Fuzzy Logic Control

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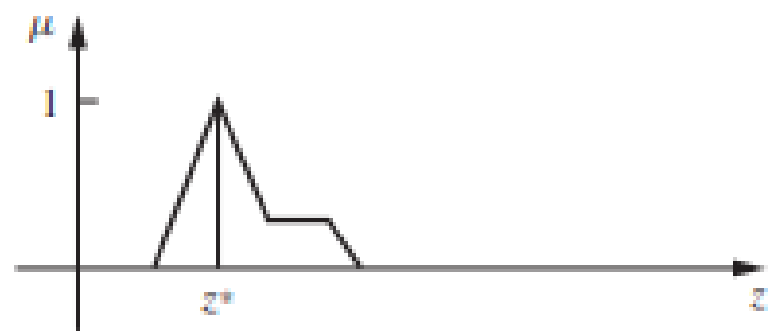
Outline

- Defuzzification Techniques
- Fuzzy Controls(Architecture)
  - Components/Steps
  - Examples

Defuzzification Techniques

1- <u>Max membership principle:</u> Also known as the height method, this scheme is limited to peaked output functions. This method is given by the algebraic expression:

- $\mu_C(z^*) \geq \mu_C(z)$, for all $z \in Z$,
- where z^* is the defuzzified value



If multiple peaks exist, the middle one is chosen.

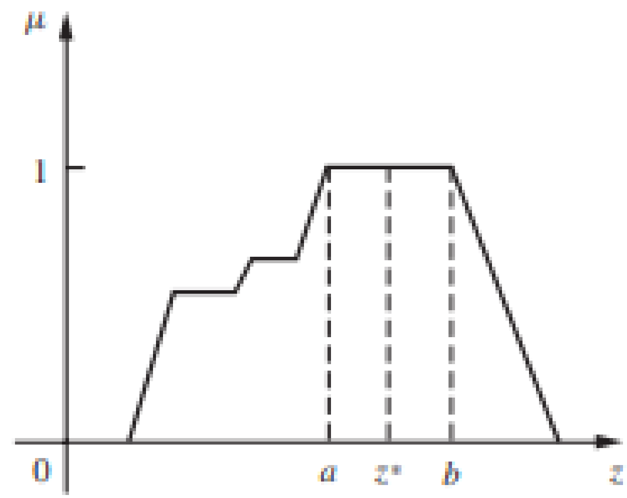
Defuzzification Techniques

2- <u>Mean Max membership:</u> This method (also called middle-of-maxima) is closely related to the first method, except that the locations of the maximum membership can be nonuniqe (i.e., the maximum membership can be a plateau rather than a single point).

This method is given by

the expression:

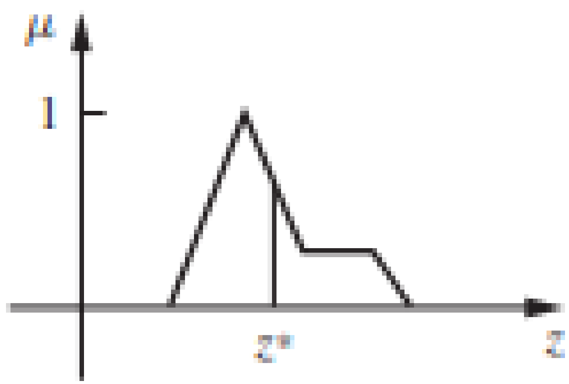
$$z^* = \frac{a+b}{2}$$



Defuzzification Techniques

3- <u>Centroid method:</u> This procedure (also called center of area or center of gravity) is the most prevalent of all the defuzzification methods (Sugeno, 1985); it is given by the algebraic expression:

$$z^* = \frac{\int \mu_{\underset{\sim}{C}}(z) \cdot z \, dz}{\int \mu_{\underset{\sim}{C}}(z) \, dz},$$



– where ∫ denotes an algebraic integration

Defuzzification Techniques

4 - <u>**Weighted average method:**</u> The weighted average method is the most frequently used in fuzzy applications since it is one of the more computationally efficient methods. Unfortunately, it is usually restricted to symmetrical output membership functions. It is given by the algebraic expression:

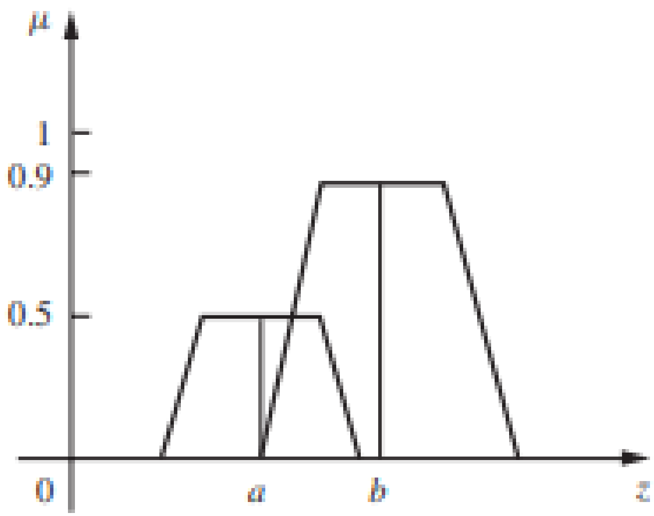
- $\mathbf{z}^* = \frac{\sum \mu_C(\bar{z}).\bar{z}}{\sum \mu_C(\bar{z})}$
- where $\sum$ denotes the algebraic sum and where $\bar{z}$ is the centroid of each symmetric membership function. The weighted average method is formed by weighting each membership function in the output by its respective membership value.

Defuzzification Techniques

4 - <u>**Weighted average method: (Example)**</u>

- The two functions shown would result in the following general form for the defuzzified value:

$$z* = \frac{a(0.5) + b(0.9)}{0.5 + 0.9}$$

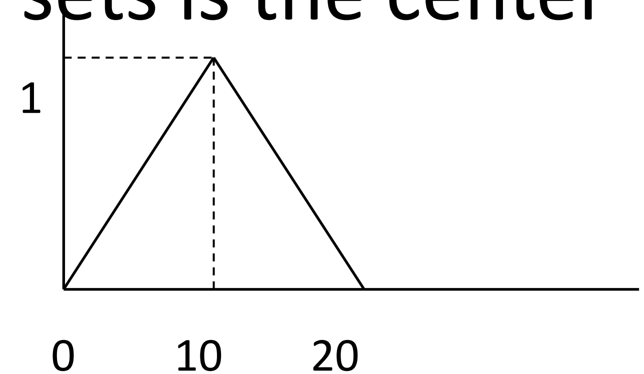


Defuzzification Techniques

4- <u>Weighted average method: (getting the centroid)</u>

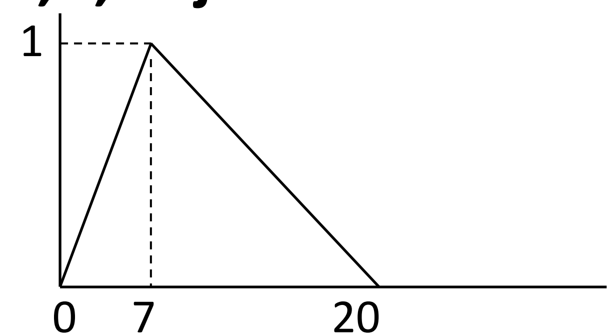
- The centroid of symmetrical fuzzy sets is the center point on the x-axis.

    Example: centroid = 10



- For non-symmetric fuzzy sets (trapezoidal or triangular):
    - Add defining points of fuzzy set and divide by their number.

    Example: centroid of triangular set {0,7,20}= (0+7+20)/3 = 9



Fuzzy Inference

- The most commonly used fuzzy inference technique is the so-called **Mamdani** method.
- **Mamdani's** model expresses the output using fuzzy terms instead of mathematical combinations of the input variable.
- All defuzzification techniques previously discussed belong to Mamdani's model.

- The Mamdani-style fuzzy inference process is performed in four steps:
  1. Fuzzification of the input variables
  2. Rule evaluation (inference)
  3. Aggregation of the rule outputs (composition)
  4. Defuzzification.

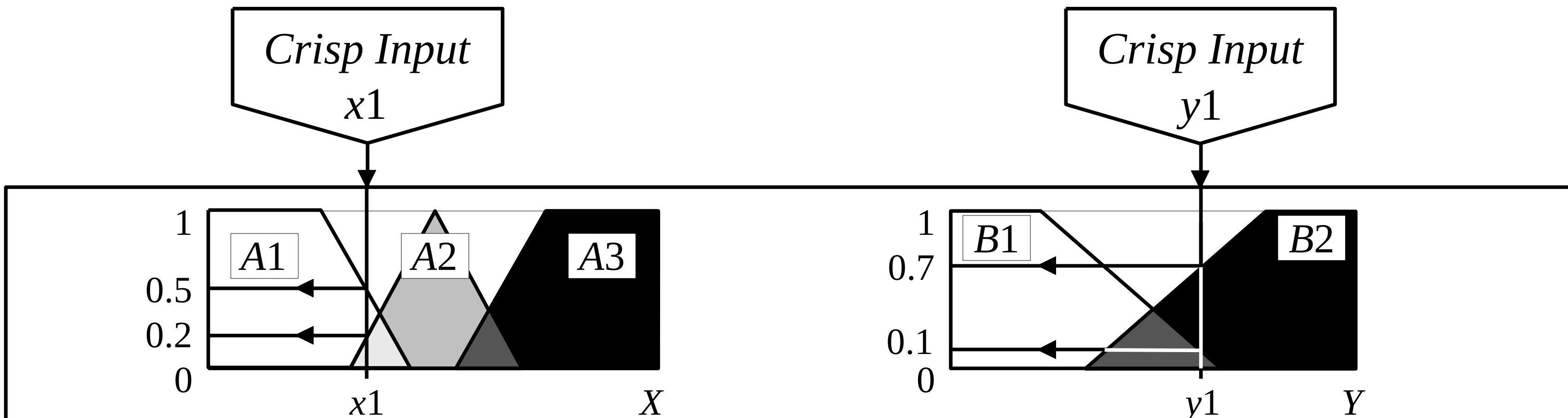
Mamdani Fuzzy Inference

We examine a simple two-input one-output problem that includes three rules:

<u>Rule: 1</u>			<u>Rule: 1</u>		
IF	x is A3		IF	project_funding	is adequate
OR	y is B1		OR	project_staffing	is small
THEN	z is C1		THEN	risk	is low
<u>Rule: 2</u>			<u>Rule: 2</u>		
IF	x is A2		IF	project_funding	is marginal
AND	y is B2		AND	project_staffing	is large
THEN	z is C2		THEN	risk	is normal
<u>Rule: 3</u>			<u>Rule: 3</u>		
IF	x is A1		IF	project_funding	is inadequate
THEN	z is C3		THEN	risk	is high

Step 1: Fuzzification

- The first step is to take the crisp inputs, x1 and y1 (*project funding* and *project staffing*) and determine the degree to which these inputs belong to each of the appropriate fuzzy sets.

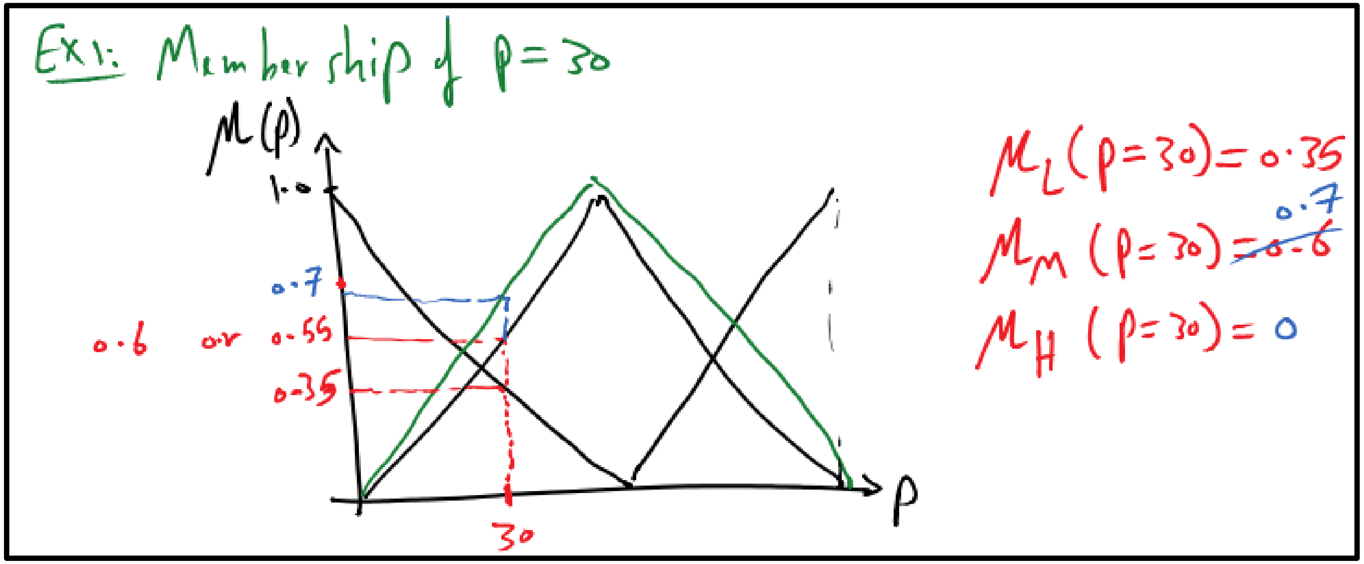


$\mu_{(x = A1)} = 0.5$
$\mu_{(x = A2)} = 0.2$

$\mu_{(y = B1)} = 0.1$
$\mu_{(y = B2)} = 0.7$

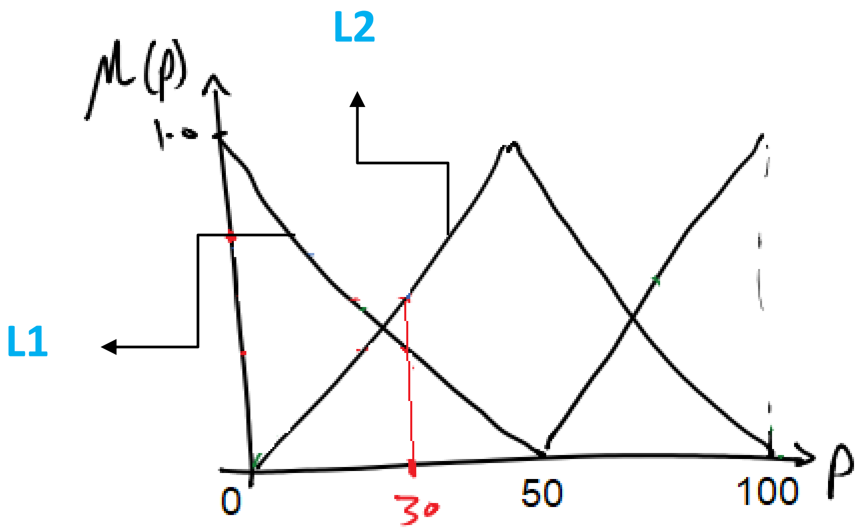
Step 1: Fuzzification

- To accurately get the membership degree of the crisp value provided, equations of lines of the different fuzzy sets should be determined. Then you can get value of y (membership) by substituting in line equations by x value (crisp value)
- values are not accurate



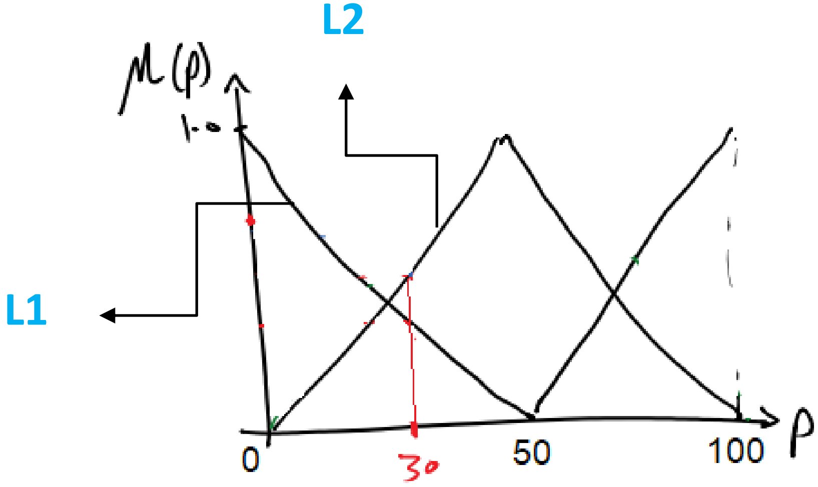
Step 1: Fuzzification

- To get membership of crisp value p=30, we need to get equations for the lines where the value 30 intersects with (L1 & L2).



Step 1: Fuzzification

- L1: 1-get 2 points on the line: (0,1), (50,0)
  2-get slope = (y2-y1)/(x2-x1) = (0-1)/(50-0)= -1/50
  3-line equation: y = ax + b (a = slope)
  4-line equation: y = (-x/50) + b, substitute by one of the points to get b.
  5- 1 = 0 + b => b=1
  6- y = (-x/50) + 1
  7- To get membership substitute by x=30,
  8- y= (-30/50)+1 = 0.4
  9- $\mathbf{\mu}_L(p = 30) = 0.4$



- L2: use same steps.

Step 2: Rule Evaluation

- The second step is to take the fuzzified inputs, $\mu_{(x=A1)} = 0.5$, $\mu_{(x=A2)} = 0.2$, $\mu_{(y=B1)} = 0.1$ and $\mu_{(y=B2)} = 0.7$, and apply them to the antecedents of the fuzzy rules.

- If a given fuzzy rule has multiple antecedents, the fuzzy operator (**AND** or **OR**) is used to obtain a single number that represents the result of the antecedent evaluation.

- This number (the truth value) is then applied to the consequent membership function.

Step 2: Rule Evaluation

RECALL:

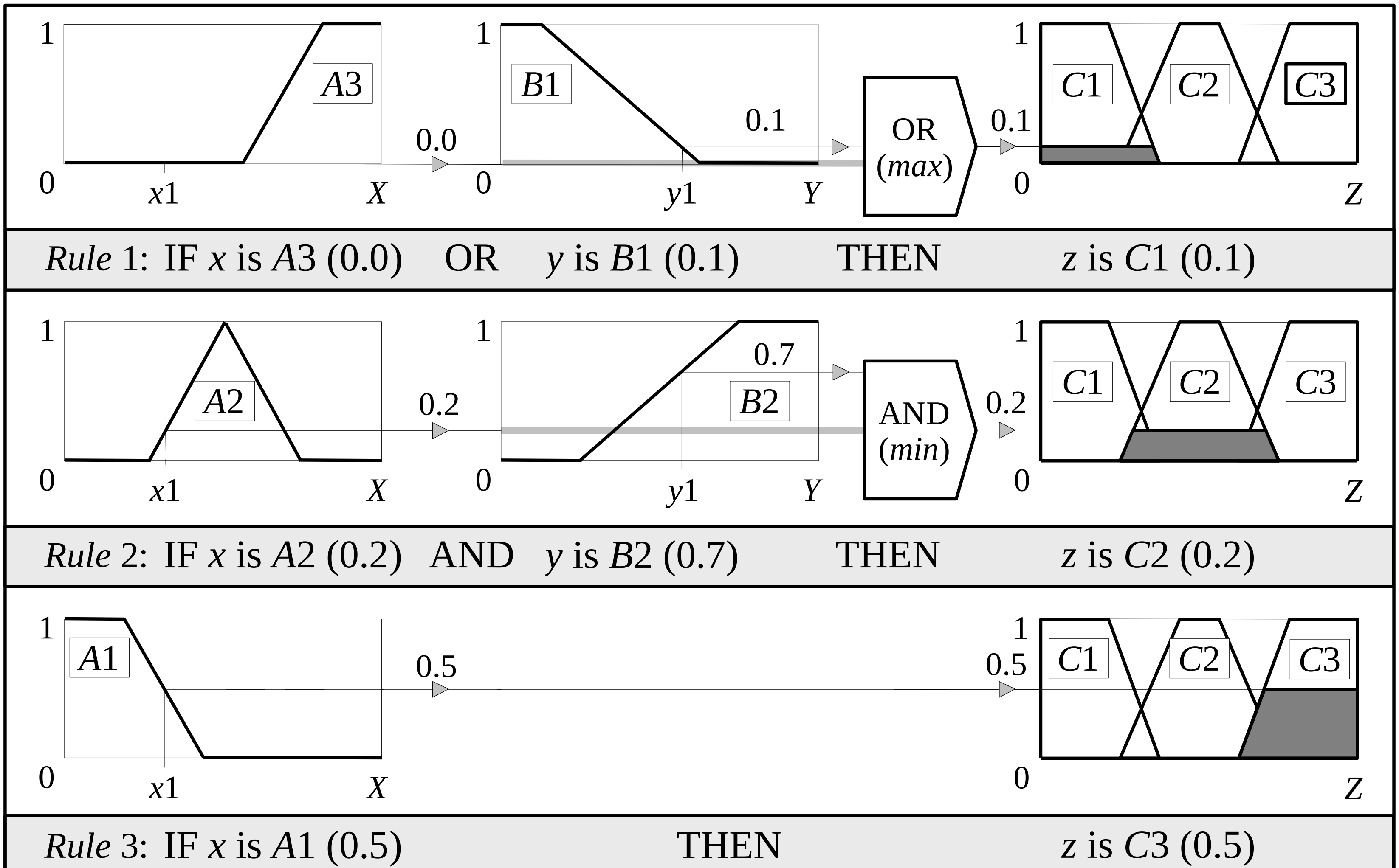
To evaluate the disjunction of the rule antecedents, we use the **OR** fuzzy operation. Typically, fuzzy expert systems make use of the classical fuzzy operation union:

$$\mu_{A \cup B}(x) = \mathbf{max}\ [\mu_A(x),\ \mu_B(x)]$$

Similarly, in order to evaluate the conjunction of the rule antecedents, we apply the **AND** fuzzy operation intersection:

$$\mu_{A \cap B}(x) = \mathbf{min}\ [\mu_A(x),\ \mu_B(x)]$$

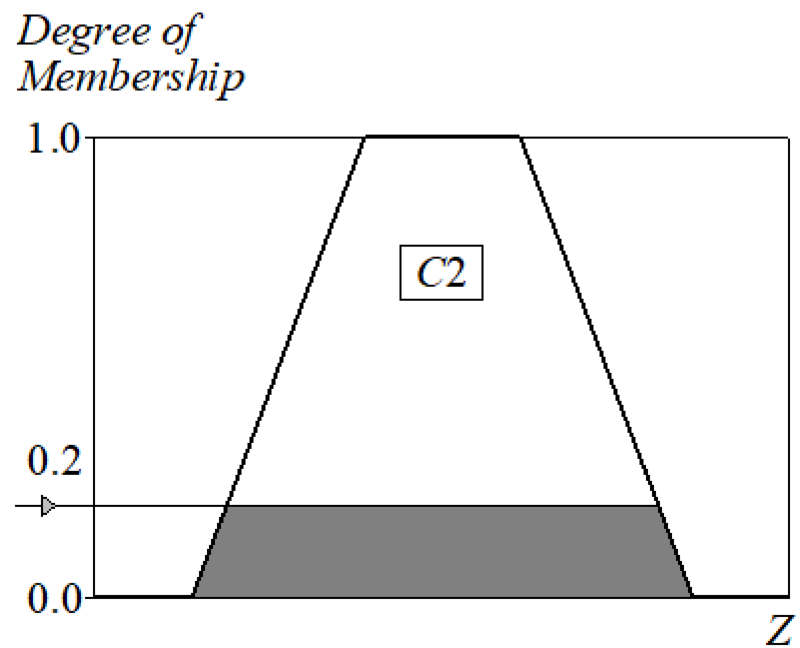
Step 2: Rule Evaluation



Step 2: Rule Evaluation

- The most common method of correlating the rule consequent with the truth value of the rule antecedent is to cut the consequent membership function at the level of the antecedent truth. This method is called **clipping** (alpha-cut).

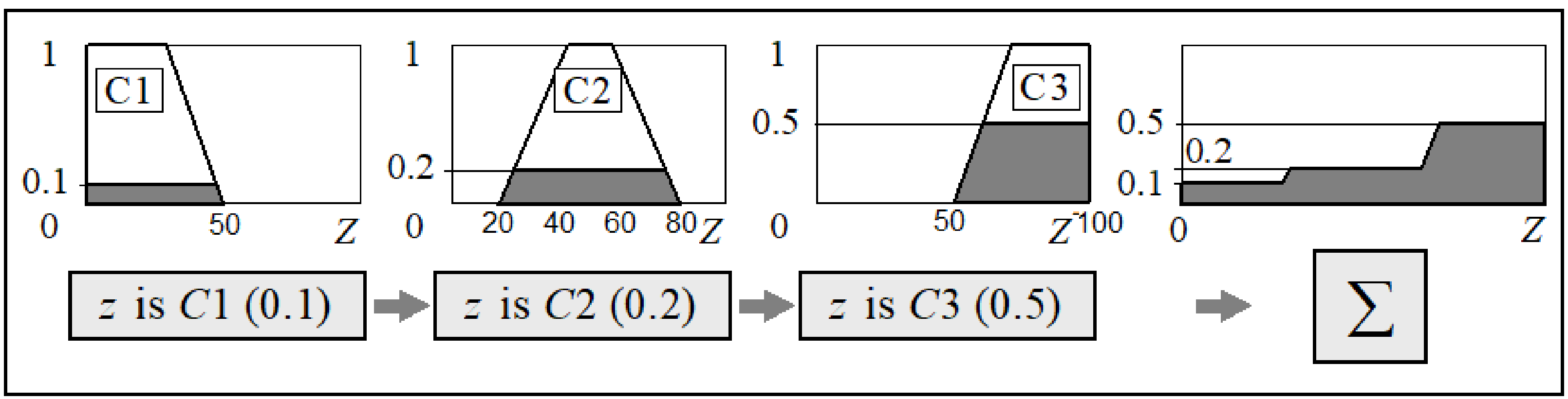
- However, clipping is still often preferred because it involves less complex and faster mathematics, and generates an aggregated output surface that is easier to defuzzify.



clipping

Step 3: Aggregation of the rule outputs

- Aggregation is the process of unification of the outputs of all rules.
- We take the membership functions of all rule consequents previously clipped and combine them into a single fuzzy set.
- The input of the aggregation process is the list of clipped consequent membership functions, and the output is one fuzzy set for each output variable.



Step 4: Defuzzification

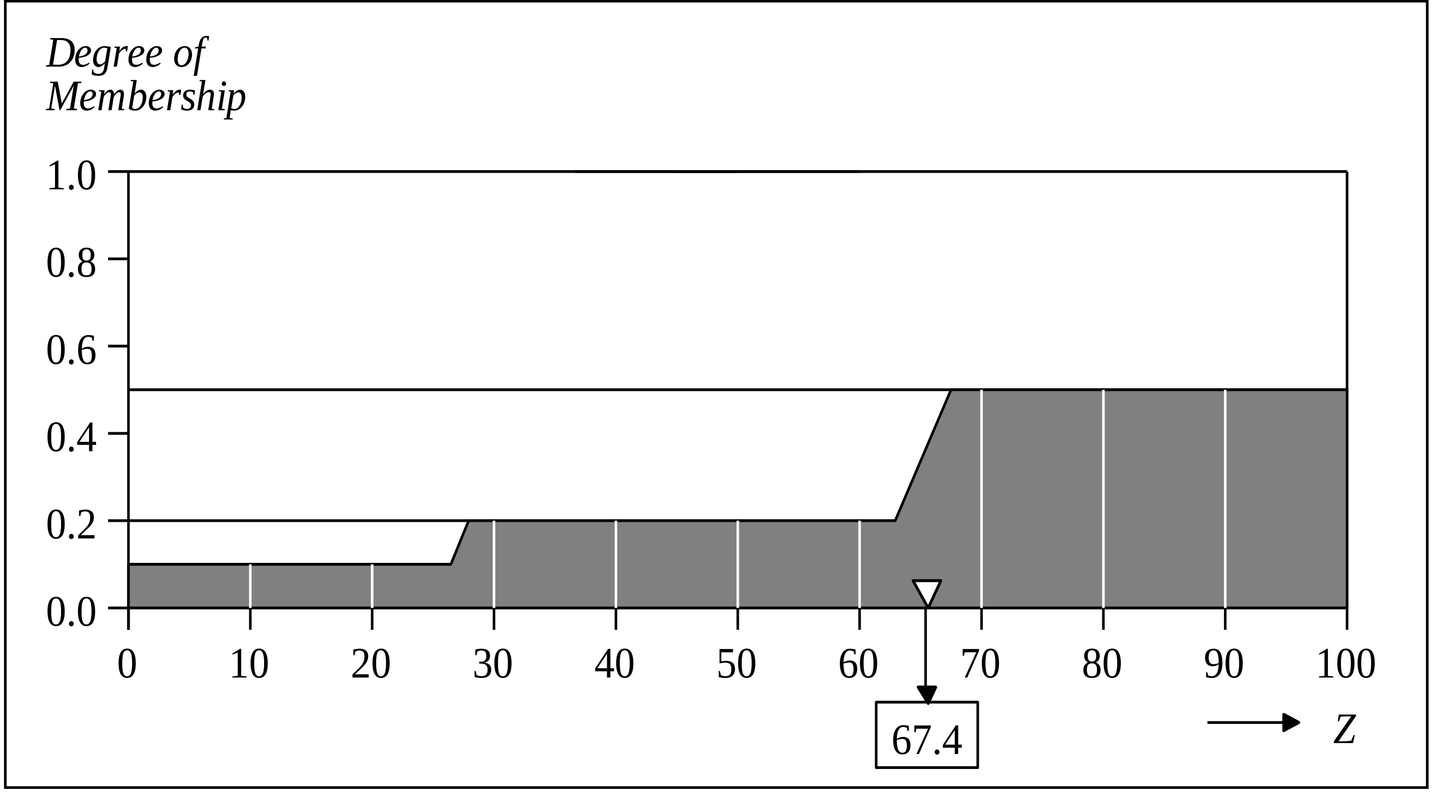
- The last step in the fuzzy inference process is defuzzification.
- Fuzziness helps us to evaluate the rules, but the final output of a fuzzy system has to be a crisp number.
- The input for the defuzzification process is the aggregate output fuzzy set and the output is a single number.

Step 4: Defuzzification (centroid)

- There are several defuzzification methods, but probably the most popular one is the **centroid technique**. It finds the point where a vertical line would slice the aggregate set into two equal masses. Mathematically this **centre of gravity** (**COG**) can be expressed as:

$$COG = \frac{\int_a^b \mu_A(x)\, x\, dx}{\int_a^b \mu_A(x)\, dx}$$

Step 4: Defuzzification (centroid)



$$COG = \frac{(0+10+20)\times 0.1+(30+40+50+60)\times 0.2+(70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$

Step 4: Defuzzification (weighted average)

- Another method is weighted average method where centroid of each of the fuzzy sets is weighted by the membership value, divided by the sum of memberships.
- Centroid(C1) = (0+0+30+50)/4 = 20
- Centroid(C2) = (20+40+60+80)/4 = 50
- Centroid(C3) = (50+70+100+100)/4 = 80
-

$$Z^* = (20*0.1 + 50*0.2 + 80*0.5) / (0.1 + 0.2 + 0.5)$$

$$Z^* = 65$$

Example

- ***Given a stock market with variables:***
  1. Opening price (**Op**)
  2. Previous day price (**Pdp**)
  3. Closing price (**Cp**)

From the domain experts, it is known that:

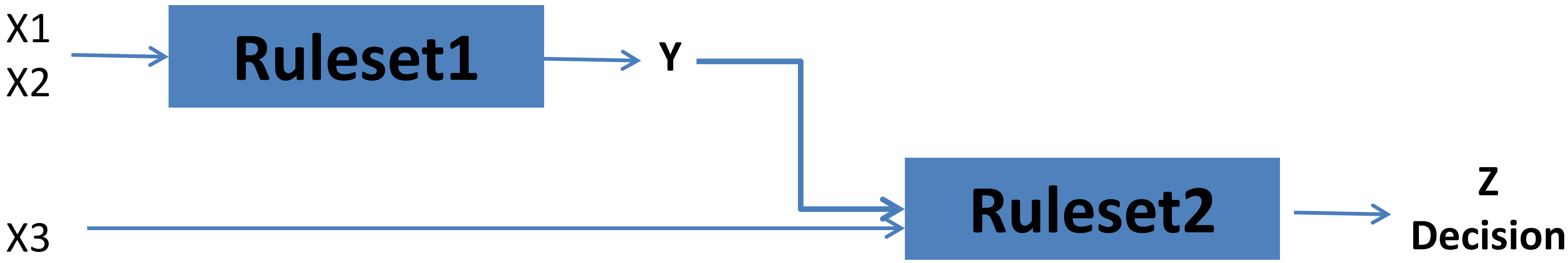
- Op has **3** fuzzy sets **L, M, H** (range 0 - 100)
- Pdp has **3** fuzzy sets **L, M, H** (range 0 - 100)
- Cp has **5** fuzzy sets **VL, L, M, H, VH** (range 0 - 100)

With the following rules:

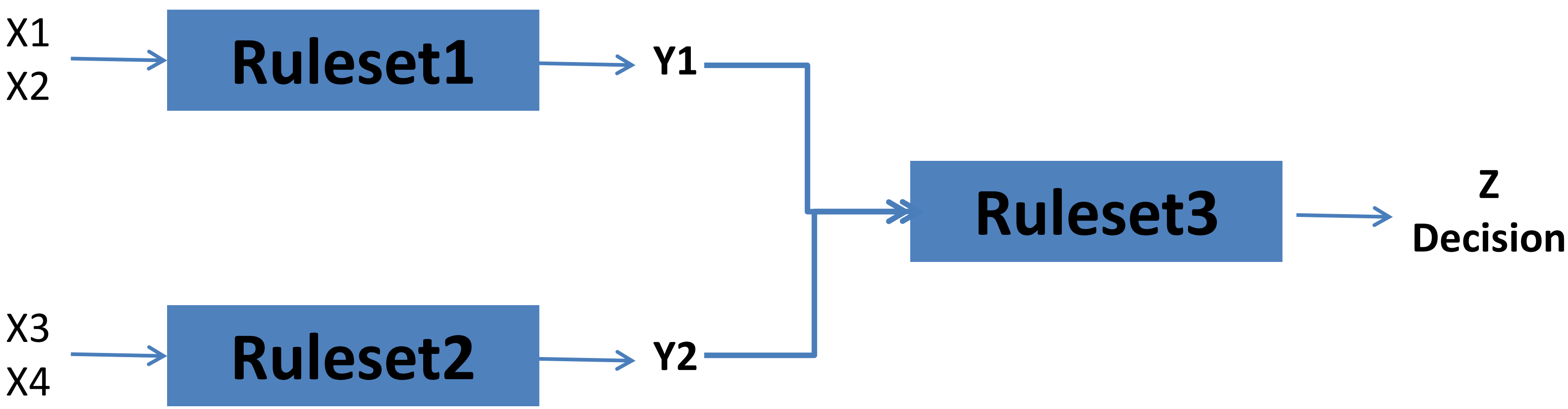
R1: If Op=L **AND** Pdp=L THEN Cp=VL
R2: If Op=L **OR** Pdp =L THEN Cp=L
R3: If Op=M **AND** Pdp=**NOT**(H) THEN Cp=M
R4: If Op=H **OR** Pdp=H THEN Cp=H
R5: If Op=H **AND** Pdp=H THEN Cp=VH

Using triangular membership function, Predict the Cp for Op = 30 and Pdp=90 ?

Multiple Rulesets



OR



Example

- ***Given a medical information system with governing variables X1, X2, X3. It is required to predict decision D which is Benign or Malignant B*** {0, 0, 10}, **M** {0, 10, 10} **.**

The following information is given: X1, X2, and X3 have **3** fuzzy sets **L, M, H** (range 0 - 100)

L {0, 0, 50}, medium **M** {0, 50, 100}, good **H** {50, 100, 100}

With the following rulesets:

RS1:

R1: If X1=L **AND** X2=M THEN Y=L

R2: If X1=M **AND** X2 =H THEN Y=H

RS2:

R3: If Y=L **AND** X3=M THEN D=B

R4: If Y=H **AND** X3=H THEN D=M

Using triangular membership function, Predict D for X1 = 30 , X2 = 80 and X3 = 90 ?