**Multi-objective Optimization**

**Notations:**

- $N$ – number of nodes
- $M$ – number of modules
- $K$ – number of dependencies
- $Z$ – number of module pairs with dependencies
- $Q$ – number of loop deadlines

- $P^{Mips}$ [€] , $1 \times N$ matrix representing the price of using processing resources in each fog node
- $P^{Ram}$ [€], $1 \times N$ matrix representing the price of using memory resources in each fog node
- $P^{Strg}$ [€], $1 \times N$ matrix representing the price of using storage resources in each fog node
- $P^{Bw}$ [€], $1 \times N$ matrix representing the price of using network resources in each fog node
- $P^{Pw}$ [€], $1 \times N$ matrix representing the power price in each fog node

- $f^{Mips}$ $[MIPS]$, $N \times 1$ matrix representing the processing capacity of each fog node
- $f^{Ram}$ $[Byte]$, $N \times 1$ matrix representing the memory capacity of each fog node
- $f^{Strg}$ $[Byte]$, $N \times 1$ matrix representing the storage capacity of each fog node
- $f^{bPw}$ $[W]$, $N \times 1$ matrix representing the busy power consumption of each fog node
- $f^{iPw}$ $[W]$, $N \times 1$ matrix representing the idle power consumption of each fog node
- $f^{Tx}$ $[W]$, $N \times 1$ matrix representing the transmitter power ($f_n^{Tx}$ is non 0 if $n$ is a mobile node)

- $m^{Mips}$ $[MIPS]$, $M \times 1$ matrix representing the processing resources needed for each module
- $m^{Ram}$ $[Byte]$, $M \times 1$ matrix representing the memory resources needed for each module
- $m^{Strg}$ $[Byte]$, $M \times 1$ matrix representing the storage resources needed for each module

- $e^{Cpu}$ $[MI]$, $K \times 1$ matrix representing the tuple CPU size needed to be processed for each dependency
- $e^{Nw}$ $[Byte]$, $K \times 1$ matrix representing the tuple network size needed to be sent for each dependency
- $e^{Pe}$ $[s]$, $K \times 1$ matrix representing the periodicity of sending the tuple for each dependency
- $e^{Prob}$ [%], $K \times 1$ matrix representing the probability of sending the tuple for each dependency
- $e^S$, $K \times 1$ matrix representing the source module for each dependency
- $e^D$, $K \times 1$ matrix representing the destination module for each dependency

- $l^S$, $Z \times 1$ matrix representing the source module for each pair of modules with dependencies
- $l^D$, $Z \times 1$ matrix representing the destination module for each pair of modules with dependencies

- $m^D$, $M \times M$ matrix representing the dependencies between modules
- $m^B$, $M \times M$ matrix representing the bandwidth needed between modules

  item $f^L$, $N \times N$ matrix representing the link latency between each two nodes
- $f^B$, $N \times N$ matrix representing the link bandwidth between each two nodes

- $D$, $N \times M$ binary matrix representing the nodes where each module can be deployed
- $C$, $N \times M$ binary matrix representing the current module placement

- $b$, constant representing the percentage of link bandwidth allocated for sending tuples
- $\epsilon$ small constant

**Variables:**

- $P$, $N \times M$ binary matrix representing the placement mapping between modules and nodes
- $R$, $Z \times N \times N$ binary matrix representing the tuple routing map between modules
- $V$, $M \times N \times N$ binary matrix representing the module migration routing map

**Preliminary computations:**

$$m_i^{Mips} = \sum_{k \in K} \frac{e_k^{Prob} e_k^{Cpu}}{e_k^{Pe}}, \; e_k^D = i$$

$$m_{i,j}^B = \sum_{k \in K} \frac{e_k^{Prob} e_k^{Nw}}{e_k^{Pe}}, \; e_k^S = i, \; e_k^D = j$$

$$m_{i,j}^D = \sum_{k \in K} \frac{e_k^{Prob}}{e_k^{Pe}}, \; e_k^S = i, \; e_k^D = j$$

**Objectives:**

**Operational Cost** $(C_O)$

$$C_O = P^{Mips} \times P \times m^{Mips} + P^{Ram} \times P \times m^{Ram} + P^{Strg} \times P \times m^{Strg} + \left( P^{Bw} \sum_{z \in Z} m^B_{l_z^S, l_z^D} \times R_z \right) 1N + C_{Pw}(P^{Pw'})$$

**Power Cost** $(C_{Pw})$

$$C_{Pw}(x) = C_P(x. \times (f^{bPw} - f^{iPw})) + C_B(x. \times f^{Tx}),$$
$$x = 1N$$

**Processing Cost** $(C_P)$

$$C_P(x) = \sum_{n \in N} x_n \frac{P_n \times m^{Mips}}{f_n^{Mips}},$$
$$x = 1N$$

**Transmission Cost** $(C_T)$

$$C_T = \sum_{i \in N} \sum_{j \in N} f_{i,j}^L \sum_{z \in Z} \left( m^D_{l_z^S, l_z^D} \times R_{z,i,j} \right)$$

**Bandwidth Cost** $(C_B)$

$$C_B(x) = \sum_{z \in Z} m^B_{l_z^S, l_z^D} \sum_{i \in N} x_i \sum_{j \in N} \frac{R_{z,i,j}}{f_{i,j}^B \times b + \epsilon},$$
$$x = 1N$$

**Migration Cost** $(C_M)$

$$C_M = \sum_{i \in N} \sum_{j \in N} \sum_{m \in M} V_{m,i,j} \times \left( \frac{m_m^{Strg} + m_m^{Ram}}{f_{i,j}^B \times (1 - b)} + f_{i,j}^L \right)$$

**Final problem:**

$$
\begin{aligned}
\underset{P,R}{\text{minimize}} \quad & C_O,\ C_E,\ C_P,\ C_T,\ C_B,\ C_M \\
\text{subject to} \quad & P \times m^{Mips} \leq f^{Mips}, \\
& P \times m^{Ram} \leq f^{Ram}, \\
& P \times m^{Strg} \leq f^{Strg}, \\
& P \leq D \\
& P_{i,j} \in \{0,1\},\ \forall i \in [0,N],\ \forall j \in [0,M] \\
& R_{z,i,j} \in \{0,1\},\ \forall z \in [0,Z],\ \forall i \in [0,N],\ \forall j \in [0,N] \\
& V_{z,i,j} \in \{0,1\},\ \forall z \in [0,M],\ \forall i \in [0,N],\ \forall j \in [0,N] \\
& \sum_{i \in N} P_{i,j} = 1,\ \forall j \in [0,M] \\
& \sum_{j \in N} R_{z,i,j} - \sum_{j \in N} R'_{z,i,j} = P_{i,l_z^S} - P_{i,l_z^D} \quad ,\forall z \in [0,Z],\ \forall i \in [0,N] \\
& \sum_{z \in Z} m^B_{l_z^S,l_z^D} \times R_{z,i,j} \leq f^B_{i,j} \times b \quad ,\ \forall i \in [0,N],\ \forall j \in [0,N] \\
& \sum_{j \in N} V_{z,i,j} - \sum_{j \in N} V'_{z,i,j} = C_{j,i} - P_{j,i} \quad ,\ \forall z \in [0,M],\ \forall i \in [0,N]
\end{aligned}
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