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A MULTILEVEL MODEL FOR COMMUNITY SEGREGATION*

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Segregation indices can be modelled statistically by using bootstrap re-sampling along with multilevel modelling. Computer software is now available to accomplish this straightforwardly. Examples are provided from measurements of social-class segregation among Scottish secondary-school pupils. The modelling shows that some of the differences in segregation between communities can be attributed to characteristics of the communities. Extensions of the model would allow the tracking of changes in segregation over time, for example to assess the impact of policies to reduce segregation.

One of the central issues in sociology concerns the effects of segregating low status groups from mainstream society. Racial and ethnic minorities, those in lower class occupations, and the unemployed tend to be segregated by their place of residence in most cities worldwide (Ginsberg, 1965; Lieberman, 1963; London and Flanagan, 1976). Residential segregation restricts their access to certain labour markets, exposes them to more crime and health risks, and generally affects their social and economic well being (see Massey, Condron, and Denton, 1987, for a review). Low and high status groups are also separated by differential access to schools, hospitals, and other social services. Residential segregation directly affects access to these institutions; it is one of the main determinants of institutional segregation. But institutional segregation can be exacerbated by structural features of the community that determine where certain programs are located, and by rules governing access. Additionally, low and high status groups are often segregated within institutions,

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not only between them. For example, minorities are disproportionately allocated to lower academic tracks within many high schools (Gamoran and Mare, 1989).

Generally segregation is measured by examining the extent to which a low status group is over-represented in some geographic areas or institutions, and under-represented in others. But the issue is complex because minorities can be segregated in a number of ways, such as being crowded in certain areas, especially a city core, or distributed such that they have little exposure to the majority culture (Massey and Denton, 1988). Sociologists have proposed at least 30 different indices to measure segregation (e.g., see James and Taeuber, 1985; Massey and Denton, 1988; White, 1986), and have conducted lively debate about their efficacy. Massey and Denton (1988) brought some order to the debate by classifying a wide array of indices into five conceptually distinct dimensions of spatial variation. In most research applications, researchers use one of the many indices as a descriptive variable to show variation across communities, or to assess change over time.

In a few cases, an index is used as a dependent variable in a regression model, with individuals' socioeconomic characteristics as predictor variables (e.g., Massey and Denton, 1985). To do this, researchers typically aggregate individual characteristics to the level at which the segregation index was estimated (e.g., census tract or school district). Conceived in this way, the model could be extended to include other factors measured at the level of the aggregate, such as characteristics of a neighbourhood or school district. For example, one may wish to ask whether units that are more segregated tend also to be larger, have poorer living conditions, or be dominated by a particular political party. Questions about which community-level factors are related to segregation can potentially have important implications for social policy, but despite the considerable effort by sociologists to identify indices of segregation, little attention has been paid to how segregation might be modelled.

Charles and Grusky (in press) suggested an approach for modelling the underlying structure of sex segregation between occupations across countries by decomposing variation using log-linear and log-multiplicative approaches. They proposed an index that does not depend on the proportions in each occupational category (i.e., "margin free"), and is therefore better suited to modelling sex segregation than most of the conventional measures. Unlike the modelling of sex segregation, where there is a one-to-one correspondence of groups—in this case occupations—across units at the second level (e.g., states or countries), the modelling of residential or school segregation is conceived differently because census tracts and school districts are defined separately for each second-level unit. Our purpose is not to specify a new segregation index; rather to provide a general approach for the modelling of any segregation index measured across a large number of second-level units. Thus our approach complements that of Charles and Grusky, and can extend it by modelling variation in sex segregation among firms within a country, and by distinguishing that variation from between-country variation.

The modelling of segregation indices is not straightforward. One problem is that they are measured with error. There is sampling error, arising from the use of samples to estimate the indices. The size of the sampling error depends not only on the size of the community, but also on the number of units and the size of the units within the community. The sampling distribution for the majority of segregation

indices is unknown.¹ In some cases, researchers use population data to estimate indices. But even with population data, the observed data are a sample from some "hyper-population"; that is, they are representative of some larger population of individuals that could be members of the communities. Thus, error of estimation is still a concern. Segregation indices also include measurement error. We can assume that factors such as race and ethnicity are measured with little error; in most cases subjects' responses to a question about their race or ethnicity would be highly reliable. But responses may be unreliable for some groups, especially those comprising a large proportion of subjects whose parents differ in their racial or ethnic background. Also, researchers sometimes wish to estimate the extent of segregation along social class lines (e.g., Willms, 1986), and measures of socioeconomic status always include measurement error.

Another problem, related to the problem of sampling and measurement error at the individual level, is that the social and political processes that produce segregation occur at several levels. At the individual level, segregation is determined in part by individuals' background characteristics and the choices they make. At the community level, segregation can be affected by any number of policy decisions, such as the location of certain kinds of programs, or by certain structural features of a community such as its size or the concentration of commerce in a city core. Researchers could calculate the segregation index for each community and then use traditional (i.e., ordinary least squares) regression techniques to fit regression models to community-level data that included aggregate individual-level variables and community-level variables. However, they would likely obtain biased estimates of the regression parameters because of aggregation bias and because the analysis would not take account of the accuracy with which each segregation index was estimated (Bryk and Raudenbush, 1992; Goldstein, 1987). And of course even with a multilevel approach, biased estimates are possible if the regression model is not adequately specified.

In this paper we suggest the use of two techniques to address these problems: one is the use of the "bootstrap" (Efron and Gong, 1983; Efron and Tibshirani, 1986; Hinkley, 1988) to estimate the standard errors of a segregation index for each community in a sample; the other is the use of multilevel regression modelling to describe variation among communities in the extent of segregation and to model that variation in terms of various community-level characteristics. The use of a computationally intensive techniques for estimating the standard errors of segregation indices is not novel; Massey (1976) used the jackknife, a related technique² for esti-

¹Cortese, Falk, and Cohen (1976) estimated the expected value and standard error of the Dissimilarity Index, D , under the assumption that the minority group was randomly distributed throughout a population, and used these estimates to compute an improved measure of D . They point out that the standard error of D (under the assumption of non-randomness) is unknown, and suggest its computation be considered.

²The jackknife technique involves repeating the estimation of a statistic of interest (in this case the segregation index) for each of n subsamples obtained by removing the i th observation ($i = 1, 2, \dots, n$) in turn from the obtained sample of size n . We expect the jackknife and bootstrap techniques would produce similar estimates of the standard errors for the common segregation indices, because the indices are not based on any order statistics (as with a median or midrange). Although the bootstrap is more computationally intensive, we prefer it because its application is straightforward even when the sample comprises schools with different sample sizes.

mating the error variance of a statistic (see Mosteller and Tukey, 1977) to compute the standard errors of an index measuring the segregation of males with disabilities. Our approach shows how such techniques can be used in conjunction with multi-level regression modelling to address more detailed questions about the segregation process. We demonstrate the use of these techniques using data describing Scottish secondary-school pupils from some 50 communities. The results suggest that there was significant variation among these communities in their segregation, and that this variation can be partially explained by features of the communities. The paper concludes with a discussion about how this application could be extended to address three questions relevant to research in this area: (1) Is there segregation along social class lines over and above the segregation associated with individual-level ability (or some other individual-level factor)? (2) To what extent do communities change in their levels of segregation over time? (3) Are indices pertaining to different social class groups (or racial and ethnic groups) correlated? (4) What is the impact of an intervention aimed at reducing levels of segregation?

Our application is also of substantive interest, particularly in the UK. Educators and policy-makers have been persistently concerned over the last few decades with school segregation along social class lines. Recently, this concern has arisen in Scotland because of the interaction of a number of significant Government policies that affected the distribution of pupils amongst Scottish schools. One policy was the reorganisation of Scottish secondary education along comprehensive lines during the 1960s and 1970s, and there is evidence that this policy reduced social class segregation in many communities (McPherson and Willms, 1987). However, some researchers have speculated that social class segregation may have increased during the 1980s, partly as a result of the UK Conservative Government's introduction of open enrollment in 1981 (Adler, 1992; Echols, McPherson, and Willms, 1990), and partly because of a slow change in the geographical dispersal of social groups (Garner and Raudenbush, 1991).

DATA AND METHODS

The data are from the 1981 Scottish School Leavers' Survey (SSLS), which describe people who left school in the summer of 1980. The sampling fraction was approximately 37%. The sample was selected quasi-randomly (i.e., those with particular birthdates) from a frame compiled by head-teachers, and ordered by birth-date. The selected pupils were sent a questionnaire in the spring after they had left school. The questionnaire asked numerous questions about their school experience, the transition from school, and their family background. A full description of the design of the survey is provided by Burnhill (1984).

Estimation of Community Segregation

The definition of "community" is seldom an easy task. This study used the definition specified by McPherson and Willms (1987),³ and employed data covering the 50

³Community "was defined in three stages. First, all schools, including private schools, were allocated to places as defined by the Registrar General Scotland (RGS, 1967). Second, all places served by more than

“multi-school communities” established by their analysis. For each community, we estimated five measures of segregation. These were based on three types of indices:

Dissimilarity index, D. This is the proportion of the minority group that would have to change schools in order to achieve an even distribution of the group across schools. It is defined by:

$$D = \frac{1}{2TP(1-P)} \sum_{i=1}^n t_i |p_i - P| \quad (1)$$

where t_i is the number of pupils in school i , p_i is the proportion in the minority group in that school, T is the total number of pupils in the community, P is the proportion in the minority group in the community, and n is the number of schools in the community. We see by the numerator of this index that segregation depends on the discrepancy between the proportion of minorities in each school and the overall proportion of minorities in the community (i.e., $p_i - P$). The denominator standardizes the index so that it ranges from 0 to 1 (see James and Taeuber, 1985).

We calculated this index twice: once with p_i being the proportion of pupils with middle-class backgrounds, and once with it being the proportion of pupils with working-class background. Middle class was defined as having an occupation in categories 1 (Professional) or 2 (Intermediate Professional) of the Registrar General's classification of occupations (Office of Population Censuses and Survey, 1970); working class comprised categories 4 (Partly Skilled), 5 (Unskilled), or unclassified. We included “unclassified” in this category because preliminary analyses showed that the education levels of this group were similar to the working class category. (Later we show how this group could be modelled separately in a multivariate model). The scale includes two middle categories, 3a (Skilled Non-manual) and 3b (Skilled Manual). Note, therefore, that the complementary group for our middle class category (i.e., *not* middle class) comprises 3a, 3b, 4, 5 and unclassified, while the complementary group for working class (i.e., *not* working class) comprises 1, 2, 3a and 3b. For the full sample, 21.6 percent of the pupils came from middle-class backgrounds (categories 1 and 2), and 33.9 percent were from working-class background (categories 4, 5, unclassified).

Isolation index, I. This index measures the extent to which members of a minority group are exposed only to each other, and thus isolated from members of its complementary group. It is conceptually distinct from the dissimilarity index because a group could be unevenly distributed across schools in a community, but, if it were a relatively small minority group, it would not necessarily be isolated from members of the majority group (Blau, 1977). An example of this situation is the distribution of Native Americans amongst schools in California (Rumberger and Willms, 1992). Unlike the dissimilarity index, therefore, the isolation index depends of the size of the minority group.

one school were then identified. Third, if the schools in that place served all pupils in that place, and only those pupils, the place was defined as a multi-school community ... Where a school in one place served pupils living in another place, the places were concatenated until a set of places were identified such that it was the minimum set within which the schools served all pupils in that set, and only those pupils. This set was then also defined as a multiple-school community.” (McPherson and Willms, 1987, p. 517).

The index is given by:

$$I = \frac{1}{TP} \sum_{i=1}^n t_i p_i^2 \quad (2)$$

where p_i , t_i , T and P are defined as in equation (1). Again we calculated the index for pupils with middle- and working-class backgrounds.

SES segregation. We define a continuous measure of socioeconomic status (SES), as the first principal component of father's occupation, mother's education, and family size (see Willms, 1986 for the scaling). This variable thus had a mean of zero and a standard deviation of one for the full sample. Then we calculate the proportion of the variance of this measure which is among schools. Let y_{ij} be the value of the measure for pupil j in school i , $Y_{i.}$ be its mean in school i , and $Y_{..}$ be the overall mean. The index is:

$$V = \frac{\sum_{i=1}^n t_i (Y_{i.} - Y_{..})^2}{\sum_{i=1}^n \sum_{j=1}^{t_i} (y_{ij} - Y_{..})^2} \quad (3)$$

V is simply the correlation ratio (i.e., the between sums of squares divided by the total sums of squares) derived from an analysis of variance of y_{ij} . The same formula could be used with a binary variable y_{ij} , indicating, for example, membership of a minority group. The proportion of minority pupils in school i (denoted above by p_i) is simply the school mean of the binary indicator. One can therefore substitute p_i for $Y_{i.}$, and P for $Y_{..}$ in the equation for V . In doing so, one can show that V and the isolation index are related through the formula (White, 1986):

$$V = \frac{I - P}{1 - P}. \quad (4)$$

For binary variables, V can be thought of as an isolation index adjusted for the population composition of the minority group (Coleman, Kelly and Moore, 1975; Massey and Denton, 1988).

The standard errors of the estimates of all the indices are calculated by bootstrapping (Efron and Tibshirani, 1986). Bootstrapping entails calculating a statistic of interest for each of a large number of samples drawn from the data available. Each sample is a 100 percent sample drawn with replacement. The standard error of the statistic is simply the standard deviation of the values of the statistic obtained from the set of samples. Thus the usual form of the bootstrap would require us to draw a large number of samples, with replacement, from the achieved SSLS sample within each school. But a simplification is possible here. The distribution of the Isolation and Dissimilarity indices depends on the repeated samples only through the proportions p_i , the distribution of which is fully described by the binomial, with sample size t_i and with probability equal to the value of p_i obtained in the original sample. Therefore all that is needed is to draw repeated samples from this binomial. Similarly, the index of SES segregation depends on the repeated samples only through the mean and variance of the SES measure within schools. Because the SES measure has approximately a Gaussian distribution within schools, the distribution of its mean and variance across repeated samples can be fully described by

repeated sampling from the Gaussian distribution with mean and variance from the original sample.

We did not use finite-population corrections in the estimates of variance, even though the SSLS sampling fraction was large (37%). The sampling variance in which we are interested is not due only to the mechanism by which pupils were selected from schools into the SSLS sample, but is due also to, for example, year-to-year random variation in the characteristics of the pupils in the schools. Thus we assume that the achieved SSLS sample came from a hyperpopulation of possible configurations of the school system, not just from the specific configuration that happened to exist in 1980.

For each index, we used 1000 bootstrap samples, and analysis of the distributions across these samples showed that the sampling distributions of the indices were all approximately Gaussian.

Taking Account of the Reliability of Estimation of the Indices

Multilevel modelling next allows us to take account of the fact that segregation indices are estimated more reliably for large communities than for small ones. We use a two-level model where the observed value of an index is modelled at the first (individual) level as a “true” value, δ_j , and a residual term, e_j , which comprises measurement and sampling errors

$$q_j = \delta_j + e_j. \quad (5)$$

The “true” values are modelled at the second (community) levels as:

$$\delta_j = \beta_0 + u_j \quad (6)$$

where β_0 is the overall average index, the u_j is a community-level residual (or error term) which has a Gaussian distribution at the community level with mean 0 and variance τ . Equations (5) and (6) can be combined into a single equation to model any index:

$$q_j = \beta_0 + u_j + e_j. \quad (7)$$

The two residual terms in this model can be distinguished from each other only because we have the independent estimates of the distribution of the e_j from the bootstrap: as mentioned above, these distributions were approximately Gaussian, with variances as exemplified in Table 1. We will test statistically whether $\tau = 0$, in other words whether the index varies significantly among communities. Because the total variance of the index is the sum of its sampling variance, σ^2 , and its “true” variance, τ , the test involves comparing the total variance of the index ($\tau + \sigma^2$) with the sampling variance embodied in the distributions of the errors e_j : if the total variance is greater than would be expected from the sampling variance, then we conclude that there really is among-community variation in the index.

Normally a two-level multilevel model entails the specification of a separate regression model at both the individual and group levels. The regression model at the individual level is fitted to data for each of J groups, thereby yielding J sets of regression parameters. These J sets become the dependent variables at the second level of the model. For example, if the within-group model was a simple linear

TABLE 1
Segregation Indices for Scottish Communities

Community	Segregation Index (and Standard Errors)				
	Dissimilarity		Isolation		V
	SC 1&2	SC 4&5	SC 1&2	SC 4&5	SES
Inverness	17.9 (6.3)	19.3 (6.3)	31.4 (3.8)	29.1 (3.8)	9.1 (3.1)
Banffshire	18.8 (6.1)	24.4 (6.2)	26.3 (3.7)	44.5 (3.8)	3.2 (2.3)
North Aberdeenshire	14.3 (5.9)	17.6 (6.3)	27.3 (3.5)	43.4 (3.7)	3.0 (2.0)
West Aberdeenshire	19.4 (6.2)	21.4 (7.7)	30.4 (3.9)	34.5 (4.1)	8.9 (3.5)
Aberdeen	37.1 (3.3)	25.4 (3.3)	38.0 (2.4)	34.1 (2.0)	19.6 (2.5)
Dundee	44.3 (3.5)	22.2 (2.8)	41.9 (2.7)	40.9 (1.9)	29.0 (2.6)
Perth	30.7 (5.2)	27.8 (5.8)	35.0 (3.8)	44.4 (3.6)	14.5 (3.9)
Edinburgh	42.2 (1.9)	26.6 (1.8)	46.6 (1.3)	38.0 (1.2)	31.8 (1.4)
Falkirk	23.4 (5.9)	12.2 (4.4)	24.7 (3.2)	31.8 (2.9)	3.6 (2.1)
Dumfries	45.8 (4.5)	30.7 (5.1)	45.4 (4.1)	49.1 (3.0)	23.1 (4.0)
Shetland	18.6 (4.1)	22.8 (6.5)	29.8 (5.2)	50.9 (4.8)	6.1 (3.0)
Western Isles	40.1 (7.3)	27.6 (8.2)	30.1 (5.4)	45.3 (5.0)	20.1 (6.0)
Argyll & Ayr	17.4 (4.1)	23.5 (5.3)	37.4 (2.9)	34.6 (3.0)	7.1 (2.5)
Kilmarnock+	21.6 (6.3)	18.3 (5.1)	20.3 (2.9)	42.7 (3.3)	6.4 (2.4)
Ayr+	31.4 (4.5)	16.3 (4.0)	33.0 (2.6)	30.3 (2.2)	15.1 (2.6)
Dunbarton	41.0 (5.7)	17.2 (5.0)	46.8 (5.7)	36.0 (3.7)	27.6 (5.0)
Glasgow	54.8 (1.9)	25.7 (1.4)	41.6 (1.6)	47.8 (.9)	34.4 (1.3)
Lanark NE	27.3 (3.1)	15.3 (2.1)	17.0 (1.4)	37.7 (1.3)	7.9 (1.3)
Blantyre and Hamilton+	32.0 (3.8)	13.6 (3.4)	26.8 (2.4)	31.9 (2.0)	10.7 (2.3)
East Kilbride	22.5 (5.5)	14.9 (4.6)	24.5 (2.8)	30.3 (2.7)	6.3 (2.3)
Paisley+	37.3 (2.9)	20.7 (2.2)	35.3 (2.1)	37.2 (1.5)	23.1 (2.0)
Greenock and Gourock+	49.2 (7.9)	22.3 (6.2)	34.7 (5.8)	39.5 (3.7)	18.7 (5.5)
All of Scotland	37.5 (0.7)	23.2 (0.6)	33.8 (0.7)	39.5 (0.4)	22.7 (0.5)

Note: Includes all Scottish communities with at least 5 schools. The Social Class 4 and 5 category includes fathers' occupations listed as "unclassified".

model providing an intercept and slope for each group, then one could specify a group-level model for the J intercepts, and another group-level model for the J slopes. In estimating the second-level models, the technique uses information on the sampling variances and covariances of the parameters estimated at the individual level. In the case of segregation indices, however, there is only one parameter at the individual level—the index itself. To model the index in multilevel fashion, one needs to know the sampling variance for each of the J estimated values. As explained above, we estimated these using the bootstrap. The multilevel modelling package HLM allows one to enter the analysis at the second level, using this information with its "Variance-Known" (or V -Known) option. Fitting equation (7) and carrying out the associated tests is then straightforward. A version of equation (7) with only one source of error could be fitted using an ordinary least squares (OLS) regression package, but OLS would not be able to distinguish between error variance (σ^2) and the true variance (τ) of the index among communities.

Explaining Variance Among Communities

The final step is to model the among-community variance in terms of community characteristics. For example, if z_j is the population size of community j , then the

above equation for q_j becomes:

$$q_j = \beta_0 + \beta_1 z_j + u_j + e_j. \quad (8)$$

If β_1 is statistically significantly different from 0, then we can conclude that community size is associated with the size of the index. We can also assess whether the addition of z_j to the model has reduced the proportion of the variance at the community level. In particular, if the among-community variance τ is now not statistically different from 0, then we can conclude that the community-size variable has fully explained the variation in the index among communities, and that the only unexplained variation in q_j is due to sampling and measurement error. Adding other variables as well as z_j is straightforward.

In our analysis we look at three community features. The first is directly related to educational policy: a measure of the proportion of the schools in the community that were selective. Overall, one-third of the pupils in the sample were attending selective schools.⁴ The second is the size of the community (as estimated by the sample size), which we take to be a crude index of urbanisation. Average community size was 375 pupils. The third is a measure of broader social policy: we used a dummy variable to identify the five "New Towns" that were created in the period between 1945 and 1970. These towns were intended to attract new industries to accommodate the "overspill" population from the cities (McPherson and Willms, 1986), and were planned with a deliberate attempt to minimize residential segregation. Later they became growth points for the expanding light-manufacturing sector, especially electronics (Randall, 1985). Our data described 1283 pupils who attended 19 schools in the five new towns.

RESULTS

The Estimation of Community Segregation

Table 1 shows estimates of the five indices of segregation for the 22 Scottish communities that had at least five schools. (Although our analysis includes all 50 communities, the ethical code governing the survey prevents us from publicly identifying groups of fewer than five schools.) The Dissimilarity index for social classes 1 and 2 ranges from 14.3 (North Aberdeenshire) to 54.8 (Glasgow). Thus, in North Aberdeenshire, 14.3 percent of the pupils from professional and intermediate professional backgrounds would need to change schools for there to be an even distribution of these pupils across schools. To achieve this in Glasgow, 55 percent of the pupils from the top two social classes would need to change schools. We notice also that the index is high for Scotland's four largest cities: Aberdeen (37.1), Dundee (44.3), Edinburgh (42.2), and Glasgow (54.8). The index for all of Scotland is 37.5. The standard errors of these estimates, shown in parentheses in the second column, range from 1.9 (Edinburgh and Glasgow) to 7.9 (Greenock, Gourock and Port Glasgow). We can say that there is a 95 percent probability that the "true" value of an

⁴Our definition of "selective" includes not only independent (private) schools, but also grant-aided schools and other state-funded schools that had not yet established fully comprehensive intakes. See McPherson and Willms (1987) for details.

index for a community lies within plus or minus two standard errors of the observed value.

The dissimilarity index for social classes 4 and 5 is smaller for most communities than the index for the top two social classes. The estimate for all of Scotland is 23.2, which is also smaller than the index for social classes 1 and 2. The range of this index—12.2 (Falkirk) to 30.7 (Dumfries)—is also smaller than the range of the index for social classes 1 and 2. Therefore, in most communities the proportion of pupils from the bottom two social classes that would need to change schools to achieve an even distribution of pupils from these classes is smaller than the proportions in the top two social classes. This seems counter-intuitive, but because we have classified pupils into three groups (the middle two groups are social classes 3 non-manual and 3 manual), it is theoretically possible to have a community with an index of zero for one group and a high value for the other group. One group could be evenly distributed among schools in the community, while another group tended to be concentrated in only a few schools. The discrepancies observed here suggest that pupils from the top two social classes tend to be concentrated in only a few schools.

The isolation index for a group could be small even though the dissimilarity index is large if members of that group tended to be congregated in a few large schools. Whether this happens depends on the values of p_i . For example, a small minority group might not be isolated from the majority group, even though its members were only attending a few schools in the community. This can be seen from Table 1 by comparing Edinburgh and Glasgow on the dissimilarity and isolation indices for social classes 1 and 2. Glasgow has a higher dissimilarity index than Edinburgh, meaning that a relatively high proportion of these classes in Glasgow would have to shift schools for them to be evenly spread. This is because these classes form a smaller proportion of the total population of Glasgow (14%) than of Edinburgh (28%): Edinburgh's economy is predominantly in the service sector, particularly government, law, education, and finance, with many opportunities for professional employment, whereas Glasgow is at the centre of an area of declining manufacturing industry. On the other hand, Glasgow has a lower isolation index for classes 1 and 2 than Edinburgh. This is partly a consequence again of the classes' being relatively smaller in Glasgow, but it is also because of the popularity in Edinburgh of fee-paying schools which are independent of the local public authority. In Edinburgh, fee-paying schools attract some 34% of middle-class pupils; in Glasgow, by contrast, they attract only 10% of the pupils from these classes. Thus middle-class pupils are relatively isolated in the independent schools in Edinburgh.

Multilevel Modelling of Community Segregation

Table 2 shows the results from model (1) for each of the five measures of segregation. The means are the values of β_0 .⁵ For the Dissimilarity index, we can see that achieving an even spread of classes 1 and 2 across schools would require a higher proportion of these classes to be moved than achieving an even spread of classes 4

⁵These means are the means of the indices across the 50 communities. The values shown for "All of Scotland" in Table 1 are not mean values, but the indices for the country as a whole.

TABLE 2
Mean, Variance, and Reliability of Segregation Indices for 50 Scottish Communities

Index of Segregation	Mean	(SE)	Variance	(χ^2)	Reliability
Dissimilarity					
Social Classes 1 and 2	24.58	(1.78)	114.57	(544.39)	0.671
Social Classes 4 and 5	18.27	(0.96)	16.85	(104.65)	0.291
Isolation					
Social Classes 1 and 2	27.87	(1.36)	76.67	(725.04)	0.811
Social Classes 4 and 5	37.34	(0.89)	25.81	(263.82)	0.594
V					
Socioeconomic Status	8.86	(1.24)	68.24	(1174.90)	0.872

Notes: The Social Class 4 and 5 category includes fathers' occupations listed as "unclassified". There were 49 degrees of freedom for the χ^2 tests. The means and variances for all indices were significantly greater than zero ($p < .001$).

and 5 would require of them. In contrast, from the Isolation index, we can see that classes 1 and 2 are less likely to be isolated from contact with other social groups than are classes 4 and 5. Putting these two comments together, we can conclude that classes 1 and 2 are fairly small, and therefore are not isolated, but also for the same reason could be evenly spread only if a large proportion of them moved. The opposite conclusions apply to classes 4 and 5.

The variances in Table 2 are τ —the variance of u_j in equation (7). The χ^2 values test whether τ is different from 0, and should be compared to a chi-square distribution with 49 degrees of freedom. In all cases there is strong evidence that the τ are different from 0, and so we conclude that the indices *do* vary among communities.

The final column measures the reliability with which β_0 is estimated (on a scale from 0 to 1). It is lower for the Dissimilarity index than for the others, and is lower for social classes 4 and 5 than for classes 1 and 2. Reliability depends on how precisely each of the β_0 are estimated, and on the "true" variation in the index which can potentially be explained by community-level influences on the indices, as in equation (8).

The analysis also allows us to calculate new estimates of the segregation in each community, adjusted to take account of the greater amount of information that is available on larger communities than on smaller ones. The new estimates are called empirical Bayes estimates (or posterior means), and shrink the originals towards the overall mean β_0 to an extent that is determined mainly by the number of schools in the community, but also by the extent of variation amongst schools. For example, Glasgow, with 67 schools, had the highest original estimate of dissimilarity—54.8—and its estimate was shrunk very little—to 53.9. By contrast, the Greenock and Gourock conurbation had the second-highest original estimate—49.2—but only the fifth-highest shrunk estimate—40.6—because it contains only 9 schools.

Explaining Differences Between Communities

The final step in the analysis is to attempt to explain some of the variation among communities by community characteristics; this is equation (8). Most among-school segregation is probably due to residential segregation. Our purpose is to assess

whether other features of the community affect segregation. Insofar as the pattern of residential segregation is a feature of the community, then we can say something about it: for example, we know that residential segregation is greater in cities than elsewhere.

The results are in Table 3. The first model for each index—model A—assesses whether the amount of segregation is associated with the proportion of selective schools in the community. Although most of the coefficients of the selectivity proportion are positive—indicating that high levels of segregation are associated with higher proportions of selective schools—none of them reach statistical significance. Contrary to some expectations, therefore, there is not strong evidence that the communities with higher proportions of pupils attending selective schools had greater segregation along social class lines.

Model B adds the two other community-level variables to model A. The dominant result concerns the effect of the measure of urbanisation: the larger the community, the greater the segregation within it. This conclusion reflects, above all, the much greater amount of residential segregation in the cities than elsewhere, but is not explained only by that: for each index, a model which picked out the four cities by means of a dummy indicator still had a statistically significant coefficient for the community-size variable. Also, in all models except the model pertaining to the Dissimilarity index for social classes 4 and 5, the residual parameter variance was statistically significant, indicating that there was remaining variation amongst communities that could potentially be explained by community-level factors. Although the coefficients of the new-towns indicator are all negative, none are statistically significant. Therefore we cannot say conclusively that the new towns did experience less social segregation among schools than the rest of the country. Overall, we feel that the model is not fully specified at the community level, and that with only 50 communities, it lacks power to detect some potentially important community-level effects. This calls for more powerful models, such as those that examine changes in community segregation stemming from policy interventions. These are discussed below.

SOME EXTENSIONS OF THE MODEL

Controlled Measure of Segregation

In some cases a researcher may wish to know whether there is segregation along social class lines (or some other factor), over and above that stemming from other factors associated with selection into the units. For example, Singapore has a highly segregated schooling system at the secondary level. Schools select pupils on the basis on their performance during elementary school, and parents choose schools on the basis of their location, their reputation, and probably several other criteria. Thus a reasonable question would be whether there is segregation along social class lines that is independent of the segregation stemming from merit-based criteria (e.g., levels of academic achievement at the end of elementary school).

In this application, the hierarchical linear model is used to calculate the standard deviations used in the estimation of the segregation index V . One could estimate a

TABLE 3
HLM Regression Results for Models Estimating the Effects of Community Size and Selectivity on Segregation

	Dissimilarity				Isolation				Eta	
	Classes 1&2		Classes 4&5		Classes 1&2		Classes 4&5		SES	
	I	II	I	II	I	II	I	II	I	II
Intercept (Std. Error)	25.39** (2.42)	23.22** (2.22)	17.56** (1.28)	16.57** (1.35)	26.73** (1.81)	26.91** (1.84)	36.16** (1.17)	35.71** (1.30)	9.02** (1.66)	8.05** (1.23)
Selectivity (Std. Error)	-2.31 (4.64)	1.90 (3.99)	2.20 (2.66)	3.68 (2.58)	3.39 (3.53)	4.05 (3.41)	3.57 (2.34)	4.41 (2.43)	-.46 (3.25)	1.84 (2.30)
Community Size (Std. Error)		11.64** (2.13)		3.42** (.96)		5.75** (2.04)		2.44 (1.29)		10.33** (1.36)
New Town (Std. Error)		-.81 (5.03)		-3.12 (2.92)		-6.31 (4.12)		-.902 (2.83)		-2.88 (2.75)

* $p < .05$.

** $p < .01$.

“null” hierarchical linear model for SES:

$$Y_{ij} = \beta_{0j} + e_{ij} \quad (9)$$

$$\beta_{0j} = \gamma_{00} + u_{0j} \quad (10)$$

where Y_{ij} is the SES score for pupil i in school j , β_{0j} is the school mean SES, e_{ij} are pupil-level residuals, γ_{00} is the grand mean of SES, and u_{0j} is the deviation of a school's mean SES from the grand mean. This is exactly analogous to a random effects ANOVA of SES with schools as the grouping factor (see Bryk and Raudenbush, 1992). The estimation procedure produces estimates of the between-school variance (i.e., $\text{Var}(u_{0j})$), and one can simply estimate the segregation index, V , as the ratio of the between-school variance to the total variance [i.e., $\tau/(\tau + \sigma^2)$]. One could then introduce a covariate to the first level of the model, equation (9), which is constrained to have a fixed effect across schools (i.e., $\beta_{0j} = \gamma_{10}$ for all j). The combined equation is:

$$Y_{ij} = \gamma_{00} + \gamma_{10}X_{ij} + u_{0j} + e_{ij} \quad (11)$$

where X_{ij} is the score on the covariate (e.g., entry-level achievement) for pupil i in school j . The u_{0j} are now residual terms, and the $\text{Var}(u_{0j})$ is a residual parameter variance. The HLM program provides an estimate of the residual parameter variance, which can be divided by the total variance to obtain an estimate of the controlled measure of segregation. We could obtain the standard error of this controlled measure of segregation for each community through bootstrap sampling. This would be computationally intensive but not impossible.

Stability Over Time

In many studies, the change or stability of segregation over time is more interesting than the absolute levels. The multilevel approach allows time to be modelled conveniently, by including it as a separate level. We follow the same general approach outlined by Willms and Raudenbush (1989) in an application pertaining to the estimation of school effects. The first level is simply a regression for each community of its index on ω_{jt} , the time of the t th observation for each community:

$$q_{jt} = \beta_{j0} + \beta_{j1}\omega_{jt} + u_{jt} + e_{jt} \quad (12)$$

where q_{jt} is the index for community j in year t , β_{j0} is the average index over the period considered, β_{j1} is the trend in the segregation (positive or negative), u_{jt} is the community residual at time t (i.e., year-to-year fluctuations in its index not accounted for by its overall trend), and e_{jt} is the error term associated with sampling variability at each time t . The next level of the model is specified with separate equations for the average index and the trend:

$$\beta_{j0} = \Phi_{00} + V_{j0} \quad (13)$$

$$\beta_{j1} = \Phi_{10} + V_{j1} \quad (14)$$

where the average index of each community has been expressed as a grand mean for all communities, and a community-level residual, V_{j0} ; the trend for each community has been expressed as an overall trend, and a community-level residual, V_{j1} . By

substituting equations 13 and 14 into 12, we obtain the full model:

$$q_{jt} = \Phi_{00} + \Phi_{10}\omega_{jt} + V_{j0} + V_{j1}\omega_{jt} + u_{jt} + e_{jt} \quad (15)$$

One can test statistically whether there is a significant overall trend in segregation (i.e., $H_0 : \Phi_{10} = 0$), whether communities varied in their average levels of segregation over the period (i.e., $H_0 : \text{Var}(V_{j0}) = 0$), and whether communities varied in their trends over the period (i.e., $H_0 : \text{Var}(V_{j1}) = 0$).

Correlation Amongst Multiple Indices of Segregation

In our computation of the Dissimilarity and Isolation indices, we defined middle class as comprising the top two categories of the Registrar General's classification of occupations, and working class as comprising the bottom two categories and unclassified group. These definitions are arbitrary, and the choice of a different "cut-point" separating middle class and *not* middle class (or working class and *not* working class) would yield different results. The multilevel model can be extended to accommodate multivariate outcomes for each group (Goldstein, 1987). Therefore one can compute a number of segregation indices for each community using different cut-points in the definition of middle or working class. Multivariate, multilevel modelling is accomplished by including a level of analysis below the community level. For example, one can define three dichotomous variables (Professional, Skilled, and Working Class), and model the three resulting segregation indices:

$$Y_{ij} = q_{1j}X_{1ij} + q_{2j}X_{2ij} + q_{3j}X_{3ij} \quad (16)$$

where Y_{ij} is the i th index for community j , X_{1ij} , X_{2ij} , and X_{3ij} are dummy variables denoting whether Y_{ij} pertain to the first, second, or third index. For example, X_{1ij} equals one if the index describes segregation of the Professional category, zero otherwise. Therefore q_{1j} is the index for professional occupations, q_{2j} is the index for skilled occupations, and q_{3j} is the index for working class occupations. Note that this intra-community model does not include an error term. At the next two levels, the three q_j 's are modelled as in equations 5 and 6.

The multivariate multilevel model not only provides estimates of the average value and variance for each index, but also the covariances (or correlations) between each pair of indices. This would be particularly useful in the U.S. context, where one may wish to determine the correlation amongst segregation indices for different racial and ethnic groups.

Estimation of an Intervention Effect

A particular case of the study of change over time is where there has been a specific policy intervention in some communities to reduce segregation. Here we can introduce a time-varying covariate z_{jt} to equation 12, which would have value 1 if community j had experienced the intervention by year t , and would have value 0 for all other cases. The coefficient of this variable would then tell us whether the intervention had been associated with a decrease of segregation. Other time-variant covariates, such as selectivity in our example, could be included as control variables

at this level. Similarly, one could include time-invariant covariates, such as community type (e.g., New town or other), to model both the overall average segregation (equation 13) and the communities' trends (equation 14).

CONCLUSIONS

This paper provides a straightforward technique for modelling segregation indices. For several decades sociologists have measured the segregation of low status groups from the mainstream of society. Generally these efforts have informed social policy by showing trends in segregation levels for communities, states or countries, or by making comparisons across units at each of these levels. But policy-makers are usually interested in addressing questions concerning the *causes* of segregation; for example, are employment equity policies effective in reducing gender inequality in the labour market, or has an increase in parental choice of schools resulted in increased between-school segregation. Generally analyses have only exposed differences across units or over time, and inferences that such variation stems from particular policies or practices are weak. We contend that the casual inferences concerning segregation can be strengthened by directly modelling segregation indices. The technique we propose can be used in either experimental or quasi-experimental designs, and can incorporate variables at both the individual and group levels.

Our approach employs bootstrap resampling to estimate the standard errors of the segregation index estimated for each unit, and multilevel regression techniques to model variation among communities in their levels of segregation. If researchers use OLS regression to model segregation indices by estimating the index for each group and regressing the estimated indices on group-level characteristics, their results will be biased and inefficient because OLS regression does not take account of the reliability with which each group's index is estimated. Other techniques, such as the log-multiplicative approach suggested by Charles and Grusky (in press) requires a separate parameter for each unit, which is practical for comparing indices across a small number of countries, but may be computationally infeasible for comparing indices across a large number of firms within a country. Thus, the approach we recommend has several advantages. First, it takes account of the reliability with which each index is estimated. Second, there is a parsimony of parameters: it is not necessary to construct separate dummy variables for each group or estimate separate parameters. Third, the model allows for the inclusion of variables at both the individual and group levels. Thus it is feasible to estimate the extent of social class segregation, net of racial and ethnic segregation, or *vice versa*, and to model variation in this segregation on a number of group-level variables. One of the group-level variables could be a dummy variable denoting whether or not a particular policy was implemented for each group. Thus, the technique can be used to assess the effect of an intervention in an experimental or quasi-experimental design, with control for the effects of other group-level factors.

Moreover, the multilevel approach provides a new framework for considering problems about segregation. For example, much of the research on sex segregation in the workplace has been concerned with comparing countries or monitoring changes over time (e.g., Blackburn, Jarman, and Siltanen, 1993; Brinton and Ngo,

1991; Jacobs, 1989). But much of the interesting variation may be between firms within a country (e.g., see Tienda, Smith, and Ortiz, 1987). The multilevel approach would start with the questions, "How much variation is there among firms within a country?"; "Is that variation statistically significant?" And if the variation were statistically significant, one would ask further, "What firm-level characteristics, such as type of industry, size of the firm, age of the firm, explain that variation?" The model could be extended to three levels to address questions like, "Does the extent of variation among firms vary among countries?" or "Has the variation among firms changed over the last decade?" Similar questions can be posed about residual or school segregation.

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