Data Structures and Algorithms

Heaps
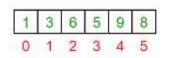
Week 9

Heaps

- A type of binary tree with

    - every level (except maybe the last) is complete (no gaps)

    - Last level has values as far left as possible

- Stored using an array by index going level by level from left to right

- Children of position i at 2i+1 and 2i+2

- Parent of position j at floor((j-1)/2) or ceil(j/2)-1

- Come in two forms - Min Heap and Max Heap

- Min heap - parents are less than both children

- Max heap - parents are greater than both children

Heaps - Activity (Self/Group-work)

- For a min-heap, find the runtimes for the following functions (no internet):

  - GetMinimum() - gets minimum of Heap

  - ExtractMinimum() - removes minimum from heap and restructures to keep it as min heap.

  - Insert(val) - insert value into heap (assume you add to end and fix if it is no longer a min heap)

  - Delete(index) - delete value at a given index from heap. Need to leave it as a min heap so think about how to move things around.

Heaps - Applications

- Why are heaps useful?
  - Max Heap -> 'Priority Queues' where element with largest priority gets dequeued first. Keeping max at index 0 allows this to happen fast.
  - Heap Sort - allows you to sort $O(n \log n)$ in worst case with $O(1)$ extra space
  - Very useful in many graphing algorithms (see: Djikstra's algorithm)
  - Efficient way of getting the 'k smallest elements' by repeatedly extracting min
- There are many types of heaps - fibonacci, binomial, leftist, pairing, brodal that

Heap Sort

- How does HeapSort work?

    - Consider your array as a heap where indices correspond to binary tree

    - Convert this heap into a max heap. How? Explained here

        - Compare each node to its children and swap putting max as parent

    - Swap values at first and last index. Last is now largest.

    - Consider heap to be all but last index.

    - Re-make into a max heap by swapping element up one at a time.

    - Repeat process again and again with max going to end again and again.

- Piazza Discussion here: https://piazza.com/class/k56ida62b2ptz?cid=68

Videos & Readings

Videos

Basic Operations of Heaps & Java Implementation

Converting Binary Tree into Heap

Using Max Heaps to Heap Sort

Readings

Course Book - Chapters 7.8 - 7.10

Geeks for Geeks - Binary Heap and Representing Heaps using Arrays

HackerEarth Explanation

Questions?....Office Hours!