Final Theoretical Examination

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Task 1

Problem description

Consider a game of two players (Alice and Bob) with the following payoff matrix:

	B1	B2	B3	B4
A1	12	24	20	19
A2	9	4	2	61

Rows of the matrix corresponds to strategies $A1$ and $A2$ of Alice, columns
– to strategies $B1$, $B2$, $B3$, $B4$ of Bob.
Firstly, characterize the game using terms and concepts introduced in the lecture notes. Then solve the game in mixed strategies.

Characteristics

Game with complete information (topic 2, slides 8-9)

Information about payoffs, strategies is available for all players.

Two players' game in the normal form (topic 2, slide 14).

Two players' game in the normal form is 2-dimential finite table
(matrix) where

- rows correspond to the strategies of the first player,
- columns correspond to the strategies of the second player,
- each row and each column produce a play and their intersection cell contains vector of individual payoffs.

Zero-sum game.

- Sum payoffs for each outcome is equal to zero: for every play *S* the overall sum $\sum_{X \text{ is a player}} \pi_X(S) = 0$ (topic 2, slide 26)
- The payoffs are represented by a matrix with payoffs of the first player (topic 2, slide 27).

Mixed game extension.

- Individual strategies are mixed individual strategies (topic 3, slide 7).

Solution

1. The pure strategy is a special case of mixed strategy (topic 3, slide 5). Let's check a given zero-sum game for Nash equilibrium in pure strategies.

a. According to the slide 35 of 2-nd topic:

A zero-sum game of two players G has Nash equilibrium iff its matrix $[G_{S_A S_B}]$ enjoys the following minimax property

$\min_{S_B} \max_{S_A} G_{S_A S_B} = \max_{S_A} \min_{S_B} G_{S_A S_B}$

b. Let's check if the matrix G for a given game corresponds to the specified property:

$$\min_{S_B \in \{B1,B2,B3,B4\}} \max_{S_A \in \{A1,A2\}} G_{S_A S_B} = 12$$

$$\max_{S_A \in \{A1,A2\}} \min_{S_B \in \{B1,B2,B3,B4\}} G_{S_A S_B} = 12$$

c. Property is satisfied, so given zero sum game has a Nash equilibrium. This game can be solved in pure strategies.

2. Let’s now reduce given payoff matrix G eliminating dominated strategies as shown in slide 44 of 2-nd topic.

a. Definition of strict domination from slide 43 of 2-nd topic:

the strategy s'_X strictly dominates strategy $s''X$ *if* $\pi_X(SX : s'X) > \pi_X(SX : s''_X)$ for every play S and any player X (in A, B, etc.)

b. Strategy *B3* strictly dominates *B2 as:*

i. $\pi_B(A1, B3) = -20 > -24 = \pi_B(A1, B2)$

ii. $\pi_B(A2, B3) = -2 > -4 = \pi_B(A2, B2)$

c. Strategy *B1* strictly dominates *B4* as:

i. $\pi_B(A1, B1) = -12 > -19 = \pi_B(A1, B4)$

ii. $\pi_B(A2, B1) = -9 > -61 = \pi_B(A2, B4)$

d. After elimination of *B2* and *B4* columns I got the following payoff matrix:

	B1	B3
A1	12	20
A2	9	2

e. Strategy A1 strictly dominates A2 as:

i. $\pi_A(A1, B1) = 12 > 9 = \pi_A(A2, B1)$

ii. $\pi_A(A1, B3) = 20 > 2 = \pi_A(A2, B3)$

f. After elimination of *A2* the payoff matrix has the following from:

	B1	B3
A1	12	20

g. Strategy B1 strictly dominates B3 as:

i. $\pi_B(A1, B1) = -12 > -20 = \pi_B(A1, B3)$

h. Finally initial payoff matrix eliminated to:

	B1
A1	12

i. Proposition: A rational player will never play a strictly dominated strategy.

Proof: No dominated strategy can ever be optimal because, by definition of strict dominance, there is another dominating strategy yielding a higher payoff regardless of the other players’ strategies. In fact, the dominating strategy yields a higher expected payoff regardless of the rational player’s beliefs regarding other players’ strategies.

Answer

- This is a zero-sum game of two players in normal form with complete information.
- Solution of the given game is Nash equilibria.
- Solution can be presented as play S in mixed strategies: ((1, 0) (1, 0, 0, 0)).
- The payoff is $\pi^{mix}(S) = (12 : -12)$

Task 2

Problem description

Consider a game of two players (Alice and Bob) with the following payoff matrix:

	B1	B2
A1	12:24	20:19
A2	9:4	2:61

Rows of the matrix corresponds to strategies $A1$ and $A2$ of Alice, columns
– to strategies $B1$ and $B2$ of Bob.

Firstly, characterize the game using terms and concepts introduced in the lecture notes. Then solve the game in mixed strategies.

Characterisitics

Game with complete information (topic 2, slides 8-9)

Information about payoffs, strategies is available for all players.

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Mixed game extension.

- Individual strategies are mixed individual strategies (topic 3, slide 7).

Solution

1. Let's reduce given payoff matrix G eliminating dominated strategies as shown in slide 44 of 2-nd topic.
   a. Definition of strict domination from slide 43 of 2-nd topic:

      the strategy s'_X strictly dominates strategy s''_X if $\pi_X\left(S_{X:s'_X}\right) > \pi_X\left(S_{X:s''_X}\right)$ for every play S and any player X (in A, B, etc.)
   b. Strategy A1 strictly dominates A*2 as:*
      i. $\pi_A(A1, B1) = 12 > 9 = \pi_A(A2, B1)$
      ii. $\pi_A(A2, B3) = -2 > -4 = \pi_A(A2, B2)$
   c. After elimination of A2 I got the following payoff matrix:

	B1	B2
A1	12:24	20:19

d. Strategy B1 strictly dominates B2 *as:*

   i. $\pi_B(A1, B1) = 24 > 19 = \pi_B(A1, B2)$

e. After elimination of B2 final payoff matrix has the following form:

	B1
A1	12:24

f. Proposition: A rational player will never play a strictly dominated strategy. (Was proved in the 1 Task)

Answer

- This is a game of two players in normal form with complete information.
- Solution can be presented as play S in mixed strategies: ((1, 0) (1, 0)).
- The payoff is $\pi^{mix}(S) = (12 : 24)$

Task 3

Problem description

Consider problem Rational Agents at the Marketplace (from lecture notes on topic 4). What are individual agents' beliefs, desires, and intentions in the model of the problem? Let agents A and B compete for a salesman, and the matrix of their game flip-or-bid game be

A\B	bid	flip
bid	-20:-2	0:-9
flip	-12:0	-12:-9

where L_A = −12 and L_B = −9 are individual (negative) losses in case of flip, F_A = −20 and F_B = −2 are individual (also negative) fins for simultaneous bidding. – Characterize and solve the flip-or-bid game.

Characteristics

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Mixed game extension.

- Individual strategies are mixed individual strategies (topic 3, slide 7).

Multiagent Research Paradigm (topic 4, slide 7)

- Agent's beliefs represent its ideas/opinion about itself, other agents and the network; this ideas/opinions may be incorrect, incomplete, and (even) inconsistent.
- Agent's desires represent its long-term aims, obligations and purposes (that may be controversial).
- Agent's intensions are used for a short-term planning (related to its desires of course).

Solution

1. Let's reduce given payoff matrix G eliminating dominated strategies as shown in slide 44 of 2-nd topic.

   Initial payoff matrix is:

A\B	B1	B2
A1	-20:-2	0:-9
A2	-12:0	-12:-9

   a. Definition of strict domination from slide 43 of 2-nd topic:

      the strategy s'_X strictly dominates strategy s''_X if $\pi_X\left(S_{X:s'_X}\right) > \pi_X\left(S_{X:s''_X}\right)$ for every play S and any player X (in A, B, etc.)

   b. Strategy B1 strictly dominates B*2 as:*

      i. $\pi_B(A1, B1) = -2 > -9 = \pi_B(A1, B2)$

      ii. $\pi_B(A2, B1) = 0 > -9 = \pi_B(A2, B2)$

   c. After elimination of B2 I got the following payoff matrix:

A\B	B1
A1	-20:-2
A2	-12:0

   d. Strategy A2 strictly dominates A1 as:

      i. $\pi_A(A1, B1) = 24 > 19 = \pi_A(A2, B1)$

   e. Final eliminated payoff matrix:

A\B	B1
A2	-12:0

   f. Proposition: A rational player will never play a strictly dominated strategy. (Was proved in the 1 Task)

Answer

- Agent's beliefs:

  All buyers are rational agents that can communicate, negotiate, make concessions, and flip (change) individually and swap (exchange) their salesmen pairwise (and only pairwise) in peer-to-peer (P2P) manner. (topic 4, slides 18-22)

- Agent's desires:

  Every salesman has a (single) indivisible piece of cake, and every buyer wants to buy exactly one. (topic 4, slide 17)

- Agent's intentions:

  A buyer never mind to remember data of other customers and always is looking for the most rational local action (i.e., just one step ahead). (topic 4, slide 22)

- This is a game of two players in normal form with complete information.

- Solution can be presented as play S in mixed strategies: ((0, 1) (1, 0)).
- The payoff is $\pi^{mix}(S) = (-12 : 0)$