Chapter 1

Hierarchy and co-evolution processes

Juste RAIMBAULT

Center for Advanced Spatial Analysis, University College London

1.1. Introduction

1.1.1. *Complexity and hierarchy*

Complex systems with emergent properties through self-organization processes are also most of the time exhibiting some kind of hierarchical structure. Although the term of hierarchy has several different definitions and uses in very different disciplines, ranging from political science (Crumley 1987) to physics (Mones *et al*. 2012), it seems to be intrinsically linked with complexity. Lane (2006) classifies four frequent uses of the term hierarchy, namely (i) order hierarchy corresponding to the existence of an order relation for a set of elements, (ii) inclusion hierarchy which is a recursive inclusion of elements within each other, (iii) control hierarchy which is the "common sense" use of the term as ranked entities controlling other entities with lower rank, and (iv) level hierarchy which captures the multi-scale nature of complex systems as ontologically distinct levels (or scales). For the particular study of social systems, he concludes that hierarchical levels may be entangled, that upward and downward causations are both essential, and that at least three levels (micro, meso, macro) are generally needed to capture the complexity of such systems. In a more philosophical account of complexity, Morin (1980-2005) constructs a hierarchical method of interdisciplinary knowledge, insists on the tension between dependancy and interdependency or between opening and closing (rejoining ideas from Holland

(2012)), and develops an implicit hierarchy of social systems when hypothesizing the emergence of third-type societies (swarm intelligence between humans).

Different types of complexity may be related to different types of hierarchy as Raimbault (2019*a*) proposes, and hierarchy would indeed be endogenous to theories of complexity. Allen *et al*. (2017) develop a multiscale information theory in which the information profile across scales, or hierarchical levels, allows quantifying the complexity of a system. The complex adaptive system theory of Holland (2012) considers complex systems as systems of boundaries that filter signals, implying an inclusion and scale hierarchy between boundaries. Theories of scaling as the one synthesized by West (2017) rely on the quantification of hierarchy in certain dimensions of systems, captured by exponents of scaling laws. Hierarchy may be endogenous to complexity, or to knowledge of the complex itself, since for example Fanelli and Glänzel (2013) provides empirical evidence of a "hierarchy of sciences", in the sense of possibility to reach theoretical and methodological consensus. This corresponds in some sense to the "ontological complexity" of Pumain (2003), which relies on the number of viewpoints needed to grasp a system, or the number of perspectives in an applied perspectivism framework (Raimbault 2018*c*). Wether linked to systems themselves or to models and theories of them, hierarchy appears to be tightly linked to complexity.

1.1.2. *Territorial systems and hierarchy*

Urban systems, and more generally territorial systems, are particularly linked to hierarchy (Pumain 2006*c*): they indeed encompass all the meanings aforementioned (order hierarchy between settlement sizes for example, inclusion hierarchy between territorial boundaries, control hierarchy through governance structure, and more importantly level hierarchy through their multi-scalar nature). Batty (2006) shows that hierarchies are inherent to urban systems, as fat tail distribution of settlement size are already produced by simple models of urban growth, and suggests also that urban design processes imply underlying overlapping hierarchies. Pumain (2006*a*) links hierarchical selection and hierarchical diffusion of innovation across cities to the long-term dynamics of urban systems. Pumain (2019) recalls that interactions in systems of cities are tightly linked to the emergence of urban hierarchies. Generally, scaling laws in urban systems can be considered as systematic manifestations of a hierarchical structure (Pumain 2004), which is more complex than a simple order hierarchy, since scaling patterns vary with the definition of cities Cottineau *et al*. (2017)

Several dimensions of urban systems exhibit hierarchical properties, as transportation networks and transportation flows (Jiang 2009), the global distribution of multinational firms (Godfrey and Zhou 1999), or governance structures (Li *et al*. 2015).

1.1.3. *Co-evolution and hierarchy*

Hierarchy in complex systems is furthermore intrinsically linked to the concept of co-evolution. Following Lane (2006), the approach to complex adaptive systems proposed by Holland (2012) integrates levels and nested hierarchies, since it considers complex systems as ensembles of boundaries that filter signals.

In the context of economic geography and the co-evolution of firms, Volberda and Lewin (2003) distinguishes between a genealogical hierarchy (evolutionary processes in the biological sense) and an ecological hierarchy (co-evolutionary economic processes).

1.1.4. *Proposed approach*

Pumain (2006*b*) recalls in the context of social systems some remaining methodological questions: how are hierarchies produced? How do hierarchies evolve? What discriminates between continuous and discrete hierarchical organisations?

Our contribution brings new elements of answer to the first two questions above, in the particular case of co-evolution of transportation networks and territories. More precisely, we systematically explore a macroscopic co-evolution model and study its properties regarding both hierarchies of cities and networks, in terms of final hierarchy produced but also in terms of dynamics of hierarchies.

1.2. Co-evolution model

1.2.1. *Context and rationale*

The issue of interactions between transportation networks and territories remains an open question for which different approaches have been proposed Offner (1993), Offner *et al*. (2014). Raimbault (2018*a*) has explored a co-evolution approach, in the sense that both dynamics have circular causal relationships. More precisely, Raimbault (2019*b*) introduces a definition of co-evolution in that particular context, based on co-evolution niches Holland (2012)

Raimbault (2019*c*)

1.2.2. *Model description*

The co-evolution model for cities and transportation networks at the macroscopic scale extends the spatial interaction model introduced by Raimbault (2018*b*) by adding dynamical speeds to network links. More precisely, (i)

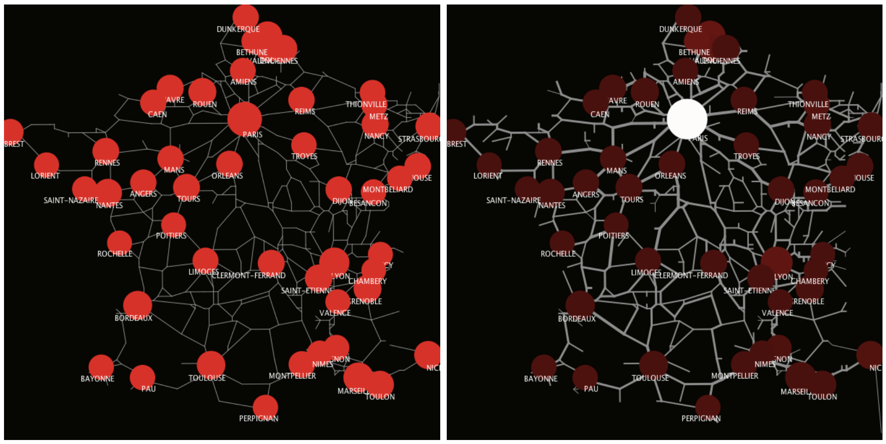

Figure 1.1. ***Application of the co-evolution model with a physical network for the French system of cities.***

1.2.3. *Quantifying hierarchy in systems of cities*

1.2.3.1. *Static quantification of hierarchy*

A simple way to quantify hierarchy is to use Zipf rank-size law, or more generally scaling laws. Let Y_i the variable for which the hierarchy is estimated. Assuming i is ordered in decreasing order, the OLS estimation of $\log(Y_i) \sim \log(i)$ gives an estimation of the rank-size slope which is a proxy of hierarchy.

The correlation between two hierarchies informs how two dimensions correspond in terms of ranks, and is computed with $r_s[X_i, Y_i]$ for two variables X_i, Y_i with r_s an estimator of Spearman rank correlation.

1.2.3.2. *Dynamical indicators*

The rank correlation between initial and final distribution of a variable will measure how much an ordering hierarchy was modified, which is different from the variation of hierarchy given the variations of previous indicators such as the rank-size slope.

Dynamical hierarchy regimes are defined the following way:

1.2.3.3. *Spatialized indicators*

A spatial non-stationary version of a scaling law would write $Y_i(\vec{x}) \sim \left(\frac{X_i(\vec{x})}{X_0(\vec{x})}\right)^{\alpha(\vec{x})}$, where $\vec{x}$ is the spatial position and assuming that samples can be defined at each point in space. In practice, a discrete version could be more relevant, for which $\vec{x}_k$ center point are defined, samples consist of points within Thiessen polygons of centers and the exponents are estimated for each center $\alpha(\vec{x}_k)$. Some heuristics should be developed to estimate such a discrete non-parametric scaling law, and remains out of the scope of this paper.

1.3. Results

1.3.1. *Implementation*

The model is implemented in NetLogo Tisue and Wilensky (2004), which is a good compromise between performance and interactivity, the former being necessary with a model with such a spatialized network.

1.3.2. *Hierarchy patterns*

1.3.3. *Hierarchy regimes*

1.4. Discussion

1.5. Bibliography

Allen, B., Stacey, B. C., Bar-Yam, Y. (2017), Multiscale information theory and the marginal utility of information, *Entropy*, 19(6), 273.

Batty, M. (2006), Hierarchy in cities and city systems, *in* Hierarchy in natural and social sciences, Springer, pp. 143–168.

Cottineau, C., Hatna, E., Arcaute, E., Batty, M. (2017), Diverse cities or the systematic paradox of urban scaling laws, *Computers, environment and urban systems*, 63, 80–94.

Crumley, C. L. (1987), A dialectical critique of hierarchy, *Power relations and state formation*, pp. 155–169.

Fanelli, D., Glänzel, W. (2013), Bibliometric evidence for a hierarchy of the sciences, *PLoS One*, 8(6), e66938.

Godfrey, B. J., Zhou, Y. (1999), Ranking world cities: multinational corporations and the global urban hierarchy, *Urban Geography*, 20(3), 268–281.

Holland, J. H. (2012), *Signals and boundaries: Building blocks for complex adaptive systems*, Mit Press.

Jiang, B. (2009), Street hierarchies: a minority of streets account for a majority of traffic flow, *International Journal of Geographical Information Science*, 23(8), 1033–1048.

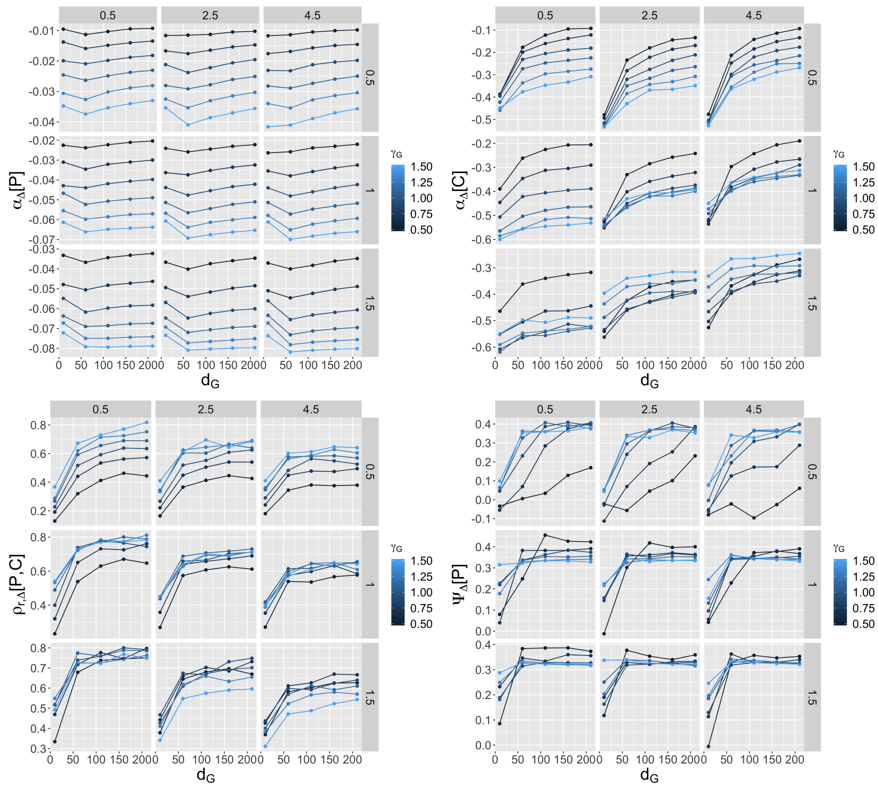

Figure 1.2. ***Patterns of hierarchy in the model with a virtual network.*** *(Top Left) Difference in the rank-size exponent between final time and initial time,*

Lane, D. (2006), Hierarchy, complexity, society, *in* Hierarchy in natural and social sciences, Springer, pp. 81–119.

Li, H., Wei, Y. D., Liao, F. H., Huang, Z. (2015), Administrative hierarchy and urban land expansion in transitional china, *Applied Geography*, 56, 177–186.

Mones, E., Vicsek, L., Vicsek, T. (2012), Hierarchy measure for complex networks, *PLOS ONE*, 7(3), 1–10.
URL: *https://doi.org/10.1371/journal.pone.0033799*

Morin, E. (1980-2005), *La Méthode, tome 2. La Vie de la Vie*, Le Seuil.

Offner, J.-M. (1993), Les "effets structurants" du transport: mythe politique, mystification scientifique, *Espace géographique*, 22(3), 233–242.

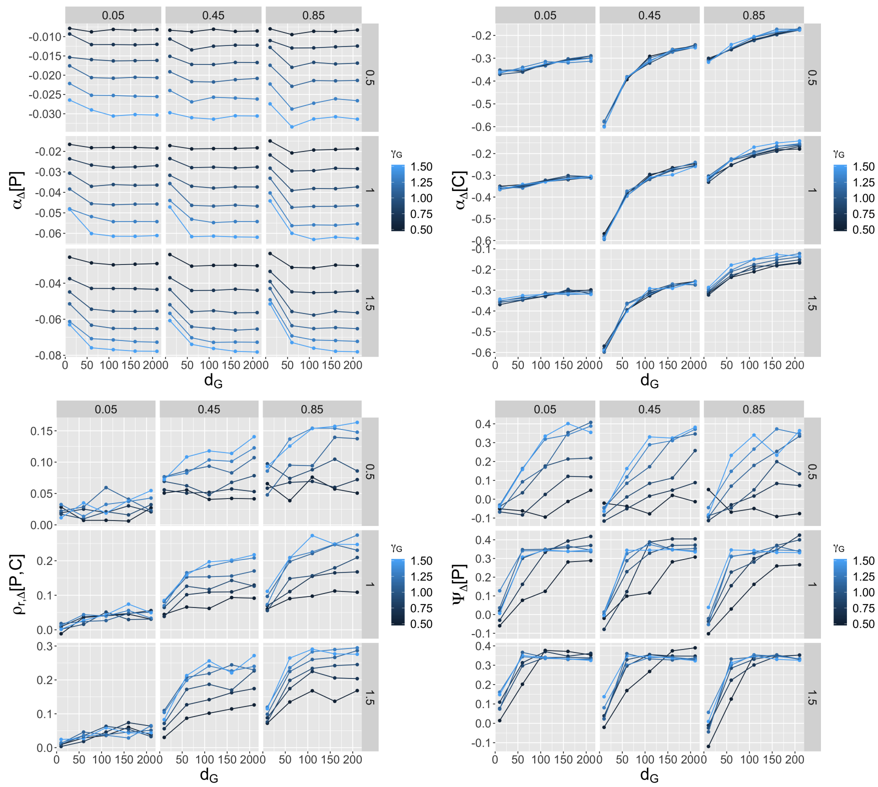

Figure 1.3. ***Patterns of hierarchy in the model with a physical network.*** *(Top Left)*

Offner, J.-M., Beaucire, F., Delaplace, M., Frémont, A., Ninot, O., Bretagnolle, A., Pumain, D. (2014), Les effets structurants des infrastructures de transport, *Espace Geographique*, (42), p–51.

Pumain, D. (2003), Une approche de la complexité en géographie, *Géocarrefour*, 78(1), 25–31.

Pumain, D. (2004), Scaling laws and urban systems, , .

Pumain, D. (2006*a*), Alternative explanations of hierarchical differentiation in urban systems, *in* Hierarchy in natural and social sciences, Springer, pp. 169–222.

Pumain, D. (2006*b*), Hierarchy in natural and social sciences, introduction, *in* Hierarchy in natural and social sciences, Springer, pp. 1–12.

Pumain, D. (2019), Les voies de l'interaction et les hiérarchies urbaines, *in* actes du colloque Voies, réseaux, paysages en Gaule en hommage à Jean-Luc Fiches, Revue

d'Archéologie Narbonnaise.
URL: *https://halshs.archives-ouvertes.fr/halshs-02303136*

Pumain, D. e. (2006*c*), *Hierarchy in Natural and Social Sciences*, Springer, Dordrecht.

Raimbault, J. (2018*a*), Caractérisation et modélisation de la co-évolution des réseaux de transport et des territoires, PhD thesis, Université Paris 7 Denis Diderot.

Raimbault, J. (2018*b*), Indirect evidence of network effects in a system of cities, *Environment and Planning B: Urban Analytics and City Science*, p. 2399808318774335.

Raimbault, J. (2018*c*), Relating complexities for the reflexive study of complex systems, *arXiv e-prints*, p. arXiv:1811.04270.

Raimbault, J. (2019*a*), De l'endogénéité des hiérarchies dans les systèmes territoriaux complexes, *in* Journée Des jeunes chercheurs de l'Institut de Géographie :Compter, classer, catégoriser, Paris, France.
URL: *https://halshs.archives-ouvertes.fr/halshs-02089520*

Raimbault, J. (2019*b*), Modeling interactions between transportation networks and territories: a co-evolution approach, *arXiv preprint arXiv:1902.04802*, .

Raimbault, J. (2019*c*), Modeling the co-evolution of cities and networks, *forthcoming in Handbook of Cities and Networks, Rozenblat C., Niel Z., eds. arXiv:1804.09430*, .

Tisue, S., Wilensky, U. (2004), Netlogo: A simple environment for modeling complexity, *in* International conference on complex systems, vol. 21, Boston, MA, pp. 16–21.

Volberda, H. W., Lewin, A. Y. (2003), Co-evolutionary dynamics within and between firms: From evolution to co-evolution, *Journal of management studies*, 40(8), 2111–2136.

West, G. B. (2017), *Scale: the universal laws of growth, innovation, sustainability, and the pace of life in organisms, cities, economies, and companies*, Penguin.