

## Homework 1 : Notation

### Problems:

1. Show the correctness of the following statements :

- $\log(n) \in O(n)$
- $n \in O(n \cdot \log(n))$
- $n \cdot \log(n) \in O(n^2)$
- $2^n \in \Omega(5 \cdot \ln(n))$

2. Sort the following functions by complexity :

$$\begin{aligned} n \cdot \log(n) &- (\log(n))^3 &- (5n^2 + 7n) &- n^{5/2} &- n! \\ n^n &- (n^n + \ln(n)) &- 5^{\log(n)} &- \log(n!) &- (\log(n))! \\ \sqrt{n} &- e^n &- (8n + 12) &- 10^n + n^{20} \end{aligned}$$

3. What is the time complexity **T(n)** of the nested loops below?

- For simplicity, you may assume that **n is a power of 2**.  
That is "**n = 2<sup>k</sup>**" for some positive integer k.

```
int j;
for(int i = 0 ; i < n ; i++){
    j = n;
    while(j >= 1){
        j = j / 2;
    }
}
```

4. Consider the following algorithm :

```
int add_them(int n , int A[]){
    int i, j, k;
    j = 0;
    for(i = 1 ; i <= n ; i++){
        j = j + A[j];
    }
    k = 1;
    for(i = 1 ; i <= n ; i++){
        k = 2 * k
    }
    return j + k;
}
```

- If  $n = 5$  and the array  $A$  contains 2, 5, 3, 7 and 8 , what's the output?
  - What's the time complexity  $T(n)$  of the algorithm?
  - Try to improve the efficiency of the algorithm.
5. Give an algorithm for the following problem. Given a list of  $n$  distinct Positive integers, partition the list into 2 sublists, each of size  $n/2$ , such That the difference between the sums of the integers in the 2 sublists is Minimized. Determine the time complexity of your algorithm. You may assume that  $n$  is a multiple of 2.
6. Suppose you have a computer that requires 1 minute to solve problem Instances of size  $n = 1,000$ . Suppose you buy a new computer that runs 1,000 times faster than the old one. What instance sizes can be run in 1 Minute, assuming the following time complexities  $T(n)$  for our algorithm?