Single-objective Optimization

Notations:

- N − number of nodes
- M − number of modules
- K − number of dependencies
- Z − number of dependencies between different pairs of modules
- Q − number of loop deadlines

- P^{Mips} − matrix $1 \times N$ representing the MIPS price of each fog node per unit
- P^{Ram} − matrix $1 \times N$ representing the memory price of each fog node per unit
- P^{Strg} − matrix $1 \times N$ representing the storage price of each fog node per unit
- P^{Bw} − matrix $1 \times N$ representing the bandwidth price of each fog node per unit
- P^{En} − matrix $1 \times N$ representing the energy price of each fog node per unit

- f^{Mips} − matrix $N \times 1$ representing the MIPS capacity of each fog node
- f^{Ram} − matrix $N \times 1$ representing the memory capacity of each fog node
- f^{Strg} − matrix $N \times 1$ representing the storage capacity of each fog node
- f^{bPw} = matrix $N \times 1$ representing the busy power consumption of each fog node per unit
- f^{iPw} − matrix $N \times 1$ representing the idle power consumption of each fog node per unit
- f^{Tx} − matrix $N \times 1$ representing the transmitter power (f_n^{Tx} is non 0 if n is a mobile node)

- m^{Mips} − matrix $M \times 1$ representing the MIPS needed for each application's module
- m^{Ram} − matrix $M \times 1$ representing the memory needed for each application's module
- m^{Strg} − matrix $M \times 1$ representing the storage needed for each application's module

- e^{Cpu} − matrix $1 \times K$ representing the tuple CPU size (MI) needed to be processed
- e^{Nw} − matrix $1 \times K$ representing the tuple network size (MB) needed to be sent
- e^{Prob} − matrix $1 \times K$ representing the probability of sending the tuple
- e^{Pe} − matrix $1 \times K$ representing the periodicity of the producer (i.e., periodic sources)
- e^{S} − matrix $1 \times K$ representing the edge source
- e^{D} − matrix $1 \times K$ representing the edge destination

- l^{S} − matrix $N \times 1$ representing the starting nodes of each pair of modules
- l^{D} − matrix $N \times 1$ representing the ending nodes of each pair of modules

- m^{D} − matrix $M \times M$ representing the dependencies between modules
- m^{B} − matrix $M \times M$ representing the bandwidth needed between modules

- f^{L} − matrix $N \times N$ representing the latency between each two nodes
- f^{B} − matrix $N \times N$ representing the bandwidth between each two nodes

- D − matrix $N \times M$ representing the nodes where each module can be deployed
- C − matrix $N \times M$ current module placement

- A^{L} − matrix $Q \times M$ representing the loop module list
- A^{D} − matrix $Q \times 1$ representing the loop deadline list

- e − matrix $1 \times N$ with all entries set to 1

- t^{Boot} − constant representing the average virtual machine boot time
- t^{Min} − constant representing the minimum time interval between events
- b − constant representing the percentage of bandwidth used for periodic tuples

Variables:

- P − matrix $N \times M$ representing the placement mapping between modules and nodes
- R − matrix $Z \times N \times N$ representing the tuple routing map between modules
- V − matrix $M \times N \times N$ representing the VM routing map

Preliminary computations:

$$m_i^{Mips} = \sum_{k \in K} \left(\frac{e_k^{Prob} e_k^{Cpu}}{e_k^{Pe}} \right), e_k^D = i$$

$$m_{i,j}^B = \sum_{k \in K} \left(\frac{e_k^{Prob} e_k^{Nw}}{e_k^{Pe}} \right), e_k^S = i,\ e_k^D = j$$

$$m_{i,j}^D = \sum_{k \in K} \left(\frac{e_k^{Prob}}{e_k^{Pe}} \right), e_k^S = i,\ e_k^D = j$$

Problem formulation:

Operational Cost (C_O)

$$C_O = P^{Mips} \times P \times m^{Mips} + P^{Ram} \times P \times m^{Ram} + P^{Strg} \times P \times m^{Strg} + \left(P^{Bw} \sum_{z \in Z} m^B_{l_z^S, l_z^D} \times R_z \right) e' + C_{Pw} \ ,$$

$$where \ :$$

$$C_{Pw} = \sum_{i \in N} P_i^{En} \left[f_i^{iPw} + (f_i^{bPw} - f_i^{iPw}) \times \frac{P_i \times m^{Mips}}{f_i^{Mips}} + f_i^{Tx} \sum_{j \in N} \sum_{z \in Z} R_{z,i,j} \times \left(\frac{m^B_{l_z^S, l_z^D}}{f_{i,j}^B \times b} + f_{i,j}^L \times m^D_{l_z^S, l_z^D} \right) \right]$$

Final problem:

$$
\begin{aligned}
\underset{P,R}{\text{minimize}} \quad & C_O \\
\text{subject to} \quad & P \times m^{Mips} \leq f^{Mips}, \\
& P \times m^{Ram} \leq f^{Ram}, \\
& P \times m^{Strg} \leq f^{Strg}, \\
& P \leq D \\
& P_{i,j} \in \{0,1\},\ \forall i \in [0,N],\ \forall j \in [0,M] \\
& R_{z,i,j} \in \{0,1\},\ \forall z \in [0,Z],\ \forall i \in [0,N],\ \forall j \in [0,N] \\
& V_{z,i,j} \in \{0,1\},\ \forall z \in [0,M],\ \forall i \in [0,N],\ \forall j \in [0,N] \\
& \sum_{i \in N} P_{i,j} = 1,\ \forall j \in [0,M] \\
& \sum_{j \in N} R_{z,i,j} - \sum_{j \in N} R'_{z,i,j} = P_{i,l_z^S} - P_{i,l_z^D} \quad ,\forall z \in [0,Z],\ \forall i \in [0,N] \\
& \sum_{z \in Z} m^B_{l_z^S,l_z^D} \times R_{z,i,j} \leq f^B_{i,j} \times b \quad ,\ \forall i \in [0,N],\ \forall j \in [0,N] \\
& \sum_{j \in N} V_{z,i,j} - \sum_{j \in N} V'_{z,i,j} = C_{j,i} - P_{j,i} \quad ,\ \forall z \in [0,M],\ \forall i \in [0,N] \\
& L_q^P + L_q^T + L_q^M < A_q^D,\ \forall q \in [0,Q]
\end{aligned}
$$

$where:$

$$
L_q^P = \sum_{i \in M} A^L_{q,i} \times \left(\frac{m_i^{Mips}}{\sum_{j \in N} \frac{f_j^{Mips} \times P_{j,i}}{\sum_{l \in M} P_{j,i} \times P_{j,l}}} + t^{Min} \right)
$$

$$
\begin{aligned}
L_q^T = \sum_{i=0}^{M-1} \sum_{j=i+1}^{M} A^L_{q,i}\ A^L_{q,j} \sum_{l \in N} \sum_{k \in N} & \left(fL_{l,k} \times R_{z,l,k} + \frac{m^B_{l_z^S,l_z^D} \times R_{z,l,k}}{f^B_{l,k} \times b - \sum_{r \in Z} m^B_{l_r^S,l_r^D} \times R_{r,l,k}} \right), \\
& i < j, \\
& \sum_{t=i+1}^{j-1} A^L_{q,t} = 0 \\
& \sum_{t_2 \in N} R_{z,t_1,t_2} - \sum_{t_2 \in N} R'_{z,t_1,t_2} = 1,\ t_1 = \sum_{t \in N} t \times P_{t,i},\ z \in Z \\
& \sum_{t_2 \in N} R_{z,t_1,t_2} - \sum_{t_2 \in N} R'_{z,t_1,t_2} = -1,\ t_1 = \sum_{t \in N} t \times P_{t,j},\ z \in Z \\
& l_r^S \neq l_z^S \cap l_r^D \neq l_z^D
\end{aligned}
$$

$$
\begin{aligned}
L_q^M = \sum_{i \in M} A^L_{q,i} \times & \left(\sum_{l \in N} \sum_{k \in N} \left(fL_{l,k} \times V_{z,l,k} + \frac{(m_z^{Strg} + \times m_z^{Ram}) \times V_{z,l,k}}{f^B_{l,k} \times (1-b)} \right) + t^{Boot} \right), \\
& \sum_{t_2 \in N} V_{z,t_1,t_2} - \sum_{t_2 \in N} V'_{z,t_1,t_2} = 1,\ t_1 = \sum_{t \in N} t \times C_{t,i},\ z \in M \\
& \sum_{t_2 \in N} V_{z,t_1,t_2} - \sum_{t_2 \in N} V'_{z,t_1,t_2} = -1,\ t_1 = \sum_{t \in N} t \times P_{t,i},\ z \in M
\end{aligned}
$$