Senior Thesis: Comparison of Elastic, Momentum, and Differential Cross Sections of Various Systems

Josiah Claypool
Under the Guidance of Dr. Quintanilla

University of North Texas

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Abstract

Elastic, momentum transfer and differential cross sections are computed for positron-hydrogen, positron-helium, electron-helium, positronium-helium, and electron-positronium using already existing phase shifts. An examination and comparison is made between the systems. All results present were obtained via Python.

Intro

Helium was chosen as the focused target because of its compromise between theoretical and experimental ease. Of particular interest for comparison is the behavior of positronium-helium scattering. Even though the composition of positronium is a bound state of an electron and the positron, it has been shown experimentally that the elastic cross section of positronium-helium is very similar to electron-helium [15] compared to the disparity between positronium-helium and positron-helium. What is also examined here is the potential similarity of these systems' differential cross sections. The range of energies examined are constrained such that only the elastic channel is open; there is no possibility of excitation. All phase shifts used throughout were obtained from the literature or private communication, none were calculated.

Theory

The total cross section, σ, is a measure of the likelihood of a scattering event and is expressed as an effective area. Incident particles go through an infinitesimal area $d\sigma$ and are then scattered though an infinitesimal solid angle $d\Omega$. The differential cross section is the ratio between them and is given by:

$$\frac{d\sigma}{d\Omega} = |f(k,\theta,\phi)|^2$$

$f(k,\theta,\phi)$ is the scattering amplitude. It can be shown that:

$$|f(k,\theta)|^2 = \frac{1}{k^2}\sum_{l=0}^{\infty}\sum_{l'=0}^{\infty}(2l{+}1)(2l'{+}1)exp(i[\delta_l(k){-}\delta_{l'}(k)])sin(\delta_l)sin(\delta_{l'})P_l(cos(\theta))P_{l'}(cos(\theta))$$

The phase shift $\delta_l(k)$ determines the interaction of each partial wave and is real. The differential cross section is a real measured quantity. It can be shown:

$$|f(k,\theta)|^2 = \frac{1}{k^2}\sum_{l=0}^{\infty}\sum_{l'=0}^{\infty}(2l{+}1)(2l'{+}1)cos(\delta_l(k){-}\delta_{l'}(k))sin(\delta_l)sin(\delta_{l'})P_l(cos(\theta))P_{l'}(cos(\theta))$$

Calculating the imaginary component of the differential cross section readily shows that it is indeed 0, within numerical error.

Integration of the differential cross section over all angles yields the total cross section.

$$\sigma_{tot}(k) = \int \frac{d\sigma}{d\Omega}d\Omega = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta sin\theta |f(k,\theta,\phi)|^2$$

Substitution of the scattering amplitude yields the total cross section.

$$\sigma_{tot}(k) = 2\pi\int_0^{\pi}\frac{d\sigma}{d\Omega}(k,\theta)sin(\theta)d\theta = \frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)sin^2\delta_l(k)$$

This is the total cross section as represented by a sum of partial waves. Since the energies here only allow elastic scattering, $\sigma_{tot} = \sigma_{el}$.

The momentum transfer cross section is:

$$\sigma_M(k) = \int \frac{d\sigma}{d\Omega}(1-cos\theta)d\Omega = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta(1-cos\theta)sin\theta|f(k,\theta,\phi)|^2$$

The momentum transfer cross section can also be represented by a sum of partial waves.

$$\sigma_M(k) = \frac{4\pi}{k^2}\sum_{l=0}^{\infty}(l+1)sin^2(\delta_{l+1}(k)-\delta_l(k))$$

Results

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1 Individual Systems

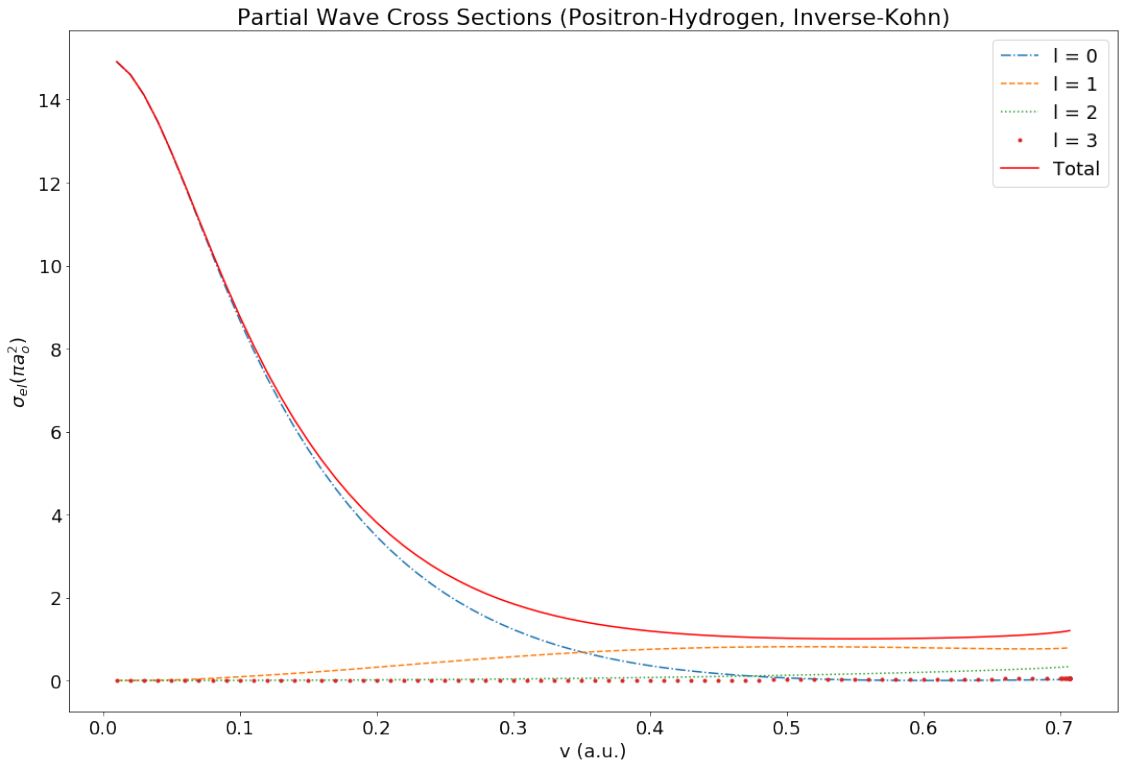
1.1 Positron-Hydrogen

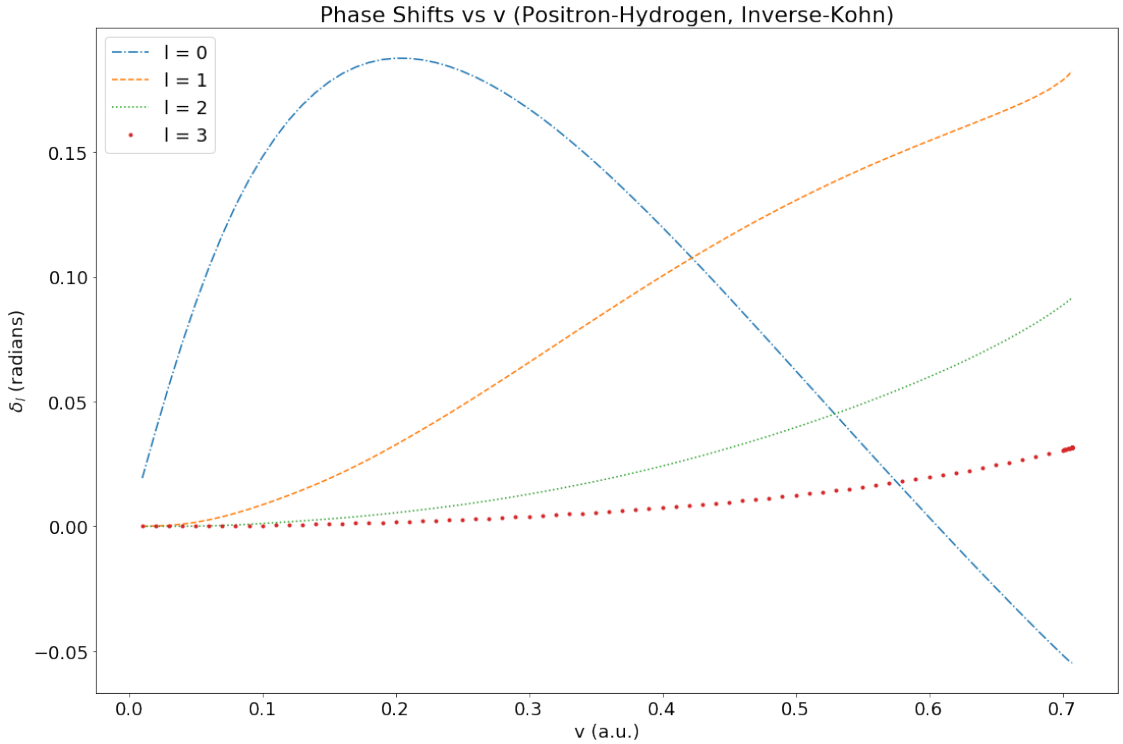
1.1.1 Inverse-Kohn Phase Shifts for Waves l < 4

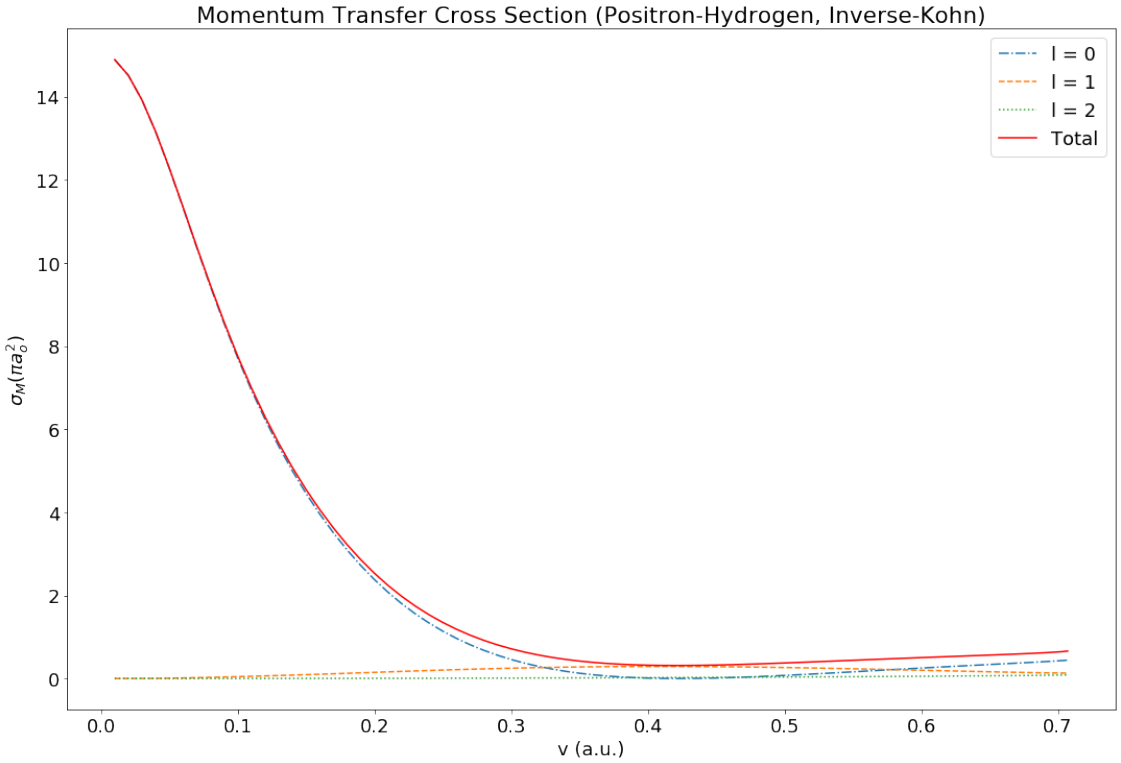
Here inverse-kohn phase shifts from P Van Reeth [12] are used for s, p, d, and f waves. The inverse-kohn phase shifts provide great stability here, with no noticeable singularities. An abundance of data points allows for very smooth graphing.

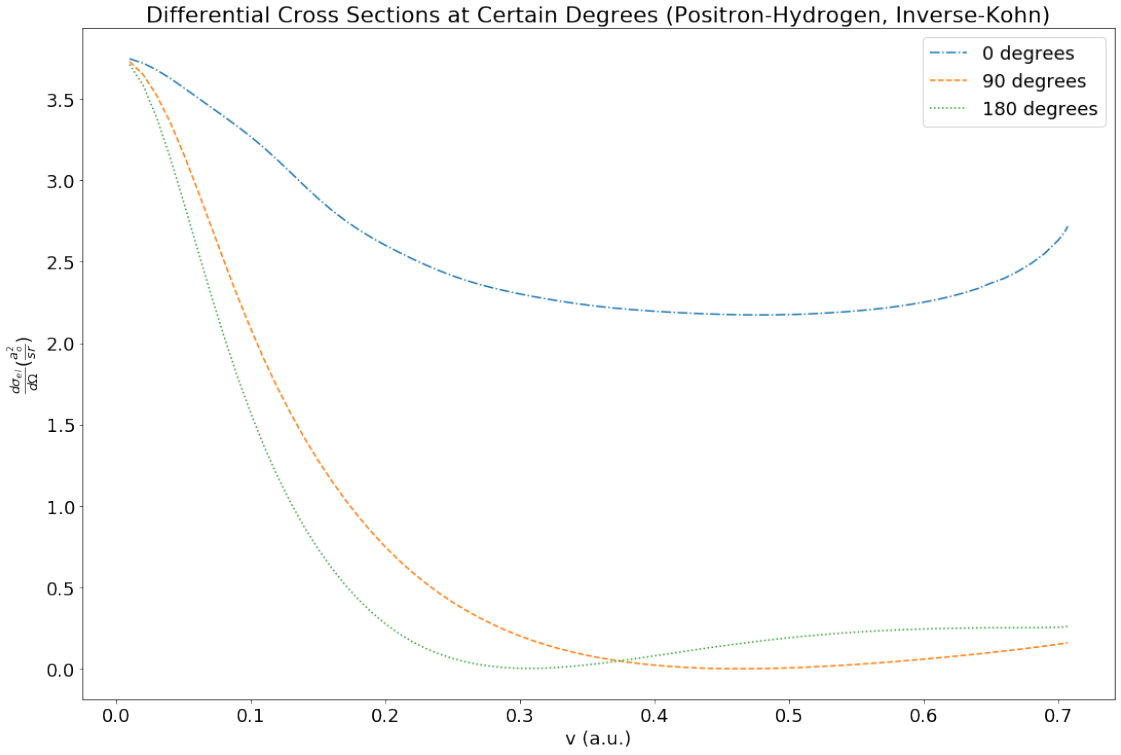
[11]:
```
#  Positron-Hydrogen using inverse-kohn phase shifts for l<4 from P Van Reeth␣
↪Private Communication 2020
positron_hyd_vr_inv_lower_system =␣
↪collision(positron_hyd_vr_inv_lower,k_list_positron_hyd_vr,␣
↪'Positron-Hydrogen, Inverse-Kohn',1)
```

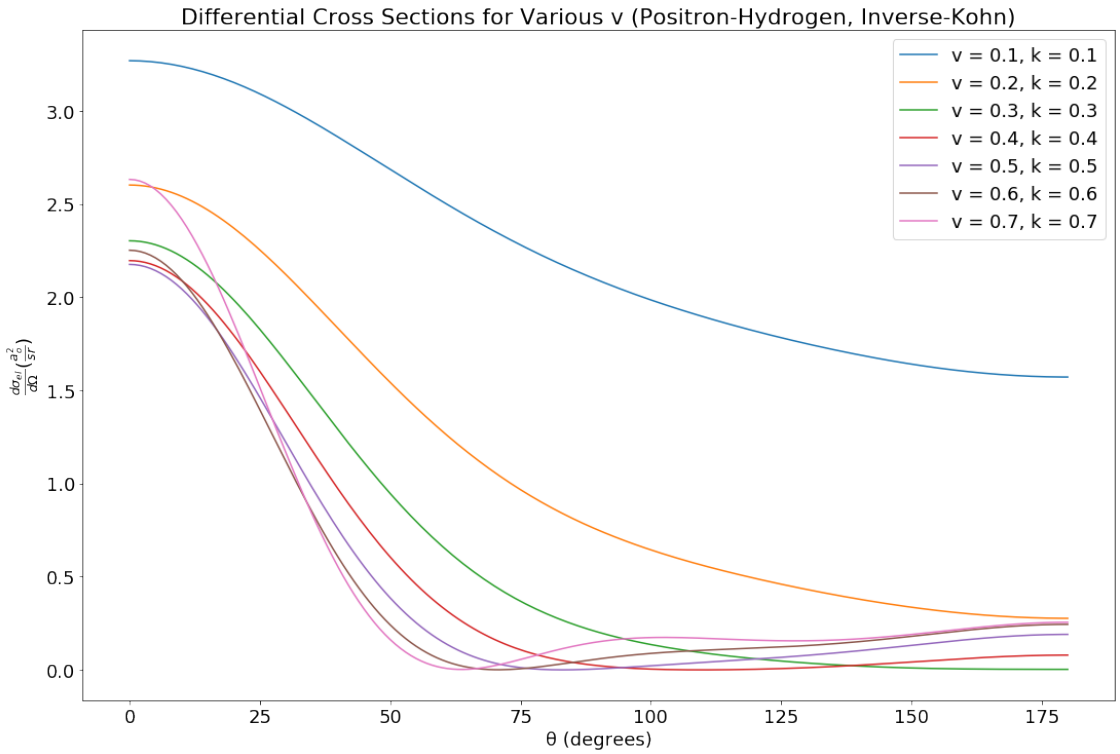
[12]:
```
positron_hyd_vr_inv_lower_system.plot_cross_section(velocity=True)
positron_hyd_vr_inv_lower_system.plot_phase_shifts(velocity=True)
positron_hyd_vr_inv_lower_system.plot_momentum_transfer(velocity=True)
positron_hyd_vr_inv_lower_system.plot_diff_cross_vs_k(r_interval,velocity=True)
positron_hyd_vr_inv_lower_system.diff_plot(interval='hydrogen',velocity=True)
positron_hyd_vr_inv_lower_system.diff_plot(sin=True,interval='hydrogen')
positron_hyd_vr_inv_lower_system.diff_plot_3d(velocity=True)
positron_hyd_vr_inv_lower_system.diff_plot_3d(density=True)
```

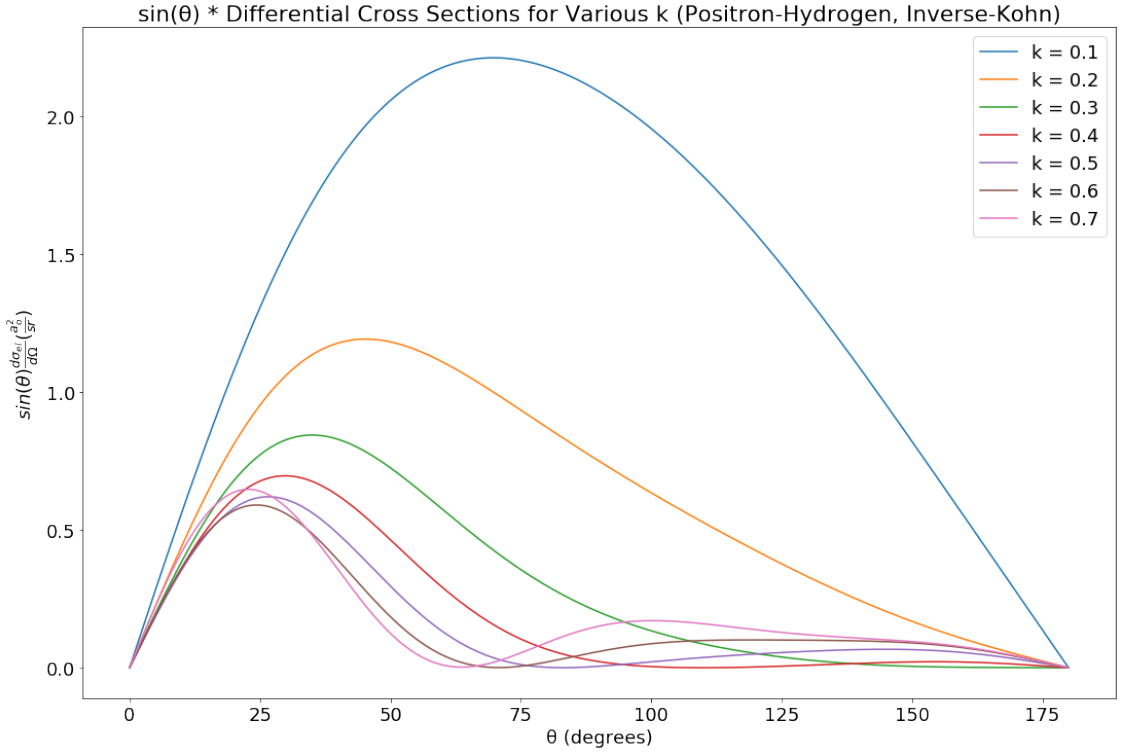

Partial Wave Cross Sections (Positron-Hydrogen, Inverse-Kohn)
l = 0
l = 1
l = 2
l = 3
Total
$\sigma_{el}(\pi a_o^2)$
v (a.u.)

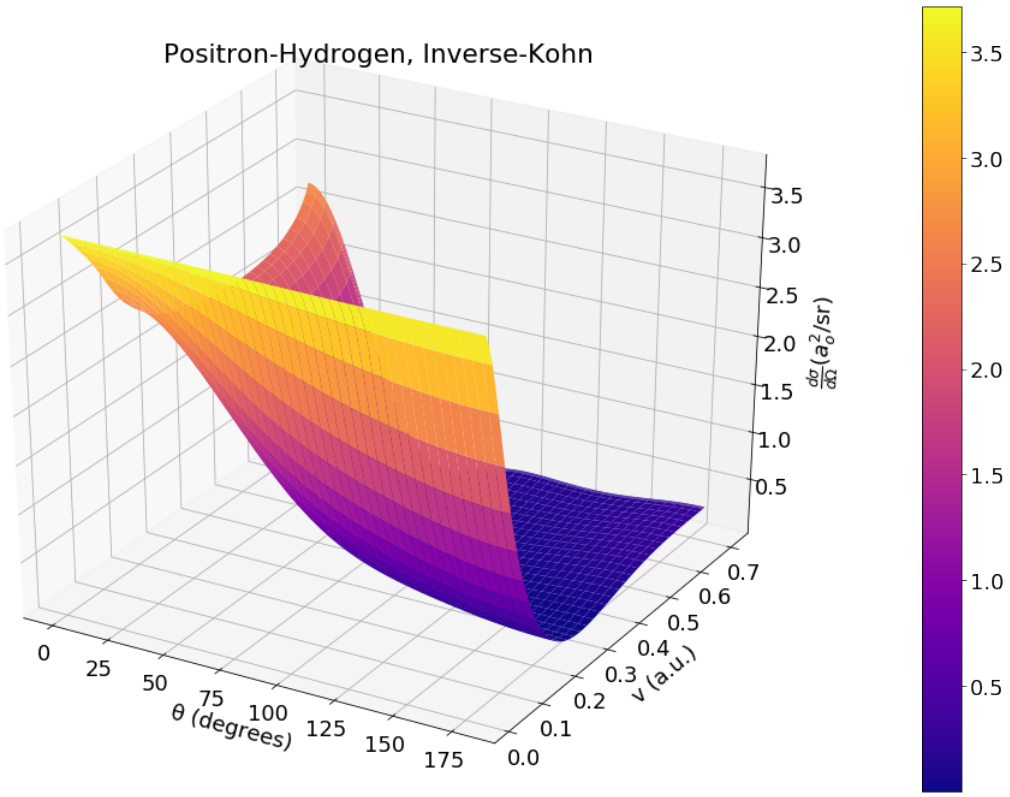

Phase Shifts vs v (Positron-Hydrogen, Inverse-Kohn)
l = 0
l = 1
l = 2
l = 3
δ_l (radians)
v (a.u.)

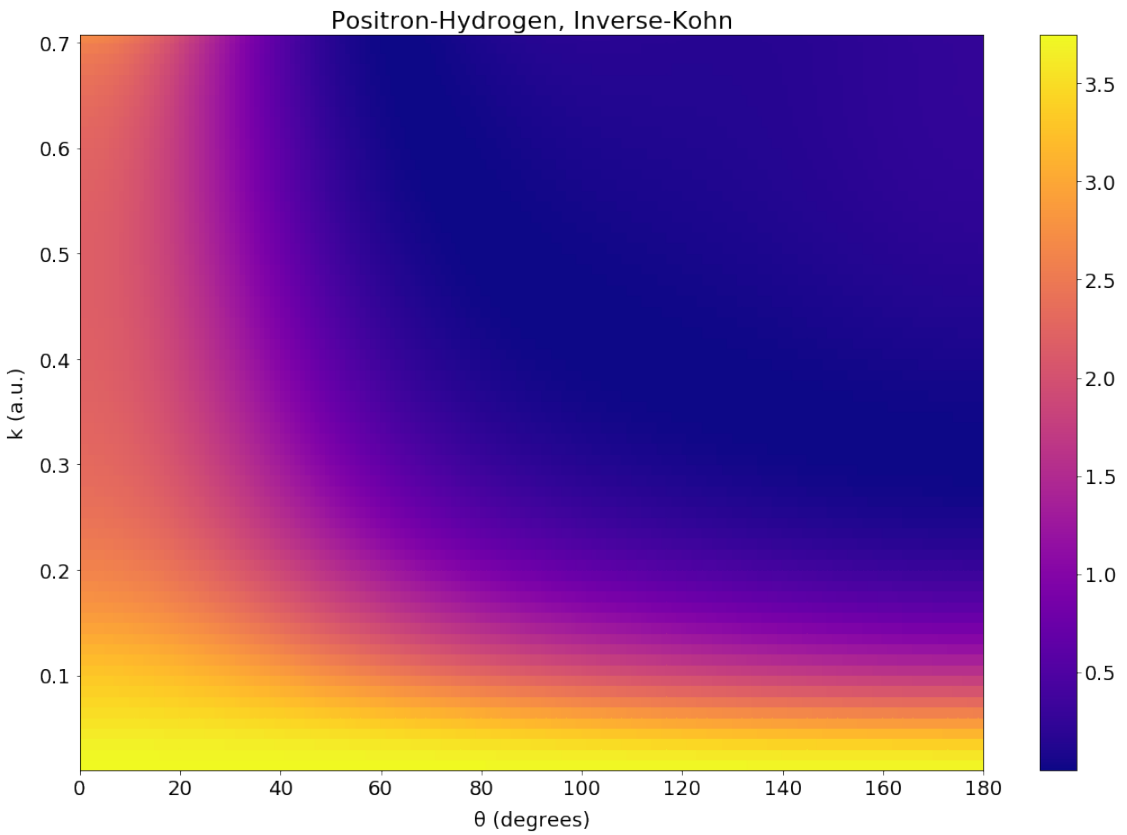

Momentum Transfer Cross Section (Positron-Hydrogen, Inverse-Kohn)
l = 0
l = 1
l = 2
Total
$\sigma_M(\pi a_o^2)$
v (a.u.)


Differential Cross Sections at Certain Degrees (Positron-Hydrogen, Inverse-Kohn)
0 degrees
90 degrees
180 degrees
$\frac{d\sigma_{el}}{d\Omega}(\frac{a_o^2}{SR})$
v (a.u.)


Differential Cross Sections for Various v (Positron-Hydrogen, Inverse-Kohn)
v = 0.1, k = 0.1
v = 0.2, k = 0.2
v = 0.3, k = 0.3
v = 0.4, k = 0.4
v = 0.5, k = 0.5
v = 0.6, k = 0.6
v = 0.7, k = 0.7
θ (degrees)


sin(θ) * Differential Cross Sections for Various k (Positron-Hydrogen, Inverse-Kohn)
k = 0.1
k = 0.2
k = 0.3
k = 0.4
k = 0.5
k = 0.6
k = 0.7
θ (degrees)

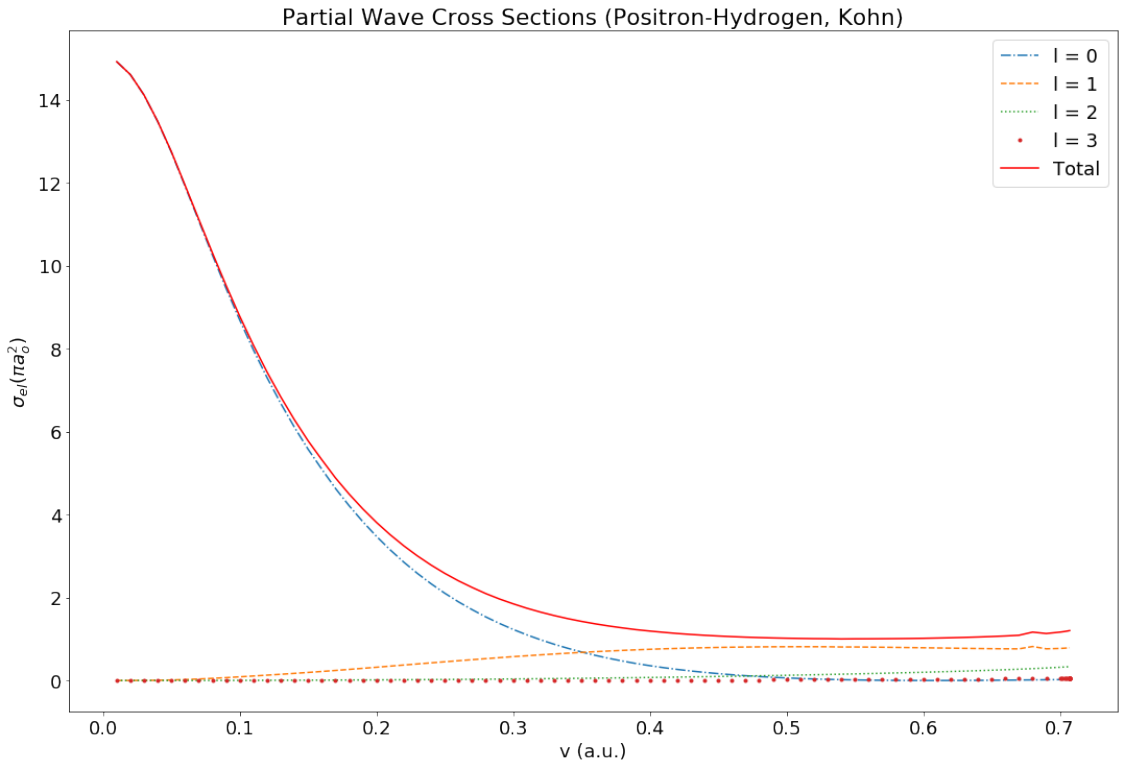

Positron-Hydrogen, Inverse-Kohn
θ (degrees)
0
25
50
75
100
125
150
175
ν (a.u.)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
$\frac{d\sigma}{d\Omega}(a_0^2/sr)$
0.5
1.0
1.5
2.0
2.5
3.0
3.5

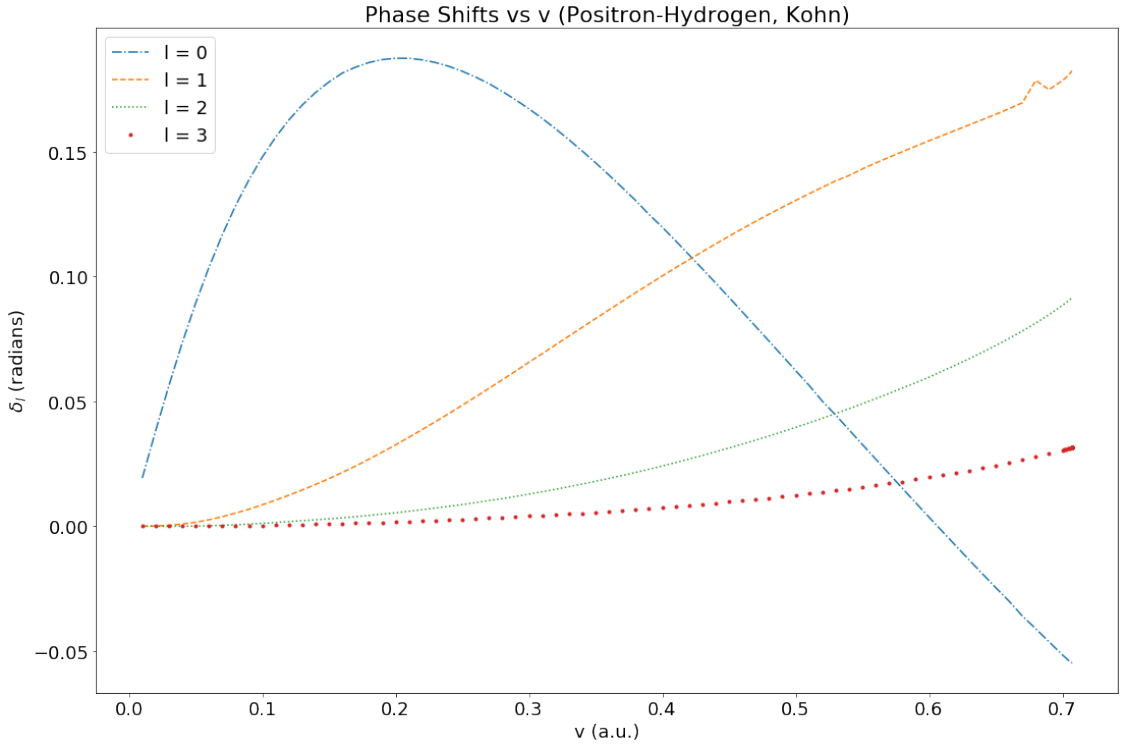


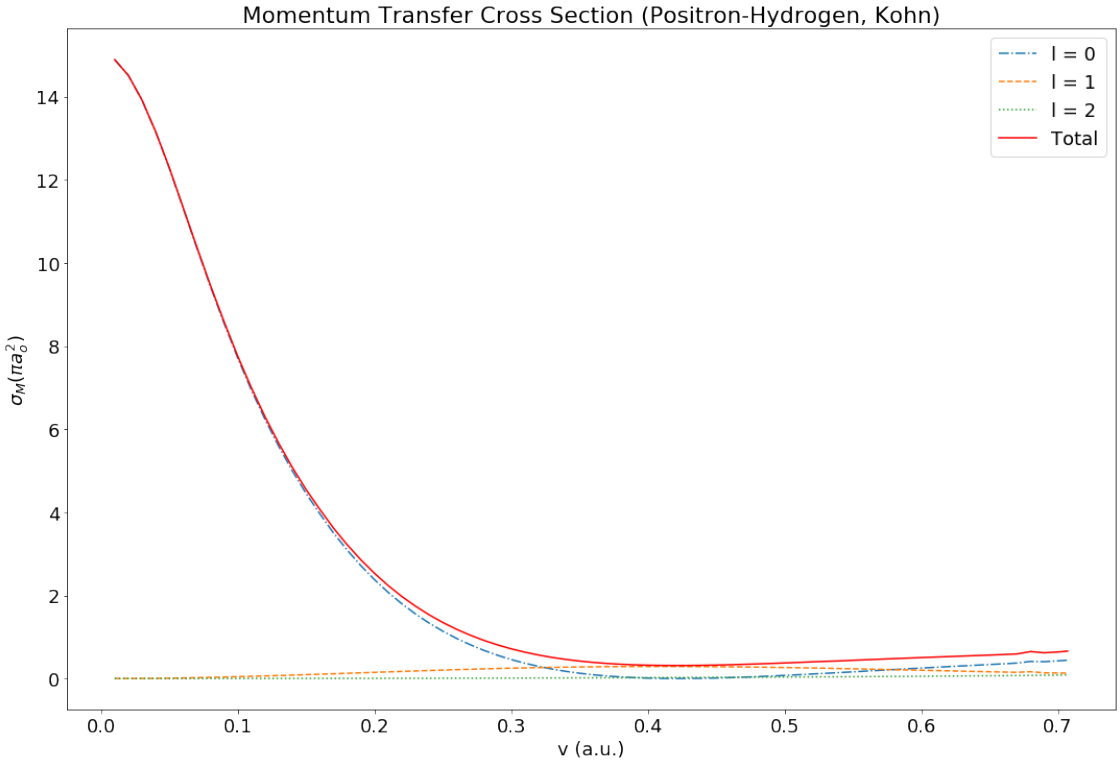
1.1.2 Kohn Phase Shifts for Waves l < 4

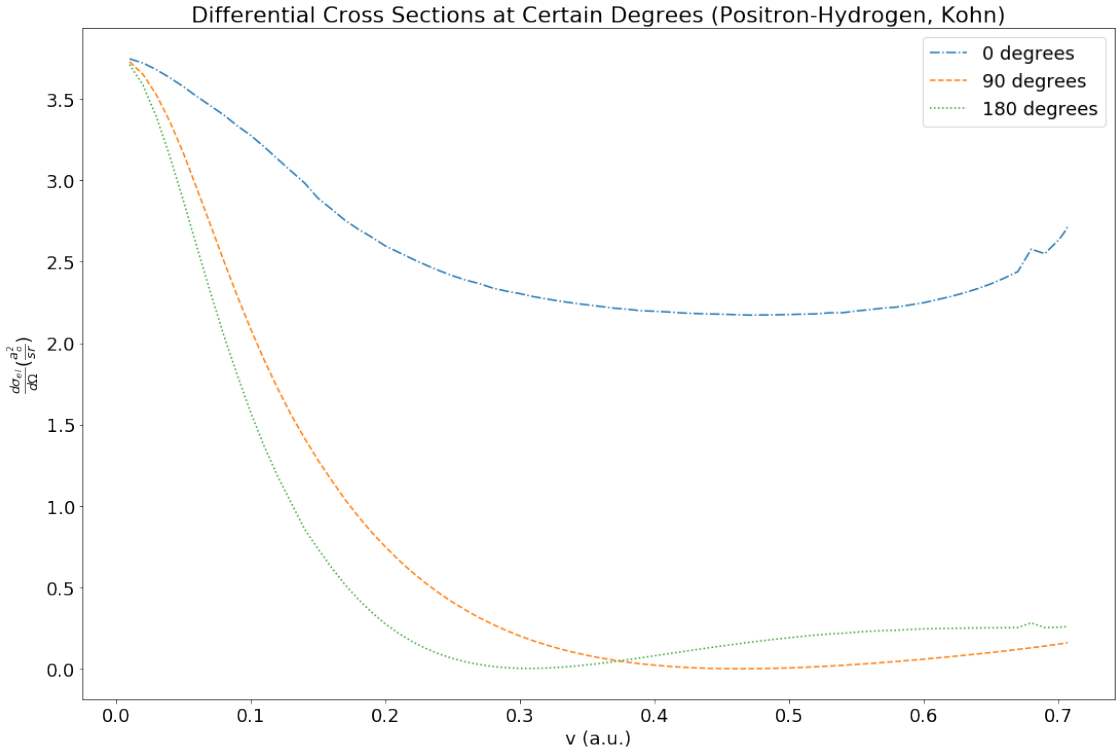
Kohn phase shifts [12] for s, p, d, and f waves are used here. A singularity is present around v = 0.7, which makes the inverse-kohn phase shifts preferable.

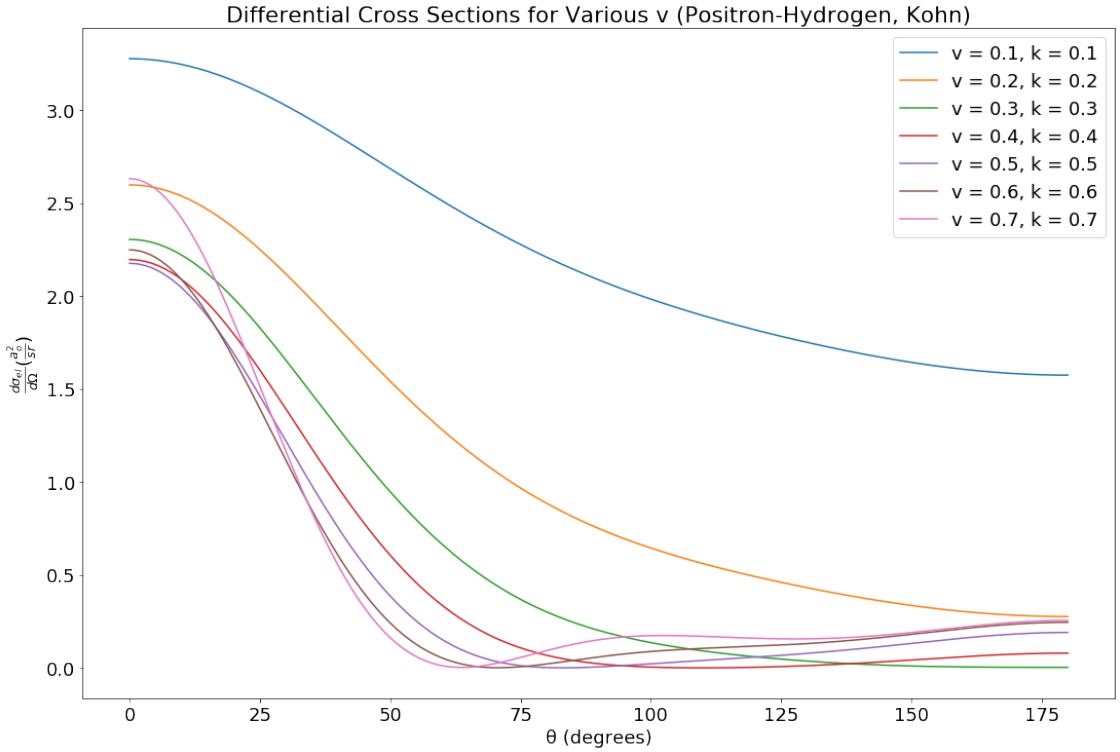
[13]:
```
#  Positron-Hydrogen using kohn phase shifts for l<4 from P Van Reeth Private␣
↪Communication 2020

positron_hyd_vr_kohn_lower_system =␣
↪collision(positron_hyd_vr_kohn_lower,k_list_positron_hyd_vr,␣
↪'Positron-Hydrogen, Kohn',1)

positron_hyd_vr_kohn_lower_system.plot_cross_section(velocity=True)
positron_hyd_vr_kohn_lower_system.plot_phase_shifts(velocity=True)
positron_hyd_vr_kohn_lower_system.plot_momentum_transfer(velocity=True)
positron_hyd_vr_kohn_lower_system.plot_diff_cross_vs_k(r_interval,velocity=True)
positron_hyd_vr_kohn_lower_system.diff_plot(interval='hydrogen',velocity=True)
positron_hyd_vr_kohn_lower_system.
↪diff_plot(sin=True,interval='hydrogen',velocity=True)
positron_hyd_vr_kohn_lower_system.diff_plot_3d(velocity=True)
positron_hyd_vr_kohn_lower_system.diff_plot_3d(density=True, velocity=True)
```

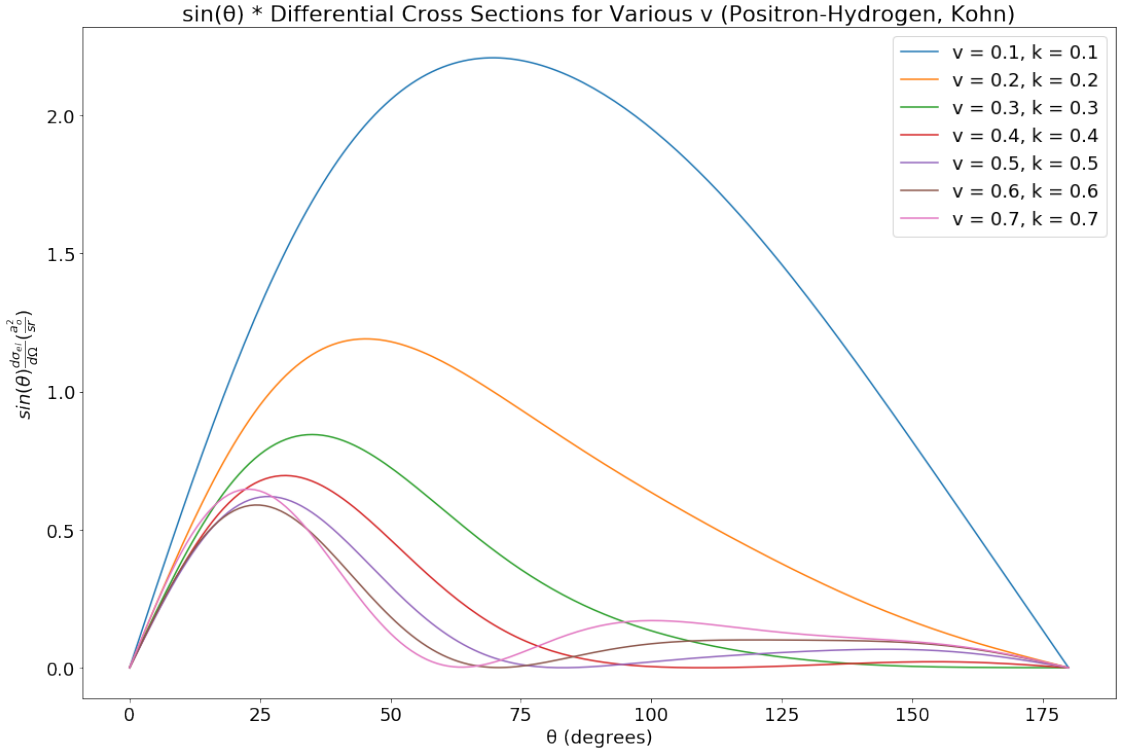

Partial Wave Cross Sections (Positron-Hydrogen, Kohn)
l = 0
l = 1
l = 2
l = 3
Total
$\sigma_{el}(\pi a_o^2)$
v (a.u.)

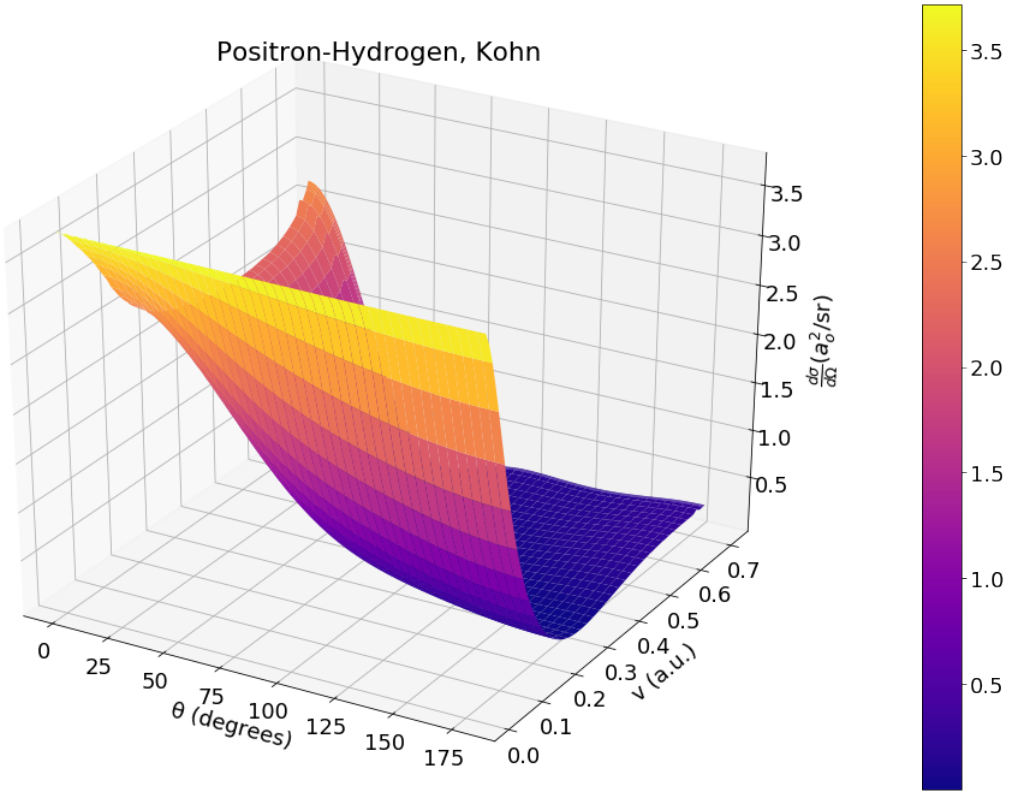

Phase Shifts vs v (Positron-Hydrogen, Kohn)
l = 0
l = 1
l = 2
l = 3
δ_l (radians)
v (a.u.)

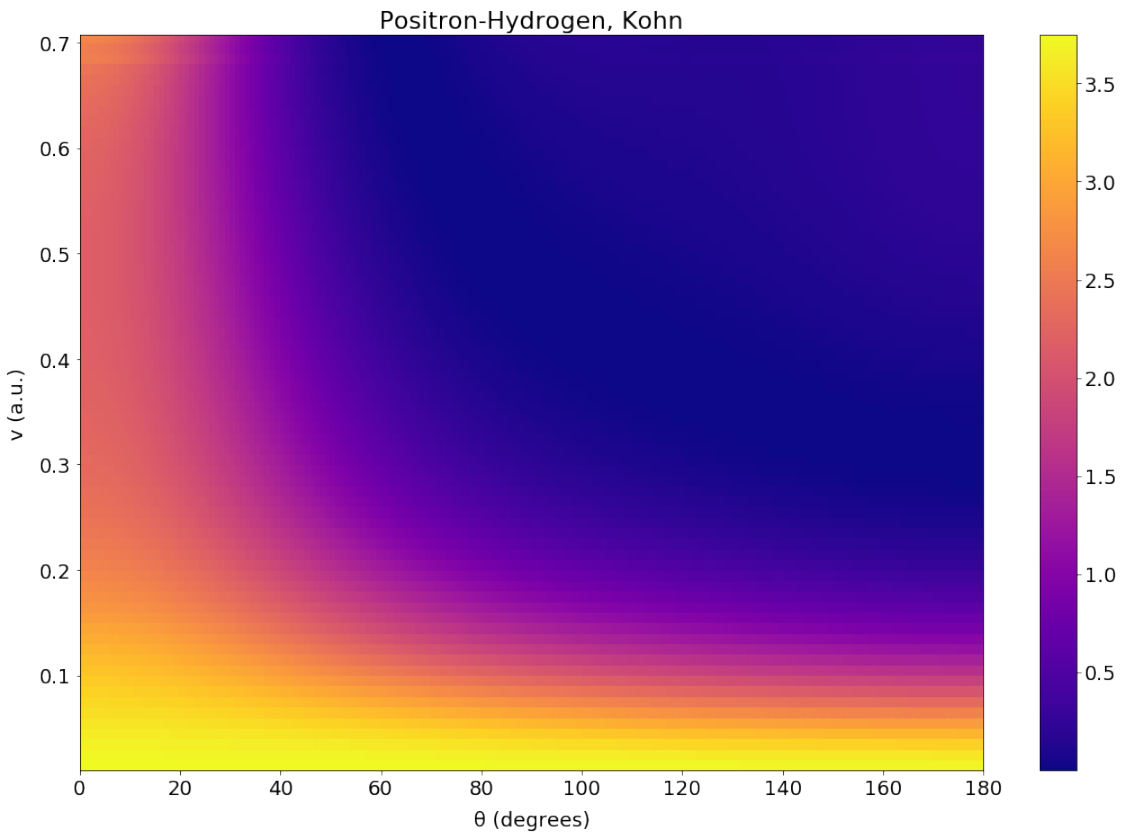

Momentum Transfer Cross Section (Positron-Hydrogen, Kohn)
l = 0
l = 1
l = 2
Total
$\sigma_M(\pi a_o^2)$
v (a.u.)


Differential Cross Sections at Certain Degrees (Positron-Hydrogen, Kohn)
0 degrees
90 degrees
180 degrees
$\frac{d\sigma_{el}}{d\Omega}(\frac{a_o^2}{SR})$
v (a.u.)


Differential Cross Sections for Various v (Positron-Hydrogen, Kohn)
v = 0.1, k = 0.1
v = 0.2, k = 0.2
v = 0.3, k = 0.3
v = 0.4, k = 0.4
v = 0.5, k = 0.5
v = 0.6, k = 0.6
v = 0.7, k = 0.7
θ (degrees)


sin(θ) * Differential Cross Sections for Various v (Positron-Hydrogen, Kohn)
v = 0.1, k = 0.1
v = 0.2, k = 0.2
v = 0.3, k = 0.3
v = 0.4, k = 0.4
v = 0.5, k = 0.5
v = 0.6, k = 0.6
v = 0.7, k = 0.7
θ (degrees)


Positron-Hydrogen, Kohn
θ (degrees)
0
25
50
75
100
125
150
175
ν (a.u.)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
$\frac{d\sigma}{d\Omega}(a_0^2/sr)$
0.5
1.0
1.5
2.0
2.5
3.0
3.5

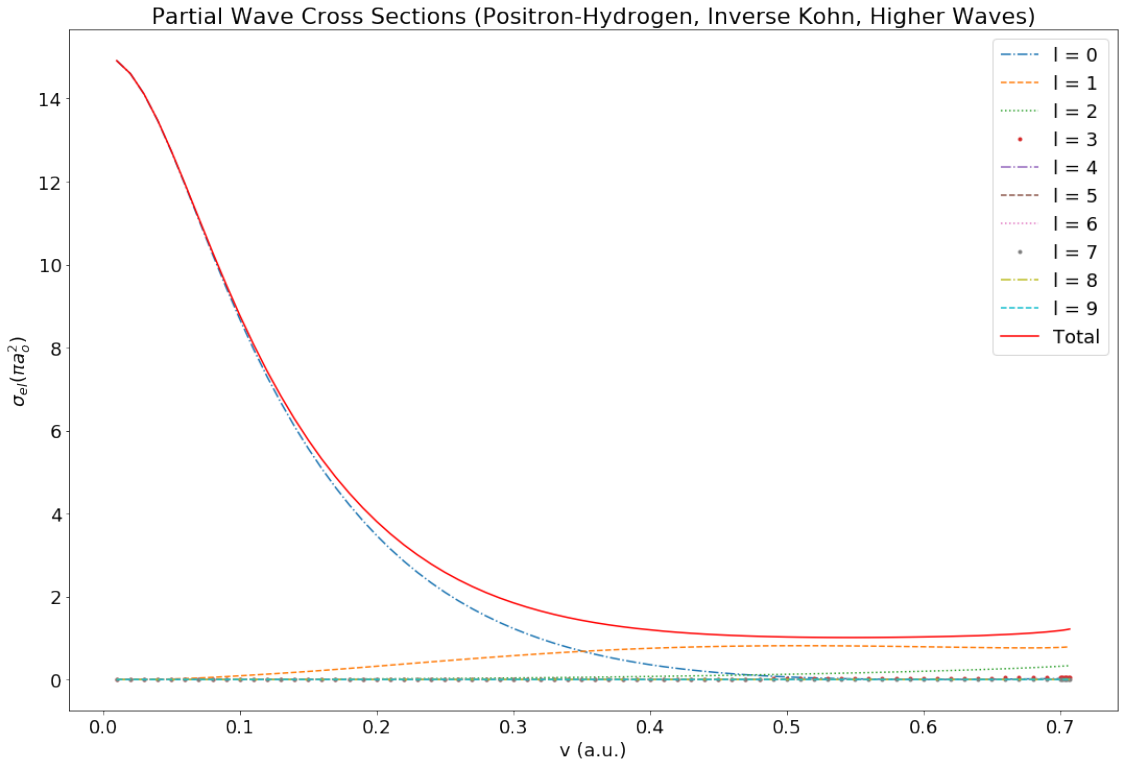


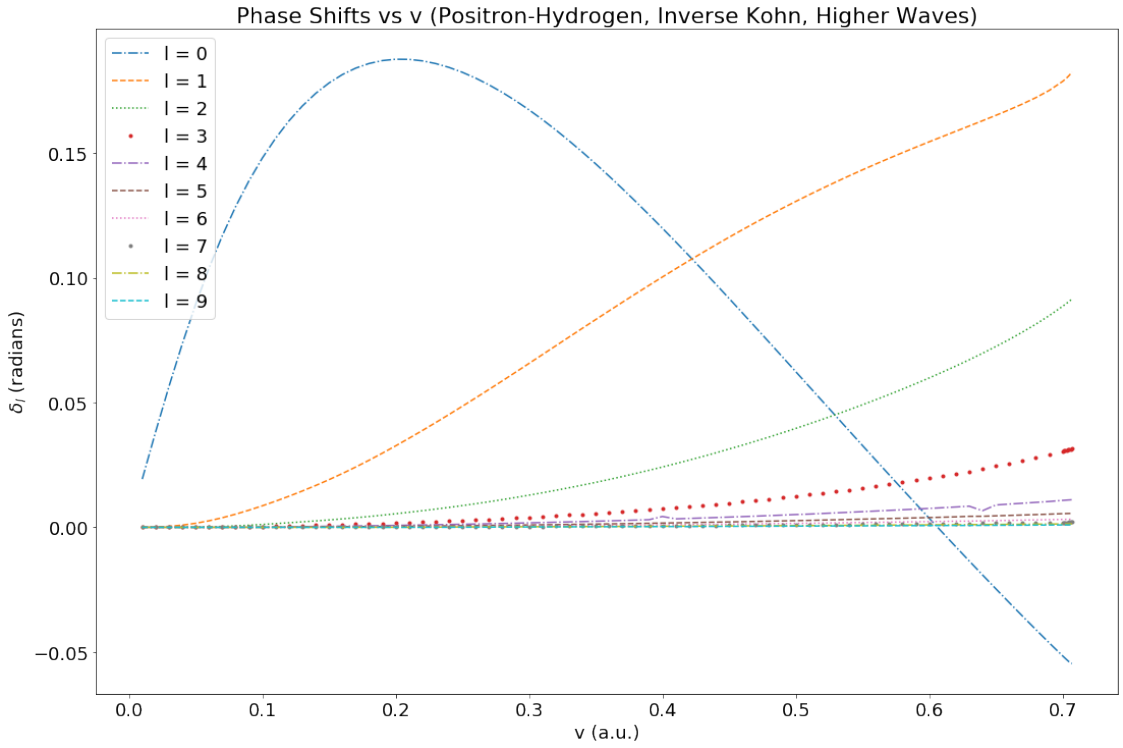
1.1.3 Inverse-Kohn Phase Shifts for Waves l $<$ 10

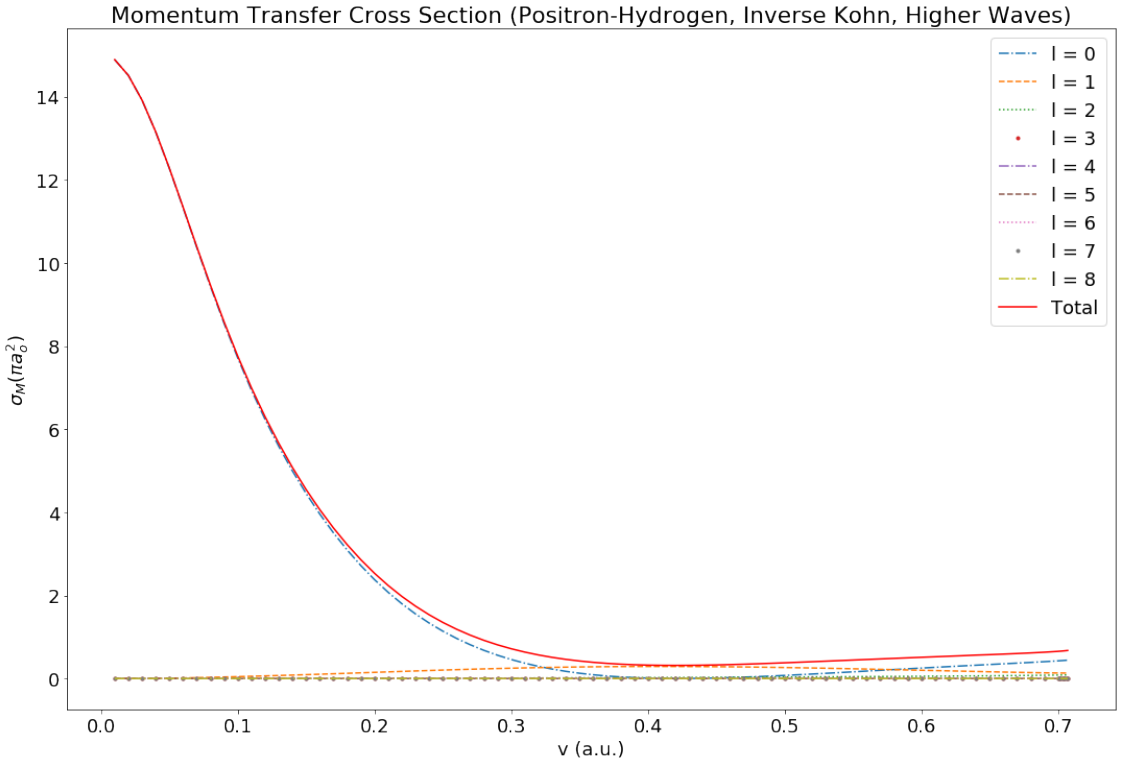
Inverse-Kohn phase shifts [12] for l $<$ 10 are used here. There are two noticeable singularities around v = 0.4, 0.65.

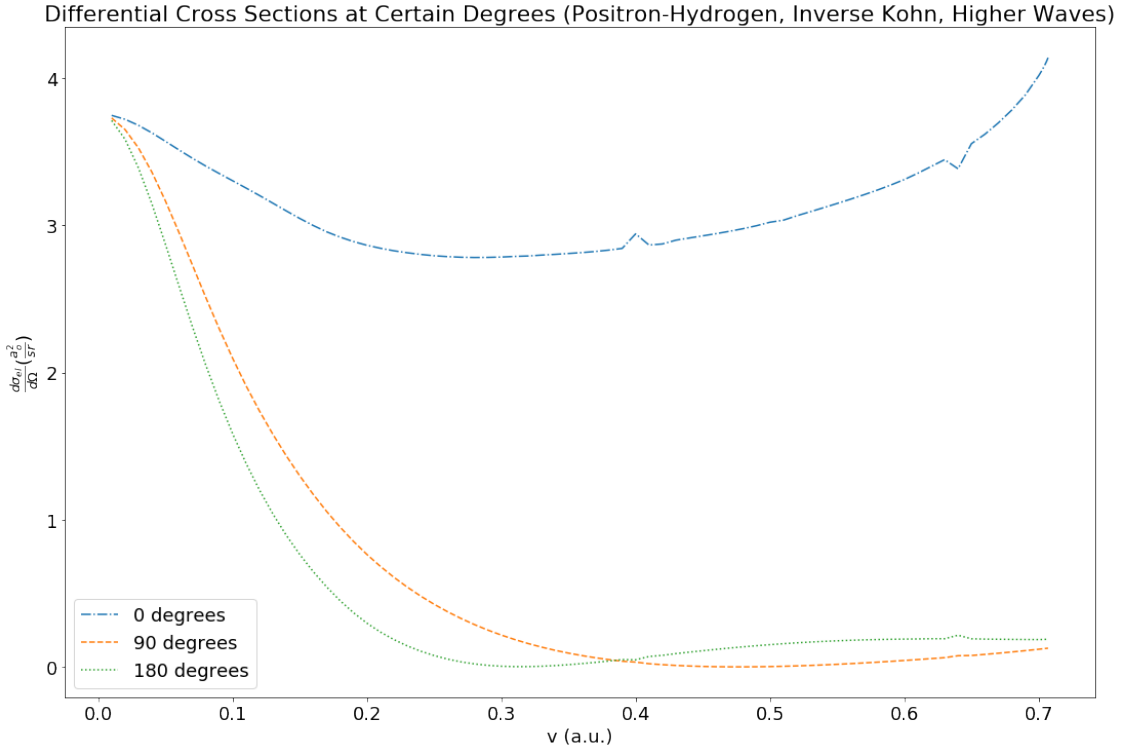
[14]:
```
#  Positron-Hydrogen using inverse-kohn phase shifts for l<10 from P Van Reeth␣
↪Private Communication 2020

positron_hyd_vr_kohn_inv_all_system =␣
↪collision(positron_hyd_vr_inv_all,k_vr_higher,'Positron-Hydrogen, Inverse␣
↪Kohn, Higher Waves',1)

positron_hyd_vr_kohn_inv_all_system.plot_cross_section(velocity=True)
positron_hyd_vr_kohn_inv_all_system.plot_phase_shifts(velocity=True)
positron_hyd_vr_kohn_inv_all_system.plot_momentum_transfer(velocity=True)
positron_hyd_vr_kohn_inv_all_system.
↪plot_diff_cross_vs_k(r_interval,velocity=True)
positron_hyd_vr_kohn_inv_all_system.diff_plot(interval='hydrogen',velocity=True)
positron_hyd_vr_kohn_inv_all_system.
↪diff_plot(sin=True,interval='hydrogen',velocity=True)
positron_hyd_vr_kohn_inv_all_system.diff_plot_3d(velocity=True)
```

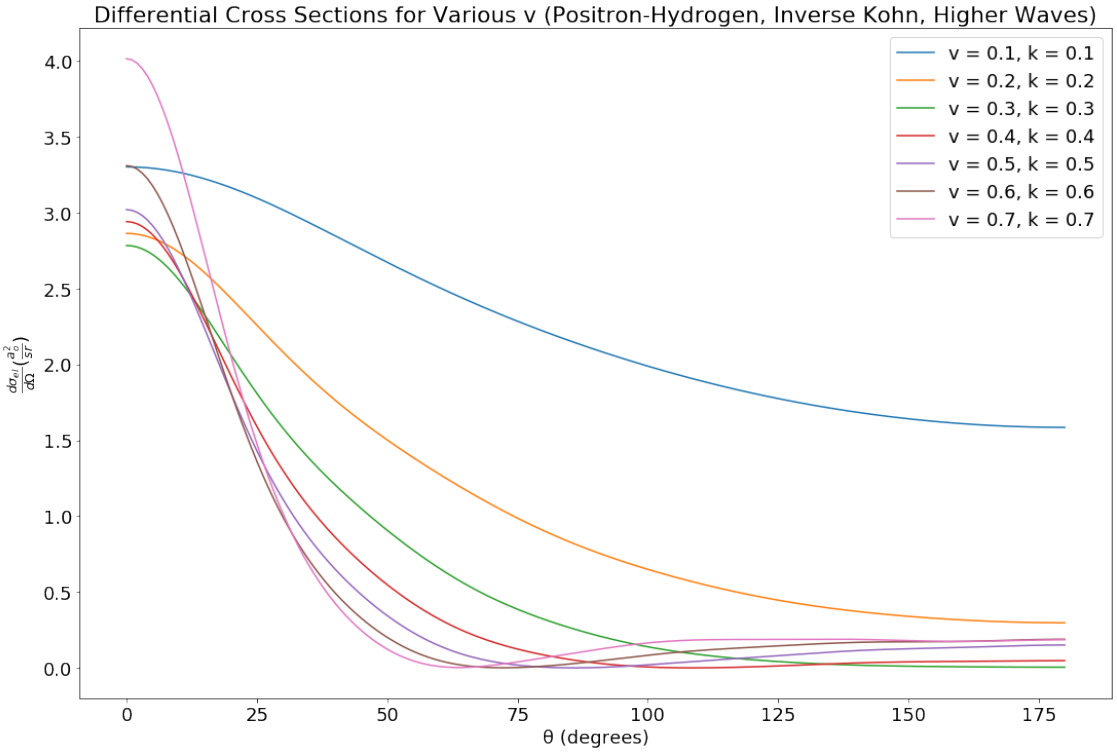
```
positron_hyd_vr_kohn_inv_all_system.diff_plot_3d(density=True,velocity=True)
```

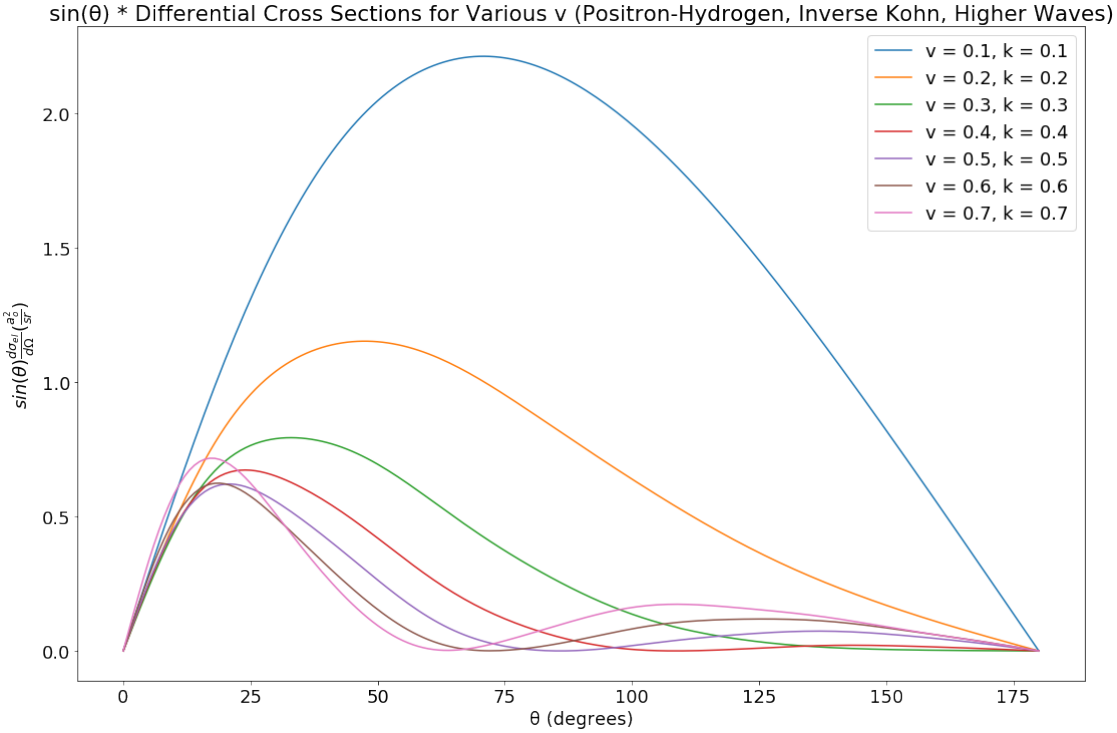


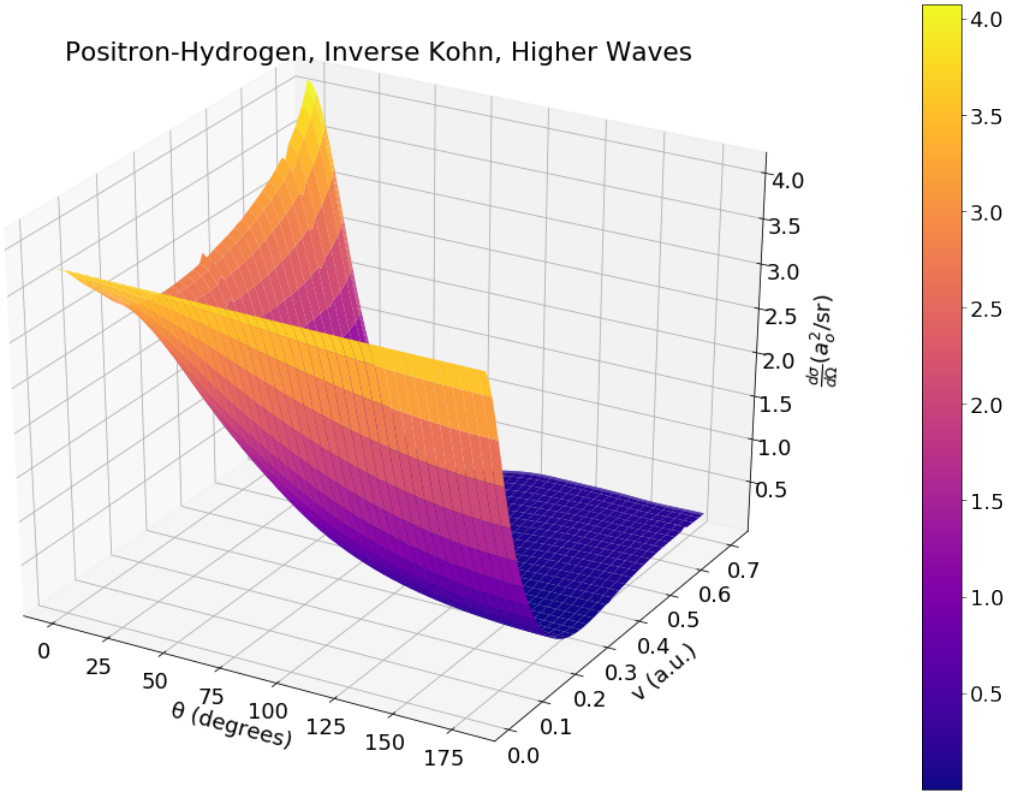


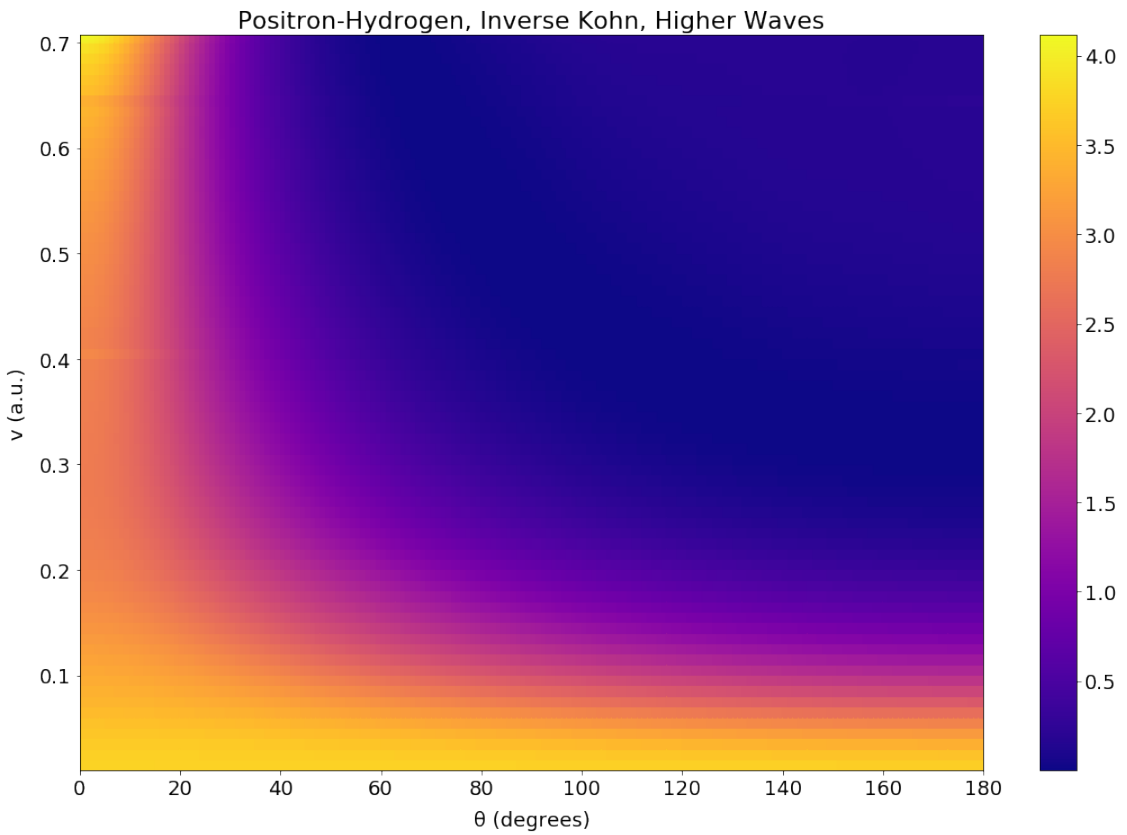

Momentum Transfer Cross Section (Positron-Hydrogen, Inverse Kohn, Higher Waves)
l = 0
l = 1
l = 2
l = 3
l = 4
l = 5
l = 6
l = 7
l = 8
Total
$\sigma_M(\pi a_o^2)$
v (a.u.)


Differential Cross Sections at Certain Degrees (Positron-Hydrogen, Inverse Kohn, Higher Waves)
0 degrees
90 degrees
180 degrees
$\frac{d\sigma_{el}}{d\Omega}(\frac{a_o^2}{sr})$
v (a.u.)


Differential Cross Sections for Various v (Positron-Hydrogen, Inverse Kohn, Higher Waves)
v = 0.1, k = 0.1
v = 0.2, k = 0.2
v = 0.3, k = 0.3
v = 0.4, k = 0.4
v = 0.5, k = 0.5
v = 0.6, k = 0.6
v = 0.7, k = 0.7
θ (degrees)


sin(θ) * Differential Cross Sections for Various v (Positron-Hydrogen, Inverse Kohn, Higher Waves)
v = 0.1, k = 0.1
v = 0.2, k = 0.2
v = 0.3, k = 0.3
v = 0.4, k = 0.4
v = 0.5, k = 0.5
v = 0.6, k = 0.6
v = 0.7, k = 0.7
θ (degrees)

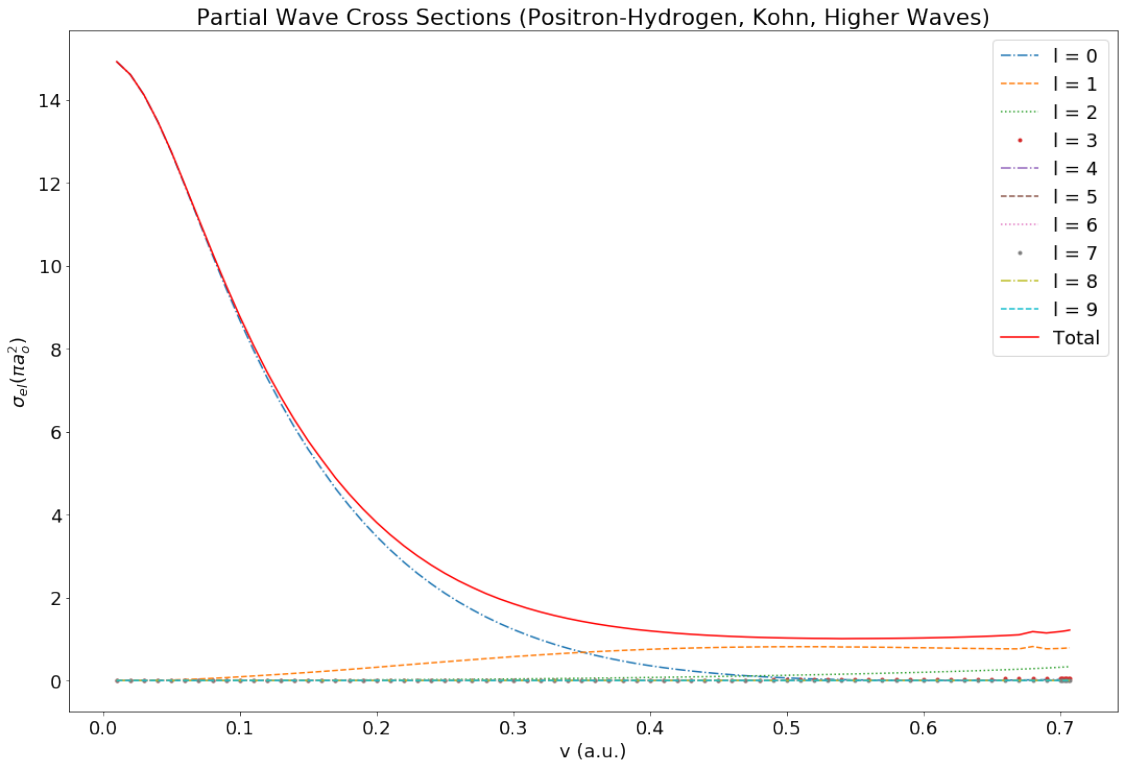

Positron-Hydrogen, Inverse Kohn, Higher Waves
θ (degrees)
0
25
50
75
100
125
150
175
ν (a.u.)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
$\frac{d\sigma}{d\Omega}(a_o^2/sr)$
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

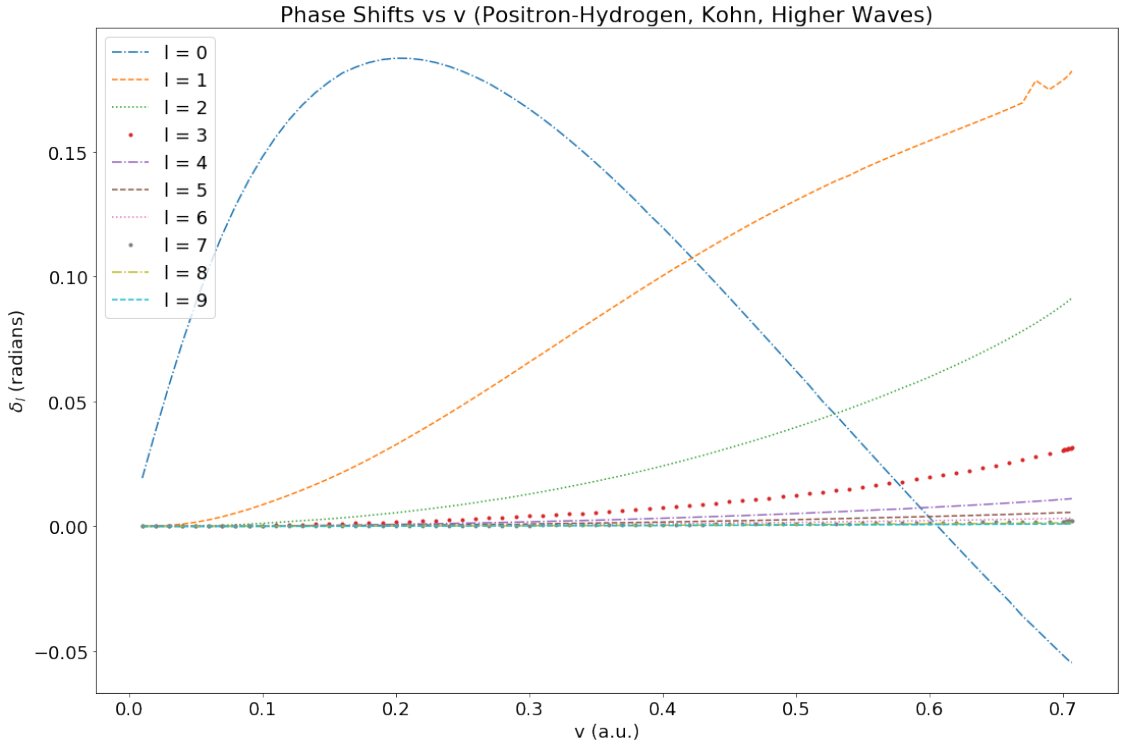


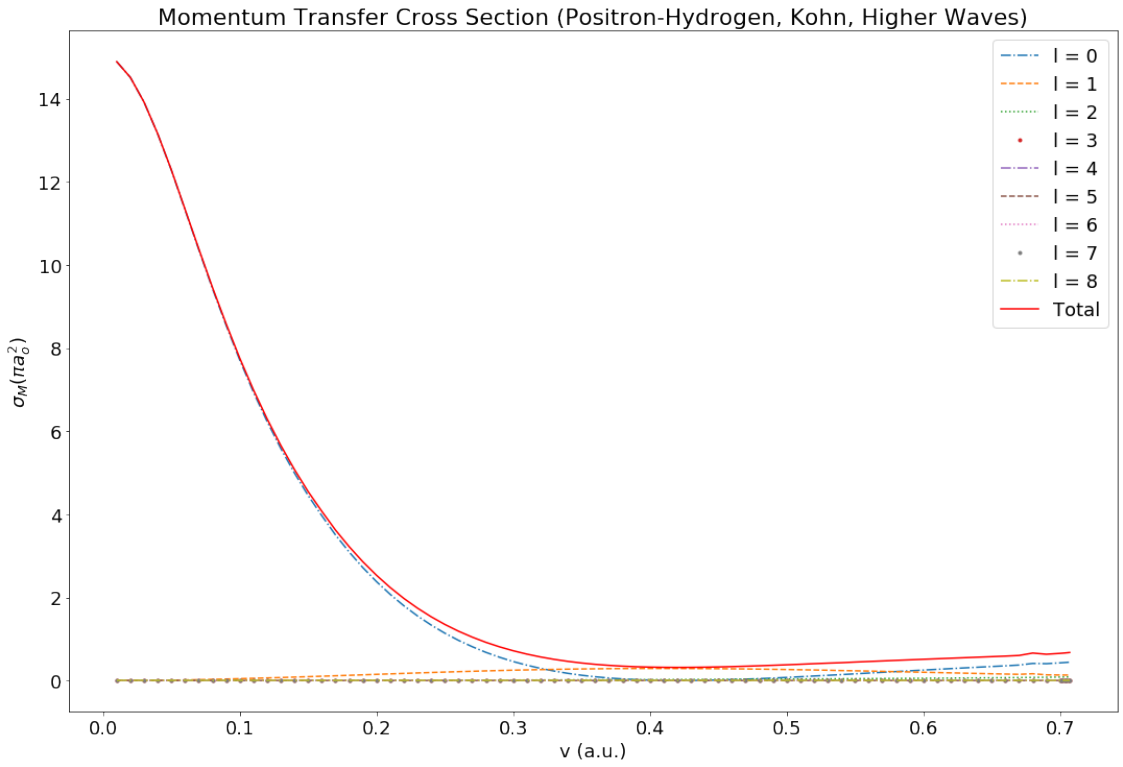
1.1.4 Kohn Phase Shifts l < 10

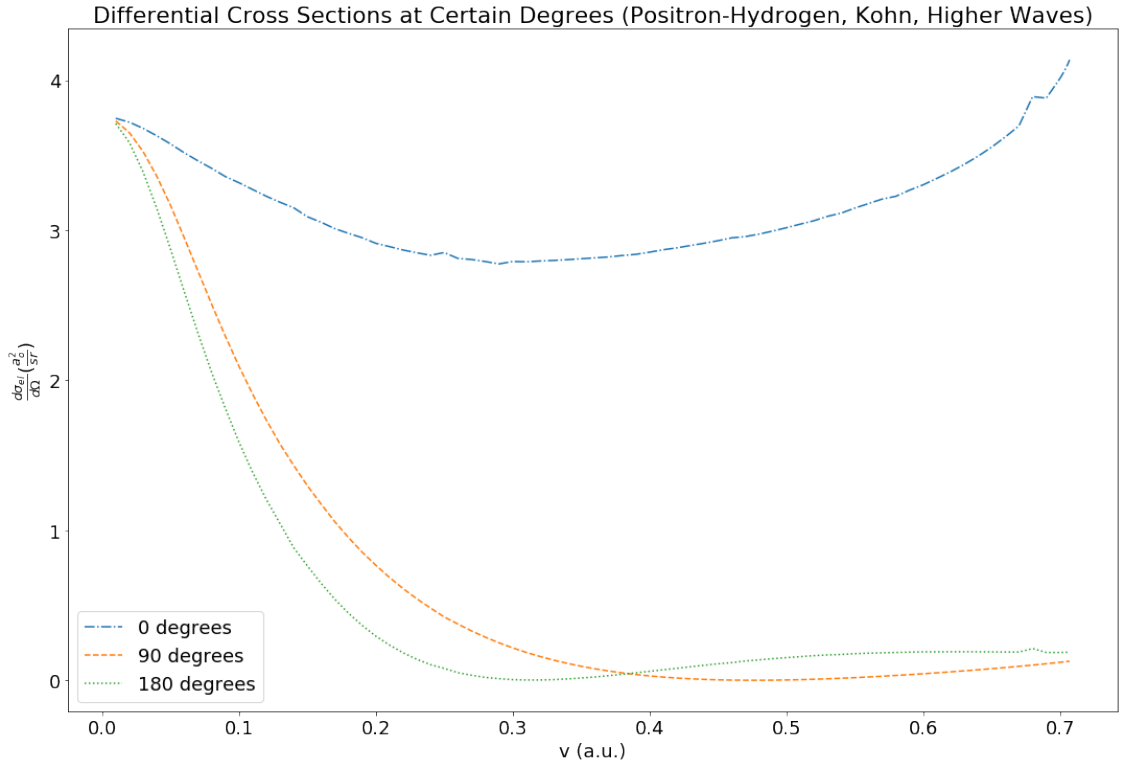
Kohn phase shifts for l < 10 [12] are used here. Several singularities are visible here, especially around v = 0.7.

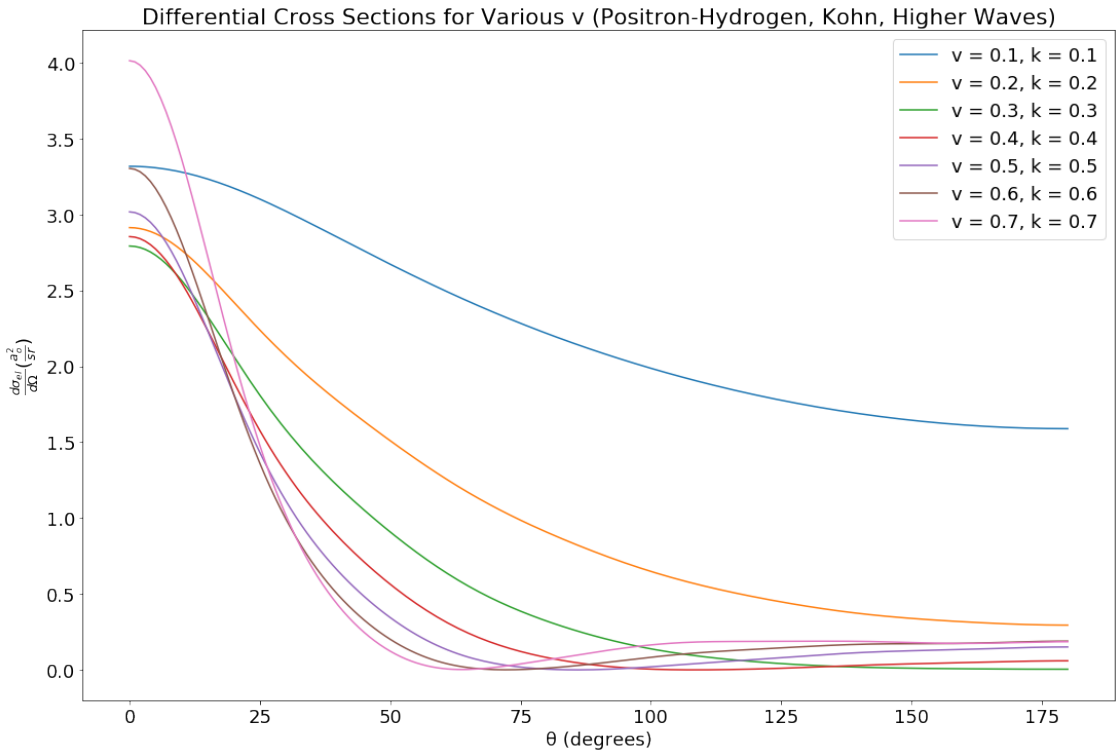
[15]:
```
#  Positron-Hydrogen using kohn phase shifts for l<10 from P Van Reeth Private␣
↪Communication 2020

positron_hyd_vr_kohn_all_system =␣
↪collision(positron_hyd_vr_kohn_all,k_vr_higher,'Positron-Hydrogen, Kohn,␣
↪Higher Waves',1)

positron_hyd_vr_kohn_all_system.plot_cross_section(velocity=True)
positron_hyd_vr_kohn_all_system.plot_phase_shifts(velocity=True)
positron_hyd_vr_kohn_all_system.plot_momentum_transfer(velocity=True)
positron_hyd_vr_kohn_all_system.plot_diff_cross_vs_k(r_interval,velocity=True)
positron_hyd_vr_kohn_all_system.diff_plot(interval='hydrogen',velocity=True)
positron_hyd_vr_kohn_all_system.
↪diff_plot(sin=True,interval='hydrogen',velocity=True)
positron_hyd_vr_kohn_all_system.diff_plot_3d(velocity=True)
positron_hyd_vr_kohn_all_system.diff_plot_3d(density=True,velocity=True)
```

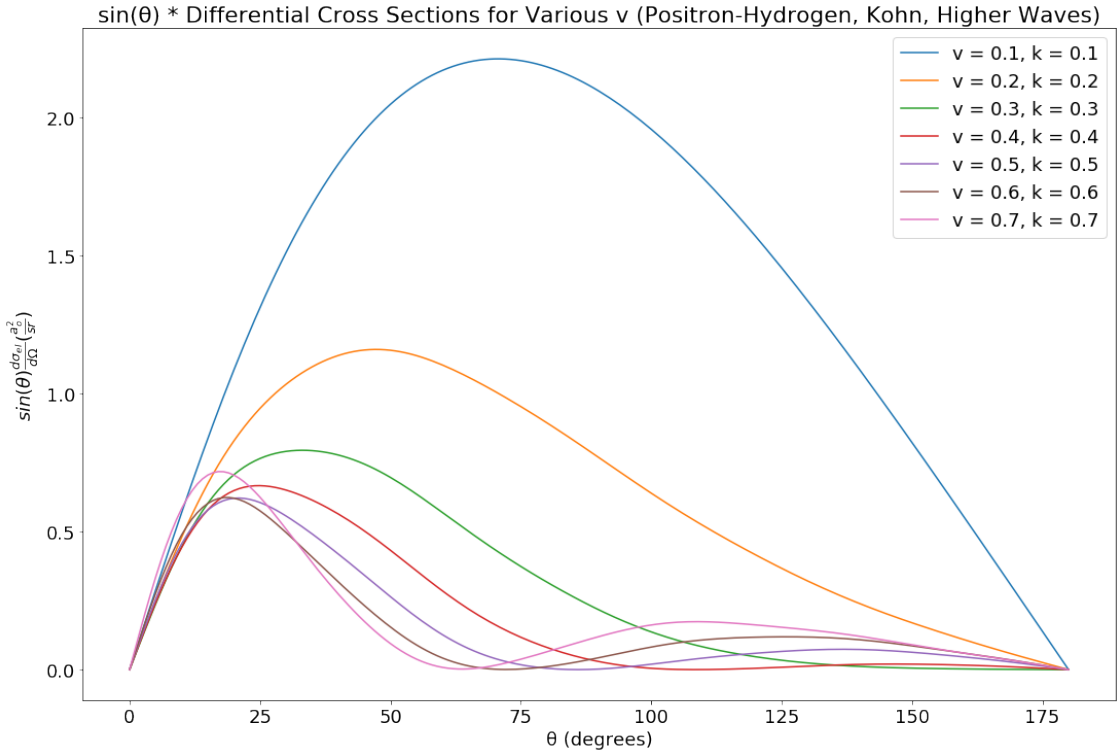

Partial Wave Cross Sections (Positron-Hydrogen, Kohn, Higher Waves)
l = 0
l = 1
l = 2
l = 3
l = 4
l = 5
l = 6
l = 7
l = 8
l = 9
Total
$\sigma_{el}(\pi a_o^2)$
v (a.u.)

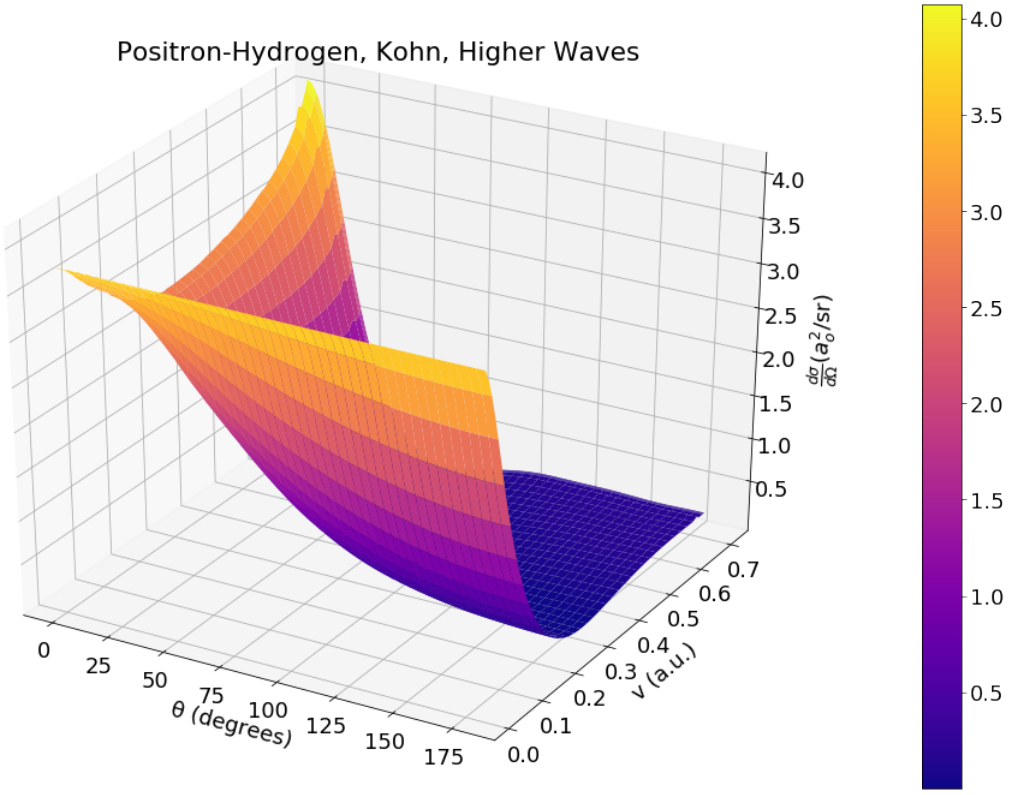

Phase Shifts vs v (Positron-Hydrogen, Kohn, Higher Waves)
l = 0
l = 1
l = 2
l = 3
l = 4
l = 5
l = 6
l = 7
l = 8
l = 9
δ_l (radians)
v (a.u.)

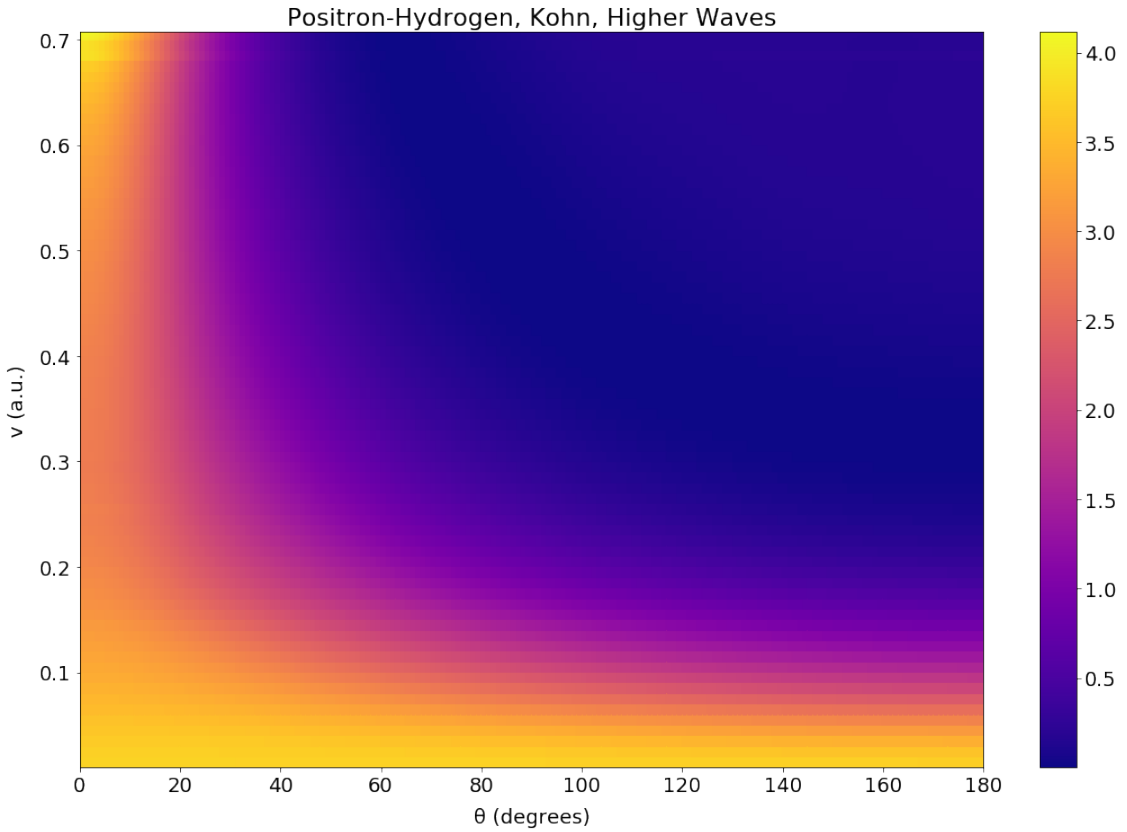

Momentum Transfer Cross Section (Positron-Hydrogen, Kohn, Higher Waves)
l = 0
l = 1
l = 2
l = 3
l = 4
l = 5
l = 6
l = 7
l = 8
Total
$\sigma_M(\pi a_o^2)$
v (a.u.)


Differential Cross Sections at Certain Degrees (Positron-Hydrogen, Kohn, Higher Waves)
0 degrees
90 degrees
180 degrees
$\frac{d\sigma_{el}}{d\Omega}(\frac{a_o^2}{sr})$
v (a.u.)


Differential Cross Sections for Various v (Positron-Hydrogen, Kohn, Higher Waves)
v = 0.1, k = 0.1
v = 0.2, k = 0.2
v = 0.3, k = 0.3
v = 0.4, k = 0.4
v = 0.5, k = 0.5
v = 0.6, k = 0.6
v = 0.7, k = 0.7
θ (degrees)


sin(θ) * Differential Cross Sections for Various v (Positron-Hydrogen, Kohn, Higher Waves)
v = 0.1, k = 0.1
v = 0.2, k = 0.2
v = 0.3, k = 0.3
v = 0.4, k = 0.4
v = 0.5, k = 0.5
v = 0.6, k = 0.6
v = 0.7, k = 0.7
θ (degrees)

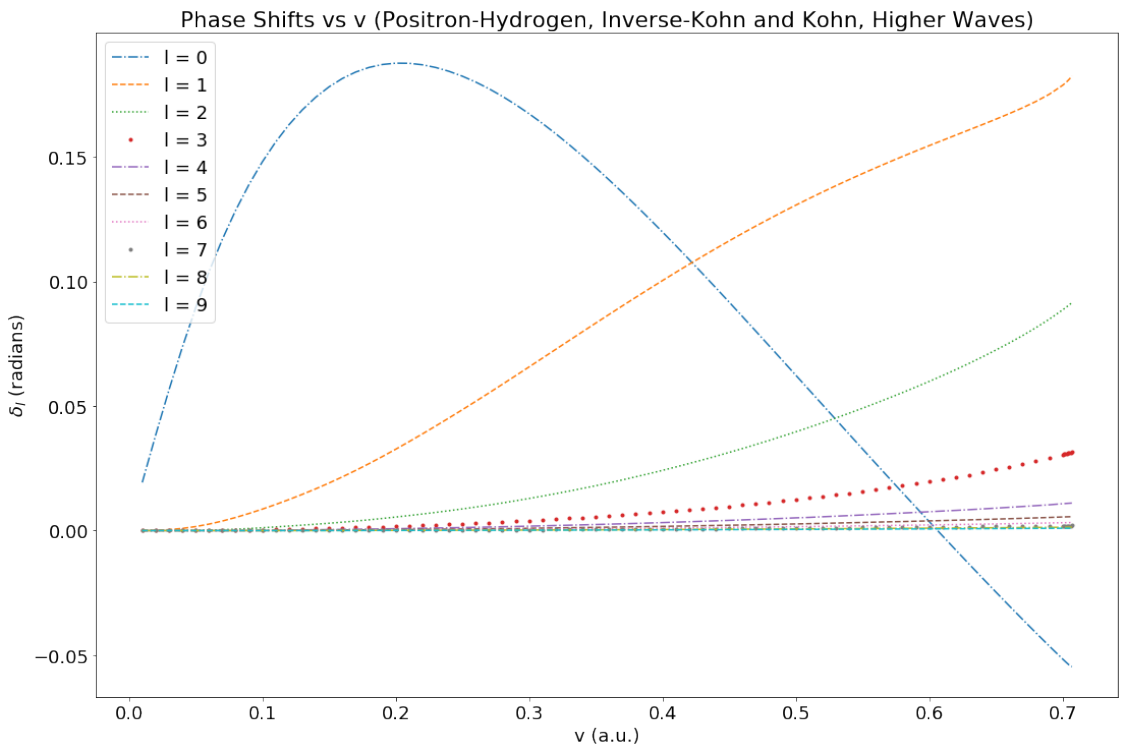

Positron-Hydrogen, Kohn, Higher Waves
θ (degrees)
v (a.u.)
$\frac{d\sigma}{d\Omega}(a_o^2/sr)$



1.1.5 Mixture of Inverse-Kohn and Kohn Phase Shifts for Waves l < 10

Here a mixture of kohn and inverse-kohn phase shifts [12] are used in an attempt to reduce singularities. These graphs are the most comprehensive and stable for the positron-hydrogen system.

[16]:
```
#  Positron-Hydrogen mixture of inverse-kohn and kohn phase shifts for l<10␣
 ↪from P Van Reeth Private Communication 2020
positron_hyd_vr_hybrid_all_system =␣
 ↪collision(positron_hyd_vr_hybrid,k_vr_higher,'Positron-Hydrogen,␣
 ↪Inverse-Kohn and Kohn, Higher Waves',1)

positron_hyd_vr_hybrid_all_system.plot_phase_shifts(velocity=True)
```

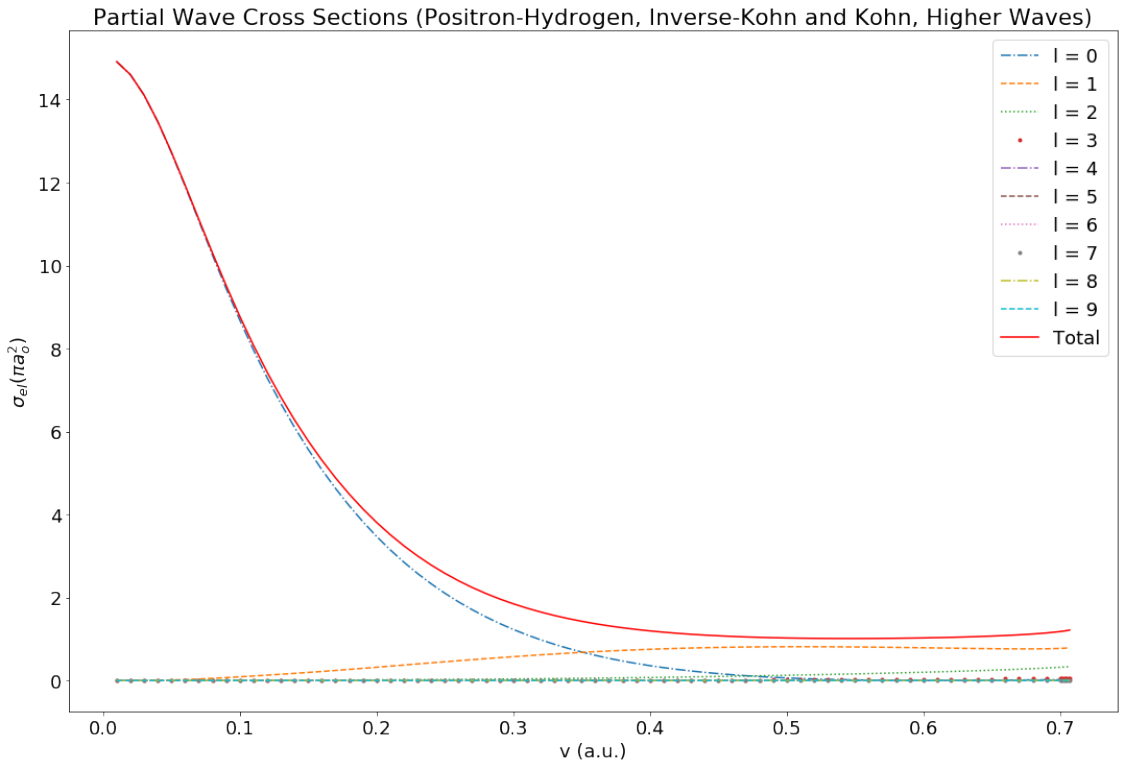


[17]:
```
pos_hyd_cross_data = pos_hyd_vr_hybrid_all_system.plot_cross_section(data=True)
sliced_pos_hyd = pos_hyd_cross_data[pos_hyd_cross_data.index % 5 == 0]
display(sliced_pos_hyd)
get_avg(pos_hyd_cross_data, 'total cross section')
data_min(pos_hyd_cross_data, 'total cross section')
data_max(pos_hyd_cross_data, 'total cross section')
positron_hyd_vr_hybrid_all_system.plot_cross_section(velocity=True)
```

	v	total cross section	l = 0 contribution	l = 1 contribution	\
0	0.010	14.914	99.9998%	0.0002%	
5	0.060	11.940	99.8095%	0.1889%	
10	0.110	8.081	98.6027%	1.3465%	
15	0.160	5.312	95.7048%	4.0949%	
20	0.210	3.516	89.5984%	9.8402%	
25	0.260	2.404	78.8536%	19.8308%	
30	0.310	1.745	63.087%	34.1912%	
35	0.360	1.369	44.3477%	50.7381%	
40	0.410	1.165	26.5916%	65.5208%	
45	0.460	1.063	13.0849%	75.4975%	
50	0.510	1.019	4.7984%	79.85%	
55	0.560	1.013	0.9111%	79.4868%	
60	0.610	1.032	0.0046%	75.9227%	
65	0.660	1.084	0.7692%	70.4237%	
70	0.701	1.184	1.856%	65.4204%	

```
75  0.706                1.209             1.958%           64.9862%

   l = 2 contribution l = 3 contribution l = 4 contribution  \
0                 0.0%               0.0%               0.0%
5              0.0016%               0.0%               0.0%
10             0.0447%            0.0054%            0.0007%
15             0.1674%             0.024%             0.006%
20             0.4682%            0.0672%            0.0166%
25             1.0994%            0.1537%            0.0388%
30             2.2824%             0.314%            0.0788%
35             4.1383%            0.5582%            0.1351%
40              6.652%            0.8982%            0.2104%
45             9.6324%            1.3117%             0.294%
50            12.9363%            1.8087%            0.3806%
55            16.4458%            2.3947%            0.4818%
60            20.1049%            3.0673%            0.5774%
65            23.9294%             3.846%            0.6649%
70            27.1023%            4.5061%            0.7251%
75            27.3875%            4.5556%            0.7241%

   l = 5 contribution l = 6 contribution l = 7 contribution  \
0                 0.0%               0.0%               0.0%
5                 0.0%               0.0%               0.0%
10             0.0001%               0.0%               0.0%
15              0.002%            0.0007%            0.0002%
20             0.0055%            0.0021%             0.001%
25             0.0134%            0.0055%            0.0029%
30             0.0267%            0.0107%            0.0051%
35             0.0467%            0.0193%            0.0091%
40             0.0719%            0.0297%             0.014%
45             0.0983%            0.0457%            0.0197%
50             0.1285%            0.0528%            0.0248%
55             0.1593%            0.0652%            0.0306%
60             0.1857%            0.0748%            0.0348%
65             0.2114%            0.0847%            0.0393%
70             0.2272%            0.0898%            0.0409%
75             0.2264%            0.0894%            0.0405%

   l = 8 contribution l = 9 contribution
0                 0.0%               0.0%
5                 0.0%               0.0%
10                0.0%               0.0%
15             0.0001%               0.0%
20             0.0005%            0.0003%
25             0.0013%            0.0007%
30             0.0026%            0.0014%
35             0.0047%            0.0026%
40             0.0072%            0.0041%
```

```
45            0.0101%           0.0056%
50            0.0128%           0.0071%
55            0.0158%           0.0088%
60            0.0178%           0.0099%
65            0.0202%           0.0112%
70            0.0208%           0.0115%
75            0.0208%           0.0114%


Average value for total cross section : 3.361
Minimum value for total cross section

       v  total cross section l = 0 contribution l = 1 contribution  \
53  0.54                1.012           2.0266%          80.0959%
54  0.55                1.012           1.4037%          79.8606%

   l = 2 contribution l = 3 contribution l = 4 contribution  \
53          15.0246%           2.1511%           0.4427%
54          15.7325%           2.2713%           0.4623%

   l = 5 contribution l = 6 contribution l = 7 contribution  \
53           0.1473%           0.0605%           0.0284%
54           0.1534%           0.0629%           0.0295%

   l = 8 contribution l = 9 contribution
53           0.0147%           0.0082%
54           0.0153%           0.0085%


Maximum value for total cross section

      v  total cross section l = 0 contribution l = 1 contribution  \
0  0.01               14.914          99.9998%           0.0002%

  l = 2 contribution l = 3 contribution l = 4 contribution l = 5 contribution  \
0              0.0%              0.0%              0.0%              0.0%

  l = 6 contribution l = 7 contribution l = 8 contribution l = 9 contribution
0              0.0%              0.0%              0.0%              0.0%
```



The elastic cross section quickly decreases before leveling out around 1. Waves beyond the f wave do not have significant contribution to the elastic cross section; at no point do any of these higher waves contribute even 1% to the total cross section. As the velocity increases, the contribution from the s wave gradually decreases, while the contribution from all other waves increases. At the highest velocity, the contribution from the s, p, d, and f wave are 2, 65, 27, 5 percent, respectively.

[18]:
```
pos_hyd_momentum_data = pos_hyd_vr_hybrid_all_system.
 ↪plot_momentum_transfer(data=True)
sliced = pos_hyd_momentum_data[pos_hyd_momentum_data.index % 5 == 0]
display(sliced)
get_avg(pos_hyd_momentum_data, 'total momentum transfer')
data_max(pos_hyd_momentum_data, 'total momentum transfer')
data_min(pos_hyd_momentum_data, 'total momentum transfer')

positron_hyd_vr_hybrid_all_system.plot_momentum_transfer(velocity=True)
```

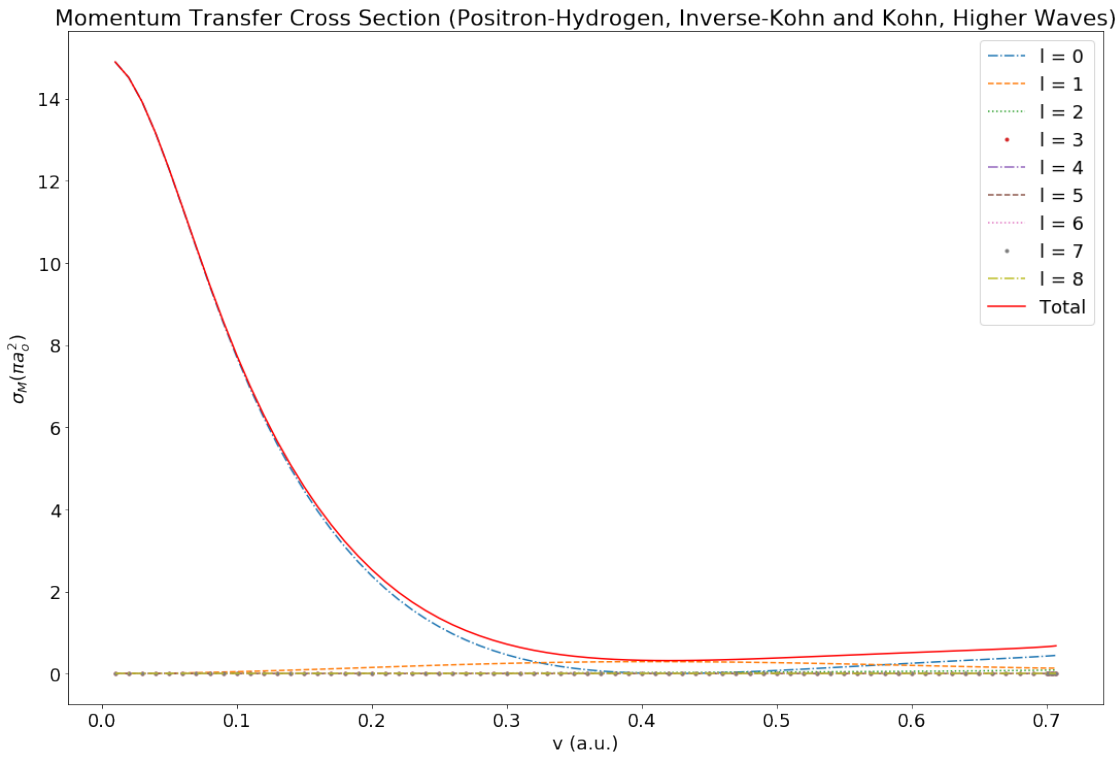
	v	total momentum transfer	l = 0 contribution	l = 1 contribution	\
0	0.010	14.891	99.9999%	0.0001%	
5	0.060	11.343	99.8849%	0.1144%	
10	0.110	6.995	99.2176%	0.765%	
15	0.160	4.063	97.3936%	2.5386%	
20	0.210	2.239	92.6545%	7.118%	
25	0.260	1.187	81.3928%	17.9136%	

30	0.310	0.635	57.9394%	40.0969%
35	0.360	0.389	23.1038%	72.3012%
40	0.410	0.317	0.8697%	91.3983%
45	0.460	0.334	6.2883%	84.0419%
50	0.510	0.390	23.1847%	66.2017%
55	0.560	0.457	39.0813%	49.5893%
60	0.610	0.523	51.2929%	36.4887%
65	0.660	0.587	60.1406%	26.4045%
70	0.701	0.656	65.0685%	20.1127%
75	0.706	0.671	65.4091%	19.6255%

	l = 2 contribution	l = 3 contribution	l = 4 contribution \
0	0.0%	0.0%	0.0%
5	0.0008%	0.0%	0.0%
10	0.0155%	0.0017%	0.0002%
15	0.0608%	0.0056%	0.001%
20	0.2039%	0.019%	0.0034%
25	0.6252%	0.0552%	0.0096%
30	1.7731%	0.1536%	0.0269%
35	4.1552%	0.3596%	0.0579%
40	6.9804%	0.6203%	0.0955%
45	8.7026%	0.8086%	0.1181%
50	9.4867%	0.957%	0.1242%
55	10.0376%	1.1071%	0.1365%
60	10.6987%	1.3188%	0.1502%
65	11.6092%	1.6277%	0.1637%
70	12.6421%	1.9415%	0.1773%
75	12.7571%	1.9735%	0.1772%

	l = 5 contribution	l = 6 contribution	l = 7 contribution	l = 8 contribution
0	0.0%	0.0%	0.0%	0.0%
5	0.0%	0.0%	0.0%	0.0%
10	0.0%	0.0%	0.0%	0.0%
15	0.0003%	0.0001%	0.0%	0.0%
20	0.0008%	0.0003%	0.0001%	0.0%
25	0.0025%	0.0007%	0.0004%	0.0001%
30	0.007%	0.002%	0.0008%	0.0003%
35	0.0149%	0.0048%	0.0018%	0.0008%
40	0.0241%	0.0077%	0.0029%	0.0012%
45	0.0237%	0.0119%	0.0035%	0.0015%
50	0.0308%	0.0097%	0.0036%	0.0015%
55	0.0326%	0.0102%	0.0038%	0.0016%
60	0.0346%	0.0106%	0.0039%	0.0016%
65	0.0372%	0.0112%	0.0041%	0.0017%
70	0.0398%	0.0121%	0.0043%	0.0018%
75	0.0396%	0.0121%	0.0042%	0.0018%

```
Average value for total momentum transfer : 2.563
Maximum value for total momentum transfer

      v  total momentum transfer l = 0 contribution l = 1 contribution  \
0  0.01                    14.891           99.9999%            0.0001%

  l = 2 contribution l = 3 contribution l = 4 contribution l = 5 contribution  \
0               0.0%               0.0%               0.0%               0.0%

  l = 6 contribution l = 7 contribution l = 8 contribution
0               0.0%               0.0%               0.0%


Minimum value for total momentum transfer

       v  total momentum transfer l = 0 contribution l = 1 contribution  \
41  0.42                     0.315            0.0274%            91.7349%

   l = 2 contribution l = 3 contribution l = 4 contribution  \
41            7.4337%            0.6649%            0.1012%

   l = 5 contribution l = 6 contribution l = 7 contribution l = 8 contribution
41            0.0254%            0.0082%            0.0031%            0.0013%
```



The elastic cross section is greater than the momentum transfer cross section throughout. The

difference between the two increases until mid-range velocities, where it then begins to shrink. The s wave maintains a majority contribution to the momentum transfer cross section except at mid-range velocity. At the highest velocity, the contribution from the s, p, d, and f wave are 65, 20, 13, 2 percent, respectively.

[19]:
```
pos_hyd_cross_momentum = cross_momentum_compare(pos_hyd_cross_data,␣
↪pos_hyd_momentum_data)

sliced = pos_hyd_cross_momentum[pos_hyd_cross_momentum.index % 5 == 0]
display(sliced)
get_avg(pos_hyd_cross_momentum, 'ratio momentum/cross')
data_max(pos_hyd_cross_momentum, 'ratio momentum/cross')
data_min(pos_hyd_cross_momentum, 'ratio momentum/cross')
```

	v	total cross section	total momentum transfer	ratio momentum/cross
0	0.010	14.914	14.891	0.998
5	0.060	11.940	11.343	0.950
10	0.110	8.081	6.995	0.866
15	0.160	5.312	4.063	0.765
20	0.210	3.516	2.239	0.637
25	0.260	2.404	1.187	0.494
30	0.310	1.745	0.635	0.364
35	0.360	1.369	0.389	0.284
40	0.410	1.165	0.317	0.272
45	0.460	1.063	0.334	0.314
50	0.510	1.019	0.390	0.383
55	0.560	1.013	0.457	0.451
60	0.610	1.032	0.523	0.507
65	0.660	1.084	0.587	0.542
70	0.701	1.184	0.656	0.554
75	0.706	1.209	0.671	0.555

```
Average value for ratio momentum/cross : 0.547
Maximum value for ratio momentum/cross
```

	v	total cross section	total momentum transfer	ratio momentum/cross
0	0.01	14.914	14.891	0.998

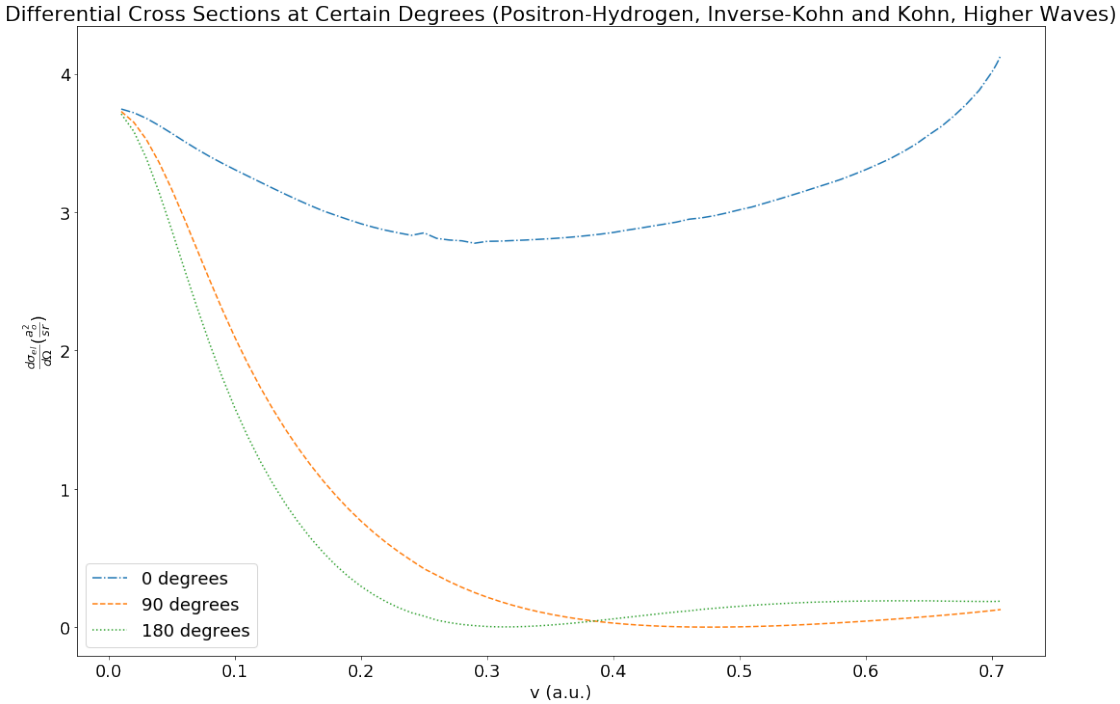
```
Minimum value for ratio momentum/cross
```

	v	total cross section	total momentum transfer	ratio momentum/cross
38	0.39	1.231	0.331	0.269
39	0.40	1.196	0.322	0.269

[20]:
```
pos_hyd_diff_degree =  positron_hyd_vr_hybrid_all_system.
↪plot_diff_cross_vs_k(r_interval,velocity=True, data=True)
sliced = pos_hyd_diff_degree[pos_hyd_diff_degree.index % 5 == 0]
```

```
display(sliced)

positron_hyd_vr_hybrid_all_system.plot_diff_cross_vs_k(r_interval,velocity=True)
```

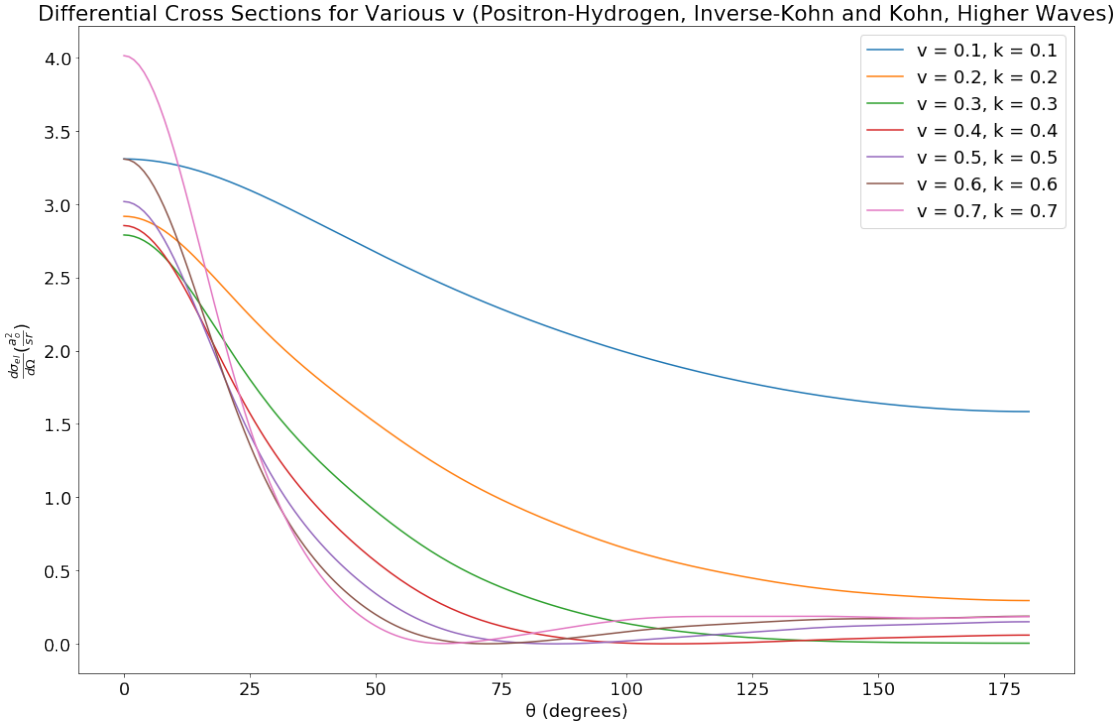
	v	0 degrees	90 degrees	180 degrees
0	0.010	3.746	3.729	3.711
5	0.060	3.512	2.953	2.591
10	0.110	3.263	1.910	1.384
15	0.160	3.048	1.173	0.645
20	0.210	2.891	0.685	0.235
25	0.260	2.811	0.375	0.051
30	0.310	2.791	0.186	0.002
35	0.360	2.815	0.077	0.022
40	0.410	2.869	0.021	0.070
45	0.460	2.950	0.001	0.118
50	0.510	3.038	0.004	0.158
55	0.560	3.177	0.022	0.181
60	0.610	3.348	0.050	0.189
65	0.660	3.620	0.085	0.188
70	0.701	4.028	0.121	0.186
75	0.706	4.114	0.126	0.187



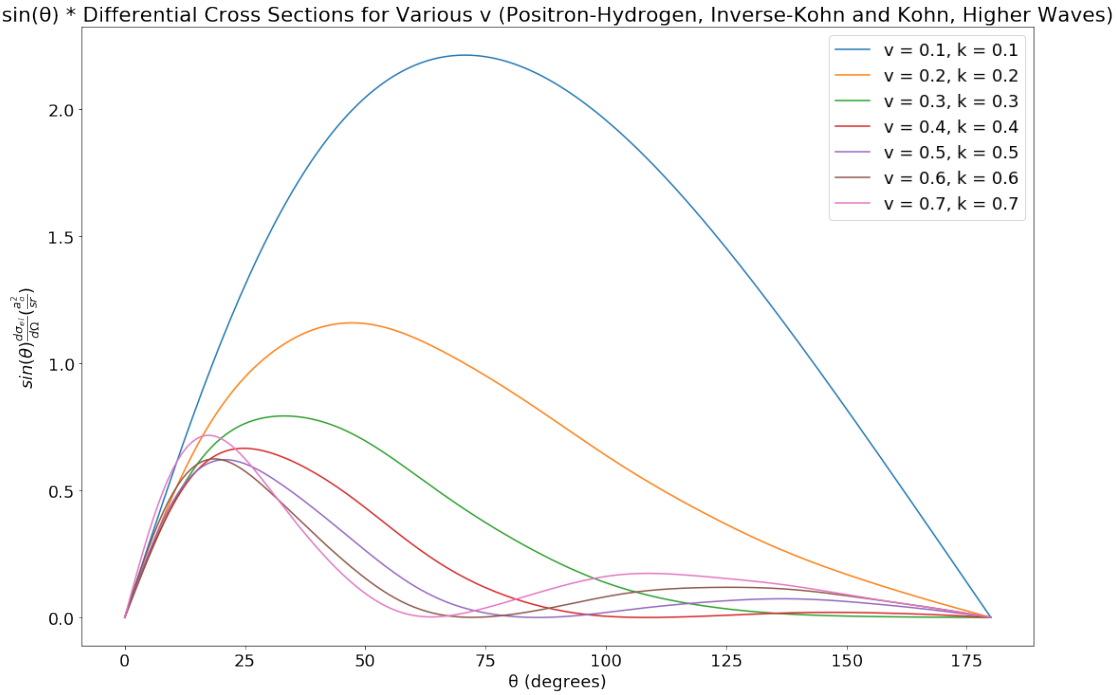
[21]:
```
pos_hyd_diff = positron_hyd_vr_hybrid_all_system.
↪diff_plot(interval='hydrogen',velocity=True, data=True)
```

```
sliced = pos_hyd_diff[pos_hyd_diff.index % 10 == 0 ]
display(sliced)

positron_hyd_vr_hybrid_all_system.diff_plot(interval='hydrogen',velocity=True)
```

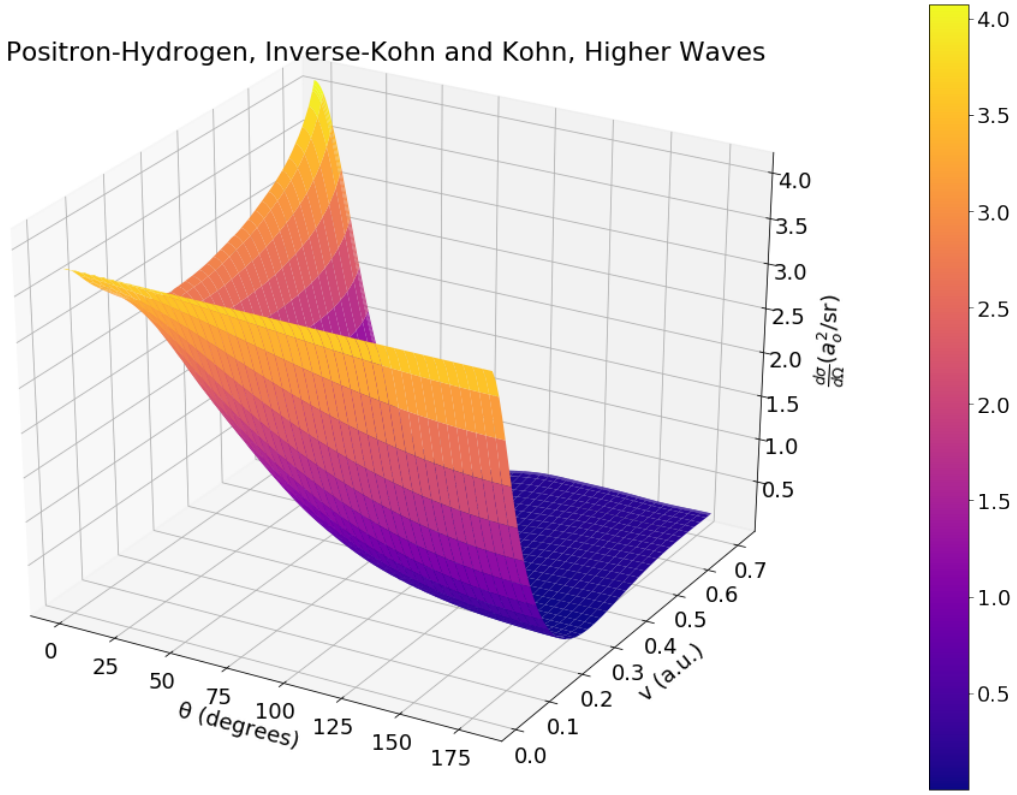
	degrees	v = 0.1	v = 0.2	v = 0.3	v = 0.4	v = 0.5	v = 0.6	v = 0.7
0	0	3.308	2.917	2.790	2.854	3.018	3.308	4.013
10	10	3.271	2.767	2.562	2.548	2.627	2.816	3.363
20	20	3.167	2.425	2.062	1.895	1.815	1.818	2.060
30	30	3.017	2.069	1.579	1.300	1.112	0.994	1.018
40	40	2.846	1.771	1.208	0.877	0.651	0.495	0.426
50	50	2.673	1.510	0.908	0.564	0.345	0.202	0.123
60	60	2.508	1.274	0.656	0.329	0.146	0.048	0.009
70	70	2.356	1.072	0.462	0.174	0.043	0.002	0.016
80	80	2.219	0.907	0.322	0.082	0.005	0.011	0.066
90	90	2.097	0.767	0.216	0.029	0.002	0.043	0.120
100	100	1.988	0.648	0.139	0.005	0.020	0.083	0.164
110	110	1.894	0.554	0.087	0.000	0.045	0.115	0.185
120	120	1.812	0.480	0.054	0.006	0.069	0.136	0.187
130	130	1.743	0.419	0.032	0.017	0.093	0.154	0.188
140	140	1.686	0.372	0.018	0.030	0.114	0.168	0.188
150	150	1.642	0.340	0.011	0.040	0.126	0.172	0.180
160	160	1.610	0.317	0.007	0.048	0.134	0.174	0.174
170	170	1.591	0.302	0.005	0.056	0.145	0.183	0.180
180	180	1.585	0.295	0.004	0.060	0.151	0.189	0.185

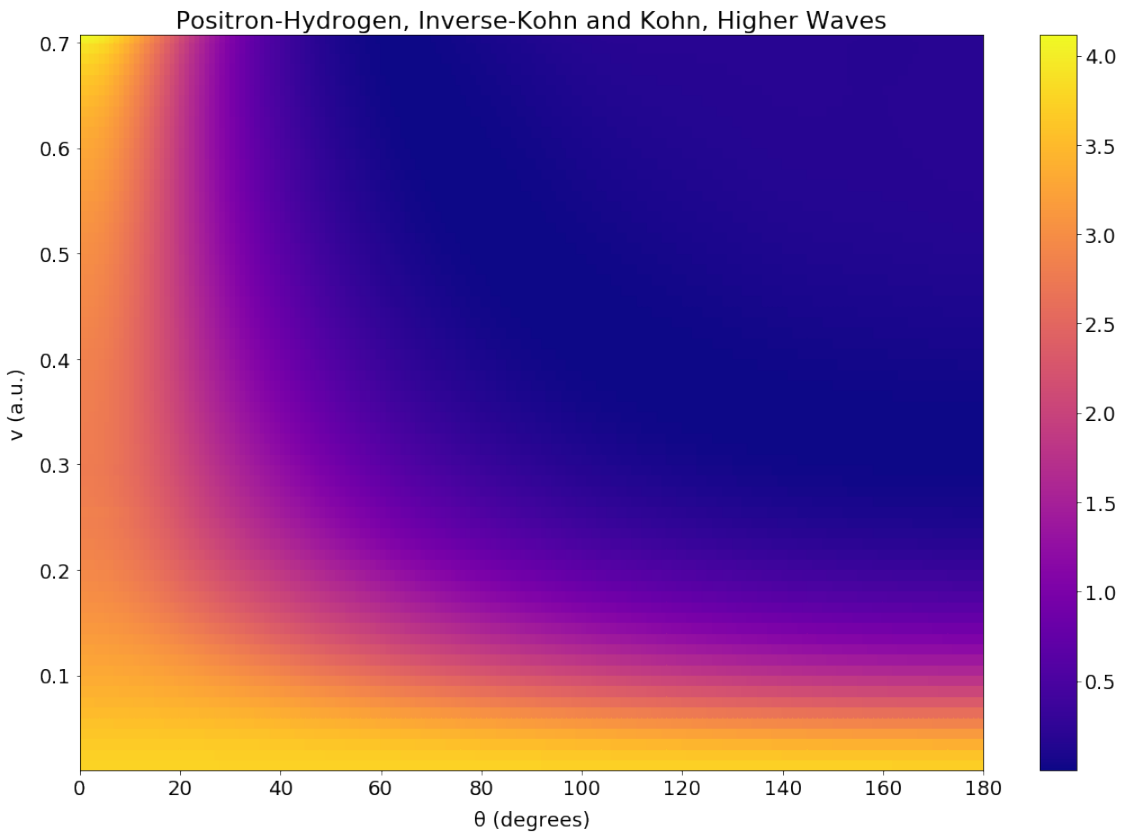


[22]:
```
positron_hyd_vr_hybrid_all_system.
↪diff_plot(sin=True,interval='hydrogen',velocity=True)
```



[23]:
```
positron_hyd_vr_hybrid_all_system.diff_plot_3d(velocity=True)
positron_hyd_vr_hybrid_all_system.diff_plot_3d(density=True,velocity=True)
```


Positron-Hydrogen, Inverse-Kohn and Kohn, Higher Waves
0
25
50
75
100
125
150
175
θ (degrees)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v (a.u.)
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
dσ/dΩ (a_0^2/sr)

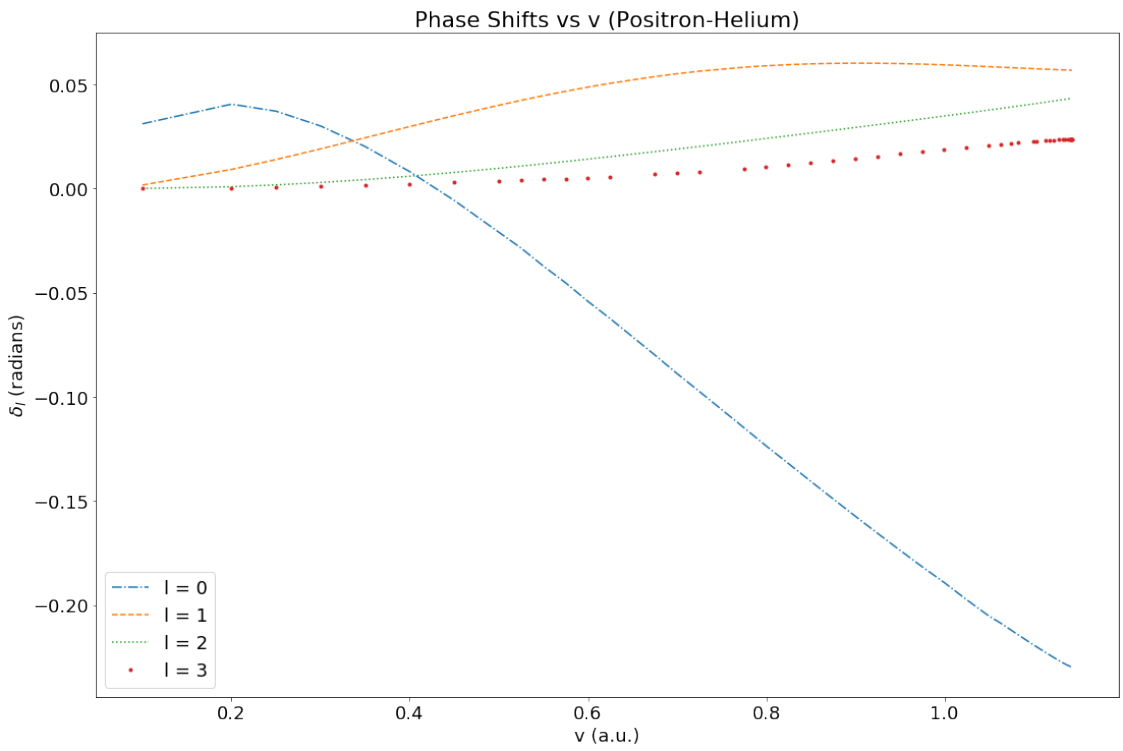


1.2 Positron-Helium

Phase shifts provided by P Van Reeth, Private Communication 2020 [13].

[24]:
```
#  Positron-Helium, data from P Van Reeth Private Communication 2020, this is␣
 ↪the most optimal data
positron_hel_system_vr = collision(pos_hel_vr_phases,k_vr_pos_hel,␣
 ↪'Positron-Helium',1)
```

[25]:
```
positron_hel_system_vr.plot_phase_shifts(velocity=True)
```

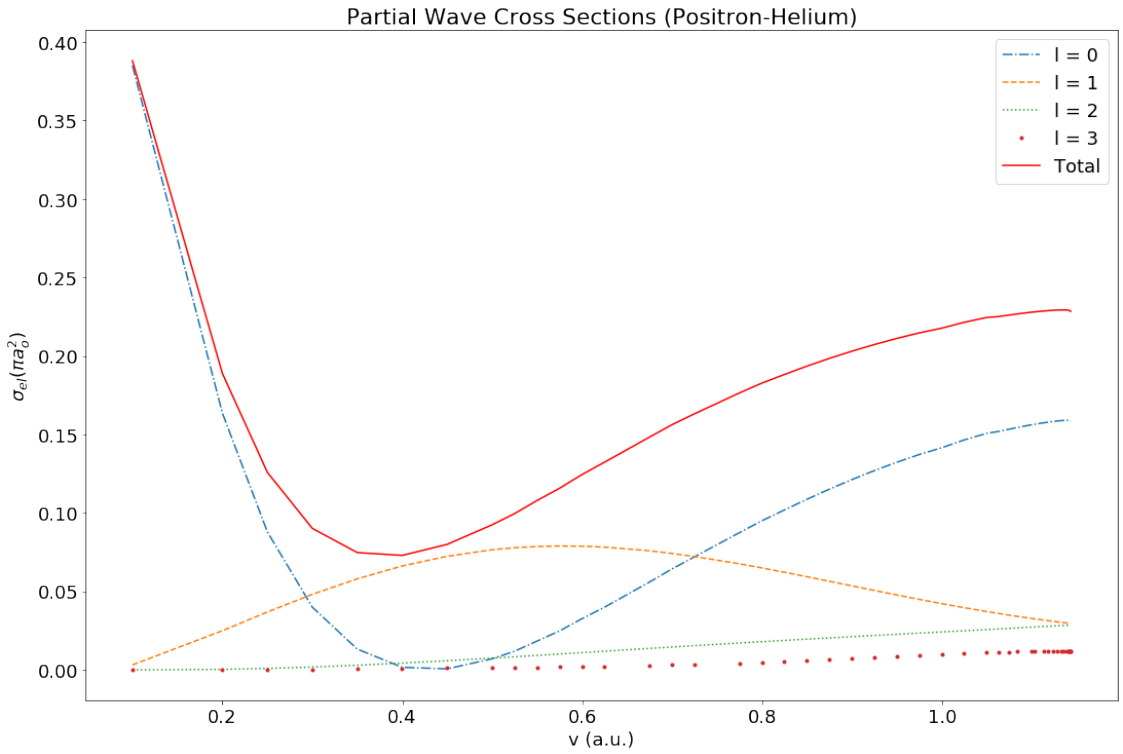


The s wave phase shifts rapidly decrease into negative values that are greater in magnitude than the other waves. The p, d, and f waves all have a slight increase with increasing energy. The higher the wave, the less this increase.

[26]:
```
pos_hel_cross_data = positron_hel_system_vr.plot_cross_section(data=True)
sliced = pos_hel_cross_data[pos_hel_cross_data.index % 5 == 0]

display(sliced)
get_avg(pos_hel_cross_data, 'total cross section')
data_max(pos_hel_cross_data, 'total cross section')
data_min(pos_hel_cross_data, 'total cross section')

positron_hel_system_vr.plot_cross_section(velocity=True)
```

	v	total cross section	l = 0 contribution	l = 1 contribution	\
0	0.100	0.388	99.1792%	0.8204%	
5	0.400	0.073	2.2413%	90.7431%	
10	0.575	0.116	21.4459%	68.2032%	
15	0.725	0.163	44.1328%	44.2743%	
20	0.875	0.198	58.0061%	28.4515%	
25	1.000	0.218	65.003%	19.3915%	
30	1.083	0.227	68.0713%	15.0621%	
35	1.123	0.229	69.0722%	13.5116%	
40	1.141	0.229	69.3567%	12.966%	

```
45  1.143                0.228           69.2947%           12.9341%

   l = 2 contribution l = 3 contribution
0              0.0002%            0.0001%
5              5.8941%            1.1214%
10             8.7352%            1.6157%
15             9.4542%            2.1388%
20            10.2636%            3.2788%
25            11.0993%            4.5061%
30            11.7671%            5.0996%
35            12.1936%            5.2226%
40            12.4373%            5.2401%
45            12.5144%            5.2568%


Average value for total cross section : 0.189
Maximum value for total cross section

     v  total cross section l = 0 contribution l = 1 contribution  \
0  0.1                0.388           99.1792%            0.8204%

  l = 2 contribution l = 3 contribution
0            0.0002%            0.0001%


Minimum value for total cross section

     v  total cross section l = 0 contribution l = 1 contribution  \
5  0.4                0.073            2.2413%           90.7431%

  l = 2 contribution l = 3 contribution
5            5.8941%            1.1214%
```



The elastic cross section rapidly decreases before beginning to gradually rise again with increasing energy after v = 0.4. The s wave is the greatest contributor until v = 0.3, where the p wave overtakes it. The s wave has its lowest contribution at v = 0.45 with only a 0.8% contribution; the p wave contributes 90.5% at this velocity. After this point, the contribution of the s wave rises while the p wave decreases. The d and f wave contributions increase with increasing energy throughout. At the highest velocity of 1.14, the s, p, d, and f wave contribute 69.9, 12.9, 12.5, and 5.3 percent, respectively.

[27]:
```
pos_hel_momentum_data = positron_hel_system_vr.
 ↪plot_momentum_transfer(velocity=True, data=True)

sliced = pos_hel_momentum_data[pos_hel_momentum_data.index % 5 == 0]

display(sliced)
get_avg(pos_hel_momentum_data, 'total momentum transfer')
data_max(pos_hel_momentum_data, 'total momentum transfer')
data_min(pos_hel_momentum_data, 'total momentum transfer')

positron_hel_system_vr.plot_momentum_transfer(velocity=True)
```

	v	total momentum transfer	l = 0 contribution	l = 1 contribution	\
0	0.100	0.348	99.3737%	0.6258%	
5	0.400	0.041	28.4088%	69.0922%	
10	0.575	0.132	77.2972%	20.8408%	

```
15  0.725                    0.202          88.4974%           9.8603%
20  0.875                    0.239          94.0959%           4.5093%
25  1.000                    0.250           96.831%           1.9331%
30  1.083                    0.252          97.8473%           0.9009%
35  1.123                    0.251          98.0898%           0.5751%
40  1.141                    0.250          98.1391%           0.4634%
45  1.143                    0.249          98.1367%           0.4503%
```

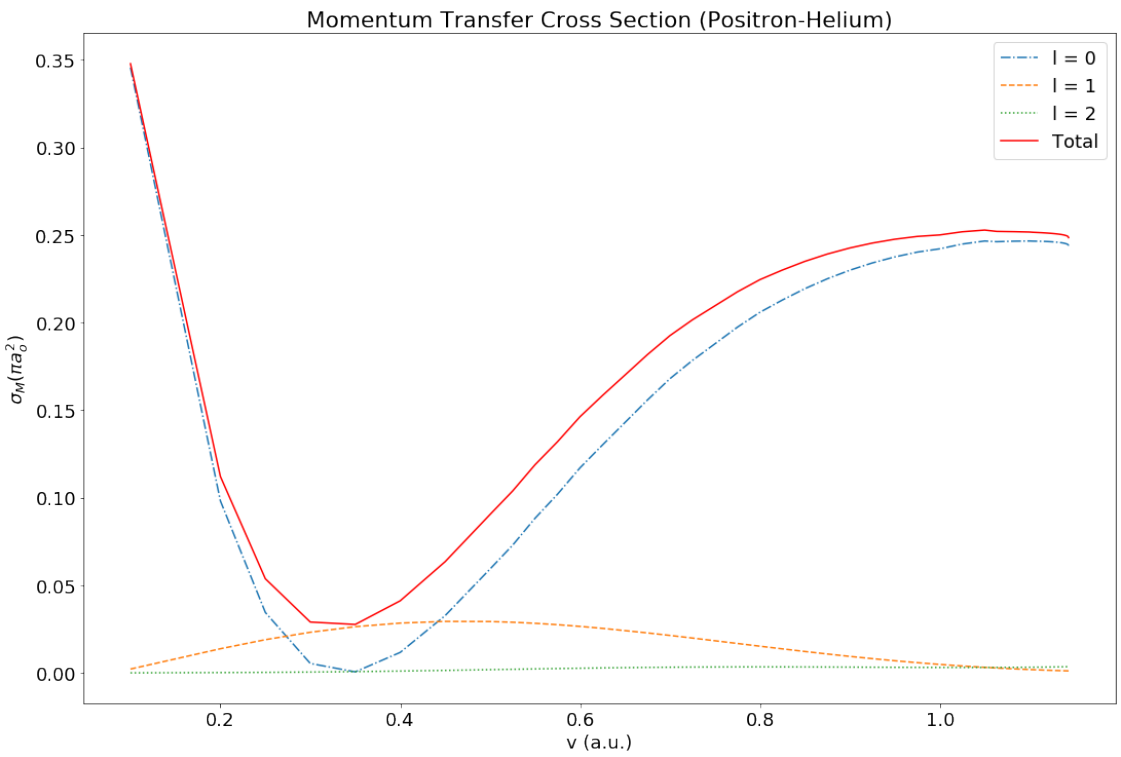
```
   l = 2 contribution
0              0.0004%
5               2.499%
10              1.862%
15             1.6423%
20             1.3948%
25             1.2358%
30             1.2518%
35             1.3351%
40             1.3975%
45             1.4129%
```

```
Average value for total momentum transfer : 0.204
Maximum value for total momentum transfer
```

```
     v  total momentum transfer l = 0 contribution l = 1 contribution  \
0  0.1                    0.348           99.3737%            0.6258%

  l = 2 contribution
0            0.0004%
```

```
Minimum value for total momentum transfer
```

```
      v  total momentum transfer l = 0 contribution l = 1 contribution  \
4  0.35                    0.028            2.0909%           95.3412%

  l = 2 contribution
4            2.5678%
```



The momentum transfer cross section is less than the elastic cross section for v < 0.52. The momentum transfer cross section rises quicker than the elastic cross section. The s wave has a greater contribution to the momentum transfer cross section than the elastic cross section.

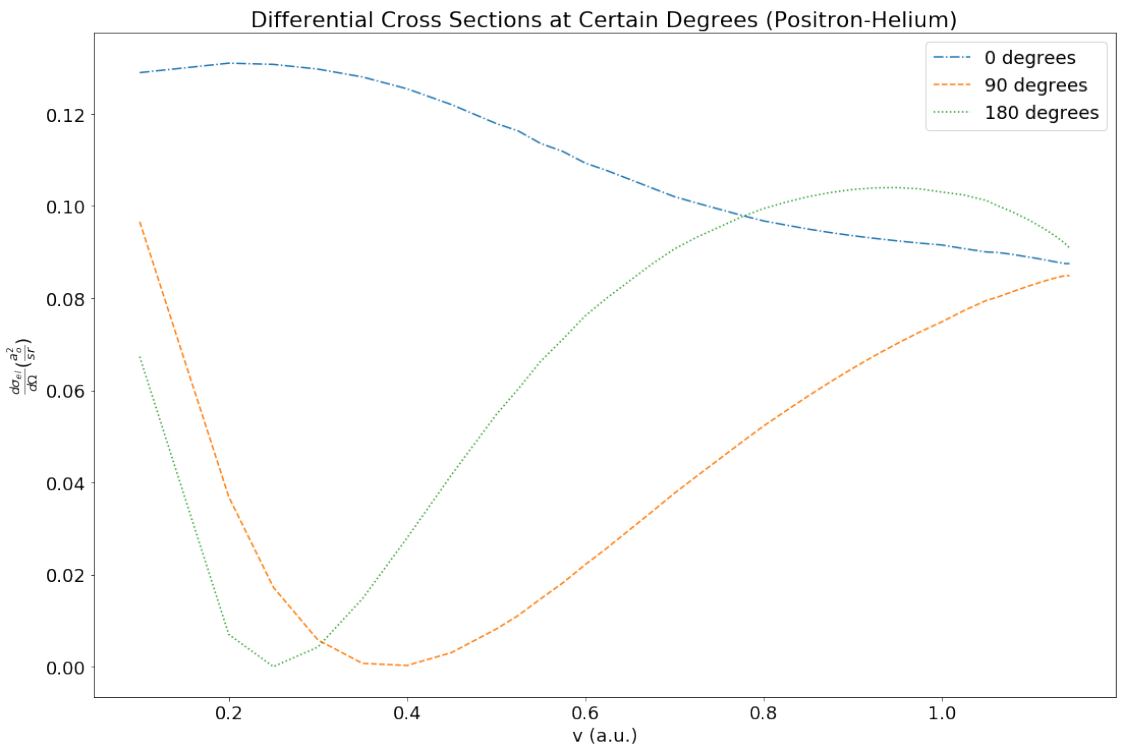
[28]:
```
pos_hel_cross_momentum = cross_momentum_compare(pos_hel_cross_data,␣
↪pos_hel_momentum_data)

sliced = pos_hel_cross_momentum[pos_hel_cross_momentum.index % 5 == 0]
display(sliced)

get_avg(pos_hel_cross_momentum, 'ratio momentum/cross')
data_max(pos_hel_cross_momentum, 'ratio momentum/cross')
data_min(pos_hel_cross_momentum, 'ratio momentum/cross')
```

	v	total cross section	total momentum transfer	ratio momentum/cross
0	0.100	0.388	0.348	0.897
5	0.400	0.073	0.041	0.562
10	0.575	0.116	0.132	1.138
15	0.725	0.163	0.202	1.239
20	0.875	0.198	0.239	1.207
25	1.000	0.218	0.250	1.147
30	1.083	0.227	0.252	1.110
35	1.123	0.229	0.251	1.096
40	1.141	0.229	0.250	1.092

```
45  1.143                0.228                     0.249                   1.092


Average value for ratio momentum/cross : 1.05
Maximum value for ratio momentum/cross

        v  total cross section  total momentum transfer  ratio momentum/cross
15  0.725                0.163                    0.202                 1.239
16  0.775                0.176                    0.218                 1.239


Minimum value for ratio momentum/cross

     v  total cross section  total momentum transfer  ratio momentum/cross
3  0.3                 0.09                    0.029                 0.322
```

[29]:
```
pos_hel_diff_degree =  positron_hel_system_vr.
 ↪plot_diff_cross_vs_k(r_interval,velocity=True, data=True)
sliced = pos_hel_diff_degree[pos_hel_diff_degree.index % 5 == 0]
display(sliced)

positron_hel_system_vr.plot_diff_cross_vs_k(r_interval,velocity=True)
```

```
        v  0 degrees  90 degrees  180 degrees
0   0.100      0.129       0.097        0.067
5   0.400      0.125       0.000        0.028
10  0.575      0.112       0.018        0.071
15  0.725      0.101       0.041        0.093
20  0.875      0.094       0.062        0.103
25  1.000      0.092       0.075        0.103
30  1.083      0.089       0.082        0.098
35  1.123      0.088       0.084        0.094
40  1.141      0.087       0.085        0.091
45  1.143      0.088       0.085        0.091
```



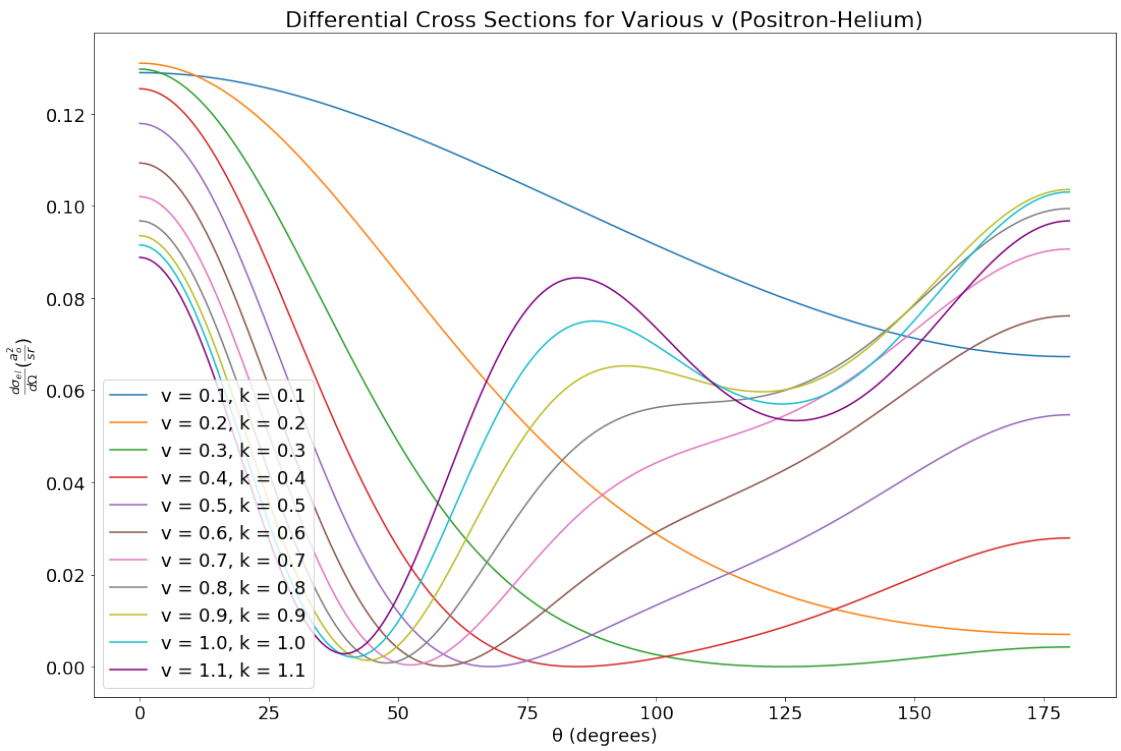
At 180 degrees the curve resembles the shape of the momentum transfer cross section. At 90 degrees it resembles the shape of the elastic cross section.

[30]:
```
pos_hel_diff = positron_hel_system_vr.
 ↪diff_plot(interval='helium',velocity=True, data=True)
sliced = pos_hel_diff[pos_hel_diff.index % 10 == 0 ]
display(sliced)

positron_hel_system_vr.diff_plot(interval='helium',velocity=True)
```

	degrees	v = 0.1	v = 0.2	v = 0.3	v = 0.4	v = 0.5	v = 0.6	v = 0.7	\
0	0	0.129	0.131	0.130	0.125	0.118	0.109	0.102	
10	10	0.128	0.129	0.125	0.118	0.110	0.100	0.092	
20	20	0.127	0.122	0.111	0.099	0.087	0.076	0.066	
30	30	0.124	0.112	0.091	0.074	0.059	0.046	0.035	
40	40	0.121	0.099	0.069	0.047	0.031	0.020	0.011	
50	50	0.116	0.085	0.049	0.026	0.012	0.004	0.001	
60	60	0.112	0.071	0.032	0.011	0.002	0.000	0.004	
70	70	0.107	0.058	0.020	0.003	0.000	0.005	0.015	
80	80	0.102	0.047	0.011	0.000	0.003	0.014	0.027	
90	90	0.097	0.037	0.006	0.000	0.008	0.022	0.038	
100	100	0.091	0.029	0.003	0.002	0.013	0.029	0.044	
110	110	0.087	0.022	0.001	0.004	0.018	0.035	0.048	
120	120	0.082	0.018	0.000	0.007	0.023	0.040	0.052	

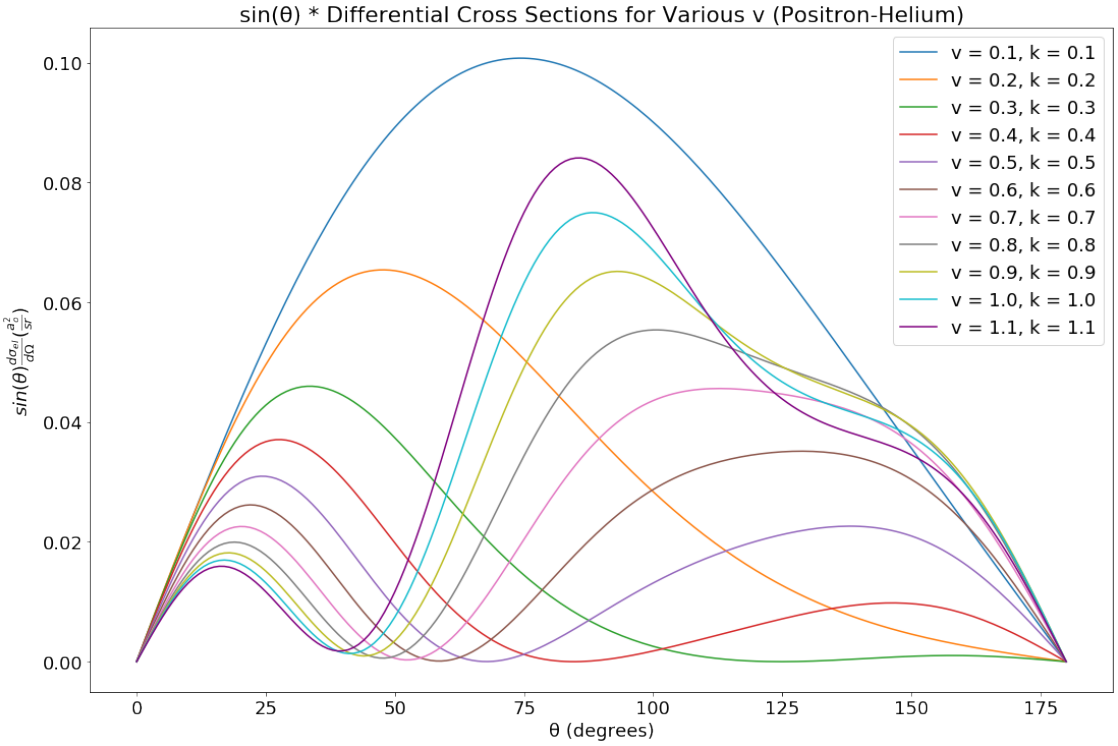
130	130	0.078	0.014	0.000	0.011	0.029	0.046	0.057
140	140	0.074	0.011	0.001	0.015	0.035	0.053	0.064
150	150	0.071	0.009	0.002	0.019	0.042	0.061	0.073
160	160	0.069	0.008	0.003	0.024	0.048	0.068	0.082
170	170	0.068	0.007	0.004	0.027	0.053	0.074	0.088
180	180	0.067	0.007	0.004	0.028	0.055	0.076	0.091

	v = 0.8	v = 0.9	v = 1.0	v = 1.1
0	0.097	0.094	0.092	0.089
10	0.086	0.082	0.079	0.076
20	0.058	0.052	0.048	0.044
30	0.027	0.021	0.017	0.014
40	0.006	0.003	0.002	0.003
50	0.001	0.005	0.010	0.015
60	0.011	0.021	0.032	0.042
70	0.027	0.042	0.056	0.069
80	0.043	0.058	0.072	0.083
90	0.052	0.065	0.075	0.083
100	0.056	0.065	0.070	0.074
110	0.057	0.061	0.062	0.062
120	0.059	0.060	0.058	0.055
130	0.062	0.062	0.058	0.054
140	0.069	0.068	0.064	0.059
150	0.078	0.079	0.075	0.069
160	0.089	0.091	0.088	0.082
170	0.096	0.100	0.099	0.093
180	0.099	0.104	0.103	0.097



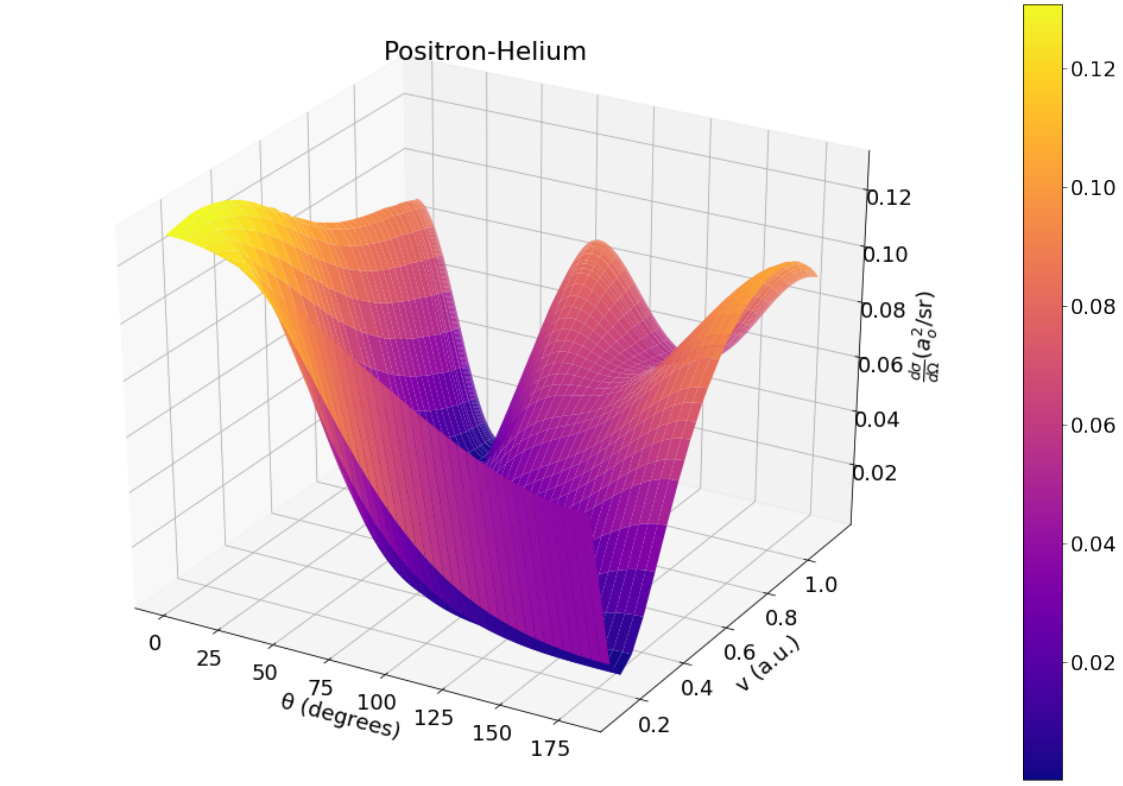
At 0 degrees the differential cross section is less the higher the energy with the exception of v = 0.1, which is slightly less than v = 0.3 . The general behavior of the differential cross section at a given energy it decreases with increasing degrees before rising again. This behavior occurs quicker for higher energy. Thus, at lower energy the curve does not begin to rise by the end of the graph. At higher energies, the differential cross section falls to a lesser extent once at local maximum is reached. It then begins to rise once more. This behavior is more evident the higher the energy.

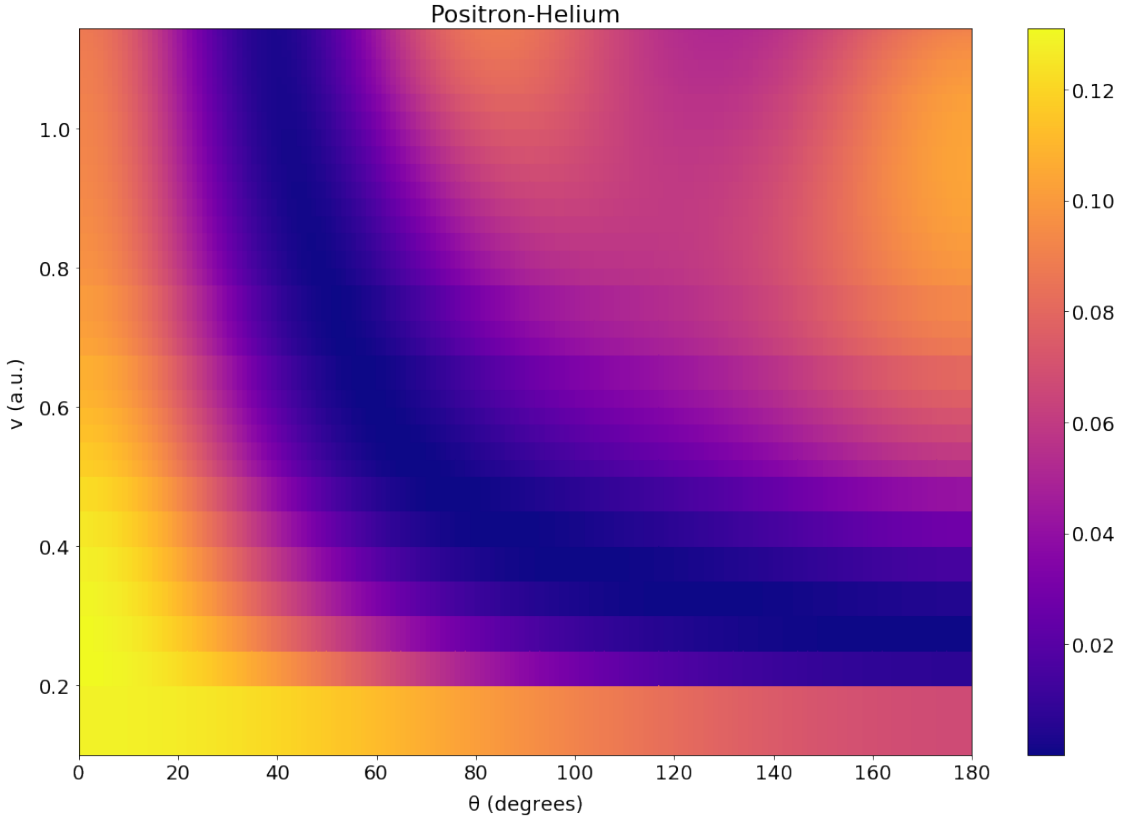
[31]:
```
positron_hel_system_vr.diff_plot(sin=True,interval='helium',velocity=True)
```



At low energies, the curve looks parabolic. As the energy increases the maximum of this first parabola becomes less, and the parabola occurs at a lower degree. Once this parabola is finished another begins to form. This is most evident at the higher energies where two parabolas are easily visible.

[32]:
```
positron_hel_system_vr.diff_plot_3d(velocity=True)
positron_hel_system_vr.diff_plot_3d(density=True,velocity=True)
```

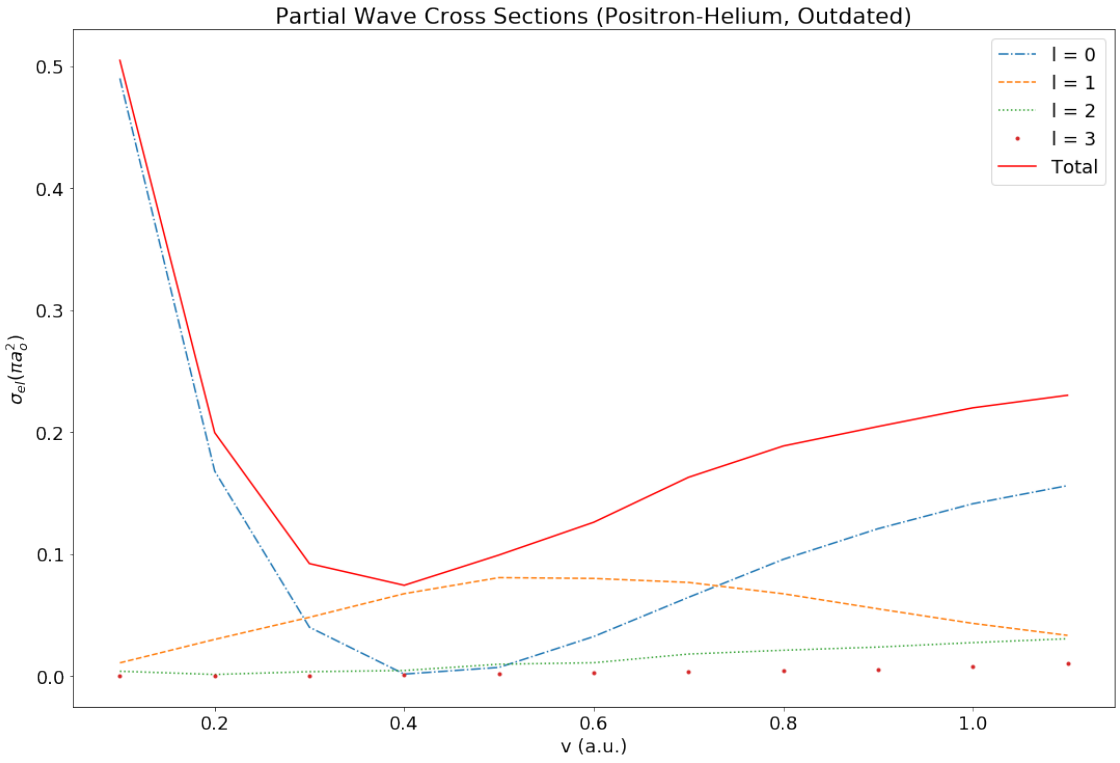

Positron-Helium
0.12
0.10
0.08
0.06
0.04
0.02
$\frac{d\sigma}{d\Omega}(a_o^2/sr)$
0
25
50
75
100
125
150
175
θ (degrees)
0.2
0.4
0.6
0.8
1.0
v (a.u.)
0.12
0.10
0.08
0.06
0.04
0.02

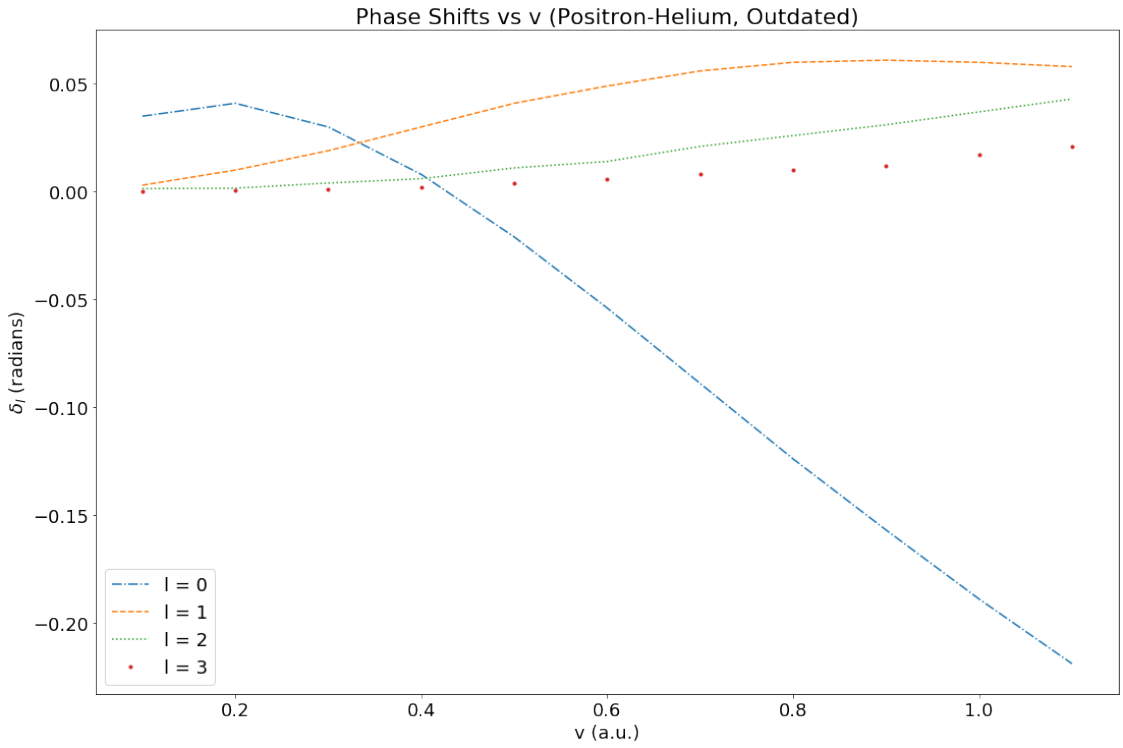

Positron-Helium
1.0
0.8
0.6
0.4
0.2
v (a.u.)
0
20
40
60
80
100
120
140
160
180
θ (degrees)
0.12
0.10
0.08
0.06
0.04
0.02

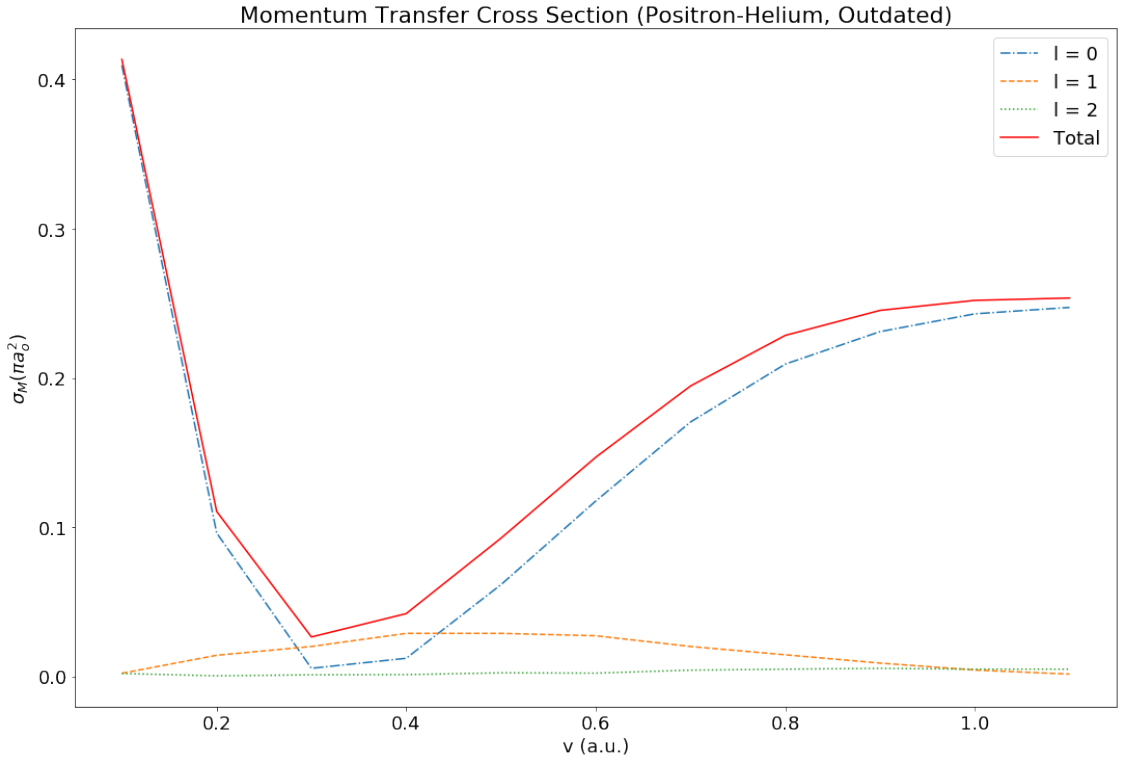
The greatest peak is in the region of lowest velocity and lowest degrees. Increasing the degrees or velocity decreases the differential cross section until the bottom of the valley is reached. Increasing the velocity or degrees beyond this valley raises the differential cross section, but it never peaks to the extent as low velocity, low degrees.

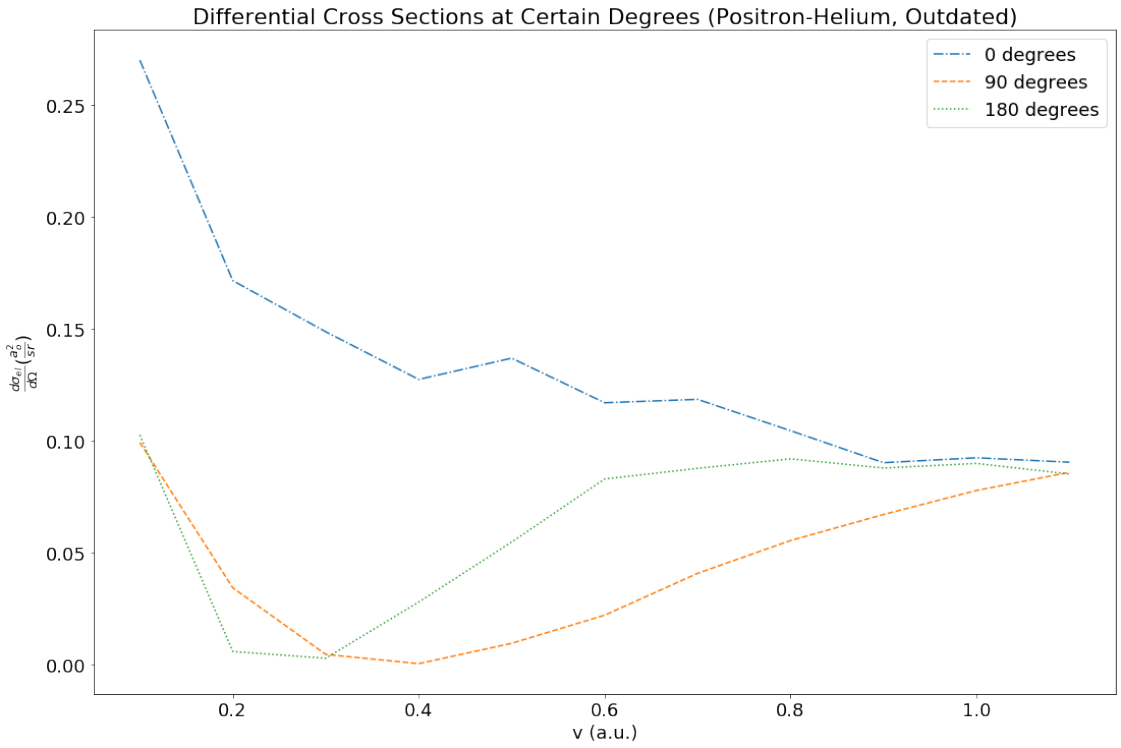
Phase shifts from P Van Reeth and J W Humberston 1999 [3] is used here. This is outdated.

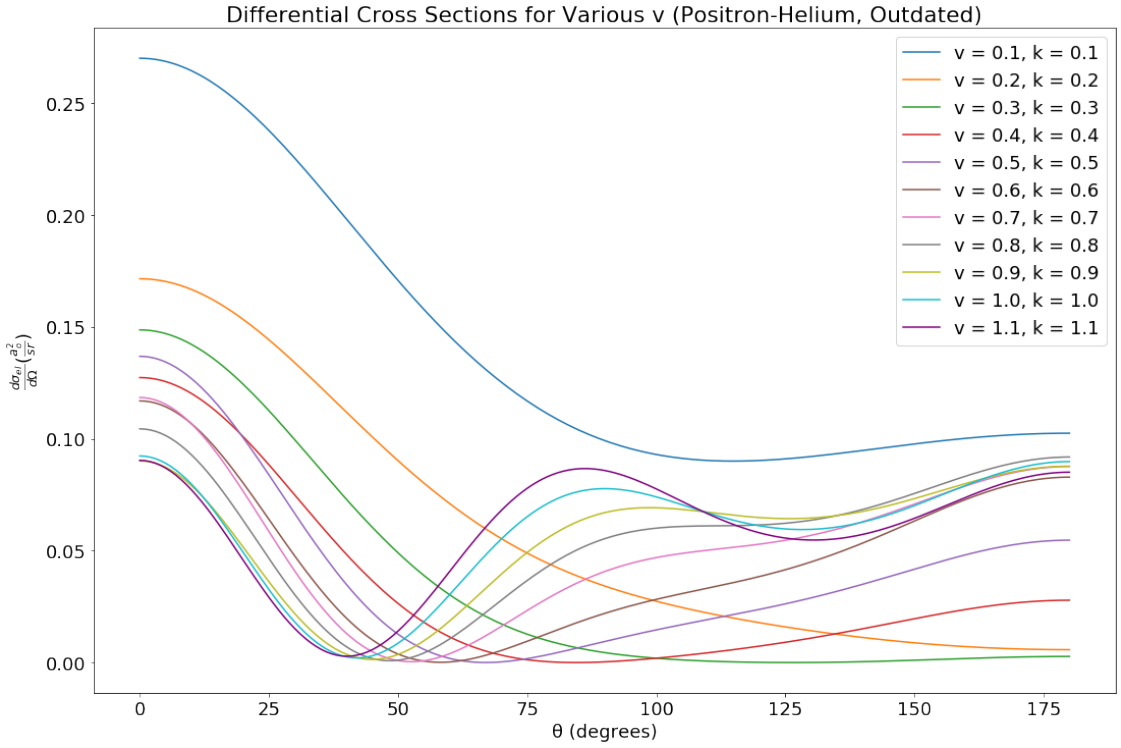
[33]:
```
#  Positron-Helium, data from P Van Reeth and J W Humberston 1999, this data is␣
↪outdated
positron_hel_system =␣
↪collision(positron_hel_phases,k_list_positron_hel,'Positron-Helium,␣
↪Outdated',1)

positron_hel_system.plot_cross_section(velocity=True)
positron_hel_system.plot_phase_shifts(velocity=True)
positron_hel_system.plot_momentum_transfer(velocity=True)
positron_hel_system.plot_diff_cross_vs_k(r_interval,velocity=True)
positron_hel_system.diff_plot(velocity=True)
positron_hel_system.diff_plot(sin=True,velocity=True)
positron_hel_system.diff_plot_3d(velocity=True)
positron_hel_system.diff_plot_3d(density=True,velocity=True)
```

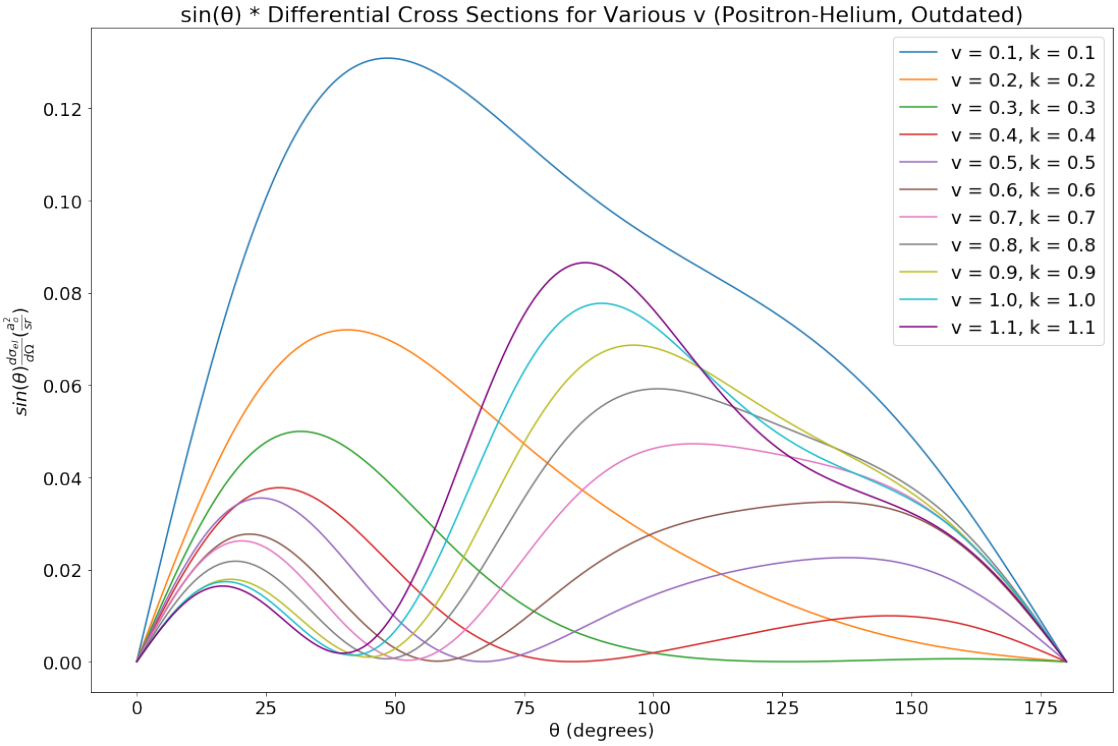


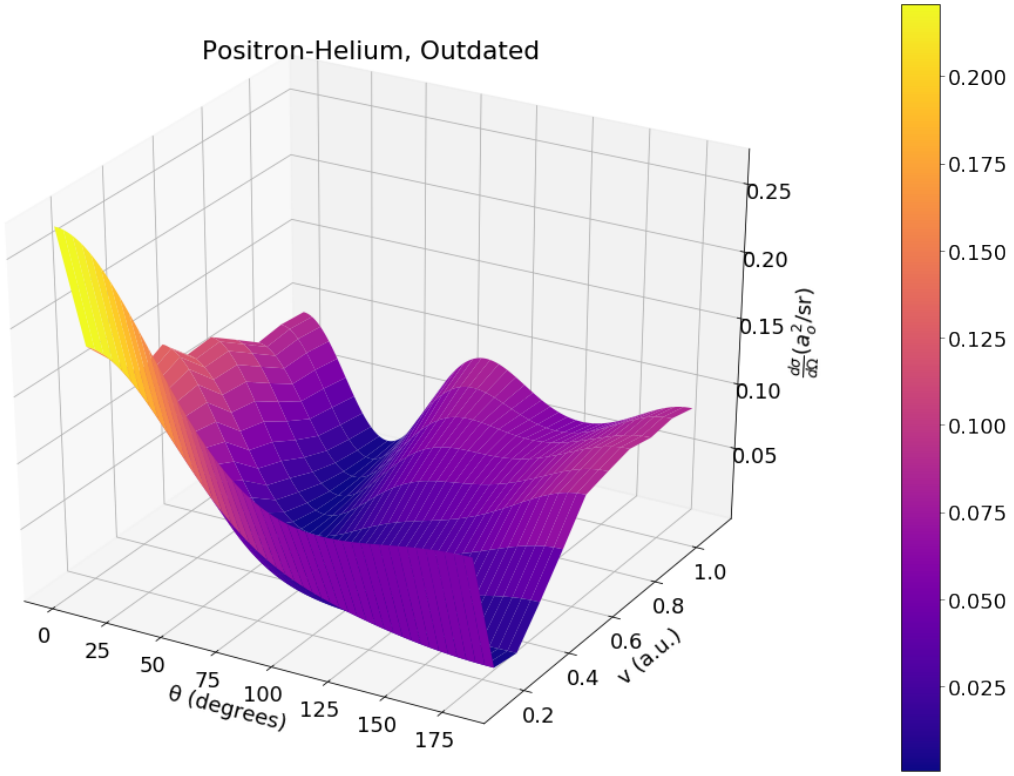

Phase Shifts vs v (Positron-Helium, Outdated)
0.05
0.00
−0.05
−0.10
−0.15
−0.20
δ_l (radians)
l = 0
l = 1
l = 2
l = 3
0.2
0.4
0.6
0.8
1.0
v (a.u.)

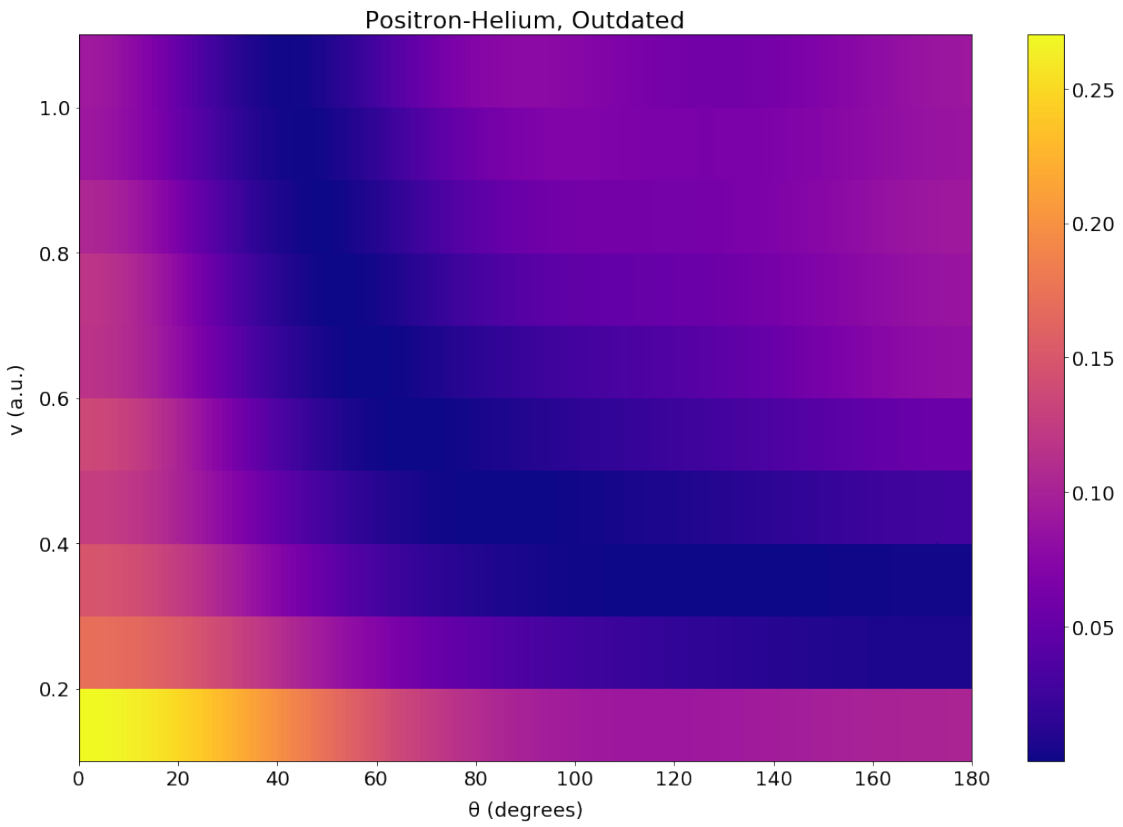

Momentum Transfer Cross Section (Positron-Helium, Outdated)
0.4
0.3
0.2
0.1
0.0
$\sigma_M(\pi a_o^2)$
l = 0
l = 1
l = 2
Total
0.2
0.4
0.6
0.8
1.0
v (a.u.)


Differential Cross Sections at Certain Degrees (Positron-Helium, Outdated)
0 degrees
90 degrees
180 degrees
0.25
0.20
0.15
0.10
0.05
0.00
0.2
0.4
0.6
0.8
1.0
v (a.u.)


Differential Cross Sections for Various v (Positron-Helium, Outdated)
v = 0.1, k = 0.1
v = 0.2, k = 0.2
v = 0.3, k = 0.3
v = 0.4, k = 0.4
v = 0.5, k = 0.5
v = 0.6, k = 0.6
v = 0.7, k = 0.7
v = 0.8, k = 0.8
v = 0.9, k = 0.9
v = 1.0, k = 1.0
v = 1.1, k = 1.1
0.25
0.20
0.15
0.10
0.05
0.00
0
25
50
75
100
125
150
175
θ (degrees)


sin(θ) * Differential Cross Sections for Various v (Positron-Helium, Outdated)
v = 0.1, k = 0.1
v = 0.2, k = 0.2
v = 0.3, k = 0.3
v = 0.4, k = 0.4
v = 0.5, k = 0.5
v = 0.6, k = 0.6
v = 0.7, k = 0.7
v = 0.8, k = 0.8
v = 0.9, k = 0.9
v = 1.0, k = 1.0
v = 1.1, k = 1.1
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0
25
50
75
100
125
150
175
θ (degrees)


Positron-Helium, Outdated
0.25
0.20
0.15
0.10
0.05
0
25
50
75
100
125
150
175
θ (degrees)
1.0
0.8
0.6
0.4
0.2
v (a.u.)
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025

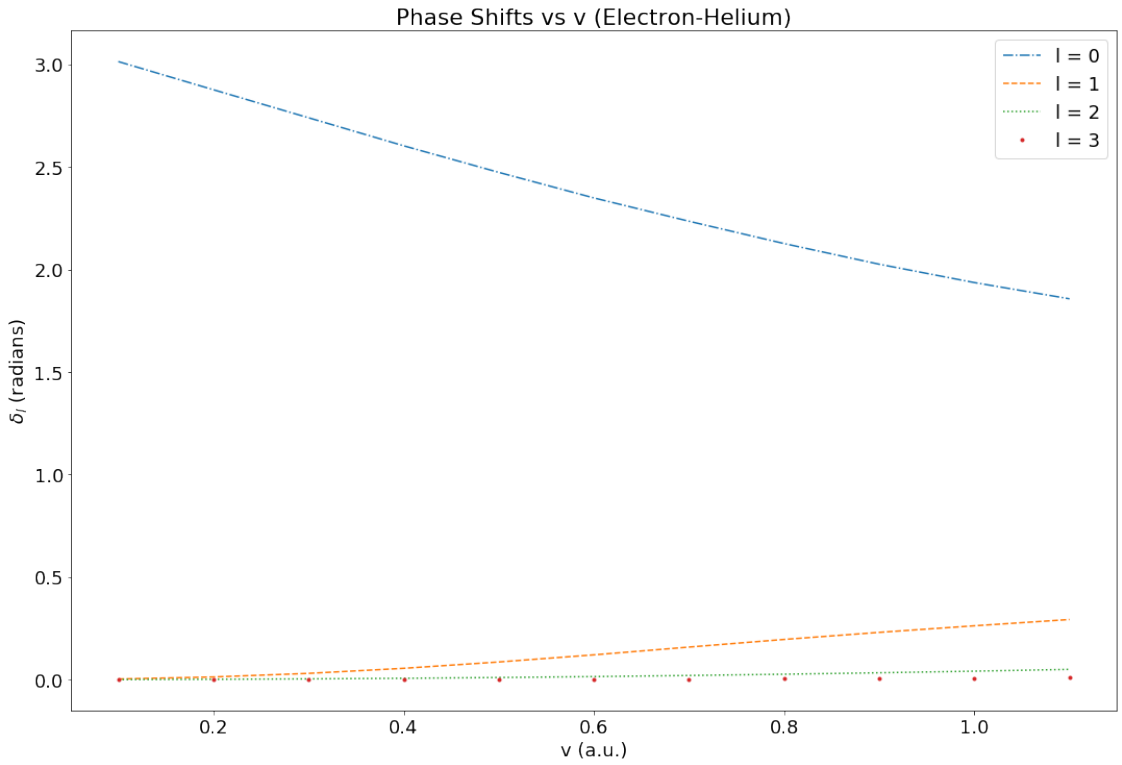


1.3 Electron-Helium

Phase shifts from both R K Nesbet 1979 [5]; A. L. Sinfailam and R K Nesbet 1972 [4].

[34]:
```
#  Electron-Helium, R K Nesbet 1979, A. L. Sinfailam and R K Nesbet 1972

electron_hel_system = collision(e_hel_phases,k_list_e_hel,'Electron-Helium',1)
```

[35]:
```
electron_hel_system.plot_phase_shifts(velocity=True)
```

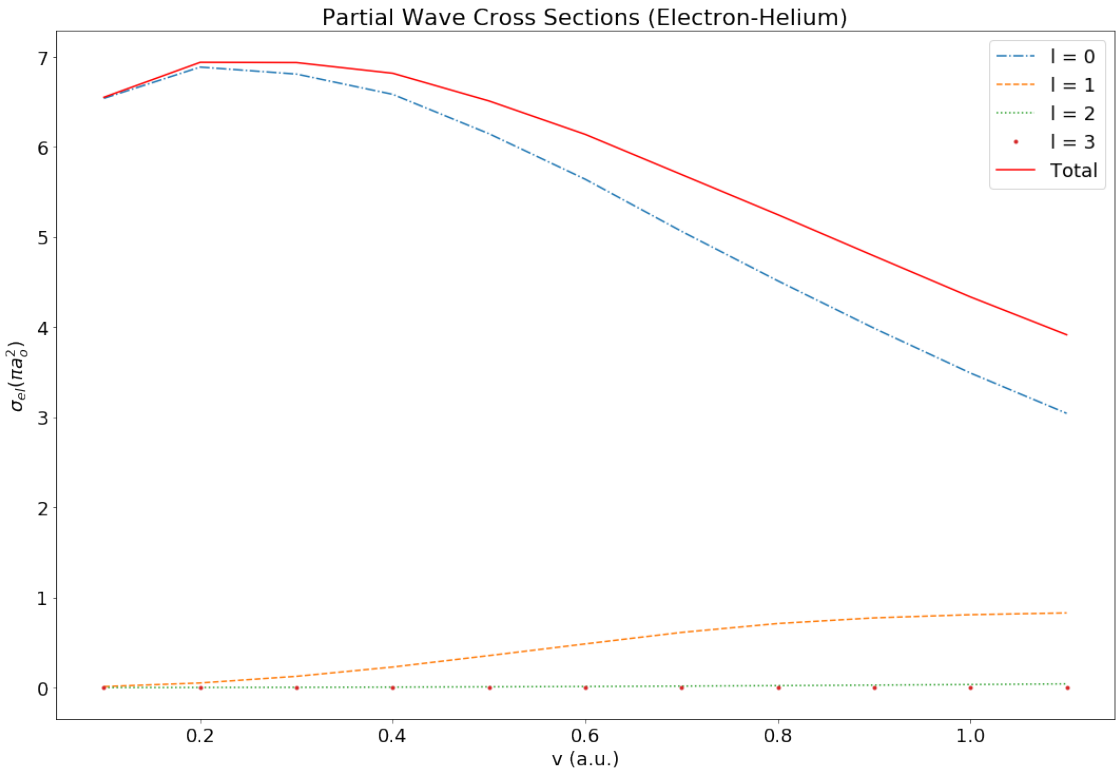


The s wave phase shifts fall with increasing energy nearly linearly. The p wave increases nearly linearly with increasing energy. The d and f waves remain close to zero.

[36]:
```
e_hel_cross_data = electron_hel_system.plot_cross_section(data=True)


display(e_hel_cross_data)
get_avg(e_hel_cross_data, 'total cross section')
data_max(e_hel_cross_data, 'total cross section')
data_min(e_hel_cross_data, 'total cross section')

electron_hel_system.plot_cross_section(velocity=True)
```

	v	total cross section	l = 0 contribution	l = 1 contribution	\
0	0.1	6.549	99.8183%	0.1761%	
1	0.2	6.938	99.2362%	0.742%	
2	0.3	6.935	98.1551%	1.7998%	
3	0.4	6.816	96.5682%	3.3496%	
4	0.5	6.509	94.4097%	5.4535%	
5	0.6	6.138	91.8925%	7.8999%	
6	0.7	5.692	88.9352%	10.7591%	
7	0.8	5.247	86.0128%	13.5536%	
8	0.9	4.789	83.2498%	16.1483%	

```
9   1.0               4.335           80.5165%           18.6559%
10  1.1               3.915           77.7232%           21.1611%

   l = 2 contribution l = 3 contribution
0              0.0049%            0.0007%
1              0.0208%             0.001%
2              0.0439%            0.0012%
3              0.0799%            0.0023%
4              0.1329%            0.0039%
5              0.2009%            0.0067%
6              0.2955%            0.0103%
7              0.4182%            0.0154%
8              0.5784%            0.0235%
9              0.7904%            0.0373%
10             1.0589%            0.0568%


Average value for total cross section : 5.806
Maximum value for total cross section

     v  total cross section l = 0 contribution l = 1 contribution  \
1  0.2               6.938           99.2362%             0.742%

  l = 2 contribution l = 3 contribution
1            0.0208%             0.001%


Minimum value for total cross section

      v  total cross section l = 0 contribution l = 1 contribution  \
10  1.1               3.915           77.7232%           21.1611%

   l = 2 contribution l = 3 contribution
10            1.0589%            0.0568%
```

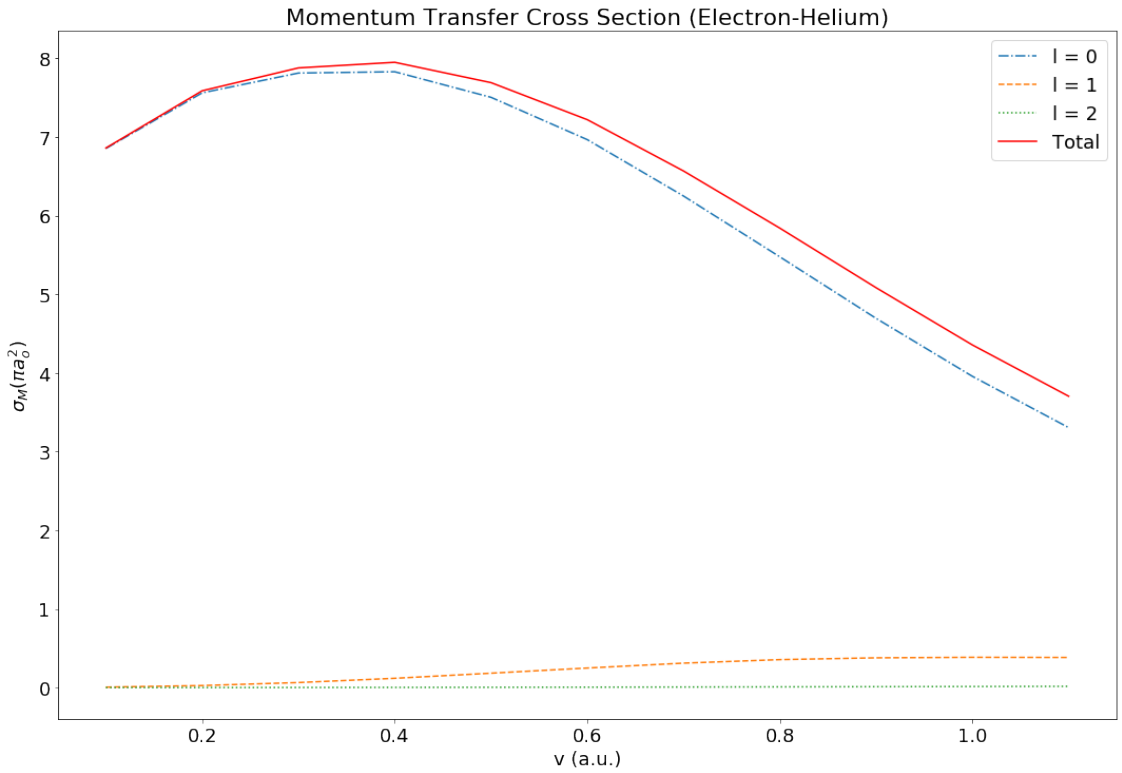


The total elastic cross section increases briefly before beginning to fall nearly linearly with increasing velocity. The greatest contributor for all velocities is the s wave, which follows almost exactly the same shape as the total elastic cross section. The contribution of the s wave gradually decreases with increasing energy, and the contribution of the p wave increases by about the amount of the s wave decreases. At the highest velocity, the s, p, d, and f wave contribute 78%, 21%, 1%, and 0.06%, respectively.

[37]:
```
e_hel_momentum_data = electron_hel_system.plot_momentum_transfer(data=True)


display(e_hel_momentum_data)
get_avg(e_hel_momentum_data, 'total momentum transfer')
data_max(e_hel_momentum_data, 'total momentum transfer')
data_min(e_hel_momentum_data, 'total momentum transfer')

electron_hel_system.plot_momentum_transfer(velocity=True)
```

	v	total momentum transfer	l = 0 contribution	l = 1 contribution	\
0	0.1	6.861	99.9137%	0.085%	
1	0.2	7.589	99.6499%	0.3425%	
2	0.3	7.880	99.1668%	0.816%	
3	0.4	7.951	98.4858%	1.4841%	
4	0.5	7.693	97.5713%	2.3792%	

```
5   0.6                   7.222           96.4822%            3.4446%
6   0.7                   6.569           95.1537%            4.7373%
7   0.8                   5.842           93.7534%            6.0885%
8   0.9                   5.084           92.3332%            7.4417%
9   1.0                   4.357           90.8468%            8.8387%
10  1.1                   3.705           89.2251%           10.3404%

   l = 2 contribution
0             0.0013%
1             0.0076%
2             0.0171%
3             0.0301%
4             0.0494%
5             0.0733%
6              0.109%
7             0.1581%
8             0.2251%
9             0.3145%
10            0.4345%


Average value for total momentum transfer : 6.432
Maximum value for total momentum transfer

     v  total momentum transfer l = 0 contribution l = 1 contribution  \
3  0.4                   7.951           98.4858%            1.4841%

  l = 2 contribution
3            0.0301%


Minimum value for total momentum transfer

      v  total momentum transfer l = 0 contribution l = 1 contribution  \
10  1.1                   3.705           89.2251%           10.3404%

   l = 2 contribution
10            0.4345%
```



The momentum transfer cross section is very close to the elastic cross section. At most, it deviants from the elastic cross section by 18% at v = 0.5. Compared to the elastic cross section, the s wave has a greater contribution, never dropping beneath 89% of the total. The maximum for the momentum transfer cross section is greater than the maximum for the elastic cross section, and the minimum is less than the minimum of the elastic cross section.

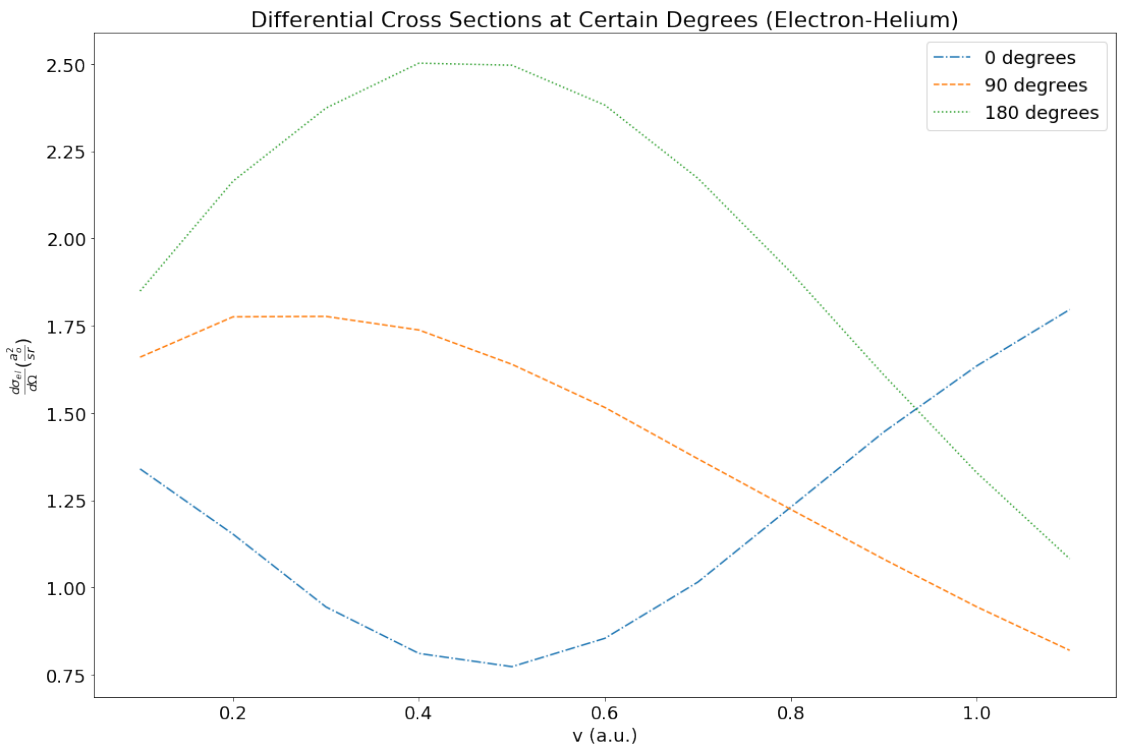
[38]:
```
e_hel_cross_momentum = cross_momentum_compare(e_hel_cross_data,␣
↪e_hel_momentum_data)


display(e_hel_cross_momentum)
get_avg(e_hel_cross_momentum, 'ratio momentum/cross')
data_max(e_hel_cross_momentum, 'ratio momentum/cross')
data_min(e_hel_cross_momentum, 'ratio momentum/cross')
```

	v	total cross section	total momentum transfer	ratio momentum/cross
0	0.1	6.549	6.861	1.048
1	0.2	6.938	7.589	1.094
2	0.3	6.935	7.880	1.136
3	0.4	6.816	7.951	1.167
4	0.5	6.509	7.693	1.182
5	0.6	6.138	7.222	1.177
6	0.7	5.692	6.569	1.154

```
7   0.8                5.247                     5.842                   1.113
8   0.9                4.789                     5.084                   1.062
9   1.0                4.335                     4.357                   1.005
10  1.1                3.915                     3.705                   0.946


Average value for ratio momentum/cross : 1.099
Maximum value for ratio momentum/cross

     v  total cross section  total momentum transfer  ratio momentum/cross
4  0.5                6.509                    7.693                 1.182


Minimum value for ratio momentum/cross

      v  total cross section  total momentum transfer  ratio momentum/cross
10  1.1                3.915                    3.705                 0.946
```

[39]:
```
e_hel_diff_degree =  electron_hel_system.
 ↪plot_diff_cross_vs_k(r_interval,velocity=True, data=True)

display(e_hel_diff_degree)

electron_hel_system.plot_diff_cross_vs_k(r_interval, velocity = True)
```

```
      v  0 degrees  90 degrees  180 degrees
0   0.1       1.340       1.660        1.849
1   0.2       1.153       1.775        2.164
2   0.3       0.944       1.777        2.373
3   0.4       0.811       1.738        2.502
4   0.5       0.773       1.639        2.496
5   0.6       0.855       1.516        2.382
6   0.7       1.016       1.369        2.172
7   0.8       1.230       1.224        1.903
8   0.9       1.445       1.081        1.609
9   1.0       1.635       0.945        1.329
10  1.1       1.796       0.820        1.082
```



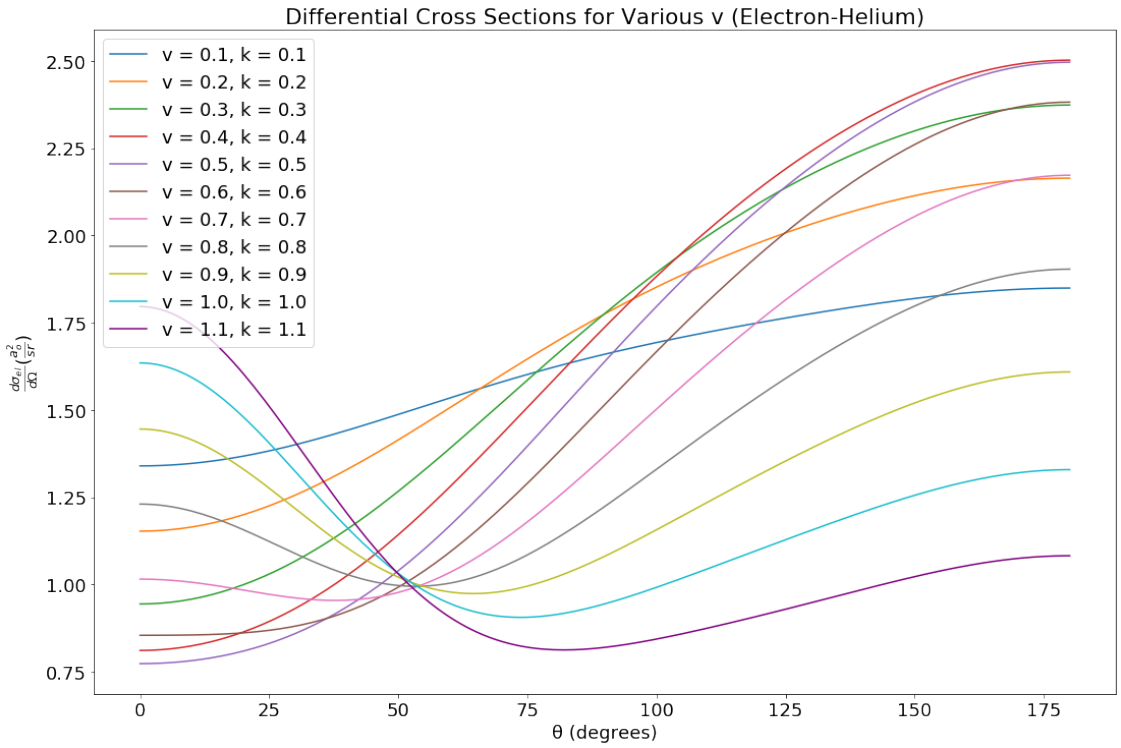
The differential cross section at 0 degrees looks similar to 180 degrees if flipped.

[40]:
```
e_hel_diff = electron_hel_system.diff_plot(velocity=True, data=True)
sliced = e_hel_diff[e_hel_diff.index % 10 == 0 ]
display(sliced)


electron_hel_system.diff_plot(velocity=True)
```

	degrees	v = 0.1	v = 0.2	v = 0.3	v = 0.4	v = 0.5	v = 0.6	v = 0.7	\
0	0	1.340	1.153	0.944	0.811	0.773	0.855	1.016	
10	10	1.347	1.165	0.958	0.824	0.782	0.856	1.006	
20	20	1.368	1.200	0.999	0.864	0.809	0.862	0.984	
30	30	1.400	1.256	1.067	0.931	0.859	0.881	0.962	
40	40	1.441	1.329	1.157	1.025	0.936	0.922	0.955	
50	50	1.487	1.414	1.267	1.143	1.040	0.992	0.977	
60	60	1.535	1.507	1.390	1.280	1.169	1.093	1.035	
70	70	1.580	1.600	1.520	1.430	1.317	1.220	1.125	
80	80	1.622	1.691	1.651	1.585	1.477	1.364	1.239	
90	90	1.660	1.775	1.777	1.738	1.639	1.516	1.369	
100	100	1.693	1.852	1.894	1.883	1.797	1.667	1.503	
110	110	1.723	1.920	2.000	2.016	1.944	1.812	1.634	
120	120	1.750	1.980	2.094	2.136	2.078	1.946	1.757	
130	130	1.776	2.033	2.175	2.241	2.197	2.067	1.871	

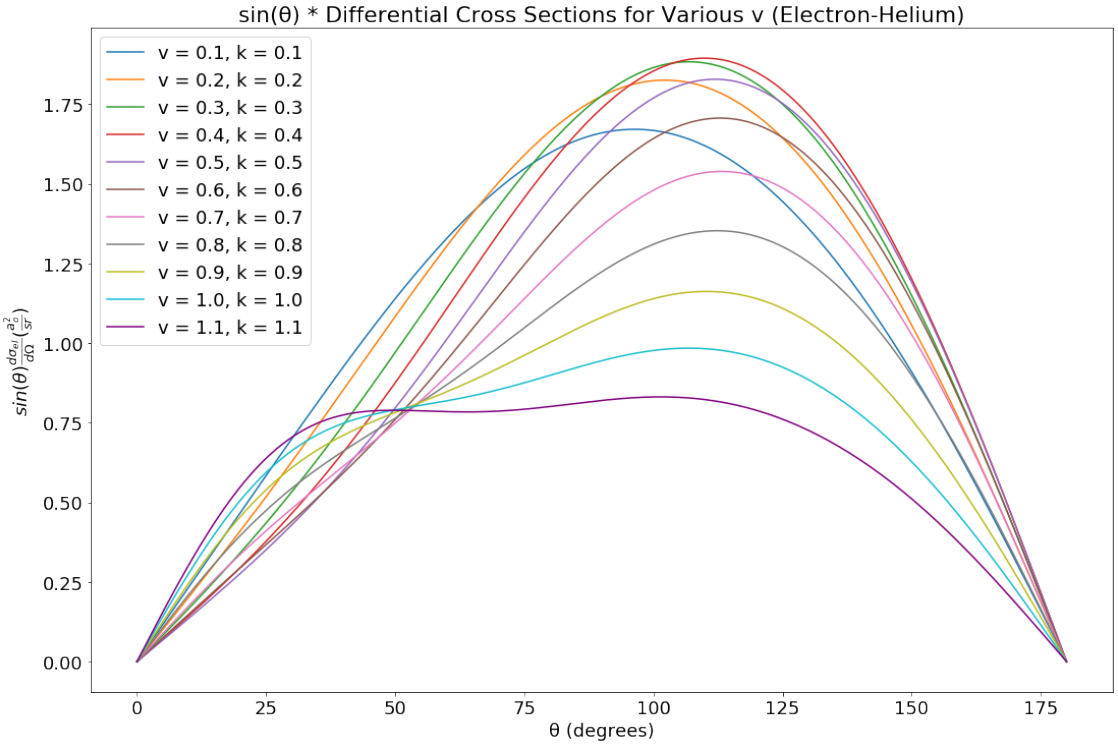
140	140	1.799	2.077	2.244	2.331	2.299	2.173	1.971
150	150	1.819	2.114	2.299	2.404	2.382	2.260	2.054
160	160	1.835	2.141	2.340	2.457	2.444	2.326	2.118
170	170	1.846	2.158	2.365	2.491	2.483	2.368	2.159
180	180	1.849	2.164	2.373	2.502	2.496	2.382	2.172

	v = 0.8	v = 0.9	v = 1.0	v = 1.1
0	1.230	1.445	1.635	1.796
10	1.210	1.414	1.593	1.745
20	1.157	1.331	1.480	1.604
30	1.089	1.218	1.325	1.409
40	1.029	1.106	1.165	1.205
50	0.997	1.021	1.032	1.030
60	1.004	0.979	0.946	0.906
70	1.050	0.980	0.909	0.837
80	1.127	1.018	0.913	0.813
90	1.224	1.081	0.945	0.820
100	1.330	1.157	0.992	0.844
110	1.438	1.237	1.046	0.875
120	1.541	1.316	1.101	0.911
130	1.638	1.391	1.156	0.948
140	1.724	1.460	1.208	0.986
150	1.798	1.520	1.256	1.023
160	1.855	1.568	1.294	1.054
170	1.891	1.598	1.320	1.075
180	1.903	1.609	1.329	1.082



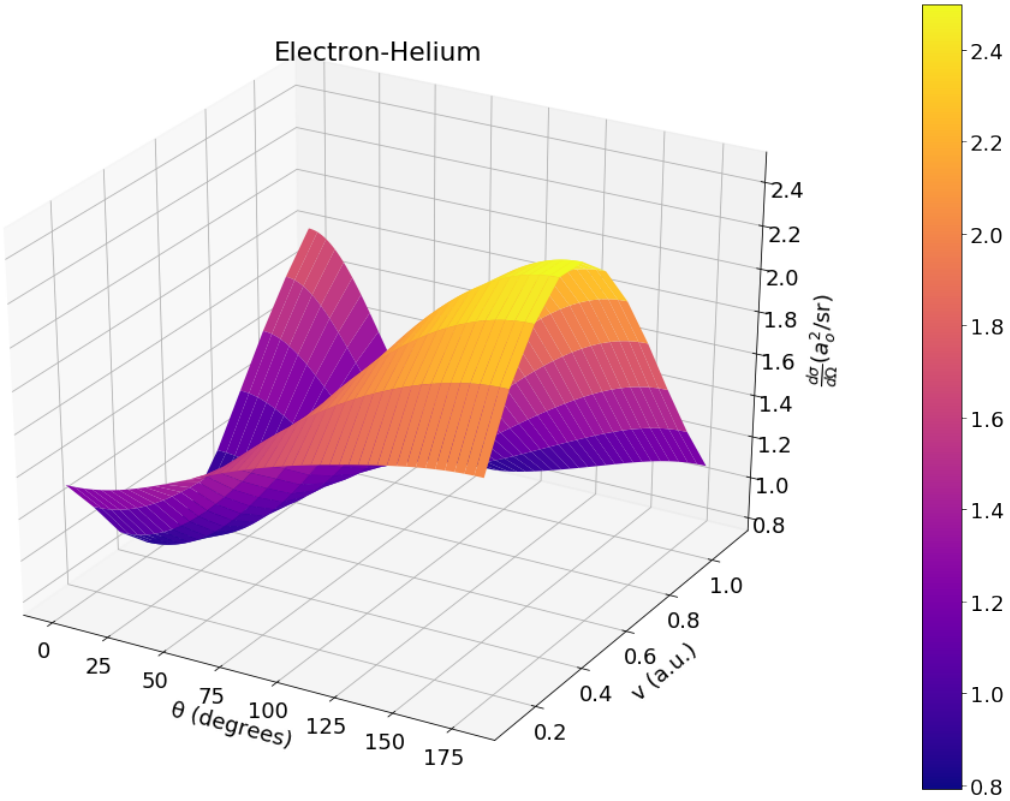
At v =< 0.6 the differential cross section increases with increasing degrees. At higher velocity, the differential cross section first decreases before increasing with increasing energy. There is a large crossing of points around 54 degrees where the differential cross section is around 1.

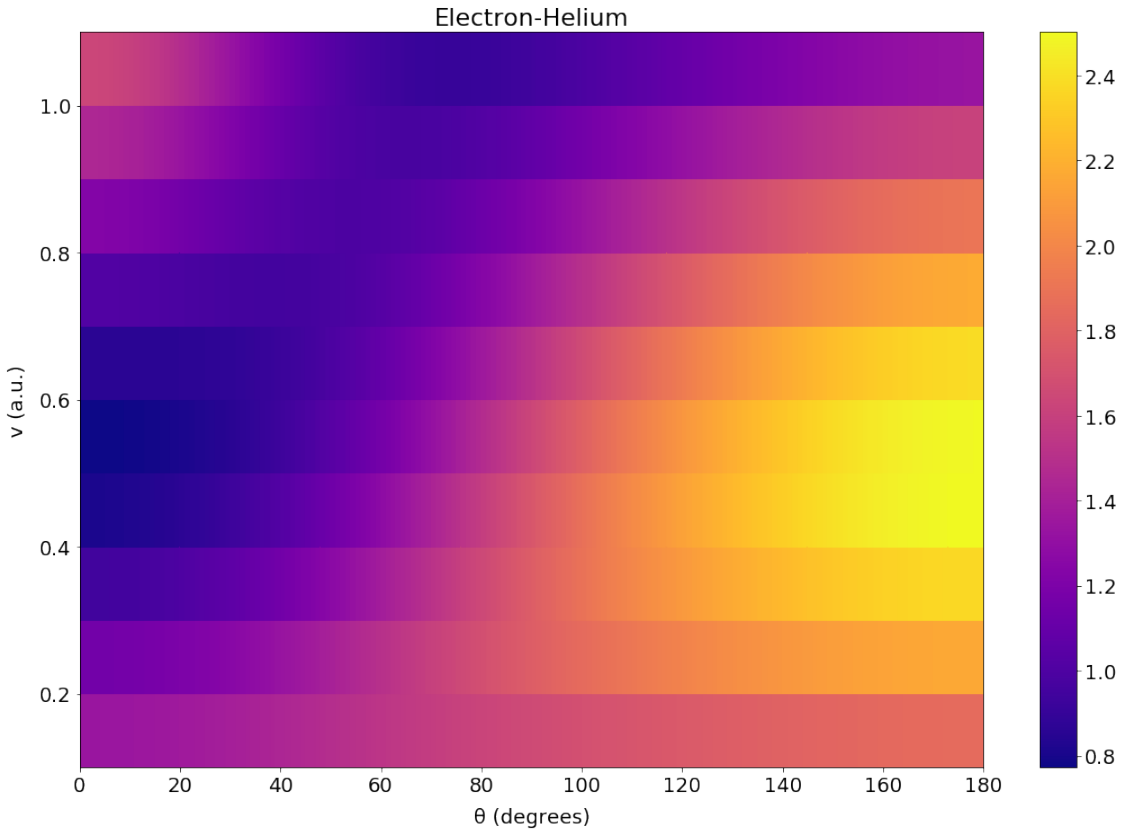
[41]:
```
electron_hel_system.diff_plot(sin=True,velocity=True)
```



The shape of the curves begins in a very parabolic shape at low velocity. As the velocity increases the peak of the parabola becomes increasing flat.

[42]:
```
electron_hel_system.diff_plot_3d(velocity=True)
electron_hel_system.diff_plot_3d(density=True,velocity=True)
```


Electron-Helium
0
25
50
75
100
125
150
175
θ (degrees)
0.2
0.4
0.6
0.8
1.0
v (a.u.)
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
$\frac{d\sigma}{d\Omega}(a_0^2/sr)$



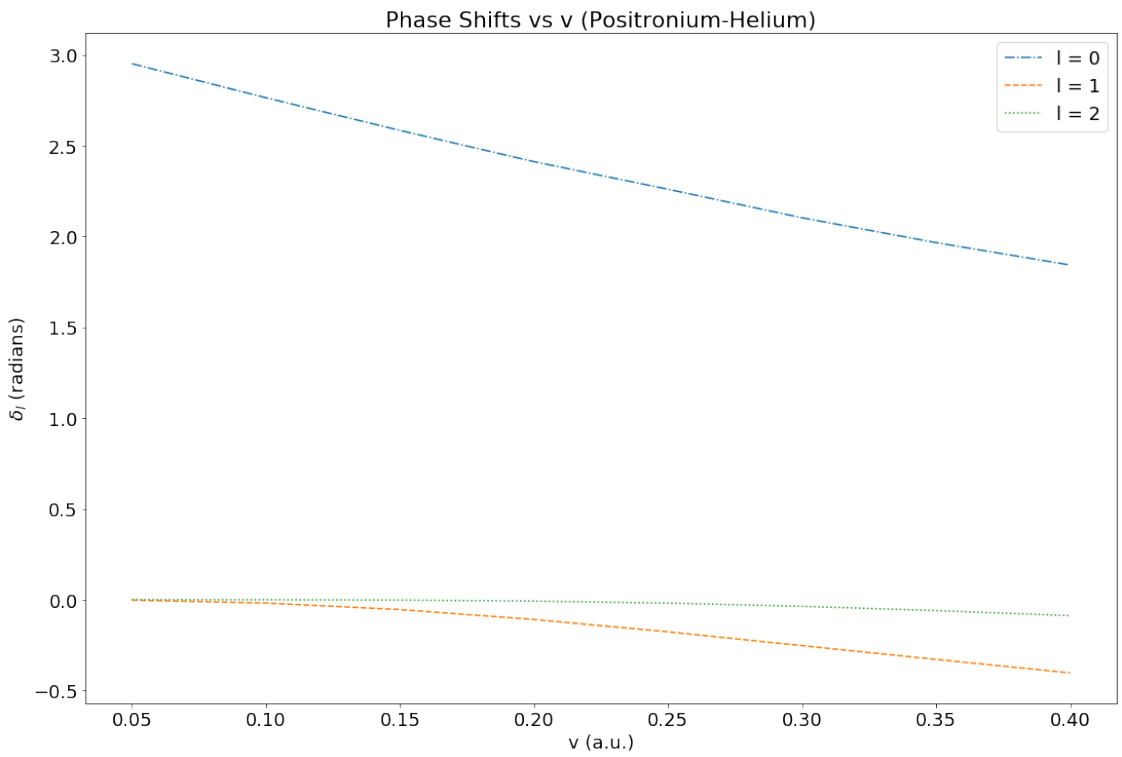
The differential cross section has a peak in the area of v = 0.5 au and 180 degrees. Moving away from this region decreases the magnitude of the differential cross section. The differential cross section then begins to rise in the region of high velocity and low degrees.

1.4 Positronium-Helium

Phase shifts used are from Nirmal K Sarkar et al 1999 [8].

[43]:
```
#  Positronium-Helium, Nirmal K Sarkar et al 1999

ps_hel_system = collision(ps_hel_phases,k_list_ps_hel,'Positronium-Helium',2)
```

[44]:
```
ps_hel_system.plot_phase_shifts(velocity=True)
```

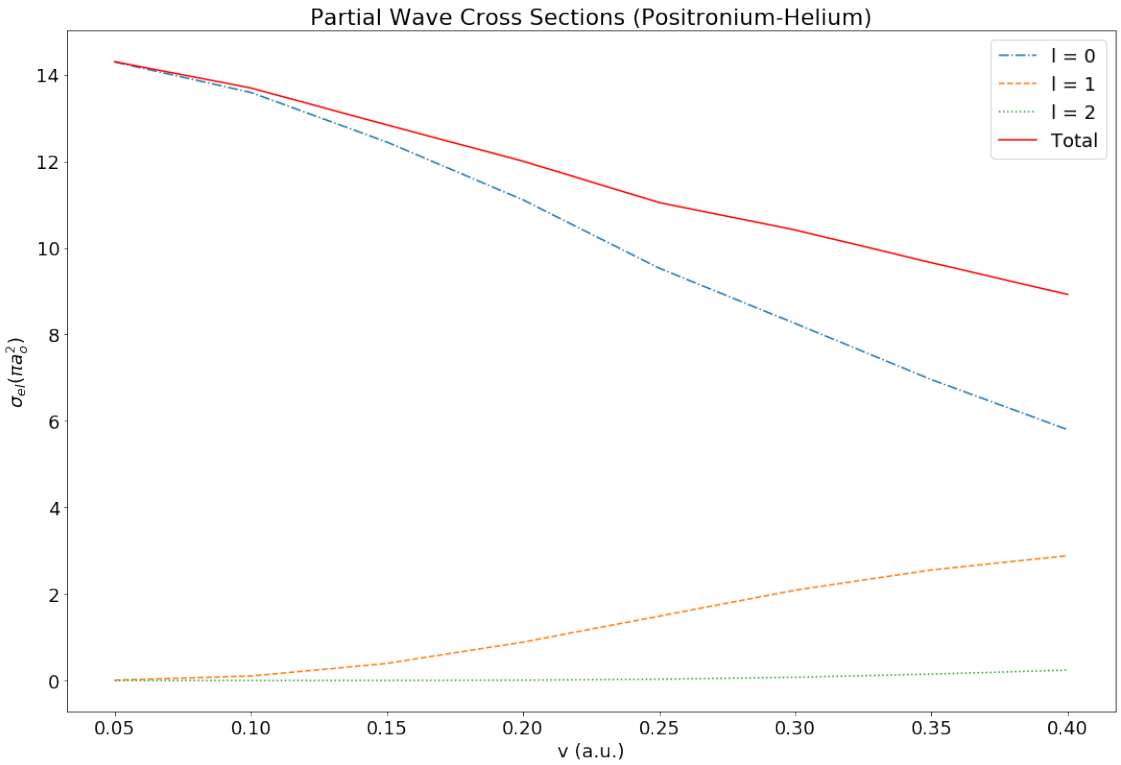


The phase shifts for the s wave decrease nearly linearly with increasing energy. The p wave phase shifts decrease largely linearly into negative values with increasing energy. The d wave phase shifts remains close to zero with a slight decrease with increasing energy.

[45]:
```
ps_hel_cross_data = ps_hel_system.plot_cross_section(data=True)


display(ps_hel_cross_data)
get_avg(ps_hel_cross_data, 'total cross section')
data_max(ps_hel_cross_data, 'total cross section')
data_min(ps_hel_cross_data, 'total cross section')

ps_hel_system.plot_cross_section(velocity=True)
```

	v	total cross section	l = 0 contribution	l = 1 contribution	\
0	0.05	14.303	99.9459%	0.0541%	
1	0.10	13.695	99.25%	0.7496%	
2	0.15	12.841	96.9105%	3.0811%	
3	0.20	12.003	92.5434%	7.3946%	
4	0.25	11.046	86.2694%	13.4719%	
5	0.30	10.413	79.2321%	20.0574%	
6	0.35	9.657	72.0021%	26.4731%	
7	0.40	8.923	64.9814%	32.3196%	

```
  l = 2 contribution
0               0.0%
1            0.0004%
2            0.0084%
3            0.0621%
4            0.2587%
5            0.7105%
6            1.5248%
7             2.699%


Average value for total cross section : 11.61
Maximum value for total cross section

      v  total cross section l = 0 contribution l = 1 contribution  \
0  0.05               14.303           99.9459%            0.0541%

  l = 2 contribution
0               0.0%


Minimum value for total cross section

     v  total cross section l = 0 contribution l = 1 contribution  \
7  0.4                8.923           64.9814%           32.3196%

  l = 2 contribution
7             2.699%
```



The elastic cross section starts at the maximum and then decreases mostly linearly with increasing velocity. As the velocity increases, the contribution from the s wave decreases, while the contribution from the higher waves, especially the p wave, increases. At the highest velocity, the s, p, d waves contribute 65, 32.3, and 2.7 percent to the total elastic cross section, respectively.

[46]:
```
ps_hel_momentum_data = ps_hel_system.plot_momentum_transfer(data=True)


display(ps_hel_momentum_data)
get_avg(ps_hel_momentum_data, 'total momentum transfer')
data_max(ps_hel_momentum_data, 'total momentum transfer')
data_min(ps_hel_momentum_data, 'total momentum transfer')

ps_hel_system.plot_momentum_transfer(velocity=True)
```

	v	total momentum transfer	l = 0 contribution	l = 1 contribution
0	0.05	13.926	99.9633%	0.0367%
1	0.10	12.416	99.4683%	0.5317%
2	0.15	10.574	97.7028%	2.2972%
3	0.20	8.965	94.2987%	5.7013%
4	0.25	7.506	89.4322%	10.5678%
5	0.30	6.583	84.4234%	15.5766%
6	0.35	5.732	79.8973%	20.1027%

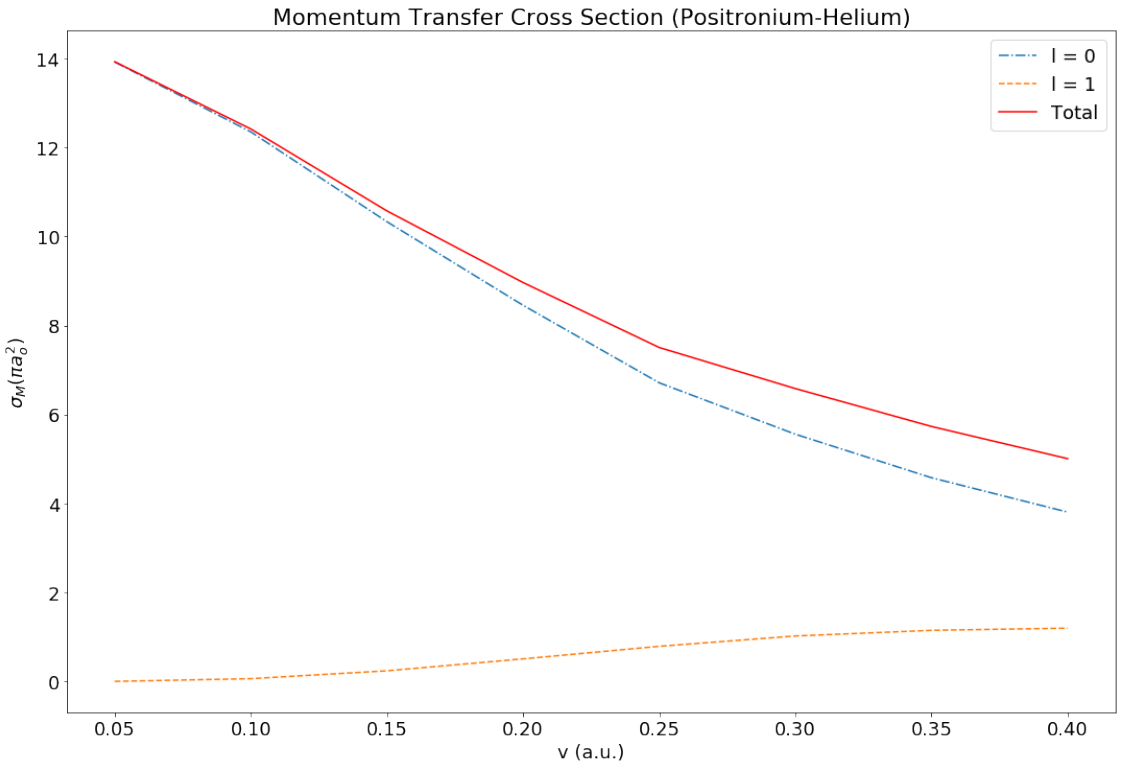
```
7  0.40                    5.009          76.0305%          23.9695%
```

```
Average value for total momentum transfer : 8.839
Maximum value for total momentum transfer
```

```
      v  total momentum transfer l = 0 contribution l = 1 contribution
0  0.05                   13.926          99.9633%           0.0367%
```

```
Minimum value for total momentum transfer
```

```
     v  total momentum transfer l = 0 contribution l = 1 contribution
7  0.4                    5.009          76.0305%          23.9695%
```



The momentum transfer cross section starts close to the elastic cross section and is always lower than the elastic cross section. The difference between the elastic cross section and momentum transfer cross section increases with increasing velocity. The contributions from the waves to the momentum transfer cross section is similar to the elastic cross section.

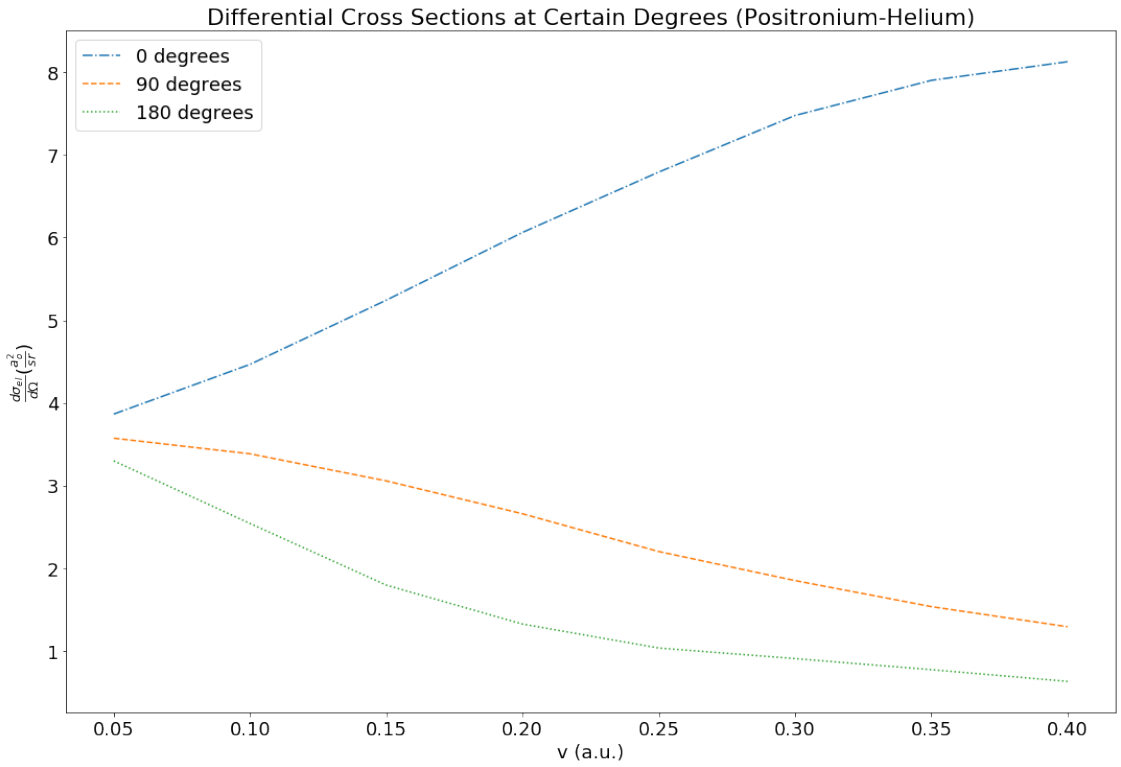
[47]:
```
ps_hel_cross_momentum = cross_momentum_compare(ps_hel_cross_data,␣
↪ps_hel_momentum_data)


display(ps_hel_cross_momentum)
```

```
get_avg(ps_hel_cross_momentum, 'ratio momentum/cross')
data_max(ps_hel_cross_momentum, 'ratio momentum/cross')
data_min(ps_hel_cross_momentum, 'ratio momentum/cross')
```

```
      v  total cross section  total momentum transfer  ratio momentum/cross
0  0.05                14.303                   13.926                 0.974
1  0.10                13.695                   12.416                 0.907
2  0.15                12.841                   10.574                 0.823
3  0.20                12.003                    8.965                 0.747
4  0.25                11.046                    7.506                 0.680
5  0.30                10.413                    6.583                 0.632
6  0.35                 9.657                    5.732                 0.594
7  0.40                 8.923                    5.009                 0.561


Average value for ratio momentum/cross : 0.74
Maximum value for ratio momentum/cross
      v  total cross section  total momentum transfer  ratio momentum/cross
0  0.05                14.303                   13.926                 0.974


Minimum value for ratio momentum/cross
     v  total cross section  total momentum transfer  ratio momentum/cross
7  0.4                 8.923                    5.009                 0.561
```

[48]:
```
ps_hel_diff_degree =  ps_hel_system.
 ↪plot_diff_cross_vs_k(r_interval,velocity=True, data=True)

display(ps_hel_diff_degree)

ps_hel_system.plot_diff_cross_vs_k(r_interval, velocity=True)
```

```
      v  0 degrees  90 degrees  180 degrees
0  0.05      3.865       3.573        3.299
1  0.10      4.466       3.384        2.541
2  0.15      5.242       3.056        1.796
3  0.20      6.062       2.659        1.324
4  0.25      6.793       2.202        1.035
5  0.30      7.477       1.850        0.907
6  0.35      7.902       1.536        0.773
7  0.40      8.127       1.292        0.631
```



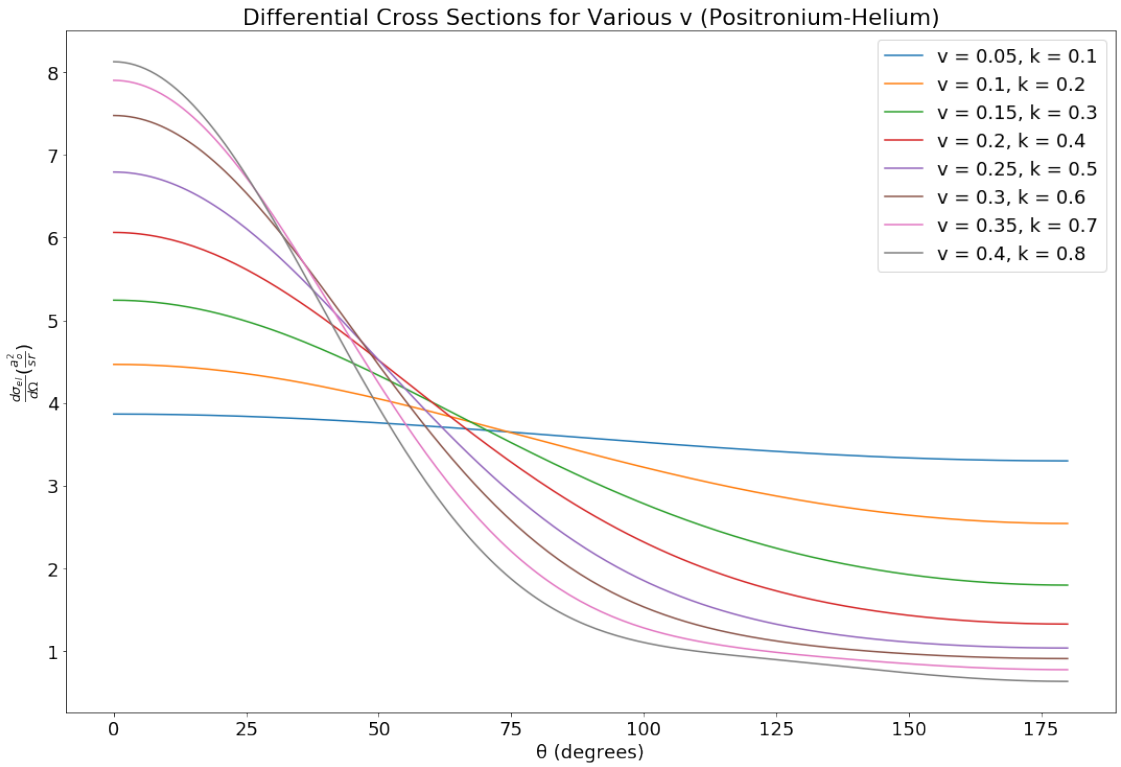
The differential cross section increases at 0 degrees with increasing energy. The curves of 90 and 180 degrees are very similar except the magnitude at 90 degrees being greater.

[49]:
```
ps_hel_diff = ps_hel_system.diff_plot(velocity=True, data=True)
sliced = ps_hel_diff[ps_hel_diff.index % 10 == 0 ]
display(sliced)

ps_hel_system.diff_plot(velocity=True)
```

	degrees	v = 0.05	v = 0.1	v = 0.15	v = 0.2	v = 0.25	v = 0.3	\
0	0	3.865	4.466	5.242	6.062	6.793	7.477	
10	10	3.860	4.448	5.200	5.987	6.677	7.317	
20	20	3.847	4.393	5.077	5.770	6.346	6.861	
30	30	3.825	4.306	4.882	5.431	5.836	6.171	
40	40	3.795	4.189	4.628	5.001	5.205	5.337	
50	50	3.759	4.049	4.332	4.515	4.517	4.458	
60	60	3.717	3.893	4.011	4.010	3.832	3.621	
70	70	3.671	3.725	3.682	3.517	3.200	2.890	
80	80	3.622	3.554	3.359	3.061	2.651	2.298	
90	90	3.573	3.384	3.056	2.659	2.202	1.850	
100	100	3.524	3.222	2.781	2.317	1.851	1.532	
110	110	3.477	3.071	2.539	2.038	1.588	1.316	
120	120	3.434	2.935	2.332	1.817	1.397	1.173	

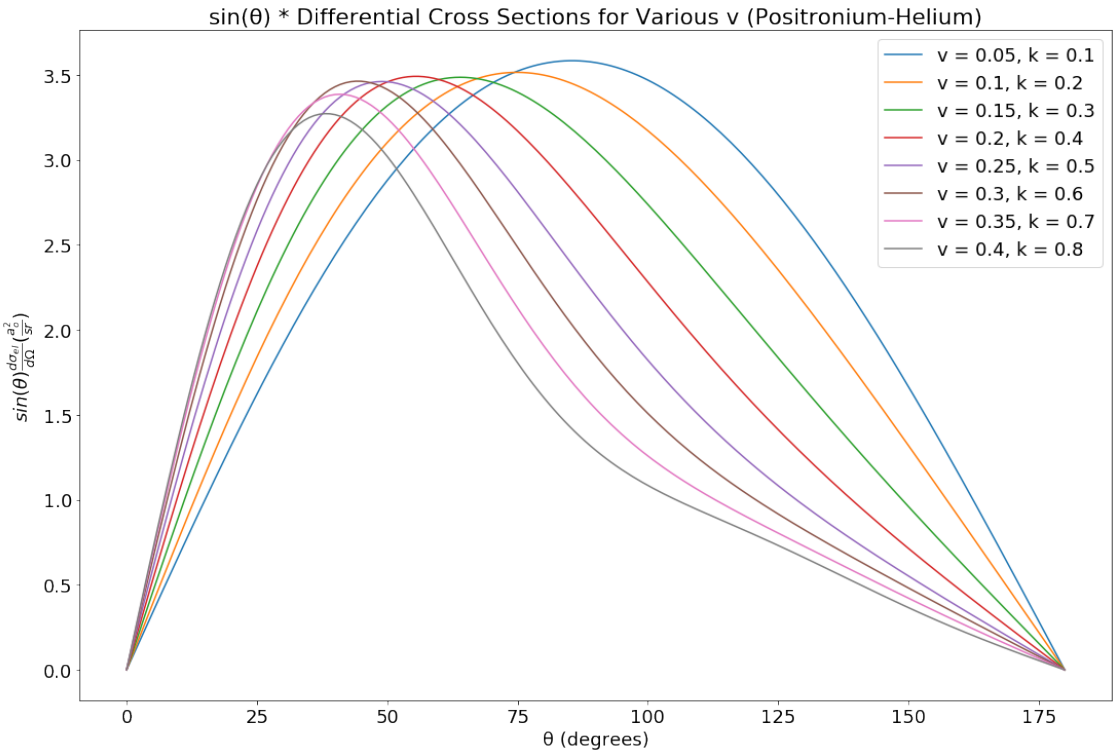
130	130	3.395	2.817	2.162	1.647	1.262	1.077
140	140	3.361	2.719	2.026	1.520	1.169	1.011
150	150	3.334	2.642	1.923	1.430	1.105	0.964
160	160	3.315	2.586	1.852	1.370	1.065	0.932
170	170	3.303	2.552	1.810	1.336	1.042	0.913
180	180	3.299	2.541	1.796	1.324	1.035	0.907

	v = 0.35	v = 0.4
0	7.902	8.127
10	7.700	7.889
20	7.128	7.217
30	6.274	6.228
40	5.266	5.082
50	4.236	3.944
60	3.296	2.948
70	2.521	2.170
80	1.938	1.629
90	1.536	1.292
100	1.278	1.102
110	1.120	0.996
120	1.020	0.926
130	0.950	0.862
140	0.892	0.797
150	0.844	0.733
160	0.806	0.679
170	0.781	0.644
180	0.773	0.631



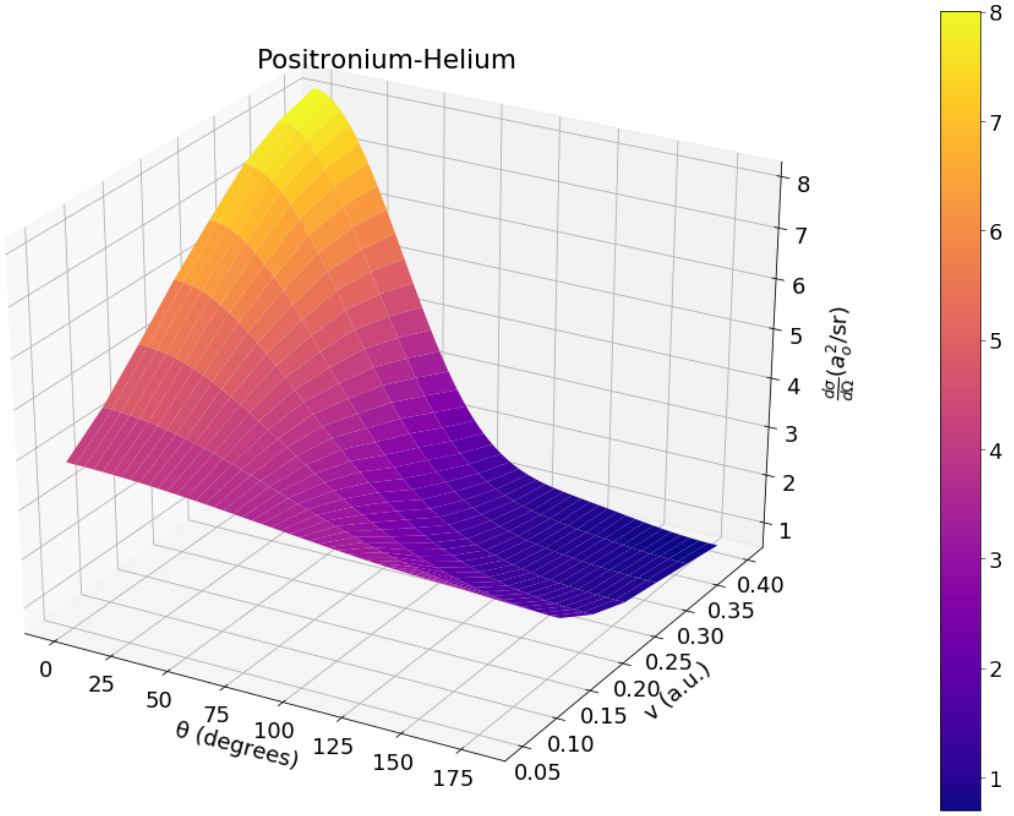
The greater the velocity the greater the drop in magnitude is for increasing degrees before flattening out. At the lowest energy, the curve is almost completely flat. At the highest velocity the curve falls very quickly before flattening out.

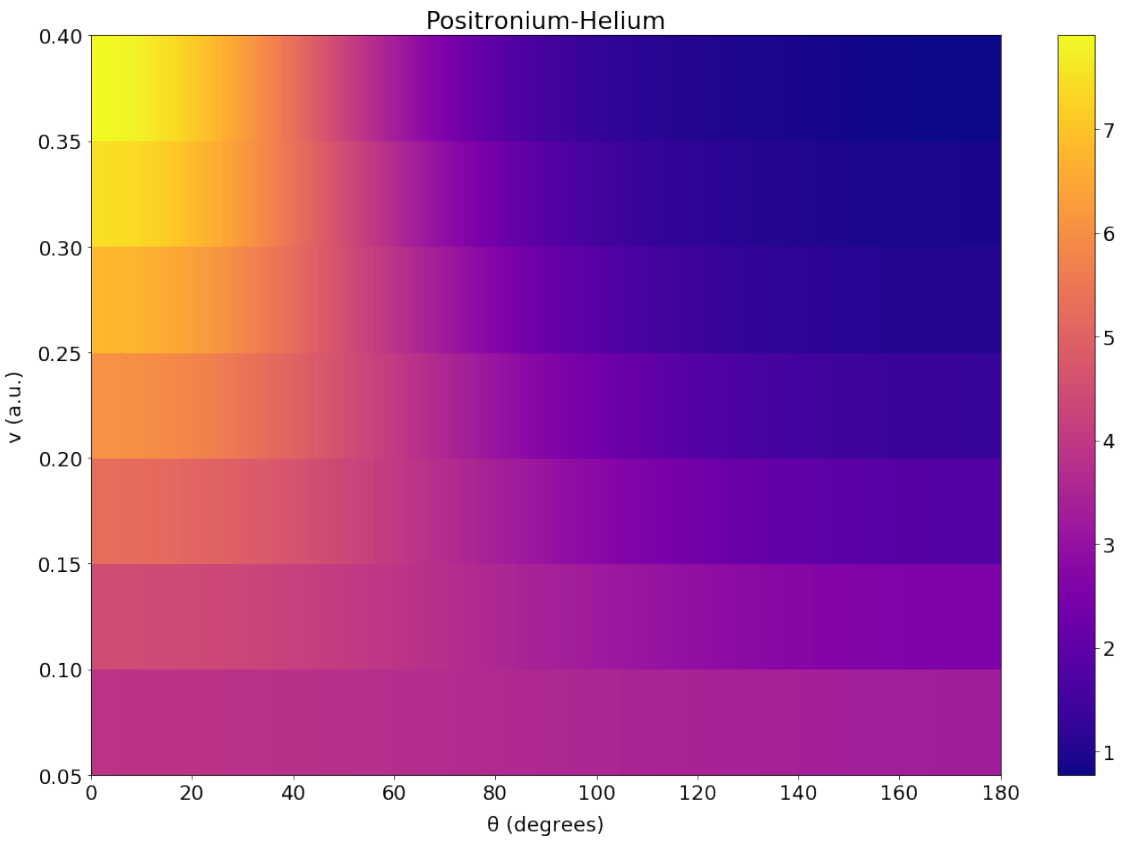
[50]:
```
ps_hel_system.diff_plot(sin=True,velocity=True)
```



The lower the energy the more parabolic the curve is. At v = 0.05, the curve is nearly completely parabolic. As the energy increases, the curve becomes more distorted, having a noticeable dip in the curve at v = 0.4.

[51]:
```
ps_hel_system.diff_plot_3d(velocity=True)
ps_hel_system.diff_plot_3d(density=True,velocity=True)
```


Positronium-Helium
θ (degrees)
0
25
50
75
100
125
150
175
v (a.u.)
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
$\frac{d\sigma}{d\Omega}(a_0^2/\mathrm{sr})$
1
2
3
4
5
6
7
8



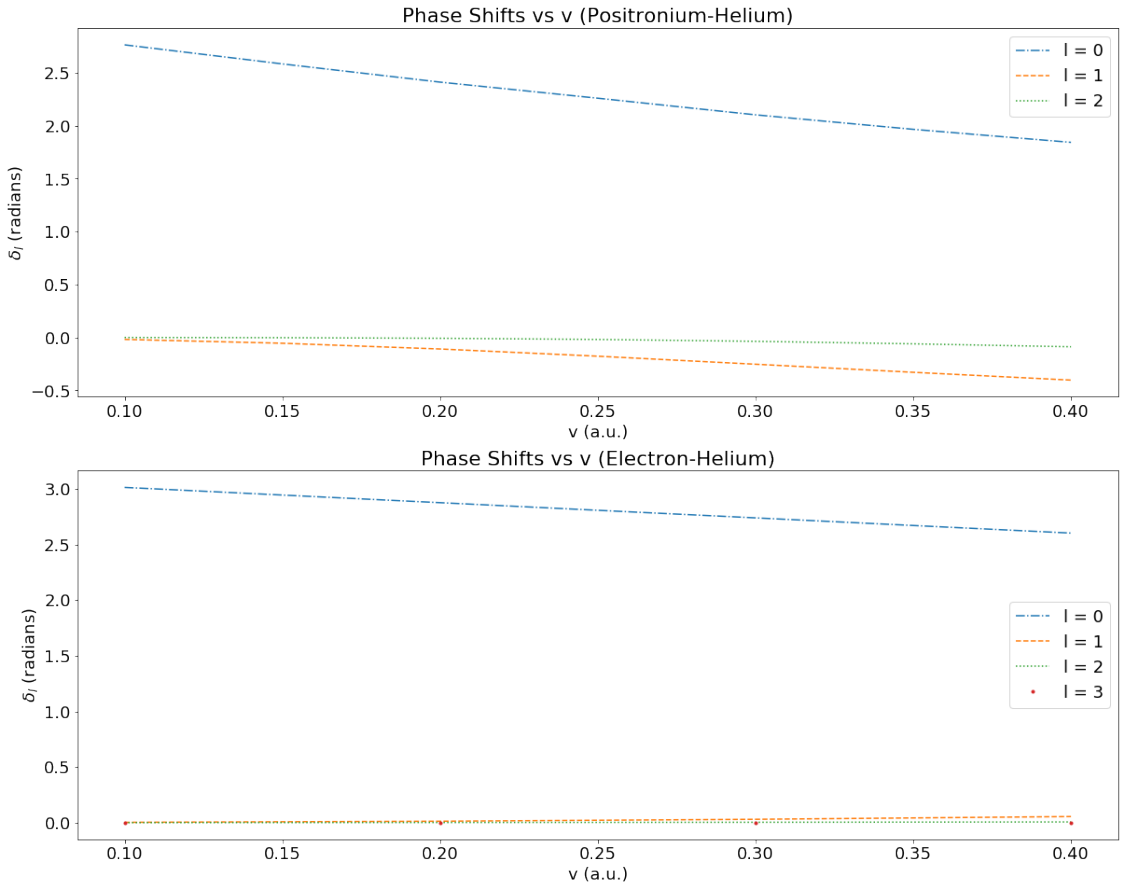
Between 0 and 40 degrees the differential cross section increases with increasing velocity. At v =< 0.15 the magnitude gradually decreases with increasing degrees. At higher energy, the magnitude decreases more with increasing degrees. The peak is at 0 degrees and the highest velocity. The minimum is at the highest velocity and 180 degrees.

2 Comparing Systems

2.1 Electron-Helium vs Positronium-Helium

Electron-Helium phase shifts from both R K Nesbet 1979 [5]; A. L. Sinfailam and R K Nesbet 1972 [4]. Positronium-Helium phase shifts are from Nirmal K Sarkar et al 1999 [8].

[52]:
```
e_ps_hel_system =␣
↪collision(ps_hel_phases,k_list_ps_hel,'Positronium-Helium',2,e_hel_phases,k_list_e_hel,'Ele

e_ps_hel_system.plot_phase_shifts(velocity=True,compare=True)
```



The phaseshifts for the electron-helium and positronium-helium systems are very similar. However, the p wave phase shifts for positronium-helium start to become inreasingly negative with increasing energy. While the magnitude of the s wave phase shifts decreases with increasing energy in both systems, the decrease is greater in the positronium-helium system.

[53]:
```
e_hel_ps_compare = compare_stuff(e_hel_cross_data, ps_hel_cross_data,␣
 ↪e_hel_momentum_data, ps_hel_momentum_data, 'e-he', 'ps-he')
display(e_hel_ps_compare)

e_ps_hel_system.plot_cross_section(velocity=True,compare=True)
```

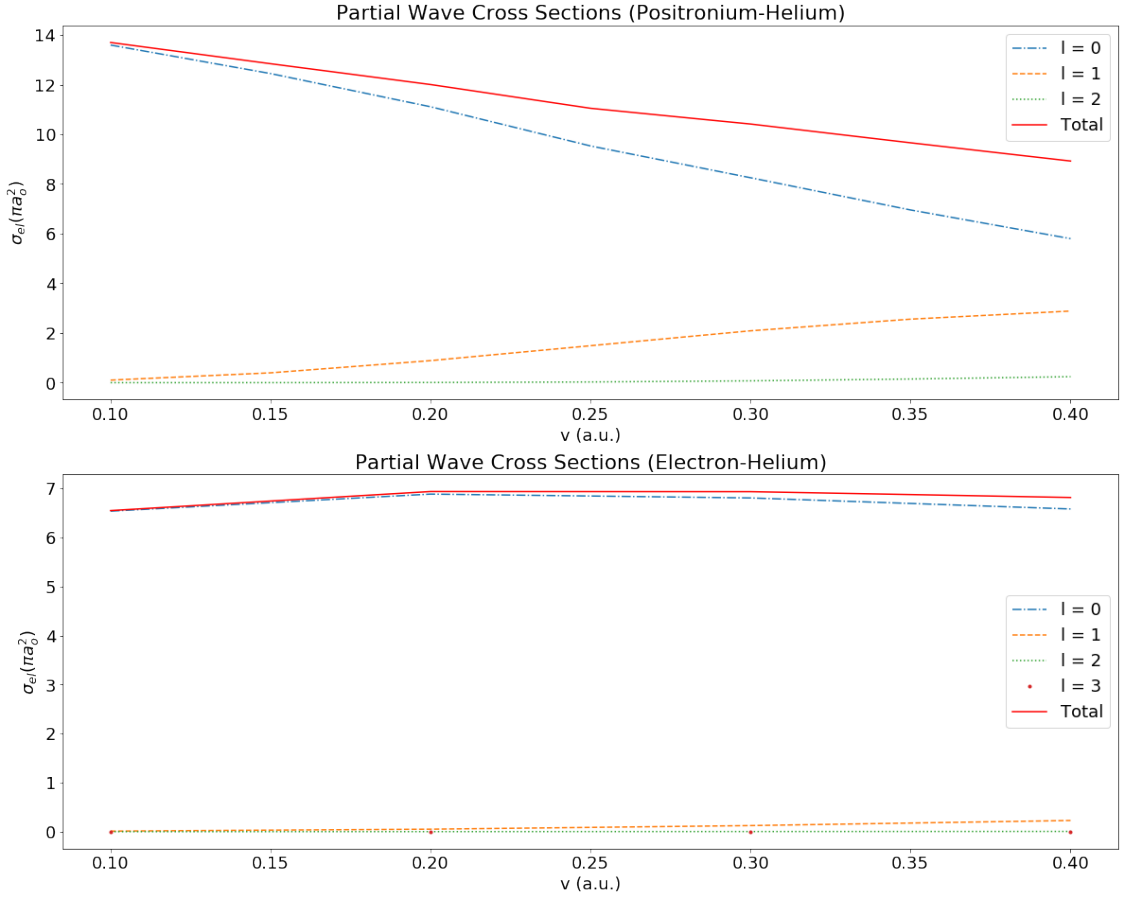
```
     v  e-he elastic cross section  ps-he elastic cross section  \
0  0.1                        6.549                        13.695
1  0.2                        6.938                        12.003
2  0.3                        6.935                        10.413
3  0.4                        6.816                         8.923

   ratio of elastic cross section e-he/ps-he  \
0                                      0.478
1                                      0.578
```

2	0.666
3	0.764

	e-he momentum transfer cross section \
0	6.861
1	7.589
2	7.880
3	7.951

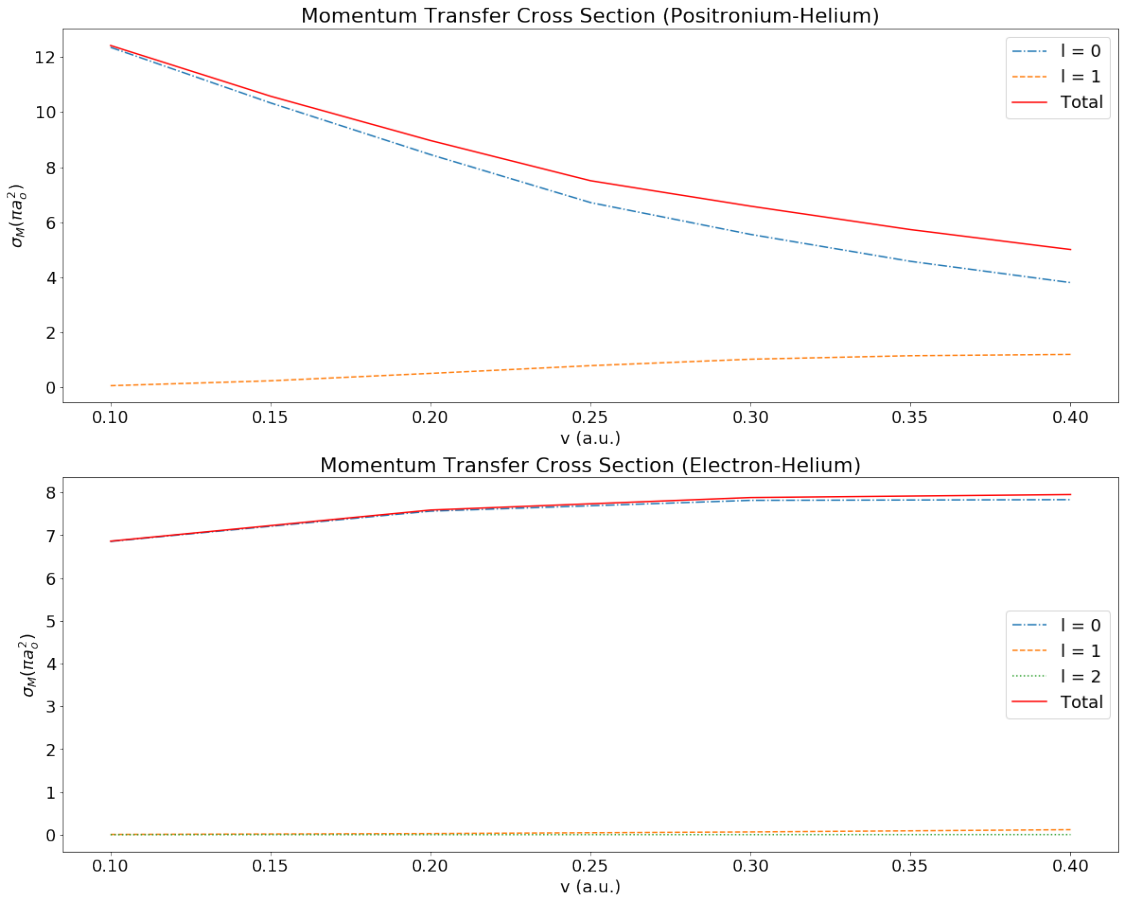
	ps-he momentum transfer cross section \
0	12.416
1	8.965
2	6.583
3	5.009

	ratio of momentum transfer cross section e-he/ps-he
0	0.553
1	0.847
2	1.197
3	1.587



The elastic cross section of the two systems become more similar with increasing energy. The elastic cross section of electron-helium is 48% and 76% that of positronium-helium at v = 0.1 au and v = 0.4 au, respectively. These are very similar compared to positron-helium vs positronium-helium, where the elastic cross section of positron-helium is not even 2% of positronium-helium's. Since the elastic cross section of electron-helium is hardly changing with increasing energy, and there is a decrease in th¬¬e magnitude for positronium-helium, this suggest that, within this energy range, the behavior of positronium approaches that of the electron. Therefore, this suggest the importance of the positron in positronium lessens with increasing energy, within this energy range. The s wave is the greatest contributor to the elastic cross section in both systems; however, while at a minimum it contributes 96% to the elastic cross section of the electron-helium system, the p wave gradually increases in influence for the positronium-helium system, finishing with a contribution of 32% for the highest velocity.

[54]:
```
e_ps_hel_system.plot_momentum_transfer(velocity=True,compare=True)
```



The shape of the momentum transfer cross section is similar to the elastic cross section, except that the decline is greater for positronium-helium, which causes the momentum transfer cross section for electron-helium to be greater than that of positronium-helium at v = 0.3 au. At the highest

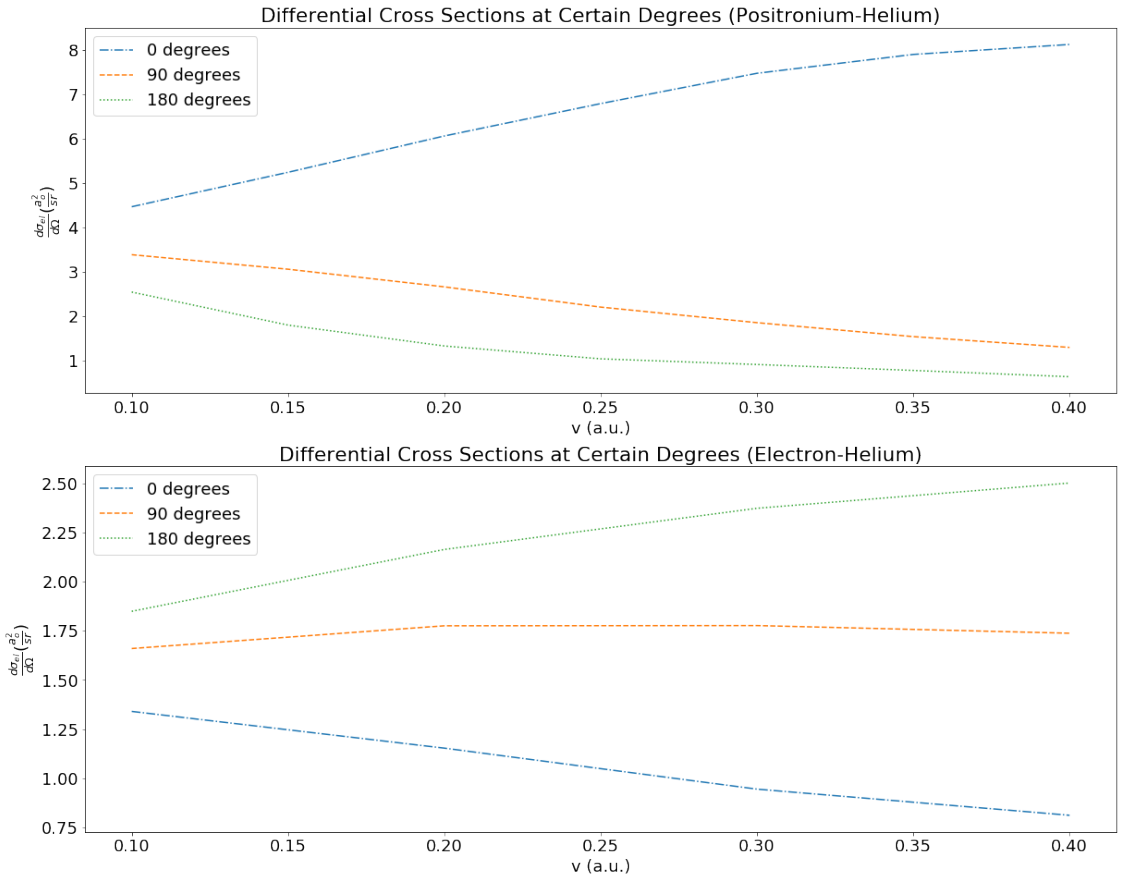
comparable velocity, it is 1.59 times greater. It is worth noting here that positronium-helium gets a larger contribution out of the higher waves, and the used dataset of positronium-helium is one wave less than that of electron-helium. Therefore, there could be a modest contribution to the momentum transfer cross section of positronium-helium from the d wave that is missing here.

[55]:
```
e_hel_ps_compare_degreee = compare_at_degree(e_hel_diff_degree,␣
↪ps_hel_diff_degree, 'e-he', 'ps-he')
display(e_hel_ps_compare_degreee)

e_ps_hel_system.plot_diff_cross_vs_k(r_interval,velocity=True,compare=True)
```

	v	e-he 0 degrees	ps-he 0 degrees	ratio e-he/ps-he at 0 degrees	\
0	0.1	1.340	4.466	0.300	
1	0.2	1.153	6.062	0.190	
2	0.3	0.944	7.477	0.126	
3	0.4	0.811	8.127	0.100	

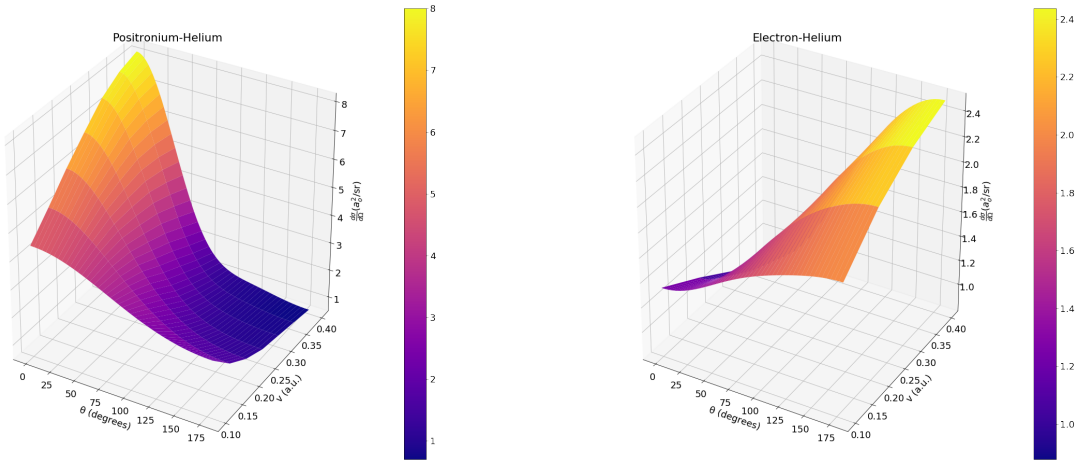
	e-he 90 degrees	ps-he 90 degrees	ratio e-he/ps-he at 90 degrees	\
0	1.660	3.384	0.491	
1	1.775	2.659	0.668	
2	1.777	1.850	0.961	
3	1.738	1.292	1.345	

	e-he 180 degrees	ps-he 180 degrees	ratio e-he/ps-he at 180 degrees
0	1.849	2.541	0.728
1	2.164	1.324	1.634
2	2.373	0.907	2.616
3	2.502	0.631	3.965

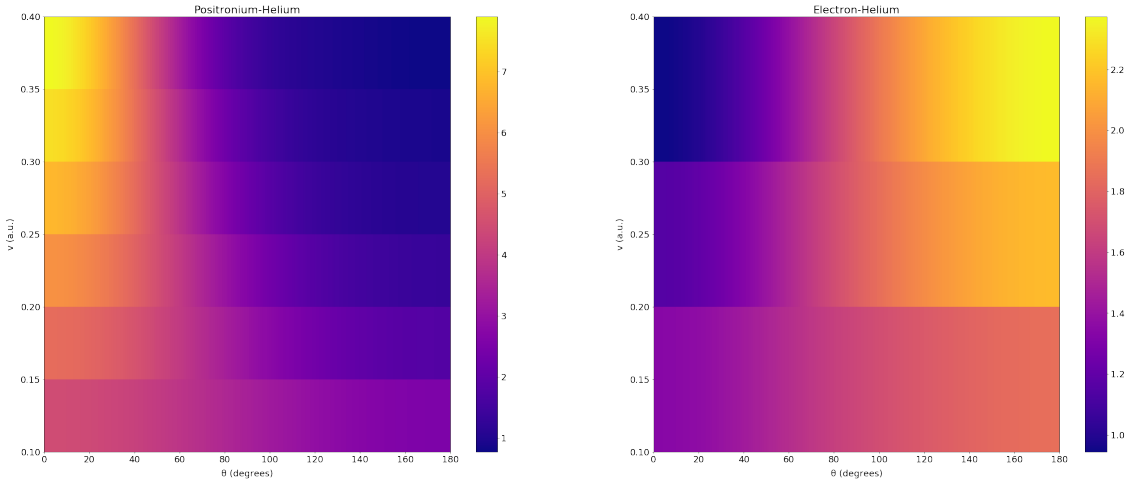


At 90 degrees the magnitude between the two systems becomes increasing comparable with increasing velocity. The change in magnitude of differential cross section as a function of velocity at 180 degrees of one system is incredibly similar to the other's at 0 degrees. For example, when comparing the change in magnitude of electron-helium at 180 degrees and positronium-helium at 0 degrees, it is seen that the magnitude of electron-helium drops to 95% of its maximum and positronium drops to 92% of its maximum. This similarity is maintained for other velocities.

[56]:
```
e_ps_hel_system.diff_plot_3d(velocity=True,compare=True)
```



[57]:
```
e_ps_hel_system.diff_plot_3d(velocity=True,compare=True, density=True)
```



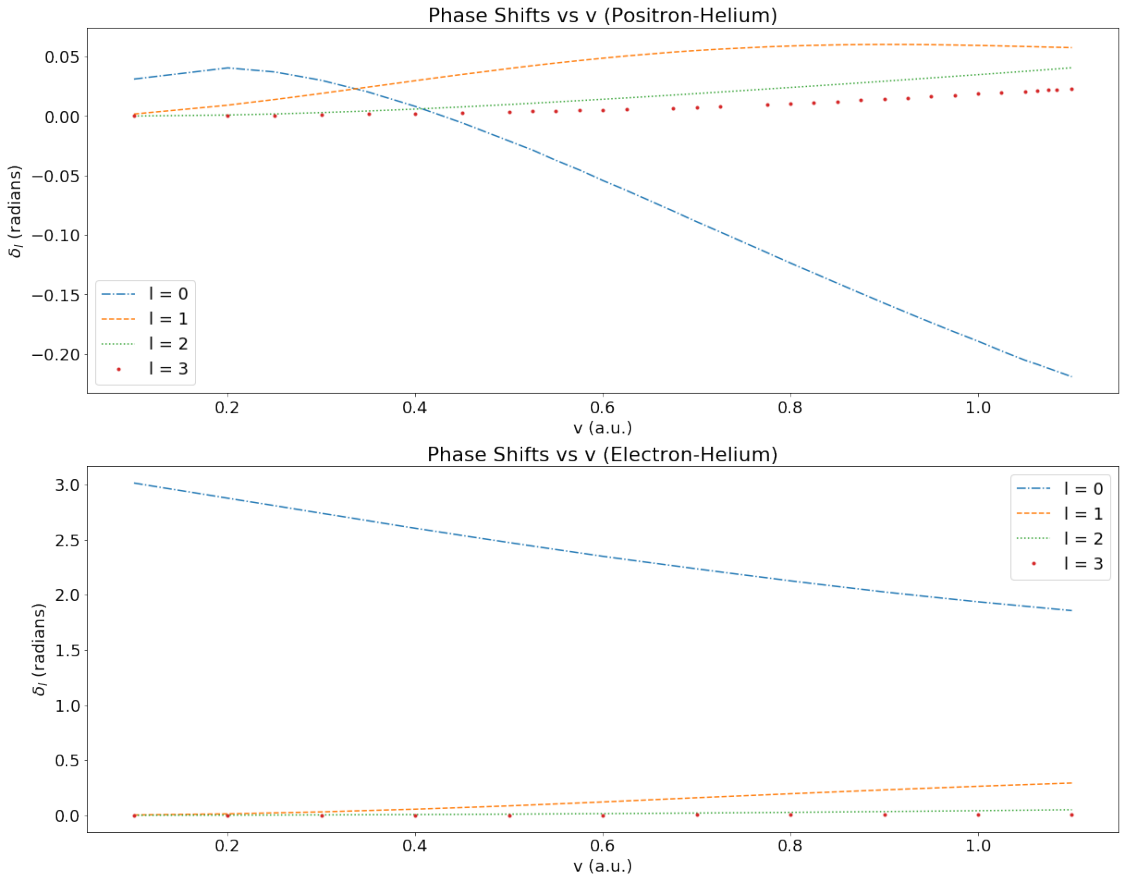
The differential cross section generally rises in one system where the other falls. Though, the magnitude for positronium-helium is generally greater than electron-helium.

2.2 Electron-Helium vs Positron-Helium

Electron-Helium phase shifts from both R K Nesbet 1979 [5]; A. L. Sinfailam and R K Nesbet 1972 [4]. Phase shifts for positron-helium provided by P Van Reeth, Private Communication 2020 [13].

[58]:
```
e_positron_hel_system =␣
↪collision(pos_hel_vr_phases,k_vr_pos_hel,'Positron-Helium',1,e_hel_phases,k_list_e_hel,'Ele
```

[59]:
```
e_positron_hel_system.plot_phase_shifts(velocity=True,compare=True)
```



The only resemblance the phaseshifts of these two systems is that the waves higher than the s wave are close to zero. The s wave for the positron system rapidly declines into negative values, but is still fairly small in magnitude compare to the s wave of the electron system. The s wave phaseshifts for the electron system linearly declines gradually with increasing energy.

[60]:
```
e_hel_pos_compare = compare_stuff(e_hel_cross_data, pos_hel_cross_data,␣
↪e_hel_momentum_data, pos_hel_momentum_data, 'e-he', 'e(+)-he')
display(e_hel_pos_compare)

e_positron_hel_system.plot_cross_section(velocity=True,compare=True)
```

	v	e-he elastic cross section	e(+)-he elastic cross section	\
0	0.1	6.549	0.388	
1	0.2	6.938	0.189	
2	0.3	6.935	0.090	
3	0.4	6.816	0.073	
4	0.5	6.509	0.092	
5	0.6	6.138	0.124	
6	0.7	5.692	0.156	
7	0.8	5.247	0.183	

8	0.9	4.789	0.203
9	1.0	4.335	0.218
10	1.1	3.915	0.228

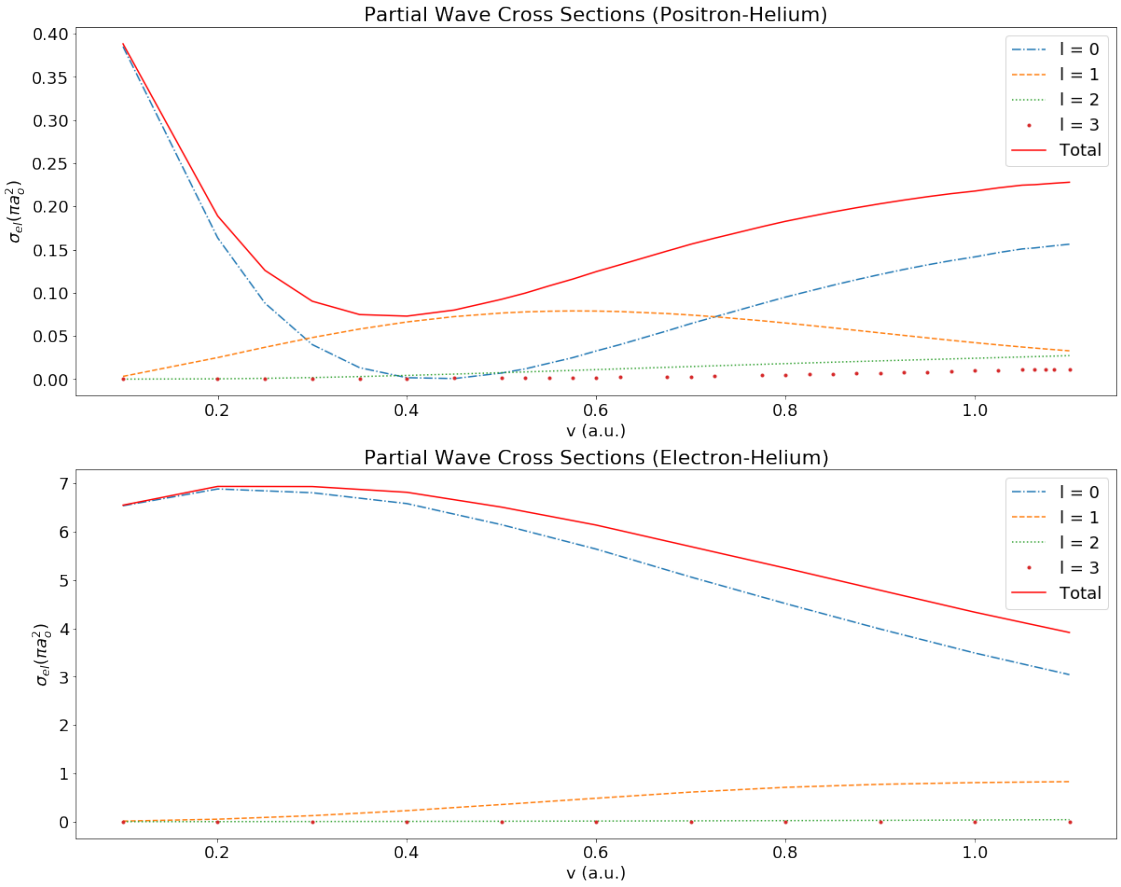
	ratio of elastic cross section e-he/e(+)-he \
0	16.879
1	36.709
2	77.056
3	93.370
4	70.750
5	49.500
6	36.487
7	28.672
8	23.591
9	19.885
10	17.171

	e-he momentum transfer cross section \
0	6.861
1	7.589
2	7.880
3	7.951
4	7.693
5	7.222
6	6.569
7	5.842
8	5.084
9	4.357
10	3.705

	e(+)-he momentum transfer cross section \
0	0.348
1	0.112
2	0.029
3	0.041
4	0.090
5	0.146
6	0.193
7	0.225
8	0.243
9	0.250
10	0.252

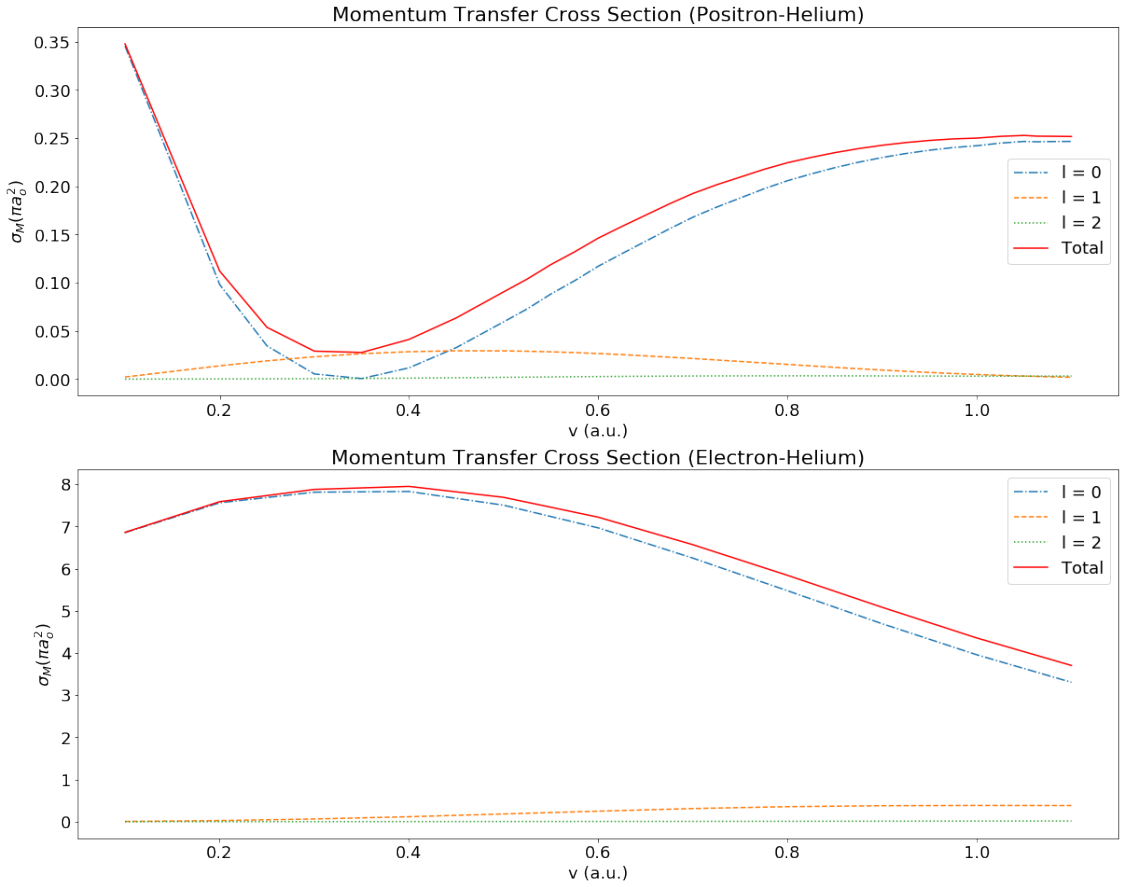
	ratio of momentum transfer cross section e-he/e(+)-he
0	19.716
1	67.759
2	271.724
3	193.927

4	85.478
5	49.466
6	34.036
7	25.964
8	20.922
9	17.428
10	14.702



The total elastic cross section of electron-helium stays around the same value before gradually decreasing with increasing energy, while for positron-helium it rapidly decreases at the beginning, before gradually increasing. This shift in slope for both systems occurs at v = 0.4 au. The magnitude of the elastic cross section of electron-helium is between 17-93 times greater than positron-helium. At the lowest and at the highest velocity, the ratio of elastic cross sections is around 17. In the electron-helium system the contribution of the s wave gradually decreases with increasing energy, and the p wave contribution mostly rises by the same amount. At the highest comparable velocity, the contribution from the s and p wave are 78% and 21%, respectively. The s wave contribution in the positron-helium system rapidly decreases; at v = 0.45 the contribution from the s, p, d and f wave are 0.8%, 90.5%, 7.3%, and 1.4%, respectively. The contribution from the s wave then increases, p wave decreases, and the d and f wave slowly increase with increasing velocity.

[61]:
```
e_positron_hel_system.plot_momentum_transfer(velocity=True, compare=True)
```



The momentum transfer cross section ratio is similar to the elastic cross section ratio, except that the maximum difference is greater. At v = 0.3 au the momentum transfer cross section is 271 times greater for electron-helium compared to positron-helium. However, at v = 1.1 au the ratio is actually smaller than that of the elastic cross section, having a value of 14.7.

[62]:
```
e_hel_pos_compare_degree = compare_at_degree(e_hel_diff_degree,␣
 ↪pos_hel_diff_degree, 'e-he', 'e(+)-he')
display(e_hel_pos_compare_degree)

e_positron_hel_system.
 ↪plot_diff_cross_vs_k(r_interval,velocity=True,compare=True)
```

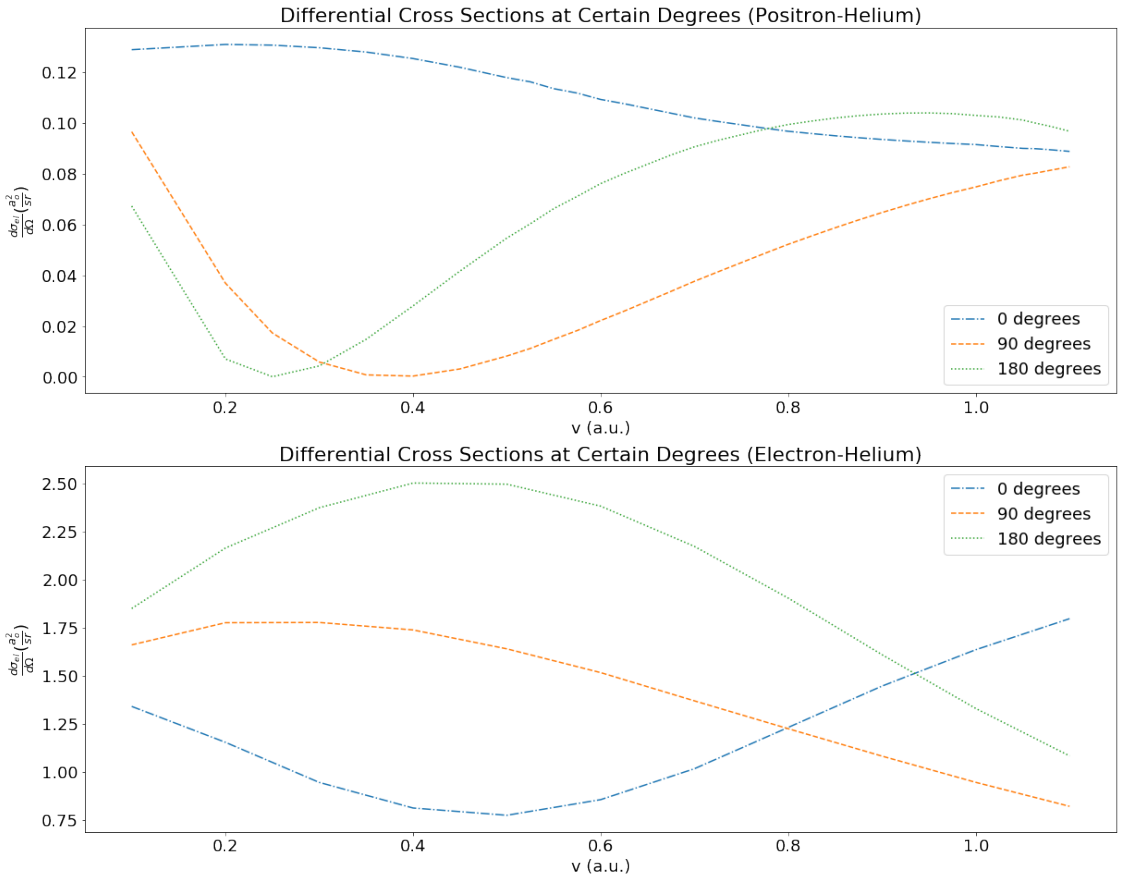
```
C:\Users\Zoruku\Anaconda3\lib\site-packages\ipykernel_launcher.py:8:
RuntimeWarning: divide by zero encountered in double_scalars
```

```
      v  e-he 0 degrees  e(+)-he 0 degrees  ratio e-he/e(+)-he at 0 degrees  \
0   0.1           1.340              0.129                            10.388
```

1	0.2	1.153	0.131	8.802
2	0.3	0.944	0.130	7.262
3	0.4	0.811	0.125	6.488
4	0.5	0.773	0.118	6.551
5	0.6	0.855	0.109	7.844
6	0.7	1.016	0.102	9.961
7	0.8	1.230	0.097	12.680
8	0.9	1.445	0.094	15.372
9	1.0	1.635	0.092	17.772
10	1.1	1.796	0.089	20.180

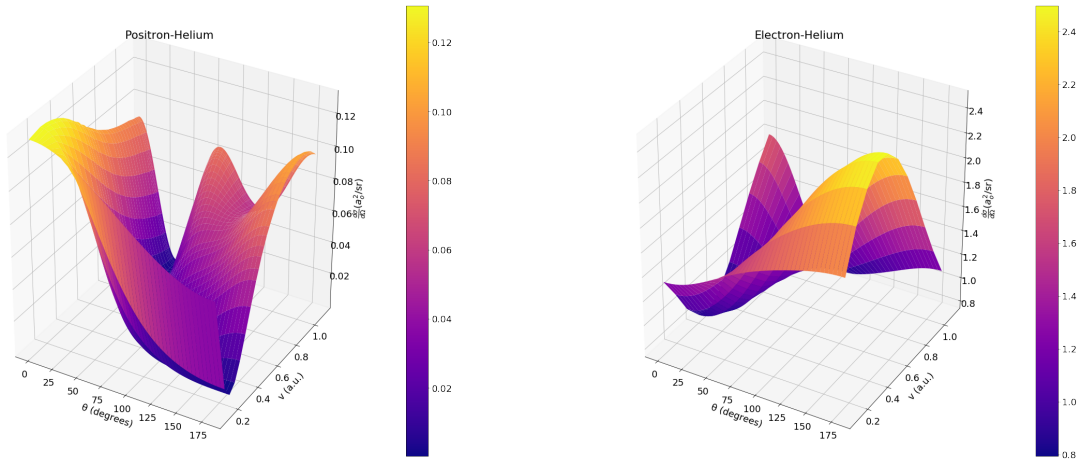
	e-he 90 degrees	e(+)-he 90 degrees	ratio e-he/e(+)-he at 90 degrees \
0	1.660	0.097	17.113
1	1.775	0.037	47.973
2	1.777	0.006	296.167
3	1.738	0.000	inf
4	1.639	0.008	204.875
5	1.516	0.022	68.909
6	1.369	0.038	36.026
7	1.224	0.052	23.538
8	1.081	0.065	16.631
9	0.945	0.075	12.600
10	0.820	0.083	9.880

	e-he 180 degrees	e(+)-he 180 degrees	ratio e-he/e(+)-he at 180 degrees
0	1.849	0.067	27.597
1	2.164	0.007	309.143
2	2.373	0.004	593.250
3	2.502	0.028	89.357
4	2.496	0.055	45.382
5	2.382	0.076	31.342
6	2.172	0.091	23.868
7	1.903	0.099	19.222
8	1.609	0.104	15.471
9	1.329	0.103	12.903
10	1.082	0.097	11.155

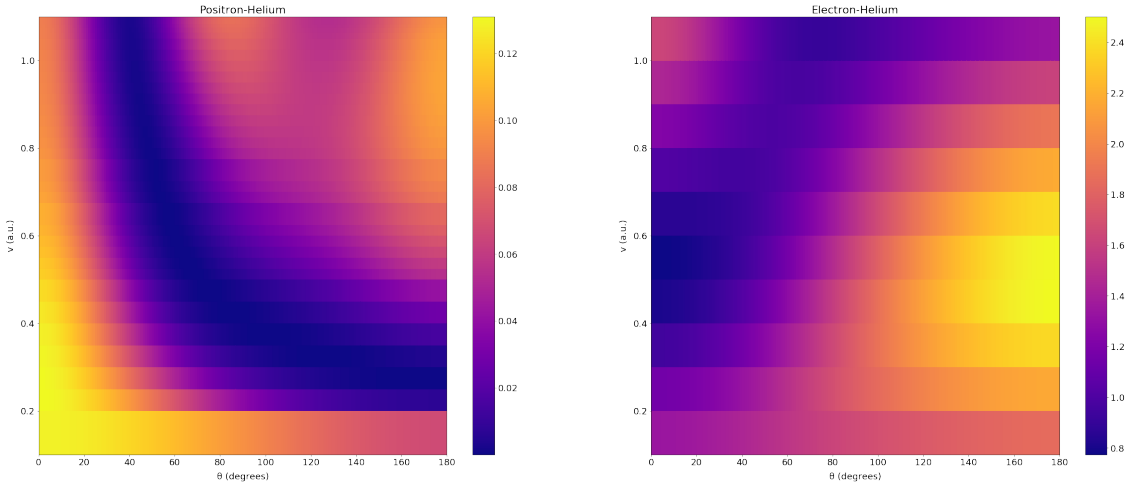


The only visible similarity here is that the differential cross section has a similar shape at 0 degrees in the electron system as at 90 degrees in the positron system. Overall, the magnitude is greater for the electron-helium system.

[63]:
```
e_positron_hel_system.diff_plot_3d(velocity=True,compare=True)
```



[64]:
```
e_positron_hel_system.diff_plot_3d(velocity=True,compare=True,density=True)
```



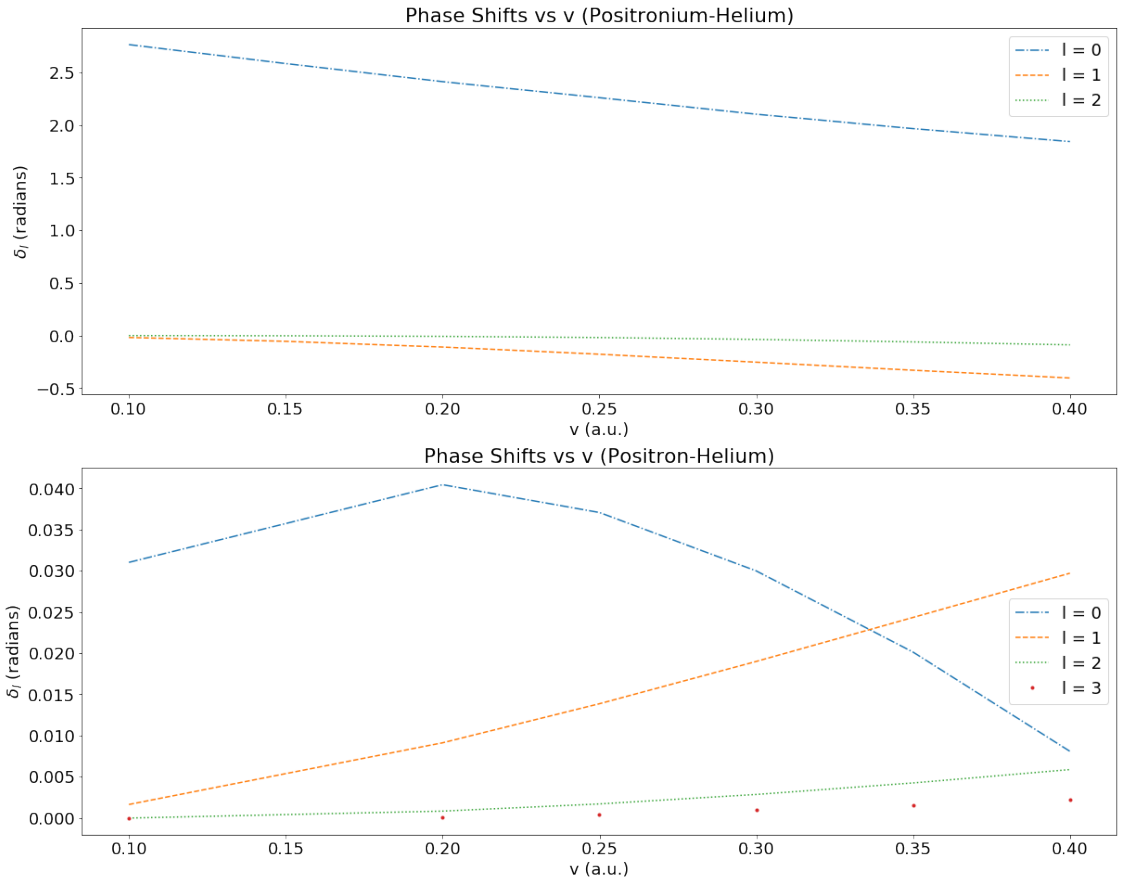
The only noticeable similarity of the differential cross section in the above graphs is that they both have a valley, though this valley is at different values. The positron-helium system peaks at the lowest energy and 0 degrees. The electron-helium system peaks at a more moderate energy at 180 degrees.

2.3 Positron-Helium vs Positronium-Helium

Phase shifts for positronium-helium are from Nirmal K Sarkar et al 1999 [8]. Phase shifts for positron-helium provided by P Van Reeth, Private Communication 2020 [13].

[65]:
```
pos_ps_hel_system =␣
↪collision(ps_hel_phases,k_list_ps_hel,'Positronium-Helium',2,pos_hel_vr_phases,k_vr_pos_hel
```

[66]:
```
pos_ps_hel_system.plot_phase_shifts(velocity=True,compare=True)
```



There are no noticeable similarities in the phaseshifts between these two systems.

[67]:
```
pos_hel_ps_compare = compare_stuff(pos_hel_cross_data, ps_hel_cross_data,␣
 ↪pos_hel_momentum_data, ps_hel_momentum_data, 'e(+)-he', 'ps-he')
display(pos_hel_ps_compare)

pos_ps_hel_system.plot_cross_section(velocity=True,compare=True)
```

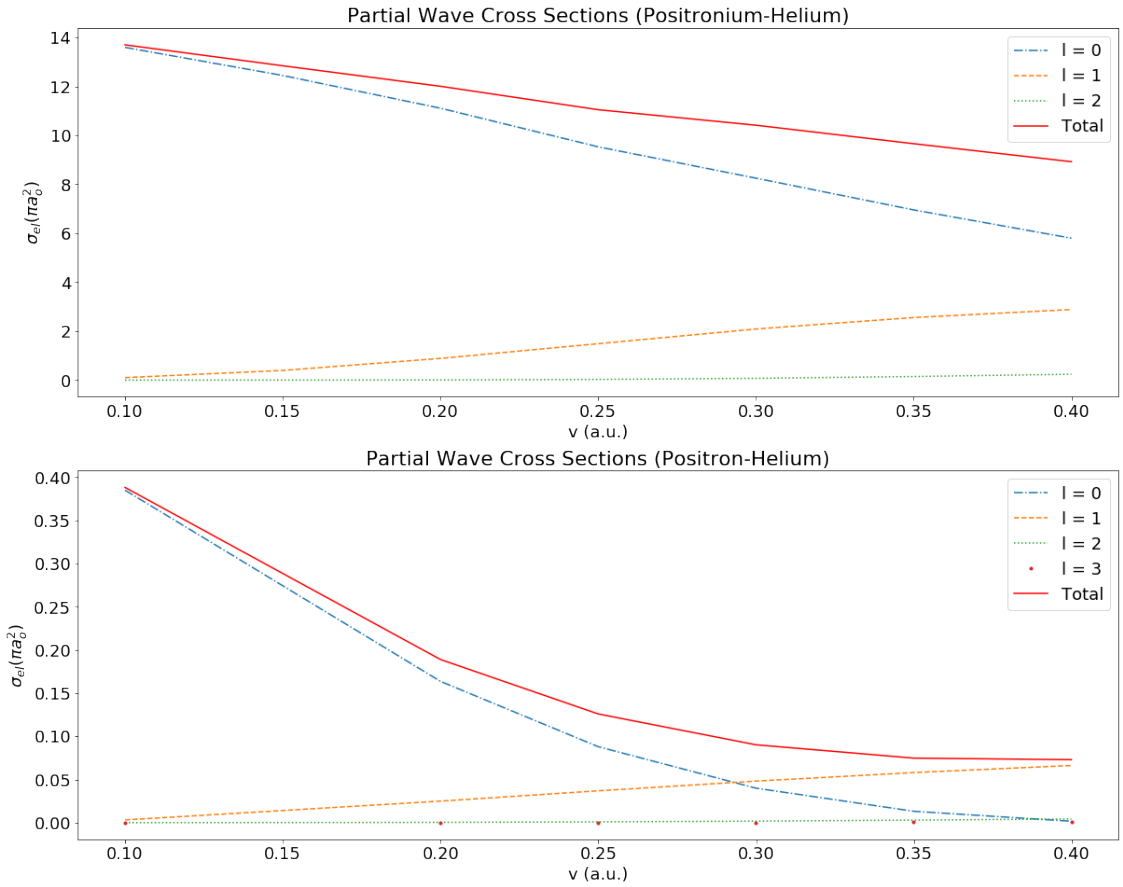
```
      v  e(+)-he elastic cross section  ps-he elastic cross section  \
0  0.10                          0.388                        13.695
1  0.20                          0.189                        12.003
2  0.25                          0.126                        11.046
3  0.30                          0.090                        10.413
4  0.35                          0.075                         9.657
5  0.40                          0.073                         8.923

   ratio of elastic cross section e(+)-he/ps-he  \
0                                         0.028
1                                         0.016
2                                         0.011
```

3	0.009
4	0.008
5	0.008

	e(+)-he momentum transfer cross section \
0	0.348
1	0.112
2	0.054
3	0.029
4	0.028
5	0.041

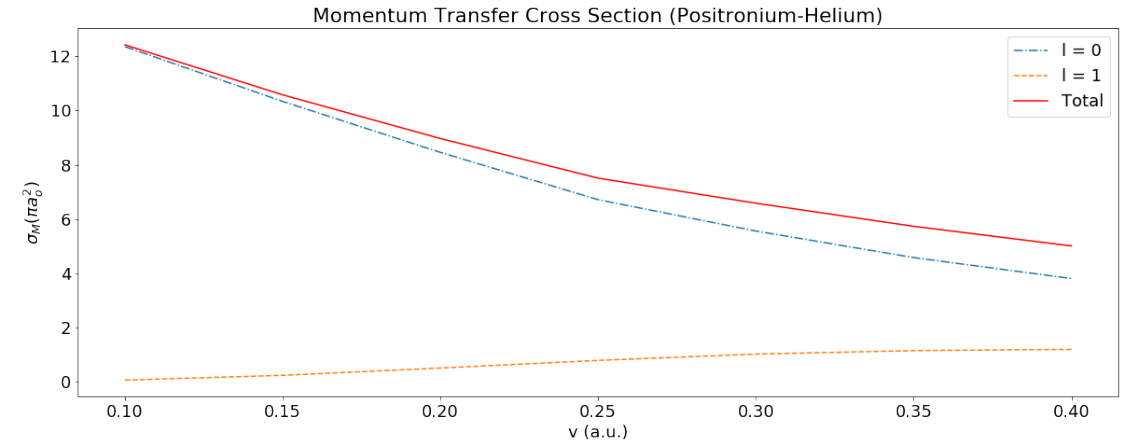
	ps-he momentum transfer cross section \
0	12.416
1	8.965
2	7.506
3	6.583
4	5.732
5	5.009

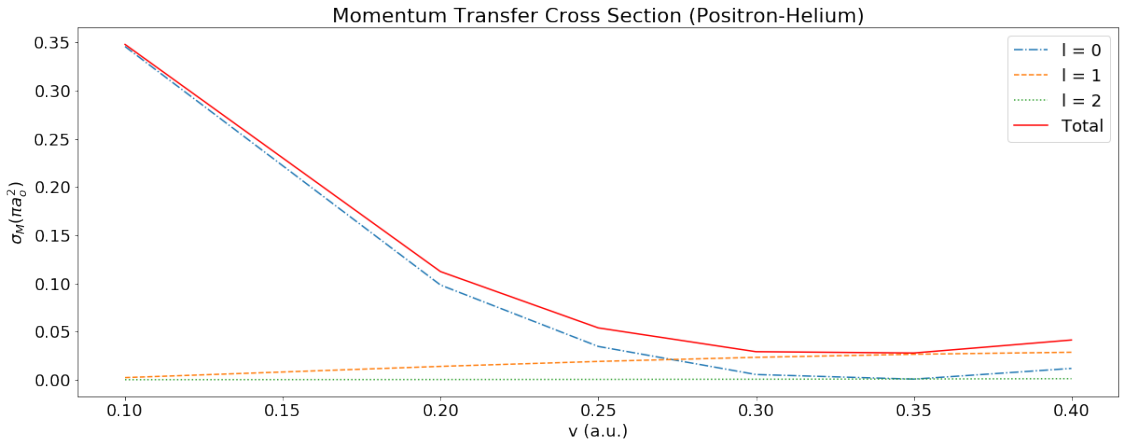
	ratio of momentum transfer cross section e(+)-he/ps-he
0	0.028
1	0.012
2	0.007
3	0.004
4	0.005
5	0.008



Both the shape and magnitude of the elastic cross section for the two systems is not comparable. The magnitude of the elastic cross section for positron-helium is less than 2% that of positronium-helium for all comparable velocities. For the comparable velocities, the s wave contribution decreases with increasing energy in both systems. However, the extent of decrease is different; the p wave contributes 90.7% of the elastic cross section at the highest comparable velocity in the positron-helium system, but the majority contributor of the elastic cross section in the positronium-helium system is still the s wave.

[68]:
```
pos_ps_hel_system.plot_momentum_transfer(velocity=True,compare=True)
```





The difference in magnitude of the momentum transfer cross section is roughly the same as the elastic cross section and is not similar between the two systems.

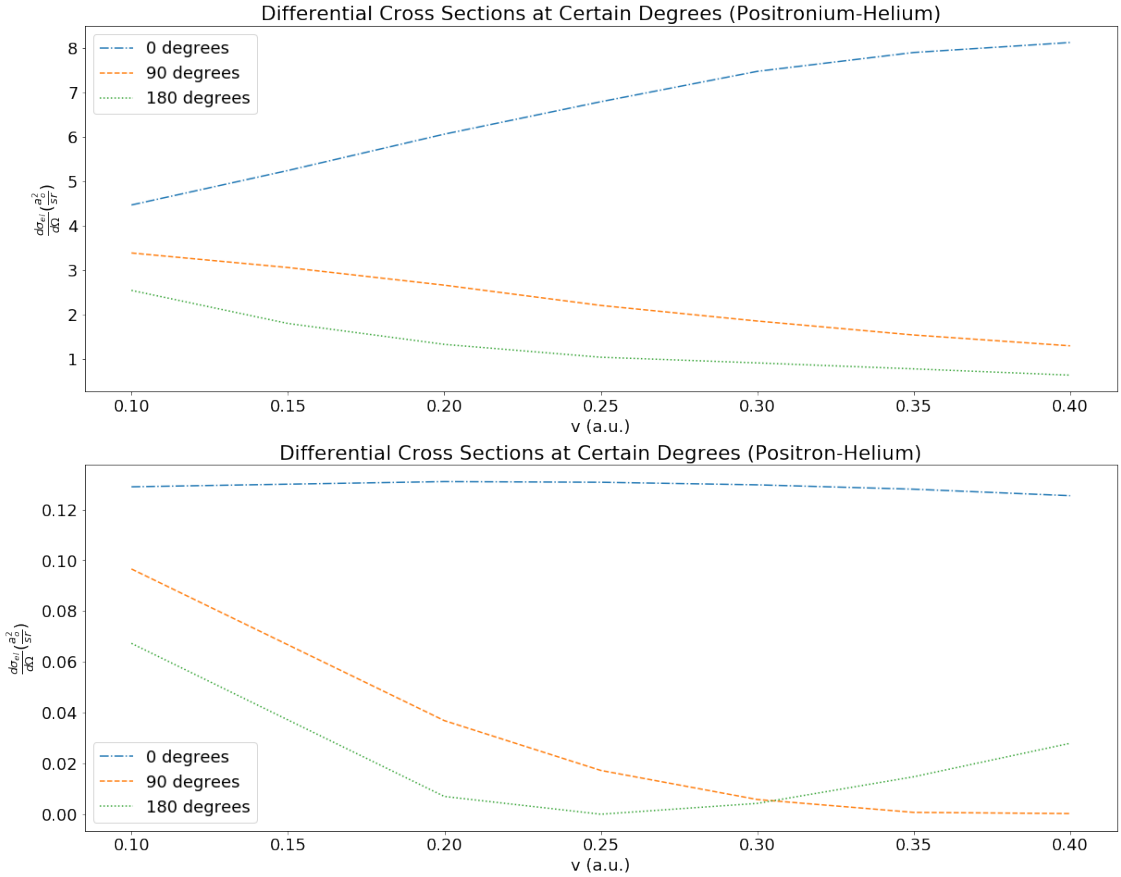
[69]:
```
pos_hel_ps_compare_degree = compare_at_degree(pos_hel_diff_degree,␣
↪ps_hel_diff_degree, 'e(+)-he', 'ps-he')
display(pos_hel_ps_compare_degree)

pos_ps_hel_system.plot_diff_cross_vs_k(r_interval,velocity=True,compare=True)
```

	v	e(+)-he 0 degrees	ps-he 0 degrees	ratio e(+)-he/ps-he at 0 degrees	\
0	0.10	0.129	4.466	0.029	
1	0.20	0.131	6.062	0.022	
2	0.25	0.131	6.793	0.019	
3	0.30	0.130	7.477	0.017	
4	0.35	0.128	7.902	0.016	
5	0.40	0.125	8.127	0.015	

	e(+)-he 90 degrees	ps-he 90 degrees	ratio e(+)-he/ps-he at 90 degrees	\
0	0.097	3.384	0.029	
1	0.037	2.659	0.014	

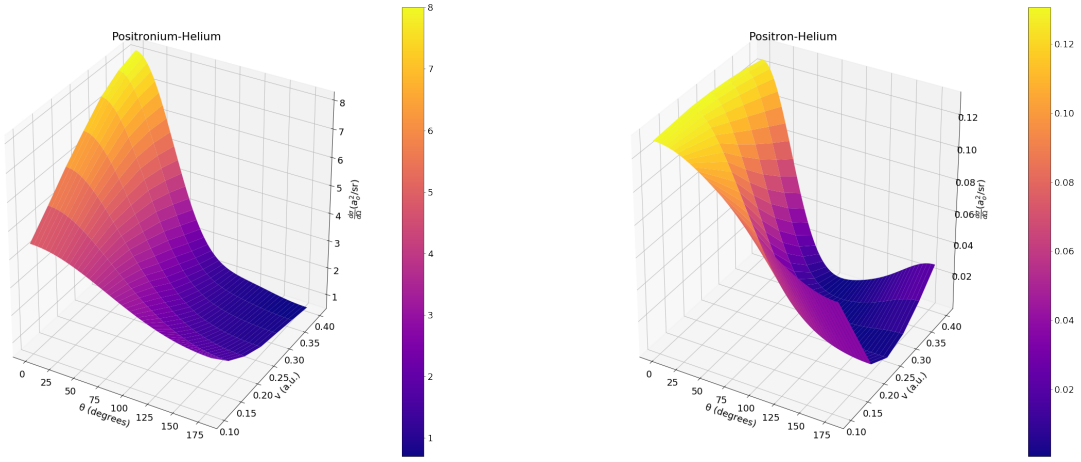
2	0.017	2.202	0.008
3	0.006	1.850	0.003
4	0.001	1.536	0.001
5	0.000	1.292	0.000

	e(+)-he 180 degrees	ps-he 180 degrees	ratio e(+)-he/ps-he at 180 degrees
0	0.067	2.541	0.026
1	0.007	1.324	0.005
2	0.000	1.035	0.000
3	0.004	0.907	0.004
4	0.015	0.773	0.019
5	0.028	0.631	0.044

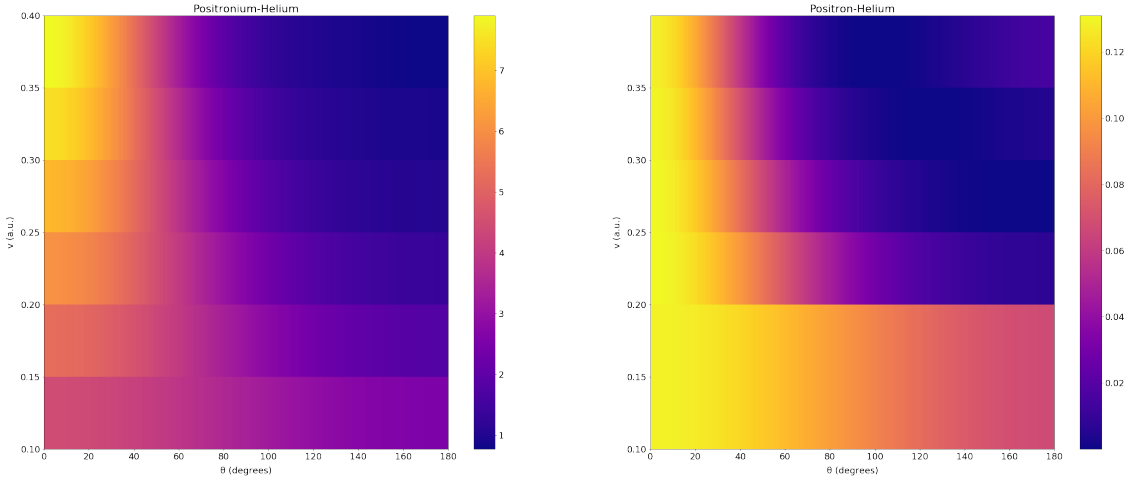


At 0 degrees the differential cross section of the positronium system increases with increasing energy while it stays mostly flat in the positron system. At 90 and 180 degrees the differential cross section decreases with increasing energy except at v = 0.30 in the positron system where at 180 degrees it begins to increase.

[70]:
```
pos_ps_hel_system.diff_plot_3d(velocity=True,compare=True)
```



[71]:
```
pos_ps_hel_system.diff_plot_3d(velocity=True,compare=True,density=True)
```



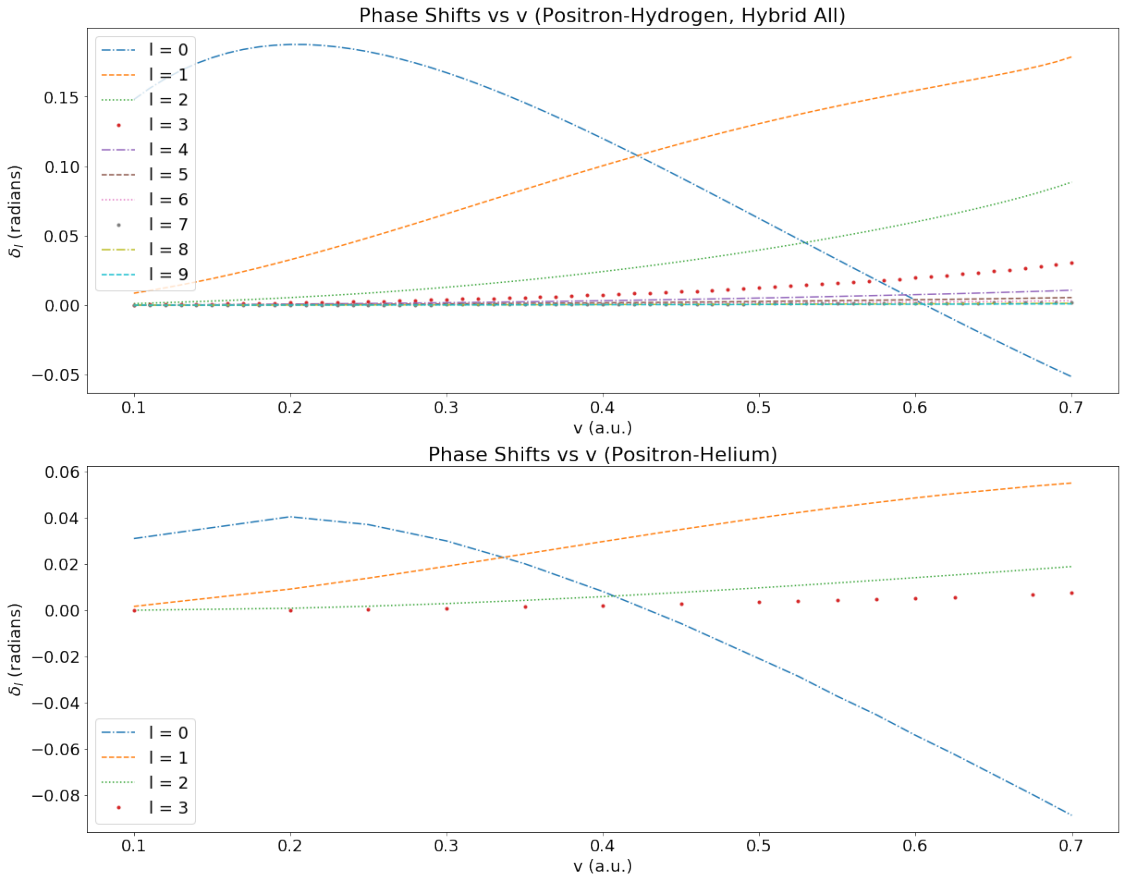
Beyond v = 0.2, the shape of the differential cross section is very similar, though the magnitude for positronium-helium is far greater.

2.4 Positron-Helium vs Positron-Hydrogen

Phase shifts for positron-helium provided by P Van Reeth, Private Communication 2020 [13].Positron-Hydrogen phase shifts from P Van Reeth, private communication 2020 [12]

[72]:
```
positron_hyd_hel_system =␣
↪collision(positron_hyd_vr_hybrid,k_vr_higher,'Positron-Hydrogen, Hybrid␣
↪All',1,pos_hel_vr_phases,k_vr_pos_hel,'Positron-Helium',1)
```

[73]:
```
positron_hyd_hel_system.plot_phase_shifts(velocity=True,compare=True)
```



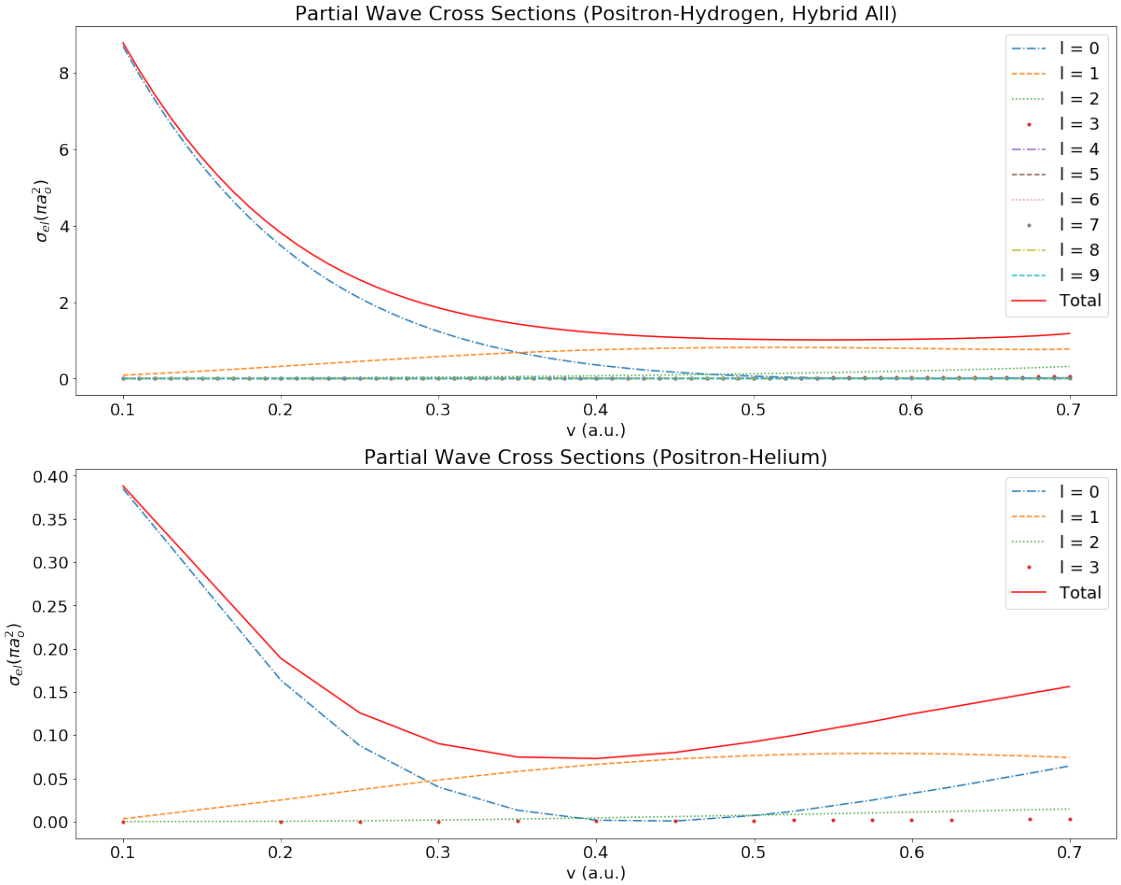
The p and d phaseshifts of the positron-hydrogen system diverge from zero much more so than in the positron-helium system.

[74]:
```
pos_hyd_pos_hel_compare = compare_stuff(pos_hyd_cross_data, pos_hel_cross_data,␣
↪pos_hyd_momentum_data, pos_hel_momentum_data, 'e(+)-h', 'e(+)-he')
display(pos_hyd_pos_hel_compare)

positron_hyd_hel_system.plot_cross_section(velocity=True,compare=True)
```

	v	e(+)-h elastic cross section	e(+)-he elastic cross section	\
0	0.10	8.779	0.388	
1	0.20	3.812	0.189	
2	0.25	2.584	0.126	
3	0.30	1.850	0.090	
4	0.35	1.427	0.075	
5	0.40	1.196	0.073	
6	0.45	1.077	0.080	
7	0.50	1.025	0.092	
8	0.55	1.012	0.108	
9	0.60	1.027	0.124	

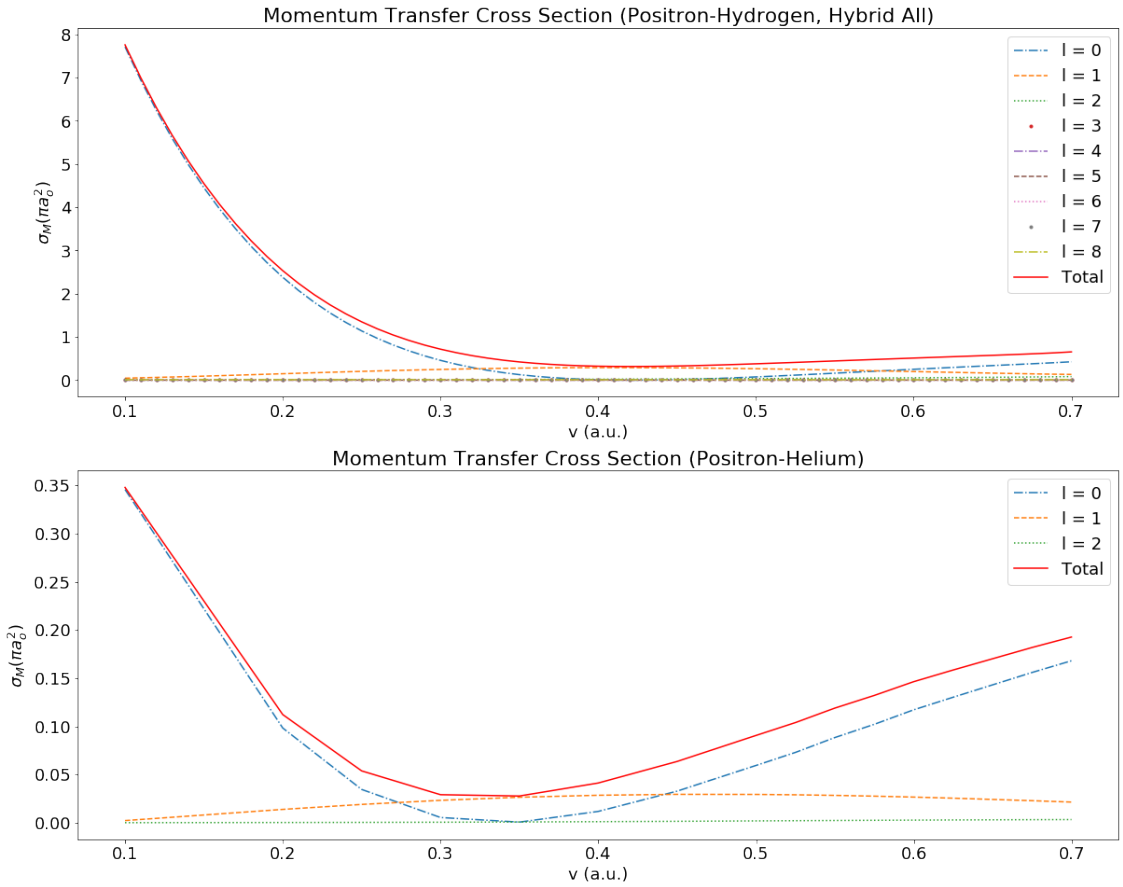
```
10  0.70                          1.180                          0.156

    ratio of elastic cross section e(+)-h/e(+)-he  \
0                                          22.626
1                                          20.169
2                                          20.508
3                                          20.556
4                                          19.027
5                                          16.384
6                                          13.462
7                                          11.141
8                                           9.370
9                                           8.282
10                                          7.564

    e(+)-h momentum transfer cross section  \
0                                    7.757
1                                    2.532
2                                    1.349
3                                    0.716
4                                    0.421
5                                    0.322
6                                    0.326
7                                    0.378
8                                    0.444
9                                    0.510
10                                   0.653

    e(+)-he momentum transfer cross section  \
0                                    0.348
1                                    0.112
2                                    0.054
3                                    0.029
4                                    0.028
5                                    0.041
6                                    0.064
7                                    0.090
8                                    0.119
9                                    0.146
10                                   0.193

    ratio of momentum transfer cross section e(+)-h/e(+)-he
0                                             22.290
1                                             22.607
2                                             24.981
3                                             24.690
4                                             15.036
5                                              7.854
```

6	5.094
7	4.200
8	3.731
9	3.493
10	3.383



The shape of the elastic cross section is very similar for the systems up to v = 0.4. Here it begins increasing for positron-helium but stays mostly flat for positron-hydrogen. Regarding the magnitude, positron-hydrogen is anywhere between 7-23 times greater than positron-helium. The s wave is the greatest contributor for low velocity in both systems before it is overtaken by the p wave. The p wave then remains the greatest contributor in both systems. In the positron-hydrogen system the s wave continues to decrease to nearly 0. In the positron-helium system the s wave only contributes slightly less than the p wave at the highest comparable velocity.

[75]:
```
positron_hyd_hel_system.plot_momentum_transfer(velocity=True,compare=True)
```



For the momentum transfer cross section, the s wave contributes the most for most velocities in both systems, briefly being overtaken by the p wave in mid-range velocities.

[76]:
```
pos_hyd_pos_hel_compare_degree = compare_at_degree(pos_hyd_diff_degree,␣
 ↪pos_hel_diff_degree,'e(+)-h', 'e(+)-he')
display(pos_hyd_pos_hel_compare_degree)

positron_hyd_hel_system.
 ↪plot_diff_cross_vs_k(r_interval,velocity=True,compare=True)
```

```
C:\Users\Zoruku\Anaconda3\lib\site-packages\ipykernel_launcher.py:8:
RuntimeWarning: divide by zero encountered in double_scalars

C:\Users\Zoruku\Anaconda3\lib\site-packages\ipykernel_launcher.py:9:
RuntimeWarning: divide by zero encountered in double_scalars
  if __name__ == '__main__':
```

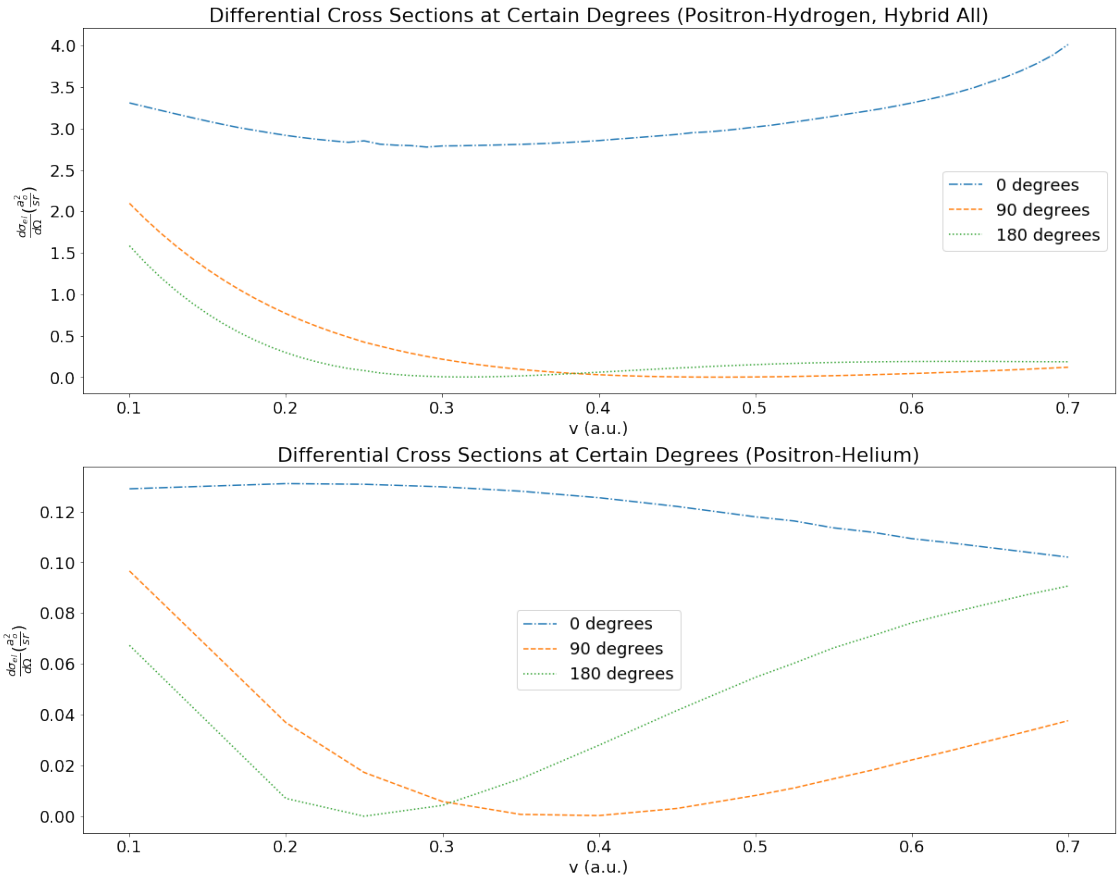
	v	e(+)-h 0 degrees	e(+)-he 0 degrees	\
0	0.10	3.308	0.129	
1	0.20	2.917	0.131	
2	0.25	2.851	0.131	

3	0.30	2.790	0.130
4	0.35	2.809	0.128
5	0.40	2.854	0.125
6	0.45	2.929	0.122
7	0.50	3.018	0.118
8	0.55	3.148	0.114
9	0.60	3.308	0.109
10	0.70	4.013	0.102

	ratio e(+)-h/e(+)-he at 0 degrees	e(+)-h 90 degrees	e(+)-he 90 degrees	\
0	25.643	2.097	0.097	
1	22.267	0.767	0.037	
2	21.763	0.422	0.017	
3	21.462	0.216	0.006	
4	21.945	0.094	0.001	
5	22.832	0.029	0.000	
6	24.008	0.003	0.003	
7	25.576	0.002	0.008	
8	27.614	0.017	0.015	
9	30.349	0.043	0.022	
10	39.343	0.120	0.038	

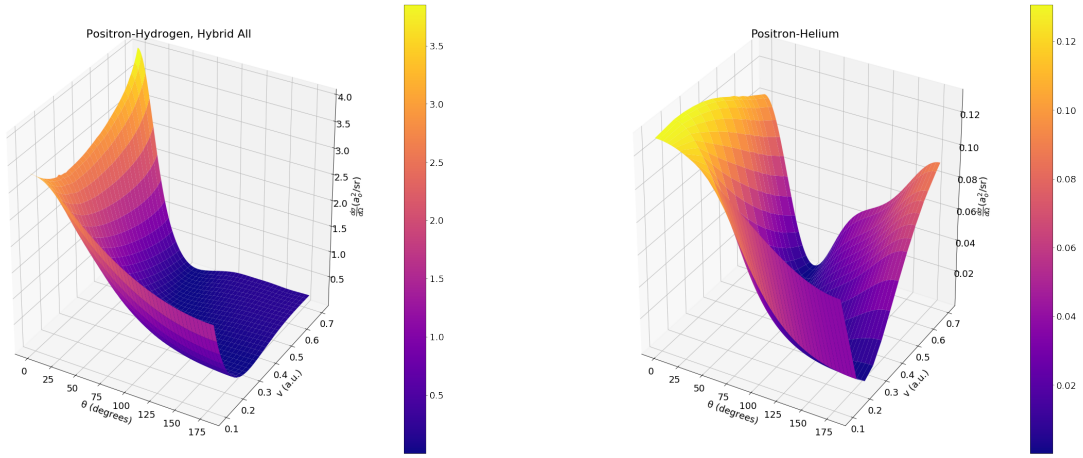
	ratio e(+)-h/e(+)-he at 90 degrees	e(+)-h 180 degrees	\
0	21.619	1.585	
1	20.730	0.295	
2	24.824	0.079	
3	36.000	0.004	
4	94.000	0.015	
5	inf	0.060	
6	1.000	0.110	
7	0.250	0.151	
8	1.133	0.177	
9	1.955	0.189	
10	3.158	0.185	

	e(+)-he 180 degrees	ratio e(+)-h/e(+)-he at 180 degrees
0	0.067	23.657
1	0.007	42.143
2	0.000	inf
3	0.004	1.000
4	0.015	1.000
5	0.028	2.143
6	0.042	2.619
7	0.055	2.745
8	0.066	2.682
9	0.076	2.487
10	0.091	2.033

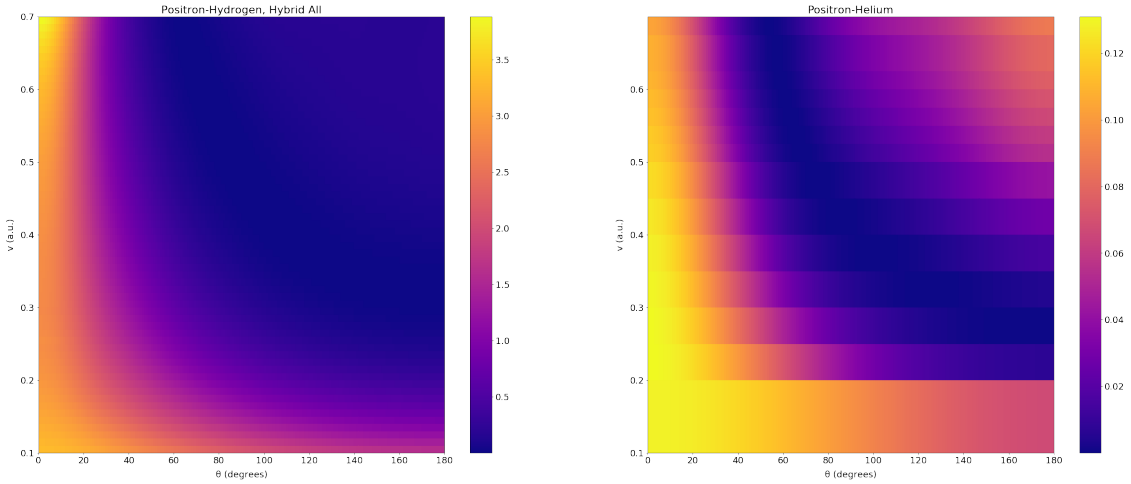


At 90 and 180 degrees the differential cross section is close for both systems. In the positron-hydrogen system at 0 degrees the differential cross section gets further away from 90 and 180 degrees in magnitude with increasing energy whereas in the positron-helium system the differential cross section at the various degrees converges with increasing energy.

[77]:
```
positron_hyd_hel_system.diff_plot_3d(velocity=True,compare=True)
```



[78]:
```
positron_hyd_hel_system.diff_plot_3d(velocity=True,compare=True,density=True)
```



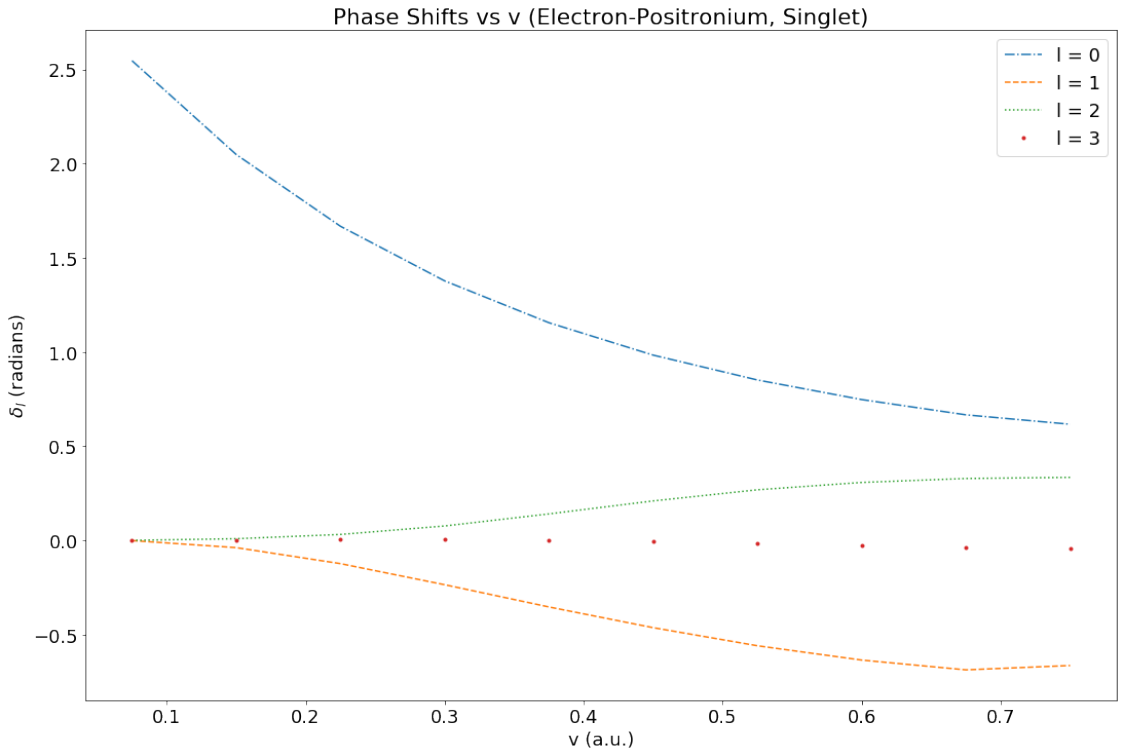
When both the velocity and degrees are low, there is a peak region maintained for both systems. Moving from this causes the differential cross section to decrease rapidly. The differential cross section never again increases in the positron-hydrogen system, but when both the velocity and degrees are high the differential cross section increases in the positron-helium system.

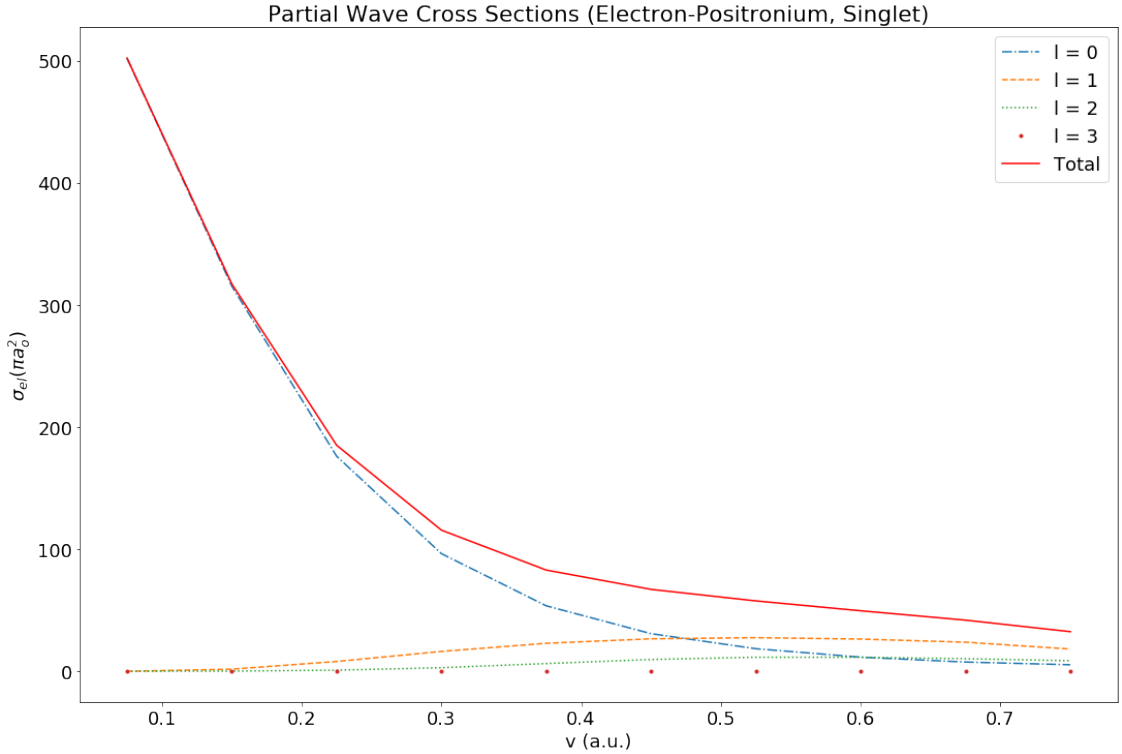
3 Electron-Positronium

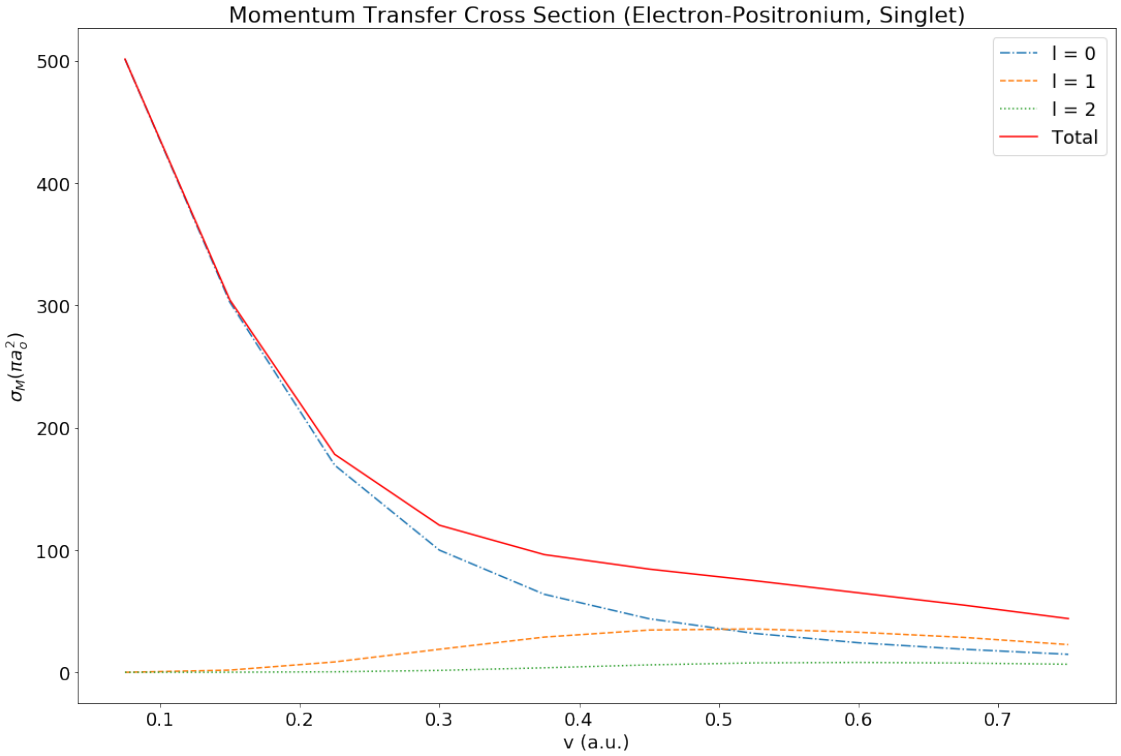
3.0.1 Ward et al 1987

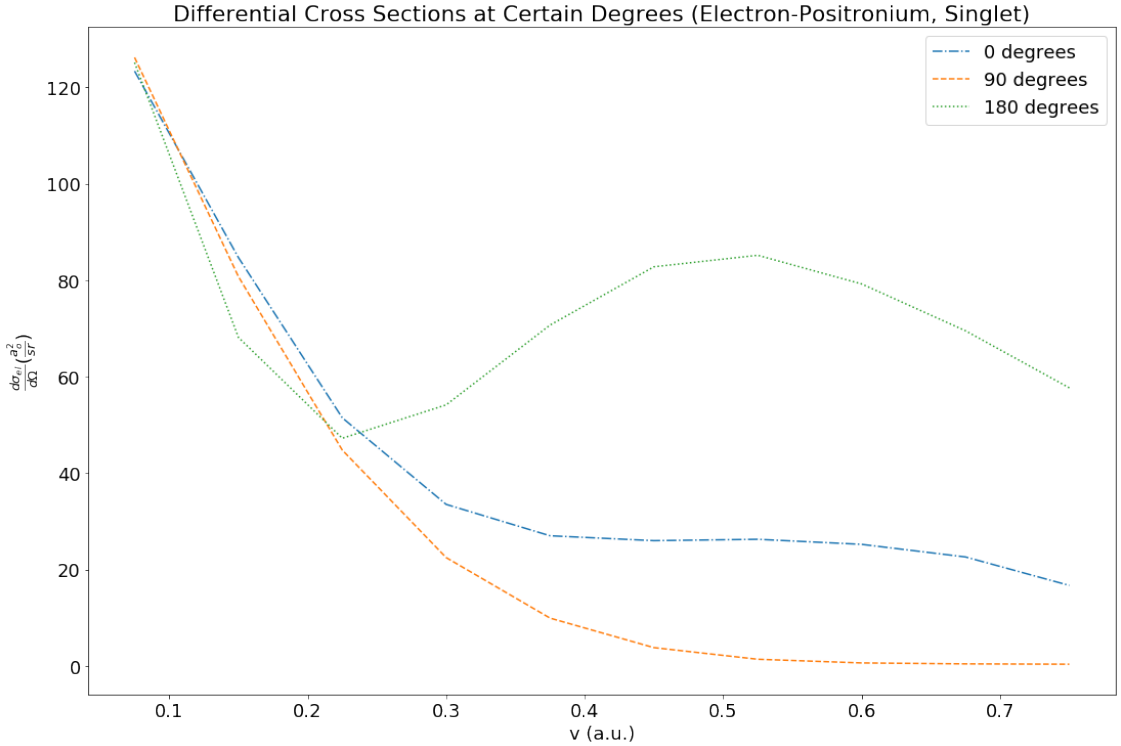
Phase shifts from Ward et al 1987 [9]

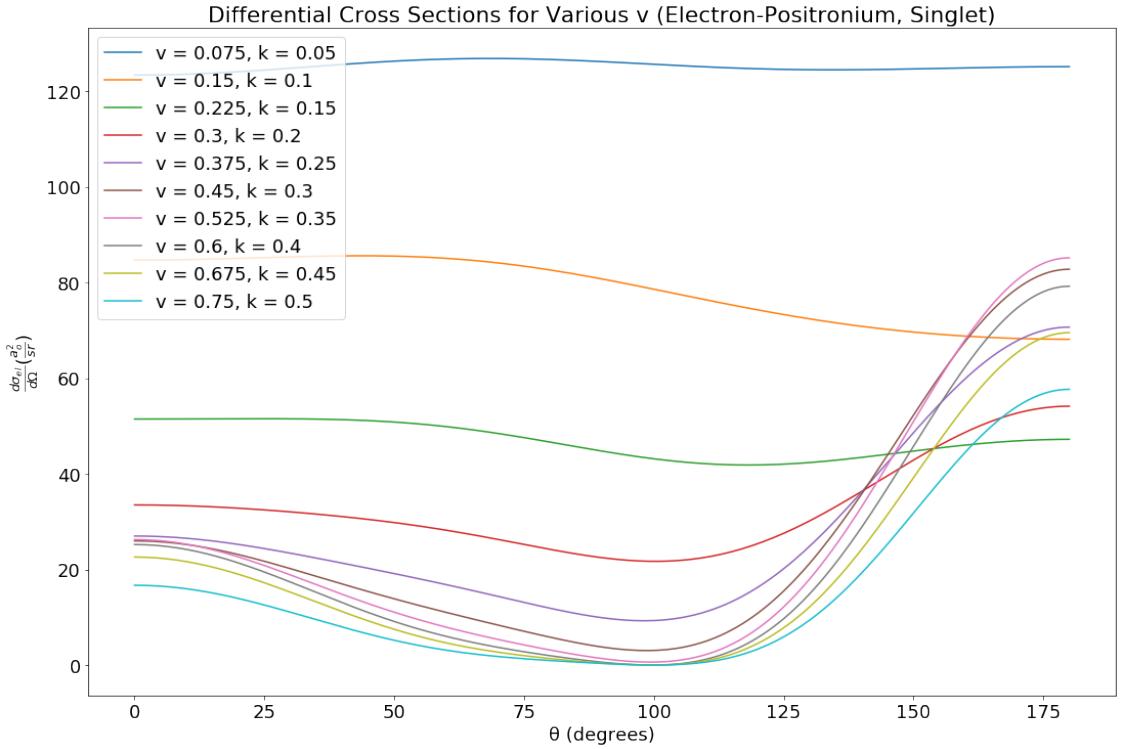
[79]:
```
e_ps_system_w = collision(e_ps_phases_w,k_e_ps_w, 'Electron-Positronium,␣
↪Singlet',(2/3))



e_ps_system_w.plot_phase_shifts(velocity=True)
e_ps_system_w.plot_cross_section(velocity=True)
e_ps_system_w.plot_momentum_transfer(velocity=True)
e_ps_system_w.plot_diff_cross_vs_k(r_interval,velocity=True)
e_ps_system_w.diff_plot(velocity=True)
e_ps_system_w.diff_plot(sin=True,velocity=True)
e_ps_system_w.diff_plot_3d(velocity=True)
e_ps_system_w.diff_plot_3d(density=True,velocity=True)
```

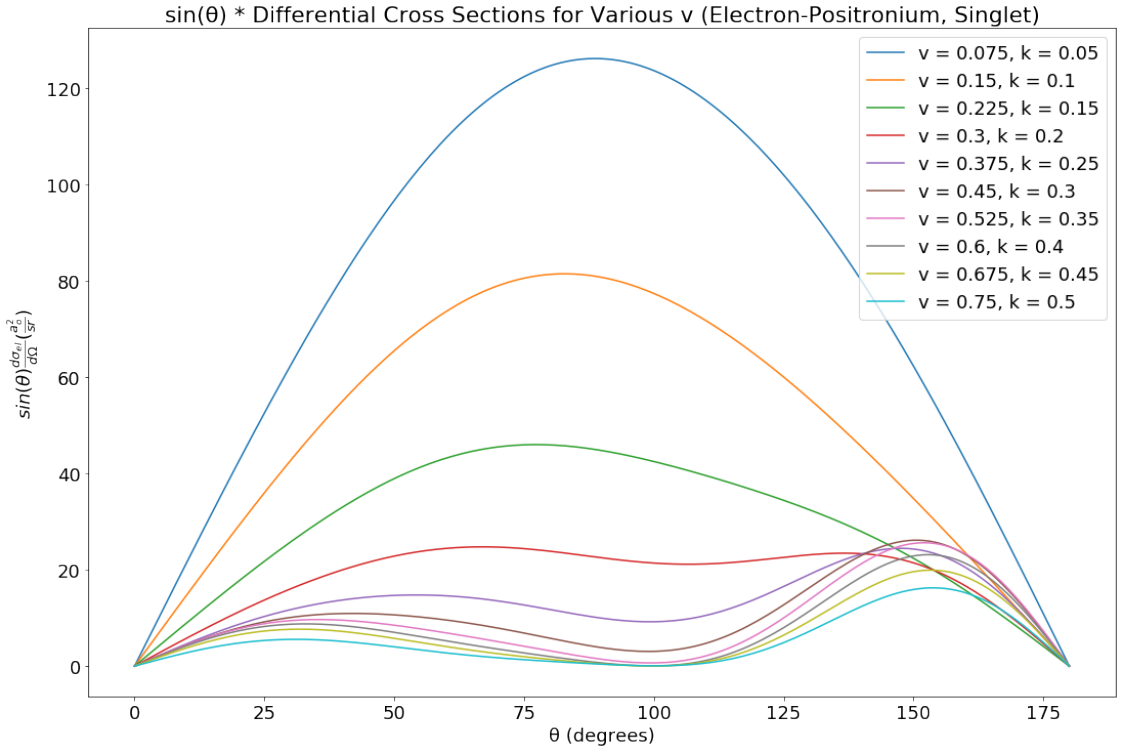

Phase Shifts vs v (Electron-Positronium, Singlet)
l = 0
l = 1
l = 2
l = 3
δ_l (radians)
v (a.u.)

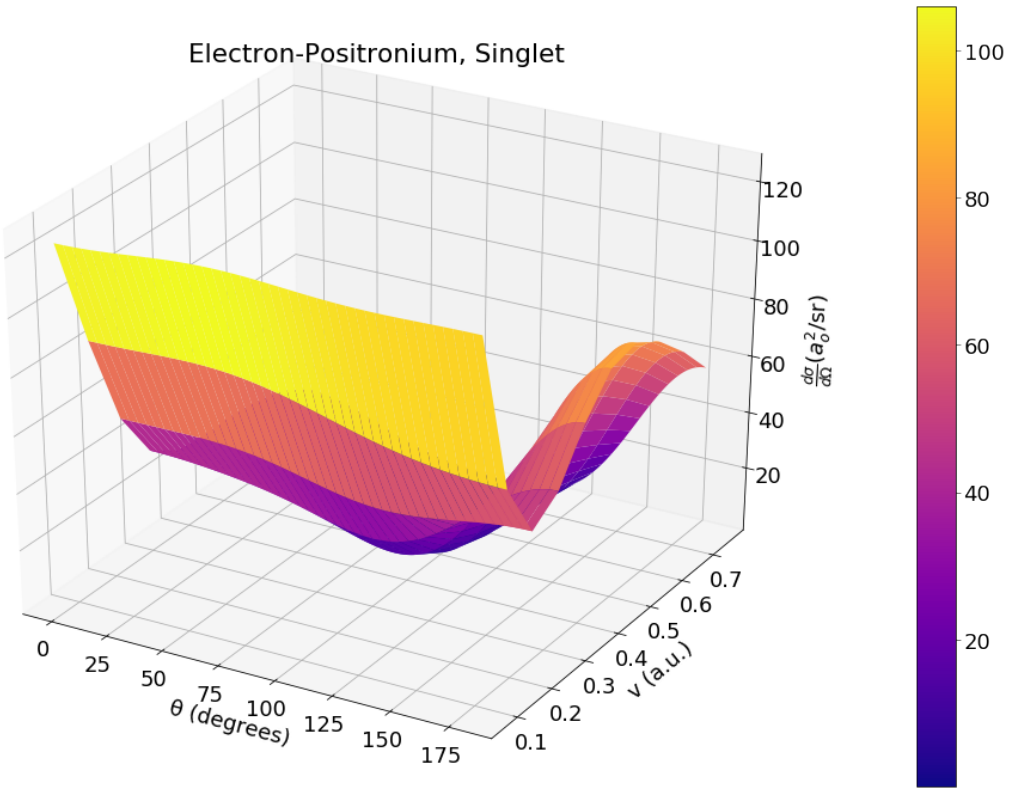

Partial Wave Cross Sections (Electron-Positronium, Singlet)
l = 0
l = 1
l = 2
l = 3
Total
$\sigma_{el}(\pi a_0^2)$
v (a.u.)

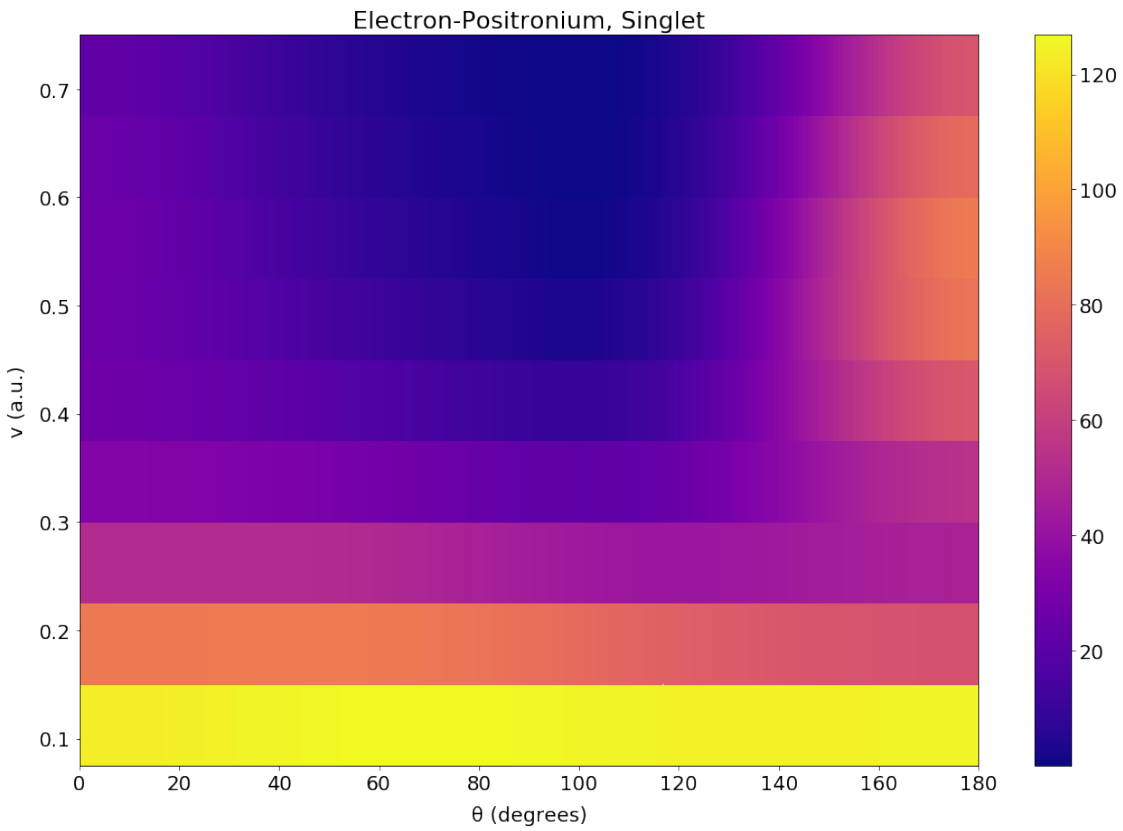

Momentum Transfer Cross Section (Electron-Positronium, Singlet)
l = 0
l = 1
l = 2
Total
$\sigma_M(\pi a_o^2)$
v (a.u.)


Differential Cross Sections at Certain Degrees (Electron-Positronium, Singlet)
0 degrees
90 degrees
180 degrees
$\frac{d\sigma_{el}}{d\Omega}(\frac{a_o^2}{Sr})$
v (a.u.)

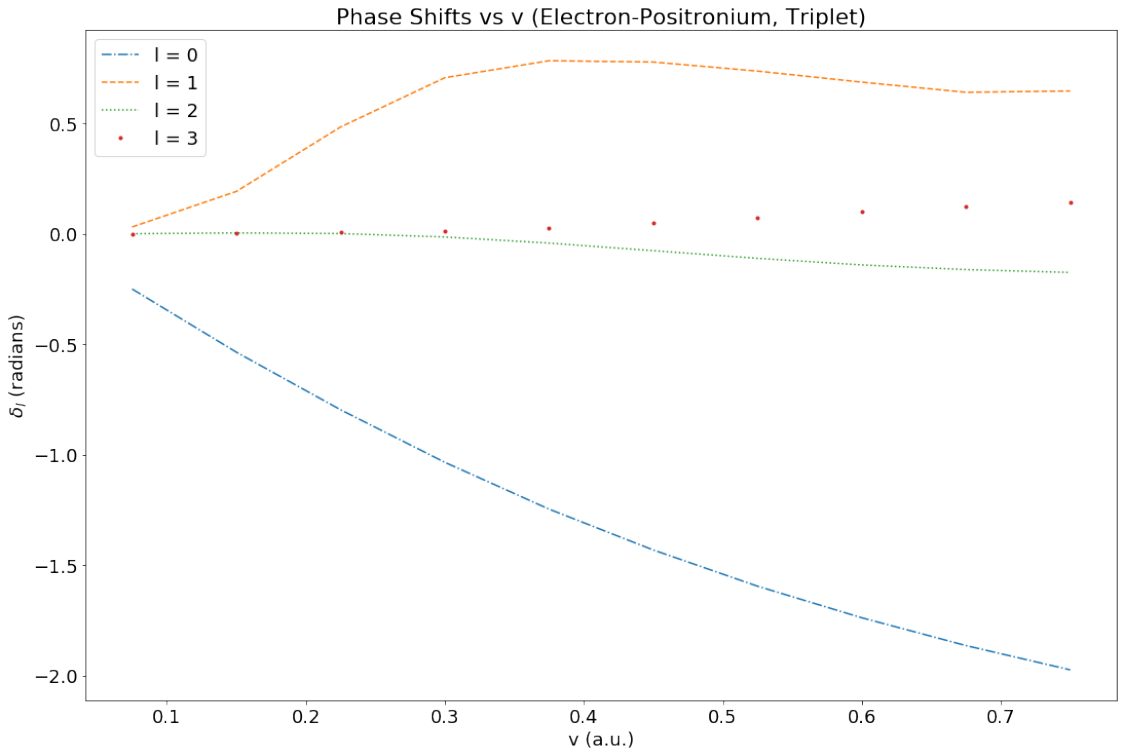

Differential Cross Sections for Various v (Electron-Positronium, Singlet)
v = 0.075, k = 0.05
v = 0.15, k = 0.1
v = 0.225, k = 0.15
v = 0.3, k = 0.2
v = 0.375, k = 0.25
v = 0.45, k = 0.3
v = 0.525, k = 0.35
v = 0.6, k = 0.4
v = 0.675, k = 0.45
v = 0.75, k = 0.5
0
20
40
60
80
100
120
0
25
50
75
100
125
150
175
θ (degrees)

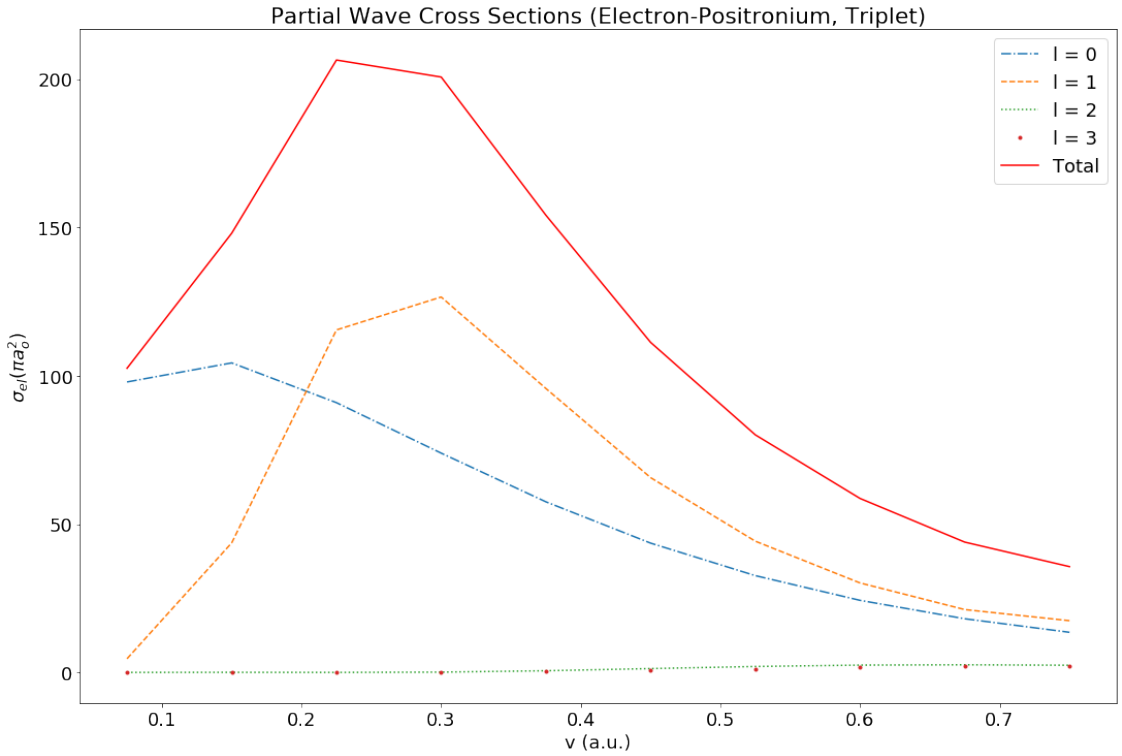

sin(θ) * Differential Cross Sections for Various v (Electron-Positronium, Singlet)
v = 0.075, k = 0.05
v = 0.15, k = 0.1
v = 0.225, k = 0.15
v = 0.3, k = 0.2
v = 0.375, k = 0.25
v = 0.45, k = 0.3
v = 0.525, k = 0.35
v = 0.6, k = 0.4
v = 0.675, k = 0.45
v = 0.75, k = 0.5
0
20
40
60
80
100
120
0
25
50
75
100
125
150
175
θ (degrees)

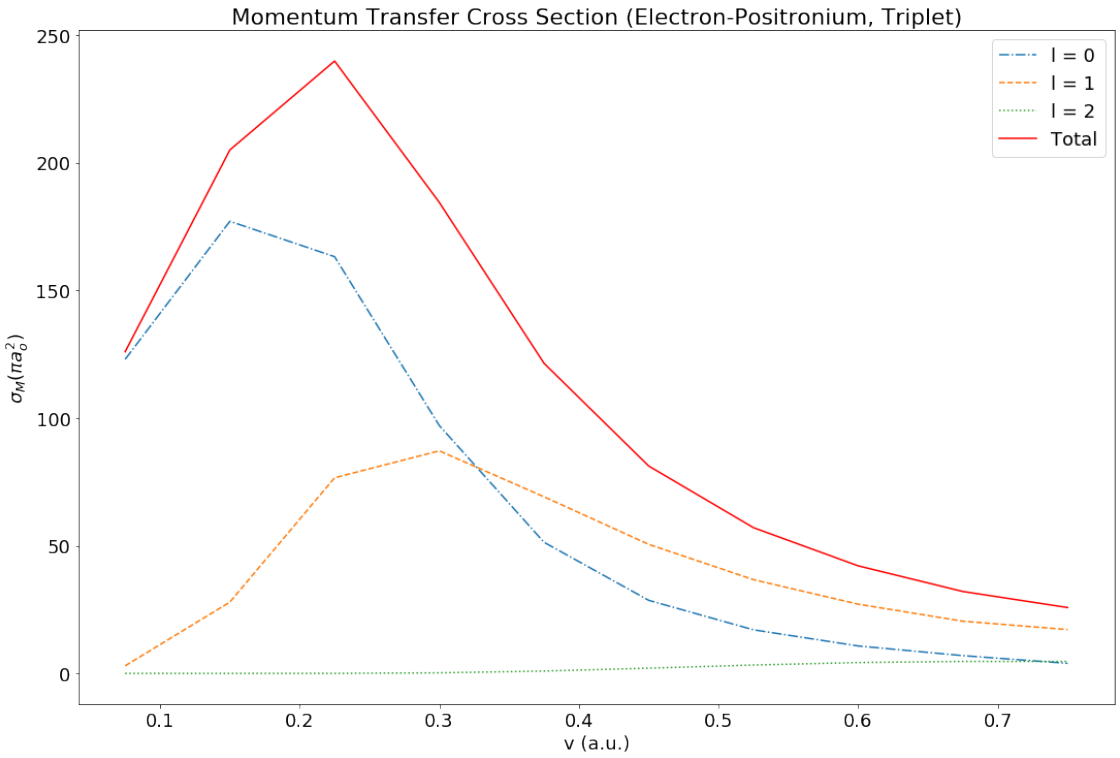

Electron-Positronium, Singlet
120
100
80
60
40
20
$\frac{d\sigma}{d\Omega}(a_0^2/sr)$
0
25
50
75
100
125
150
175
θ (degrees)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v (a.u.)
100
80
60
40
20

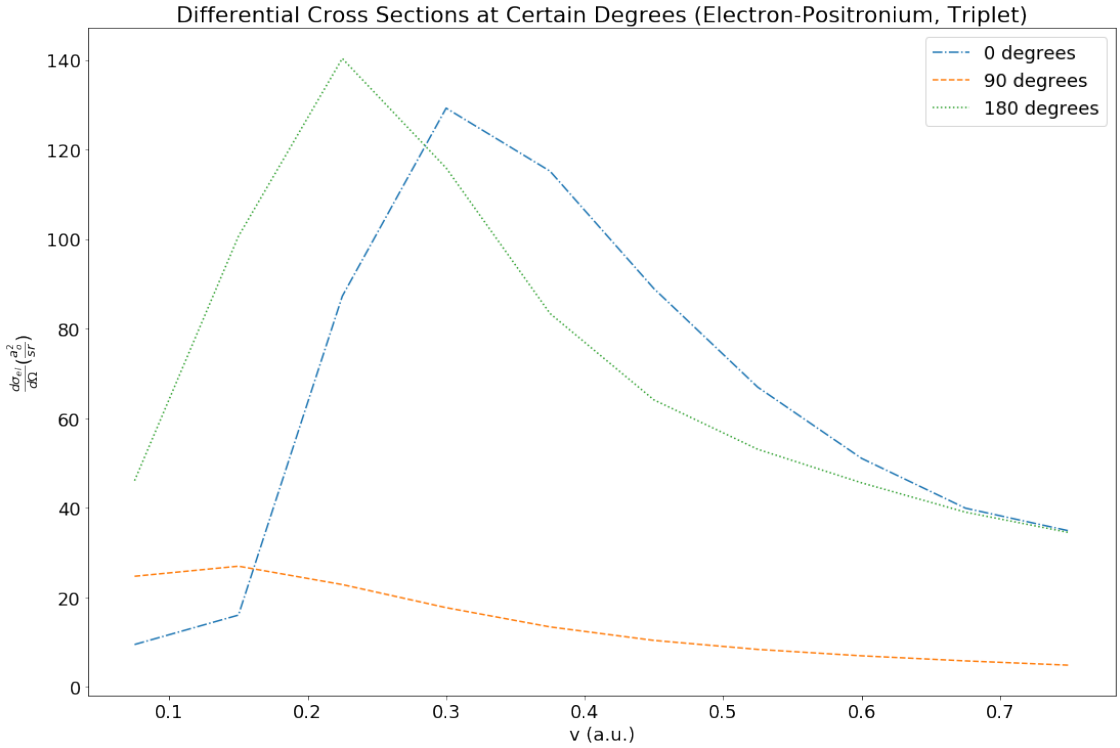


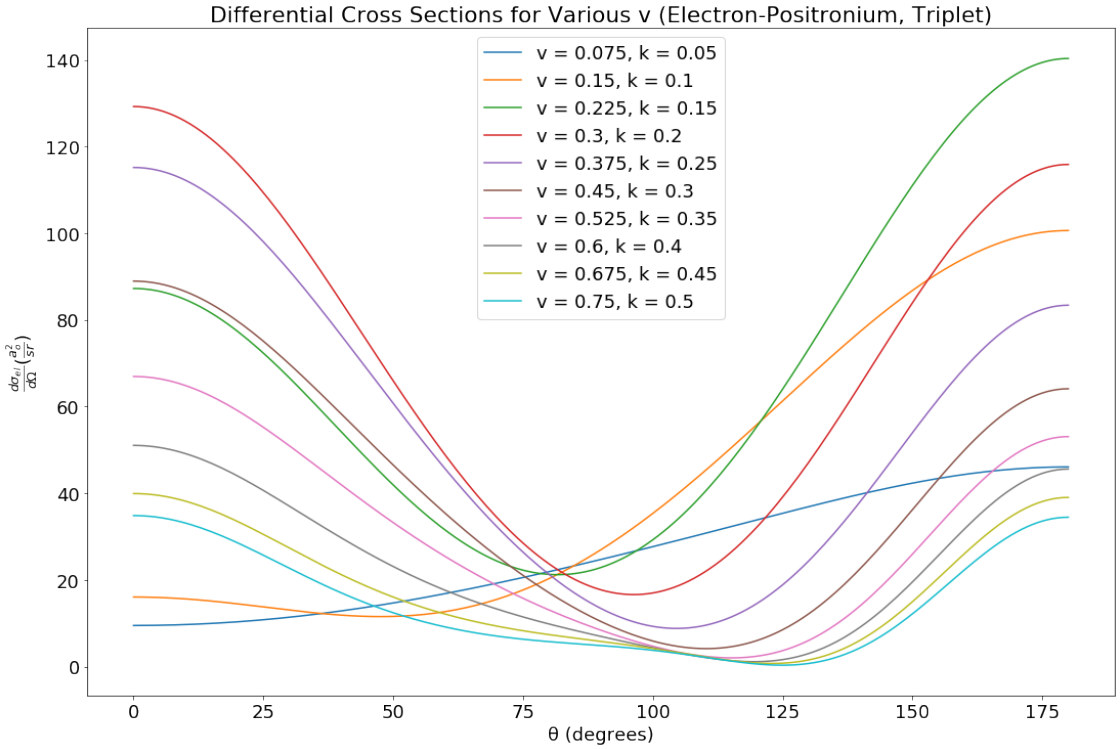
[80]:
```
e_ps_triplet_system_w = collision(e_ps_phases_trip_w,k_e_ps_w,␣
↪'Electron-Positronium, Triplet',(2/3))


e_ps_triplet_system_w.plot_phase_shifts(velocity=True)
e_ps_triplet_system_w.plot_cross_section(velocity=True)
e_ps_triplet_system_w.plot_momentum_transfer(velocity=True)
e_ps_triplet_system_w.plot_diff_cross_vs_k(r_interval,velocity=True)
e_ps_triplet_system_w.diff_plot(velocity=True)
e_ps_triplet_system_w.diff_plot(sin=True, velocity=True)
e_ps_triplet_system_w.diff_plot_3d(velocity=True)
e_ps_triplet_system_w.diff_plot_3d(density=True, velocity=True)
```

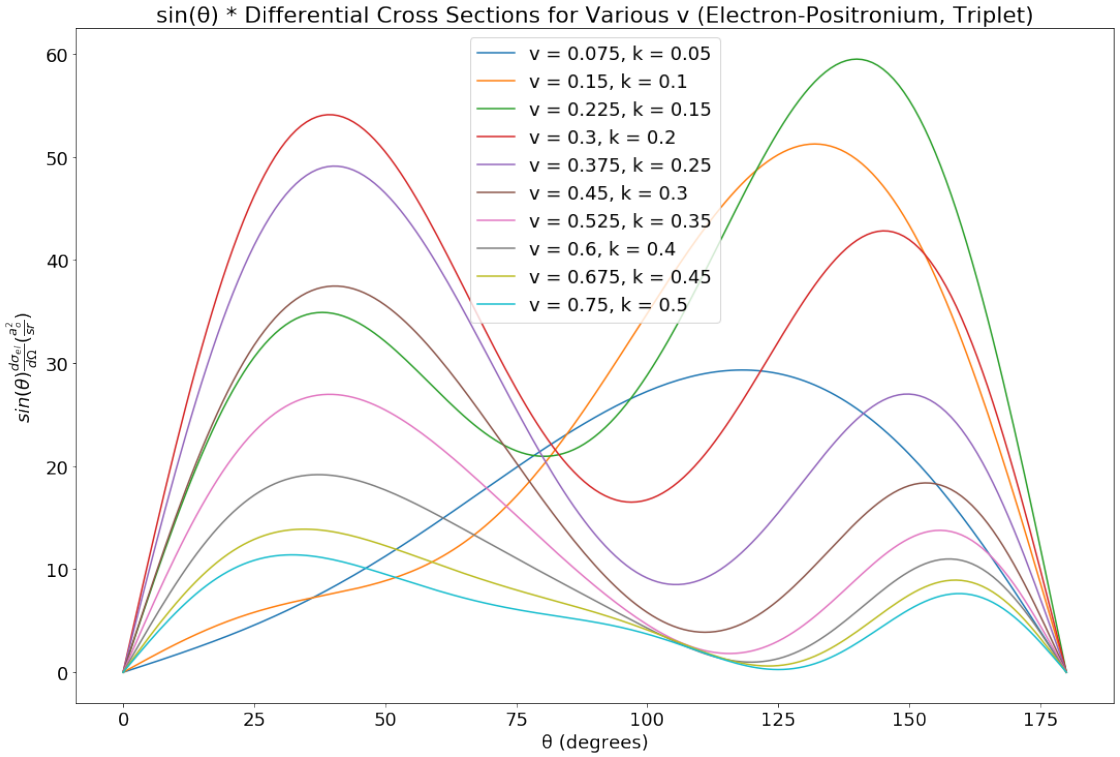

Phase Shifts vs v (Electron-Positronium, Triplet)
l = 0
l = 1
l = 2
l = 3
0.5
0.0
−0.5
−1.0
−1.5
−2.0
δ_l (radians)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v (a.u.)

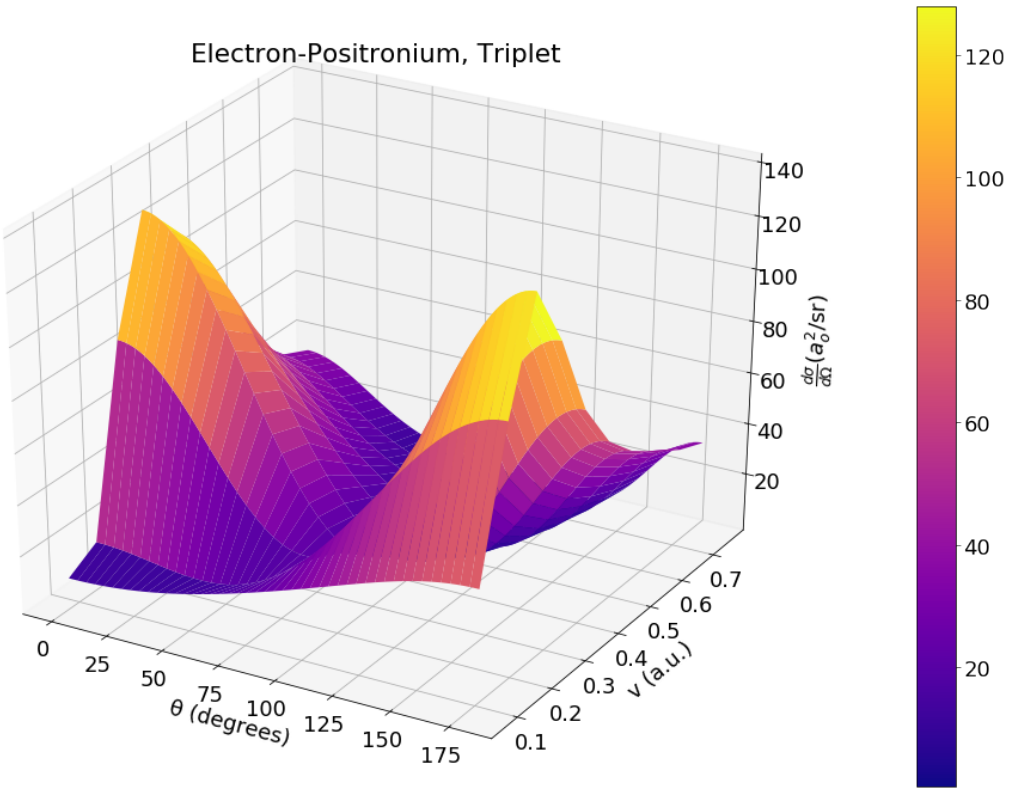

Partial Wave Cross Sections (Electron-Positronium, Triplet)
l = 0
l = 1
l = 2
l = 3
Total
200
150
100
50
0
$\sigma_{el}(\pi a_0^2)$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v (a.u.)

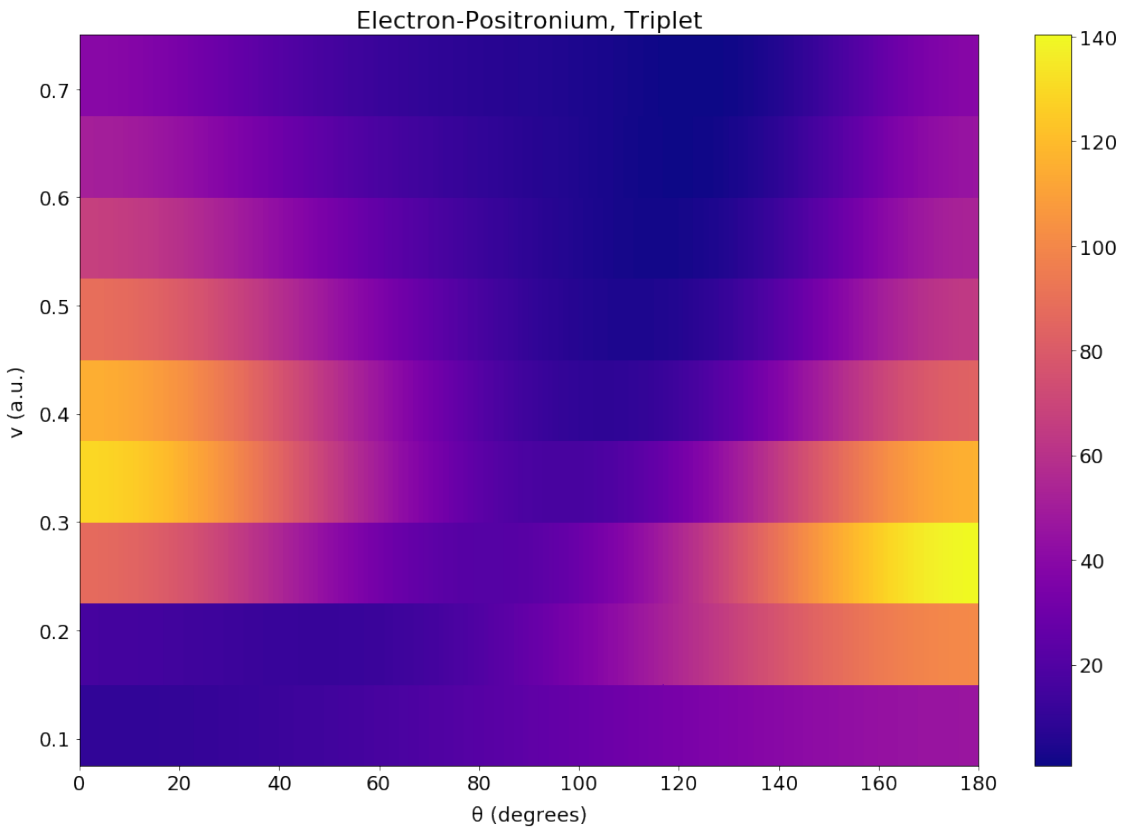

Momentum Transfer Cross Section (Electron-Positronium, Triplet)
l = 0
l = 1
l = 2
Total
250
200
150
100
50
0
$\sigma_M(\pi a_o^2)$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v (a.u.)


Differential Cross Sections at Certain Degrees (Electron-Positronium, Triplet)
0 degrees
90 degrees
180 degrees
140
120
100
80
60
40
20
0
$\frac{d\sigma_{el}}{d\Omega}(\frac{a_o^2}{sr})$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v (a.u.)

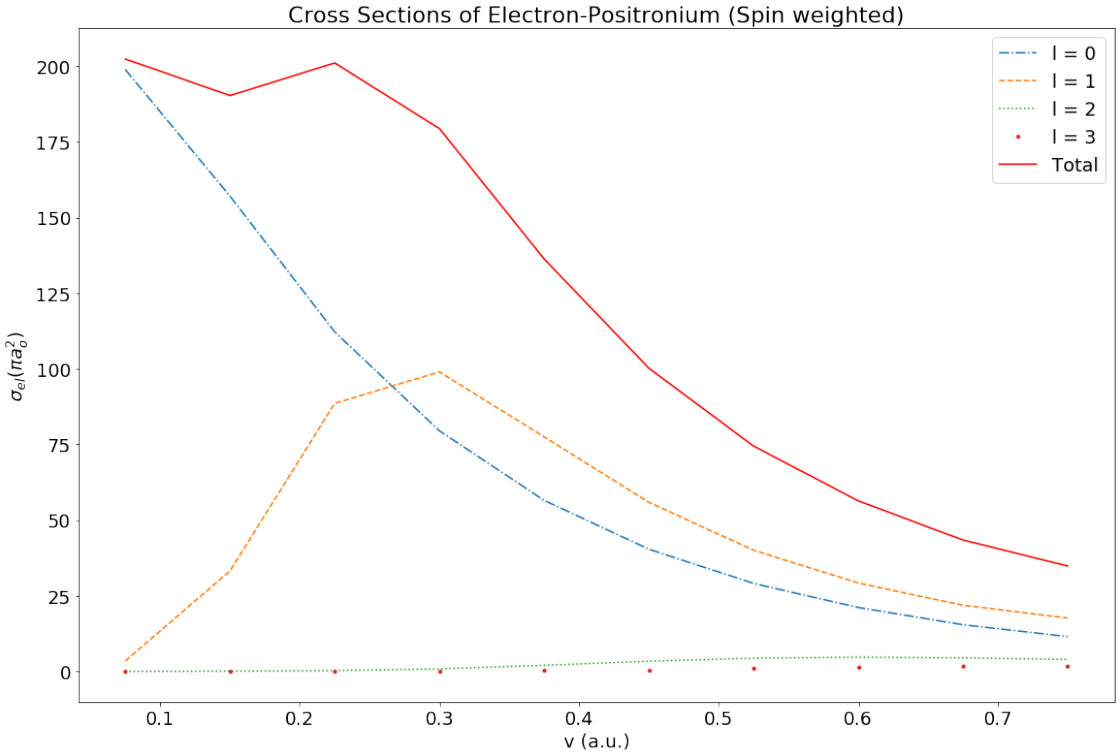

Differential Cross Sections for Various v (Electron-Positronium, Triplet)
v = 0.075, k = 0.05
v = 0.15, k = 0.1
v = 0.225, k = 0.15
v = 0.3, k = 0.2
v = 0.375, k = 0.25
v = 0.45, k = 0.3
v = 0.525, k = 0.35
v = 0.6, k = 0.4
v = 0.675, k = 0.45
v = 0.75, k = 0.5
θ (degrees)

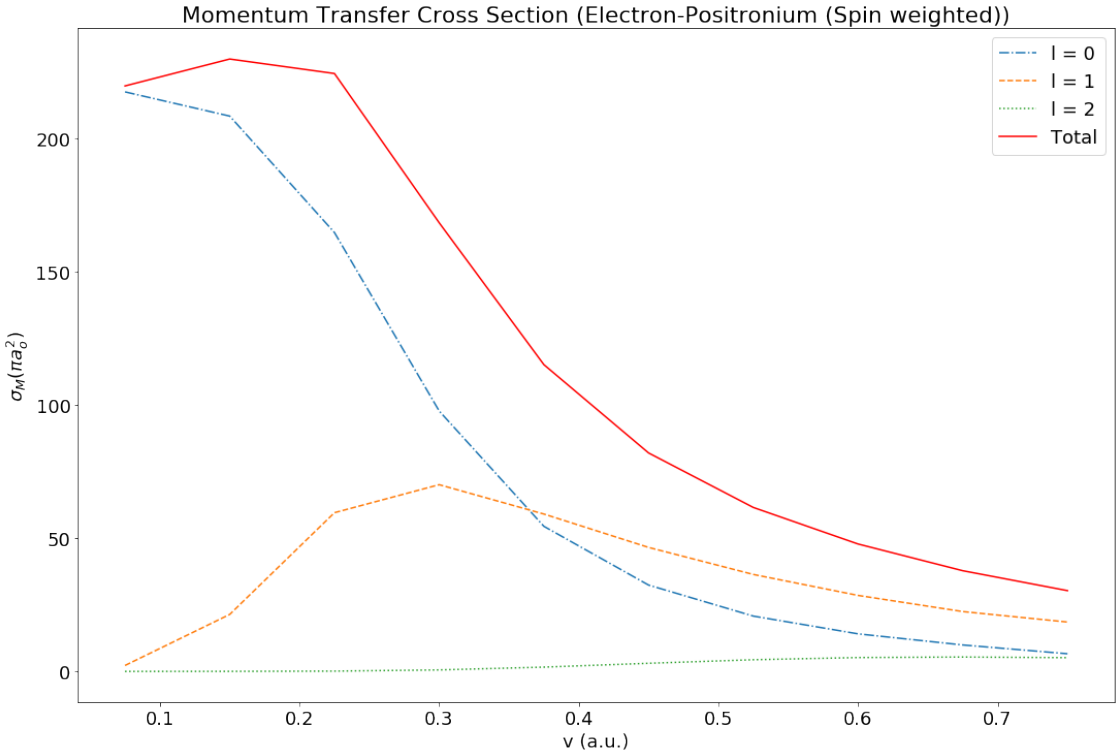

sin(θ) * Differential Cross Sections for Various v (Electron-Positronium, Triplet)
v = 0.075, k = 0.05
v = 0.15, k = 0.1
v = 0.225, k = 0.15
v = 0.3, k = 0.2
v = 0.375, k = 0.25
v = 0.45, k = 0.3
v = 0.525, k = 0.35
v = 0.6, k = 0.4
v = 0.675, k = 0.45
v = 0.75, k = 0.5
θ (degrees)

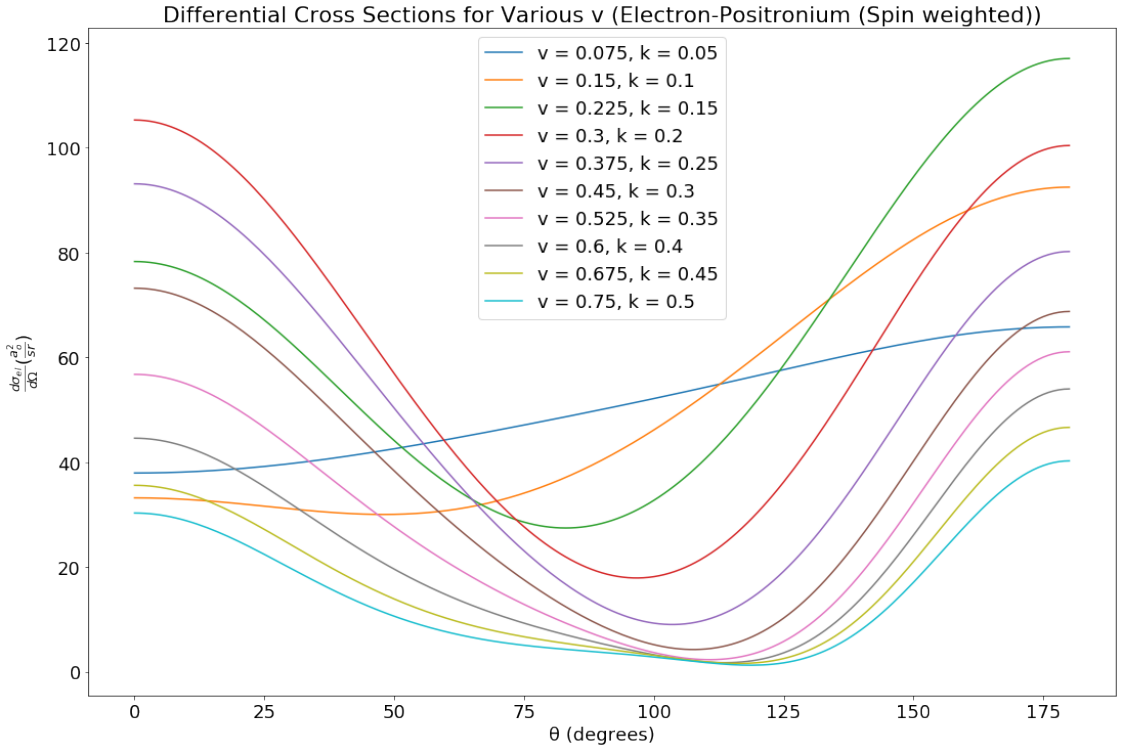

Electron-Positronium, Triplet
140
120
100
80
60
40
20
$\frac{d\sigma}{d\Omega}(a_0^2/sr)$
0
25
50
75
100
125
150
175
θ (degrees)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v (a.u.)
120
100
80
60
40
20

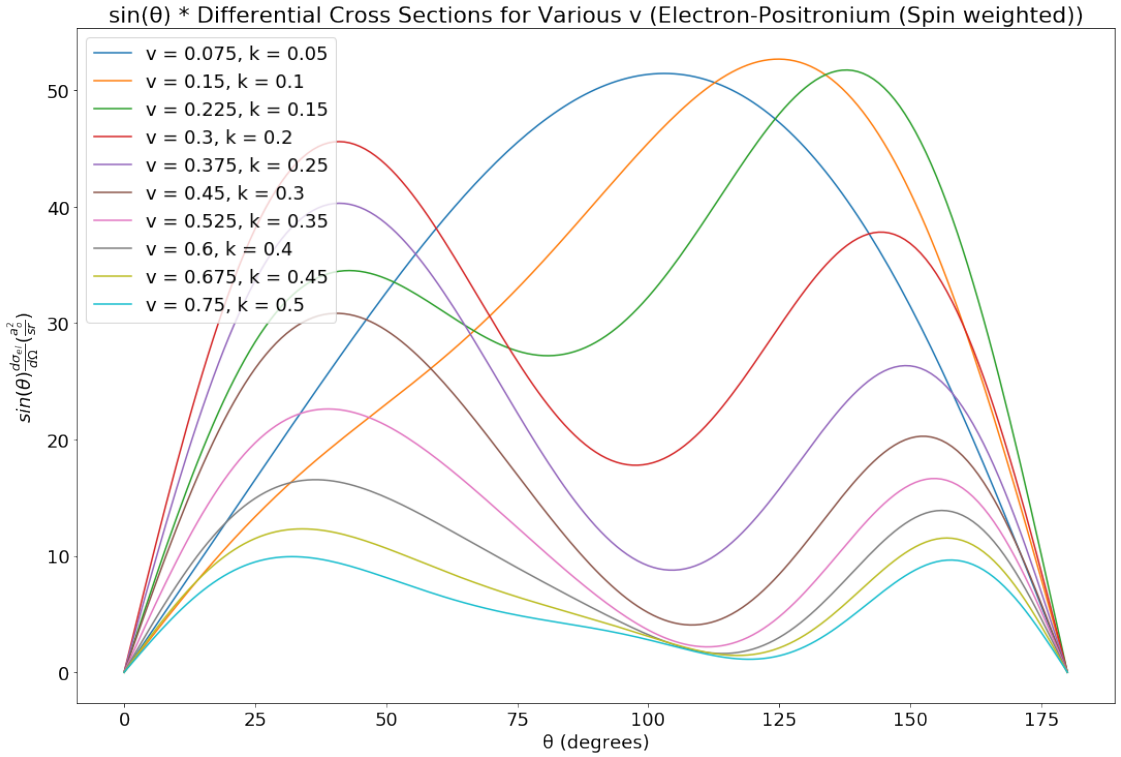


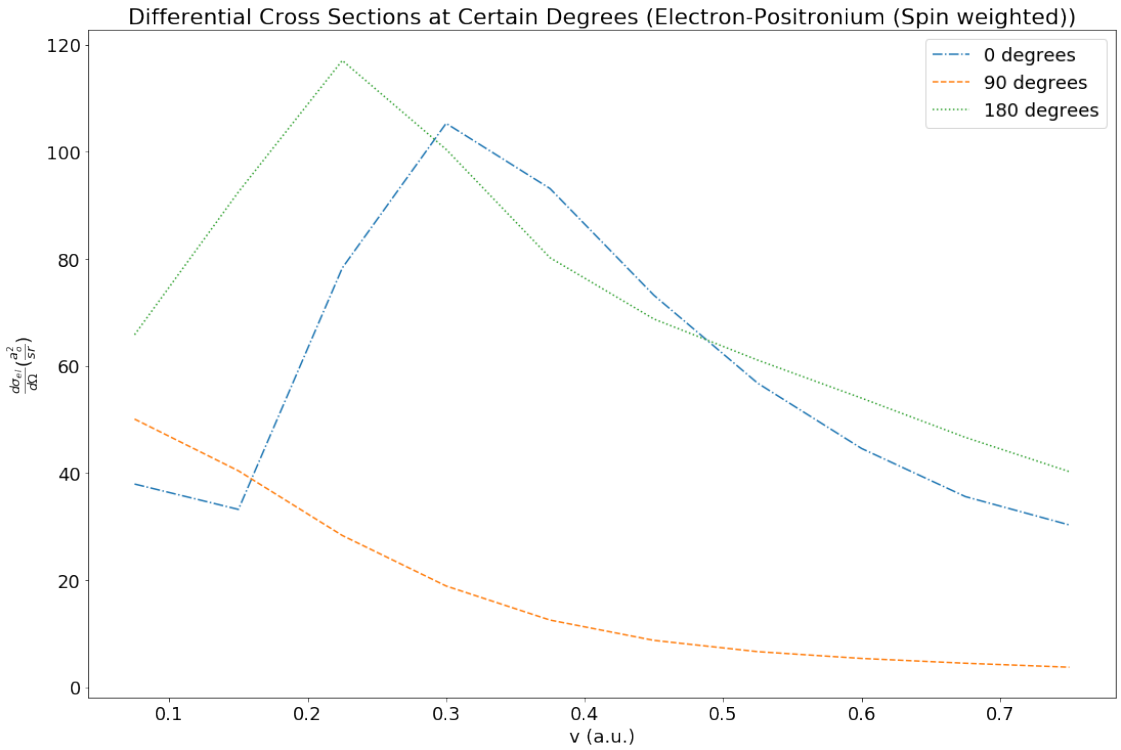
[81]:
```
e_ps_weighted_w = collision(e_ps_phases_w,k_e_ps_w,'Electron-Positronium (Spin␣
↪weighted)',(2/3),e_ps_phases_trip_w,k_e_ps_w,(2/3))


e_ps_weighted_w.plot_cross_section(spin_weight=True, velocity=True)
e_ps_weighted_w.plot_momentum_transfer(spin_weight=True, velocity=True)
e_ps_weighted_w.diff_plot(spin_weight=True, velocity=True)
e_ps_weighted_w.diff_plot(spin_weight=True, sin=True, velocity=True)
e_ps_weighted_w.plot_diff_cross_vs_k(r_interval,spin_weight=True, velocity=True)
e_ps_weighted_w.diff_plot_3d(spin_weight=True, velocity=True)
e_ps_weighted_w.diff_plot_3d(spin_weight=True, density=True, velocity=True)
```

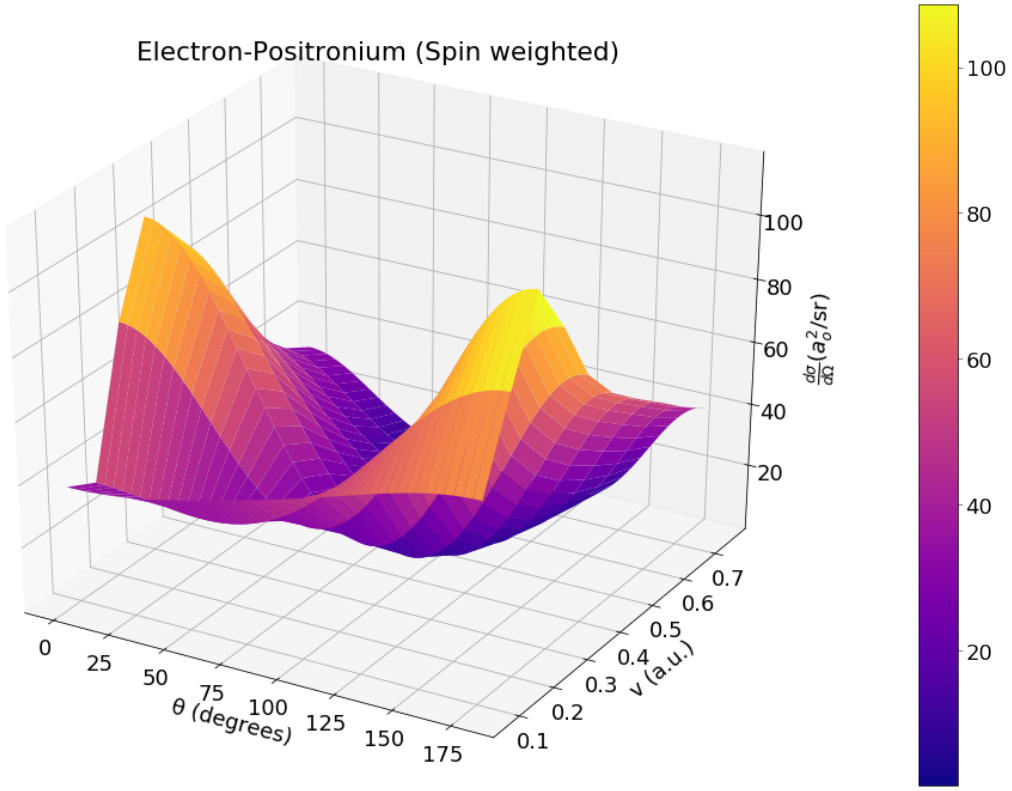

Cross Sections of Electron-Positronium (Spin weighted)
l = 0
l = 1
l = 2
l = 3
Total
$\sigma_{el}(\pi a_o^2)$
v (a.u.)

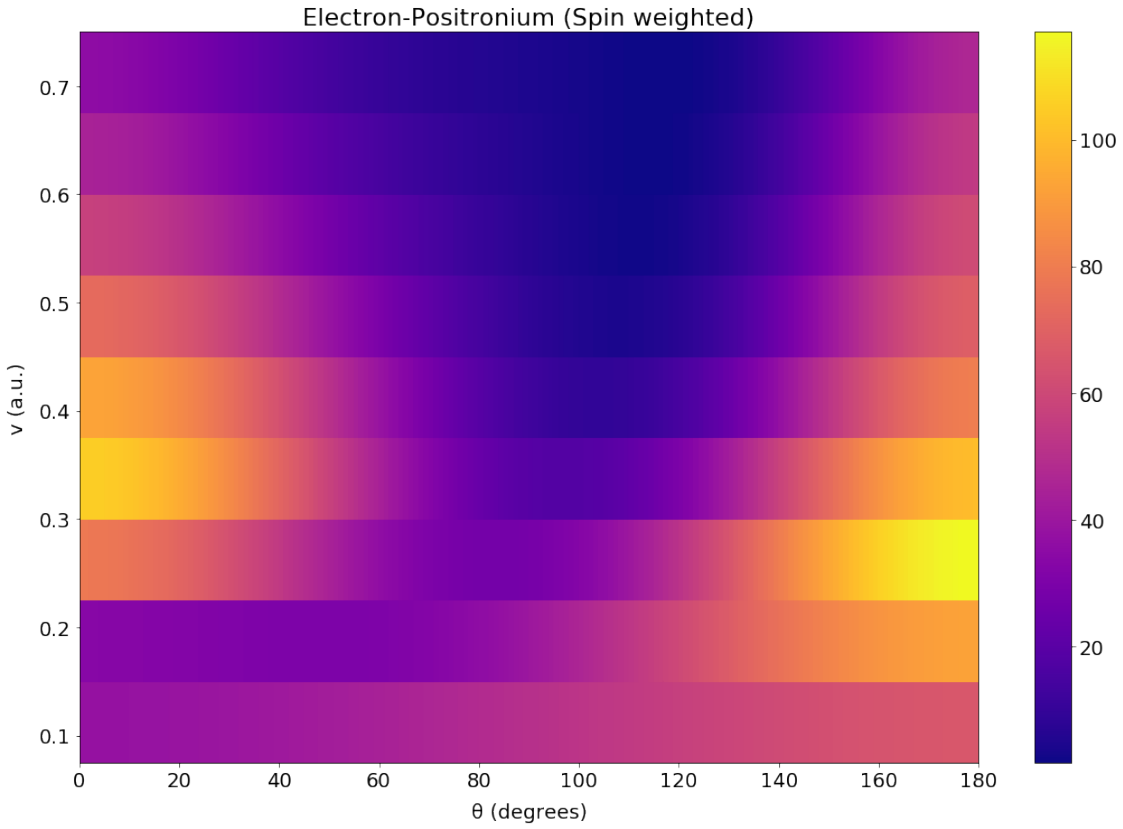

Momentum Transfer Cross Section (Electron-Positronium (Spin weighted))
l = 0
l = 1
l = 2
Total
$\sigma_M(\pi a_o^2)$
v (a.u.)


Differential Cross Sections for Various v (Electron-Positronium (Spin weighted))
v = 0.075, k = 0.05
v = 0.15, k = 0.1
v = 0.225, k = 0.15
v = 0.3, k = 0.2
v = 0.375, k = 0.25
v = 0.45, k = 0.3
v = 0.525, k = 0.35
v = 0.6, k = 0.4
v = 0.675, k = 0.45
v = 0.75, k = 0.5
θ (degrees)


sin(θ) * Differential Cross Sections for Various v (Electron-Positronium (Spin weighted))
v = 0.075, k = 0.05
v = 0.15, k = 0.1
v = 0.225, k = 0.15
v = 0.3, k = 0.2
v = 0.375, k = 0.25
v = 0.45, k = 0.3
v = 0.525, k = 0.35
v = 0.6, k = 0.4
v = 0.675, k = 0.45
v = 0.75, k = 0.5
θ (degrees)


Differential Cross Sections at Certain Degrees (Electron-Positronium (Spin weighted))
0 degrees
90 degrees
180 degrees
120
100
80
60
40
20
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v (a.u.)

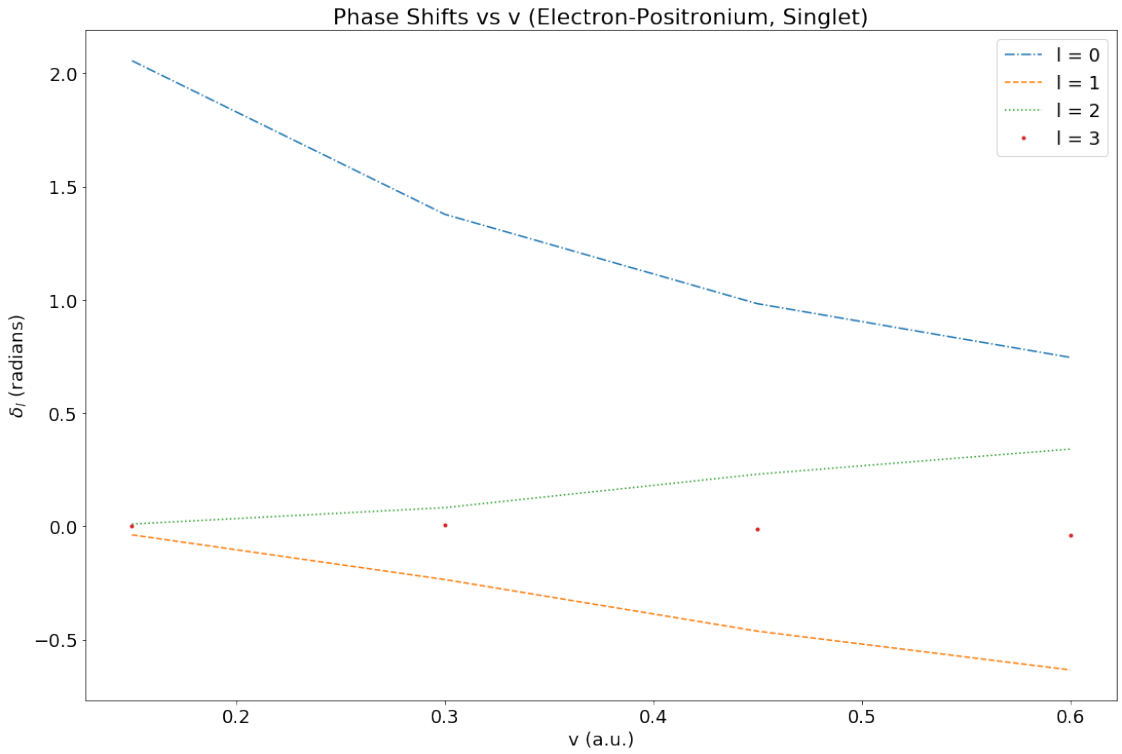

Electron-Positronium (Spin weighted)
100
80
60
40
20
0
25
50
75
100
125
150
175
θ (degrees)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v (a.u.)
100
80
60
40
20

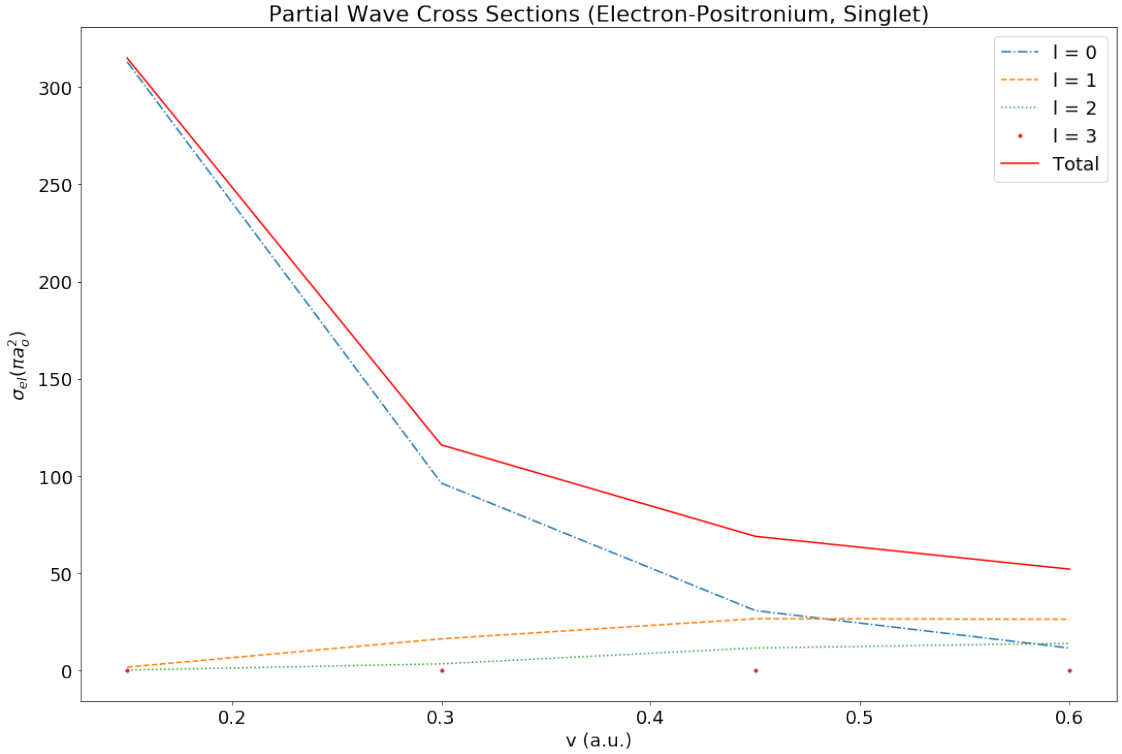


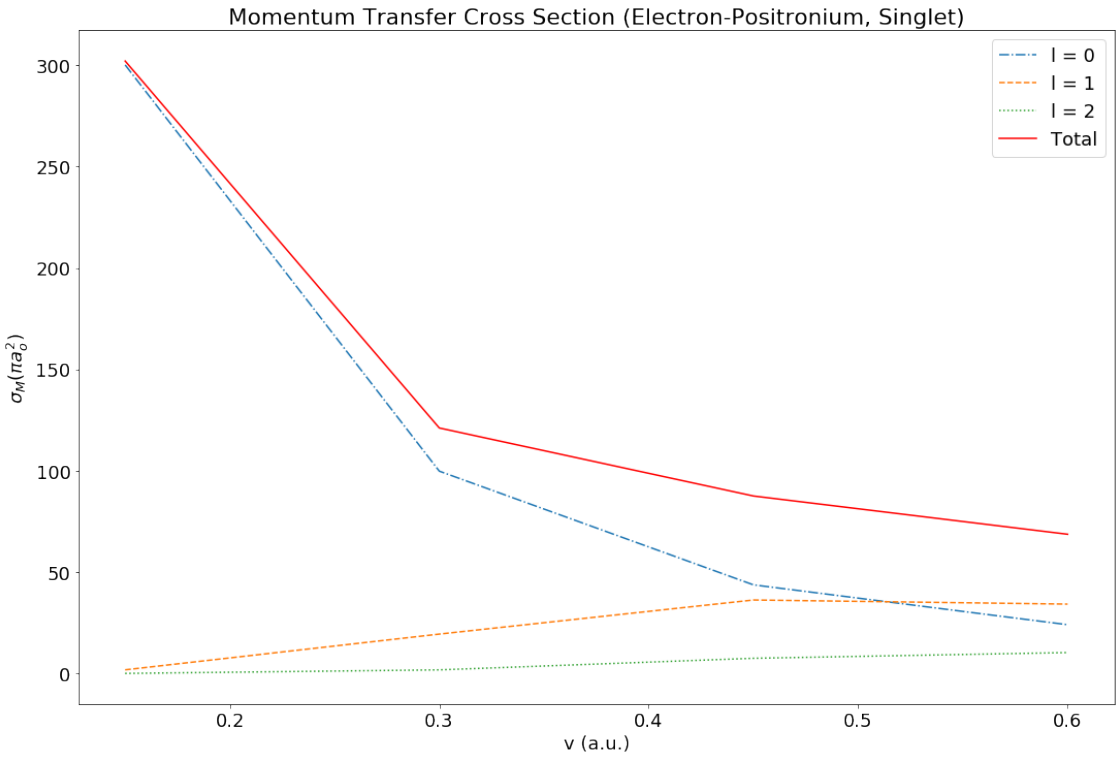
3.0.2 A Igarashi et al 2000

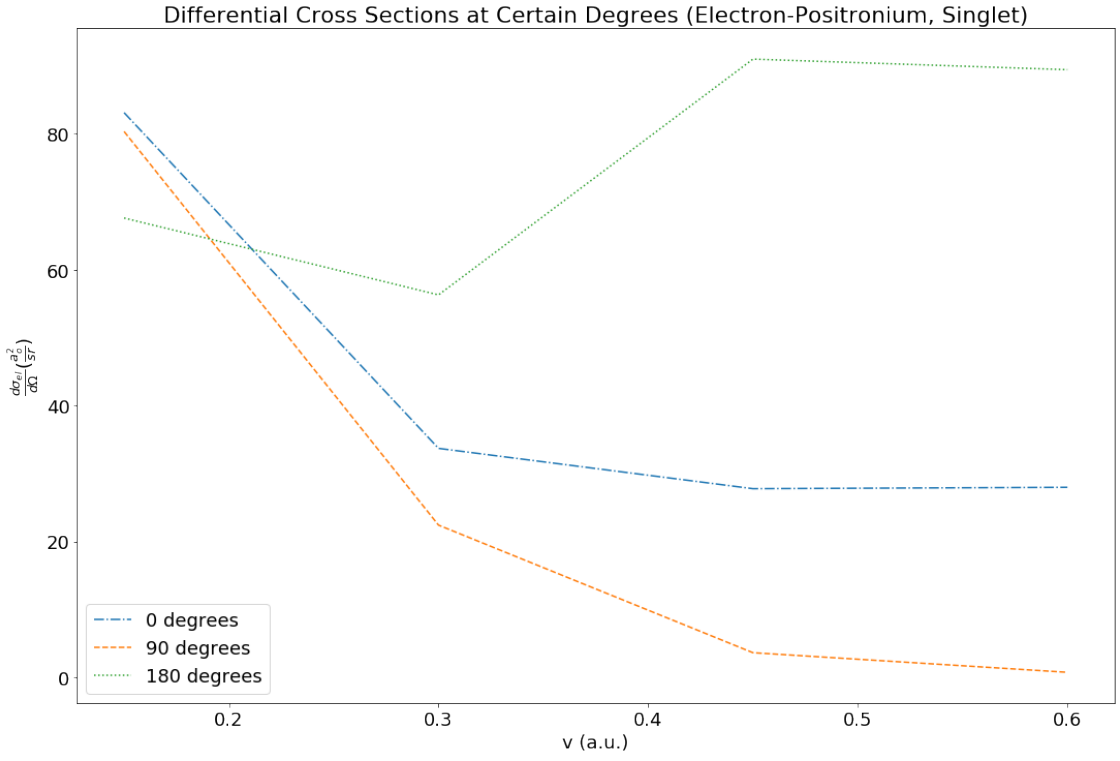
Phase shifts from A Igarashi et al 2000 [10]

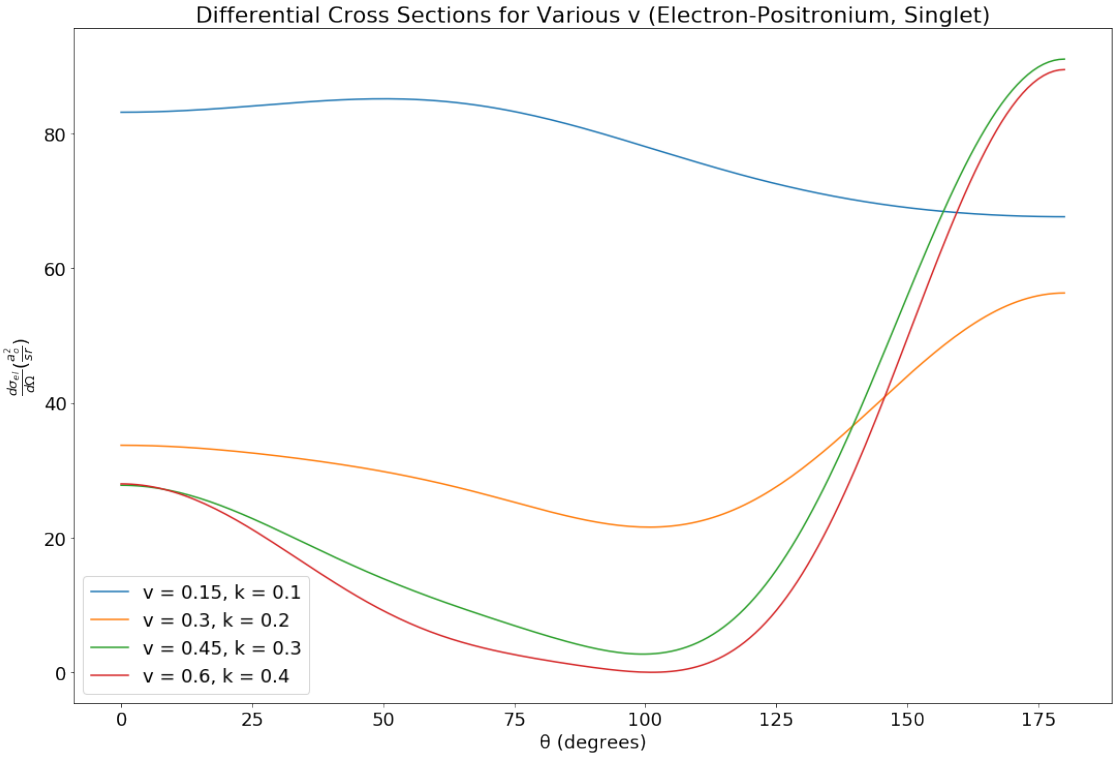
[82]:
```
e_ps_system = collision(e_ps_phases,k_e_ps, 'Electron-Positronium, Singlet',(2/
↪3))



e_ps_system.plot_phase_shifts(velocity=True)
e_ps_system.plot_cross_section(velocity=True)
e_ps_system.plot_momentum_transfer(velocity=True)
e_ps_system.plot_diff_cross_vs_k(r_interval,velocity=True)
e_ps_system.diff_plot(velocity=True)
e_ps_system.diff_plot(sin=True,velocity=True)
e_ps_system.diff_plot_3d(velocity=True)
e_ps_system.diff_plot_3d(density=True,velocity=True)
```

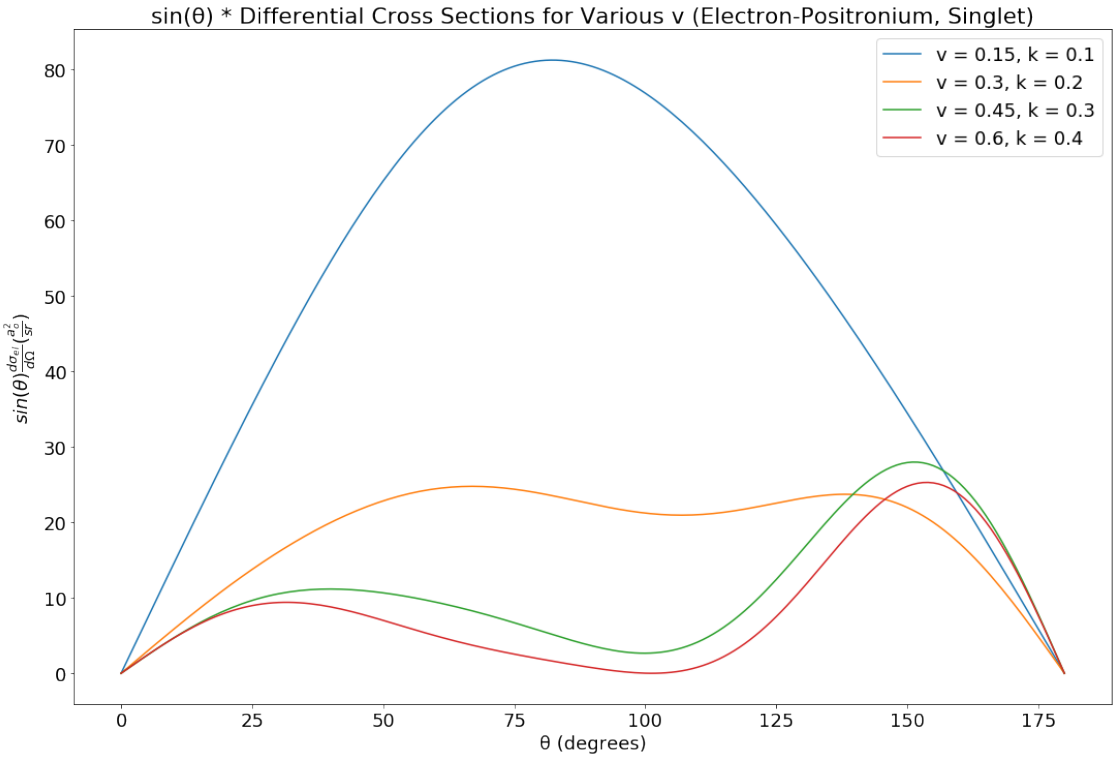

Phase Shifts vs v (Electron-Positronium, Singlet)
l = 0
l = 1
l = 2
l = 3
2.0
1.5
1.0
0.5
0.0
−0.5
δ_l (radians)
0.2
0.3
0.4
0.5
0.6
v (a.u.)

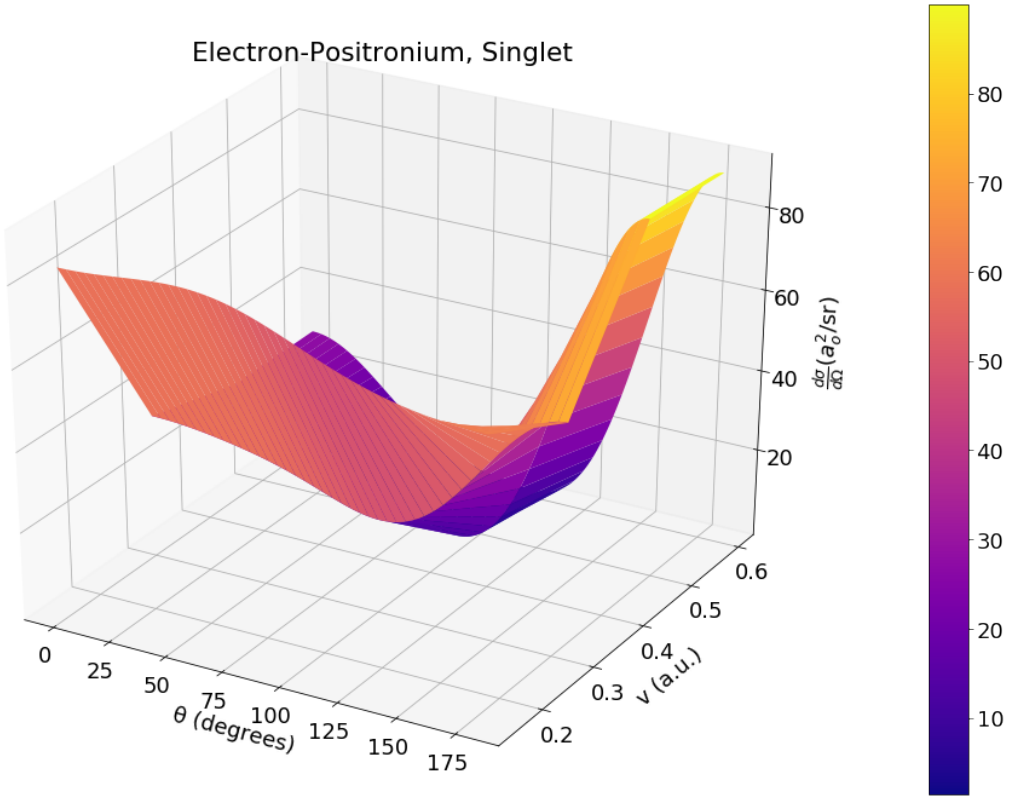

Partial Wave Cross Sections (Electron-Positronium, Singlet)
l = 0
l = 1
l = 2
l = 3
Total
300
250
200
150
100
50
0
$\sigma_{el}(\pi a_o^2)$
0.2
0.3
0.4
0.5
0.6
v (a.u.)

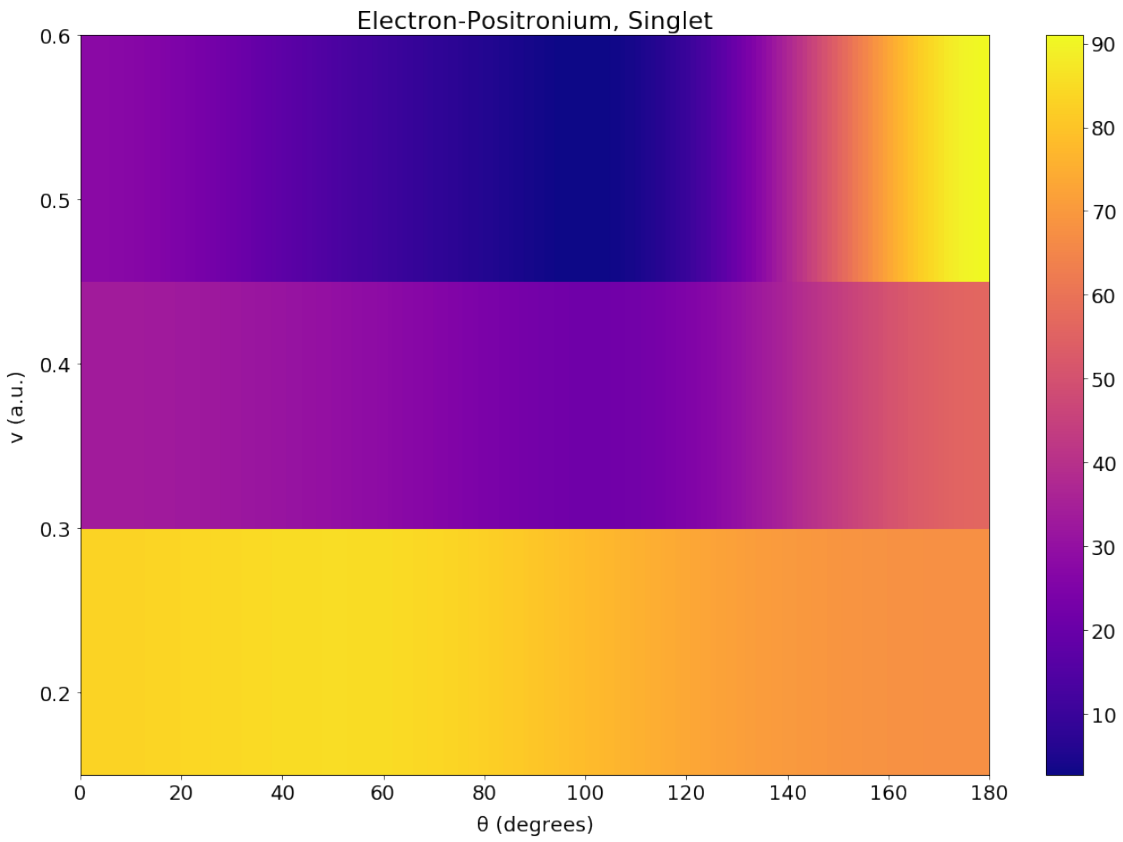

Momentum Transfer Cross Section (Electron-Positronium, Singlet)
l = 0
l = 1
l = 2
Total
$\sigma_M(\pi a_o^2)$
v (a.u.)


Differential Cross Sections at Certain Degrees (Electron-Positronium, Singlet)
0 degrees
90 degrees
180 degrees
v (a.u.)

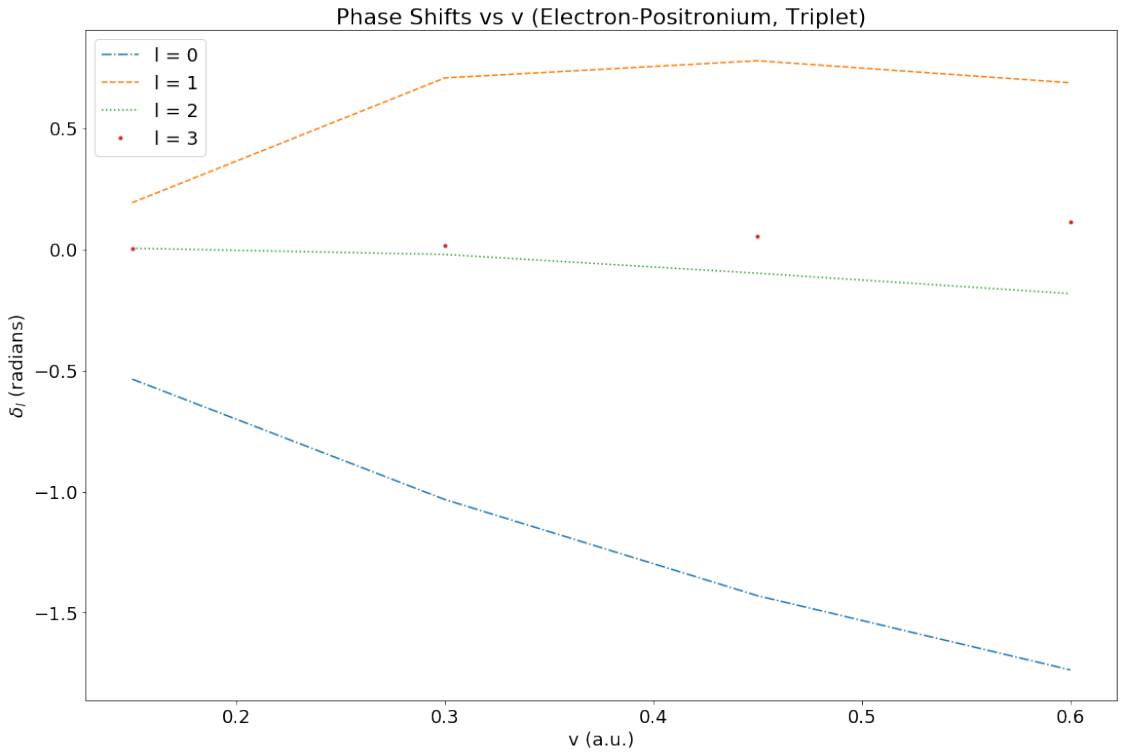

Differential Cross Sections for Various v (Electron-Positronium, Singlet)
$\frac{d\sigma_{el}}{d\Omega}(\frac{a_0^2}{sr})$
θ (degrees)
v = 0.15, k = 0.1
v = 0.3, k = 0.2
v = 0.45, k = 0.3
v = 0.6, k = 0.4

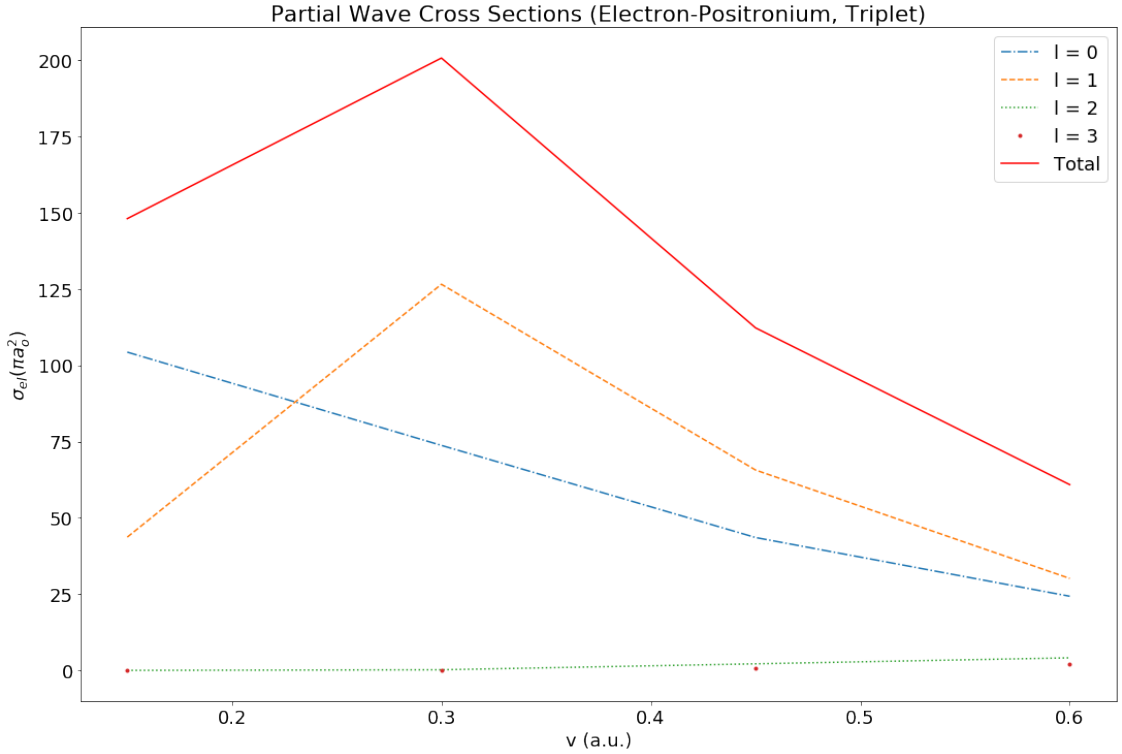

sin(θ) * Differential Cross Sections for Various v (Electron-Positronium, Singlet)
$sin(\theta)\frac{d\sigma_{el}}{d\Omega}(\frac{a_0^2}{sr})$
θ (degrees)
v = 0.15, k = 0.1
v = 0.3, k = 0.2
v = 0.45, k = 0.3
v = 0.6, k = 0.4

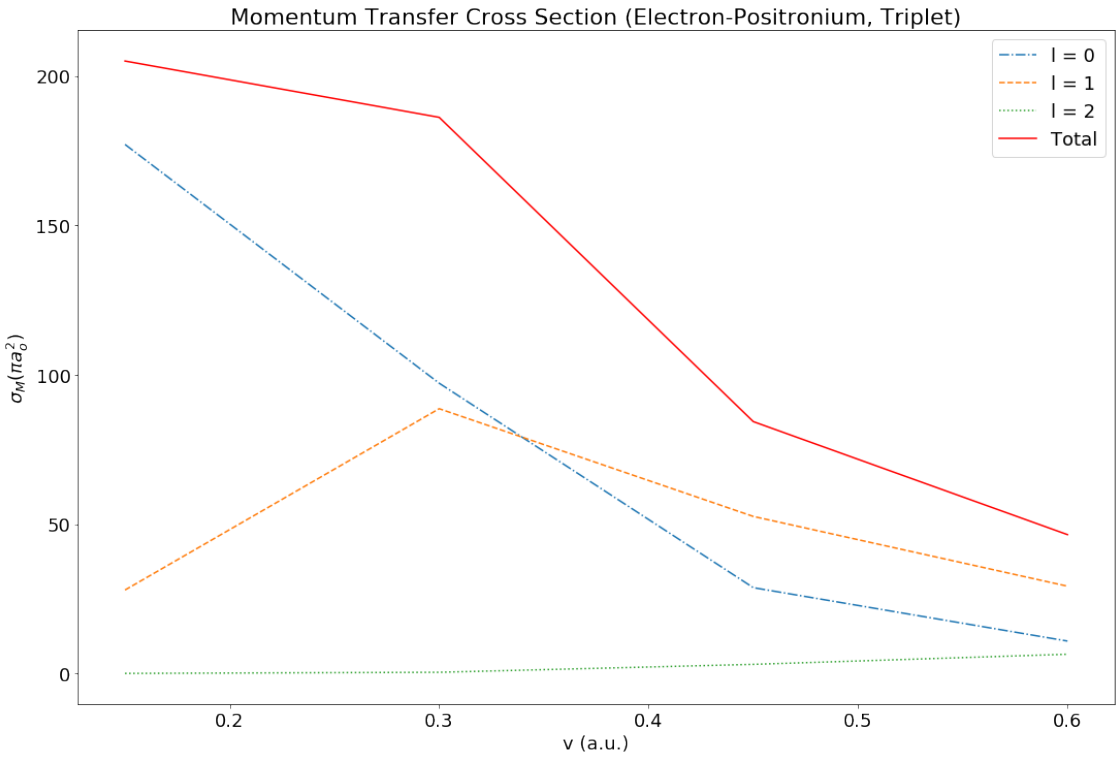

Electron-Positronium, Singlet
θ (degrees)
v (a.u.)
$\frac{d\sigma}{d\Omega}(a_o^2/sr)$

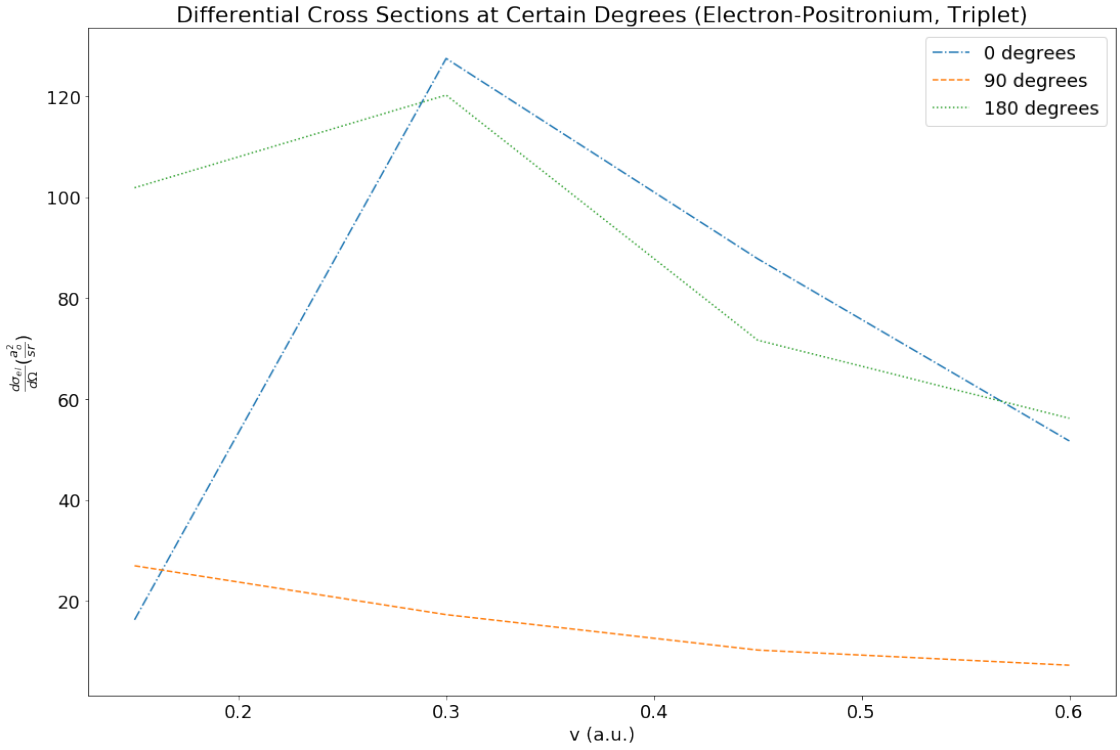


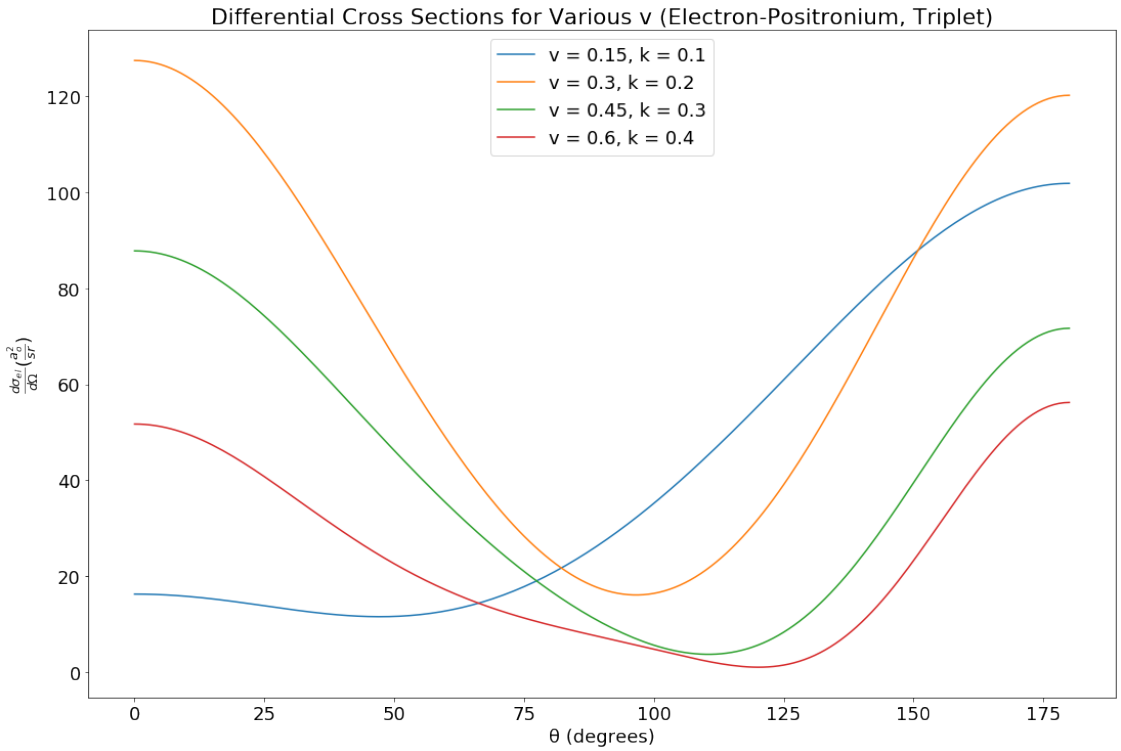
[83]:
```
e_ps_triplet_system = collision(e_ps_phases_trip,k_e_ps, 'Electron-Positronium,␣
↪Triplet',(2/3))


e_ps_triplet_system.plot_phase_shifts(velocity=True)
e_ps_triplet_system.plot_cross_section(velocity=True)
e_ps_triplet_system.plot_momentum_transfer(velocity=True)
e_ps_triplet_system.plot_diff_cross_vs_k(r_interval,velocity=True)
e_ps_triplet_system.diff_plot(velocity=True)
e_ps_triplet_system.diff_plot(sin=True, velocity=True)
e_ps_triplet_system.diff_plot_3d(velocity=True)
e_ps_triplet_system.diff_plot_3d(density=True, velocity=True)
```

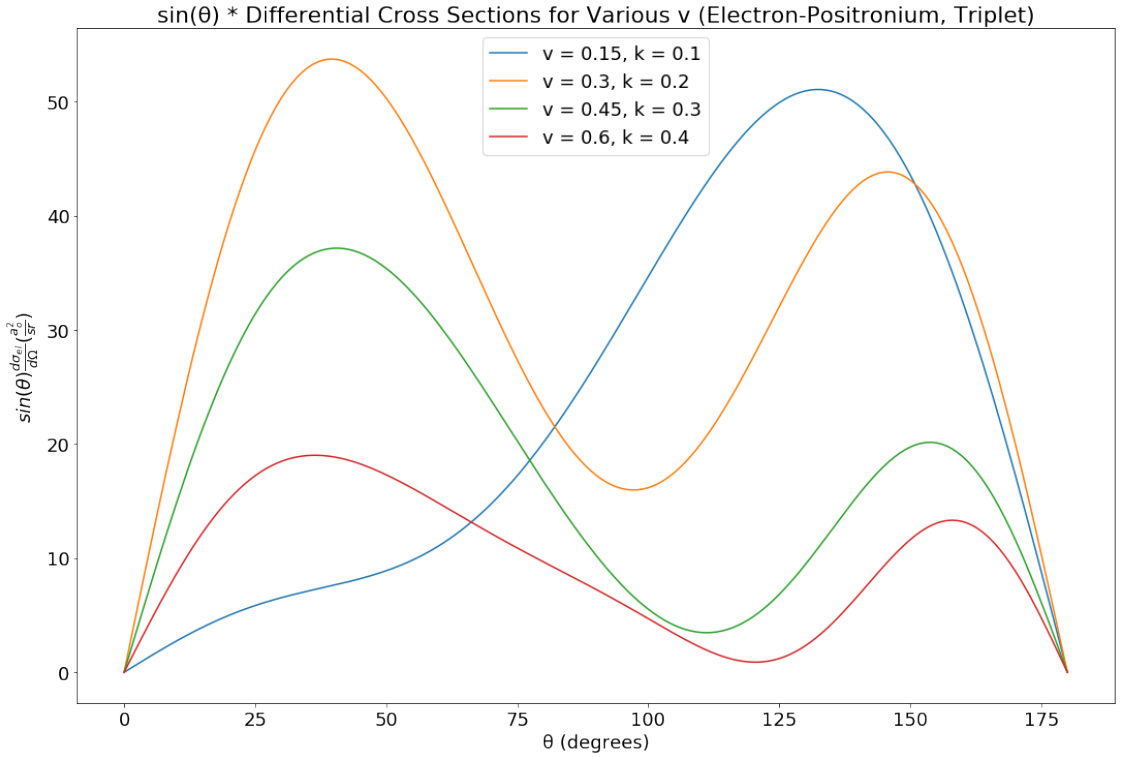

Phase Shifts vs v (Electron-Positronium, Triplet)
l = 0
l = 1
l = 2
l = 3
0.5
0.0
−0.5
−1.0
−1.5
δ_l (radians)
0.2
0.3
0.4
0.5
0.6
v (a.u.)

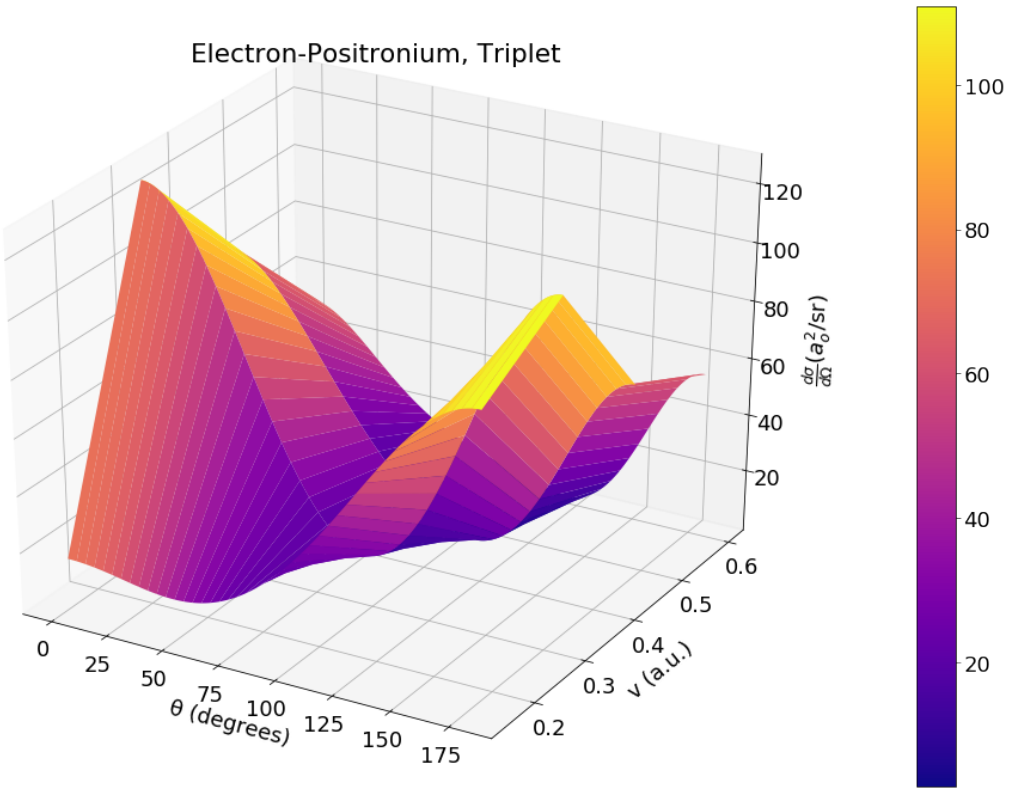

Partial Wave Cross Sections (Electron-Positronium, Triplet)
l = 0
l = 1
l = 2
l = 3
Total
200
175
150
125
100
75
50
25
0
$\sigma_{el}(\pi a_0^2)$
0.2
0.3
0.4
0.5
0.6
v (a.u.)

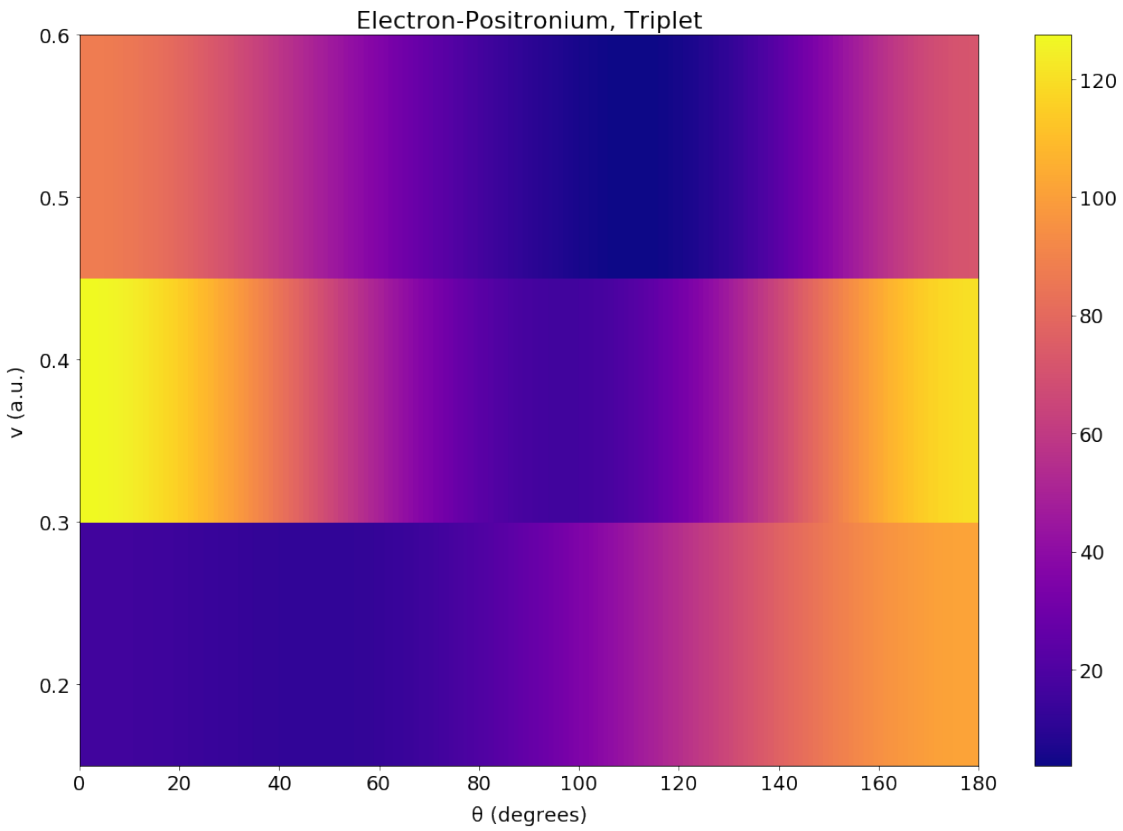

Momentum Transfer Cross Section (Electron-Positronium, Triplet)
l = 0
l = 1
l = 2
Total
200
150
100
50
0
$\sigma_M(\pi a_o^2)$
0.2
0.3
0.4
0.5
0.6
v (a.u.)


Differential Cross Sections at Certain Degrees (Electron-Positronium, Triplet)
0 degrees
90 degrees
180 degrees
120
100
80
60
40
20
$\frac{d\sigma_{el}}{d\Omega}(\frac{a_o^2}{sr})$
0.2
0.3
0.4
0.5
0.6
v (a.u.)

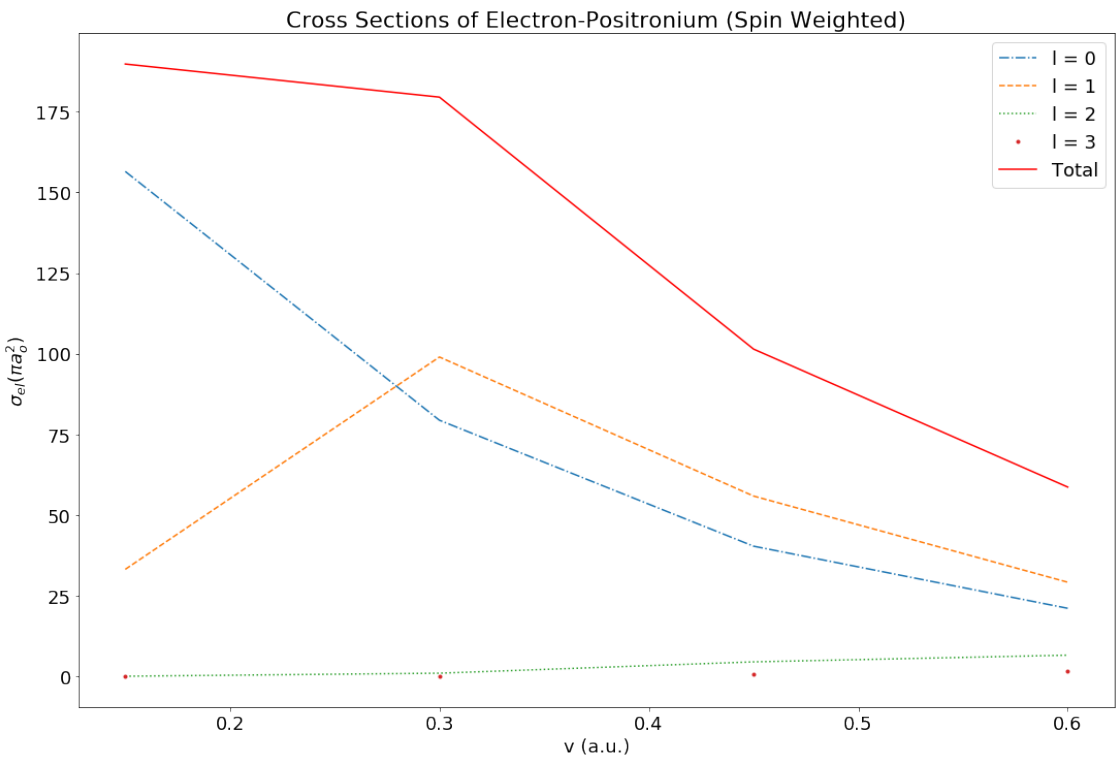

Differential Cross Sections for Various v (Electron-Positronium, Triplet)
v = 0.15, k = 0.1
v = 0.3, k = 0.2
v = 0.45, k = 0.3
v = 0.6, k = 0.4
$\frac{d\sigma_{el}}{d\Omega}(\frac{a_0^2}{sr})$
θ (degrees)

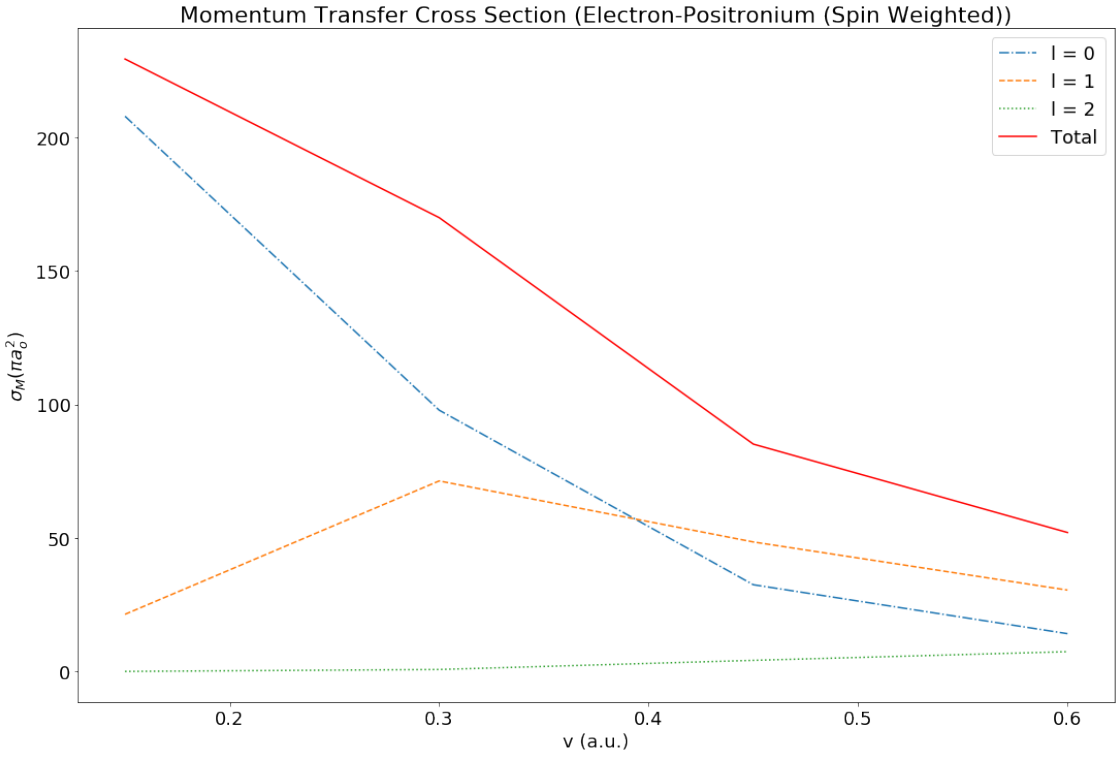

sin(θ) * Differential Cross Sections for Various v (Electron-Positronium, Triplet)
v = 0.15, k = 0.1
v = 0.3, k = 0.2
v = 0.45, k = 0.3
v = 0.6, k = 0.4
$sin(\theta)\frac{d\sigma_{el}}{d\Omega}(\frac{a_0^2}{sr})$
θ (degrees)

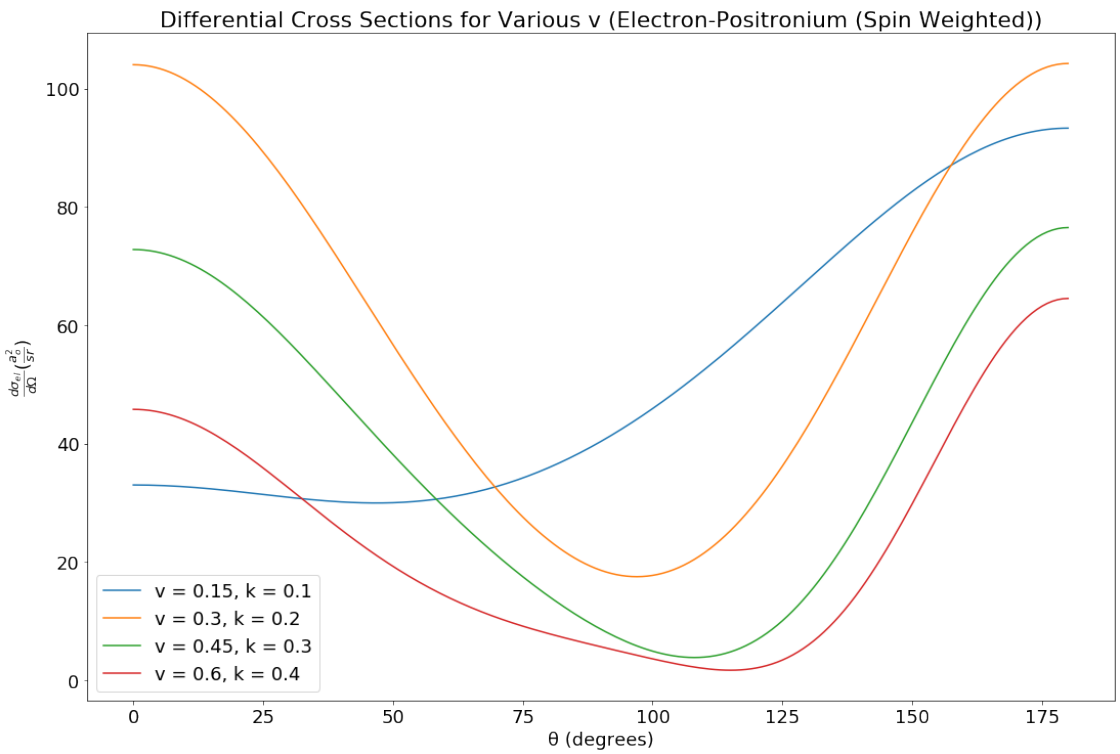

Electron-Positronium, Triplet
120
100
80
60
40
20
$\frac{d\sigma}{d\Omega}(a_0^2/sr)$
0
25
50
75
100
125
150
175
θ (degrees)
0.2
0.3
0.4
0.5
0.6
v (a.u.)
100
80
60
40
20

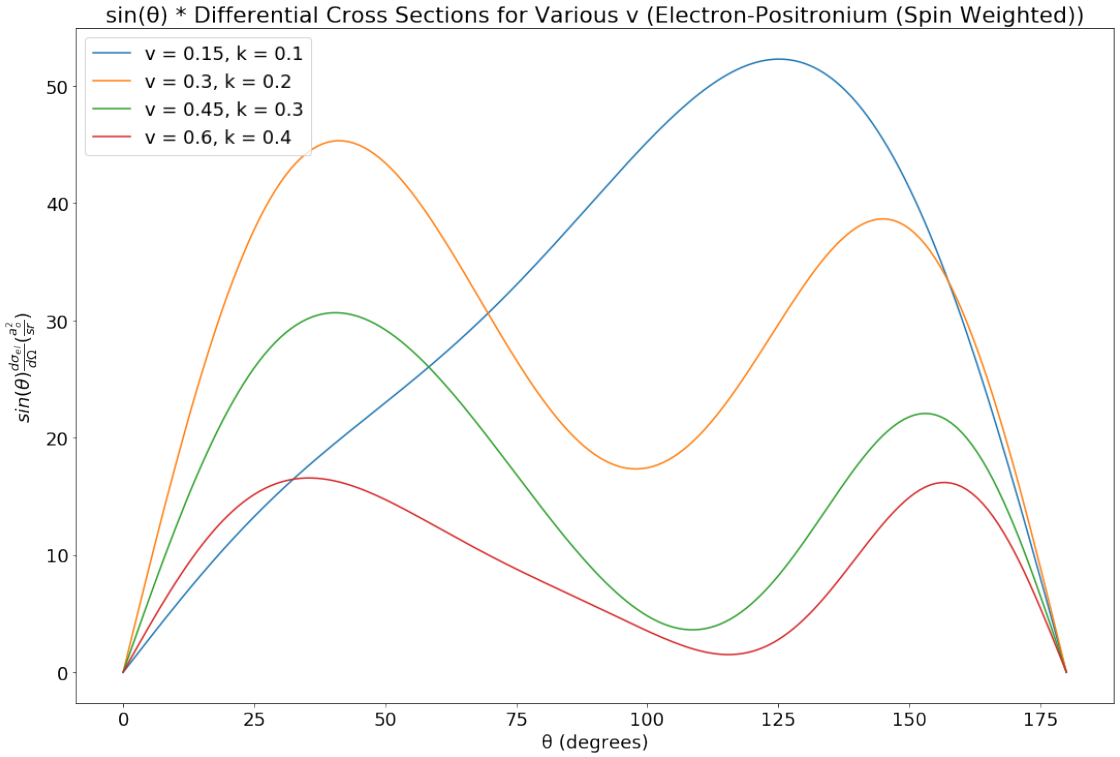


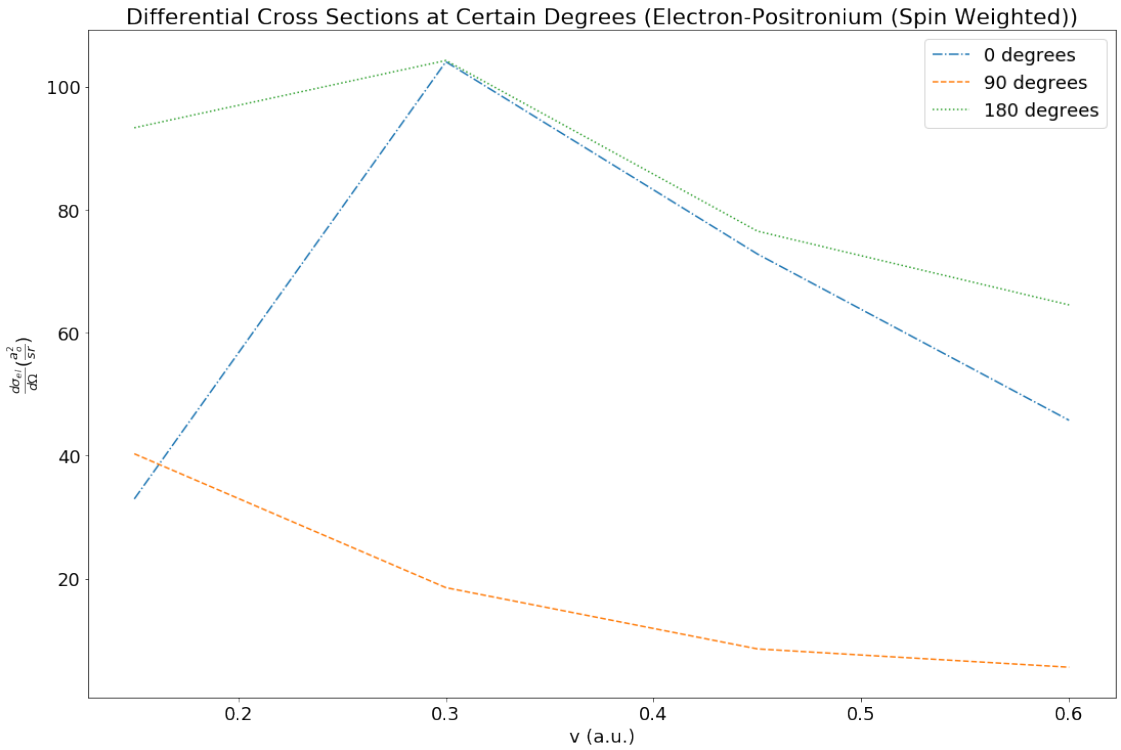
[84]:
```
e_ps_weighted_a = collision(e_ps_phases,k_e_ps,'Electron-Positronium (Spin␣
↪Weighted)',(2/3),e_ps_phases_trip,k_e_ps,(2/3))


e_ps_weighted_a.plot_cross_section(spin_weight=True, velocity=True)
e_ps_weighted_a.plot_momentum_transfer(spin_weight=True, velocity=True)
e_ps_weighted_a.diff_plot(spin_weight=True, velocity=True)
e_ps_weighted_a.diff_plot(spin_weight=True, sin=True, velocity=True)
e_ps_weighted_a.plot_diff_cross_vs_k(r_interval,spin_weight=True, velocity=True)
e_ps_weighted_a.diff_plot_3d(spin_weight=True, velocity=True)
e_ps_weighted_a.diff_plot_3d(spin_weight=True, density=True, velocity=True)
```

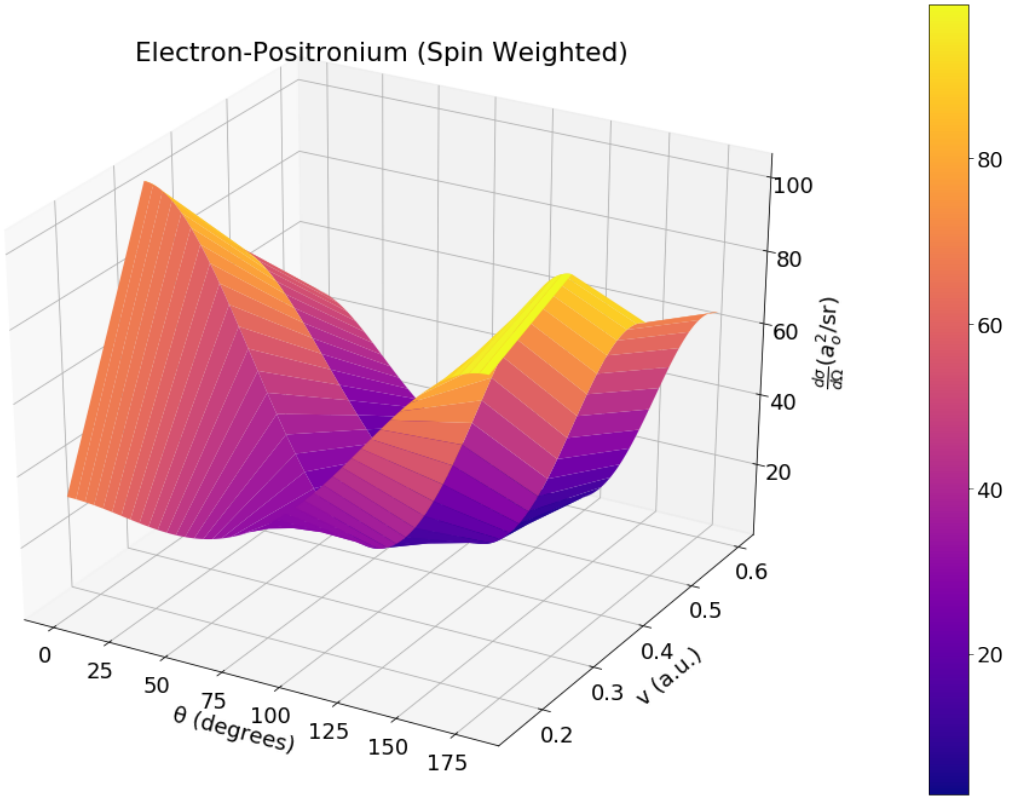

Cross Sections of Electron-Positronium (Spin Weighted)
l = 0
l = 1
l = 2
l = 3
Total
175
150
125
100
75
50
25
0
$\sigma_{el}(\pi a_o^2)$
0.2
0.3
0.4
0.5
0.6
v (a.u.)

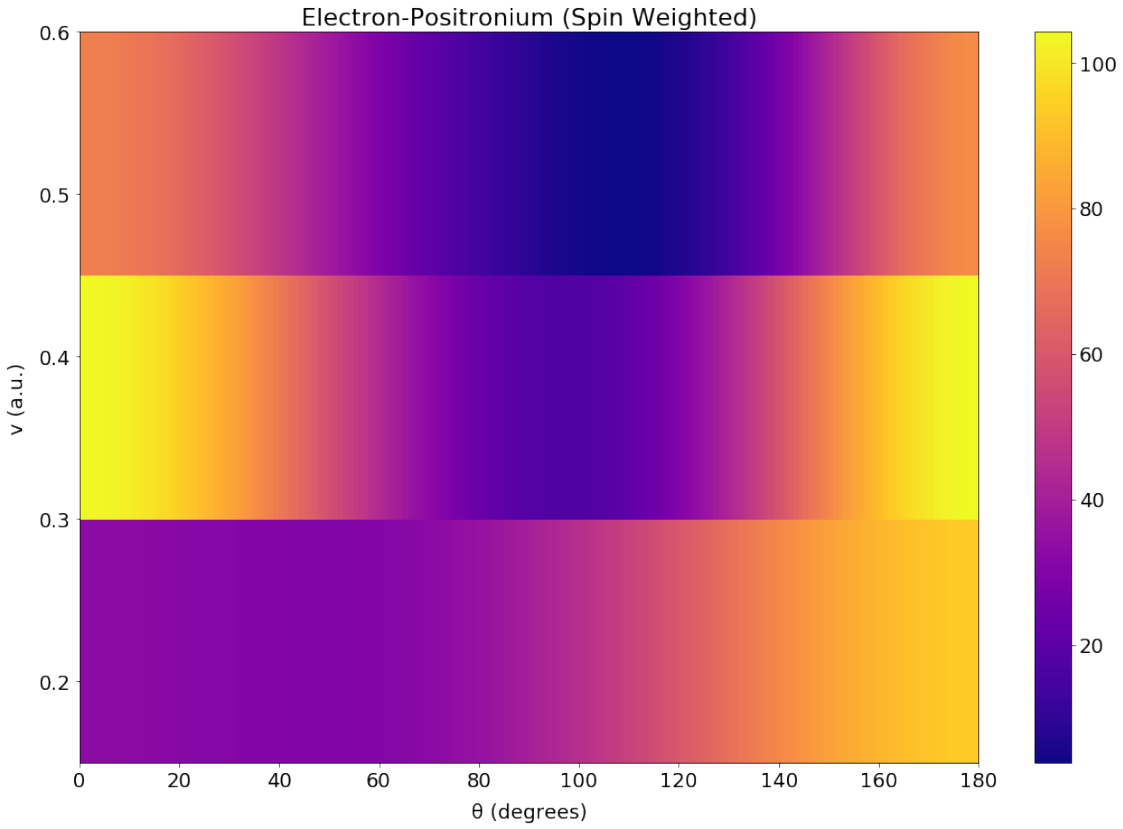

Momentum Transfer Cross Section (Electron-Positronium (Spin Weighted))
l = 0
l = 1
l = 2
Total
200
150
100
50
0
$\sigma_M(\pi a_o^2)$
0.2
0.3
0.4
0.5
0.6
v (a.u.)


Differential Cross Sections for Various v (Electron-Positronium (Spin Weighted))
v = 0.15, k = 0.1
v = 0.3, k = 0.2
v = 0.45, k = 0.3
v = 0.6, k = 0.4
θ (degrees)


sin(θ) * Differential Cross Sections for Various v (Electron-Positronium (Spin Weighted))
v = 0.15, k = 0.1
v = 0.3, k = 0.2
v = 0.45, k = 0.3
v = 0.6, k = 0.4
θ (degrees)


Differential Cross Sections at Certain Degrees (Electron-Positronium (Spin Weighted))
0 degrees
90 degrees
180 degrees
100
80
60
40
20
0.2
0.3
0.4
0.5
0.6
v (a.u.)

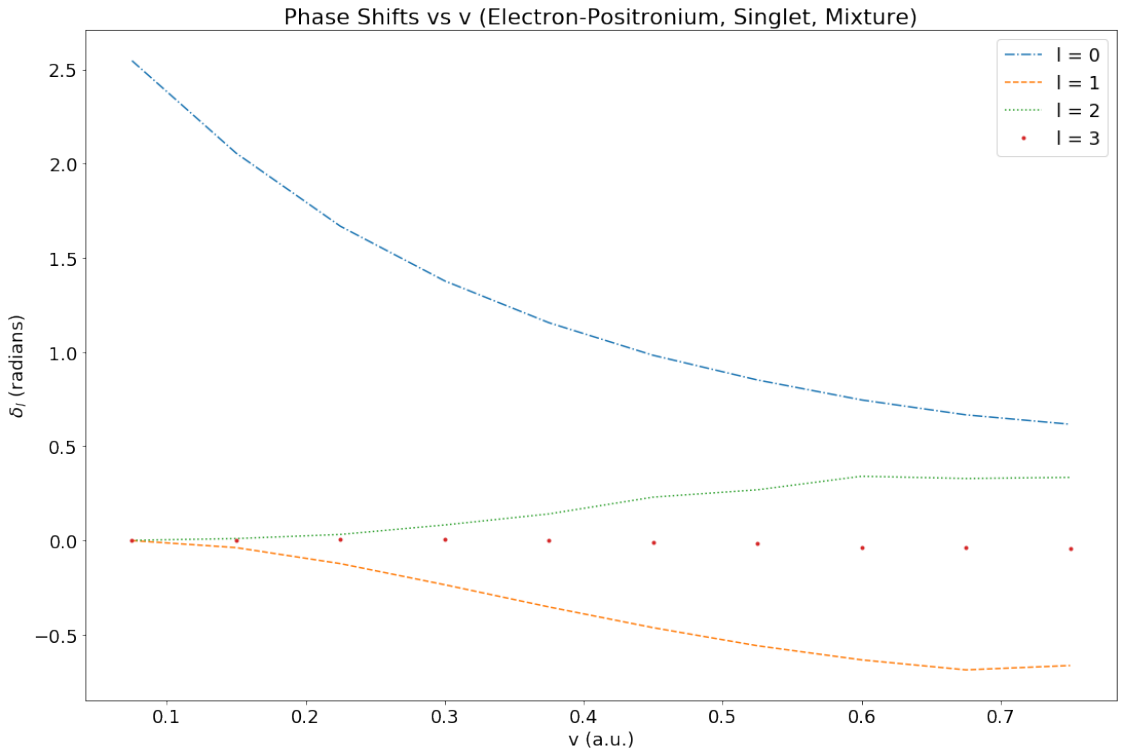

Electron-Positronium (Spin Weighted)
100
80
60
40
20
$\frac{d\sigma}{d\Omega}(a_o^2/sr)$
0
25
50
75
100
125
150
175
θ (degrees)
0.2
0.3
0.4
0.5
0.6
v (a.u.)
80
60
40
20

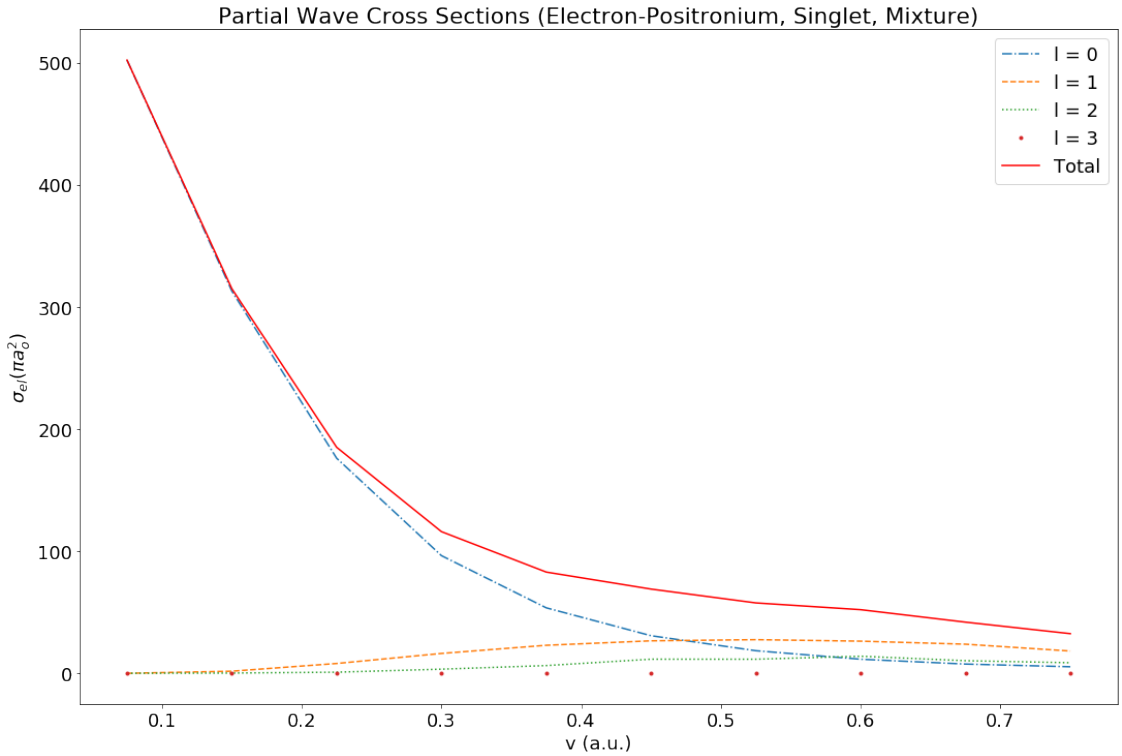


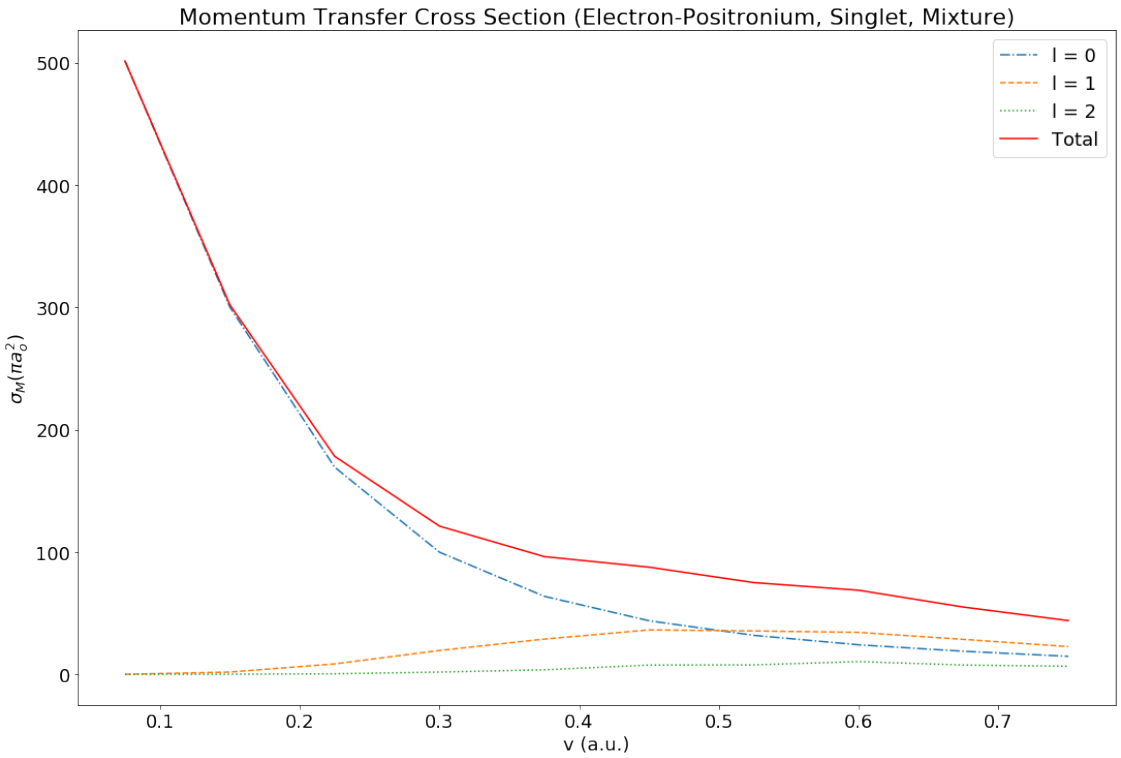
3.0.3 Mixture from Ward et al 1987 and A Igarashi et al 2000

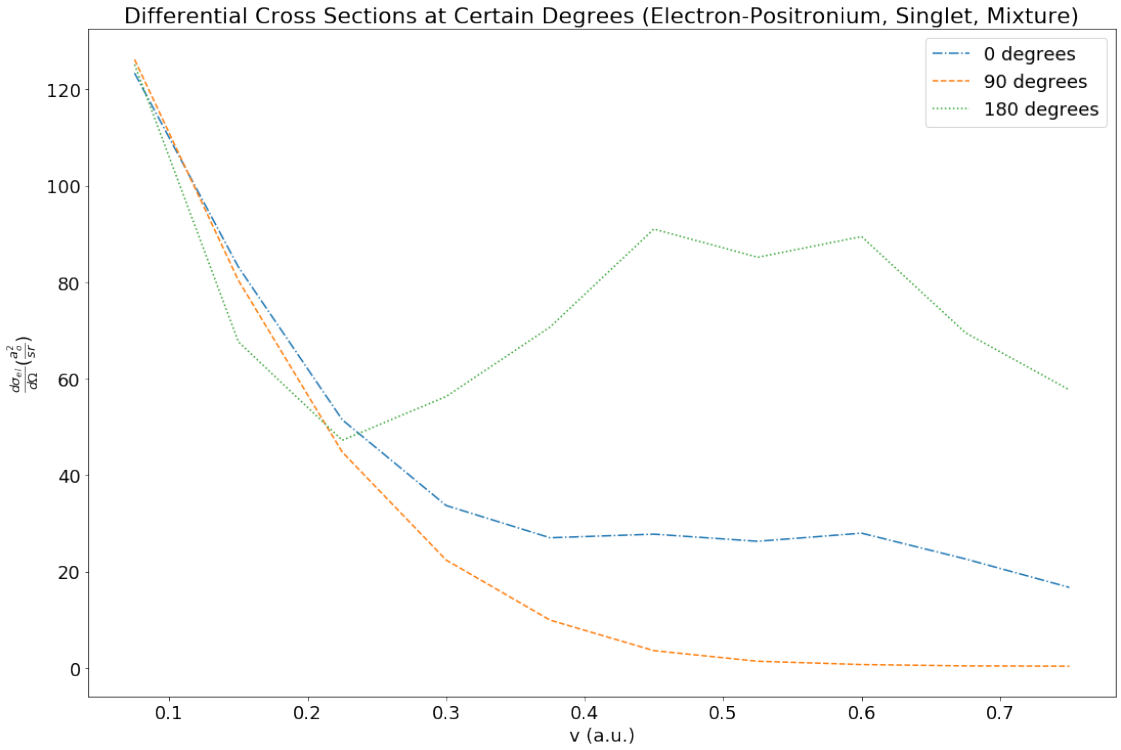
Here phase shifts from Ward et al 1987 [9] are used unless available from A Igarashi et al 2000 [10]

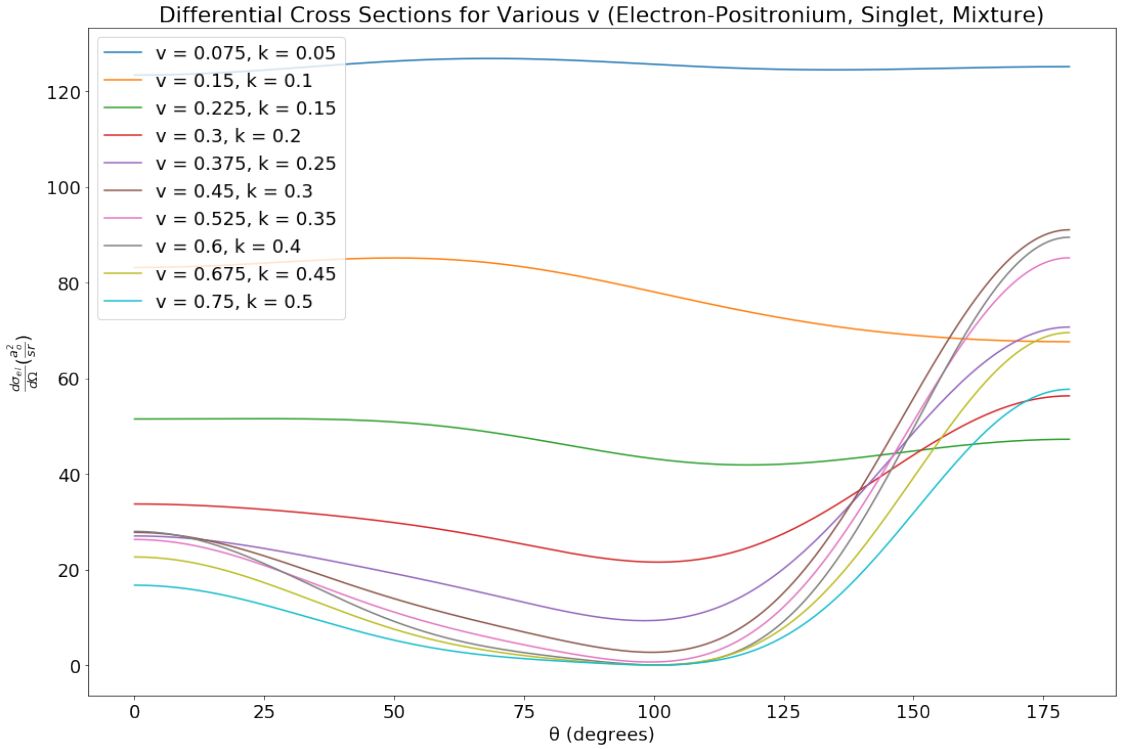
[85]:
```
e_ps_system_m = collision(e_ps_phases_m,k_e_ps_m, 'Electron-Positronium,␣
↪Singlet, Mixture',(2/3))


e_ps_system_m.plot_phase_shifts(velocity=True)
e_ps_system_m.plot_cross_section(velocity=True)
e_ps_system_m.plot_momentum_transfer(velocity=True)
e_ps_system_m.plot_diff_cross_vs_k(r_interval,velocity=True)
e_ps_system_m.diff_plot(velocity=True)
e_ps_system_m.diff_plot(sin=True,velocity=True)
e_ps_system_m.diff_plot_3d(velocity=True)
e_ps_system_m.diff_plot_3d(density=True,velocity=True)
```

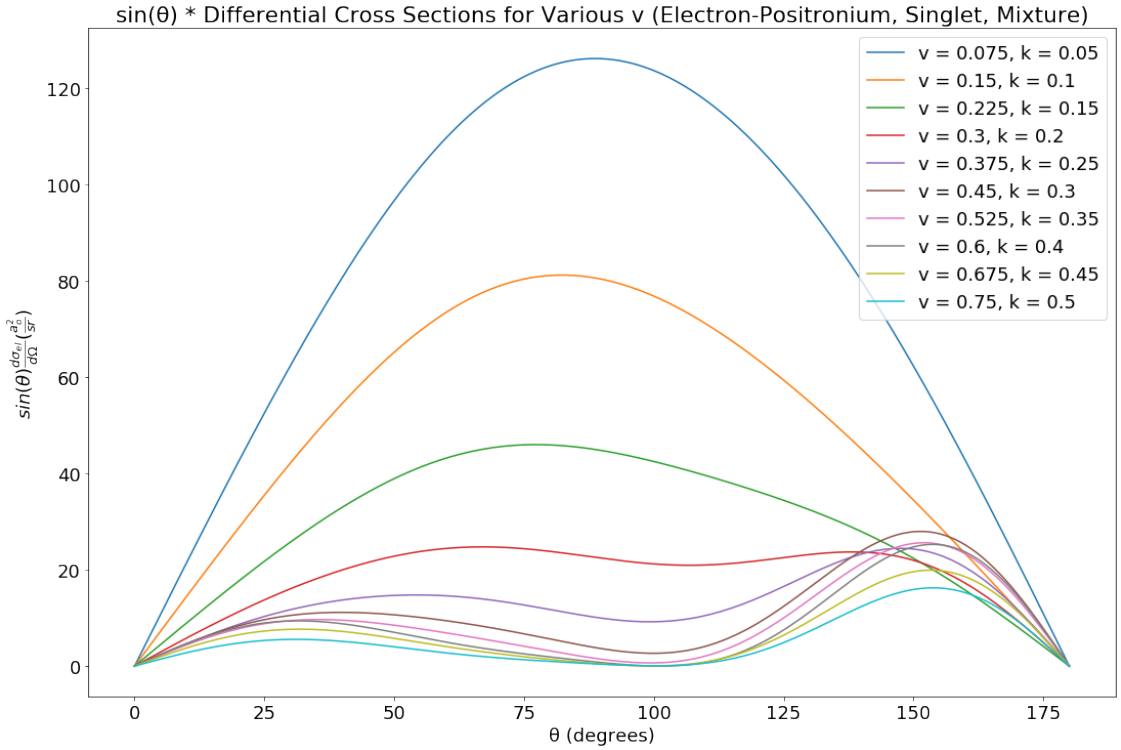

Phase Shifts vs v (Electron-Positronium, Singlet, Mixture)
l = 0
l = 1
l = 2
l = 3
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
δ_l (radians)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v (a.u.)

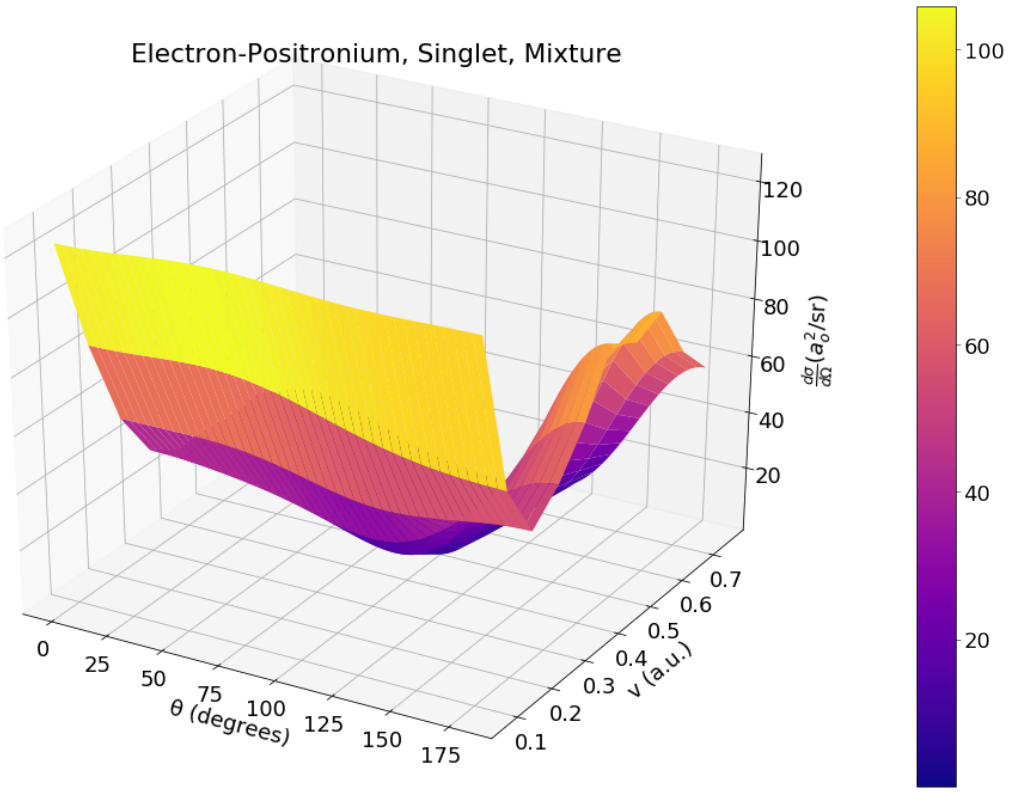

Partial Wave Cross Sections (Electron-Positronium, Singlet, Mixture)
l = 0
l = 1
l = 2
l = 3
Total
500
400
300
200
100
0
$\sigma_{el}(\pi a_0^2)$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v (a.u.)

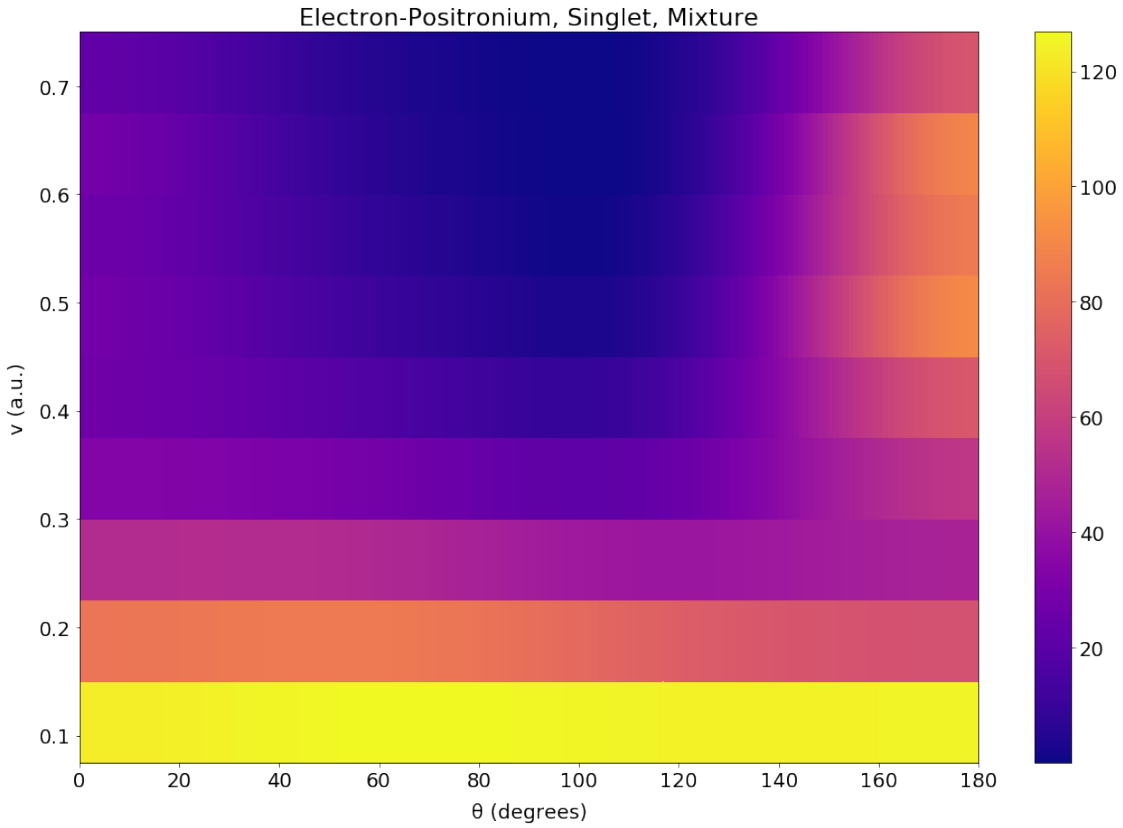

Momentum Transfer Cross Section (Electron-Positronium, Singlet, Mixture)
l = 0
l = 1
l = 2
Total
$\sigma_M(\pi a_o^2)$
v (a.u.)


Differential Cross Sections at Certain Degrees (Electron-Positronium, Singlet, Mixture)
0 degrees
90 degrees
180 degrees
$\frac{d\sigma_{el}}{d\Omega}(\frac{a_o^2}{sr})$
v (a.u.)

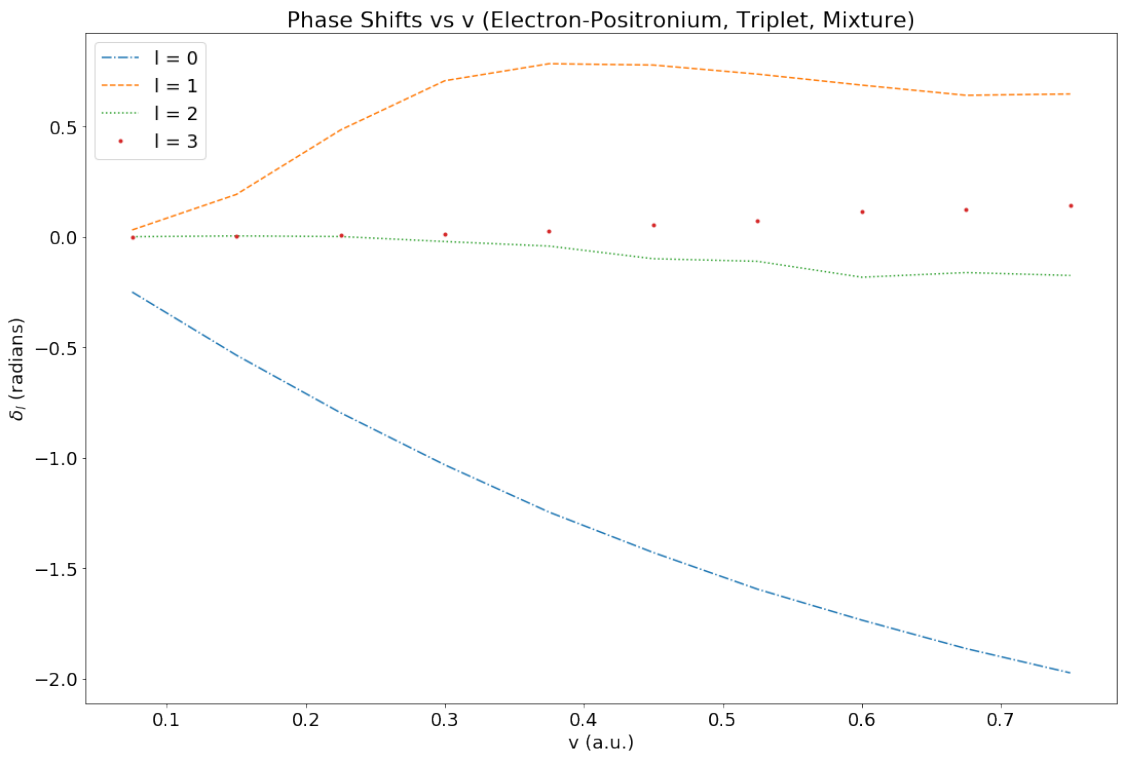

Differential Cross Sections for Various v (Electron-Positronium, Singlet, Mixture)
v = 0.075, k = 0.05
v = 0.15, k = 0.1
v = 0.225, k = 0.15
v = 0.3, k = 0.2
v = 0.375, k = 0.25
v = 0.45, k = 0.3
v = 0.525, k = 0.35
v = 0.6, k = 0.4
v = 0.675, k = 0.45
v = 0.75, k = 0.5
0
20
40
60
80
100
120
0
25
50
75
100
125
150
175
θ (degrees)

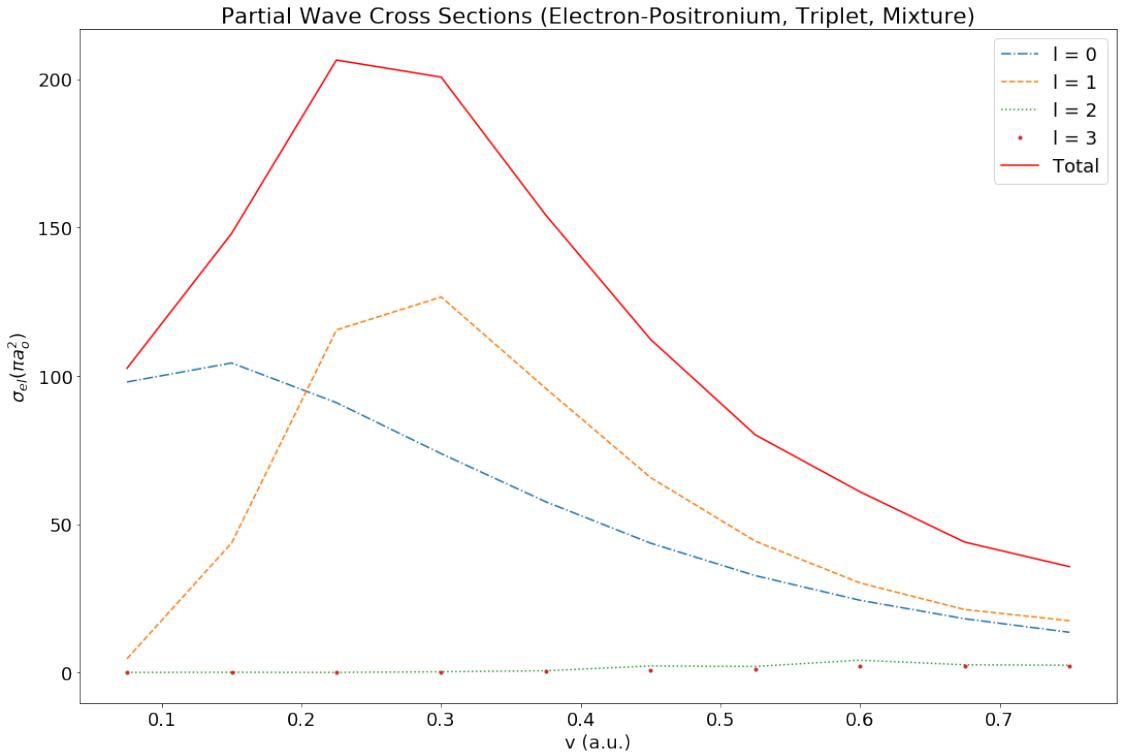

sin(θ) * Differential Cross Sections for Various v (Electron-Positronium, Singlet, Mixture)
v = 0.075, k = 0.05
v = 0.15, k = 0.1
v = 0.225, k = 0.15
v = 0.3, k = 0.2
v = 0.375, k = 0.25
v = 0.45, k = 0.3
v = 0.525, k = 0.35
v = 0.6, k = 0.4
v = 0.675, k = 0.45
v = 0.75, k = 0.5
0
20
40
60
80
100
120
0
25
50
75
100
125
150
175
θ (degrees)

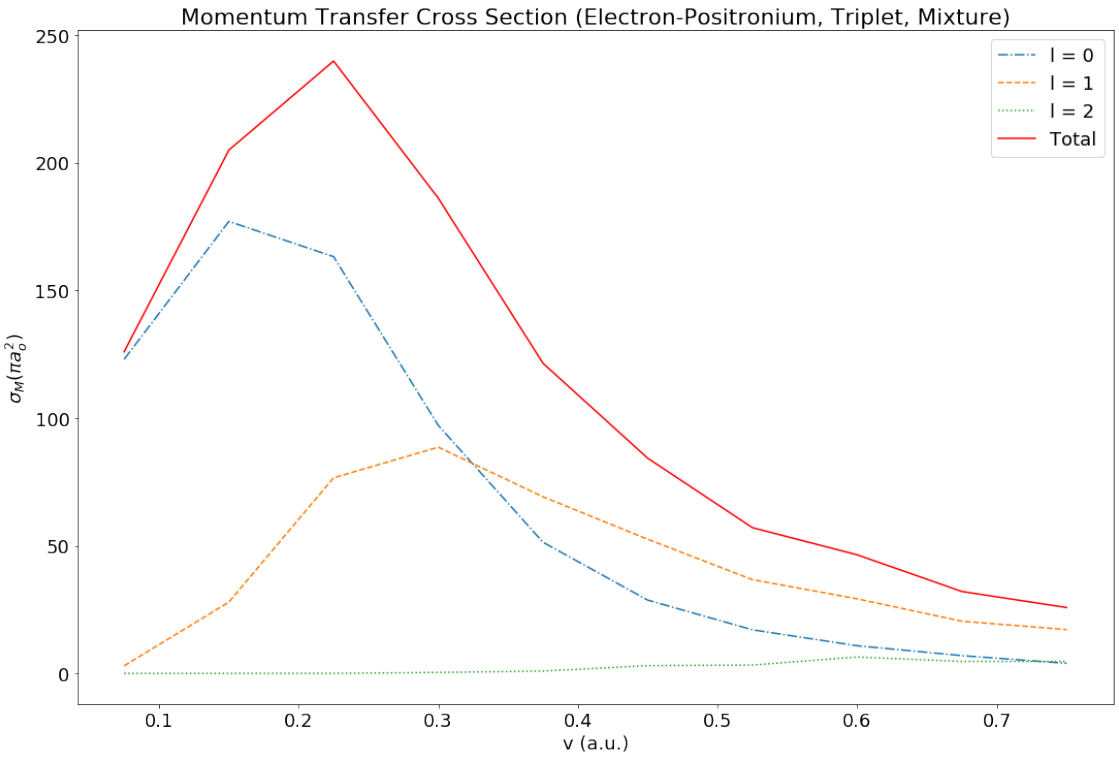

Electron-Positronium, Singlet, Mixture
120
100
80
60
40
20
$\frac{d\sigma}{d\Omega}(a_0^2/sr)$
0
25
50
75
100
125
150
175
θ (degrees)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v (a.u.)
100
80
60
40
20

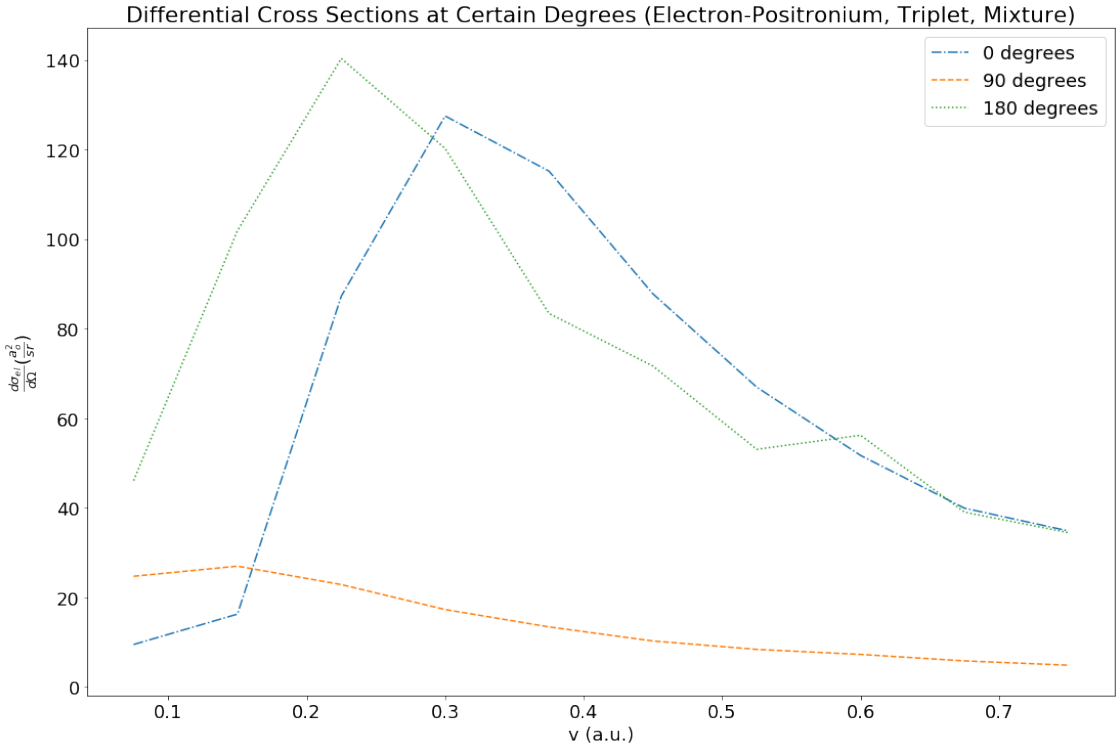


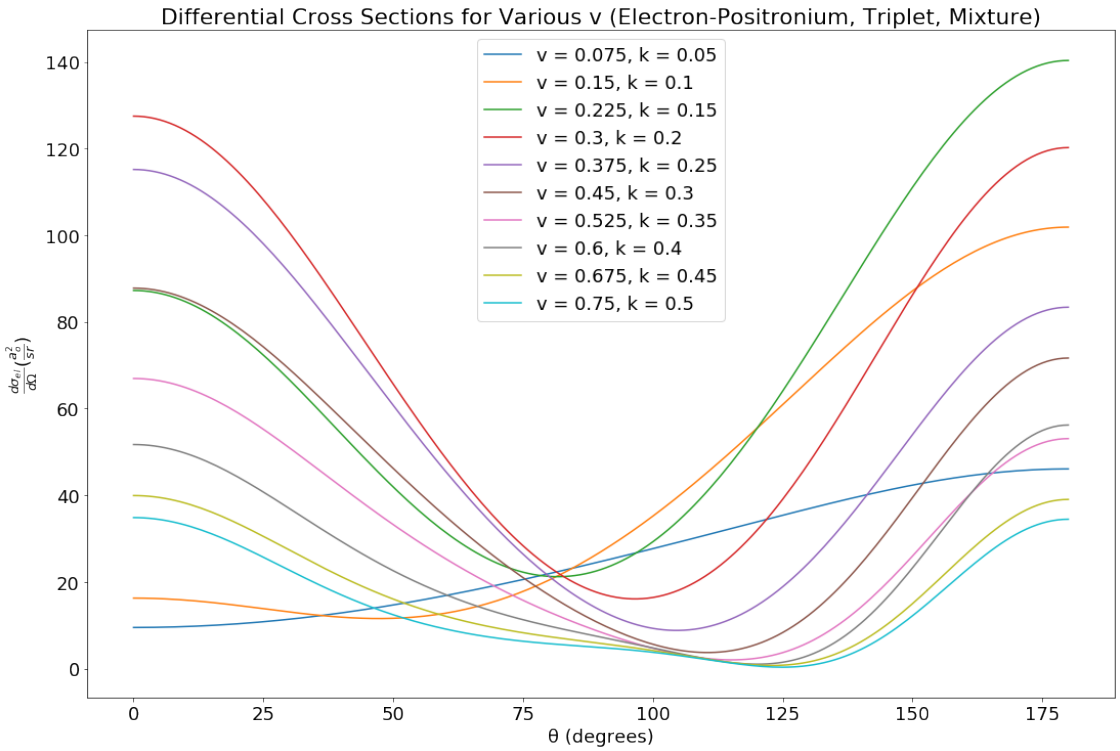
[86]:
```
e_ps_triplet_system_m = collision(e_ps_phases_trip_m,k_e_ps_m,␣
↪'Electron-Positronium, Triplet, Mixture',(2/3))


e_ps_triplet_system_m.plot_phase_shifts(velocity=True)
e_ps_triplet_system_m.plot_cross_section(velocity=True)
e_ps_triplet_system_m.plot_momentum_transfer(velocity=True)
e_ps_triplet_system_m.plot_diff_cross_vs_k(r_interval,velocity=True)
e_ps_triplet_system_m.diff_plot(velocity=True)
e_ps_triplet_system_m.diff_plot(sin=True, velocity=True)
e_ps_triplet_system_m.diff_plot_3d(velocity=True)
e_ps_triplet_system_m.diff_plot_3d(density=True, velocity=True)
```

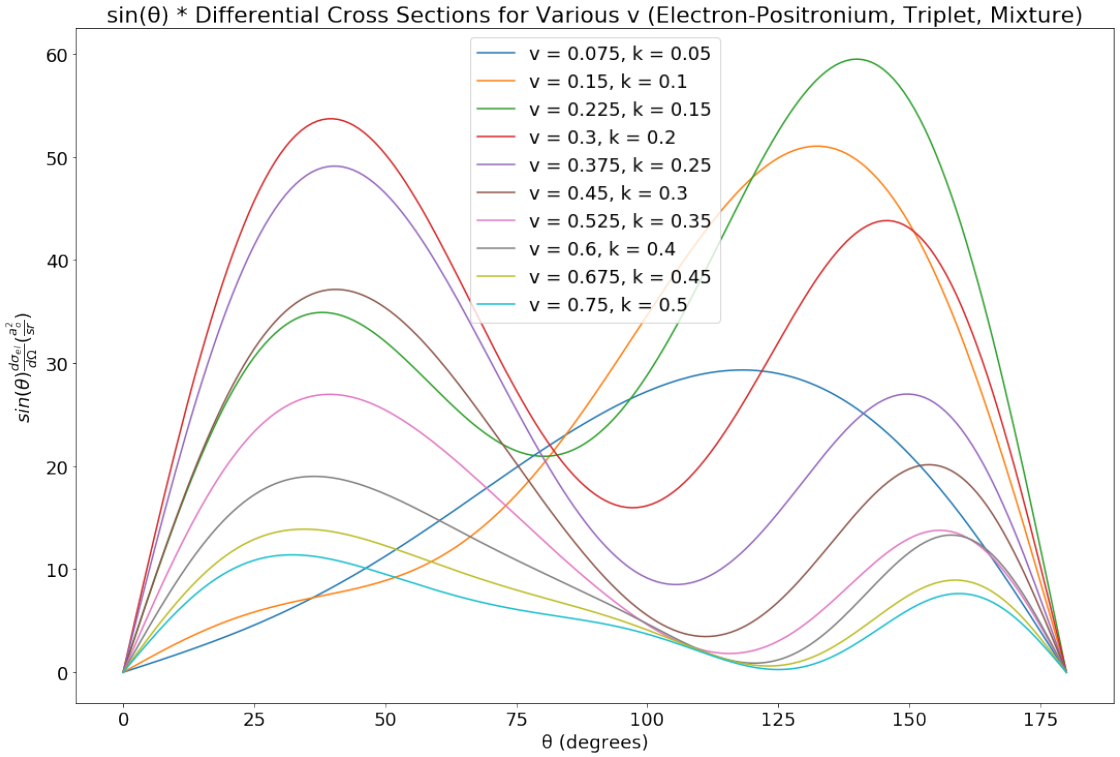

Phase Shifts vs v (Electron-Positronium, Triplet, Mixture)
l = 0
l = 1
l = 2
l = 3
0.5
0.0
−0.5
−1.0
−1.5
−2.0
δ_l (radians)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v (a.u.)

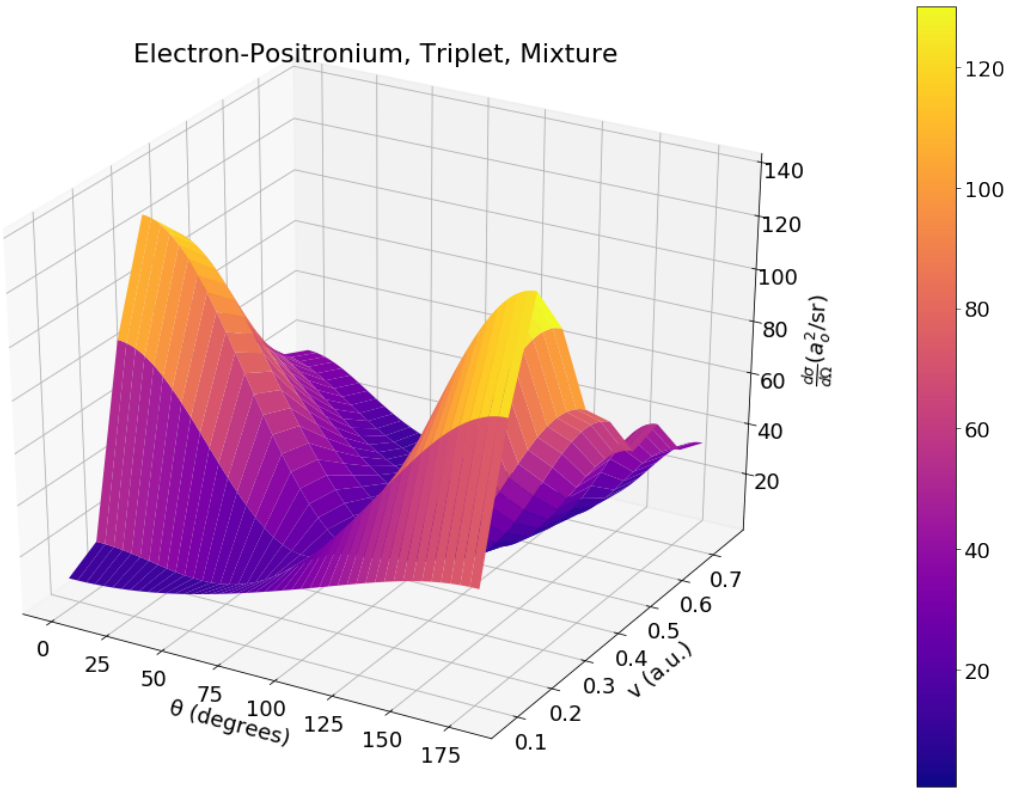

Partial Wave Cross Sections (Electron-Positronium, Triplet, Mixture)
l = 0
l = 1
l = 2
l = 3
Total
200
150
100
50
0
$\sigma_{el}(\pi a_0^2)$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v (a.u.)

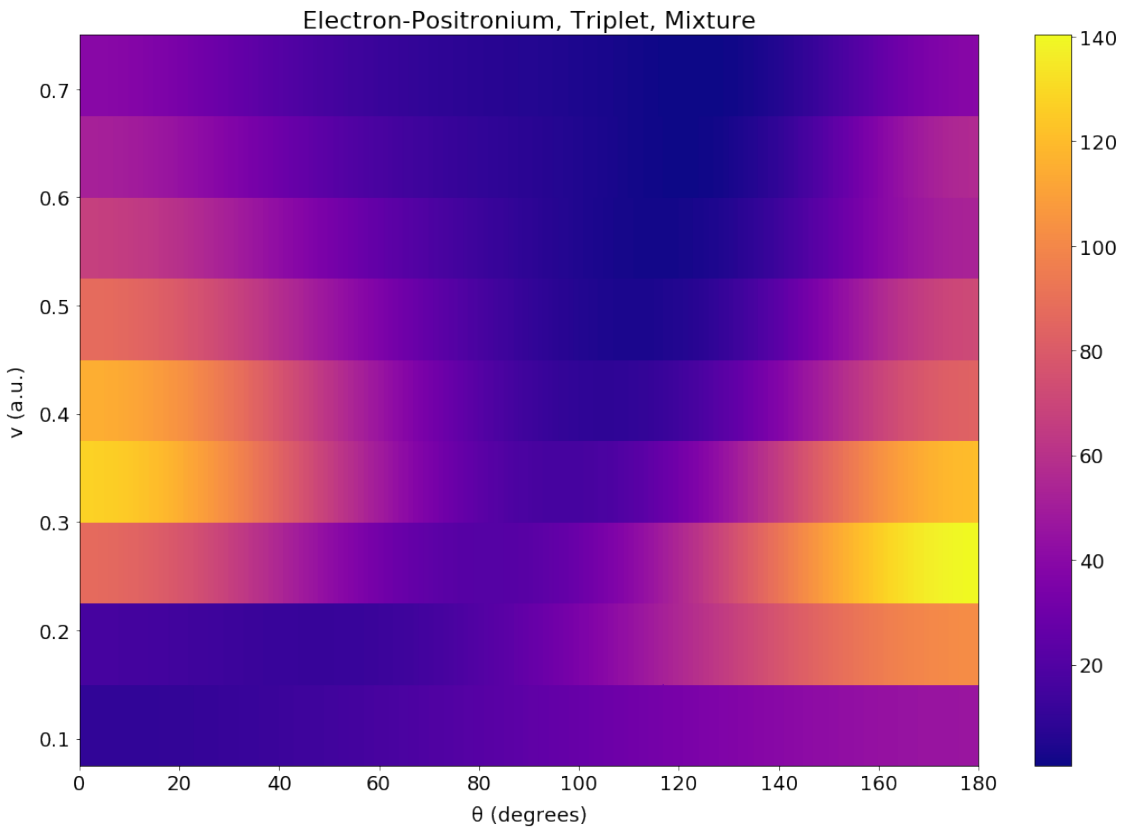

Momentum Transfer Cross Section (Electron-Positronium, Triplet, Mixture)
l = 0
l = 1
l = 2
Total
$\sigma_M(\pi a_o^2)$
v (a.u.)


Differential Cross Sections at Certain Degrees (Electron-Positronium, Triplet, Mixture)
0 degrees
90 degrees
180 degrees
$\frac{d\sigma_{el}}{d\Omega}(\frac{a_o^2}{sr})$
v (a.u.)


Differential Cross Sections for Various v (Electron-Positronium, Triplet, Mixture)
v = 0.075, k = 0.05
v = 0.15, k = 0.1
v = 0.225, k = 0.15
v = 0.3, k = 0.2
v = 0.375, k = 0.25
v = 0.45, k = 0.3
v = 0.525, k = 0.35
v = 0.6, k = 0.4
v = 0.675, k = 0.45
v = 0.75, k = 0.5
θ (degrees)


sin(θ) * Differential Cross Sections for Various v (Electron-Positronium, Triplet, Mixture)
v = 0.075, k = 0.05
v = 0.15, k = 0.1
v = 0.225, k = 0.15
v = 0.3, k = 0.2
v = 0.375, k = 0.25
v = 0.45, k = 0.3
v = 0.525, k = 0.35
v = 0.6, k = 0.4
v = 0.675, k = 0.45
v = 0.75, k = 0.5
θ (degrees)


Electron-Positronium, Triplet, Mixture
140
120
100
80
60
40
20
$\frac{d\sigma}{d\Omega}(a_0^2/sr)$
0
25
50
75
100
125
150
175
θ (degrees)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v (a.u.)
120
100
80
60
40
20



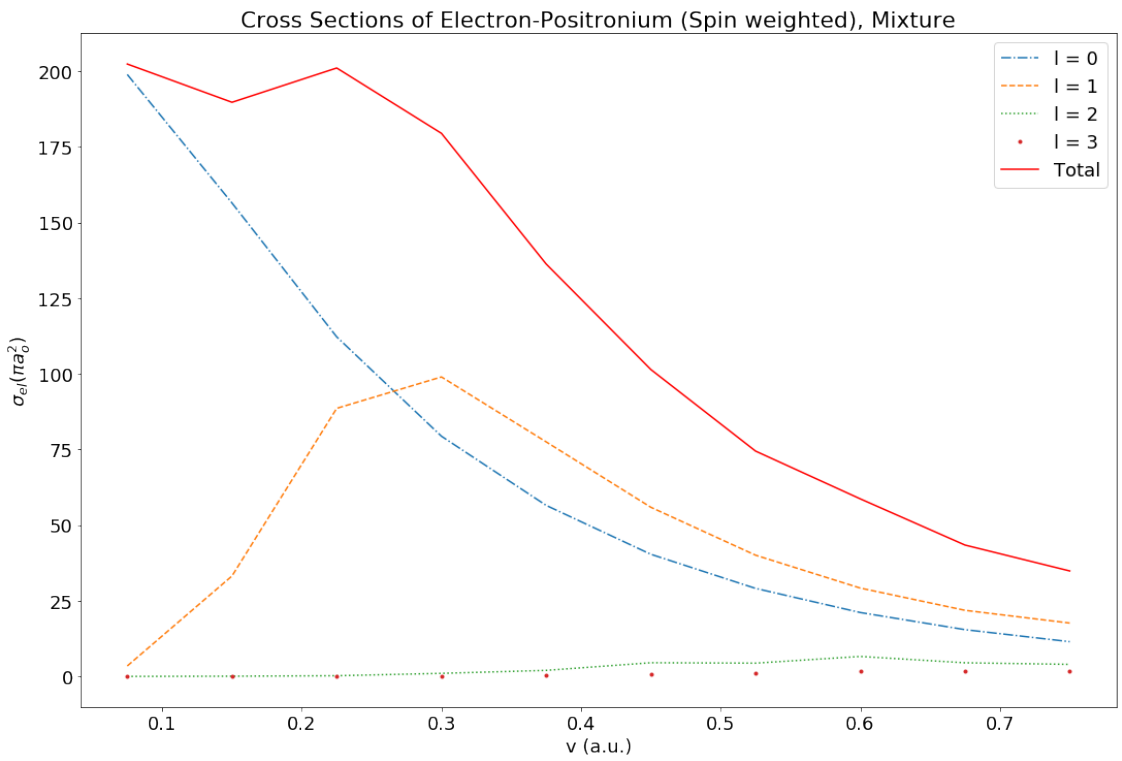
[87]:
```
e_ps_weighted_m = collision(e_ps_phases_m,k_e_ps_m,'Electron-Positronium (Spin␣
↪weighted), Mixture',(2/3),e_ps_phases_trip_m,k_e_ps_m,(2/3))
```

[88]:
```
e_ps_cross_data = e_ps_weighted_m.plot_cross_section(spin_weight=True,␣
↪velocity=True, data=True)


display(e_ps_cross_data)
get_avg(e_ps_cross_data, 'total cross section')
data_max(e_ps_cross_data, 'total cross section')
data_min(e_ps_cross_data, 'total cross section')
```

	v	total cross section	l = 0 contribution	l = 1 contribution	\
0	0.075	202.433	98.2881%	1.7087%	
1	0.150	189.787	82.457%	17.4947%	
2	0.225	201.124	55.8059%	44.061%	
3	0.300	179.528	44.2284%	55.1438%	
4	0.375	136.337	41.4723%	56.8703%	
5	0.450	101.446	39.7958%	55.1126%	
6	0.525	74.483	39.0667%	53.8009%	
7	0.600	58.694	35.9866%	49.7586%	

```
8  0.675               43.422           35.6015%           50.3253%
9  0.750               34.862           32.9383%           50.5952%

  l = 2 contribution l = 3 contribution
0            0.0012%             0.002%
1            0.0398%            0.0085%
2            0.1136%            0.0194%
3            0.5662%            0.0615%
4            1.4692%            0.1882%
5            4.4577%            0.6338%
6            5.8879%            1.2445%
7           11.2444%            3.0104%
8            10.374%            3.6993%
9           11.4181%            5.0484%


Average value for total cross section : 122.212
Maximum value for total cross section

       v  total cross section l = 0 contribution l = 1 contribution  \
0  0.075              202.433           98.2881%            1.7087%

  l = 2 contribution l = 3 contribution
0            0.0012%             0.002%


Minimum value for total cross section

      v  total cross section l = 0 contribution l = 1 contribution  \
9  0.75               34.862           32.9383%           50.5952%

  l = 2 contribution l = 3 contribution
9           11.4181%            5.0484%
```

[89]:
```
e_ps_weighted_m.plot_cross_section(spin_weight=True, velocity=True)
```



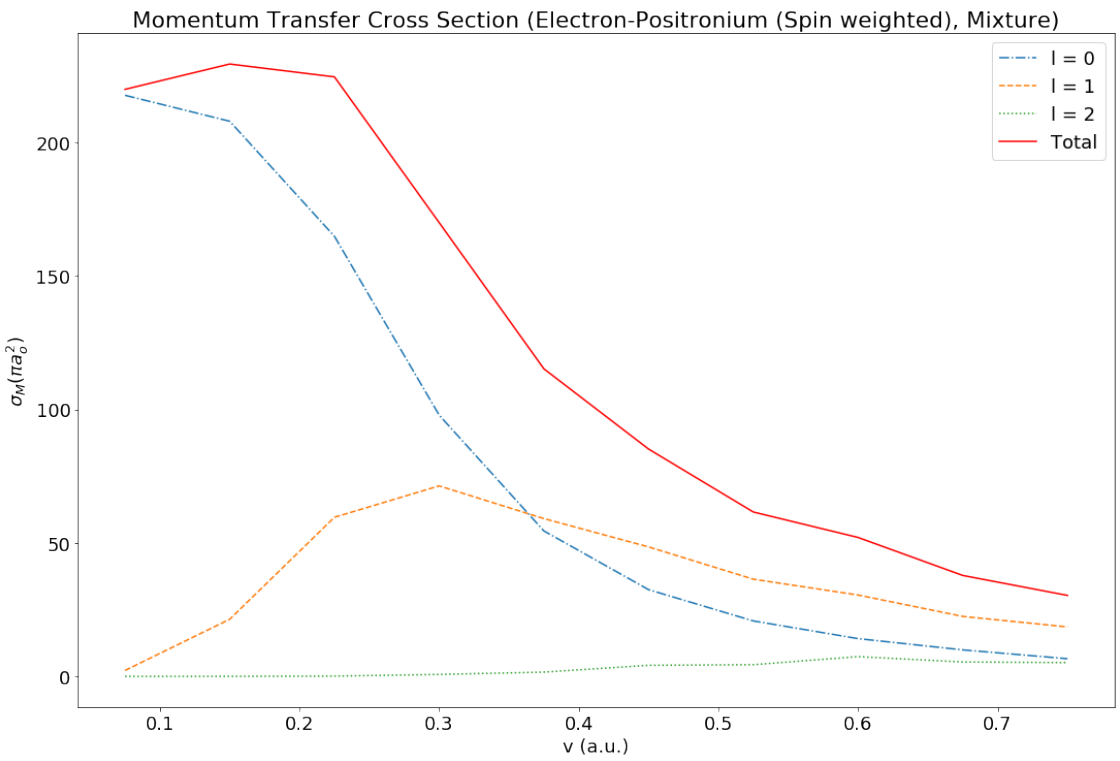
The elastic cross section decreases shortly then rises briefly before gradually decreasing with increasing velocity. The contribution from the s wave quickly decreases while the p wave contribution quickly increases. At v = 0.30 the p wave becomes the majority contributor. Beyond this point, the s and p wave contributions become increasingly similar. At the highest velocity, the contributions from the s, p, d, and f waves are 33, 51, 11, and 5 percent, respectively.

[90]:
```
e_ps_momentum_data = e_ps_weighted_m.plot_momentum_transfer(spin_weight=True,␣
↪velocity=True, data=True)


display(e_ps_momentum_data)
get_avg(e_ps_momentum_data, 'total momentum transfer')
data_max(e_ps_momentum_data, 'total momentum transfer')
data_min(e_ps_momentum_data, 'total momentum transfer')
```

	v	total momentum transfer	l = 0 contribution	l = 1 contribution	\
0	0.075	219.813	98.984%	1.016%	
1	0.150	229.289	90.6546%	9.3363%	
2	0.225	224.477	73.4015%	26.5485%	
3	0.300	169.907	57.5776%	41.9852%	
4	0.375	115.171	47.2956%	51.3042%	
5	0.450	85.120	38.1501%	56.9649%	
6	0.525	61.557	33.7397%	59.1415%	
7	0.600	52.027	27.1998%	58.5674%	

```
8  0.675                   37.806           26.2681%           59.4408%
9  0.750                   30.328            21.867%           61.1148%

  l = 2 contribution
0               0.0%
1              0.009%
2               0.05%
3             0.4371%
4             1.4002%
5              4.885%
6             7.1188%
7            14.2329%
8            14.2911%
9            17.0182%


Average value for total momentum transfer : 122.55
Maximum value for total momentum transfer

      v  total momentum transfer l = 0 contribution l = 1 contribution  \
1  0.15                  229.289           90.6546%            9.3363%

  l = 2 contribution
1              0.009%


Minimum value for total momentum transfer

      v  total momentum transfer l = 0 contribution l = 1 contribution  \
9  0.75                   30.328            21.867%           61.1148%

  l = 2 contribution
9            17.0182%
```

[91]:
```
e_ps_weighted_m.plot_momentum_transfer(spin_weight=True, velocity=True)
```



The momentum transfer cross section starts greater than the elastic cross section but becomes less than at v = 0.30. This difference increases with increasing velocity. The momentum transfer cross section maintains majority contribution from the s wave longer than the elastic cross section does. At v = 0.375, the p wave becomes the majority contributor.

[92]:
```
e_ps_cross_momentum = cross_momentum_compare(e_ps_cross_data,␣
↪e_ps_momentum_data)


display(e_ps_cross_momentum)
get_avg(e_ps_cross_momentum, 'ratio momentum/cross')
data_max(e_ps_cross_momentum, 'ratio momentum/cross')
data_min(e_ps_cross_momentum, 'ratio momentum/cross')
```

	v	total cross section	total momentum transfer	ratio momentum/cross
0	0.075	202.433	219.813	1.086
1	0.150	189.787	229.289	1.208
2	0.225	201.124	224.477	1.116
3	0.300	179.528	169.907	0.946
4	0.375	136.337	115.171	0.845
5	0.450	101.446	85.120	0.839
6	0.525	74.483	61.557	0.826
7	0.600	58.694	52.027	0.886
8	0.675	43.422	37.806	0.871

```
9  0.750               34.862                    30.328                  0.870
```

```
Average value for ratio momentum/cross : 0.949
Maximum value for ratio momentum/cross
```

```
      v  total cross section  total momentum transfer  ratio momentum/cross
1  0.15              189.787                  229.289                 1.208
```

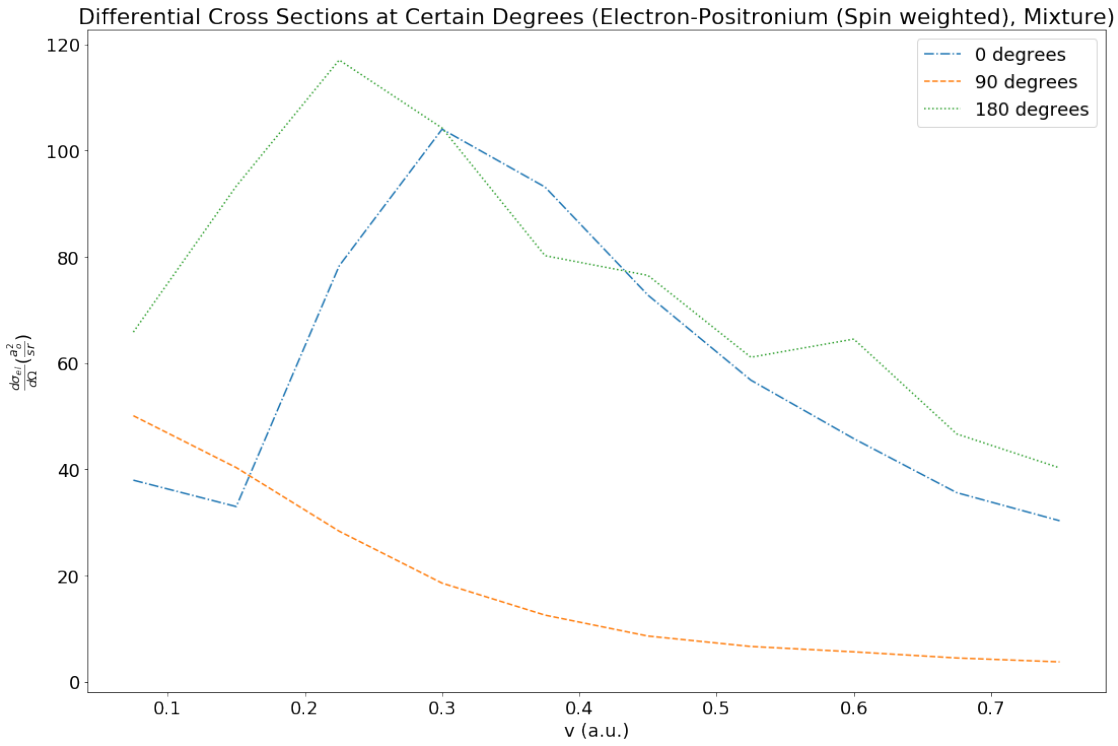
```
Minimum value for ratio momentum/cross
```

```
       v  total cross section  total momentum transfer  ratio momentum/cross
6  0.525               74.483                   61.557                 0.826
```

[93]:
```
e_ps_diff_degree =  e_ps_weighted_m.
↪plot_diff_cross_vs_k(r_interval,spin_weight=True, velocity=True, data=True)

display(e_ps_diff_degree)
```

```
       v  0 degrees  90 degrees  180 degrees
0  0.075     37.954      50.085       65.835
1  0.150     32.984      40.323       93.308
2  0.225     78.307      28.346      117.050
3  0.300    104.035      18.569      104.242
4  0.375     93.135      12.550       80.216
5  0.450     72.789       8.612       76.503
6  0.525     56.781       6.651       61.095
7  0.600     45.772       5.638       64.519
8  0.675     35.605       4.485       46.661
9  0.750     30.318       3.747       40.271
```

[94]:
```
e_ps_weighted_m.plot_diff_cross_vs_k(r_interval,spin_weight=True, velocity=True)
```



At 0 and 180 degrees there is a quick increase before a decrease with increasing velocity. At 180 degrees the slope changes repeatedly. At 90 degrees it is a simple gradual decrease with increasing energy.

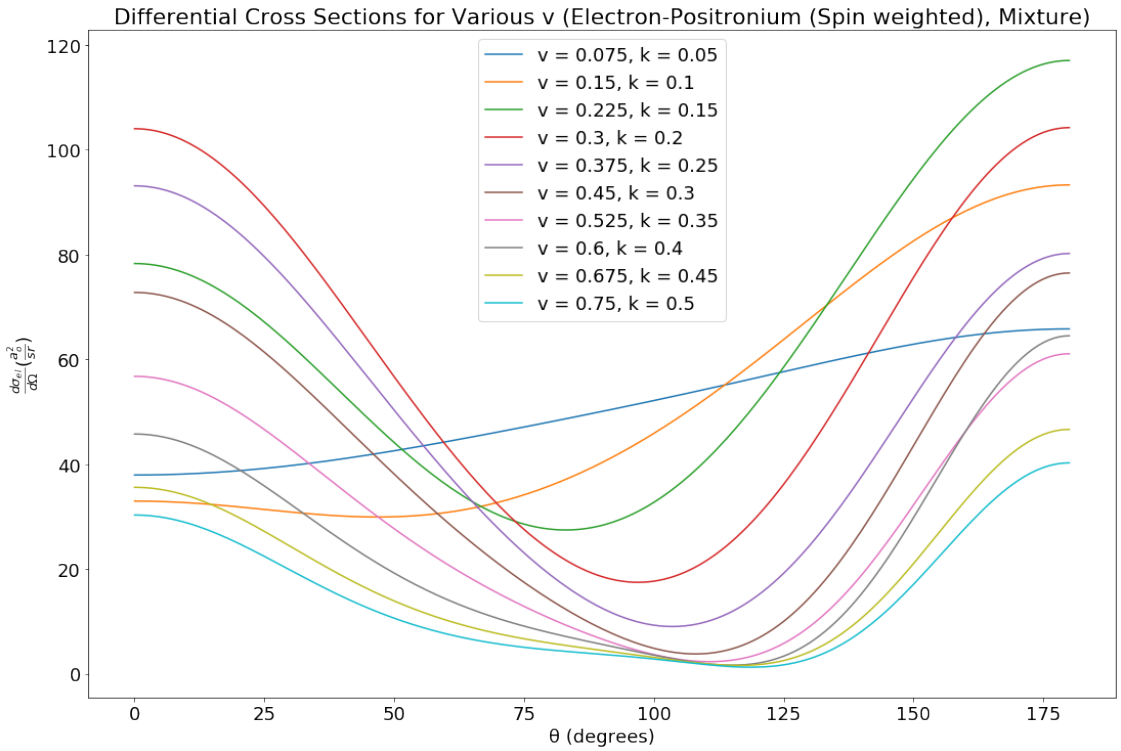
[95]:
```
e_ps_diff = e_ps_weighted_m.diff_plot(velocity=True, data=True)
sliced = e_ps_diff[e_ps_diff.index % 10 == 0 ]
display(sliced)
```

	degrees	v = 0.075	v = 0.15	v = 0.225	v = 0.3	v = 0.375	v = 0.45	\
0	0	123.342	83.128	51.464	33.699	27.017	27.763	
10	10	123.528	83.296	51.484	33.498	26.550	26.882	
20	20	124.044	83.751	51.522	32.938	25.264	24.457	
30	30	124.779	84.352	51.513	32.111	23.437	21.057	
40	40	125.574	84.890	51.336	31.076	21.356	17.366	
50	50	126.260	85.124	50.838	29.808	19.168	13.910	
60	60	126.699	84.845	49.888	28.222	16.864	10.890	
70	70	126.815	83.932	48.453	26.294	14.403	8.206	
80	80	126.612	82.392	46.656	24.203	11.928	5.718	
90	90	126.167	80.355	44.771	22.399	9.948	3.614	
100	100	125.606	78.039	43.155	21.550	9.351	2.702	
110	110	125.067	75.684	42.128	22.349	11.192	4.397	
120	120	124.665	73.495	41.872	25.253	16.307	10.280	
130	130	124.467	71.612	42.376	30.263	24.887	21.347	
140	140	124.475	70.103	43.447	36.832	36.208	37.234	

```
150      150    124.635    68.975     44.778   43.939     48.632    55.824
160      160    124.855    68.206     46.031   50.310     59.931    73.545
170      170    125.038    67.763     46.912   54.722     67.842    86.343
180      180    125.109    67.618     47.229   56.299     70.686    91.014
```

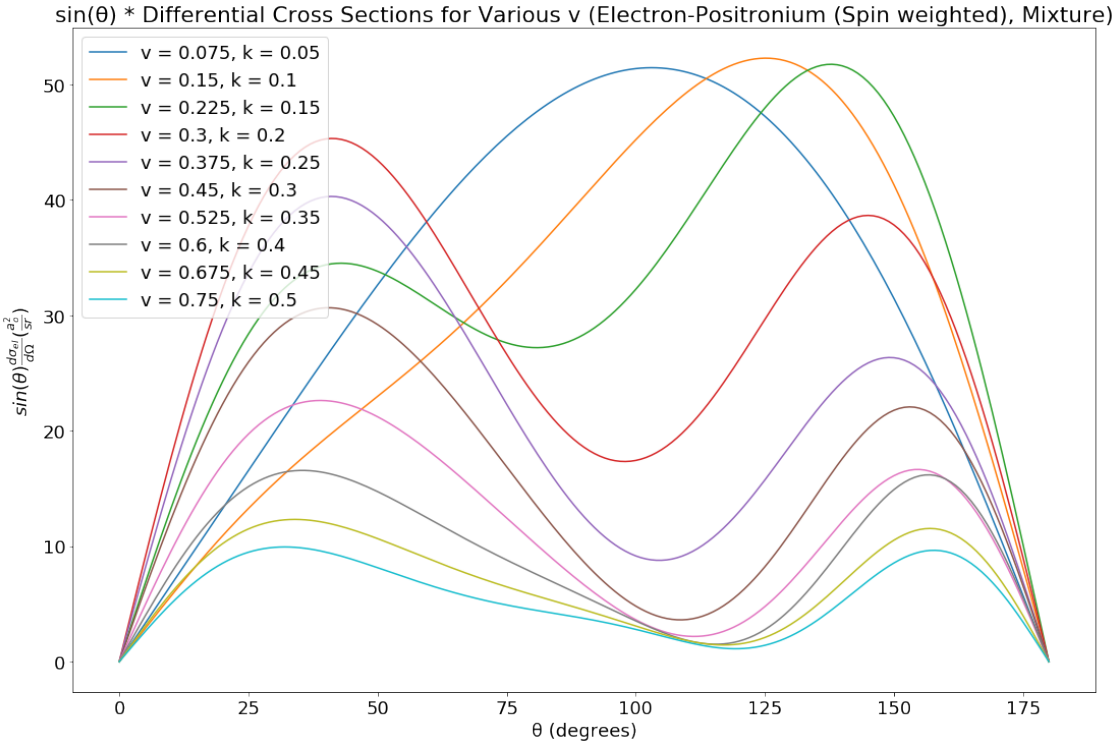
```
     v = 0.525  v = 0.6  v = 0.675  v = 0.75
0       26.285   27.968     22.623    16.726
10      25.315   26.769     21.670    15.995
20      22.644   23.448     19.027    13.968
30      18.893   18.753     15.280    11.098
40      14.831   13.680     11.204     7.992
50      11.093    9.156      7.526     5.226
60       7.971    5.739      4.688     3.161
70       5.404    3.460      2.737     1.832
80       3.200    1.925      1.412     0.997
90       1.412    0.729      0.456     0.387
100      0.671    0.013      0.020     0.058
110      2.247    0.857      0.934     0.663
120      7.678    5.127      4.569     3.390
130     18.036   14.656     12.222     9.452
140     33.114   30.016     24.219    19.274
150     50.964   49.533     39.222    31.829
160     68.129   69.230     54.217    44.560
170     80.597   83.977     65.383    54.124
180     85.161   89.456     69.520    57.682
```

[96]:
```
e_ps_weighted_m.diff_plot(spin_weight=True, velocity=True)
```

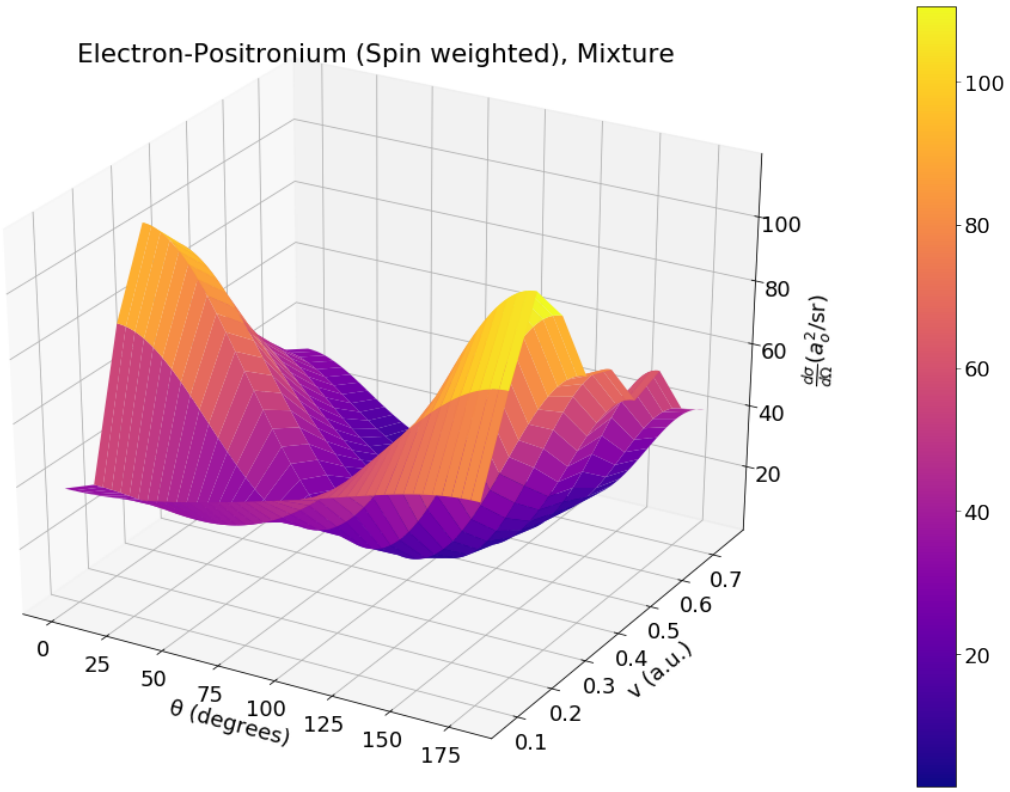


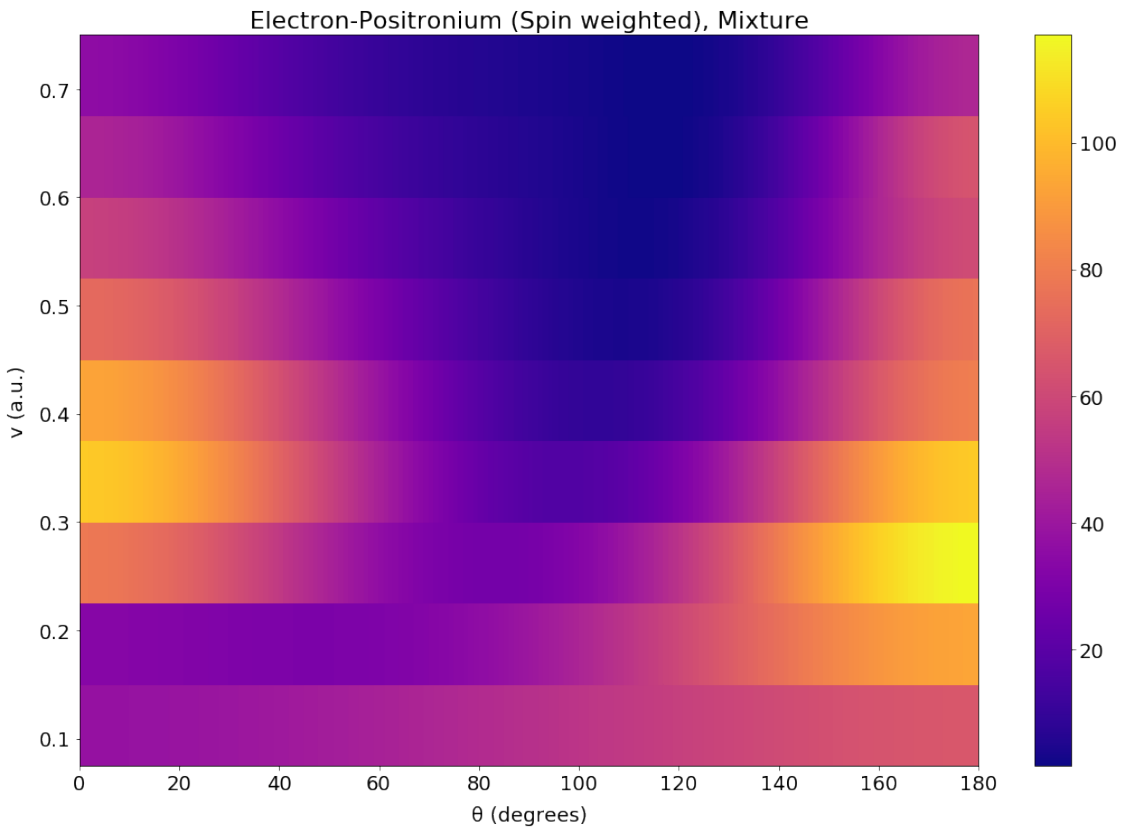
At low velocity, the curve is a mixture between linear and curved. At mid-range velocities the differential cross section quickly falls and then quickly rises. At higher velocities the decrease is slower but the increase is mostly just as quick as mid-range.

[97]:
```
e_ps_weighted_m.diff_plot(spin_weight=True, sin=True, velocity=True)
```



[98]:
```
e_ps_weighted_m.diff_plot_3d(spin_weight=True, velocity=True)
e_ps_weighted_m.diff_plot_3d(spin_weight=True, density=True, velocity=True)
```


Electron-Positronium (Spin weighted), Mixture
100
80
60
40
20
$\frac{d\sigma}{d\Omega}(a_o^2/sr)$
0
25
50
75
100
125
150
175
θ (degrees)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v (a.u.)
100
80
60
40
20



At degrees less than 60, the differential cross section quickly rises to a peak with increasing energy before falling again. At mid-range degrees, the differential cross section falls and remains low for increasing energy. At high degrees, the differential cross section increases even more rapidly than at low degrees before gradually decreasing with increasing velocity.

4 Conclusions

The elastic cross section of positronium-helium becomes increasingly similar to that of electron-helium with increasing velocity, within the velocity range examined. This is especially similar when compared to the massive difference when comparing the elastic cross section of positronium-helium to positron helium. This suggest that the positron in positronium has little influence for scattering at certain velocities. The p wave makes a significant contribution to the elastic cross section with increasing energy of positronium-helium, while it makes little contribution to that of electron-helium. The momentum transfer cross section declines faster than the elastic cross section for positronium-helium, which results in the momentum transfer of electron-helium being greater after v = 0.3. The shape of the differential cross section of one system is very similar to the other's inverted, though overall the magnitude for that of positronium-helium is greater. None of the cross sections for positron-helium are similar to electron-helium. The elastic cross section of positron-helium is never even 2% that of positronium-helium for all compared velocities. The p wave makes a greater contribution to the elastic cross section of positron-helium than that of positronium-helium. For velocity greater than 0.2, the shape of the differential cross section is

similar for the two systems, but the magnitude for positronium-helium is far greater. The shape of the elastic cross section for positron-hydrogen is similar to that of positron-helium at velocities less than 0.4. Outside of when both degrees and velocity is high, the shape of the differential cross section for positron-hydrogen and positron-helium are similar. However, at the intersection of high degrees and velocity, the positron-hydrogen's stays at a minimum while it increases in the positron-helium system.

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