

Multi-objective Optimization

Features:

- Distributed Data Flow
- Fog nodes mobility
- IoT devices mobility
- Migration support
- Partitioning techniques
- Data placement optimization
- Migration Optimization

Variables:

- QoS
 - latency
- Cost
 - CPU
 - RAM
 - MEM
 - BW
- Energy
 - busyPower
 - idlePower
- Bandwidth

Notations:

- N – fog nodes $S = \{s_1, \dots, s_n\}$
- M – modules $U = \{u_1, \dots, u_m\}$
- fP^{Mips} – matrix $1 \times N$ representing the MIPS price of each fog node per unit
- fP^{Ram} – matrix $1 \times N$ representing the memory price of each fog node per unit
- fP^{Strg} – matrix $1 \times N$ representing the storage price of each fog node per unit
- fP^{Bw} – matrix $1 \times N$ representing the bandwidth price of each fog node per unit
- f^{Mips} – matrix $N \times 1$ representing the MIPS capacity of each fog node
- f^{Ram} – matrix $N \times 1$ representing the memory capacity of each fog node
- f^{Strg} – matrix $N \times 1$ representing the storage capacity of each fog node
- f^{bPw} – matrix $N \times 1$ representing the busy power consumption of each fog node per unit
- f^{iPw} – matrix $N \times 1$ representing the idle power consumption of each fog node per unit
- f^{wPw} – matrix $N \times 1$ representing the weight of energy consumption (*willing_to_waste_energy*⁻¹)
- m^{Mips} – matrix $M \times 1$ representing the MIPS needed for each application's module
- m^{Ram} – matrix $M \times 1$ representing the memory needed for each application's module
- m^{Strg} – matrix $M \times 1$ representing the storage needed for each application's module
- m^{Bw} – matrix $M \times 1$ representing the bandwidth needed for each application's module
- K – total number of dependencies/edges
- e^{Cpu} – matrix $1 \times K$ representing the tuple CPU size (MI) needed to be processed
- e^{Nw} – matrix $1 \times K$ representing the tuple network size (MB) needed to be sent
- e^{Prob} – matrix $1 \times K$ representing the probability of sending the tuple
- e^{Pe} – matrix $1 \times K$ representing the periodicity of the producer (i.e., periodic sources)
- e^S – matrix $1 \times K$ representing the edge source
- e^D – matrix $1 \times K$ representing the edge destination
- Z – total number of dependencies/edges between different pairs of nodes
- l^S – matrix $N \times 1$ representing the starting nodes of each pair of nodes
- l^D – matrix $N \times 1$ representing the ending nodes of each pair of nodes
- mD – matrix $M \times M$ representing the dependencies between modules
- mB – matrix $M \times M$ representing the bandwidth needed between modules
- fL – matrix $N \times N$ representing the latency between each two direct nodes
- fB – matrix $N \times N$ representing the bandwidth between each two direct nodes
- D – matrix $N \times M$ representing the nodes where each module can be deployed
- P – matrix $N \times M$ representing the placement mapping between modules and nodes
- R – matrix $Z \times N \times N$ representing the routing map between modules
- e – matrix $1 \times N$, with all entries to 1
- α – operational weight
- β – energetic weight
- γ – processing weight
- δ – latency weight
- ζ – bandwidth weight

Preliminary computations:

$$\begin{aligned}
m_i^{Mips} &= \sum_{k \in K} \left(\frac{e_k^{Prob} e_k^{Cpu}}{e_k^{Pe}} \right), e_k^D = i \\
m_i^{Bw} &= \sum_{k \in K} \left(\frac{e_k^{Prob} e_k^{Nw}}{e_k^{Pe}} \right), e_k^S = i \\
m_{B_{i,j}} &= \sum_{k \in K} \left(\frac{e_k^{Prob} e_k^{Nw}}{e_k^{Pe}} \right), e_k^S = i, e_k^D = j \\
m_{D_{i,j}} &= \sum_{k \in K} \left(\frac{e_k^{Prob}}{e_k^{Pe}} \right), e_k^S = i, e_k^D = j
\end{aligned}$$

Problem formulation:

The cost function is mainly characterized by two components: Operational Cost and Service Quality Cost.

Operational Cost (C_O) is characterized by the resources allocated in each fog node to support all users' computations, namely: CPU, memory, storage, and bandwidth.

$$C_O = fP^{Mips} \times P \times m^{Mips} + fP^{Ram} \times P \times m^{Ram} + fP^{Strg} \times P \times m^{Strg} + \left(fP^{Bw} \sum_{z \in Z} m_{l_z^S, l_z^D} \times R_z \right) e'$$

Power Cost (C_{Pw}) is characterized by the busy/idle power in each fog node to support all users' computations as well as the willing to waste energy to prevent clients to process the whole application.

$$C_{Pw} = \sum_{n \in N} \left(f_n^{iPw} + f_n^{wPw} (f_n^{bPw} - f_n^{iPw}) \times \frac{P_n \times m^{Mips}}{f_n^{Mips}} \right)$$

Processing Cost (C_P) is characterized by the percentage of unused MIPS using the Jain's fairness index.

$$C_P = \frac{\left(\sum_{n \in N} \frac{P_n \times m^{Mips}}{f_n^{Mips}} \right)^2}{N \times \sum_{n \in N} \left(\frac{P_n \times m^{Mips}}{f_n^{Mips}} \right)} - OR - C_P = \sum_{n \in N} \frac{P_n \times m^{Mips}}{f_n^{Mips}}$$

Latency Cost (C_L) is characterized by the total latency on the tuple transmission.

$$C_L = e \left(fL. * \sum_{z \in Z} m_{l_z^S, l_z^D} \times R_z \right) e'$$

Bandwidth Cost (C_B) is characterized by the bandwidth usage on the tuple transmission.

$$C_B = e \left(1./(fB + \epsilon) . * \sum_{z \in Z} m_{l_z^S, l_z^D} \times R_z \right) e', \epsilon = 1^{-9}$$

Final problem:

$$\begin{aligned}
& \underset{P,R}{\text{minimize}} && C = \alpha C_O + \beta C_{Pw} + \gamma C_P + \delta C_L + \zeta C_B \\
& \text{subject to} && P \times m^{Mips} \leq f^{Mips}, \\
& && P \times m^{Ram} \leq f^{Ram}, \\
& && P \times m^{Strg} \leq f^{Strg}, \\
& && P \leq D \\
& && P_{i,j} \in \{0, 1\}, \forall i \in [0, N], \forall j \in [0, M] \\
& && \sum_{i \in N} P_{i,j} = 1, \forall j \in [0, M] \\
& && R_{z,i,j} \in \{0, 1\}, \forall z \in [0, Z], \forall i \in [0, N], \forall j \in [0, N] \\
& && \sum_{j \in N} R_{z,i,j} - \sum_{j \in N} R'_{z,i,j} = P_{i,l_z^S} - P_{i,l_z^D}, \forall z \in [0, Z], \forall i \in [0, N] \\
& && \sum_{z \in Z} mB_{l_z^S, l_z^D} \times R_{z,i,j} \leq fB_{i,j}, \forall i \in [0, N], \forall j \in [0, N]
\end{aligned}$$