

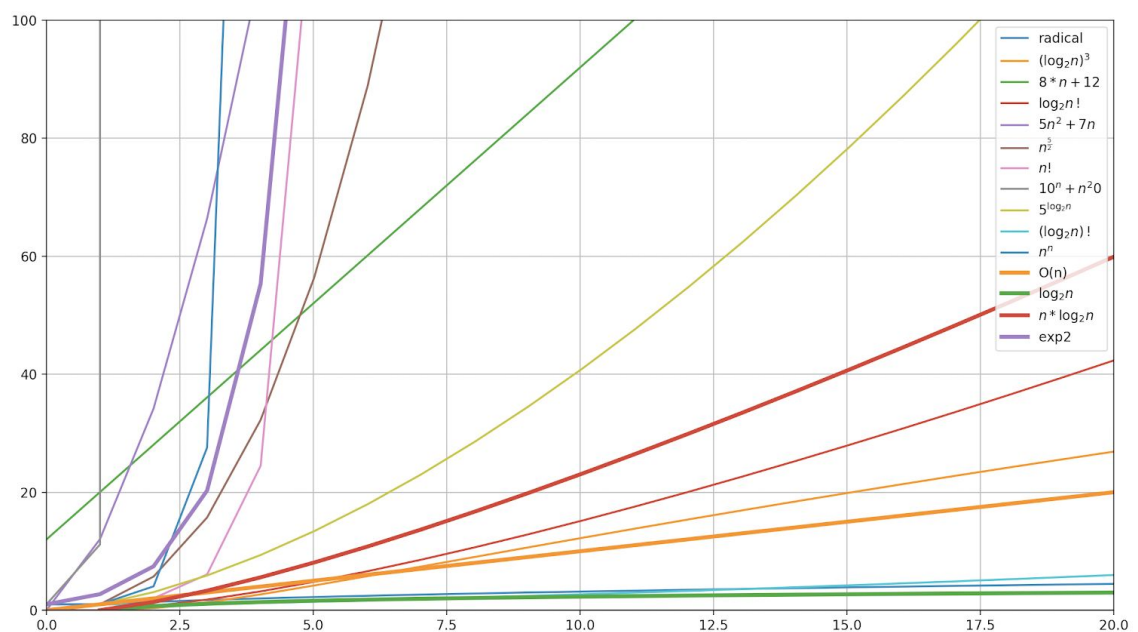
# Homework 1 : Notation

## Solution:

1. quick way to compare growth of two given functions  $f(x)$  and  $g(x)$  :

$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0 \leftrightarrow$  growth of  $f(x)$  is less than  $g(x)$  ,  $\infty \leftrightarrow$  growth of  $f(x)$  is more than  $g(x)$ ,  $k \neq 0 \leftrightarrow$  they both grow equally

- $\lim_{n \rightarrow \infty} \frac{\log(n)}{n} = 0 \rightarrow \log(n) \in O(n)$
  - $\lim_{n \rightarrow \infty} \frac{n}{n \log(n)} = 0 \rightarrow n \in O(n \log(n))$
  - $n \log(n) \leq n^2$   $N = 0$  ,  $c = 1$  ,  $n \geq 0$
  - $2^n \geq 5^{\ln(n)}$   $N = 0$  ,  $c = 1$  ,  $n \geq 0$
2.  $\log n < \sqrt{n} < (\log n)^3 < 8n + 12 < (\log n)! < \log(n!) < n \cdot \log n < 5n^2 + 7n < n^{\frac{5}{2}} < n^3 < 5^{\log n} < e^n 10n + n^{20} < n! < n^n < n^n + \ln(n)$



3. The first loop is going from 0 to  $n$  , so repetition of " $i < n$ " will be  $n$  times .  
second loop is going to execute for  $2^k = n$  , so the " $j > 1$ " will be executed  $\log n$  times. In the end the whole code is going to run in  $O(n \log(n))$  order .
4. Answers:

- a.  $25+32 = 57$
- b. Each loop is going to execute  $n$  times and they're not nested so the function runs in  $O(n)$  order .
- c. We can do the  $k$ 's multiplication in the first loop .

5.

```
exercise > r.py > ...
1  minimum = 1e7
2  def rec(arr):
3      print(arr)
4      if len(arr) == 1:
5          return [arr[0]]
6
7      mid = int(len(arr)/2)
8      left = rec(arr[:mid])
9      right = rec(arr[mid:])
10
11     if isinstance(left, list):
12         left_sum = sum(left)
13     else:
14         left_sum = left
15
16     if isinstance(right, list):
17         right_sum = sum(right)
18     else:
19         right_sum = right
20
21     global minimum
22     minimum = min(minimum, abs(left_sum - right_sum))
23
24     print("min is: ", minimum)
25     return [*left, *right]
26
27
28 print(rec([1, 4, 4, 6, 4, 5, 7, 8]))
29 print("minimum is: ", minimum)
```

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```
[4]
[6]
min is: 2
min is: 2
[4, 5, 7, 8]
[4, 5]
[4]
[5]
min is: 1
[7, 8]
[7]
[8]
min is: 1
min is: 1
min is: 1
[1, 4, 4, 6, 4, 5, 7, 8]
minimum is: 1
(env) → exercise
```

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6. The first computer will solve the problem of  $n=1000$  size in one minute . the new computer will solve  $n=1000$  size problem in  $\frac{1}{1000}$  minute , so it can solve a problem of size  $n = 10^6$  in one minute .

- a)  $T(n) \in \theta(n)$      $n=1000$      $n = 1000*1000 = 1000000$      $n^* = 10^6$   
b)  $T(n) \in \theta(n^3)$      $n=100$      $n = 100*100*100 = 1000000$      $n^* = 10^6$   
c)  $T(n) \in \theta(10^n)$      $n=6$      $n=10^6$      $10^n = 10^6$