1 Theory Test

Andrey Vagin 12.09.2002

Question 1:

Starting with a definition, represent the game as a finite position game (FPG).

Answer:

As presented in the Lecture 1 slide 15 the finite position game can be defined as a tuple $G = (P_A, P_B, M_A, M_B, F_A, F_B)$, where

- P_E and P_A are disjoint finite sets of positions for Eloise and Abelard.
- $M_E \subseteq P_E \times (P_E \cup P_A)$ and $M_A \subseteq P_A \times (P_A \cup P_E)$ are admissible moves of Eloise and Abelard.
- $F_E \subseteq (P_E \cup P_A)$ and $F_A \subseteq (P_A \cup P_E)$ are disjoint final positions (where Eloise and Abelard won already respectively).

Taking into account the description of FPG from the task description and applying
my date of birth, the sets will be as follows:

$P_E = \{(p, 1) : p \in \mathbb{Z} \wedge p \geq 1 \wedge p \leq 2023\}$

$P_A = \{(p, 2) : p \in \mathbb{Z} \wedge p \geq 1 \wedge p \leq 2023\}$

$M_E = \{((p, 1), (q, 2)) : (p, 1) \in P_E \wedge (q, 2) \in P_A \wedge \exists n(q = p + n \wedge ((n \in [2002] \wedge p + n \leq 2023) \vee ((q, 2) \in F_E \wedge n \leq 2002)))\}$

$M_A = \{((p, 2), (q, 1)) : (p, 2) \in P_A \wedge (q, 1) \in P_E \wedge \exists n(q = p + n \wedge ((n \in [2002] \wedge p + n \leq 2023) \vee ((q, 1) \in F_A \wedge n \leq 2002)))\}$

$F_E = \{(2023, 2)\}$

$F_A = \{(2023, 1)\}$

Question 2:

Give a (general) definition for a winning strategy for a player in FPG.

Answer:

- A strategy for a player $X \in \{A, B\}$ is any subset $S_X \subseteq M_X$.
- A strategy S_X for a player $X \in \{A, B\}$ is said to be a winning strategy, if the player eventually wins every play by applying this strategy, i.e., every complete play where X applies S_X is finite and winning for X.

Wining strategies for Eloise and Abelard are defined as

$S_E = \{((p, 1), (q, 2)) \in M_E : (\nexists(m, 1) \in P_E)[((q, 2), (m, 1)) \in S_A]\} \cup \{((p, 1), (q, 2)) \in M_E : (q, 2) \in F_E\}$

$S_A = \{((q, 2), (p, 1)) \in M_A : (\nexists(m, 2) \in P_A)[((p, E), (m, 2)) \in S_E]\} \cup \{((q, 2), (p, 1)) \in M_A : (p, 1) \in F_A\}$

Question 3:

Describe in the set-theoretic terms backward induction for Eloise and compute (using the description) all initial game positions where Eloise has a winning strategy.

Answer:

Eloise has a winning strategy in a position p iff she has a move $(p, q) \in M_E$ that leads to a position q such that

- is a final position (i.e., $q \in F_E$) or
- any Aberald's move $(q, r) \in M_A$ leads to a position r where Eloise has a winning strategy.

Now, I will define a function with help of which I will compute all initial game positions where Eloise has a wining strategy:

$\mathcal{F}(W) = \{p : (p + i) \in [2023] \wedge i \leq 2002\} \cup \{p : (p + 2002 + 2002) \notin W\}$ on $[1..2023]$

After computations I got the following initial game positions where Eloise has a winning strategy:

[21..2022]

Question 4:

List all initial game positions where Abelard has a winning strategy. (Explain your answer.)

Solution:

Now, I will define a function with help of which I will compute all initial game positions where Abelard has a wining strategy:

$\mathcal{F}(W) = [2023] \cup \{p : (p + 2002 + i) \in [2023] \wedge i \leq 2002\} \cup \{p : (p + 2002 + 2002) \notin W\}$

Answer:

[1..20, 2023]

Question 5:

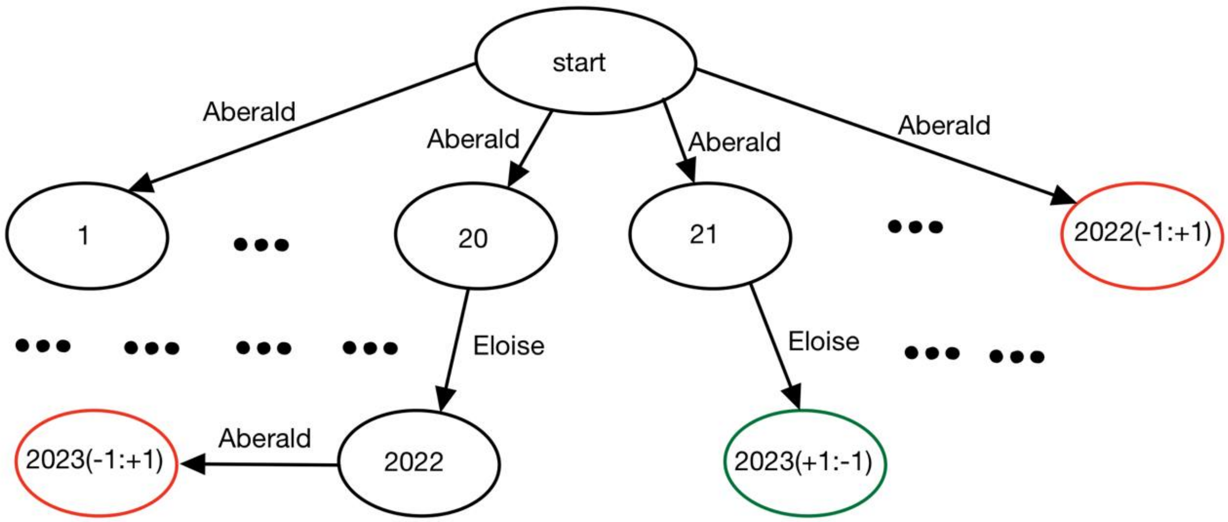
Starting with definition, draw (sketch) the game in the extensive form (using +1 for individual
win and −1 for individual loss).

Answer:

Game (with complete information) in the extensive form:

- A game (with complete information) in extensive form is a directed labeled tree (infinite maybe) where
  - the root is the initial position, nodes are positions, and the leaves are final positions
  - positions specify turns, all edges are moves by participating players according to their turns
  - all final positions are marked (by a vector of) individual payoffs for each player.
- A game in the extensive form is also known as a game decision tree.
- A game in the extensive form is finite if the game tree is finite; the game horizon is the depth of the tree in this case.
- A strategy of a player is a sequence of the player's moves along a path from the initial to a final position.
- Strategies S_A, S_B … of (different) players A, B, … are compatible if there exists a path from the initial to a final position with these strategies.
- A play is a set of strategies S_A, S_B … of (different) players A, B, … that defines a path from the initial to a final position in the game tree.

Game three for Eloise and Aberald:



Question 6:

Starting with definition, list all plays of the game in the extensive form that are Pareto optimal. (Explain your answer.)

Answer:

Pareto efficiency or Pareto optimality is a situation where no individual or preference criterion can be better off without making at least one individual or preference criterion worse off or without any loss thereof.

The game between Eloise and Abelard is the zero-sum game with possible payoffs (+1: -1) and (-1: +1). There is no way to change one strategy to another without decreasing the payoff of one of the players.

All the plays presented in the game tree from Question 5 are Pareto optimal.

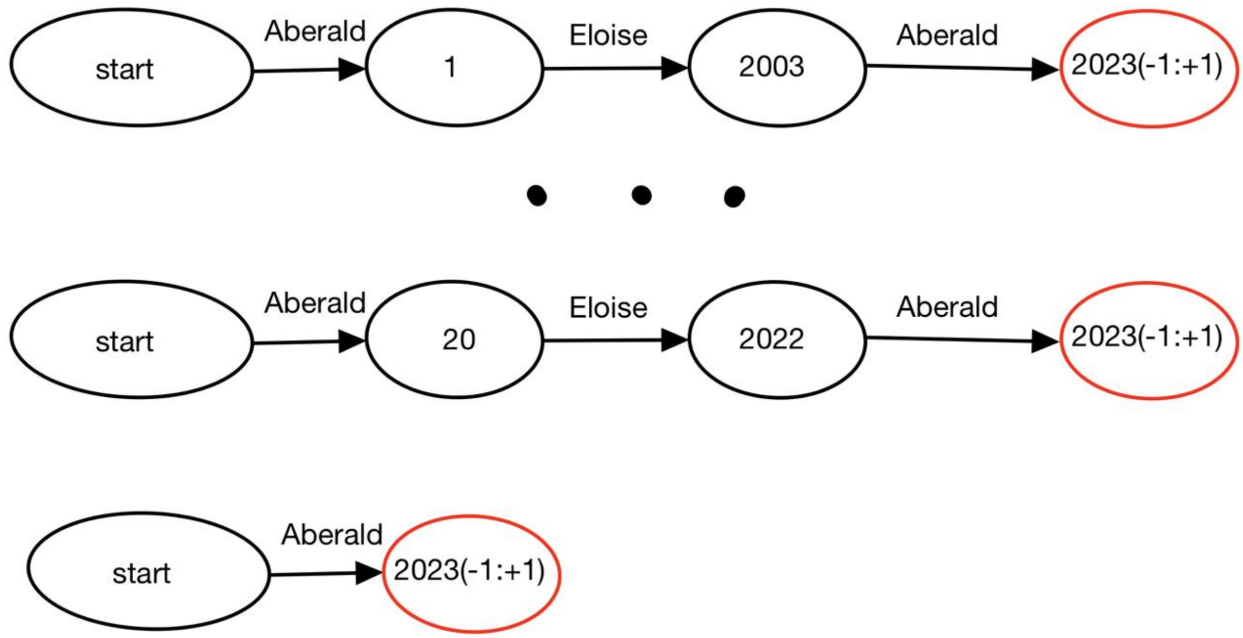
Question 7:

Starting with definition, list all plays of the game in the extensive form that are Nash equilibria. (Explain your answer.)

Answer:

Nash equilibrium is any play where no player has an incentive to deviate from their chosen strategy after considering an opponent's choice.

The game between Eloise and Abelard is the zero-sum game. Technique presented in the Lecture 2 slide 36 can be applied for finding Nash equilibria.



Question 8:

Starting with definition, represent the game in the normal form.

Answer:

- In the game in the normal form it is assumed that each player has decided what strategy to apply/use.
- Under this assumption
  - each strategy of each player collapse into a single big-step move (corresponding to this strategy)
  - turns of players disappear and individual big-step moves become simultaneous
  - each play has an appropriate vector of the individual payoffs.
- Two players' game in the normal form is 2-dimensional finite table (matrix) where
  - rows correspond to the strategies of the first player
  - columns correspond to the strategies of the second player,
  - each row and each column produce a play and their intersection cell contains vector of individual payoffs.

The only Eloise's strategy is to make a feasible move (+n) where n is the maximal integer in [1..2002] such that the next position is in the admissible range [1..2023].

Abelard's strategies are to choose the initial position and then make a feasible move (+t) where t is the maximal integer in [1..2002] such that the next position is in the admissible range [1..2023] or not to make a move if Eloise has finished the play.

Aberald / Eloise	(=1, +t)	...	(=20, +t)	(=21)	...	(=2022)	(=2023)
(+n)	-1	...	-1	+1	...	+1	-1