Homework 1 : Notation
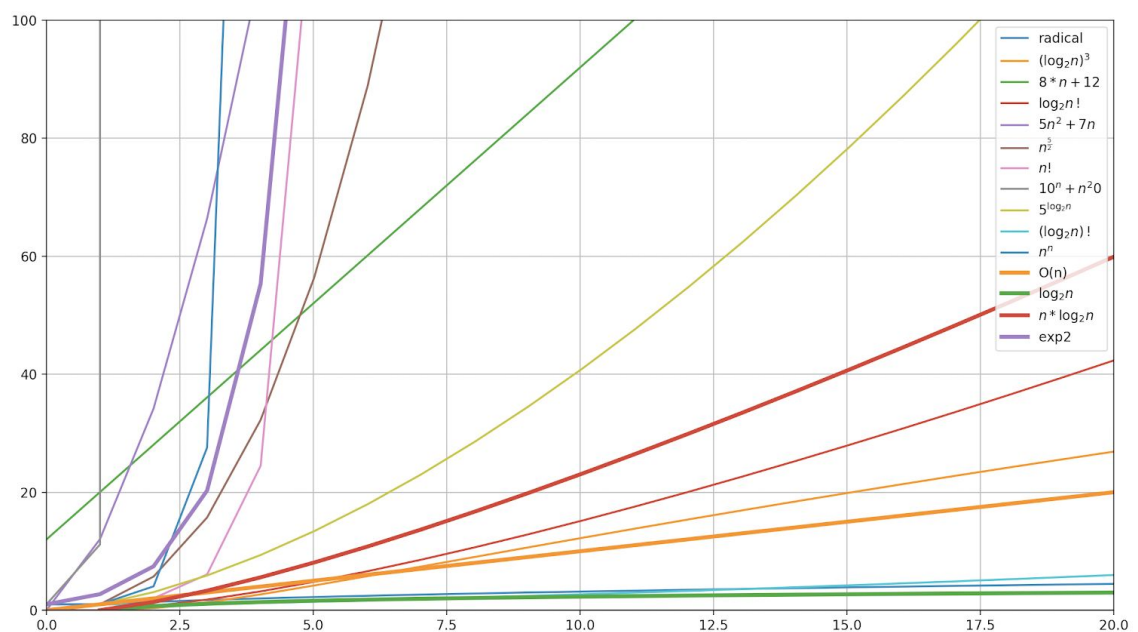
Solution:

1. quick way to compare growth of two given functions f(x) and g(x) :

   $\lim\limits_{x\to\infty} \frac{f(x)}{g(x)} =$ *if* $0 \leftrightarrow$ *growth of* $f(x)$ *is less than* $g(x)$, $\infty \leftrightarrow$ *growth of* $f(x)$
   is more than $g(x)$, $k \neq 0 \leftrightarrow$ *they both grow equally*

   - $\lim\limits_{n\to\infty} \frac{log(n)}{n} = 0 \rightarrow log(n) \;\varepsilon\; O(n)$
   - $\lim\limits_{n\to\infty} \frac{n}{nlog(n)} = 0 \rightarrow n \;\varepsilon\; O(nlog(n))$
   - $nlog(n) \leq n^2 \; N = 0 , \; c = 1, \; n \geq 0$
   - $2^n \geq 5^{ln(n)} \;\; N = 0 , \; c = 1 , \; n \geq 0$

2. $logn < (logn)^3 < \sqrt{n} \; < 8n + 12 < (logn)! < log(n!) < n.logn < 5n^2 + 7n$
   $< n^{\frac{5}{2}} < n^3 < 5^{logn} < e^n 10n + n^{20} < n! < n^n < n^n + ln(n)$

3. The first loop is going from 0 to n , so repetition of "i<n" will be n times .
   second loop is going to execute for $2^k = n$, so the "j>1" will be executed $logn$
   times. In the end the hole code is going to run in $O(nlog(n))$ order .

4. Answers:

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       a.   25+32 = 57
       b.   Each loop is going to execute n times and they're not nested so the function runs in O(n) order .
       c.   We can do the k's multiplication in the first loop .

5.

```python
exercise > r.py > ...
1    minimum = 1e7
2    def rec(arr):
3        print(arr)
4        if len(arr) == 1:
5            return [arr[0]]
6
7        mid = int(len(arr)/2)
8        left = rec(arr[:mid])
9        right = rec(arr[mid:])
10
11       if isinstance(left, list):
12           left_sum = sum(left)
13       else:
14           left_sum = left
15
16       if isinstance(right, list):
17           right_sum = sum(right)
18       else:
19           right_sum = right
20
21       global minimum
22       minimum = min(minimum, abs(left_sum - right_sum))
23
24       print("min is: ", minimum)
25       return [*left, *right]
26
27
28   print(rec([1, 4, 4, 6, 4, 5, 7, 8]))
29   print("minimum is: ", minimum)
```

```
TERMINAL    SQL CONSOLE: MESSAGES    DEBUG CONSOLE    PROBLEMS    OUTPUT

[4]
[6]
min is:  2
min is:  2
[4, 5, 7, 8]
[4, 5]
[4]
[5]
min is:  1
[7, 8]
[7]
[8]
min is:  1
min is:  1
min is:  1
[1, 4, 4, 6, 4, 5, 7, 8]
minimum is:  1
(env) → exercise
```

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6. The first computer will solve the problem of n=1000 size in one minute . the new computer will solve n=1000 size problem in $\frac{1}{1000}$ minute , so it can solve a problem of size n = 10^6 in one minute .

   a) T(n) $\in$ $\theta(n)$ n=1000   n = 1000*1000 = 1000000 n* = 10^6
   b) T(n) $\in$ $\theta(n^3)$ n=100   n = 100*100*100 = 1000000 n* = 10^6
   c) T(n) $\in$ $\theta(10^n)$ n=6   n=10^6   10^n = 10^6