QCX Class E Amplifier Design: Practical Equations and Theoretical Derivation

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1 Intro

The QCX is a great amateur radio QRP platform developed by `https://qrp-labs.com` with a unique design philosophy. It performs remarkably well ($> 80\%$ efficiency and $> 3\,\mathrm{W}$ output power) while also not following traditional design rules. Instead, the inventor (Hans Summers, G0UPL) used a trial-and-error approach to make a working amplifier. This document shows how theoretical analysis arrives at the correct (and even slightly more efficient) values right away without trial and error. First, I'll show the conclusions followed by the detailed derivation for those that are interested.

2 Comparing Original QCX Design to the New Theory

Nearly all amateur radio QRP Class E amplifiers look like Figure 1. Some square wave signal modulates the gate of a MOSFET. The drain is biased through an inductor L_1 and connected to ground through C_1. Gate modulation causes voltage modulation at the drain. Filters are used to extract the RF signal you would like to apply to your load resistor R_L (i.e., your antenna).

The QCX Class E amplifier uses the filter seen in Figure 2. The filter was chosen to be a well known filter from the G-QRP club's technical pages, originally designed by Ed Whetherhold (W3NQN). Table 1 shows the values used for a 7.1 MHz (40 m) QCX transmitter. The "QCX Start" is the design philosophy in the QCX manual, "QCX Final" is what was finally realized after experimental optimization, "NEW Theory" is based on the new theory presented here. In these tables, ω is the design frequency in radians/sec.

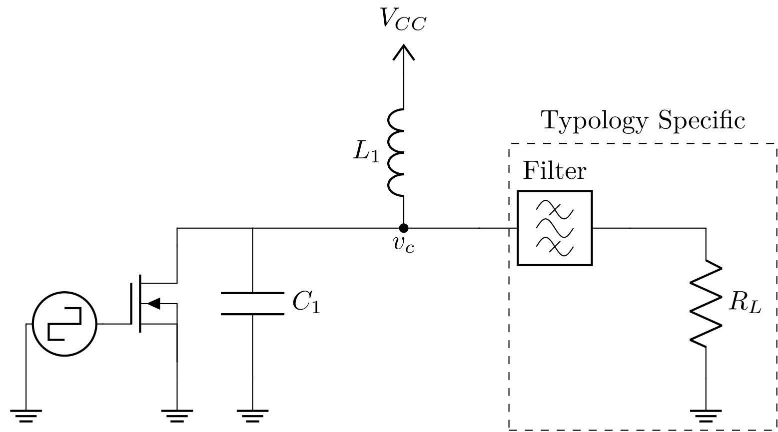
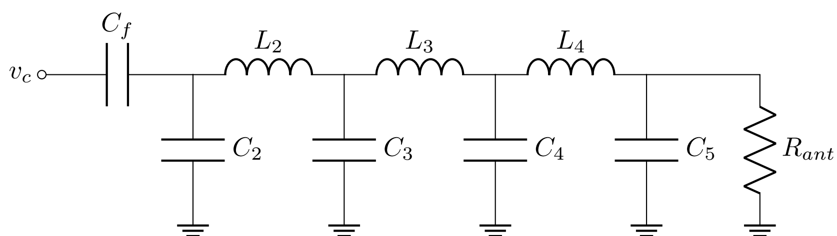


Figure 1: Generic Class E Amplifier



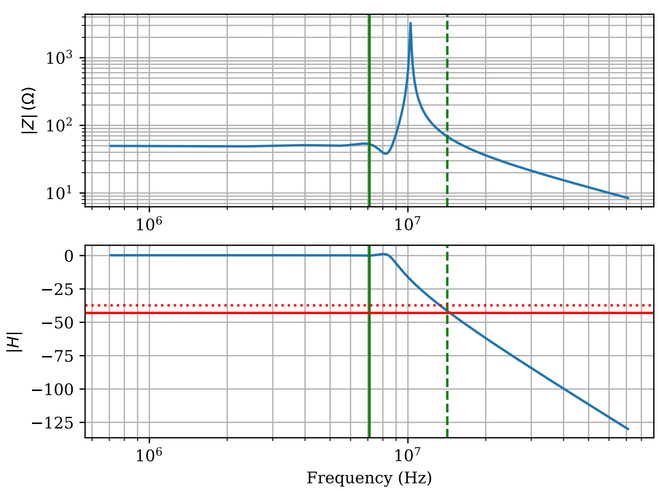


Figure 2: QCX LPF Class E for 7.1 MHz. It is completely real at ω (green line) and 2ω (dashed green line).

	QCX Start	QCX Final	NEW Theory
η	72.2%	84.7%	87.6%
$\langle P_{out} \rangle$	2.3 W	3.5 W	3.0 W
L_1(µH)	1.12	1.00	1.12
C_1(pF)	448	56	112
C_f(nF)	100	100	100
C_2(pF)	270	270	270
L_2(µH)	1.38	1.38	1.38
C_3(pF)	680	680	680
L_3(µH)	1.7	1.7	1.7
C_4(pF)	680	680	680
L_4(µH)	1.38	1.38	1.38
C_5(pF)	270	270	270

Table 1: ngSpice models results of Class E amplifiers (using 3 parallel BS170 transistors). $f = 7.1\,\mathrm{MHz}$ $V_{CC} = 12\,\mathrm{V}$, $R_{ant} = 50\,\Omega$. The theoretical starting point according to the QCX manual is "QCX Start." After the starting point goes through experimental optimization, the "QCX Final" is the final is what is sold. "NEW Theory" is based on the derivations in this document. The filters are based on W3NQN's filters from G-QRP

	Std. Class E Theory	NEW QCX Theory
$\langle P_{out} \rangle \times R_L/V_{CC}^2$	0.58	1.44
$L_1 \times \omega/R_L$	$\gg 2.84$	1.18
$C_1 \times \omega R_L$	0.184	0.212
$C_f \times \omega R_L$	Q	$\gg 1$

Table 2: Comparison of Design Values. NEW Theory is for a perfect filter. See text for equations for realistic filters.

3 The New QCX Analytical Design Equations

QCX Class E amplifier is fundamentally different than pretty much any standard QRP Class E amplifier you'll find. As such, it has different design equations. Table 2 compares the design equations for a traditional R_f choke Class E amplifier to the QXC amplifier.

Table 2 illustrates the differences between design values. The table assumes an infinitely good filter. More formally, the QCX theoretical design values are based on the impedance of the filter at the second harmonic $Z(2\omega) = \Re\{Z(2\omega\} + i\Im\{Z(2\omega)\}$ and are derived in Section 4 and summarized below.

If $Z(2\omega)$ is entirely real (which is the case in the QCX design on `https://qrp-labs.com` and $Z(2\omega) = \Re\{Z(2\omega)\} \approx 68\,\Omega$) and the load resistance is

Impedance	Std. Class E	NEW QCX Theory
DC	∞	∞
ω	Complex inductive load $R_L + i\omega\Delta L$	Real load R_L
2ω	∞	Completely real or ∞
$n\omega, n > 2$	∞	Capacitive and small

Table 3: Comparison of Filter Design Requirements

just the antenna $R_L = R_{ant}$:

$$L_1 = \frac{3\pi R_L Z(2\omega)}{[2R_L + 8Z(2\omega)]\omega} \qquad C_1 = \frac{1}{L_1(2\omega)^2}$$

$$C_f \gg \frac{1}{\omega R_L} \text{ and } C_f \gg C_1 \qquad \langle P_{out}\rangle = \frac{128}{9\pi^2}\frac{V_{CC}^2}{R_L}$$

There's actually a different set of design equations for when $C_1 \ll C_f \ll 1/(\omega R_L)$ that does some cool stuff. The current through the load becomes a cosine (since the load becomes capacitive at ω, and you get increased efficiency $\eta \approx 90\%$ at the cost of lower power ($\approx 1\,\mathrm{W}$ total output on average). It's not derived here, but it's possible to do derive changing some steps later on. It's possibly a much less stable circuit, so I won't spend time on it, but theoretically possible nonetheless.

Finally, the theory presents a completely different strategy for what's required of filters. See Table 3. It turns out that the filter in Figure 2 is one of the few filter designs that actually meets the requirements! It has an entirely real impedance at ω and 2ω and presents as a capacitive load for $n\omega$ for $n > 2$!

4 QCX Design

There are a lot of Class E explanation documents on the internet, but I haven't found a caparison document intended for people just wanting to figure out what's the deal with different Class E amplifiers they are seeing.

This was motivated by discussions at the μSDX project where different builds were using different Class E designs that were incompatible with each other (one required inductive loads, one required only real loads). Having a base transceiver design with completely different PA filter requirements could cause problems if people tried to "mix-and-match" filters between the two.

When considering this, I saw that there was no theoretical description of the qrp-labs QCX Class E amplifier that was used by several people. The qrp-labs.com's "QCX" transceiver[1] has an unconventional Class E amplifier design. The designer, Hans Summers (G0UPL) described the design as sort of a trial and error process guided by some ideas. It turned out that it worked pretty

[1] `https://qrp-labs.com/qcxp`

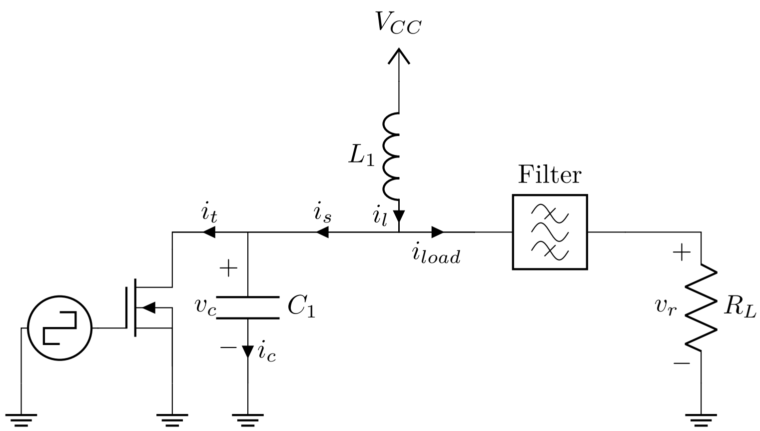


Figure 3: Generic Class E Amplifier

well, but it sort of violated some "traditional" class E amplifier design rules. I decided to model it in ngspice to figure out what's going on, and found that it is actually a beautiful and elegant design that is fundamentally different than the strategies of other class E amplifiers. Some may call it "ghetto," but there's an elegance to it how it performs.

4.1 QRP-Labs Design

The QCX manual says to use the following approach to choose components in Figure 3. Although not grounded in theory, but in experimental trial and error - and produces excellent results!

1. Choose $L_1 = R_{ant}/\omega$
2. Choose $C_1 = 1/(\omega R_{ant})$
3. For the filter, use Figure 2 and values from G-QRP club's technical pages, originally designed by Ed Whetherhold (W3NQN).
4. Tweak the value of C_1 to get the highest efficiency. This is the key point - and why it works so well. Experimental tinkering is often is superior to theory!

4.2 Analytical Approach to QCX Class E Design

Modelling the QCX Class E with the values qrp-labs uses allows you to analyze the voltages at the different nodes in order to come up with a theoretical approach to help design and optimize this style of Class E amplifier. Below is the outcome of that approach:

1. The starting point is the expression for v_c. The qrp-labs v_c is symmetric, which is needed because there is no LC bandpass filter. The voltage at v_c actually closely follows:

$$\begin{aligned} t = 0 \to \pi/\omega : \qquad & v_c = 0 \\ t = \pi/\omega \to 2\pi/\omega : \qquad & v_c = A \sin^2(\omega t) \\ & \quad = A \left(\frac{1}{2} - \frac{1}{2} \cos(2\omega t) \right) \end{aligned}$$

Acknowledging that the average voltage at $\langle v_c \rangle = V_{CC}$, you can find A:

$$\begin{aligned} V_{CC} &= \frac{1}{2\pi/\omega} \int_0^{2\pi/\omega} v_c \, dt' \\ V_{CC} &= \frac{1}{2\pi/\omega} \int_{\pi/\omega}^{2\pi/\omega} A \left(\frac{1}{2} - \frac{1}{2} \cos(2\omega t) \right) dt \\ 4V_{CC} &= A \end{aligned}$$

Allowing us to write v_c as a simple expression seen in Figure 4 and equations below. It's evident that v_c is zero at switch turn on and off, with zero derivative at turn on and off as well.

$$\begin{aligned} t = 0 \to \pi/\omega : \qquad & v_c = 0 \\ t = \pi/\omega \to 2\pi/\omega : \qquad & v_c = 2V_{CC} - 2V_{CC} \cos(2\omega t) \end{aligned}$$

2. Now we can find the current through the capacitor for the time period $t = \pi/\omega \to 2\pi/\omega$ as seen in Figure 5 and below:

$$i_c = C_1 \frac{dv_c}{dt} = 4\omega C_1 V_{CC} \sin(2\omega t)$$

3. Next we'll find the current through the load (antenna). QCX Class E amplifiers use a filter that presents an entirely real load to the amplifier, and the blocking capacitor C_f in Figure 2 is chosen such that $C_f < \frac{1}{R_L \omega}$ (i.e., it's a high pass filter with a corner frequency $\ll \omega$). To find the current through the load we represent v_c as a Fourier series:

$$v_c = V_{CC} - \frac{16}{3\pi} V_{CC} \sin(\omega t) - V_{CC} \cos(2\omega t) + \frac{16}{15\pi} V_{CC} \sin(3\omega t) + \cdots$$

We also can present R_L as a complex impedance as a function of frequency ($Z(\omega')$) since it is the combined impedance of the filter with the antenna.

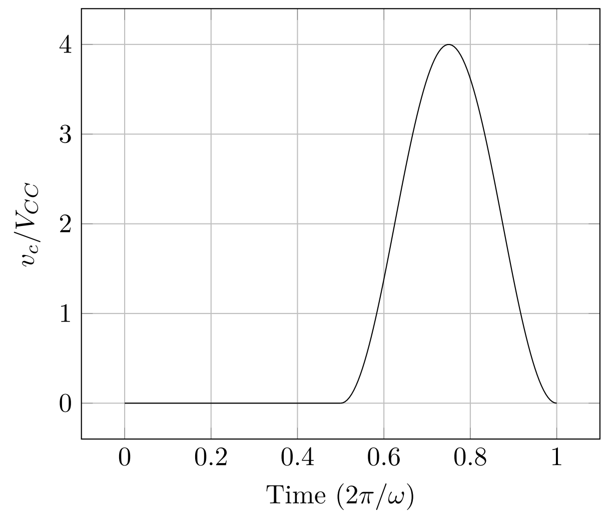


Figure 4: QCX capacitor voltage versus time for one switch period.

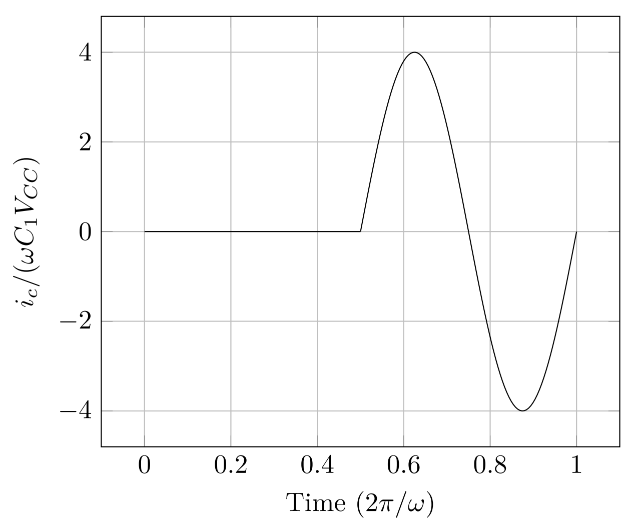


Figure 5: QCX capacitor current versus time for one switch period.

The filter presents a real load at the fundamental frequency (ω), so $Z(\omega) = R_{ant}$. The DC blocking capacitor makes $Z(0) = \infty$, but Z can by anything at other frequencies. Typically, Class E amplifiers want filters that have $Z(n\omega) = \infty$ for all $n \neq 1$, however, QCX Class E is "special" as we will see.

$$
\begin{aligned}
i_{load} &= \frac{v_c}{Z(\omega')} \\
&= -\frac{16}{3\pi}\frac{V_{CC}}{R_{ant}}\sin(\omega t) - \frac{V_{CC}}{Z(2\omega)}\cos(2\omega t) + \frac{16}{15\pi}\frac{V_{CC}}{Z(3\omega)}\sin(3\omega t) + \cdots
\end{aligned}
$$

From which you can find the average RF power emitted:

$$
\langle P_{out}\rangle = \frac{128}{9\pi^2}\frac{V_{CC}^2}{R_{ant}} \approx 1.44\frac{V_{CC}^2}{R_{ant}}
$$

4. Next we find the current through the inductor, which has the form as seen in Figure 6 and derived below:

$$
\begin{aligned}
& & i_l &= \frac{1}{L_1}\int_{-\infty}^{t} V_{CC} - v_c \, dt' \\
t = 0 \to \pi/\omega : & & i_l &= i_{l0} + \frac{1}{L_1}\int_0^t V_{CC}\, dt' \\
& & &= i_{l0} + \frac{V_{CC}}{\omega L_1}\omega t \\
t = \pi/\omega \to 2\pi/\omega : & & i_l &= i_{l0} + \frac{1}{L_1}\int_0^{\pi/\omega} V_{CC}\, dt' + \frac{1}{L_1}\int_{\pi/\omega}^{t} V_{CC} - v_c\, dt' \\
& & &= i_{l0} + \frac{V_{CC}}{\omega L_1}\left(2\pi - \omega t + \sin(2\omega t)\right)
\end{aligned}
$$

Which has the form as seen in Figure 6.

The i_{l0} term can be found be equating in the input and output average powers. The input voltage is V_{CC}, and the input current is $I_{DC} = \frac{1}{2\pi/\omega}\int_0^{2\pi/omega} i_l \, dt'$

$$
\begin{aligned}
\langle P_{in}\rangle &= \langle P_{out}\rangle \\
\frac{V_{CC}}{2\pi/\omega}\int_0^{2\pi/\omega} i_l \, dt' &= \frac{128}{9\pi^2}\frac{V_{CC}^2}{R_{ant}} \\
i_{l0} &= \frac{V_{CC}}{R_{ant}}\left(\frac{128}{9\pi^2} - \frac{\pi R}{2L\omega}\right)
\end{aligned}
$$

5. To bring it all together, we have to make sure the currents all equal to each other. We'll have to find the values of L_1, C_1 that make it happen. When the switch is closed:

$$
t = 0 \to \pi/\omega : \qquad i_l = i_{load} + i_t
$$

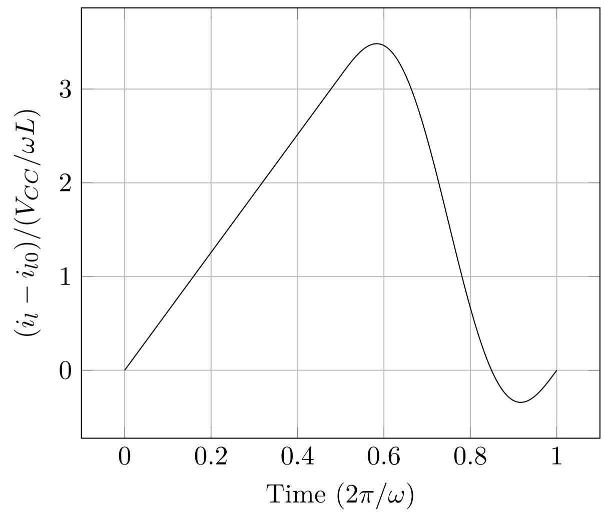


Figure 6: QCX inductor current versus time for one switch period.

And since i_t can take any current (it's a short to ground!) we don't have to worry about that. The interesting thing is when the switch is closed:

$$t = \pi/\omega \to 2\pi/\omega : \qquad i_l = i_{load} + i_c$$

To match these currents, we'll make the fundamental component of the Fourier series of each current equal to each other when the switch is open. When the switch is closed, $i_l - i_c - i_{load} - i_t = 0$ is guaranteed since i_t can provide enough current to balance the other three. However, when the switch is closed, $i_t = 0$ and the three other currents must be equal. Therefore, we'll make sure the fundamental component of the Fourier series of $i_l - i_c - i_{load} - i_t = 0$ at all time.

$$t = 0 \to \pi/\omega : \qquad 0 = i_{l1} - i_{c1} - i_{load1} - i_{t1}$$
$$t = \pi/\omega \to 2\pi/\omega : \qquad 0 = i_{t2} - i_{c2} - i_{load2}$$

Where 1 and 2 denote real values for the first or second half of the cycle, respectively, and zeros for the other half. For the second half of the cycle, we must find L_1, C_1 that makes the following true.

$$\begin{aligned}
I(t) = 0 = i_{l0} &+ \frac{V_{CC}}{\omega L_1}\left(2\pi - \omega t + \sin(2\omega t)\right) \\
&- 4\omega C_1 V_{CC} \sin(2\omega t) \\
&+ \Big(\frac{16 V_{CC}}{3\pi R_{ant}} \sin(\omega t) + \frac{V_{CC}\Re\left\{Z(2\omega)\right\}}{|Z(2\omega)|^2} \cos(2\omega t) \\
&\quad + \frac{V_{CC}\Im\left\{Z(2\omega)\right\}}{|Z(2\omega)|^2} \sin(2\omega t) + \cdots\Big)
\end{aligned}$$

If $Z(2\omega)$ is large $Z(2\omega) \gg \frac{R_{ant}}{4}$:

$$\begin{aligned}
\frac{1}{L_1 C_1} &= (2\omega)^2 \\
L_1 &= \frac{3\pi R_{ant}}{8\omega}
\end{aligned}$$

If Z2w is real (like in QCX!):

$$\begin{aligned}
\frac{1}{L_1 C_1} &= (2\omega)^2 \\
L_1 &= \frac{3\pi R_{ant} Z(2\omega)}{[2R_{ant} + 8Z(2\omega)]\omega}
\end{aligned}$$

Generally:

$$\begin{aligned}
C_1 &= \frac{\Im\{Z(2\omega)\} L_1 \omega + |Z(2\omega)|}{4 L_1 |Z(2\omega)| w^2} \\
L_1 &= \frac{3\pi R_{ant} \Re\{Z(2\omega)\} |Z(2\omega)|}{[2R_{ant}|Z(2\omega)| + 8\Re\{Z(2\omega)\}]\omega}
\end{aligned}$$

Finally, there is a DC component to $I(t)$ when the switch is closed that needs to be accounted for. How this is handled is by making sure $C_f \gg C_1$ and $C_f \gg C_2$. This means that the current will charge up the capacitor C_f while not affecting v_c that much. When the switch is closed, the current will discharge out of C_f to ground. A $C_f \approx 100\,\mathrm{nF}$ seems to work well.