

1. Let P, Q, and R be the propositions:

P: You get an A on the final exam.

Q: You do every exercise in the book.

R: You get an A in this course.

Write the following propositions using P, Q, and R and logical connectives.

- You get an A in this course but you do not do every exercise in the book.
- To get an A in this course, it is necessary for you to get an A on the final exam.
- You will get an A in this course if and only if you do every exercise in the book or you get an A on the final.
- Getting an A on the final exam and doing every exercise in the book is sufficient for getting an A in this course.

**Answers:**

- $R \wedge \neg Q$
- $R \rightarrow P$
- $R \leftrightarrow Q \vee P$
- $P \wedge Q \rightarrow R$

1. (2 marks) Express the following in predicate logic:

- Susan bought everything that Maryam bought.
- If Maryam bought everything, so did Susan.
- There is a computer which is not used by any student
- There is exactly one thing with Ahmad.

**Solution:**

- $\forall x (\text{Bought}(\text{Maryam}, x) \rightarrow \text{Bought}(\text{Susan}, x))$
- $\forall x (\text{Bought}(\text{Maryam}, x) \rightarrow \forall x \text{ Bought}(\text{Susan}, x))$
- $\exists x (\text{Computer}(x) \wedge \neg \text{UsedbyStudent}(x))$   
Or  
 $\exists x (\text{Computer}(x) \wedge \forall y \text{ Student}(y) \rightarrow \neg \text{Used}(y, x))$
- $\forall x \forall y (\text{With}(\text{Ahmed}, x) \wedge \text{With}(\text{Ahmed}, y) \leftrightarrow x=y)$   
Or  
 $\exists x (\text{With}(\text{Ahmed}, x) \wedge \forall y (\text{With}(\text{Ahmed}, y) \rightarrow x=y))$

2. Consider the following sentences:

- The members of Giza Club are Mohamed, Amir, and Iman.
- Mohamed is married to Mona.
- Amir is Iman's brother.
- The partner (husband/wife) of every married person in the Club is also a member in the Club.
- The last meeting was at Mohamed's house.

- a) (2 ½ marks) Translate these sentences into Predicate Logic.  
 b) (3 ½ marks) Prove, by using resolution, the following statement:

The last meeting of the Club was at Mona's house.

Note that this statement should be proved to be true. If you can not reach this conclusion, then add the necessary statement(s) to your knowledge base and then construct the proof.

**Answer:**

**a) FOPL:**

1.  $\text{Member}(\text{Mohamed}) \wedge \text{Member}(\text{Mona}) \wedge \text{Member}(\text{Iman})$ .
2.  $\text{Married}(\text{Mohamed}, \text{Mona})$ .
3.  $\text{Brother}(\text{Iman}, \text{Amir})$ .
4.  $\forall x \forall y (\text{Married}(y, x) \wedge \text{Member}(y) \rightarrow \text{Member}(x))$
5.  $\forall x \forall y (\text{Married}(y, x) \wedge \text{Member}(x) \rightarrow \text{Member}(y))$
6.  $\text{LastMeetingHouse}(\text{Mohamed})$ .

**b) Proof by resolution:**

Transform to CNF (1 ½ marks):

1.  $\text{Member}(\text{Mohamed}) \wedge \text{Member}(\text{Mona}) \wedge \text{Member}(\text{Iman})$ .
2.  $\text{Married}(\text{Mohamed}, \text{Mona})$ .
3.  $\text{Brother}(\text{Iman}, \text{Amir})$ .
4.  $\neg \text{Married}(y, x) \vee \neg \text{Member}(y) \vee \text{Member}(x)$
5.  $\neg \text{Married}(y, x) \vee \neg \text{Member}(x) \vee \text{Member}(y)$
6.  $\text{LastMeetingHouse}(\text{Mohamed})$ .

Also we will add (1 mark):

7.  $\forall x \forall y (\text{Married}(y, x) \wedge \text{LastMeetingHouse}(y) \rightarrow \text{LastMeetingHouse}(x))$

i.e.

$$\neg \text{Married}(y, x) \vee \neg \text{LastMeetingHouse}(y) \vee \text{LastMeetingHouse}(x)$$

The proof (1 mark):

**Q: LastMeetingHouse(Mona).**

2+7=8 {y/Mohamed, x/Mona}:

$$\neg \text{LastMeetingHouse}(\text{Mohamed}) \vee \text{LastMeetingHouse}(\text{Mona})$$

$$6+8 = 9: \text{LastMeetingHouse}(\text{Mona})$$