

Homework 1 : Notation

Problems:

1. Show the correctness of the following statements :

- $\log(n) \in O(n)$
- $n \in O(n \cdot \log(n))$
- $n \cdot \log(n) \in O(n^2)$
- $2^n \in \Omega(5 \cdot \ln(n))$

2. Sort the following functions by complexity :

$$\begin{aligned} n \cdot \log(n) &- (\log(n))^3 &- (5n^2 + 7n) &- n^{5/2} &- n! \\ n^n &- (n^n + \ln(n)) &- 5^{\log(n)} &- \log(n!) &- (\log(n))! \\ \sqrt{n} &- e^n &- (8n + 12) &- 10^n + n^{20} \end{aligned}$$

3. What is the time complexity $T(n)$ of the nested loops below?

- For simplicity, you may assume that **n is a power of 2**.
That is " $n = 2^k$ " for some positive integer k.

```
int j;  
for(int i = 0 ; i < n ; i++){  
    j = n;  
    while(j >= 1){  
        j = j / 2;  
    }  
}
```

4. Consider the following algorithm :

```
int add_them(int n , int A[]){
    int i, j, k;
    j = 0;
    for(i = 0 ; i < n ; i++){
        j = j + A[i];
    }
    k = 1;
    for(i = 0 ; i < n ; i++){
        k = 2 * k
    }
    return j + k;
}
```

- If $n = 5$ and the array A contains 2, 5, 3, 7 and 8 , what's the output?
 - What's the time complexity $T(n)$ of the algorithm?
 - Try to improve the efficiency of the algorithm.
5. Give an algorithm for the following problem. Given a list of n distinct Positive integers, partition the list into 2 sublists, each of size $n/2$, such That the difference between the sums of the integers in the 2 sublists is Minimized. Determine the time complexity of your algorithm. You may assume that n is a multiple of 2.
6. Suppose you have a computer that requires 1 minute to solve problem Instances of size $n = 1,000$. Suppose you buy a new computer that runs 1,000 times faster than the old one. What instance sizes can be run in 1 Minute, assuming the following time complexities $T(n)$ for our algorithm?