Multi-objective Optimization

Features:

- Distributed Data Flow
- Fog nodes mobility
- IoT devices mobility
- Migration support
- Partitioning techniques
- Data placement optimization
- Migration Optimization

Variables:

- QoS
    - latency
- Cost
    - CPU
    - RAM
    - MEM
    - BW
- Energy
    - busyPower
    - idlePower
- Bandwidth

Notations:

- N − fog nodes $S = \{s_1, ..., s_n\}$
- M − modules $U = \{u_1, ..., u_m\}$

- fP^{Mips} − matrix $1 \times N$ representing the MIPS price of each fog node per unit
- fP^{Ram} − matrix $1 \times N$ representing the memory price of each fog node per unit
- fP^{Strg} − matrix $1 \times N$ representing the storage price of each fog node per unit
- fP^{Bw} − matrix $1 \times N$ representing the bandwidth price of each fog node per unit

- f^{Mips} − matrix $N \times 1$ representing the MIPS capacity of each fog node
- f^{Ram} − matrix $N \times 1$ representing the memory capacity of each fog node
- f^{Strg} − matrix $N \times 1$ representing the storage capacity of each fog node
- f^{Bw} − matrix $N \times 1$ representing the bandwidth capacity of each fog node
- f^{bPw} = matrix $N \times 1$ representing the busy power consumption of each fog node per unit
- f^{iPw} − matrix $N \times 1$ representing the idle power consumption of each fog node per unit
- f^{wPw} − matrix $N \times 1$ representing the weight of energy consumption ($willing\_to\_waste\_energy^{-1}$)

- m^{Mips} − matrix $M \times 1$ representing the MIPS needed for each application's module
- m^{Ram} − matrix $M \times 1$ representing the memory needed for each application's module
- m^{Strg} − matrix $M \times 1$ representing the storage needed for each application's module
- m^{Bw} − matrix $M \times 1$ representing the bandwidth needed for each application's module

- K − total number of dependencies/edges
- e^{Cpu} − matrix $1 \times K$ representing the tuple CPU size (MI) needed to be processed
- e^{Nw} − matrix $1 \times K$ representing the tuple network size (MB) needed to be sent
- e^{Prob} − matrix $1 \times K$ representing the probability of sending the tuple
- e^{Pe} − matrix $1 \times K$ representing the periodicity of the producer (i.e., periodic sources)
- e^{S} − matrix $1 \times K$ representing the edge source
- e^{D} − matrix $1 \times K$ representing the edge destination

- Z − total number of dependencies/edges between different pairs of nodes
- l^{S} − matrix $N \times 1$ representing the starting nodes of each pair of nodes
- l^{D} − matrix $N \times 1$ representing the ending nodes of each pair of nodes

- mD − matrix $M \times M$ representing the dependencies between modules
- mB − matrix $M \times M$ representing the bandwidth needed between modules

- fL − matrix $N \times N$ representing the latency between each two direct nodes
- fB − matrix $N \times N$ representing the bandwidth between each two direct nodes

- D − matrix $N \times M$ representing the nodes where each module can be deployed

- P − matrix $N \times M$ representing the placement mapping between modules and nodes
- R − matrix $K \times N \times N$ representing the routing map between modules

- e − matrix 1xN, with all entries to 1

- α − operational weight
- β − energetic weight
- γ − processing weight
- δ − transmission weight
- ζ − transition cost

Preliminary computations:

$$
\begin{aligned}
m_i^{Mips} &= \sum_{k \in K} \left(\frac{e_k^{Prob} e_k^{Cpu}}{e_k^{Pe}} \right), e_k^D = i \\
m_i^{Bw} &= \sum_{k \in K} \left(\frac{e_k^{Prob} e_k^{Nw}}{e_k^{Pe}} \right), e_k^S = i \\
mB_{i,j} &= \sum_{k \in K} \left(\frac{e_k^{Prob} e_k^{Nw}}{e_k^{Pe}} \right), e_k^S = i,\ e_k^D = j \\
mD_{i,j} &= \sum_{k \in K} \left(\frac{e_k^{Prob}}{e_k^{Pe}} \right), e_k^S = i,\ e_k^D = j
\end{aligned} \tag{1}
$$

Problem formulation:

The cost function is mainly characterized by two components: Operational Cost and Service Quality Cost.

Operational Cost (C_O) is characterized by the resources allocated in each fog node to support all users' computations, namely: CPU, memory, storage, bandwidth.

$$
C_O = fP^{Mips} \times P \times m^{Mips} + fP^{Ram} \times P \times m^{Ram} + fP^{Strg} \times P \times m^{Strg} + e \sum_{z \in Z} R_z \times (fP^{Bw})' \times mB_{l_z^S, l_z^D} \tag{2}
$$

Energetic Cost (C_E) is characterized by the busy/idle power in each fog node to support all users' computations as well as the willing to wast energy to prevent users to process the whole application.

$$
C_E = \left(f^{wPw} .* \left(f^{bPw} - f^{iPw}\right)\right)' \times \frac{P \times m^{Mips}}{f^{Mips}}
$$

Processing Cost (C_P) is characterized by the percentage of unused MIPS.

$$
C_P = e \frac{P \times m^{Mips}}{f^{Mips}}
$$

Transmission Cost (C_T) is characterized by the total latency of tuple transmission.

$$
C_T = e \left(\sum_{z \in Z} mD_{l_z^S, l_z^D} \times R_z .* fL + mB_{l_z^S, l_z^D} \times R_z ./ (fL + \epsilon) \right) e' \tag{3}
$$

Final problem:

$$
\begin{aligned}
\underset{P,R}{\text{minimize}} \quad & C = \alpha C_O + \beta C_E + \gamma C_P + \delta C_T + \zeta \times R \\
\text{subject to} \quad & P \times m^{Mips} \leq f^{Mips}, \\
& P \times m^{Ram} \leq f^{Ram}, \\
& P \times m^{Strg} \leq f^{Strg}, \\
& P \times m^{Bw} \leq f^{Bw}, \\
& P \leq D \\
& P_{i,j} \in \{0,1\}, \forall i \in [0,N], \forall j \in [0,M], \\
& \sum_{i \in N} P_{i,j} = 1, \forall j \in [0,M] \\
& R_{z,i,j} \in \{0,1\}, \forall z \in [0,Z], \forall i \in [0,N], \forall j \in [0,N], \\
& \sum_{j \in N} R_{z,i,j} - \sum_{j \in N} R'_{z,i,j} = P_{i,l_z^S} - P_{i,l_z^D} \quad, \forall z \in [0,Z], \forall i \in [0,N]
\end{aligned}
$$