Multibody dynamics (3)

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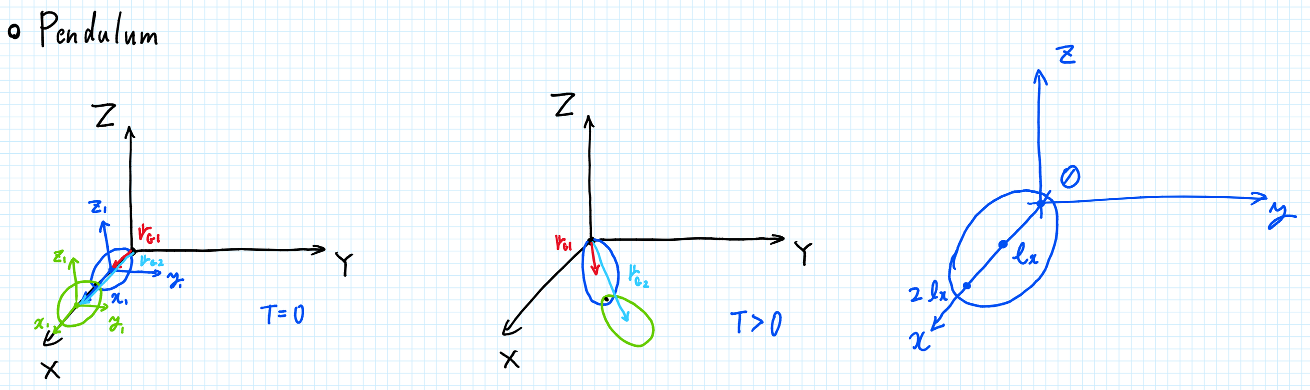
- Pendulum



$$\left.\begin{array}{l}
\bullet\ \mathbb{M}_G d_T^2\bar{r}_G + \partial_{\bar{r}_G}\mathbb{C}^T\cdot\lambda = \mathbb{F}_G,\\
\bullet\ 4\mathbb{L}_G^T\mathbb{J}_G^\triangle\mathbb{L}_G\cdot d_T^2\bar{\varepsilon} + \partial_{\bar{\varepsilon}}\mathbb{C}^T\cdot\lambda + \partial_{\bar{\varepsilon}}\mathbb{C}_E^T\cdot\lambda_E = 8\, d_T\mathbb{L}_G^T\mathbb{J}_G^\triangle d_T\mathbb{L}_G\bar{\varepsilon} + 2\mathbb{L}_G n_G^\triangle,\\
\bullet\ \partial_{\bar{r}_G}\mathbb{C}\cdot d_T^2\bar{r}_G + \partial_{\bar{\varepsilon}}\mathbb{C}\cdot d_T^2\bar{\varepsilon} = \gamma,\ \left[\gamma := -d_T(\partial_T\mathbb{C}) - d_T(\partial_{\bar{r}_G}\mathbb{C})d_T\bar{r}_G - d_T(\partial_{\bar{\varepsilon}}\mathbb{C})\cdot d_T\bar{\varepsilon}\right]\\
\bullet\ \partial_{\bar{\varepsilon}}\mathbb{C}_E\cdot d_T^2\bar{\varepsilon} = \gamma_E,\ \left[\gamma_E := -2\left[\dot{\varepsilon}_1^T\dot{\varepsilon}_1 \ldots \dot{\varepsilon}_N^T\dot{\varepsilon}_N\right]\right]
\end{array}\right\}$$

$$\boxed{\begin{bmatrix} \mathbb{M}_G & 0 & \partial_{\bar{r}_G}\mathbb{C}^T & 0\\ 0 & 4\mathbb{L}_G^T\mathbb{J}_G^\triangle\mathbb{L}_G & \partial_{\bar{\varepsilon}}\mathbb{C}^T & \partial_{\bar{\varepsilon}}\mathbb{C}_E^T\\ \partial_{\bar{r}_G}\mathbb{C} & \partial_{\bar{\varepsilon}}\mathbb{C} & 0 & 0\\ 0 & \partial_{\bar{\varepsilon}}\mathbb{C}_E & 0 & 0\end{bmatrix}\cdot\begin{bmatrix}\ddot{\bar{r}}_G\\ \ddot{\bar{\varepsilon}}\\ \lambda\\ \lambda_E\end{bmatrix} = \begin{bmatrix}\mathbb{F}_G\\ 8\, d_T\mathbb{L}_G^T\mathbb{J}_G^\triangle d_T\mathbb{L}_G\bar{\varepsilon} + 2\mathbb{L}_G^T n_G^\triangle\\ \gamma\\ \gamma_E\end{bmatrix}}$$

$$\Rightarrow\ \mathbb{S}(\bar{r}_G,\bar{\varepsilon})\begin{bmatrix}\ddot{\bar{r}}_G\\ \ddot{\bar{\varepsilon}}\\ \lambda\\ \lambda_E\end{bmatrix} = \mathbb{b}(\bar{\varepsilon}, d_T\bar{r}_G, d_T\bar{\varepsilon}),\quad \mathbb{S}(\bar{r}_G,\bar{\varepsilon}) := \begin{bmatrix} \mathbb{M}_G & 0 & \partial_{\bar{r}_G}\mathbb{C}^T & 0\\ 0 & 4\mathbb{L}_G^T\mathbb{J}_G^\triangle\mathbb{L}_G & \partial_{\bar{\varepsilon}}\mathbb{C}^T & \partial_{\bar{\varepsilon}}\mathbb{C}_E^T\\ \partial_{\bar{r}_G}\mathbb{C} & \partial_{\bar{\varepsilon}}\mathbb{C} & 0 & 0\\ 0 & \partial_{\bar{\varepsilon}}\mathbb{C}_E & 0 & 0\end{bmatrix}.$$

- $\frac{\partial}{\partial\varepsilon}(\mathbb{A}\,\mathbf{l}) = \frac{\partial}{\partial\varepsilon}\left\{(2p_0^2-1)\mathbb{I} + 2(\mathbf{p}\mathbf{p}^T + p_0\tilde{\mathbf{p}})\right\}\mathbf{l} = \frac{\partial}{\partial\varepsilon}\left\{(2p_0^2-1)\mathbf{l} + 2(\mathbf{p}\mathbf{p}^T + p_0\tilde{\mathbf{p}})\mathbf{l}\right\} = \left[4p_0\mathbf{l} + 2\tilde{\mathbf{p}}\mathbf{l}\quad 2(\underbrace{\mathbf{p}^T\mathbf{l}\,\mathbb{I} + \mathbf{p}\mathbf{l}^T}) + 2p_0\underbrace{\partial_{\mathbf{p}}(\tilde{\mathbf{p}}\cdot\mathbf{l})}\right]$

$\left[\frac{\partial}{\partial p_i}(p_j p_k l_k) = \delta_{ij}p_k l_k + p_j\delta_{ik}l_k = \delta_{ij}p_k l_k + p_j l_i = \mathbf{p}^T\mathbf{l}\,\mathbb{I} + \mathbf{p}\mathbf{l}^T\right]$ $\left[\partial_{\mathbf{p}}(\tilde{\mathbf{p}}\mathbf{l}) = [\partial_1(\tilde{\mathbf{p}}\mathbf{l})\ \partial_2(\tilde{\mathbf{p}}\mathbf{l})\ \partial_3(\tilde{\mathbf{p}}\mathbf{l})] = \ldots = -\tilde{\mathbf{l}}\right]$

$$= 2\left[2p_0\mathbf{l} + \tilde{\mathbf{p}}\mathbf{l}\quad (\mathbf{p}^T\mathbf{l}\,\mathbb{I} + \mathbf{p}\mathbf{l}^T) - p_0\tilde{\mathbf{l}}\right] = 2\left[2p_0\mathbf{l} + \tilde{\mathbf{p}}\mathbf{l}\quad \begin{bmatrix} p_1l_1+p_2l_2+p_3l_3 & 0 & 0\\ 0 & p_1l_1+p_2l_2+p_3l_3 & 0\\ 0 & 0 & p_1l_1+p_2l_2+p_3l_3\end{bmatrix} + \mathbf{p}\mathbf{l}^T - p_0\tilde{\mathbf{l}}\right]$$

$$= 2\left[2p_0\mathbf{l} + \tilde{\mathbf{p}}\mathbf{l}\quad \begin{bmatrix} l_1p_1 & l_1p_2 & l_1p_3\\ l_2p_1 & l_2p_2 & l_2p_3\\ l_3p_1 & l_3p_2 & l_3p_3\end{bmatrix} - \begin{bmatrix} -l_2p_2-l_3p_3 & l_1p_2 & l_1p_3\\ l_2p_1 & -l_1p_1-l_3p_3 & l_2p_3\\ l_3p_1 & l_3p_2 & -l_1p_1-l_2p_2\end{bmatrix} + \mathbf{p}\mathbf{l}^T - p_0\tilde{\mathbf{l}}\right] = 2\left[2p_0\mathbf{l} + \tilde{\mathbf{p}}\mathbf{l}\quad \begin{bmatrix} l_1p_1 & l_1p_2 & l_1p_3\\ l_2p_1 & l_2p_2 & l_2p_3\\ l_3p_1 & l_3p_2 & l_3p_3\end{bmatrix} - \begin{bmatrix} 0 & -p_3 & p_2\\ p_3 & 0 & -p_1\\ -p_2 & p_1 & 0\end{bmatrix}\begin{bmatrix} 0 & -l_3 & l_2\\ l_3 & 0 & -l_1\\ -l_2 & l_1 & 0\end{bmatrix} + \mathbf{p}\mathbf{l}^T - p_0\tilde{\mathbf{l}}\right]$$

$$= 2\left[2p_0\mathbf{l} + \tilde{\mathbf{p}}\mathbf{l}\quad \mathbf{l}\mathbf{p}^T - \tilde{\mathbf{p}}\tilde{\mathbf{l}} + \mathbf{p}\mathbf{l}^T - p_0\tilde{\mathbf{l}}\right] = 2\left[p_0\mathbf{l} + \tilde{\mathbf{p}}\mathbf{l}\quad -\tilde{\mathbf{p}}\tilde{\mathbf{l}} + \mathbf{p}\mathbf{l}^T - p_0\tilde{\mathbf{l}}\right] + 2\left[p_0\mathbf{l}\quad \mathbf{l}\mathbf{p}^T\right] = 2\left[(p_0\mathbb{I} + \tilde{\mathbf{p}})\mathbf{l}\quad -(p_0\mathbb{I} + \tilde{\mathbf{p}})\tilde{\mathbf{l}} + \mathbf{p}\mathbf{l}^T\right] + 2\,\mathbf{l}\left[p_0\quad \mathbf{p}^T\right]$$

$$= 2\left[-\mathbf{p}\quad p_0\mathbb{I} + \tilde{\mathbf{p}}\right]\begin{bmatrix} 0 & -\mathbf{l}^T\\ \mathbf{l} & -\tilde{\mathbf{l}}\end{bmatrix} + 2\,\mathbf{l}\,\varepsilon^T =: 2\mathbb{E}\,\mathbb{B}^- + 2\,\mathbf{l}\,\varepsilon^T,\quad \mathbb{B}^- := \begin{bmatrix} 0 & -\mathbf{l}^T\\ \mathbf{l} & -\tilde{\mathbf{l}}\end{bmatrix}.$$

- $\mathbb{M}_G = m\mathbb{I},\quad \mathbb{J}_G^\triangle = \mathbb{J}^\triangle,$
- $\bar{r}_G = r_G,\ \bar{\varepsilon} = \varepsilon,\ \mathbb{F}_G = \mathbb{F},\ n_G^\triangle = n^\triangle,$
- $\mathbb{L}_G = \mathbb{L}(\varepsilon) = [-\mathbf{p}\quad p_0\mathbb{I} - \tilde{\mathbf{p}}],\ d_T\mathbb{L}_G = d_T\mathbb{L}(d_T\varepsilon) = [-\dot{\mathbf{p}}\quad \dot{p}_0\mathbb{I} - \dot{\tilde{\mathbf{p}}}],$
- $\mathbb{C}(\bar{r}_G,\bar{\varepsilon}) = \mathbb{C}(r_G,\varepsilon) = r^T r = \left(r_G + \mathbb{A}\begin{bmatrix}-l_x\\0\\0\end{bmatrix}\right)^T\left(r_G + \mathbb{A}\begin{bmatrix}-l_x\\0\\0\end{bmatrix}\right) = (r_G + \mathbb{A}s_1)^T(r_G + \mathbb{A}s_1) = r_G^Tr_G + 2r_G^T\mathbb{A}s_1 + s_1^T\underbrace{\mathbb{A}^T\mathbb{A}}_{[=\mathbb{I}]}s_1 = r_G^Tr_G + 2r_G^T\mathbb{A}s_1 + s_1^Ts_1$

$\Rightarrow$
- $\partial_{\bar{r}_G}\mathbb{C} = 2r_G^T + 2s_1^T\mathbb{A}^T,$
- $\partial_{\bar{\varepsilon}}\mathbb{C} = 2r_G^T\partial_{\bar{\varepsilon}}(\mathbb{A}s_1) = 4r_G^T\mathbb{E}\begin{bmatrix}0 & -s_1^T\\ s_1 & -\tilde{s}_1\end{bmatrix} + 4r_G^Ts_1\varepsilon^T = 4r_G^T\mathbb{E}\,\mathbb{B}^-(s_1) + 4r_G^Ts_1\varepsilon^T$

- $\gamma = -d_T(\partial_T\mathbb{C}) - d_T(\partial_{\bar{r}_G}\mathbb{C})d_T\bar{r}_G - d_T(\partial_{\bar{\varepsilon}}\mathbb{C})\cdot d_T\bar{\varepsilon} = -d_T(2r_G^T + 2s_1^T\mathbb{A}^T)d_Tr_G - d_T(\partial_{\bar{\varepsilon}}\mathbb{C})\cdot d_T\bar{\varepsilon} = -2d_Tr_G^Td_Tr_G - 4s_1^T\underbrace{\mathbb{L}\dot{\mathbb{E}}^T}_{[\dot{\mathbb{A}}=2\dot{\mathbb{E}}\mathbb{L}^T]}d_Tr_G - 4\left\{(d_Tr_G^T\mathbb{E} + r_G^T\dot{\mathbb{E}})\begin{bmatrix}0 & -s_1^T\\ s_1 & -\tilde{s}_1\end{bmatrix} + (d_Tr_G^Ts_1\varepsilon^T + r_G^Ts_1d_T\varepsilon^T)\right\}\cdot d_T\varepsilon$

- $\mathbb{C}_E(\bar{\varepsilon}) = \mathbb{C}_E(\varepsilon) = \varepsilon^T\varepsilon - 1 = 0 \Rightarrow \bullet\ \partial_{\bar{\varepsilon}}\mathbb{C}_E = 2\varepsilon^T,$
- $\gamma_E = -2\dot{\varepsilon}^T\dot{\varepsilon}.$

- $\mathbb{S}(\bar{r}_G,\bar{\varepsilon})\cdot\begin{bmatrix}\ddot{\bar{r}}_G\\ \ddot{\bar{\varepsilon}}\\ \lambda\\ \lambda_E\end{bmatrix} = \mathbb{b}(\bar{\varepsilon}, d_T\bar{r}_G, d_T\bar{\varepsilon}) \Leftrightarrow \begin{bmatrix}\ddot{\bar{r}}_G\\ \ddot{\bar{\varepsilon}}\\ \lambda\\ \lambda_E\end{bmatrix} = \mathbb{S}^{-1}\cdot\mathbb{b}$
- $q_1 := [\bar{r}_G^T\ \ \bar{\varepsilon}^T]^T,\quad q_2 = \dot{q}_1$

$$\Rightarrow\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \lambda\\ \lambda_E\end{bmatrix} = \begin{bmatrix} q_2\\ \hline \mathbb{S}^{-1}\cdot\mathbb{b}\end{bmatrix}$$

- Double pendulum

- $\mathbb{M}_G = \begin{bmatrix} m_1\mathbb{I} & \\ & m_2\mathbb{I}\end{bmatrix},\quad \mathbb{J}_G^\triangle = \begin{bmatrix}\mathbb{J}_1^\triangle & \\ & \mathbb{J}_2^\triangle\end{bmatrix},$
- $\bar{r}_G = \begin{bmatrix} r_{G1}\\ r_{G2}\end{bmatrix},\quad \bar{\varepsilon} = \begin{bmatrix}\varepsilon_1\\ \varepsilon_2\end{bmatrix},\quad \mathbb{F}_G = \begin{bmatrix}\mathbb{F}_1\\ \mathbb{F}_2\end{bmatrix},\quad n_G^\triangle = \begin{bmatrix} n_1^\triangle\\ n_2^\triangle\end{bmatrix},$
- $\mathbb{L}_G(\bar{\varepsilon}) = \begin{bmatrix}\mathbb{L}(\varepsilon_1) & \\ & \mathbb{L}(\varepsilon_2)\end{bmatrix},\quad d_T\mathbb{L}_G(\bar{\varepsilon}) = \begin{bmatrix}\mathbb{L}(d_T\varepsilon_1) & \\ & \mathbb{L}(d_T\varepsilon_2)\end{bmatrix},$
- $\mathbb{C}(\bar{r}_G,\bar{\varepsilon}) = \begin{bmatrix}(r_{G1} + \mathbb{A}(\varepsilon_1)s_1)^T(r_{G1} + \mathbb{A}(\varepsilon_1)s_1)\\ \{r_{G1} + \mathbb{A}(\varepsilon_1)s_2 - (r_{G2} + \mathbb{A}(\varepsilon_2)s_3)\}^T\{r_{G1} + \mathbb{A}(\varepsilon_1)s_2 - (r_{G2} + \mathbb{A}(\varepsilon_2)s_3)\}^T\end{bmatrix}$

$$= \begin{bmatrix} r_{G1}^Tr_{G1} + 2r_{G1}\mathbb{A}(\varepsilon_1)s_1 + s_1^Ts_1\\ \{(r_{G1} - r_{G2}) + (\mathbb{A}(\varepsilon_1)s_2 - \mathbb{A}(\varepsilon_2)s_3)\}^T\{(r_{G1} - r_{G2}) + (\mathbb{A}(\varepsilon_1)s_2 - \mathbb{A}(\varepsilon_2)s_3)\}\end{bmatrix}$$

$$= \begin{bmatrix} r_{G1}^Tr_{G1} + 2r_{G1}\mathbb{A}(\varepsilon_1)s_1 + s_1^Ts_1\\ \{(r_{G1} - r_{G2})^T(r_{G1} - r_{G2}) + 2(r_{G1} - r_{G2})^T(\mathbb{A}(\varepsilon_1)s_2 - \mathbb{A}(\varepsilon_2)s_3) + (\mathbb{A}(\varepsilon_1)s_2 - \mathbb{A}(\varepsilon_2)s_3)^T(\mathbb{A}(\varepsilon_1)s_2 - \mathbb{A}(\varepsilon_2)s_3)\end{bmatrix}$$

$$= \begin{bmatrix} r_{G1}^Tr_{G1} + 2r_{G1}^T\mathbb{A}(\varepsilon_1)s_1 + s_1^Ts_1\\ r_{G1}^Tr_{G1} - 2r_{G1}^Tr_{G2} + r_{G2}^Tr_{G2} + 2(r_{G1} - r_{G2})^T(\mathbb{A}(\varepsilon_1)s_2 - \mathbb{A}(\varepsilon_2)s_3) + s_2^Ts_2 - 2s_2^T\mathbb{A}(\varepsilon_1)^T\mathbb{A}(\varepsilon_2)s_3 + s_3^Ts_3\end{bmatrix}$$

$$\Rightarrow\bullet\ \partial_{\bar{r}_G}\mathbb{C} = \frac{\partial}{\partial[r_{G1}^T\ \ r_{G2}^T]}\mathbb{C} = \begin{bmatrix} 2r_{G1}^T + s_1^T\mathbb{A}(\varepsilon_1)^T & 0\\ 2r_{G1}^T - 2r_{G2}^T + 2(s_2^T\mathbb{A}(\varepsilon_1)^T - s_3^T\mathbb{A}(\varepsilon_2)^T) & -2(r_{G1}^T - r_{G2}^T) - 2(s_2^T\mathbb{A}(\varepsilon_1)^T - s_3^T\mathbb{A}(\varepsilon_2)^T)\end{bmatrix}$$

$$\bullet\ \partial_{\bar{\varepsilon}}\mathbb{C} = \frac{\partial}{\partial[\varepsilon_1^T\ \ \varepsilon_2^T]}\mathbb{C} = \begin{bmatrix} 2r_{G1}^T\partial_{\varepsilon_1}(\mathbb{A}(\varepsilon_1)s_1) & 0\\ 2(r_{G1} - r_{G2})^T\partial_{\varepsilon_1}(\mathbb{A}(\varepsilon_1)s_2) - 2s_3^T\mathbb{A}(\varepsilon_2)^T\partial_{\varepsilon_1}(\mathbb{A}(\varepsilon_1)s_2) & -2(r_{G1} - r_{G2})^T\partial_{\varepsilon_2}(\mathbb{A}(\varepsilon_2)s_3) - 2s_2^T\mathbb{A}(\varepsilon_1)^T\partial_{\varepsilon_2}(\mathbb{A}(\varepsilon_2)s_3)\end{bmatrix}$$

$$= \begin{bmatrix} 4r_{G1}^T(\mathbb{E}(\varepsilon_1)\mathbb{B}^-(s_1) + s_1\varepsilon_1^T) & 0\\ 4(r_{G1} - r_{G2})^T(\mathbb{E}(\varepsilon_1)\mathbb{B}^-(s_2) + s_2\varepsilon_1^T) - 4s_3^T\mathbb{A}(\varepsilon_2)^T(\mathbb{E}(\varepsilon_1)\mathbb{B}^-(s_1) + s_1\varepsilon_1^T) & -4(r_{G1} - r_{G2})^T(\mathbb{E}(\varepsilon_2)\mathbb{B}^-(s_3) + s_3\varepsilon_2^T) - 4s_2^T\mathbb{A}(\varepsilon_1)^T(\mathbb{E}(\varepsilon_2)\mathbb{B}^-(s_3) + s_3\varepsilon_2^T)\end{bmatrix}$$

$$\Rightarrow\bullet\ d_T(\partial_{\bar{r}_G}\mathbb{C}) = \begin{bmatrix} 2\dot{r}_{G1}^T + 2s_1^T\mathbb{L}(\varepsilon_1)\dot{\mathbb{E}}(\varepsilon_1)^T & 0\\ 2\dot{r}_{G1}^T - 2\dot{r}_{G2}^T + 4(s_2^T\mathbb{L}(\varepsilon_1)\dot{\mathbb{E}}(\varepsilon_1)^T - s_3^T\mathbb{L}(\varepsilon_2)\dot{\mathbb{E}}(\varepsilon_2)^T) & 2\dot{r}_{G2}^T - 4(s_2^T\mathbb{L}(\varepsilon_1)\dot{\mathbb{E}}(\varepsilon_1)^T - s_3^T\mathbb{L}(\varepsilon_2)\dot{\mathbb{E}}(\varepsilon_2)^T)\end{bmatrix}$$

$$\bullet\ d_T(\partial_{\bar{\varepsilon}}\mathbb{C}) = \begin{bmatrix} 4\dot{r}_{G1}^T(\mathbb{E}(\varepsilon_1)\mathbb{B}^-(s_1) + s_1\varepsilon_1^T) + 4r_{G1}^T(\dot{\mathbb{E}}(\varepsilon_1)\mathbb{B}^-(s_1) + s_1\dot{\varepsilon}_1^T) & 0\\ 4(\dot{r}_{G1} - \dot{r}_{G2})^T(\mathbb{E}(\varepsilon_1)\mathbb{B}^-(s_2) + s_2\varepsilon_1^T) - 4s_3^T\dot{\mathbb{A}}(\varepsilon_2)^T(\mathbb{E}(\varepsilon_1)\mathbb{B}^-(s_1) + s_1\varepsilon_1^T) + 4(r_{G1} - r_{G2})^T(\dot{\mathbb{E}}(\varepsilon_1)\mathbb{B}^-(s_2) + s_2\dot{\varepsilon}_1^T) - 4s_3^T\mathbb{A}(\varepsilon_2)^T(\dot{\mathbb{E}}(\varepsilon_1)\mathbb{B}^-(s_1) + s_1\dot{\varepsilon}_1^T) & -4(\dot{r}_{G1} - \dot{r}_{G2})^T(\mathbb{E}(\varepsilon_2)\mathbb{B}^-(s_3) + s_3\varepsilon_2^T) - 4s_2^T\dot{\mathbb{A}}(\varepsilon_1)^T(\mathbb{E}(\varepsilon_2)\mathbb{B}^-(s_3) + s_3\varepsilon_2^T) - 4(r_{G1} - r_{G2})^T(\dot{\mathbb{E}}(\varepsilon_2)\mathbb{B}^-(s_3) + s_3\dot{\varepsilon}_2^T) - 4s_2^T\mathbb{A}(\varepsilon_1)^T(\dot{\mathbb{E}}(\varepsilon_2)\mathbb{B}^-(s_3) + s_3\dot{\varepsilon}_2^T)\end{bmatrix}$$