

## Homework 1 : Notation

### Problems:

1. Show the correctness of the following statements :

- $\log(n) \in O(n)$
- $n \in O(n \cdot \log(n))$
- $n \cdot \log(n) \in O(n^2)$
- $2^n \in \Omega(5 \cdot \ln(n))$

2. Group the following functions by complexity category :

$$\begin{aligned} & n \cdot \log(n) \quad - \quad (\log(n))^3 \quad - \quad (5n^2 + 7n) \quad - \quad n^{5/2} \quad - \quad n! \\ & n^n \quad - \quad (n^n + \ln(n)) \quad - \quad 5^{\log(n)} \quad - \quad \log(n!) \quad - \quad (\log(n))! \\ & \sqrt{n} \quad - \quad e^n \quad - \quad (8n + 12) \quad - \quad 10^n + n^{20} \end{aligned}$$

3. What is the time complexity  $T(n)$  of the nested loops below?

- For simplicity, you may assume that **n is a power of 2**.  
That is " $n = 2^k$ " for some positive integer k.

```
int j;  
for(int i = 0 ; i < n ; i++){  
    j = n;  
    while(j >= 1){  
        j = j / 2;  
    }  
}
```

4. Consider the following algorithm :

```
int add_them(int n , int A[]){  
    int i, j, k;  
    j = 0;  
    for(i = 1 ; i <= n ; i++){  
        j = j + A[j];  
    }  
    k = 1;  
    for(i = 1 ; i <= n ; i++){  
        k = 2 * k  
    }  
    return j + k;  
}
```

- If  $n = 5$  and the array  $A$  contains 2, 5, 3, 7 and 8 , what's the output?
  - What's the time complexity  $T(n)$  of the algorithm?
  - Try to improve the efficiency of the algorithm.
5. Give an algorithm for the following problem. Given a list of  $n$  distinct Positive integers, partition the list into 2 sublists, each of size  $n/2$ , such That the difference between the sums of the integers in the 2 sublists is Minimized. Determine the time complexity of your algorithm. You may assume that  $n$  is a multiple of 2.
6. Suppose you have a computer that requires 1 minute to solve problem Instances of size  $n = 1,000$ . Suppose you buy a new computer that runs 1,000 times faster than the old one. What instance sizes can be run in 1 Minute, assuming the following time complexities  $T(n)$  for our algorithm?