Multi-objective Optimization

Features:

- Distributed Data Flow
- Fog nodes mobility
- IoT devices mobility
- Migration support
- Partitioning techniques
- Data placement optimization
- Migration Optimization

Variables:

- QoS
    - latency
- Cost
    - CPU
    - RAM
    - MEM
    - BW
- Energy
    - busyPower
    - idlePower
- Bandwidth

Notations:

- N − fog nodes $S = \{s_1, ..., s_n\}$
- M − modules $U = \{u_1, ..., u_m\}$

- fP^{Mips} − matrix $1 \times N$ representing the MIPS price of each fog node per unit
- fP^{Ram} − matrix $1 \times N$ representing the memory price of each fog node per unit
- fP^{Strg} − matrix $1 \times N$ representing the storage price of each fog node per unit
- fP^{Bw} − matrix $1 \times N$ representing the bandwidth price of each fog node per unit

- f^{Mips} − matrix $N \times 1$ representing the MIPS capacity of each fog node
- f^{Ram} − matrix $N \times 1$ representing the memory capacity of each fog node
- f^{Strg} − matrix $N \times 1$ representing the storage capacity of each fog node
- f^{Bw} − matrix $N \times 1$ representing the bandwidth capacity of each fog node
- f^{bPw} = matrix $N \times 1$ representing the busy power consumption of each fog node per unit
- f^{iPw} − matrix $N \times 1$ representing the idle power consumption of each fog node per unit
- f^{wPw} − matrix $N \times 1$ representing the weight of energy consumption ($willing\_to\_waste\_energy^{-1}$)

- m^{Mips} − matrix $M \times 1$ representing the MIPS needed for each application's module
- m^{Ram} − matrix $M \times 1$ representing the memory needed for each application's module
- m^{Strg} − matrix $M \times 1$ representing the storage needed for each application's module
- m^{Bw} − matrix $M \times 1$ representing the bandwidth needed for each application's module

- K − total number of dependencies/edges
- e^{Cpu} − matrix $1 \times K$ representing the tuple CPU size (MI) needed to be processed
- e^{Nw} − matrix $1 \times K$ representing the tuple network size (MB) needed to be sent
- e^{Prob} − matrix $1 \times K$ representing the probability of sending the tuple
- e^{Pe} − matrix $1 \times K$ representing the periodicity of the producer (i.e., periodic sources)
- e^{S} − matrix $1 \times K$ representing the edge source
- e^{D} − matrix $1 \times K$ representing the edge destination

- Z − total number of dependencies/edges between different pairs of nodes
- l^{S} − matrix $N \times 1$ representing the starting nodes of each pair of nodes
- l^{D} − matrix $N \times 1$ representing the ending nodes of each pair of nodes

- mD − matrix $M \times M$ representing the dependencies between modules
- mB − matrix $M \times M$ representing the bandwidth needed between modules

- fL − matrix $N \times N$ representing the latency between each two direct nodes
- fB − matrix $N \times N$ representing the bandwidth between each two direct nodes

- D − matrix $N \times M$ representing the nodes where each module can be deployed

- P − matrix $N \times M$ representing the placement mapping between modules and nodes
- R − matrix $K \times N \times N$ representing the routing map between modules

- e − matrix 1xN, with all entries to 1

- α − operational weight
- β − energetic weight
- γ − processing weight
- δ − latency weight
- ζ − bandwidth weight

Preliminary computations:

$$m_i^{Mips} = \sum_{k\in K}\left(\frac{e_k^{Prob}e_k^{Cpu}}{e_k^{Pe}}\right), e_k^D = i$$

$$m_i^{Bw} = \sum_{k\in K}\left(\frac{e_k^{Prob}e_k^{Nw}}{e_k^{Pe}}\right), e_k^S = i$$

$$mB_{i,j} = \sum_{k\in K}\left(\frac{e_k^{Prob}e_k^{Nw}}{e_k^{Pe}}\right), e_k^S = i,\ e_k^D = j$$

$$mD_{i,j} = \sum_{k\in K}\left(\frac{e_k^{Prob}}{e_k^{Pe}}\right), e_k^S = i,\ e_k^D = j$$

Problem formulation:

The cost function is mainly characterized by two components: Operational Cost and Service Quality Cost.

Operational Cost (C_O) is characterized by the resources allocated in each fog node to support all users' computations, namely: CPU, memory, storage, and bandwidth.

$$C_O = fP^{Mips}\times P\times m^{Mips} + fP^{Ram}\times P\times m^{Ram} + fP^{Strg}\times P\times m^{Strg} + \left(fP^{Bw}\sum_{z\in Z} mB_{l_z^S,l_z^D}\times R_z\right)e'$$

Power Cost (C_{Pw}) is characterized by the busy/idle power in each fog node to support all users' computations as well as the willing to wast energy to prevent clients to process the whole application.

$$C_{Pw} = \sum_{n\in N}\left(f_n^{iPw} + f_n^{wPw}(f_n^{bPw} - f_n^{iPw})\times\frac{P_n\times m^{Mips}}{f_n^{Mips}}\right)$$

Processing Cost (C_P) is characterized by the percentage of unused MIPS using the Jain's fairness index.

$$C_P = \frac{\left(\sum_{n\in N}\frac{P_n\times m^{Mips}}{f_n^{Mips}}\right)^2}{N\times\sum_{n\in N}\left(\frac{P_n\times m^{Mips}}{f_n^{Mips}}\right)^2}$$

Latency Cost (C_L) is characterized by the total latency on the tuple transmission.

$$C_L = e\left(fL.*\sum_{z\in Z} mD_{l_z^S,l_z^D}\times R_z\right)e'$$

Bandwidth Cost (C_B) is characterized by the bandwidth usage on the tuple transmission.

$$C_B = e\left(1./(fB+\epsilon)\ .*\sum_{z\in Z} mB_{l_z^S,l_z^D}\times R_z\right)e'\ , \epsilon = 1^{-9}$$

Final problem:

$$\begin{aligned}
\underset{P,R}{\text{minimize}}\quad & C = \alpha C_O + \beta C_{Pw} + \gamma C_P + \delta C_L + \zeta C_B\\
\text{subject to}\quad & P\times m^{Mips} \le f^{Mips},\\
& P\times m^{Ram} \le f^{Ram},\\
& P\times m^{Strg} \le f^{Strg},\\
& P \le D\\
& P_{i,j}\in\{0,1\}, \forall i\in[0,N], \forall j\in[0,M],\\
& \sum_{i\in N} P_{i,j} = 1, \forall j\in[0,M]\\
& R_{z,i,j}\in\{0,1\}, \forall z\in[0,Z], \forall i\in[0,N], \forall j\in[0,N],\\
& \sum_{j\in N} R_{z,i,j} - \sum_{j\in N} R'_{z,i,j} = P_{i,l_z^S} - P_{i,l_z^D}\quad, \forall z\in[0,Z], \forall i\in[0,N]
\end{aligned}$$