**Multi-objective Optimization**

**Features:**

- Distributed Data Flow
- Fog nodes mobility
- IoT devices mobility
- Migration support
- Partitioning techniques
- Data placement optimization
- Migration Optimization

**Variables:**

- QoS
  - latency
- Cost
  - CPU
  - RAM
  - MEM
  - BW
- Energy
  - busyPower
  - idlePower
- Bandwidth

**Notations:**

- $N$ − fog nodes $S = \{s_1, ..., s_n\}$
- $M$ − modules $U = \{u_1, ..., u_m\}$

- $fP^{Mips}$ − matrix $1 \times N$ representing the MIPS price of each fog node per unit
- $fP^{Ram}$ − matrix $1 \times N$ representing the memory price of each fog node per unit
- $fP^{Strg}$ − matrix $1 \times N$ representing the storage price of each fog node per unit
- $fP^{Bw}$ − matrix $1 \times N$ representing the bandwidth price of each fog node per unit

- $f^{Mips}$ − matrix $N \times 1$ representing the MIPS capacity of each fog node
- $f^{Ram}$ − matrix $N \times 1$ representing the memory capacity of each fog node
- $f^{Strg}$ − matrix $N \times 1$ representing the storage capacity of each fog node
- $f^{bPw}$ = matrix $N \times 1$ representing the busy power consumption of each fog node per unit
- $f^{iPw}$ − matrix $N \times 1$ representing the idle power consumption of each fog node per unit

- $m^{Mips}$ − matrix $M \times 1$ representing the MIPS needed for each application's module
- $m^{Ram}$ − matrix $M \times 1$ representing the memory needed for each application's module
- $m^{Strg}$ − matrix $M \times 1$ representing the storage needed for each application's module

- $K$ − total number of dependencies/edges
- $e^{Cpu}$ − matrix $1 \times K$ representing the tuple CPU size (MI) needed to be processed
- $e^{Nw}$ − matrix $1 \times K$ representing the tuple network size (MB) needed to be sent
- $e^{Prob}$ − matrix $1 \times K$ representing the probability of sending the tuple
- $e^{Pe}$ − matrix $1 \times K$ representing the periodicity of the producer (i.e., periodic sources)
- $e^S$ − matrix $1 \times K$ representing the edge source
- $e^D$ − matrix $1 \times K$ representing the edge destination

- $Z$ − total number of dependencies/edges between different pairs of modules
- $l^S$ − matrix $N \times 1$ representing the starting nodes of each pair of modules
- $l^D$ − matrix $N \times 1$ representing the ending nodes of each pair of modules

- $mD$ − matrix $M \times M$ representing the dependencies between modules
- $mB$ − matrix $M \times M$ representing the bandwidth needed between modules

- $fL$ − matrix $N \times N$ representing the latency between each two direct nodes
- $fB$ − matrix $N \times N$ representing the bandwidth between each two direct nodes

- $D$ − matrix $N \times M$ representing the nodes where each module can be deployed
- $C$ − matrix $N \times M$ current module placement

- $e$ − matrix $1 \times N$ with all entries set to 1

- $\alpha$ − operational weight
- $\beta$ − energetic weight
- $\gamma$ − processing weight
- $\delta$ − transmission weight
- $\zeta$ − bandwidth weight
- $\eta$ − migration weight

**Variables:**

- $P$ – matrix $N \times M$ representing the placement mapping between modules and nodes
- $R$ – matrix $Z \times N \times N$ representing the tuple routing map between modules
- $V$ – matrix $M \times N \times N$ representing the VM routing map

**Preliminary computations:**

$$m_i^{Mips} = \sum_{k \in K} \left( \frac{e_k^{Prob} e_k^{Cpu}}{e_k^{Pe}} \right), e_k^D = i$$

$$mB_{i,j} = \sum_{k \in K} \left( \frac{e_k^{Prob} e_k^{Nw}}{e_k^{Pe}} \right), e_k^S = i,\ e_k^D = j$$

$$mD_{i,j} = \sum_{k \in K} \left( \frac{e_k^{Prob}}{e_k^{Pe}} \right), e_k^S = i,\ e_k^D = j$$

**Problem formulation:**

**Operational Cost** ($C_O$)

$$C_O = fP^{Mips} \times P \times m^{Mips} + fP^{Ram} \times P \times m^{Ram} + fP^{Strg} \times P \times m^{Strg} + \left( fP^{Bw} \sum_{z \in Z} mB_{l_z^S, l_z^D} \times R_z \right) e'$$

**Power Cost** ($C_{Pw}$)

$$\begin{aligned} C_{Pw} = &\sum_{n \in N} \left( f_n^{iPw} + (f_n^{bPw} - f_n^{iPw}) \times \frac{P_n \times m^{Mips}}{f_n^{Mips}} \right) + \\ &\sum_{i \in N} \sum_{j \in N} f_{i,j}^{Tx} \left[ fL_{i,j} \sum_{z \in Z} \left( mD_{l_z^S, l_z^D} \times R_{z,i,j} \right) + \frac{\sum_{z \in Z} \left( mB_{l_z^S, l_z^D} \times R_{z,i,j} \right)}{fB_{i,j}} \right] \\ &\sum_{i \in N} \sum_{j \in N} f_{i,j}^{Tx} \left[ \frac{\sum_{z \in M} \left( m_z^{Mips} + \times m_z^{Ram} \right) \times V_{z,i,j}}{fB_{i,j}} + fL_{i,j} \sum_{z \in M} \times V_{z,i,j} \right] \end{aligned}$$

**Processing Cost** ($C_P$)

$$C_P = \frac{\left( \sum_{n \in N} \frac{P_n \times m^{Mips}}{f_n^{Mips}} \right)^2}{N \times \sum_{n \in N} \left( \frac{P_n \times m^{Mips}}{f_n^{Mips}} \right)^2} \quad -OR- \quad C_P = \sum_{n \in N} \frac{P_n \times m^{Mips}}{f_n^{Mips}}$$

**Transmission Cost** ($C_T$)

$$C_T = \sum_{i \in N} \sum_{j \in N} fL_{i,j} \sum_{z \in Z} \left( mD_{l_z^S, l_z^D} \times R_{z,i,j} \right)$$

**Bandwidth Cost** ($C_B$)

$$C_B = \sum_{i \in N} \sum_{j \in N} \frac{\sum_{z \in Z} \left( mB_{l_z^S, l_z^D} \times R_{z,i,j} \right)}{fB_{i,j}}$$

**Migration Cost** ($C_M$)

$$C_M = \sum_{i \in N} \sum_{j \in N} \left[ \frac{\sum_{z \in M} \left( m_z^{Mips} + \times m_z^{Ram} \right) \times V_{z,i,j}}{fB_{i,j}} + fL_{i,j} \sum_{z \in M} \times V_{z,i,j} \right]$$

**Final problem:**

$$
\begin{aligned}
\underset{P,R}{\text{minimize}} \quad & C = \alpha C_O + \beta C_{Pw} + \gamma C_P + \delta C_T + \zeta C_B + \eta C_M \\
\text{subject to} \quad & P \times m^{Mips} \leq f^{Mips}, \\
& P \times m^{Ram} \leq f^{Ram}, \\
& P \times m^{Strg} \leq f^{Strg}, \\
& P \leq D \\
& P_{i,j} \in \{0,1\}, \forall i \in [0,N], \forall j \in [0,M] \\
& \sum_{i \in N} P_{i,j} = 1, \forall j \in [0,M] \\
& R_{z,i,j} \in \{0,1\}, \forall z \in [0,Z], \forall i \in [0,N], \forall j \in [0,N] \\
& \sum_{j \in N} R_{z,i,j} - \sum_{j \in N} R'_{z,i,j} = P_{i,l_z^S} - P_{i,l_z^D} \quad, \forall z \in [0,Z], \forall i \in [0,N] \\
& \sum_{z \in Z} mB_{l_z^S, l_z^D} \times R_{z,i,j} \leq fB_{i,j} \quad, \forall i \in [0,N], \forall j \in [0,N] \\
& \sum_{j \in N} V_{z,i,j} - \sum_{j \in N} V'_{z,i,j} = C_{j,i} - P_{j,i} \quad, \forall z \in [0,M], \forall i \in [0,N]
\end{aligned}
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