the diagonal are instances where a member of group A is fired and a member of group B is hired. Thus, the number of Group A employed decreases by one.

With this in mind consider steady state unemployment. For a steady state to occur it must be the case that the joint probability of a Group A worker being fired, F_A, and a Group B worker being hired, H_B, given a Group A worker was fired, $Pr(F_A)Pr(H_B|F_A)$, is equal to the probability that a Group B worker is fired, F_B, and a Group A is hired, H_A, given the group B was fired, $Pr(F_B)Pr(H_A|F_B)$. These are given by the off diagonal elements of the transition matrix conditioned by the fraction of firms in the respective states:

$$Pr(F_B)Pr(H_A|F_B) = d_0sq + \frac{d_1}{2}[1 - s(1-q)] \tag{4}$$

and

$$Pr(F_A)Pr(H_B|F_A) = \frac{d_1}{2}(1 - sq) + d_2 s(1 - q) \tag{5}$$

As mentioned above, in steady state these two joint probabilities must be equal:

$$d_0 sq + \frac{d_1}{2}[1 - s(1 - q)] = \frac{d_1}{2}(1 - sq) + d_2 s(1 - q) \tag{6}$$

Solving for q yields:

$$d_2 + \frac{d_1}{2} = q \tag{7}$$

By definition γN equals the Group A population and $d_2 + \frac{d_1}{2}$ is the fraction of the employed population that belong to Group A. Knowing q we can write the following equation that specifies the fraction of the Group A population belonging to each state of employment:

$$\gamma N = (d_2 + \frac{d_1}{2})E + qU \tag{8}$$

Substituting from Equation 7 yields:

$$N = (d_2 + \frac{d_1}{2})E + (d_2 + \frac{d_1}{2})U \tag{9}$$

Therefore, $\gamma = d_2 + \frac{d_1}{2} = q$ and $1 - \gamma = d_0 + \frac{d_1}{2} = 1 - q$.

The fraction of unemployed workers who belong to Group A is equal to the fraction of Group A workers in the population. As will be shown below this result carries forward to the general case as well. The result shows that even in the presence of referral hiring unemployment rates are equal across the two population groups for any $s > 0$. The result holds even though the probability of a given type being hired directly depends on the group make-up of the hiring firm. As will be seen below the result occurs partly because of the segregation of workers across firms. As Group A becomes predominant at a set of firms that creates more opportunities for Group B at the firms not predominantly occupied by Group A members.

Continuing with the example, rewrite equation 1 as:

$$d_0 = \frac{d_1(1 - sq)}{2sq} \tag{10}$$

and equation 3 as:

$$d_2 = \frac{d_1[1 - s(1 - q)]}{2s(1 - q)} \tag{11}$$

Similarly re-write equation 2 as:

$$d_1 = \frac{d_0 sq + d_2 s(1 - q)}{1 - \frac{s}{2}} \tag{12}$$

Now substitute equation 12 into equation 10 to yield:

$$2(1-\frac{s}{2})d_0sq = d_0sq(1-sq) + d_2s(1-q)(1-sq) \tag{13}$$

Simplifying and substituting γ for q yields an equation stating the relationship between d_0, d_2, γ and s.

$$d_0\gamma(1-s+s\gamma) = d_2(1-\gamma)(1-s\gamma) \tag{14}$$

which implies:

$$\frac{d_0}{d_2} = \frac{(1-\gamma)(1-s\gamma)}{\gamma(1-s+s\gamma)} \tag{15}$$

I now consider how the level of non-referral hiring, s, affects the distribution of workers across firms.

2.1.1 Special Case 1

Consider the special case of only traditionally hiring where $s = 1$. Equation 15 yields: $d_0\gamma^2 = d_2(1-\gamma)^2$. Therefore: $\frac{d_0}{d_2} = \frac{(1-\gamma)^2}{\gamma^2}$. The ratio of the Group B and Group A populations equals the ratio of the all Group B and all Group A firms. In addition there is a positive measure of mixed firms, $d_1 > 0$ from equation 12. Note that since $s = 1$ all hiring occurs from random draws from the unemployed. This is the distribution that occurs when no social network effects are present. It will be a binomial distribution with parameter γ.

2.1.2 Special Case 2

In contrast to special case 1, consider the case of the limit as $s \to 0$. As $s \to 0$ equation 2 approaches 0. Thus $d_1 = 0$.

If $d_1 = 0$, this implies that $d_2 = 1 - d_0$. Again note that $\gamma = d_2 + \frac{d_1}{2}$ which implies that $d_2 = \gamma$ and $d_0 = 1 - \gamma$ since $d_1 = 0$. The distribution as $s \to 0$ consists of a fraction γ of the firms with all Group A workers and a fraction $1 - \gamma$ having all Group B workers, with no firms having a mix. In other words, as $s \to 0$ the worker groups become completely segregated across firms.

As can be seen in the above two special cases referral based hiring shifts the distribution of workers toward greater levels of segregation as s decreases. This gives the intuition for why the equality result above holds. The distribution yields a fraction of firms where Group B (the majority group) is advantaged but at the same time another group of firms is created where Group A (the minority group) is advantaged. In equilibrium the aggregate size of the advantage of each group must be proportional to the population sizes of the groups.

As a final special case consider the example where the population sizes are equal, $\gamma = 1/2$, there is no minority or majority in this case.

2.1.3 Special Case 3

Suppose $\gamma = \frac{1}{2}$. We know that $\gamma = q = d_2 + \frac{d_1}{2}$ and $1 - \gamma = d_0 + \frac{d_1}{2}$. Solving each for d_1 yields: $d_1 = 1 - 2d_2$ and $d_1 = 1 - 2d_0$. Therefore $d_0 = d_2$, the steady state distribution is symmetric for equal populations.

Surprisingly this result does not hold in the general case. If the population sizes are equal the equality result still holds and the level of segregation is affected by the level of referral hiring but the distribution does not have to be symmetric except in the limit as $s \to 0$. As this occurs the expected result of complete segregation holds with one-half the firms having all group A workers and the other half having all group B workers.

2.2 The General Model

Now turn to the general model with firms of arbitrary size Θ. For notational convenience label the rows of the transition matrix T such that they correspond to the firm states. Call the first row of T "row 0" since it contains transition probabilities for firms in state 0. Label the second row of T "row 1" etc... Each row $i \subset \{1, 2, \ldots, \Theta - 1\}$ of the transition matrix consists of three non zero values. The diagonal element is:

$$t_d(i) = \frac{\Theta - i}{\Theta}[\frac{\Theta - i - 1}{\Theta - 1}(1 - s) + s(1 - q)] + \frac{i}{\Theta}[\frac{\Theta - i}{\Theta - 1}(1 - s) + s(1 - q)] \tag{16}$$

The element to the left of the diagonal is:

$$t_l(i) = \frac{\Theta - i}{\Theta}[\frac{i}{\Theta - 1}(1 - s) + sq] \tag{17}$$

And, the element to the right of the diagonal is:

$$t_r(i) = \frac{i}{\Theta}[\frac{\Theta - i}{\Theta - 1}(1 - s) + s(1 - q)] \tag{18}$$

The remaining elements of rows 1 to $\Theta - 1$ are 0.

Rows 0 and Θ only have two elements; a firm that already has 0 (Θ) minorities can not lose (gain) more.

T is thus a matrix of the form:

$T =$

$$\begin{vmatrix} t_d(0) & t_r(0) & 0 & 0 & 0 & ... & 0 \\ t_l(1) & t_d(1) & t_r(1) & 0 & 0 & ... & 0 \\ 0 & t_l(2) & t_d(2) & t_r(2) & 0 & ... & 0 \\ & & & & & & \\ 0 & ... & & & & t_l(\Theta) & t_d(\Theta) \end{vmatrix}$$

A fixed point, $DT = D$ is given by the solution to the following set of $\Theta + 1$ equations:

$$\begin{aligned} d_i = d_{i-1}\frac{\Theta - i + 1}{\Theta}[\frac{i-1}{\Theta - 1}(1-s) + sq] \\ + d_i\frac{\Theta - i}{\Theta}[\frac{\Theta - i - 1}{\Theta - 1}(1-s) + s(1-q)] \\ + d_i\frac{i}{\Theta}[\frac{\Theta - i}{\Theta - 1}(1-s) + s(1-q)] + d_{i+1}\frac{i+1}{\Theta}[\frac{\Theta - i}{\Theta - 1}(1-s) + s(1-q)] \end{aligned} \tag{19}$$

for $i = 1, 2, ..., \Theta - 1$.

$$\begin{aligned} d_i = d_i\frac{\Theta - i}{\Theta}[\frac{\Theta - i - 1}{\Theta - 1}(1-s) + s(1-q)] \\ + d_i\frac{i}{\Theta}[\frac{\Theta - i}{\Theta - 1}(1-s) + s(1-q)] + d_{i+1}\frac{i+1}{\Theta}[\frac{\Theta - i}{\Theta - 1}(1-s) + s(1-q)] \end{aligned} \tag{20}$$

for $i = 0$.

And,

$$
\begin{aligned}
d_i = d_{i-1}\frac{\Theta-i+1}{\Theta}[\frac{i-1}{\Theta-1}(1-s)+sq] \\
+ d_i\frac{\Theta-i}{\Theta}[\frac{\Theta-i-1}{\Theta-1}(1-s)+s(1-q)] + d_i\frac{i}{\Theta}[\frac{\Theta-i}{\Theta-1}(1-s)+s(1-q)]
\end{aligned}
\tag{21}
$$

for $i = \Theta$.

Now consider steady state unemployment. As above, the unique level of unemployment is equal across groups.

Proposition 1 *In steady state, the unique level of unemployment is equal across groups,* $q = \gamma$*, for all* $s > 0$*.*

(Proofs are provided in the appendix.)

One may find this result surprising. Remember, the likelihood that a member of group i being hired is a direct function of the number of members of group i employed. Even though one group may have more employed members at a given point in time the advantage is not stable for any $s > 0$. As long as some arbitrarily small amount of non-referral hiring occurs group equality is the only stable state.

Additionally note that the proposition is only true for $s > 0$. If $s = 0$ then each firm would eventually converge to employing workers from only one group. Once this happens the firm would never transition to employing workers from the other group in any future time period. Some firms will have all group A workers and some will have all group B workers. But there is no guarantee that the number of firms of each type will be proportional to the worker population. But as long as there is an arbitrarily small probability of hiring traditionally, $s > 0$, the equality result holds. The limit as $s \to 0$ differs from the case where $s =$.

Again consider the case of the limit as all hiring becomes referral hiring or the limit as $s \to 0$. As in the example above, as $s \to 0$ the population approaches complete segregation across firms.

Proposition 2 *In the limit as $s \to 0$ workers will be completely segregated such that measure γ of firms have only group A workers and measure $1-\gamma$ have only group B workers: $d_0 = 1-\gamma$, $d_\Theta = \gamma$, and $d_i = 0$ for $i = [1, 2, ..., \Theta - 1]$.*

As in the special case above as hiring moves toward more referral hiring more workplace segregation occurs. The feedback in the referral process biases firms to specialize in hiring from one particular group. Referring back to Proposition 1, it is interesting that the fraction of firms which have a bias to group i is equal to the proportion of group i member in the population. Thus in aggregate each group has equal opportunities in the labor market but not equal opportunity at each individual firm.

I discuss the intuition for the proofs of Propositions 1 and 2 more fully as I discuss results from the numerical experiments contained in the next section.

3 Numerical Experiments

Using the special cases of $s = 1$ and the limit as $s \to 0$ as benchmarks, a set of numerical experiments for intermediate values, $s \in (0, 1)$ can be obtained for comparison. I compare two measures: the shape of the distribution and a measure of segregation across various values of s.

I simulate the hiring process described above with a set of 20 firms each with 20 jobs. The population of eligible workers consists of 500 agents. Thus there will be 400 employed and 100 unemployed agents. The results presented will be averages over 50 runs of the simulation.

3.1 Measuring Segregation

Before I present the results I need to define a measure of segregation in order to compare the results across values of s. One standard measure that suits the analysis here is the dissimilarity index of Duncan and Duncan (1955). Their measure is defined as: $\psi = \frac{1}{2}\sum_{i=1}^{M} |\frac{N_{1,i}}{N_1} - \frac{N_{2,i}}{N_2}|$, where M is the number of firms in the population, $N_{j,i}$ is the number of employees of type j at firm i, and N_j is the number of total members of type j in the population. Note that $\psi \in [0, 1]$. This measure of segregation can be interpreted as the proportion of population members that would have to change jobs in order for them to be distributed evenly across the firms.[5]

3.2 Results of Experiments

I begin by presenting a visual comparison of the distribution of minority workers across firms. The resulting distribution when there is no referral hiring, $s = 1$, is shown in Figure 1. (Figures are provided in the appendix.) In this example the worker distribution is a binomial distribution with parameter γ.

Figures [2-5] show the minority worker distribution for levels of s ranging from .50 to .001. As s decreases the distribution becomes less single peaked and eventually reaches a level of almost complete segregation when $s = .001$. As predicted by the model as $s \to 0$ the distribution contains only firms with either all minority workers or no minority workers. As s decreases it is as if pressure was put on the top of the distribution and the peak is pushed down with the frequencies being pushed out to the edges. If you push all the way down there is only space in the tail of each side of the distribution.

As this decrease in s occurs note that as more zero minority firms are created more

[5] While there are many other measures that may be chosen that would yield different absolute levels of segregation I am interested in relative changes in segregation as a function of s. Thus limiting the analysis to this one measure is sufficient for my purpose.

firms with large numbers of minorities are also created. As minority opportunities are cut off at some firms; other firms become more likely to hire a minority. This is what allows the unemployment rate for minorities to stay constant over all levels of s. Suppose that $\gamma = 1/2$ and that there are 11 firms with all group A workers and 9 firms with all group B workers. Then even for an arbitrarily small s the probability that one of the all A firms becomes an all B firm goes to one in time because there are more group B workers in the unemployed pool.

We can compare these simulation sets directly by looking at segregation using the disimilarity index. Figure 6 shows the level of segregation and the unemployment rate of the minority group. The model was run for s =.001,.01,.05, and .10 and then in intervals of .10 up to 1.0. As can be seen in the figure, the unemployment rate is constant at γ as predicted by the model. At $s = .001$ there is a very high level of segregation. As expected from the analytical discussion above, excepting a few random firms, all firms have either all minority or all majority workers. There is close to complete segregation at this value of s.

As s approaches 0, segregation approaches 1. But as s increases even slightly segregation drops rapidly. With $s = .10$ segregation has already dropped over 40% to approximately 0.55. And at $s = .20$ segregation is about 0.45. Further increases in s result in much smaller changes, leaving segregation at approximately 0.20 for the full search case, $s = 1$.[6]

The result may be thought of in the spirit of Schelling. He found that a very small bias in preferences for similar neighbors creates large amounts of residential segregation. I find that small amounts of traditional search-based hiring greatly lower the amount of segregation across firms. In my case any small increase in non-referral

[6] Even though there is no bias toward referral hiring the Duncan and Duncan segregation measure may still be greater than 0 due to random placement of workers. Therefore the segregation results should be compared to the base case of $s = 1$ and not to a segregation level of 0.

hiring feeds back upon itself. For any $s < 1$ once a worker from an under-represented group is hired into a firm the probability of hiring another worker of that type increase. This feedback in the referral process allows small increases in s to have large effects on the levels of segregation in a population.

4 Two Extensions

In the examples above firms are homogeneous in that they all have the same number of workers and no structure. Additionally, the firms do not have any relation to each other except for the fact that they all hire from the same pool of unemployed workers. I next consider two simple extensions to the structure of firm relations: In the first there are multiple industries that all hire from the same pool of workers. In the second firms are organized as a hierarchy of H levels where new workers for level h must come from level $h-1$ instead of the unemployed pool. This extension is especially important because it considers the effect of hiring from other firms instead of the unemployed pool. The second model represents a situation where workers must acquire on the job skills or have some representative experience before being promoted to a higher level (perhaps managers who need to be familiar with the workings of the company.) Both extensions are similar to the base model in that equality between groups still attains but segregation only occurs between firms within a segment or industry not across segments.

4.1 Horizontal Industries

I begin with the economy organized into K segments or industries. As before there are M firms each with Θ jobs in each segment. Let the population, N, be larger than $M\Theta K$. The dynamics of the model work as before. Each period one worker is fired and a new worker is hired randomly from the unemployed pool with probability s or with probability $1-s$ the firm chooses the type of worker according to the proportion of workers it currently employs.

Changing notation slightly from above let q_u be the number of unemployed minority members as a fraction of the total unemployed population. And define q_k as the fraction of workers in segment k who are minority workers. I ask, how will the minority workers be distributed across the segments of the economy and the unemployed pool?

Proposition 3 *In steady state, the unique level of unemployment is equal across groups, $q_u = \gamma$, and each segment of the economy employs the same fraction of minority workers, $q_k = \gamma$ for all k and for all $s > 0$.*

Proposition 3 claims that the minority workers will all be spread uniformly across the segments if the segments hire from the same pool of workers. Note however that there will still be firm level segregation inside the segments as a function of referral hiring but there is not segregation across segments. Therefore if segregation occurs across industries it is likely that the industries are hiring from different pools of workers.

4.2 Vertical Industries

The second extension again considers firm segments but this time they are arranged in a hierarchy. Again each firm has Θ jobs. However, here workers must "work their way up" to be eligible for employment. A firm in segment $h+1$ must hire a new worker from segment h.[7] The dynamics of this model are also slightly different than

[7] A better interpretation of this model may be that there are M types of jobs (instead of firms) at each level in the hierarchy but that any worker in job type m at level h is a perfect substitute for another worker at job type m' at level h.

in the previous two. As before a random worker is chosen. The chosen worker leaves her job and enters the unemployed pool and the firm from which she left chooses a worker from the level below,$h-1$, as a function of s and the number of minorities it employs. Once the firm chooses a new worker this creates a vacancy at another firm (unless the originally fired worker was from level 1. In which case the firm chooses from the unemployed pool and the dynamics are the same as in the previous cases.) This firm then chooses a new worker from level $h-2$ and the process continues until a level 1 firm chooses a worker from the unemployed pool. This extension is especially relevant because it deviates from the assumption above that only workers are hire from the unemployed population.

Again let q_u be the number of unemployed minority members as a fraction of the total unemployed population. And define q_h as the fraction of workers in hierarchy level h who are minority workers. Analogous to Proposition 3 organizing firms into segments with a hierarchy also does not change the mix of workers at each segment.

Proposition 4 *In steady state, the unique level of unemployment is equal across groups, $q_u = \gamma$, and each segment of the economy employs the same fraction of minority workers, $q_h = \gamma$ for all h and for all $s > 0$.*

Proposition 4 indicates that one should not expect vertical segregation as a consequence of referral hiring alone. The same logic used for proposition 1 holds at each level here. If there are disproportionately many workers of one type in the unemployed pool then they will be absorbed into the level one firms. Once the level one firms have the equilibrium level of workers the same logic holds for level two firms hiring from level one, etc... Thus in steady state one would expect there to be equality in terms of employees at each level.

Both Proposition 3 and 4 appear in agreement with observed levels of employment segregation. Becker (1980) studies EEOC data covering over 7,000 places of employment. He finds that when segregation is decomposed into segregation by workplace and segregation between occupations and industries "most of the segregation of black and white workers. . . is segregation by place of work [85%] rather than stratification into different occupational categories [15%]." While this is far from a confirmation of the model, his result suggests that the predictions of this paper agree with observed patterns of workplace segregation.

5 Referral Hiring, Public Policy, and Inequality

In the model above I show that the two groups have equal unemployment rates in equilibrium. Thus the result of Proposition 1 suggests that referral hiring should not be considered important for public policy related to persistent inequality. However, there may be important factors concerning the out of equilibrium behavior of a labor market where referrals play an important role in the hiring process. For instance, how quickly and in what fashion does the economy reach steady state? Elsewhere I show that convergence to steady state equality may be greatly slowed if a large fraction of hiring is done by referrals and a minority group begins in a state of inequality (Tassier 2002). Referral hiring can perpetuate existing levels of inequality longer than would be the case without referral hiring. This is especially true if an industry has low rates of turnover in their work force. Since we view persistent inequality for some minority groups as a function of referral hiring it may be that we are still approaching steady state. Limiting referral hiring may not change the final level of equality between groups but reductions in referral hiring may speed the rate at which we attain equality.

In addition the segregation of workers may affect other decisions related to employment such as education prior to the economy attaining steady state. For instance the opportunity cost of attending school may be different across the groups in the presence of inequality. Imagine the choice of education by a member of an under-represented group in the labor market similar to the final model presented. Suppose referrals and education are both important in obtaining "high level jobs" and at a given point in time some individual does not have friends that are able to provide a needed referral. In this case it may be optimal for the individual to forgo investing in education. Once these education differences occur equality is no longer the expected result and the steady state outcome changes to one with persistent inequality. As this example suggests there may be effects from mixing referral based hiring and other social processes prior to the attainment of steady state.

Thus one should be careful in interpreting the results of this paper and other models of referral hiring. The purpose of this paper with regard to inequality is to show that referral hiring by itself may not directly cause group-level inequality. Therefore, public policy efforts aimed at reducing persistent inequality as it pertains to referral hiring should focus on the mixing of referral hiring with other social interactions, agent decisions, or norm creation prior to attaining steady state. Understanding the mix of these effects will better reveal the relationships between referral hiring and inequality.

With regard to segregation one important point to consider is that the firms may adjust their rate of referral hiring in order to either create a more or less diverse work force or to discriminate against workers depending on their current work force. With regard to the first point much work has been done in regard to the performance of diverse versus homogeneous working groups[8] Firms may prefer diverse collections of workers and thus lower their rate of referral hiring if they their work force becomes too homogeneous. Or second firms may have a preference for a specific type of worker and

[8]For an example see **?**).

use referral hiring to discriminate. For instance if Firm A prefers workers of gender X (as opposed to Y) then they may use referral hiring if their work force contains mostly group X workers but use non-referral methods if they employ few group X workers. But note that these effects only control the amount of segregation across firms; it does not effect the level of equality between the worker types.

Lastly note that there are at least two reasons we may view levels of segregation in a work force. First is the idea that some firms discriminate against certain worker types by not hiring them. This is the "taste for discrimination" story (?). Second is the story told in this paper: referrals can generate segregation if social networks are segregated. Note that a specific level of segregation and a specific worker distribution is predicted given a level of referral hiring. Thus if one knows the level of referral hiring in an industry one can compare the distribution of workers across firms to that which would be predicted by a model of referral hiring. And if one found more segregation than predicted that would be strong evidence that discrimination was present in the industry. Thus it is possible to use this model to identify when discrimination occurs.

6 Concluding Remarks

The model presented in this paper studies two potential effects of referral hiring: First, by itself, referral hiring does not produce population level inequality. Second, referral hiring creates segregation of worker groups across firms in industries but not across industries.

Taken together the results highlight the difference between viewing the effect of referral hiring at population and individual levels. The probability of obtaining a job at a given firm will be influenced by the labor market status of members of the individual's network. Thus there is a short run individual effect of referrals on labor market success. But this effect is aggregated out at the population level in the long run; producing equality between groups in the population in equilibrium.

One limitation of this paper is that it only considers the steady state distribution of workers; I leave the consideration of the paths which lead to steady state, and the rate of convergence to steady state to other work. I address these topics in Tassier (2002) and Tassier and Menczer (2002) and show that referral hiring greatly slows convergence to steady state if one of the groups begins in a disadvantaged position relative to the other. Thus the main effects of referral hiring on inequality are likely to be most apparent in dynamic models of referral hiring.

In addition to the main results, this paper highlights two significant items of interest in the referral hiring literature: First, one segment of the referral hiring literature looks primarily at the implications for individuals. These models concentrate on individual agents and the likelihood of labor market success given the success or failure of members an agent's network. For instance if agent i has a good job and she is connected to agent j, it is likely that agent j also has a good job. But, these statements may not carry forward to the population level. My work shows it is not the case that from these individual level correlations that one can automatically claim "group A people have bad jobs and group B people have have good jobs because of referral hiring." Thus there can be a difference between implications of referral hiring at the individual level and the population level as long as some non-referral hiring occurs. Looking at correlations between the labor market outcomes of connected individuals does not necessarily generalize to population level statements about referral hiring and inequality between groups.

Second, referrals are sometimes modeled by use of a proxy such that search intensity of firms may be directed at a particular group (for example Mailath, Samuelson, and Shaked (2000) mentioned above.) This type of model introduces an element of statistical discrimination that makes it difficult to isolate the effect of referrals from statistical discrimination. The model I investigate endogenously creates a referral structure in the economy that removes this underlying element of statistical discrimination. In my model firms do not target a specific group in their search to fill jobs; instead there exists a feedback in the hiring process that is a function of the relative success of each group at the particular hiring firm. The more successful a given group is the more likely it is that a member of that group receives a job offer. Thereby a direct connection exists between labor market outcomes and referral job offers that is isolated from elements of statistical discrimination by firms. That equality across population groups occurs in my model highlights a main difference in using firm search intensity as a proxy for referral hiring through networks as opposed to directly modeling networks.

The work in this paper should be understood as an attempt to isolate the population level, long run effects of referral hiring in the simplest possible way. However, as discussed in Section 6, a full understanding of the relationship between referral hiring and inequality needs to consider the interactions between referral hiring, segregation, social norms, and education decisions. From an understanding of these interactions will come the best public policy prescriptions for addressing the effect of referral hiring on inequality.

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7 Proofs and Figures

Proof of Proposition 1: As in the case above, for a steady state to occur it must be the case that the likelihood of a Group A worker being fired and a Group B being hired, $Pr(H_B)Pr(F_A|H_B)$, is equal to the likelihood that a Group B worker is fired and a Group A worker is hired $Pr(H_A)Pr(F_B|H_A)$.

Here this means that, $Pr(H_B)Pr(F_A|H_B) = \sum_{i=0}^{\Theta} d_i \frac{i}{\Theta}[\frac{\Theta-i}{\Theta-1}(1-s) + s(1-q)]$ and $Pr(H_A)Pr(F_B|H_A) = \sum_{i=0}^{\Theta} d_i \frac{\Theta-i}{\Theta}[\frac{i}{\Theta-1}(1-s) + sq]$.

Thus steady state unemployment is satisfied by the following:

$\sum_{i=0}^{\Theta} d_i \frac{i}{\Theta}[\frac{\Theta-i}{\Theta-1}(1-s) + s(1-q)] - \sum_{i=0}^{\Theta} d_i \frac{\Theta-i}{\Theta}[\frac{i}{\Theta-1}(1-s) + sq] = 0$

$\implies \sum_{i=0}^{\Theta}[d_i(\frac{\Theta i-i^2}{\Theta^2-\Theta}) - (\frac{\Theta i-i^2}{\Theta^2-\Theta})(1-s) + d_i(\frac{i}{\Theta}s(1-q) - \frac{\Theta-i}{\Theta}sq] = 0$

$\implies \sum_{i=0}^{\Theta}[\frac{d_i}{\Theta}(is - iqs - \Theta sq + isq] = 0$

$\implies s\sum_{i=0}^{\Theta} d_i(\frac{i}{\Theta} - q) = 0$

$\implies \sum_{i=0}^{\Theta} d_i \frac{i}{\Theta} = q\sum_{i=0}^{\Theta} d_i = q$ since $\sum_{i=0}^{\Theta} d_i = 1$.

As above $\gamma N = \sum_{i=0}^{\Theta} d_i \frac{i}{\Theta} E + qU$ Since $q = \sum_{i=0}^{\Theta} d_i \frac{i}{\Theta}$ This implies $q = \gamma$ for all $s > 0$. This completes the proof.

Proof of Proposition 2: To prove I first show that the steady state distribution as $s \to 0$ contains only firms in state d_0 or d_Θ. I then show the measure of firms in these states is $d_0 = 1-\gamma$ and $d_\Theta = \gamma$

Re-write Equation 19 as:

$$\begin{aligned} d_i = d_{i-1}\frac{\Theta-i+1}{\Theta}[\frac{i-1}{\Theta-1}(1-s) + sq] + d_i\frac{\Theta-i}{\Theta}[\frac{\Theta-i-1}{\Theta-1}(1-s) + s(1-q)] \\ + d_i\frac{i}{\Theta}[\frac{\Theta-i}{\Theta-1}(1-s) + s(1-q)] + d_{i+1}\frac{i+1}{\Theta}[\frac{\Theta-i}{\Theta-1}(1-s) + s(1-q)] \end{aligned} \tag{22}$$

Taking the limit as $s \to 0$ implies:

$$d_i = d_{i-1}\frac{(\Theta - i + 1)(i-1)}{\Theta(\Theta-1)} + d_i\frac{(\Theta - i)(\Theta - i - 1) + i(\Theta - 1)}{\Theta(\Theta-1)} + d_{i+1}\frac{(i+1)(\Theta - i)}{\Theta(\Theta-1)} \tag{23}$$

Now,

$\frac{(\Theta-i)(\Theta-1-1)+i(i-1)}{\Theta(\Theta-1)} = 1 + \frac{i(1-\Theta)}{\Theta(\Theta-1)}$.

So rewrite Equation 23 as:

$$d_i\frac{i(1-\Theta)}{\Theta(\Theta-1)} = d_{i-1}\frac{(\Theta - i + 1)(i-1)}{\Theta(\Theta-1)} + d_{i+1}\frac{(i+1)(\Theta - i)}{\Theta(\Theta-1)} \tag{24}$$

Which can be rewritten as:

$$d_i = d_{i-1}\frac{(\Theta - i + 1)(i-1)}{i(1-\Theta)} + d_{i+1}\frac{(i+1)(\Theta - i)}{i(1-\Theta)} \tag{25}$$

for $i = \{1, 2, ..., \Theta - 1\}$.

For $i = 0$ rewrite Equation 20 as:

$$d_0 = d_0 + d_1\frac{1}{\Theta - 1} \tag{26}$$

This implies $d_1 = 0$.

Given this write Equation 25 for $i = 1$:

$$d_1 = 0 = d_0 \frac{(\Theta - 1 + 1)(1 - 1)}{i(1 - \Theta)} + d_2 \frac{(1 + 1)(\Theta - 1)}{i(1 - \Theta)} \tag{27}$$

Which implies $d_2 = 0$ since $\frac{(\Theta-1+1)(1-1)}{i(1-\Theta)} = 0$.

Now, for $i = 2$ we have:

$$d_2 = 0 = d_1 \frac{(\Theta - 2 + 1)(2 - 1)}{2(2)(\Theta - 2)} + d_3 \frac{(2 + 1)(\Theta - 2)}{2(2)(\Theta - 2)} \tag{28}$$

Which implies $d_3 = 0$ since $d_1 = 0$.

In this manner it can be shown recursively that $d_1 = d_2 = ... = d_{\Theta-1} = 0$ in the limit as $s \to 0$.

By definition, $\sum_{i=0}^{\Theta} d_i \frac{\Theta-i}{\Theta} = 1 - \gamma$.

So, $d_0 \frac{\Theta}{\Theta} + 0 + ... + d_\Theta \frac{\Theta-\Theta}{\Theta} = 1 - \gamma$.

$\implies d_0 = 1 - \gamma$ and $d_\Theta = \gamma$. This completes the proof.

Proof of Proposition 3: Define the state of each of the m firms in segment k as the number of minorities at the firm. Let the fraction of firms in the state with 0 minorities in segment k be given by $d_{k,0}$, the fraction with 1 by $d_{k,1}$,..., the fraction with Θ by $d_{k,\theta}$. This defines a distribution $D^K = [d_{1,0}, d_{1,1}, ..., d_{1,\theta}, ..., d_{k,\theta}]$.

Define a transition matrix T^k as above. In this case there will be $K(\Theta + 1)$ rows and columns. Also note that a firm from segment k can not transition to another segment k'. Therefore each row will have either two or three non-zero elements. Rows $R_{k,0}$ and $R_{k,\Theta}$ will each have two non-zero elements for all k and the other rows will all have three non-zero elements.

Again find a steady state distribution. This will yield a set of $K(\Theta+1)$ equations where each equation contains only one industry segment. One can therefore solve for each of the k segments independently. From Proposition 1 it is known that each segment k will have $q_k = q_u$ for all $s > 0$. From here it is obvious that $q_u = q_k = \gamma$ for all k and for all $s > 0$. This completes the proof.

Proof of Proposition 4: Define the state of each of the m firms in hierarchy level h as the number of minorities at the firm. Let the fraction of firms in the state with 0 minorities in level h be given by $d_{h,0}$, the fraction with 1 by $d_{h,1}$,..., the fraction with Θ by $d_{h,\theta}$. This defines a distribution $D^H = [d_{1,0}, d_{1,1}, ..., d_{1,\theta}, ..., d_{H,\theta}]$.

Define a transition matrix T^h as above. In this case there will be $H(\Theta + 1)$ rows and columns. Also note that a firm from segment h can not transition to another segment h'. Therefore each row will have either two or three non-zero elements. Rows $R_{h,0}$ and $R_{h,\Theta}$ will each have two non-zero elements for all h and the other rows will all have three non-zero elements.

To prove I need to show that $q_h = q_h'$ for all h, h'. Begin with the level H. For level H the only loss of workers that occurs is from random firing. Therefore, the setup is the same as in Proposition 1. We want to find q_H which solves:

$$\sum_{i=0}^{\Theta} d_{H,i} \frac{i}{\Theta}[\frac{\Theta-i}{\Theta-1}(1-s) + s(1-q_{H-1})] - \sum_{i=0}^{\Theta} d_{H,i} \frac{\Theta-i}{\Theta}[\frac{i}{\Theta-1}(1-s) + sq_{H-1}] = 0$$

As in Proposition 1 this implies:

$$\sum_{i=0}^{\Theta} d_{H,i} \frac{i}{\Theta} = q_{H-1} \sum_{i=0}^{\Theta} d_{H,i}$$

$$\implies \frac{\sum_{i=0}^{\Theta} d_{H,i} \frac{i}{\Theta}}{\sum_{i=0}^{\Theta} d_{H,i}} = q_{H-1}$$

Now note that the left hand side is simply the fraction of minorities in the population of firms at level H which is q_H by definition. So, $q_H = q_{H-1}$.

For levels $H - 1$ to 1 we need to consider workers who leave because of random draws as well as workers who leave because a firm above them hires a worker. Thus the steady state requirement is:

$$\sum_{i=0}^{\Theta} d_{h,i} \frac{i}{\Theta}[\frac{\Theta-i}{\Theta-1}(1-s) + s(1-q_{h-1})] - \sum_{i=0}^{\Theta} d_{h,i} \frac{\Theta-i}{\Theta}[\frac{i}{\Theta-1}(1-s) + sq_{h-1}] + \bar{d}_h[q_{h+1}(1-q_h) - (1-q_{h+1})q_h] = 0$$

Where $\bar{d}_h$ represents the probability that the initially fired worker was from a level higher than h. The first and second term are the standard steady state condition from above. The last term is the likelihood that the originally fired worker was from a level higher than h (meaning level $h+1$ will have to hire a worker in this period from level h) multiplied by the likelihood the worker lost to level $h+1$ is a minority and the worker hired at level h is a majority member less the likelihood the worker hired by level $h+1$ is a majority member multiplied by the likelihood the worker hired by level h is a minority member.

This can be re-written as:

$s\sum_{i=0}^{\Theta} d_i(\frac{i}{\Theta} - q_{h-1}) + \bar{d}_h[q_{h+1} - q_h] = 0$

$\implies s\sum_{i=0}^{\Theta} d_i(q_h - q_{h-1}) + \bar{d}_h[q_{h+1} - q_h] = 0$

since $\frac{\sum_{i=0}^{\Theta} d_{h,i}\frac{i}{\Theta}}{\sum_{i=0}^{\Theta} d_{h,i}} = q_h$ by definition.

Since $s\sum_{i=0}^{\Theta} d_i$ and $\bar{d}_h$ are both positive there are three possible results: 1) $q_{h+1} = q_h = q_{h-1}$. The second and third possibilities are related: In these two either 2) $q_h > q_{h-1}$ and $q_h > q_{h+1}$ or 3) $q_h < q_{h-1}$ and $q_h < q_{h+1}$. Suppose the second solution is true. Now take the sequence $q_h, q_h + 1, ..., q_{H-2}, q_{H-1}, q_H$. If solution 2) is true then either $q_{H-2} < q_{H-1} > q_H$ or $q_{H-2} > q_{H-1} < q_H$. But this contradicts that $q_H = q_{H-1}$ derived above. Similarly solution 3) can be ruled out. Therefore solution 1) is the only possible outcome, $q_{h+1} = q_h = q_{h-1}$ for all $h \in [1, 2, ..., H]$. As shown above this implies that $q_h = \gamma$ for all h. This completes the proof.

Figure 1: Distribution of minority workers across firms when $s = 1$.

Figure 2: Distribution of minority workers across firms when $s = .50$.

Figure 3: Distribution of minority workers across firms when $s = .30$.

Figure 4: Distribution of minority workers across firms when $s = .10$.

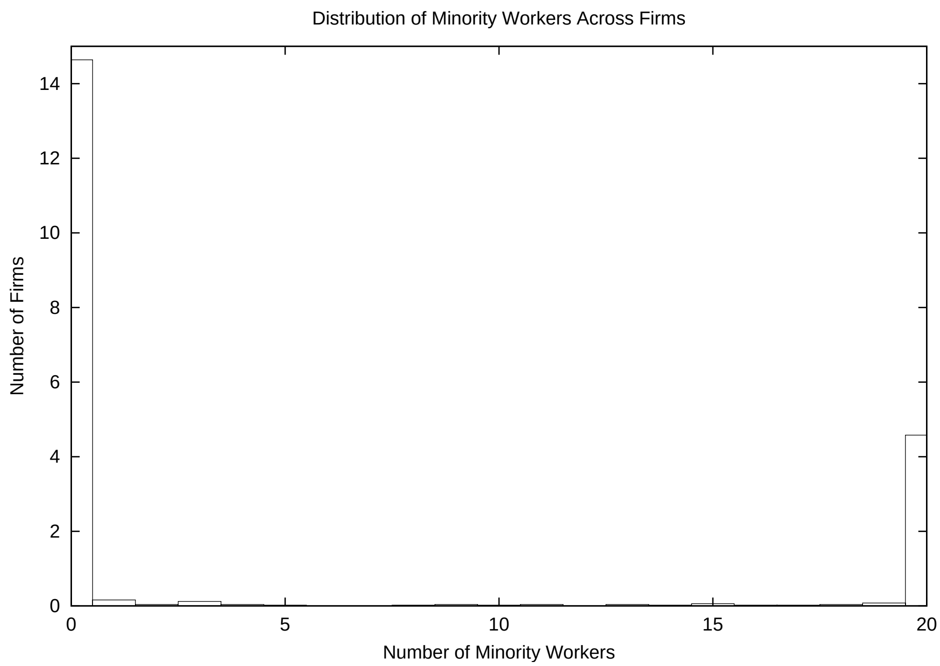

Figure 5: Distribution of minority workers across firms when $s = .001$. When almost all hiring is by referral ($s \to 0$)there is complete segregation.

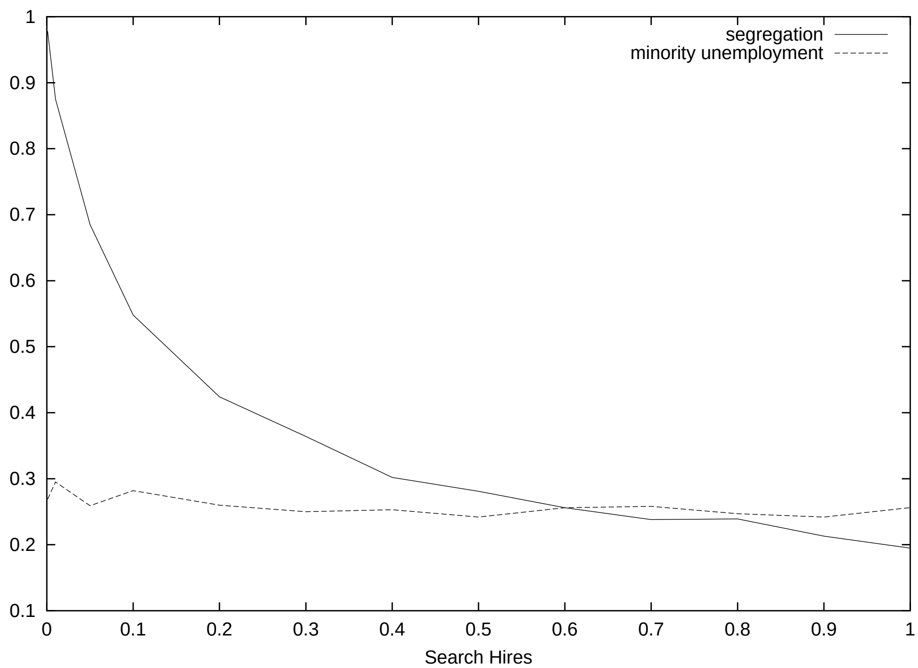

Figure 6: Level of segregation and percent of unemployed who are minorities for $\gamma = .25$. Note that the level of segregation falls rapidly as s increases from 0.