Homework 1 : Notation

Problems:

1. Show the correctness of the following statements :
   - $log(n)\ \varepsilon\ O(n)$
   - $n\ \varepsilon\ O(n.log(n))$
   - $n.log(n)\ \varepsilon\ O(n^2)$
   - $2^n\ \varepsilon\ \Omega(5.ln(n))$

2. Group the following functions by complexity category :

$$n.log(n) \quad - \quad (log(n))^3 \quad - \quad (5n^2 + 7n) \quad - \quad n^{5/2} \quad - \quad n!$$
$$n^n \quad - \quad (n^n + ln(n)) \quad - \quad 5^{log(n)} \quad - \quad log(n!) \quad - \quad (log(n))!$$
$$\sqrt{n} \quad - \quad e^n \quad - \quad (8n + 12) \quad - \quad 10^n + n^{20}$$

3. What is the time complexity **T(n)** of the nested loops below?
   - For simplicity, you may assume that **n is a power of 2.** That is "**n = 2k**" for some positive integer k.

```
int j;
for(int i = 0 ; i < n ; i++){
    j = n;
    while(j >= 1){
        j = j / 2;
    }
}
```

4. Consider the following algorithm :

```
int add_them(int n , int A[]){
    int i, j, k;
    j = 0;
    for(i = 1 ; i <= n ; i++){
        j = j + A[j];
    }
    k = 1;
    for(i = 1 ; i <= n ; i++){
        k = 2 * k
    }
    return j + k;
}
```

   a. If n = 5 and the array A contains 2, 5, 3, 7 and 8 , what's the output?
   b. What's the time complexity T(n) of the algorithm?
   c. Try to improve the efficiency of the algorithm.

5. Give an algorithm for the following problem. Given a list of n distinct Positive integers, partition the list into 2 sublists, each of size n/2, such That the difference between the sums of the integers in the 2 sublists is Minimized. Determine the time complexity of your algorithm. You may assume that n is a multiple of 2.

6. Suppose you have a computer that requires 1 minute to solve problem Instances of size n = 1,000. Suppose you buy a new computer that runs 1,000 times faster than the old one. What instance sizes can be run in 1 Minute, assuming the following time complexities T(n) for our algorithm?