

Notation:

- N – number of nodes
- M – number of modules
- K – number of dependencies
- Z – number of module pairs with dependencies
- Q – number of loop deadlines
- E – number of network links

- P^{Mips} [€], $1 \times N$ matrix representing the price of using processing resources in each node
- P^{Mem} [€], $1 \times N$ matrix representing the price of using memory resources in each node
- P^{Strg} [€], $1 \times N$ matrix representing the price of using storage resources in each node
- P^{Bw} [€], $1 \times N$ matrix representing the price of using network resources in each node
- P^{Pw} [€], $1 \times N$ matrix representing the power price in each node

- f^{Fog} , $N \times 1$ binary matrix representing whether each node is a client or not (f_n^{Fog} is 0 if n is a client)
- f^{Mips} [MIPS], $N \times 1$ matrix representing the processing capacity of each node
- f^{Mem} [Byte], $N \times 1$ matrix representing the memory capacity of each node
- f^{Strg} [Byte], $N \times 1$ matrix representing the storage capacity of each node
- f^{bPw} [W], $N \times 1$ matrix representing the busy power consumption of each node
- f^{iPw} [W], $N \times 1$ matrix representing the idle power consumption of each node
- f^{Tx} [W], $N \times 1$ matrix representing the transmitter power (f_n^{Tx} is non 0 if n is a mobile node)

- m^{Mips} [MIPS], $M \times 1$ matrix representing the processing resources needed for each module
- m^{Mem} [Byte], $M \times 1$ matrix representing the memory resources needed for each module
- m^{Strg} [Byte], $M \times 1$ matrix representing the storage resources needed for each module
- m^{MigD} [s], $M \times 1$ matrix representing the migration deadline for each module

- e^{Cpu} [MI], $K \times 1$ matrix representing the tuple CPU size needed to be processed for each dependency
- e^{Nw} [Byte], $K \times 1$ matrix representing the tuple network size needed to be sent for each dependency
- e^{Pe} [s], $K \times 1$ matrix representing the periodicity of sending the tuple for each dependency
- e^{Prob} , $K \times 1$ matrix representing the probability of sending the tuple for each dependency
- e^S , $K \times 1$ matrix representing the source module for each dependency
- e^D , $K \times 1$ matrix representing the destination module for each dependency

- l^S , $Z \times 1$ matrix representing the source module for each pair of modules with dependencies
- l^D , $Z \times 1$ matrix representing the destination module for each pair of modules with dependencies

- m^{Bw} [Byte/s], $M \times M$ matrix representing the bandwidth needed between modules
- m^{CPU} [MI], $M \times M$ matrix representing the CPU size of dependencies between modules
- m^{NW} [Byte], $M \times M$ matrix representing the network size of dependencies between modules

- E^S , $E \times 1$ matrix representing the source node for each network link
- E^D , $E \times 1$ matrix representing the destination node for each network link
- E^L [s], $E \times 1$ matrix representing the link latency between each two nodes
- E^{Bw} [Byte/s], $E \times 1$ matrix representing the link bandwidth between each two nodes

- D , $N \times M$ binary matrix representing the nodes where each module can be deployed
- C , $N \times M$ binary matrix representing the current module placement

- A^L , $Q \times M \times M$ binary matrix representing the loop module list
- A^D [s], $Q \times 1$ matrix representing the loop deadline list
- A^P [€], $Q \times 1$ matrix representing the price for not accomplishing the loop deadline
- α^p , the percentage of processing capacity which is not used for control operations
- α^m , the percentage of memory capacity which is not used for control operations
- α^s , the percentage of storage capacity which is not used for control operations
- α^b , the percentage of bandwidth capacity which is not used for control operations
- t^{Boot} [s], constant representing the average virtual machine boot time

Variables:

- P , $N \times M$ binary matrix representing the module placement
- R , $Z \times E$ binary matrix representing the tuple routing map
- V , $M \times E$ binary matrix representing the module migration routing map

Preliminary computations:

$$m_i^{Mips} = \sum_{k \in K} \frac{e_k^{Prob} e_k^{Cpu}}{e_k^{Pe}}, \quad e_k^D = i$$

$$m_{i,j}^{Bw} = \sum_{k \in K} \frac{e_k^{Prob} e_k^{Nw}}{e_k^{Pe}}, \quad e_k^S = i, \quad e_k^D = j$$

$$m_{i,j}^{CPU} = \sum_{k \in K} e_k^{Cpu}, \quad e_k^S = i, \quad e_k^D = j$$

$$m_{i,j}^{NW} = \sum_{k \in K} e_k^{Nw}, \quad e_k^S = i, \quad e_k^D = j$$

Objectives:

Operational Cost:

$$C_O = P^{Mips} \times P \times m^{Mips} + P^{Mem} \times P \times m^{Mem} + P^{Strg} \times P \times m^{Strg} + \left(P^{Bw} \sum_{z \in Z} m_{l_z^S, l_z^D}^{Bw} \times R_z \right) 1N + C_{Pw}(P^{Pw'}) + C_Q(A^P)$$

QoS Cost:

$$C_Q(x) = \sum_{q \in Q} a_q x_q, \\ a_q = \begin{cases} 1, & \text{if } L_q^P + L_q^T > A_q^D; \\ 0, & \text{otherwise.} \end{cases}, \quad \forall q \in [0, Q]$$

Power Cost:

$$C_{Pw}(x) = C_P(x \times (f^{bPw} - f^{iPw})) + C_B(x \times f^{Tx})$$

Processing Cost:

$$C_P(x) = \sum_{n \in N} f_n^{Fog} x_n \frac{P_n \times m^{Mips}}{\alpha^P \times f_n^{Mips}}$$

Bandwidth Cost:

$$C_B(x) = \sum_{z \in Z} m_{l_z^S, l_z^D}^{Bw} \sum_{e \in E} f_i^{Fog} x_i \frac{R_{z,e}}{\alpha^b \times E_e^{Bw}} \\ i = E_e^S$$

Migration Cost:

$$C_M(x) = \sum_{m \in M} s_m \sum_{e \in E} f_i^{Fog} x_i \frac{V_{m,e}}{(1 - \alpha^b) \times E_e^{Bw}} \\ i = E_e^S, \quad s_m = m_m^{Strg} + m_m^{Mem}$$

Multiple-objective (Fog service provider):

$$F = \{C_Q(1N), C_{Pw}(1N), C_P(1N), C_B(1N), C_M(1N)\}$$

Single-objective (Broker):

$$F = C_O$$

Final problem:

$$\underset{P,R,V}{\text{minimize}} \quad F$$

$$\text{subject to} \quad P_{n,m} \in \{0,1\}, \forall n \in [0, N], \forall m \in [0, M]$$

$$R_{z,e} \in \{0,1\}, \forall z \in [0, Z], \forall e \in [0, E]$$

$$V_{m,e} \in \{0,1\}, \forall m \in [0, M], \forall e \in [0, E]$$

$$P \times m^{Mips} \leq \alpha^p \times f^{Mips}$$

$$P \times m^{Mem} \leq \alpha^m \times f^{Mem}$$

$$P \times m^{Strg} \leq \alpha^s \times f^{Strg}$$

$$\sum_{z \in Z} m_{l_z^S, l_z^D}^{Bw} \times R_{z,e} \leq \alpha^b \times E_e^{Bw}, \forall e \in [0, E]$$

$$\sum_{n \in N} P_{n,m} = 1, \forall m \in [0, M]$$

$$P \leq D$$

$$\sum_{i \in E} R_{z,i} - \sum_{j \in E} R_{z,j} = P_{n,l_z^S} - P_{n,l_z^D}, \forall z \in [0, Z], \forall n \in [0, N], E_i^S = n, E_j^D = n$$

$$\sum_{i \in E} V_{m,i} - \sum_{j \in E} V_{m,j} = C_{n,m} - P_{n,m}, \forall m \in [0, M], \forall n \in [0, N], E_i^S = n, E_j^D = n$$

$$L_m^M \leq m_m^{MigD}, \forall m \in [0, M]$$

where :

$$L_q^P = \sum_{i \in M} \sum_{j \in M} A_{q,i,j}^L \sum_{n \in N} P_{n,j} \frac{\sum_{l \in M} \sum_{k \in M} m_{l,k}^{CPU} \times P_{n,k}}{\alpha^p \times f_n^{Mips}}$$

$$L_q^T = \sum_{i \in M} \sum_{j \in M} A_{q,i,j}^L \sum_{e \in E} R_{z',e} \left(\frac{\sum_{z \in Z} R_{z,e} \times m_{l_z^S, l_z^D}^{NW}}{\alpha^b \times E_e^{Bw}} + E_e^L \right)$$

$$l_{z'}^S = i, l_{z'}^D = j$$

$$L_m^M = b \times t^{Boot} + \sum_{e \in E} V_{m,e} \left(\frac{m_m^{Strg} + m_m^{Mem}}{(1 - \alpha^b) \times E_e^{Bw}} + E_e^L \right)$$

$$b = C_{n,m} - P_{n,m}, C_{n,m} = 1, \sum_{n' \in N} C_{n',m} = 1$$