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Proof-of-Search: Combining Blockchain Consensus Formation With Solving Optimization Problems

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ABSTRACT To address the large amount of energy wasted by blockchains, we propose a decentralized consensus protocol for blockchains in which the computation can be used to search for good approximate solutions to any optimization problem. Our protocol allows the wasted energy to be used for finding approximate solutions to problems submitted by any nodes (called clients). Our protocol works in a similar way to proof-of-work, and it makes nodes evaluate a large number of solution candidates to add a new block to the chain. A client provides a search program that implements any search algorithm that finds a good solution by evaluating a large number of solution candidates. The node that finds the best approximate solution is rewarded by the client. Our analysis shows that the probability of a fork and the variance in the block time with our protocol are lower than those in proof-of-work.

INDEX TERMS Peer-to-peer computing, distributed computing, grid computing.

I. INTRODUCTION

Since the introduction of Bitcoin [1], there have been many cryptocurrencies built on distributed public ledgers called blockchains. A blockchain is a growing list of blocks of data items that are designed to be resistant to modification of the data. In a cryptocurrency, each transaction is registered as an item in a block. Bitcoin uses a proof-of-work (PoW) system to decide which outcome is the correct outcome of the latest transactions and prevent double-spending of coins. In a PoW system, the participating nodes are asked to do some computational task to make a majority decision on a one-CPU-one-vote basis. While PoW works very robustly against misbehavior and malicious participants on a network where impersonation is easy, the very large amount of electricity expended by PoW systems is becoming a problem. The estimated energy consumed by Bitcoin was at least 2.55 gigawatts in 2018, which is comparable to the electricity consumed in countries such as Ireland (3.1 gigawatts) [2].

To address this problem, various alternatives to PoW and distributed public ledgers based on different working principles have been proposed. Although many of the alternative methods exhibit better energy efficiency than PoW, some of these methods introduce a single point of failure, or they have

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to trust an external party. The main advantage of PoW is that it does not need to trust anything except that it requires a majority of CPU power to be controlled by honest nodes. There are only a few alternative methods that satisfy this condition. Bitcoin and cryptocurrencies based on PoW are still predominant because of their unparalleled security and robustness.

In this paper, we propose a consensus protocol for blockchains, named proof-of-search (PoS). Our protocol allows computation for making a consensus to be used for finding a good approximate solution of an instance of any optimization problem. Our protocol allows a blockchain to be used as a batch processing system for solving optimization problems. Any user, called a client, can submit a job for finding a solution of an optimization problem. Our protocol has functionalities for submitting a problem instance and delivering the found solution.

The rest of this paper is organized as follows. Section II introduces related work, including Bitcoin and proof-of-work, along with alternatives to proof-of-work. Section III explains our proposed consensus protocol, including the main idea, job requests and execution, environment, and method of compacting a blockchain. Section IV discusses properties of our protocol including the requirements for the evaluator, probability of fork occurrence, and variance in block time. This section also describes the properties of PoS and

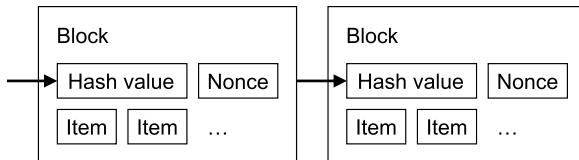


FIGURE 1. Distributed timestamp server in Bitcoin [1].

security considerations. Finally, Section V presents our conclusions, including some thoughts on uses for a blockchain with our protocol and the benefits of using our protocol.

II. RELATED WORK

A. BITCOIN AND PROOF-OF-WORK

Bitcoin [1] is a robust, secure and decentralized cryptocurrency. It is built on a peer-to-peer distributed timestamp server that generates computational proof of the chronological order of transactions. This proof is provided by a PoW system. The idea of a PoW was first proposed as a way to deter spam e-mails [3], [4]. In Bitcoin, PoW is used to enforce majority decision making on a one-CPU-one-vote basis on peer-to-peer networks where a user can allocate many IP addresses and one-IP-address-one-vote does not work.

The distributed timestamp server (Fig. 1) in Bitcoin works by forming a linked list of blocks of data items to be timestamped. This linked list is called a blockchain. Each block contains the hash value of the last block and the data items. Each time a new block is added to the chain, a hash value of the new block is computed and widely published. The PoW in Bitcoin involves finding a value whose hash value begins with a required number of zero bits. Each block has an entry for an integer value called a nonce, and only a block that has a nonce that makes the block's hash value begin with the required number of zero bits is accepted as a valid block. An incentive is provided to nodes that support the network such that a new coin is given to a node that succeeds in adding a new block. Network nodes that try to add blocks are called miners. Honest miners try to add a new block to the longest chain. As long as a majority of CPU power is controlled by honest miners, an honest chain will grow the fastest. In this way, a majority decision is made. The time required for a block to be added is called the block time. The required number of zero bits is automatically adjusted to make the expected block time 10 minutes.

Bitcoin is not a mere online payment system; it aims at replacing a currency. For this purpose, the system has to be extremely robust, and it does not depend on any party or computing node. The Bitcoin entity is the data in the network. It will never vanish into nothing as long as there are enough honest nodes. Bitcoin has the following properties:

- Decentralized and self-regulated
- No need to trust any node or any party
- Hard to modify the data in the blocks
- Immune to Sybil attacks
- The winning probability for each miner is proportional to its computational power

- Legitimacy of blocks can be quickly verified at anytime by any node
- Any node can join at anytime without preregistration

However, Bitcoin requires miners to use their computational resources for PoW, which is basically the repeated calculation of hash values. This requirement is a waste of computational resources and electricity, which could be applied to useful work.

B. ALTERNATIVES TO PROOF-OF-WORK

To address the wasted computational resources and energy used by Bitcoin, various alternative consensus protocols for blockchains have been proposed.

Proof-of-stake and proof-of-burn [5] are techniques used in some of the recently developed cryptocurrencies.

Proof-of-stake, which was first implemented in Peercoin [6], is a consensus protocol that chooses the creator of the next block based on combinations of random selection and wealth or age. With this protocol, the node that has more coins will create blocks more often, and thus more coins are granted. In this protocol, the designated creator of the next block has to be trusted. The richest nodes are more likely to be selected, and thus they have control of the network. Proof-of-activity [7] is a combination of PoW and proof-of-stake. In this scheme, miners work on PoW to add an empty block header. In this header, a random group of validators are designated in the same way as in proof-of-stake. These validators are asked to sign the new block. If the new block is signed by all the chosen validators, it is added to the chain. The advantage of proof-of-activity is that it requires both a majority of CPU power and a majority of coins to take control of the cryptocurrency.

Proof-of-burn [5] is a consensus protocol that gives a right to vote to a node if it sends its coin to a special address where the coin cannot be redeemed. Obviously, nodes that have the right to vote have to be trusted in this scheme. There is an interesting use of proof-of-burn that moves coins from one cryptocurrency to another cryptocurrency. To do so, the coins are sent to the address specific to the destination cryptocurrency where the coins cannot be redeemed with the original cryptocurrency. Then, a new transaction is added to the destination cryptocurrency, indicating that the user received the equivalent amount of coins, which refers to the transaction in the original cryptocurrency.

With proof-of-useful-work [8], Gridcoin [9] and Perma-coin [10], computational resources in consensus formation can be used for more meaningful purposes.

In proof-of-useful-work [8] and Primecoin [11], the computation for making a consensus is used for more meaningful purposes than the calculation of hash values. Proof-of-useful-work uses the computation to solve the Orthogonal Vectors problems, while Primecoin makes a consensus by searching for chains of prime numbers. However, it is not clear how much public demand there is to find solutions to such problems.

TABLE 1. Strengths and weaknesses of consensus protocols.

Protocol	Strength	Weakness
Proof-of-work	Extremely robust	Wastes electricity and computational resources
Proof-of-stake	No waste of electricity	The richest nodes have control of the network
Proof-of-activity	Requires both majority of CPU power and majority of coins to take control of the network	In-between PoW and proof-of-stake
Proof-of-burn	No waste of electricity	Nodes that have the right to vote have to be trusted
Proof-of-useful-work	Consensus is made by solving the Orthogonal Vectors problems	Unclear public demand for solving such problems
Primecoin	Consensus is made by searching for chains of prime numbers	Small public demand for finding prime numbers
Gridcoin	Rewards miners who perform computation on BOINC	Depends on BOINC
Permacoin	Mining resources can be used for distributed storage of archival data	High redundancy of data storage is required
Tendermint	No waste of electricity	Utilizes proof-of-stake approach to prevent Sybil attacks
Proof-of-space	Allows mining by storing data	Exhibits unique weaknesses
Proof-of-luck	No waste of electricity	Requires trusted execution environment
Proof-of-elapsed-time	No waste of electricity	Requires trusted execution environment
Proof-of-search	Consensus is made by solving optimization problems	Falls back to PoW if there is no problem to solve

Gridcoin [9] implements a proof-of-research scheme, which rewards miners who perform computations on the Berkeley Open Infrastructure for Network Computing (BOINC) [12]. The computation for the consensus protocol is used for scientific computations, so the energy is used for very meaningful purposes; however, the cryptocurrency system has to depend on the BOINC system. This means that Gridcoin will cease to work if the BOINC system goes down, and thus, Gridcoin is less robust than Bitcoin.

In Permacoin [10], mining resources are used for distributed storage of archival data. Successfully minting money in this system requires random access to a local copy of a file. To mine coins, a miner needs to prove that the archive file is intact with a proof-of-retrievability, which is an interactive protocol for cryptographically proving the retrievability of data. However, in order to guarantee access to stored data at any time, high redundancy of data storage is required.

Proof-of-space [13] is a protocol between a prover and a verifier where the prover is supposed to store some large amount of data. A verifier asks a prover to send a piece of data to check whether it is still storing the data. The protocol is designed to make the computation, storage requirements and communication complexity of the verifier small. To use proof-of-space in a decentralized blockchain, a way of determining the winning node and a way for each miner to know how likely it is to win are required. The probability of winning should be proportional to the space allocated for data storage in each node. These practical considerations are discussed in [14]. As mentioned in the paper, proof-of-space has its own weaknesses. One of the problems is that nodes can mine on multiple chains simultaneously. Miners can also try creating many different blocks with a single proof-of-space by altering the block contents slightly and announcing the most favorable one.

Proof-of-luck [15] and proof-of-elapsed-time [16] utilize a trusted execution environment (TEE) to form a consensus. A TEE is special hardware that executes software securely with respect to confidentiality and integrity. With this kind of hardware, user interference with the consensus process can

be avoided, and thus a consensus protocol can be realized relatively easily. However, only software signed by a trusted party can be executed on a TEE.

Tendermint [17] is a consensus protocol for blockchains without mining. This protocol uses a Byzantine fault tolerance algorithm to form a consensus among a known set of participants. This is resilient to up to 1/3 of Byzantine participants being dishonest. This protocol uses a proof-of-stake approach to prevent Sybil attacks.

Nano [18] utilizes distributed acyclic graphs (DAGs) to store transactions. In a DAG, transactions are stored in nodes where each node corresponds to a single transaction. A conflict is resolved by a majority vote among representatives chosen by the participants. Each vote has a weight calculated as the sum of all balances of the participants who chose this representative.

The main contribution of this study is that we propose a truly decentralized consensus protocol for blockchains that allows the computational power wasted in proof-of-work to be used for a more meaningful purpose than proof-of-useful-work [8] and Primecoin [11]. Unlike Gridcoin [9], our protocol does not depend on any party or computing node, and thus, our protocol is more robust. The working principle of our protocol is close to that of PoW, and thus it does not need to trust any node to work correctly.

The strengths and weaknesses of the consensus protocols are summarized in Table 1.

III. PROPOSED PROTOCOL

In this paper, we propose a new decentralized consensus protocol for blockchains, called proof-of-search (PoS), which has all the advantages of proof-of-work explained in II-A. PoS allows the computational power wasted in proof-of-work to be used for searching for an approximate solution of an instance of an optimization problem. It allows any node to submit a job that includes a program called an evaluator that evaluates a solution candidate of a problem to be solved. In our protocol, a concatenation of a solution candidate and its evaluation value is used as a nonce instead of an integer.

To generate a valid nonce, a node has to execute the evaluator to evaluate some solution candidate. Since a large number of nonces have to be generated in the consensus process, a large number of solution candidates have to be evaluated. To prevent collusion between nodes, our protocol has two separate ways of rewarding nodes. A node that succeeds in adding a new block is rewarded in the same way as in PoW. A node that succeeds in finding the best solution of an optimization problem is given the charge paid by the corresponding client. A client can submit a job without mining, and a miner is not required to submit a job. If no job is submitted to a PoS-based blockchain, it automatically adds an empty job, which makes PoS work in a similar way to PoW.

In this paper, a solution means an approximate solution. To solve a problem is to find an approximate solution of the problem.

A. KEY IDEA

We first introduce the concepts of an *evaluator*, a *client* and a *job*. An *evaluator* is a computer program that deterministically computes the evaluation value of a given solution candidate of an instance of an optimization problem. An evaluator has to always output the same value if the same input is given regardless of the platform it runs on. An evaluator includes an instance of a problem. A *job* is data that represents an execution request for a search for a solution of an optimization problem. A job includes an evaluator and all necessary information regarding the search request. Any node can submit a job to a PoS-based blockchain, and the node submitting a job is called a *client*. A job can be submitted to the system by registering the job on the blockchain. The ID of the client is also included in a job. For example, a client can implement an evaluator to evaluate a solution candidate of an instance of the traveling salesman problem (TSP). In this case, an order for visiting the cities is an input for the evaluator, and the evaluator outputs the total length of the tour.

In PoS, a concatenation of a solution candidate and its evaluation value is used as a nonce. A PoS-based blockchain chooses an evaluator from submitted jobs and specifies which evaluator is used to generate a valid nonce to add the next block. To add a new block, a mining node has to find a nonce that makes the block's hash value begin with a required number of zero bits, in the same way as in a PoW. However, unlike PoW, not every nonce is valid. To generate a valid nonce, a node executes the specified evaluator to evaluate some solution candidate. A valid nonce has to contain a solution candidate and its evaluation value. Since miners have to generate a large number of hash values to add a new block, we can enforce that miners evaluate a large number of solution candidates. By requiring a valid nonce that makes the block's hash value begin with a required number of zero bits, the system provides a probabilistic proof that the miners have evaluated a large number of solution candidates. To verify a hash value means to execute the evaluator with the solution candidate in the nonce and confirm that the resulting evaluation value matches the one included in the nonce. Then,

the hash value of the block is also checked to see if it begins with the specified number of zero bits. To make verification quick, evaluation also has to be quick.

There are two objectives for evaluation. One is to find a nonce that begins with a required number of zero bits. Another objective is to find a good solution with a good evaluation. In PoS, we assume that a large number of solution candidates have to be evaluated to find a good solution. A node that succeeds in finding the best solution among all nodes will be rewarded by the client. To make this search efficient, a client provides a computer program called a *searcher* that implements a randomized search algorithm such as a genetic algorithm. A searcher is included in a job, and executed by miners. Internally, it calls the evaluator many times. Each time an evaluation is made inside the searcher, it automatically calculates the hash value of the block to check whether it begins with the required number of zero bits and broadcasts it if it does. The search continues until a new block is added.

In PoW, there is no need to guard against reuse of the results of computations from the past. This is because the only way to obtain a hash value is to compute the hash function and because the ID of the miner and the last block are associated with the hash value. In PoS, however, we need to guarantee that an evaluation is made each time a block's hash value is generated. Because the amount of computation in an evaluation can be substantially larger than that for calculating a hash value, miners might try to reuse the results of evaluations by sharing them among different nodes. To prevent this, we have to associate the result of evaluation with the ID of the miner and other data. To do this, we make the evaluator take the hash value of all the items in a block except the nonce itself as the second argument. A tiny amount of error is introduced in the output of an evaluator to make the output depend on the second argument. The algorithm for introducing this error has to be devised and implemented differently by each client. This is like using an evaluator as a substitute for a hash function. However, such a property is not strictly required for an evaluator. If an evaluator is executed twice with the same solution candidate given for the first argument and different values for the second argument, then it should be infrequent that the same value is returned. However, this is not strictly prohibited. This will be further discussed in IV-A.

Fig. 2 shows the data structure of a blockchain with a minimal PoS scheme. This minimal scheme works with a single fixed evaluator, and it does not have a functionality for searching for a good solution. A blockchain with this scheme works in a similar way to a blockchain with PoW. In the next subsections, we describe how to enhance this minimal scheme to add functionalities for submission of jobs, payment and execution of multiple jobs.

B. PLACING A JOB REQUEST

We want the following three properties in PoS.

- A miner has a financial incentive to find and provide a good solution.

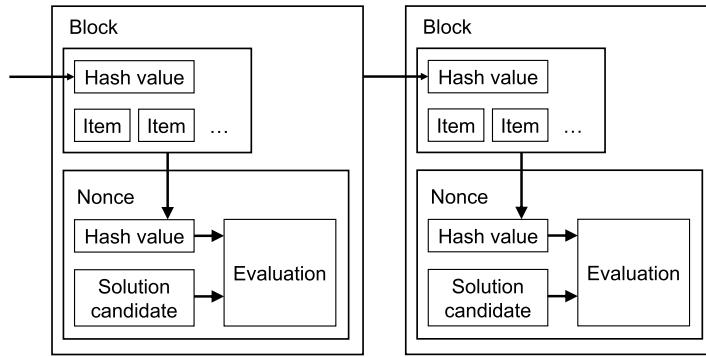


FIGURE 2. Distributed timestamp server in a minimal PoS scheme.

- A node is not incentivized to submit a problem instance for which it already knows a good solution.
- Submitting a problem instance that is not worth solving is financially discouraged.

We especially need to prepare for the case where a client knows a good solution for its job. The possible motivations for submitting such a job are listed below.

- 1) It is advantageous in adding a block.
- 2) It is rewarded by minted coin.
- 3) The node can make a profit by finding a good solution.

We need to give honest miners a financial incentive to find a good solution while preventing malicious miners from making a profit. In PoS, this is realized by making a client pay a charge for its job. Finding the solution with the best evaluation is only rewarded by this charge, and in this way, PoS will have all the properties listed above. For item 1, knowing or finding a good solution is not advantageous in adding a block, as explained in III-A. For item 2, finding a good solution is not rewarded by minted coins. For item 3, a client cannot make a profit by finding a good solution with the job it submitted. Since a client has to pay the charge for its job, a client will only submit a problem instance that is worth the charge.

We want to make the payment process fully automated without trusting any node. To ensure that the charge is paid, it is collected before the execution of a job. A client first submits a job including the charge. Then, the PoS-based blockchain automatically deducts the charge before execution of the job and pays it to the winner after completion of the job.

The found solution has to be sent to the client. If a node simply broadcasts its solution to the network, this solution can be stolen by another node. If a node encrypts the solution with the public key of the client, the corresponding private key has to be published afterward, and the client has to be trusted to do that. To make sure that the node that found the best solution is paid automatically without having the solution stolen by other nodes and without trusting any node, each node first registers the evaluation value of the found solution and the hash value of a concatenation of its ID and the solution on the blockchain. Then, in the next block time, each node checks whether its solution is the best. The winning node registers its

solution on the blockchain, and then the charge is paid to the winning node after confirming that the solution is genuine.

C. SIMULTANEOUS EXECUTION OF MULTIPLE JOBS

To make the charge reasonable, we make miners execute multiple jobs at a time. We also want to make the winning probability of each node proportional to the computational power spent for the job. A simple way of realizing these properties might be adjusting the block time according to the job size. However, this method has the problem of an increased probability of fork occurrence because the probability depends on the block time. In order not to increase the probability of fork occurrence, we add miniblocks between blocks without changing the block time. Each miniblock corresponds to a job.

A miniblock consists of only a nonce, the ID of the mining node, and the hash value of the last block, as shown in Fig. 3. Miners try to find a valid nonce of any miniblock that makes the miniblock's hash value begin with the required number of zero bits. As explained in III-A, a valid nonce contains a solution candidate for the corresponding job and its evaluation value. Each time such a nonce is found, the node adds the corresponding miniblock by broadcasting the miniblock with that nonce. A block is added when all the miniblocks are added for all the jobs specified by the PoS-based blockchain. New coins are awarded to all the nodes that add the miniblocks. Verification of hash values requires repeating the process explained in III-A for each miniblock.

Because of message delivery delay in a network, there could be a difference in the sets of messages received by two different nodes. Consequently, two mining nodes may have different sets of items to be included in a new block. Thus, two different nodes may be executing the jobs to add different blocks. In other words, these nodes are executing their jobs to add miniblocks that contain the hash values of different blocks. To prevent a mini-fork and make a larger number of nodes work to add the same block, we make miners execute jobs on the longest chain whenever possible. Here, one chain is longer than another if it has more blocks. If two chains have the same number of blocks, the chain with more miniblocks after the last block is longer. When a node receives a new miniblock added by another node, it checks whether the chain

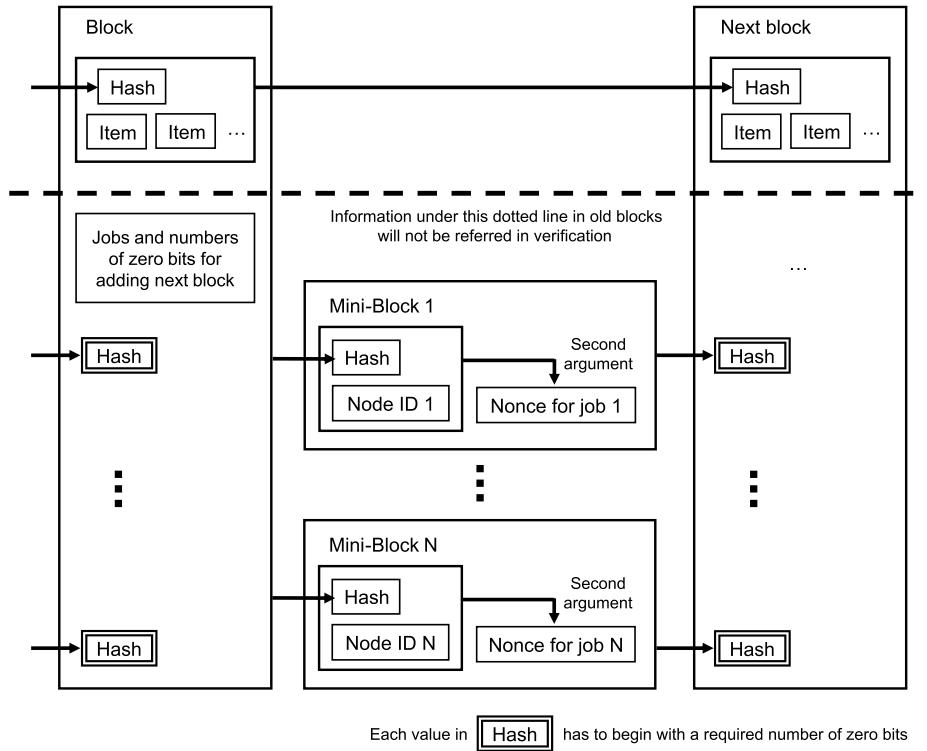


FIGURE 3. Distributed timestamp server in the proposed method.

associated with the new miniblock is longer. If this chain is valid and longer, the node immediately starts executing jobs on it. When a node adds a new miniblock, it also broadcasts the corresponding block and all the added miniblocks that come after the block.

We make the expected amount of computation for each job proportional to the charge. It is fairly straightforward to realize this by making the PoS-based blockchain adjust the required numbers of zero bits in the hash values according to the charge. We now assume that the same amount of computation is required in an evaluation for each job. The system knows the average total charge C of incoming job requests per block time by scanning the requests placed in the past. The system also knows the average number E of evaluations made by all miners per block time by checking the block times and the numbers of zero bits of the past miniblocks. Let c_j denote the charge for job j . The system should choose the combination of jobs to satisfy $\sum_j c_j \approx C$. Then, the number z_j of zero bits for job j should be set to satisfy $2^{z_j} \approx c_j \cdot E/C$. The node that succeeds in finding the nonce that makes a miniblock's hash value begin with the required number of zero bits is rewarded in the same way as in PoW. However, the total amount of reward for adding new miniblocks in a block time has to be kept constant, which we denote by R . In order to do this, the reward for adding the miniblock corresponding to job j is set to $R \cdot 2^{z_j} / \sum_k 2^{z_k}$.

D. EXECUTION ENVIRONMENT

To implement PoS, an execution environment is needed for evaluators and searchers. To make the amount of computation

for each job proportional to the charge, we need to compensate for the difference in the amount of computation of evaluation for each job. For example, if the evaluator for job j_1 takes two times the execution time that the evaluator for job j_2 takes, then 1 more zero bit should be required in the hash value with j_2 . Since execution of the evaluator has to be deterministic, we need a platform-independent way of counting the number of steps. The number of steps can be, for example, the number of bytecode instructions executed on a virtual machine. The execution environment has to have a functionality to count the number of steps in the execution of an evaluator. The required number of zero bits for each miniblock has to be adjusted according to the measured number of steps in the execution.

Since any user can submit a job, an evaluator can be inappropriately implemented. In case the evaluator takes too many steps for execution, there must be a way to terminate this execution after a specified number of steps. To make execution of the evaluator deterministic, this step count has to be exact. If an evaluator crashes or is terminated, it is regarded as returning the worst evaluation.

It is possible that a searcher takes too much computation compared to the evaluator. This can be easily detected by checking the number of steps. In this case, a miner is allowed to switch the search algorithm to a simple random search. By doing this, the node can increase its hash rate while it would be less likely to find the best solution.

In summary, the execution environment has to satisfy the following conditions.

- It allows safe execution of untrusted code.
- It provides a way to guarantee that an evaluator runs deterministically.
- It counts the number of steps of execution in a platform-independent way.
- It returns the number of steps after execution.
- It terminates execution after a specified number of steps.

Implementing an interpreter-based virtual machine for the execution environment satisfying all the above conditions should be straightforward. Execution can be made deterministic by not providing APIs that make execution non-deterministic.

E. COMPACTING BLOCKCHAIN

With the method explained above, all the evaluators recorded in a blockchain have to be executed to verify a chain. However, the size of an evaluator will be significantly larger than a hash value, and the storage space for keeping all the evaluators can become a heavy burden in managing a blockchain. The required storage size can be reduced by relaxing the requirements for verification of a chain. If the participants think that it is sufficient to verify a certain number of blocks, then the information regarding the jobs recorded in the older blocks can be discarded. In Fig. 3, only the hash value above the dotted line is checked during verification of old blocks, and therefore the information below the dotted line can be discarded for old blocks. Even with this relaxed method of verification, it is very hard to modify the items in the old blocks without redoing all the PoS for the new blocks.

F. PUTTING THEM ALL TOGETHER

Our protocol is an enhancement of PoW, and thus, it uses common techniques with PoW. A blockchain with our protocol is structured as a peer-to-peer network. The entire network is loosely connected without a fixed topology. In order for a node to join a network, it has to know one of the nodes that is already part of the network. Each node connects to several random nodes. A message is broadcast with a gossip protocol. Each node retains a copy of the entire information of a blockchain.

Our protocol allows any client to submit a job and receive the solution. This is very simple from the point of view of a client, as shown in Algorithm 1.

Executing a job and making the resulting payment take at least 4 block times. The following is how a job is processed in the fastest scenario.

Algorithm 1 Place a Job Execution Request

Input: Job request q

Output: Best found solution s

- 1: Register q to the blockchain.
 - 2: Wait until a solution s is received.
 - 3: **return** s
-

Block time 0 A job is broadcast by the client.

Block time 1 Validity of the job is inspected. The charge is deducted from the client's account. This job is chosen for execution in block time 2.

Block time 2 The job is executed.

Block time 3 Each node broadcasts the hash value of the found solution. These hash values are registered on the blockchain.

Block time 4 The charge is paid to the node that found the best solution.

Job execution and payments are all processed by miners. Miners process jobs in a pipelined manner. Algorithm 2 shows how they are processed from the miner's point of view.

There are two threads running in parallel, and Algorithm 2 runs on one of them. In the other thread, the received items are enqueued in the list of newly received items. To register an item on the blockchain, it has to be broadcast to the network. When these items are received by a miner, they are enqueued in the list.

After Algorithm 2 starts, the miner first chooses the chain to work on (line 1-2). There can be multiple valid chains, and they all begin with the same block. One specific sequence of block, or a chain, can be specified by the last block. The miner starts working on the chain associated with the first block received.

At line 5, the balance of the client's account is checked.

At line 8, to make all the miners work on the same set of jobs, a consensus has to be made on the set of jobs and the corresponding numbers of zero bits before the miners start working on them.

At line 16, a searcher is executed in the execution environment explained in III-D. As an example of a searcher, a random search algorithm is shown in Algorithm 4. As explained in III-A, this searcher internally calls the corresponding evaluator. Each time an evaluator is called, it automatically creates the nonce with the result of evaluation and checks the hash value to see if it begins with the required number of zero bits. If it does, the new miniblock is broadcast, and the execution of the searcher is terminated. The execution of the searcher also terminates if a new miniblock is added by another mining node. Please note that when a new block is added, the last miniblock for that block is also added.

At line 18, the chain associated with a miniblock is verified. This procedure is shown in Algorithm 3. This algorithm works as explained in III-C and III-E. There is no need to verify blocks and miniblocks if exactly the same blocks or miniblocks are included in a chain previously verified by that node.

At line 21, the items contained in orphaned blocks are moved to the list of newly received items. An orphaned block is a block that was a part of the chain worked on by the miner, but is no longer a part of the chain because the miner is now working on another chain.

Algorithm 2 Mine

```

1: Wait until a new block is added.
2: Set activeChain to the added block.
3: while true do
4:   {Process for the jobs that will be executed in the next
   block time}
5:   Create the list of valid unexecuted jobs.
6:   Choose the jobs that will be executed in the next block
   time.
7:   Deduct the charge of these jobs from the client.
8:   Include these jobs in the block being added.

9:   {Process for the jobs that is executed in the current
   block time}
10:  Verify newly received items (transactions) and put
    them in the block being added.

11:  {Process for the jobs that were executed two block
    times before}
12:  Check the blockchain to see whether the solutions
    found two block times before were the best solutions,
    as explained in III-B. Register the solutions to the
    blockchain if they were the best.

13:  {Process for the jobs that were executed three block
    times before}
14:  Check whether the registered solutions are genuine.
    Process payment of the corresponding charge.

15: repeat
16:   Execute one of unfinished jobs listed in the previous
    block, until a new miniblock is received.
17:   Register the hash value of the found solution to
    the blockchain (see III-B). This hash value will be
    included in the next block.
18:   Verify that the chain associated with the received
    miniblock by calling Algorithm 3.
19:   if the miniblock is valid and the corresponding chain
    is longer then
20:     Change activeChain to the chain with the
    miniblock.
21:     Move the items in orphaned blocks to the list of
    newly received items.
22:   end if
23:   until a new block is added
24: end while

```

G. ILLUSTRATIVE EXAMPLE

We now explain how a blockchain with PoS can be utilized from the users' point of view.

Suppose that a client has an instance of an optimization problem to be solved. He implements an evaluator and a searcher for the problem and decides the amount of charge to pay for solving this problem. He then constructs a job by combining the evaluator, the searcher and the charge and

Algorithm 3 Verify

Input: Miniblock *b*

Output: Verifies the chain corresponding to *b* and returns *true* iff it succeeds.

```

1: for all blocks k in the chain pointed by b in chronological
   order do
2:   if the same block as k is included in a previously
   verified chain then
3:     continue
4:   end if
5:   Check all the transactions in k and return false if it
   fails.
6:   if k is old then
7:     Do relaxed verification explained in III-E and return
     false if it fails.
8:   else
9:     for all miniblocks m between k and the next block
     do
10:    if the same miniblock as m is included in a previ-
        ously verified chain then
11:      continue
12:    end if
13:    Execute the evaluator and check whether the
        nonce and all the hash values satisfy the corre-
        sponding requirements and return false if it fails.
14:  end for
15: end if
16: end for
17: return true

```

submits this job as explained in Algorithm 1. The charge is automatically withdrawn after registering the job. The expected amount of computation for this job is proportional to the charge. Miners work for the blockchain to find a good solution to the problem. The found solution is eventually registered to the blockchain, and thus, the client will obtain the found solution in exchange for the charge.

We assume that there are always many miners trying to add a new block to the chain. Suppose that a new block time for the longest chain has just started. The chain provides multiple jobs to work on, and a miner chooses a job from these. If there is no job submitted by clients, it falls back to PoW, and empty jobs for finding a nonce that makes the block's hash value begin with the required number of zero bits are automatically inserted. The miner extracts a searcher and an evaluator from the job and executes the searcher. Upon execution of the searcher, the searcher calls its evaluator many times, and the searcher generates and evaluates many solution candidates. When a miner finds a good solution candidate, it keeps that solution candidate for later use. Each time a miner evaluates a solution candidate, it also generates a valid nonce, and the miner calculates the hash value of the miniblock including this nonce. If the hash value begins with the required number of zero bits, then the miner broadcasts the miniblock

Algorithm 4 Random Search

Input: Hash h of the block to be added, node ID id , evaluator ev

Output: The best solution candidate found in the search

```

1: bests := null, beste = null
2: repeat
3:   Generate a random solution candidate  $s$ .
4:    $e := ev(s, hash(h, id))$ 
5:   if  $e$  is better than beste then
6:     beste :=  $e$ , bests :=  $s$ 
7:   end if
8:    $h := hash(h, id, s, e)$ 
9:   if  $h$  begins with the required number of zero bits then
10:    broadcast the miniblock.
11:    break
12:  end if
13: until a new miniblock is received
14: return [bests, beste]

```

containing the found nonce, in addition to the last block. A miner that succeeds in adding a new miniblock is rewarded in the same way as in PoW. When a miner notices that a new miniblock is added, the miner starts working on a new miniblock that comes after the previous block broadcast with the new miniblock in order to prevent a mini-fork. A block time continues until all the miniblocks are added. After a block time ends, a miner registers his best found solution to the blockchain. The node that succeeded in finding the best solution for a job is given the charge included in the job.

Now, suppose that a client and a miner are colluding to obtain an unfair amount of coins. In order for this miner to add a new block, he has to find a valid nonce that makes the hash value begin with the required number of zero bits. This is essentially picking a random solution candidate and calculating the hash value. The problem and its solution do not matter for this, and therefore there is no merit in a client and a miner to colluding. On the other hand, a client can carry out computation for its job before submitting it and tell the solution to a colluding miner. However, there is no benefit for doing this since the resulting reward is paid by the client to the colluding miner. This is effectively the client sending its coin on the colluding miner, which is a normal transaction in a cryptocurrency.

We now summarize the flow of coins and the incentive mechanism.

Incentive for finding the best solution to a job: Each client pays the charge when it registers its job. This charge is automatically withdrawn upon registration and kept by the system until the completion of the job. The registered job is eventually executed by miners. After execution of the job,

the system knows which miner found the best solution for each job. The system pays the kept charge to the account of the miner who found the best solution.

Incentive for adding a new miniblock: The system decides the required number of zero bits in the hash value for each miniblock according to the charge paid for the job, as explained in III-C. When a block time begins, each miner chooses a miniblock to work on, and executes the corresponding searcher. A large number of nonces are generated upon execution of the searcher. Each time a nonce is generated, the miner checks whether the hash value of the miniblock including the generated nonce begins with the required number of zero bits. If it does, the miner succeeds in adding a new miniblock, and the miner receives the reward for the new miniblock. The amount of this reward is set according to the required number of zero bits in the hash value, as explained in III-C.

IV. CONSIDERATION

A. REQUIREMENTS FOR EVALUATOR

The whole scheme of PoS depends on the way it enforces an evaluation each time a hash value is generated.

We now consider an attack where one of the items in Fig. 2 is altered by a miner. Altering an item in a block is not always easy, but here we assume a pessimistic scenario in which the miner can alter an item freely. Suppose that an evaluator returns the same value at a rate of once per u times if the same first argument and a different second argument are given. The miner could try calculating the hash value of a block faster than it should in the following way.

- 1) The miner alters the item.
- 2) Calculate the hash value of the block assuming that the evaluator returns the same value.
- 3) Check whether the hash value begins with the required number of zero bits.
- 4) If it does, then execute the evaluator to check whether it returns the assumed value.

The above attempt succeeds at a rate of once per u times. The rate of generating the valid hash value of a block is higher with this method if evaluation is very slow and u is small. To prevent it from paying off, the following condition has to be satisfied.

B. PROBABILITY OF FORK OCCURRENCE

A fork in a blockchain is a situation where there are two or more valid chains with the same length. In Bitcoin, a fork occurs when a mining node adds a block before knowing that another node has already added a block. This can happen because of message delivery delay in a network.

Now, we discuss how the probability of fork occurrence is affected by introducing miniblocks. For the sake of sim-

$$u > \frac{\text{Amount of computation in evaluation and hash calculation}}{\text{Amount of computation in hash calculation}}$$

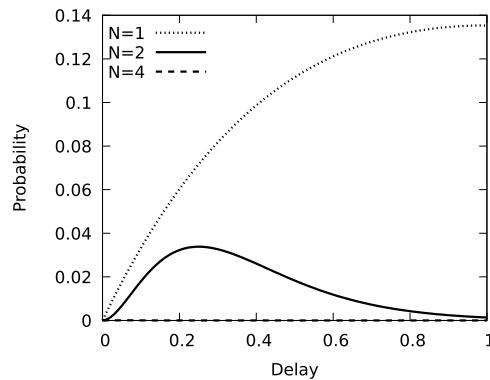


FIGURE 4. Probability of a fork after a block is added, with N miniblocks.

plicity, we suppose that there are only two mining nodes in the network. A communication channel with a constant delay d connects them. We can model block creation as a Poisson process. Now, suppose that node A has just created a block right now. Then, the probability p_1 of a fork occurring within time d is the probability that the other node B creates another block within this time period, while A does not create a block in this period.

$$p_1 = e^{-\lambda d} (e^{-\lambda d} \lambda d),$$

where λ is the average number of block creations per interval . node.

We now assume that all miniblocks require the same expected amount of computation to be added. The probability p_N of a fork with length N after a creation of some block is the probability that this event happens N times in succession.

$$p_N = \left\{ e^{-\lambda d} (e^{-\lambda d} \lambda d) \right\}^N \quad (1)$$

In the proposed method, N miniblocks are required to be added before adding a new block. Therefore, the above event has to succeed N times in order for a fork to occur.

We now assume that all miniblocks have the same creation rate $\lambda = N$, where N is the number of miniblocks required to add a block. Equation 1 is plotted in Fig. 4. It is shown that the probability of a fork can be significantly lowered by increasing the number of miniblocks.

C. VARIANCE OF BLOCK TIME

Now we discuss how the variance in the block time is affected by introducing miniblocks. The probability $b(t)$ that a block is added within time period t is the probability that N or more miniblocks are added within this period. We have N miniblock creations per interval on average; thus,

$$b(t) = 1 - e^{-Nt} \sum_{i=0}^{N-1} \frac{(Nt)^i}{i!}. \quad (2)$$

Fig. 5 shows the probability that a block will be added within a time period. The expected block time is 1, and the variance is $1/N$. This means that the variance in the block time decreases as the number of miniblocks increases.

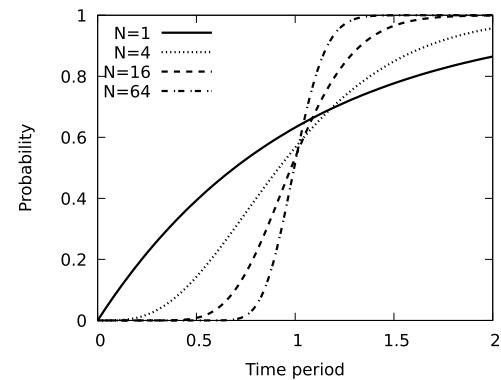


FIGURE 5. Probability that a block is added within a time period with N miniblocks.

D. WINNING PROBABILITY OF A NODE

In Bitcoin, the winning probability for each miner is proportional to its computational power. This is because when a block is added, the probability that each node is the node that added the block is proportional to the number of hash values generated by that node.

Now, we discuss the winning probability of each node to add each miniblock. Suppose that there is no message delivery delay and that all nodes share the latest information. When a new block is added, the total computation needed for adding all the corresponding miniblocks is the same as the total computational power spent by miners in that block time. The probability $w_{n,k}$ that node n adds miniblock k is as follows.

$$w_{n,k} = \frac{\text{Computation power spent by } n \text{ for } k}{\text{Total computational power spent for } k}$$

E. PROPERTIES OF POS

The following is the list of properties that PoS has.

- All the properties of PoW listed in II-A are preserved.
- Any node can submit a job and become a client.
- A client can specify any instance of an optimization problem in a job.
- A client can implement any search algorithm for any optimization problem for a job.
- A client pays a charge for its job.
- Miners have a financial incentive to find a good solution for the instance in a job.
- Miners have a financial incentive to provide the best found solution to the client.
- The charge is automatically paid to the node who provides the best solution to the client.
- A probabilistic proof that the miners have evaluated a large number of solution candidates is provided.
- Multiple jobs can be executed at a time.
- The winning probability for each miner to find the best solution is proportional to the computational power spent by the miner for the job.
- The expected amount of computation for a job is proportional to the charge paid for the job.

- The variance in block time is lower than that with PoW.
- The probability of a fork is lower than that with PoW.
- The storage capacity required to manage a blockchain based on PoS is not too large.
- A blockchain based on PoS accepts jobs that run on computers with different architectures.

PoS has built-in countermeasures against the following cases.

- A node submits a problem instance for which the node already knows a good solution.
- A node has a very effective way of searching for a solution to some specific problems.
- A node tries to steal a solution found by another node.

The requirements for an evaluator are as follows.

- It takes a solution candidate as the first argument and a hash value as the second argument.
- It returns the evaluation of the solution candidate with a tiny amount of error depending on both the arguments.
- It has to satisfy the condition explained in IV-A.

F. SECURITY CONSIDERATION

Irrespective of search strategies, there is no difference in the difficulty of generating a valid nonce. Thus, the following items are allowed for nodes.

- Using its own search method, instead of the provided searcher.
- Sharing intermediate results of a search among other nodes.
- Participating in a search for only a particular job, instead of trying to add a whole block.
- Starting the search immediately after a job is submitted.

There is always a possibility that an evaluator will be reverse-engineered. If some malicious party comes to know how error is introduced in some specific evaluator, they could try to quickly compute the output of the evaluator with a different second argument by reusing the result of previous evaluation. This is a potential flaw in PoS. However, it can be prevented by introducing the error in an early stage of evaluation rather than in the last step. For example, an evaluator for the TSP could be implemented to introduce error by slightly changing the positions of cities.

When a miniblock is added, the corresponding nonce with a hash value beginning with the required number of zero bits is broadcast to the network. A solution candidate is included in this nonce, and we can think of a cheat where a miner tries to find a better solution by taking advantage of this solution candidate. There is approximately one block time to carry out a search after a nonce is received before the deadline of registering a hash value for the found solution. However, such an attempt would be hardly advantageous because this solution candidate is found in the middle of a search, and therefore it is unlikely to be the best solution found by the node that added the miniblock. It is also not very likely that the node that adds the block also finds the best solution among all nodes.

In another cheat, a client and miners can collude by submitting problem instances which do not have a solution and forcing others to work on these problems as the colluding miners work on instances submitted by other clients. If a problem instance does not have a solution, the corresponding evaluator always returns the same worst evaluation value. If multiple miners find the best solutions with the same evaluation value, the reward can be divided equally. This will financially discourage this cheat because the colluding client will have to pay other miners.

We can think of another cheat where a malicious client implements an evaluator in such a way that it returns an unfairly high evaluation when a special solution candidate is given. The client would implement the searcher in such a way that it does not evaluate such a solution candidate. By submitting a job with such an evaluator, the client can almost always find the best solution and collect the charge. However, this client cannot obtain the solutions found by other nodes because a node publishes its solution only if it is the winning node. The client is still be able to obtain the solution candidate contained in the nonce. However, this would not be a very good solution candidate, as explained above. Such a cheat can be financially discouraged by giving a part of the charge to the node that adds the new block.

In order to load up the CPUs of rival miners, a client and a miner might collude and submit jobs for which they already know a good solution. In this case, there is no chance for rival miners to find the best solution for the job since the colluded miner will submit the best solution and collect the charge. This cheat can be financially discouraged by giving a part of the charge to the node that adds the new block. Rival miners will have no problems in adding a new block with this cheat.

There are several ways of making a denial-of-service (DoS) attack that prevents execution or payment for jobs; although, most of them are not very effective. We can think of a DoS attack where a malicious node registers a false good evaluation with a false hash value on the blockchain when the best found solution is chosen and provided to the client. This prevents the client from receiving the true solution. If there is a node that makes such an attack, that node has to be banned from the network and the payment process has to be restarted.

G. APPLICABLE OPTIMIZATION PROBLEMS

Our protocol allows a blockchain to be used as a batch processing system for solving optimization problems. For the society to have demand for such a system, a large number of problem instances satisfying the following two properties have to be provided. First, the problem, the instance and the solution should not contain privacy-sensitive data so that it is legal to publish them. Second, solving an instance should require considerable amount of computational resources. If a user can easily carry out the computation on a small computer, there will be no reason to bother executing the search on a batch processing system. The success of BOINC projects [12] has demonstrated that there is continued demand for such computation. The computation involved in part of the BOINC

projects and other distributed computation projects shown below is optimization, and therefore we believe that such projects can be ported to the computing platform provided by our protocol.

Protein folding is simulated in Folding@Home [19] to examine the causes of protein misfolding. To predict the folded structure of a protein, a target protein sequence is deconstructed into small fragments. Then, the qualities of fragments and their assemblies are assessed by using some form of scoring function that aims to select more native-like protein structures from among the many possible combinations [20].

MilkyWay@home [21] is a project that aims to generate accurate three-dimensional dynamic models of stellar streams in the immediate vicinity of the Milky Way. It runs two kinds of applications. The first kind of application fits the spatial density profile of tidal streams using statistical photometric parallax. The other application finds the N-body simulation parameters that produce tidal streams that best match the measured density profile of known tidal streams.

In addition, many optimization problems have to be solved repeatedly in fintech. For example, the Markowitz model is used for constructing portfolios to optimize or maximize expected return based on a given level of market risk [22], [23]. Genetic algorithms are used for index fund management [24].

Optimization is also a key technique in deep learning [25], and we believe that there will be continued demand for optimizing neural networks.

V. CONCLUSION

We have proposed a consensus protocol for blockchains with which the wasted energy in the proof-of-work system can be used for solving optimization problems submitted by any user. Cryptocurrencies based on proof-of-work are already very popular, and to replace those cryptocurrencies, we designed our protocol to be robust, secure and decentralized. Our protocol is better than Gridcoin in that sense since our protocol does not depend on any external organization or system.

The motivation of this research is to apply the huge amount of electricity and computational power to useful work. The proposed system automatically adjusts the charge according to the balance between the demand and the total amount of computing power provided by the miners. If the demand is very small, the price for the optimization service will drop significantly. In such a situation, we can expect an increase in demand according to the law of supply. Therefore, fallback to PoW should not happen frequently, and the proposed method is expected to effectively prevent energy from being wasted.

There will be three kinds of users of a PoS-based blockchain, and these three user groups can be independent of each other. The first kind of users is those who want to use a PoS-based blockchain as a payment method. The second kind of users is those who spend their computational resources to

earn e-coins. The existing blockchains already have these two kinds of users, and PoS adds a third kind of user who uses a PoS-based blockchain as a grid computing infrastructure. The computational service provided by our protocol would be beneficial for computation tasks whose intermediate results can be published. Considering the popularity of BOINC projects and public clouds, we expect that there is a large public demand for such a computational service.

The computation in our protocol is ASIC resistant. This would make mining with ordinary CPUs more profitable. Unlike cloud computing, the computers used in mining need not be reliable. Since the computational resources are no longer wasted, public organizations and more general users could be expected to join. This would make a blockchain with our protocol more decentralized than existing ones.

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