

Notation:

- N – number of nodes
- M – number of modules
- K – number of dependencies
- Z – number of module pairs with dependencies
- Q – number of loop deadlines
- $P^{Mips} [\text{€}]$, $1 \times N$ matrix representing the price of using processing resources in each fog node
- $P^{Mem} [\text{€}]$, $1 \times N$ matrix representing the price of using memory resources in each fog node
- $P^{Strg} [\text{€}]$, $1 \times N$ matrix representing the price of using storage resources in each fog node
- $P^{Bw} [\text{€}]$, $1 \times N$ matrix representing the price of using network resources in each fog node
- $P^{Pw} [\text{€}]$, $1 \times N$ matrix representing the power price in each fog node
- f^F , $N \times 1$ binary matrix representing whether nodes are clients or not (f_n^F is 0 if n is a client)
- $f^{Mips} [MIPS]$, $N \times 1$ matrix representing the processing capacity of each fog node
- $f^{Mem} [Byte]$, $N \times 1$ matrix representing the memory capacity of each fog node
- $f^{Strg} [Byte]$, $N \times 1$ matrix representing the storage capacity of each fog node
- $f^{bPw} [W]$, $N \times 1$ matrix representing the busy power consumption of each fog node
- $f^{iPw} [W]$, $N \times 1$ matrix representing the idle power consumption of each fog node
- $f^{Tx} [W]$, $N \times 1$ matrix representing the transmitter power (f_n^{Tx} is non 0 if n is a mobile node)
- $m^{Mips} [MIPS]$, $M \times 1$ matrix representing the processing resources needed for each module
- $m^{Mem} [Byte]$, $M \times 1$ matrix representing the memory resources needed for each module
- $m^{Strg} [Byte]$, $M \times 1$ matrix representing the storage resources needed for each module
- $m^{MigD} [s]$, $M \times 1$ matrix representing the migration deadline for each module
- $e^{Cpu} [MI]$, $K \times 1$ matrix representing the tuple CPU size needed to be processed for each dependency
- $e^{Nw} [Byte]$, $K \times 1$ matrix representing the tuple network size needed to be sent for each dependency
- $e^{Pe} [s]$, $K \times 1$ matrix representing the periodicity of sending the tuple for each dependency
- $e^{Prob} [\%]$, $K \times 1$ matrix representing the probability of sending the tuple for each dependency
- e^S , $K \times 1$ matrix representing the source module for each dependency
- e^D , $K \times 1$ matrix representing the destination module for each dependency
- l^S , $Z \times 1$ matrix representing the source module for each pair of modules with dependencies
- l^D , $Z \times 1$ matrix representing the destination module for each pair of modules with dependencies
- m^D , $M \times M$ matrix representing the dependencies between modules
- m^B , $M \times M$ matrix representing the bandwidth needed between modules
- m^{CPU} , $M \times M$ matrix representing the CPU size of dependencies between modules
- m^{NW} , $M \times M$ matrix representing the network size of dependencies between modules
- f^L , $N \times N$ matrix representing the link latency between each two nodes
- f^B , $N \times N$ matrix representing the link bandwidth between each two nodes
- D , $N \times M$ binary matrix representing the nodes where each module can be deployed
- C , $N \times M$ binary matrix representing the current module placement

- A^L , $Q \times M \times M$ binary matrix representing the loop module list
- A^D , $Q \times 1$ matrix representing the loop deadline list
- α^p , the percentage of processing capacity which is not used for control operations
- α^m , the percentage of memory capacity which is not used for control operations
- α^s , the percentage of storage capacity which is not used for control operations
- α^b , the percentage of bandwidth capacity which is not used both for control operations and migrations
- t^{Boot} , constant representing the average virtual machine boot time
- ϵ small positive constant

Variables:

- P , $N \times M$ binary matrix representing the placement mapping between modules and nodes
- R , $Z \times N \times N$ binary matrix representing the tuple routing map between modules
- V , $M \times N \times N$ binary matrix representing the module migration routing map

Preliminary computations:

$$m_i^{Mips} = \sum_{k \in K} \frac{e_k^{Prob} e_k^{Cpu}}{e_k^{Pe}}, \quad e_k^D = i$$

$$m_{i,j}^B = \sum_{k \in K} \frac{e_k^{Prob} e_k^{Nw}}{e_k^{Pe}}, \quad e_k^S = i, \quad e_k^D = j$$

$$m_{i,j}^D = \sum_{k \in K} \frac{e_k^{Prob}}{e_k^{Pe}}, \quad e_k^S = i, \quad e_k^D = j$$

$$m_{i,j}^{CPU} = \sum_{k \in K} e_k^{Cpu}, \quad e_k^S = i, \quad e_k^D = j$$

$$m_{i,j}^{NW} = \sum_{k \in K} e_k^{Nw}, \quad e_k^S = i, \quad e_k^D = j$$

Objectives:

Operational Cost:

$$C_O = P^{Mips} \times P \times m^{Mips} + P^{Mem} \times P \times m^{Mem} + P^{Strg} \times P \times m^{Strg} + \left(P^{Bw} \sum_{z \in Z} m_{l_z^S, l_z^D}^B \times R_z \right) 1N + C_{Pw}(P^{Pw'})$$

Power Cost:

$$C_{Pw}(x) = C_P(x \cdot (f^{bPw} - f^{iPw})) + C_B(x \cdot f^{Tx})$$

Processing Cost:

$$C_P(x) = \sum_{n \in N} f_n^F x_n \frac{P_n \times m^{Mips}}{\alpha^p \times f_n^{Mips}}$$

Bandwidth Cost:

$$C_B(x) = \sum_{z \in Z} m_{l_z^S, l_z^D}^B \sum_{i \in N} f_i^F x_i \sum_{j \in N} \frac{R_{z,i,j}}{\alpha^b \times f_{i,j}^B + \epsilon}$$

Migration Cost:

$$C_M(x) = \sum_{m \in M} s_m \sum_{i \in N} f_i^F x_i \sum_{j \in N} \frac{V_{m,i,j}}{(1 - \alpha^b) \times f_{i,j}^B + \epsilon},$$

$$s_m = m_m^{Strg} + m_m^{Mem}$$

Multiple-objective (Fog service provider):

$$F = \{C_{Pw}(1N), C_P(1N), C_B(1N), C_M(1N)\}$$

Single-objective (Broker):

$$F = C_O$$

Final problem:

$$\begin{aligned}
& \underset{P, R, V}{\text{minimize}} && F \\
& \text{subject to} && P_{i,j} \in \{0, 1\}, \forall i \in [0, N], \forall j \in [0, M] \\
& && R_{z,i,j} \in \{0, 1\}, \forall z \in [0, Z], \forall i \in [0, N], \forall j \in [0, N] \\
& && V_{z,i,j} \in \{0, 1\}, \forall z \in [0, M], \forall i \in [0, N], \forall j \in [0, N] \\
& && P \times m^{Mips} \leq \alpha^p \times f^{Mips} \\
& && P \times m^{Mem} \leq \alpha^m \times f^{Mem} \\
& && P \times m^{Strg} \leq \alpha^s \times \leq f^{Strg} \\
& && \sum_{z \in Z} m_{l_z^S, l_z^D}^B \times R_{z,i,j} \leq \alpha^b \times f_{i,j}^B, \forall i \in [0, N], \forall j \in [0, N] \\
& && \sum_{i \in N} P_{i,j} = 1, \forall j \in [0, M] \\
& && P \leq D \\
& && \sum_{j \in N} R_{z,i,j} - \sum_{j \in N} R'_{z,i,j} = P_{i,l_z^S} - P_{i,l_z^D}, \forall z \in [0, Z], \forall i \in [0, N] \\
& && \sum_{j \in N} V_{z,i,j} - \sum_{j \in N} V'_{z,i,j} = C_{j,i} - P_{j,i}, \forall z \in [0, M], \forall i \in [0, N] \\
& && L_q^P + L_q^T \leq A_q^D, \forall q \in [0, Q] \\
& && L_m^M \leq m_m^{MigD}, \forall m \in [0, M]
\end{aligned}$$

where :

$$\begin{aligned}
L_q^P &= \sum_{i \in M} \sum_{j \in M} A_{q,i,j}^L \sum_{n \in N} P_{n,j} \frac{\sum_{l \in M} \sum_{k \in M} m_{l,k}^{CPU} \times P_{n,k}}{\alpha^p \times f_n^{Mips}} \\
L_q^T &= \sum_{i \in M} \sum_{j \in M} A_{q,i,j}^L \sum_{l \in N} \sum_{k \in N} R_{z',l,k} \left(\frac{\sum_{z \in Z} R_{z,l,k} \times m_{l_z^S, l_z^D}^{NW}}{\alpha^b \times f_{l,k}^B + \epsilon} + f_{l,k}^L \right), \\
l_{z'}^S &= i, \quad l_{z'}^D = j \\
L_m^M &= b \times t^{Boot} + \sum_{i \in N} \sum_{j \in N} V_{m,i,j} \times \left(\frac{m_m^{Strg} + m_m^{Mem}}{(1 - \alpha^b) \times f_{i,j}^B + \epsilon} + f_{i,j}^L \right) \\
b &= C_{n,i} - P_{n,i}, \quad C_{n,i} = 1, \quad \sum_{n' \in N} C_{n',i} = 1
\end{aligned}$$

Abandoned ideas

Transmission Cost:

$$C_T = \sum_{i \in N} \sum_{j \in N} f_{i,j}^L \sum_{z \in Z} \left(m_{l_z^S, l_z^D}^D \times R_{z,i,j} \right)$$

Constraint:

$$L_q^P = \sum_{i \in M} \sum_{j \in M} A_{q,i,j}^L \times \frac{m_{i,j}^{CPU}}{m_j^{Mips}}, \quad m_j^{Mips} \neq 0$$

$$L_q^T = \sum_{i \in M} \sum_{j \in M} A_{q,i,j}^L \sum_{l \in N} \sum_{k \in N} R_{z,l,k} \left(\frac{m_{i,j}^{NW}}{m_{i,j}^B} + f_{l,k}^L \right), \quad l_z^S = i, \quad l_z^D = j, \quad m_{i,j}^B \neq 0$$

$$L_q^M = \sum_{i \in M} A_i \times \left(\sum_{l \in N} \sum_{k \in N} V_{i,l,k} \left(\frac{m_i^{Strg} + m_i^{Mem}}{f_{l,k}^B + \epsilon} + f_{l,k}^L \right) + B_i \times t^{Boot} \right),$$

$$A_i = \min \left\{ \sum_{j \in M} A_{q,i,j}^L + A_{q,j,i}^L, \quad 1 \right\},$$

$$B_i = C_{n,i} - P_{n,i}, \quad C_{n,i} = 1, \quad \sum_{n' \in N} C_{n',i} = 1$$