Regular Languages Equivalence of RE, NFAs and DFAs

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Why do we care about Regular Languages?

- Formally describe tokens in the language
  - Regular Expressions
  - NFA
  - DFA
- Regular Expressions ➔ finite automata
- Tools assist in the process

Regular Expressions

The **regular expressions** over finite Σ are the strings over the alphabet Σ + {), (, |, * } such that:

1. { } (empty set) is a regular expression for the empty set
2. ε is a regular expression denoting { ε }
3. *a* is a regular expression denoting set { *a* } for any *a* in Σ

Regular Expressions

4. If P and Q are regular expressions over Σ, then so are:

- **P | Q (union)**

  If P denotes the set {*a*,…,*e*}, Q denotes the set {*0*,…,*9*} then
  P | Q denotes the set {*a*,…,*e*,*0*,…,*9*}

- **PQ (concatenation)**

  If P denotes the set {*a*,…,*e*}, Q denotes the set {*0*,…,*9*} then
  PQ denotes the set {*a0*,…,*e0*,*a1*,…,*e9*}

- **Q* (closure)**

  If Q denotes the set {*0*,…,*9*} then Q* denotes the set
  {ε,*0*,…,*9*,*00*,…*99*,…}

Examples

If $\Sigma = \{a,b\}$

- (a | b)(a | b)
- (a | b)*b
- a*b*a*
- a*a (also known as a+)
- (ab*)|(a*b)

Nondeterministic Finite Automata

A **nondeterministic finite automaton** (NFA) is a mathematical model that consists of

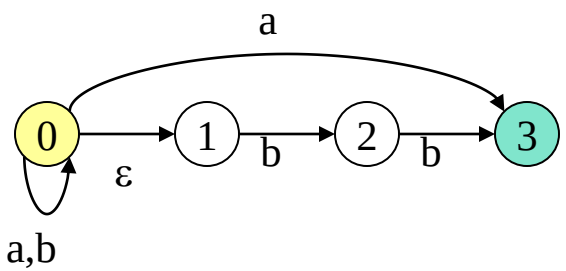
1. A set of states S
2. A set of input symbols Σ
3. A transition function that maps state/symbol pairs to a set of states:

   S x {Σ + ε} → set of S
4. A special state s_0 called the start state
5. A set of states F (subset of S) of final states

INPUT: string

OUTPUT: yes or no

Example NFA



Transition Table:

STATE	a	b	ε
0	0,3	0	1
1		2	
2		3	
3			

$S = \{0,1,2,3\}$

$S_0 = 0$

$\Sigma = \{a,b\}$

$F = \{3\}$

NFA Execution

An NFA says 'yes' for an input string if there is some path from the start state to some final state where all input has been processed.

Deterministic Finite Automata

A **deterministic finite automaton** (DFA) is a mathematical model that consists of

1. A set of states S
2. A set of input symbols Σ
3. A transition function that maps state/symbol pairs to a state:

   S x Σ → S
4. A special state s_0 called the start state
5. A set of states F (subset of S) of final states

INPUT: string

OUTPUT: yes or no

Regular Languages

1. There is an algorithm for converting any RE into an NFA.
2. There is an algorithm for converting any NFA to a DFA.
3. There is an algorithm for converting any DFA to a RE.

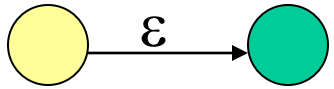
These facts tell us that REs, NFAs and DFAs have equivalent expressive power. All three describe the class of regular languages.

Converting Regular Expressions to NFAs

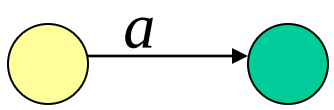
The **regular expressions** over finite Σ are the strings over the alphabet Σ + {), (, |, *} such that:

- { } (empty set) is a regular expression for the empty set

- Empty string ε is a regular expression denoting { ε }



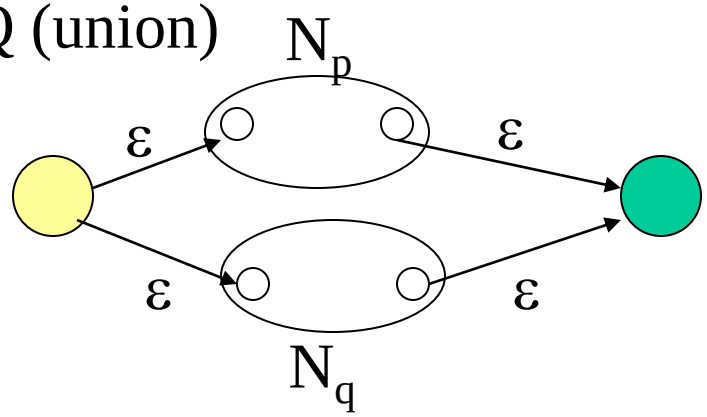
- *a* is a regular expression denoting {*a* } for any *a* in Σ



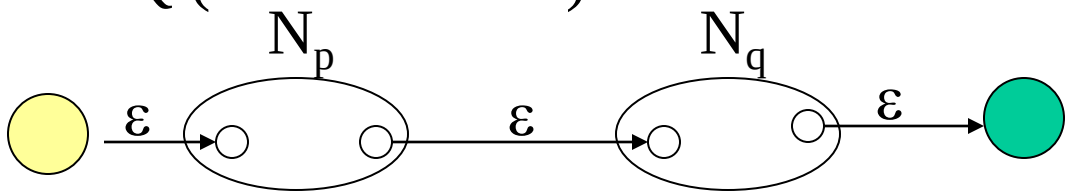
Converting Regular Expressions to NFAs

If P and Q are regular expressions with NFAs N_p, N_q:

P | Q (union)



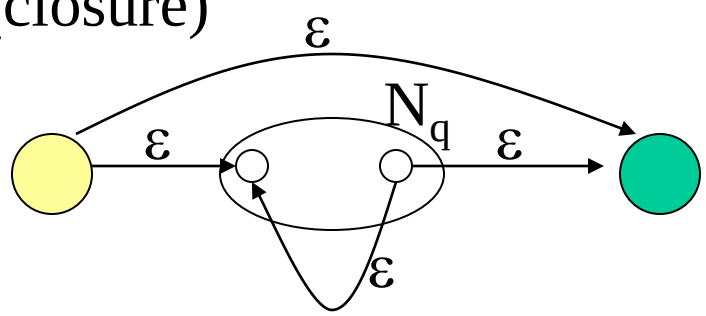
PQ (concatenation)



Converting Regular Expressions to NFAs

If Q is a regular expression with NFA N_q:

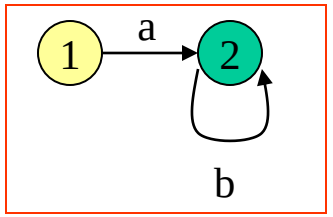
Q* (closure)



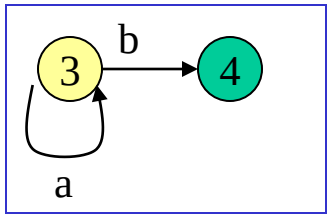
Example (ab* | a*b)*

Starting with:

ab*



a*b



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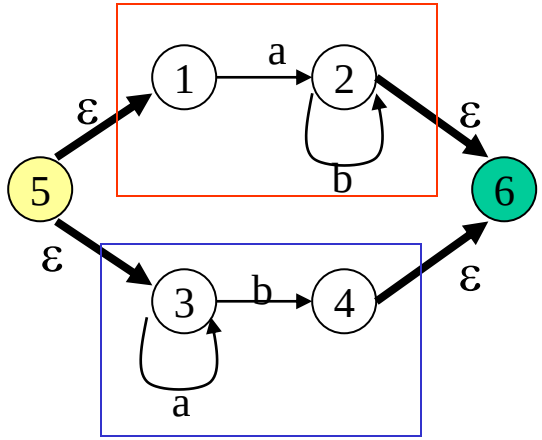
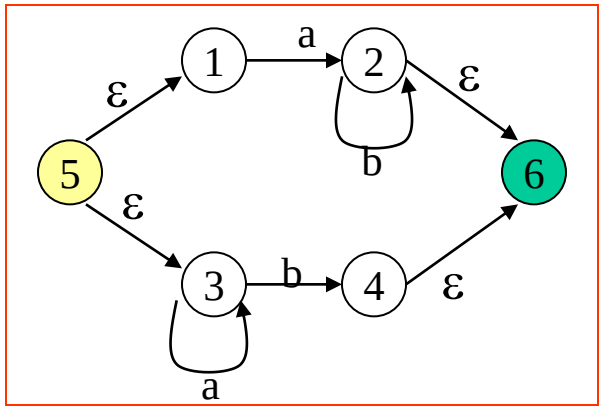
ab* | a*b

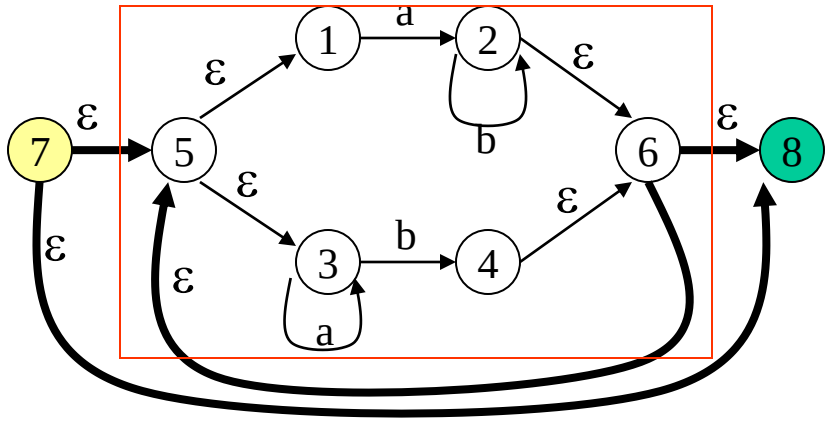


Example (ab* | a*b)*

ab* | a*b



(ab* | a*b)*



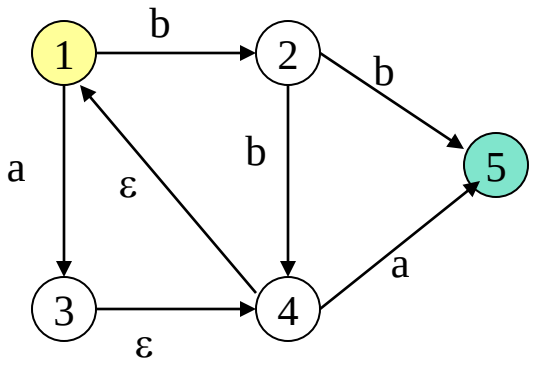
Converting NFAs to DFAs

- **Idea**: Each state in the new DFA will correspond to some set of states from the NFA. The DFA will be in state $\{s_0,s_1,\ldots\}$ after input if the NFA could be in *any* of these states for the same input.

- **Input**: NFA N with state set S_N, alphabet Σ, start state s_N, final states F_N, transition function T_N: $S_N \times \Sigma + \{\varepsilon\} \rightarrow$ set of S_N
- **Output**: DFA D with state set S_D, alphabet Σ, start state $s_D = \varepsilon\text{-closure}(s_N)$, final states F_D, transition function T_D: $S_D \times \Sigma \rightarrow S_D$

ε-closure()

Defn: ε-closure(T) = T + all NFA states reachable from any state in T using only ε transitions.



ε-closure({1,2,5}) = {1,2,5}
ε-closure({4}) = {1,4}
ε-closure({3}) = {1,3,4}
ε-closure({3,5}) = {1,3,4,5}

Algorithm: Subset Construction

s_D = ε-closure(s_N) -- create start state for DFA

S_D = {s_D} (unmarked)

while there is some unmarked state **R** in S_D

    mark state **R**

    for all *a* in Σ do

        s = ε-closure(T_N(**R**,*a*));

        if s not already in S_D then add it (unmarked)

        T_D(**R**,*a*) = s;

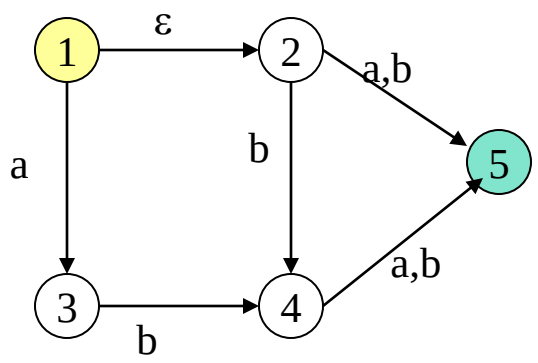
    end for

end while

F_D = any element of S_D that contains a state in F_N

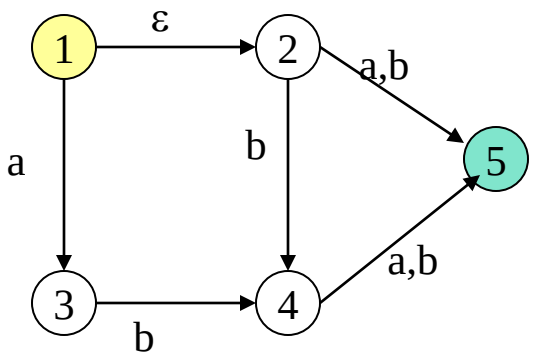
Example 1: Subset Construction

NFA



Example 1: Subset Construction

NFA

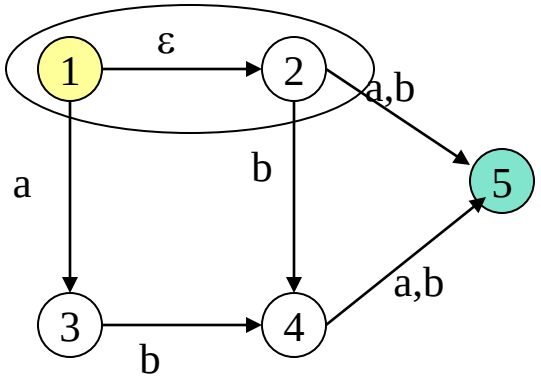


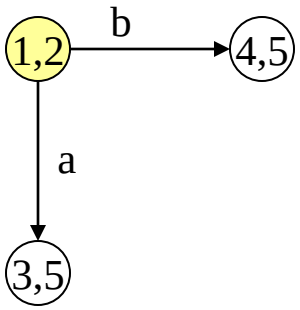
1,2

	a	b
{1,2}		

Example 1: Subset Construction

NFA

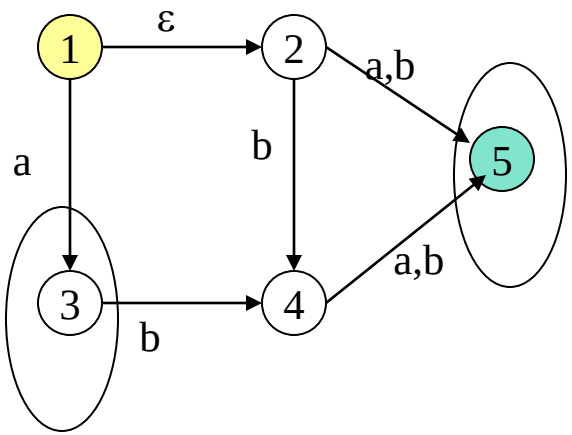


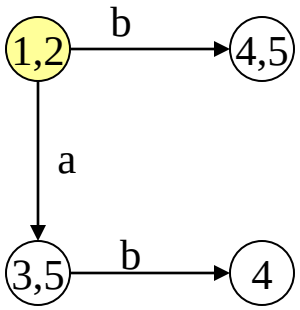


	a	b
{1,2}	{3,5}	{4,5}
{3,5}		
{4,5}		

Example 1: Subset Construction

NFA

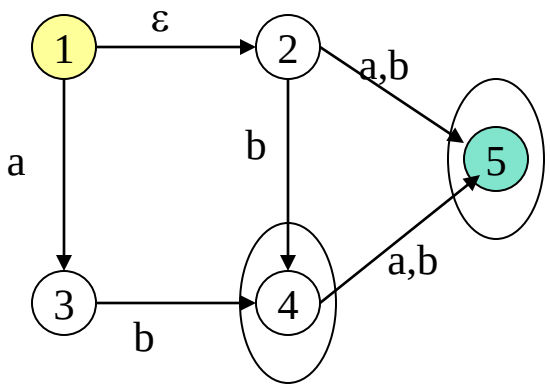


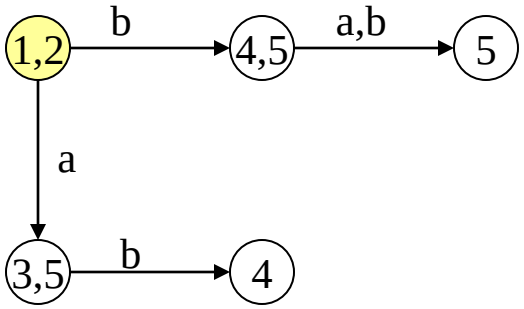


	a	b
{1,2}	{3,5}	{4,5}
{3,5}	-	{4}
{4,5}		
{4}		

Example 1: Subset Construction

NFA

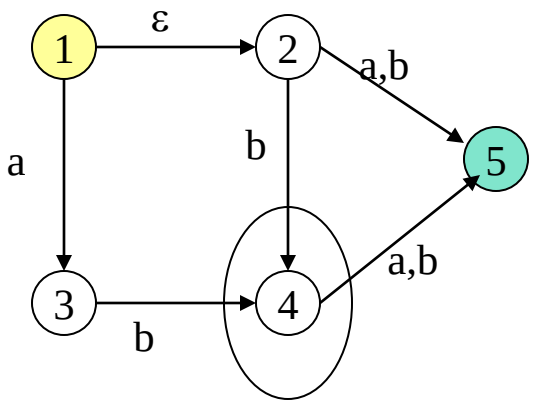


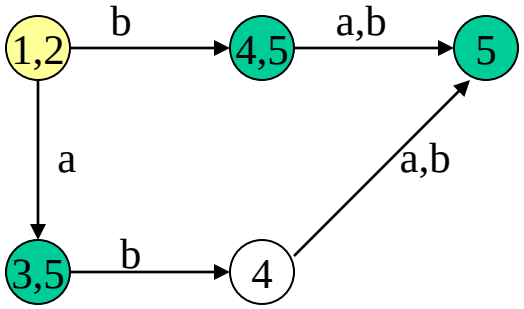


	a	b
{1,2}	{3,5}	{4,5}
{3,5}	-	{4}
{4,5}	{5}	{5}
{4}		
{5}		

Example 1: Subset Construction

NFA



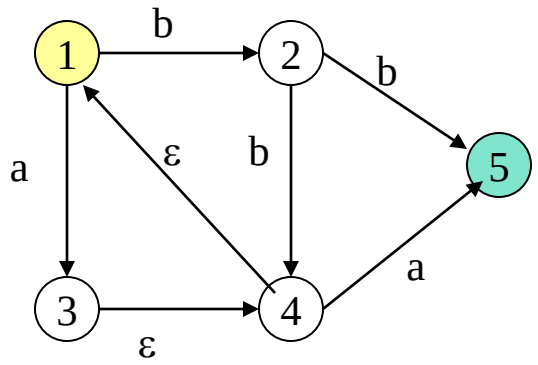


All final states since the NFA final state is included

	a	b
{1,2}	{3,5}	{4,5}
{3,5}	-	{4}
{4,5}	{5}	{5}
{4}	{5}	{5}
{5}	-	-

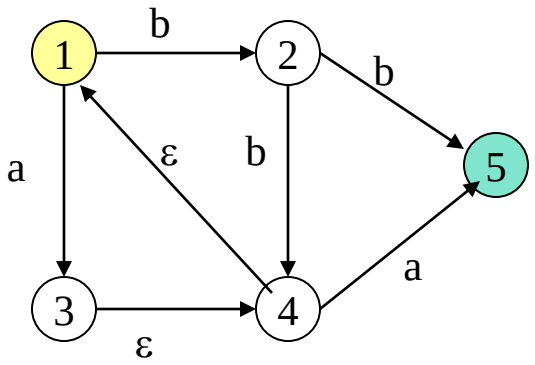
Example 2: Subset Construction

NFA

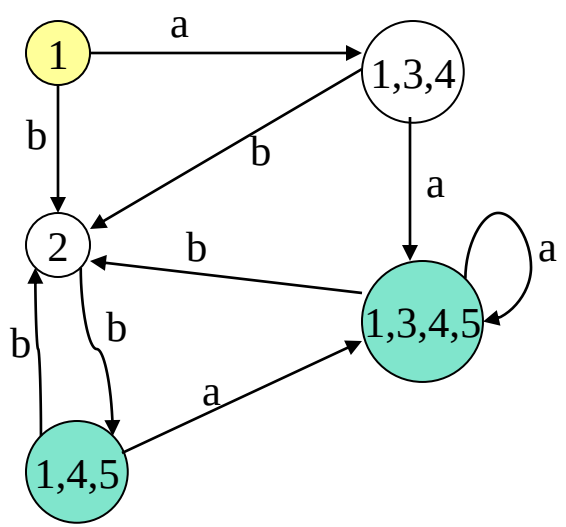


Example 2: Subset Construction

NFA

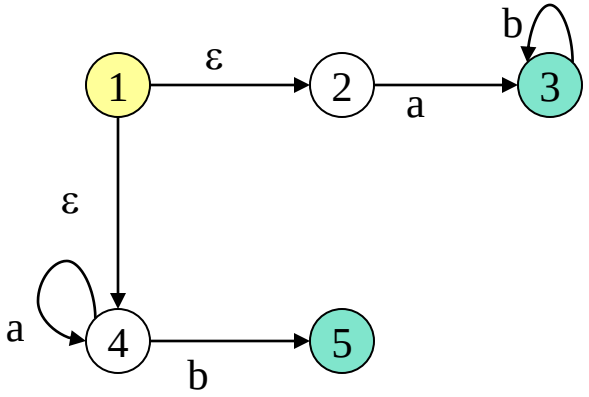


DFA

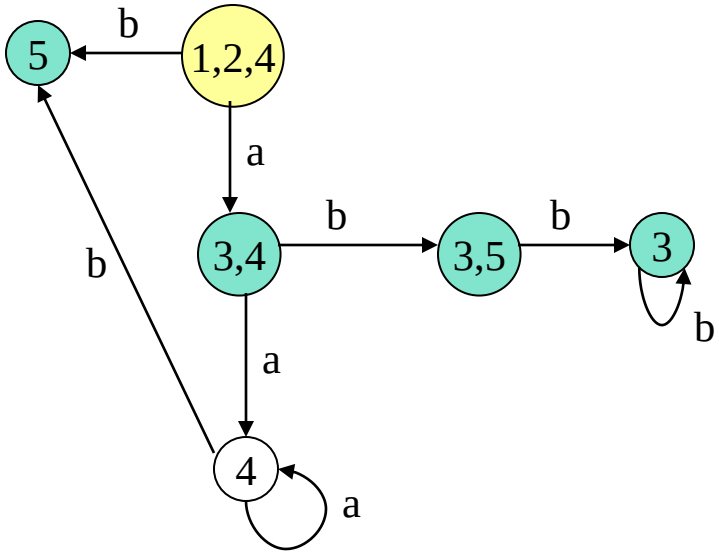


Example 3: Subset Construction

NFA



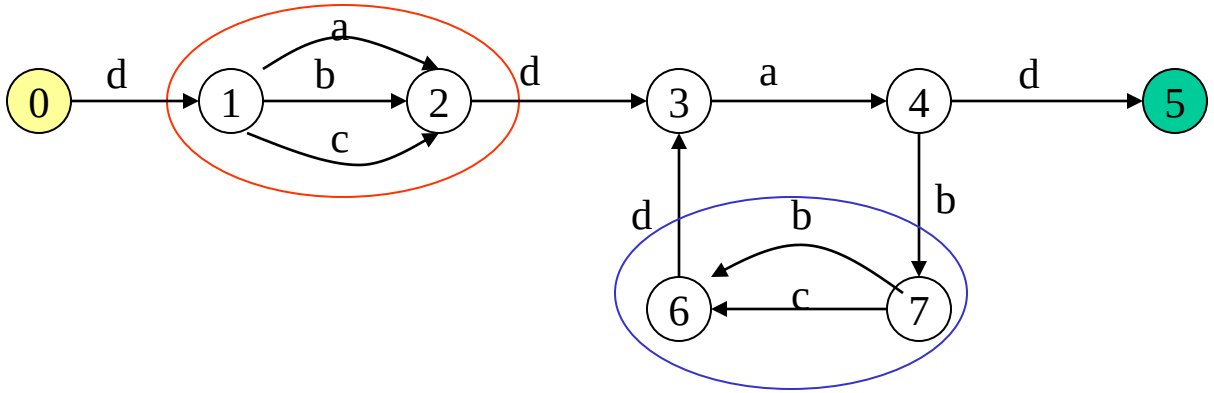
DFA



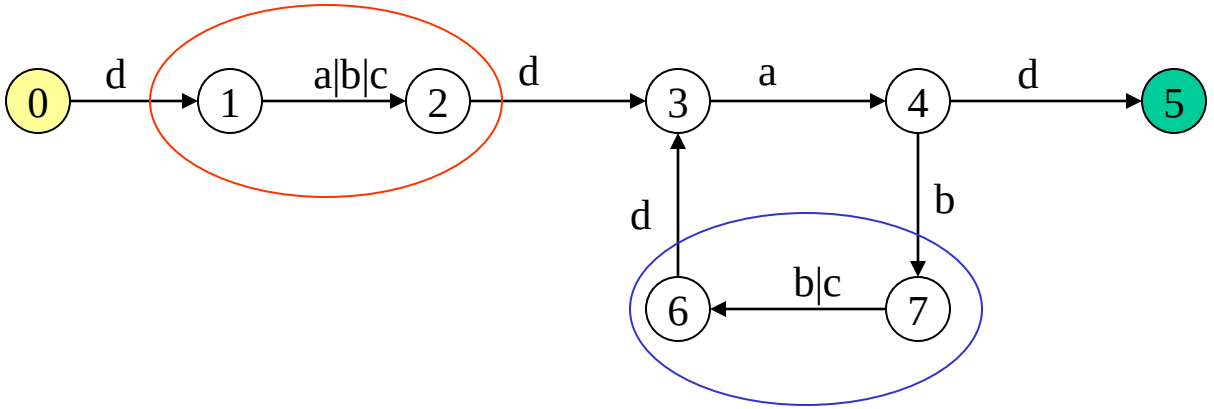
Converting DFAs to REs

1. Combine serial links by concatenation
2. Combine parallel links by alternation
3. Remove self-loops by Kleene closure
4. Select a node (other than initial or final) for removal. Replace it with a set of equivalent links whose path expressions correspond to the in and out links
5. Repeat steps 1-4 until the graph consists of a single link between the entry and exit nodes.

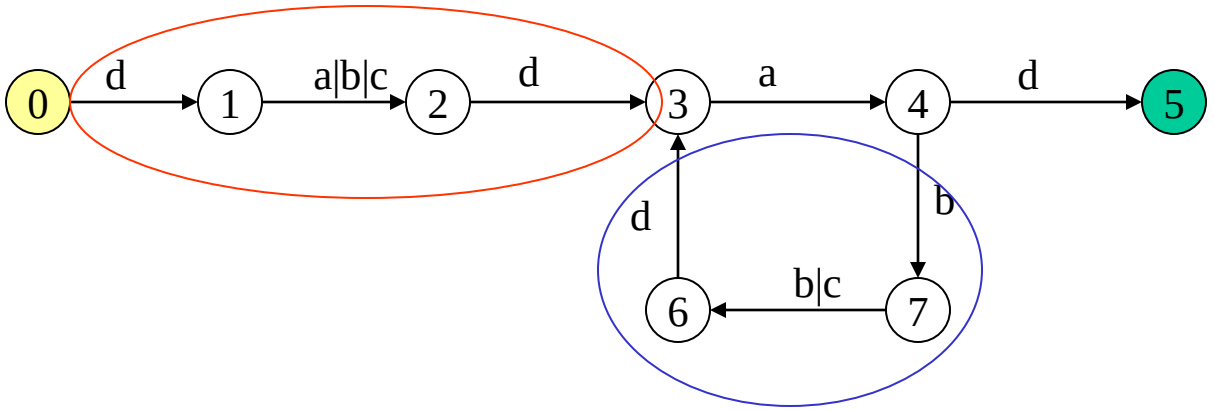
Example



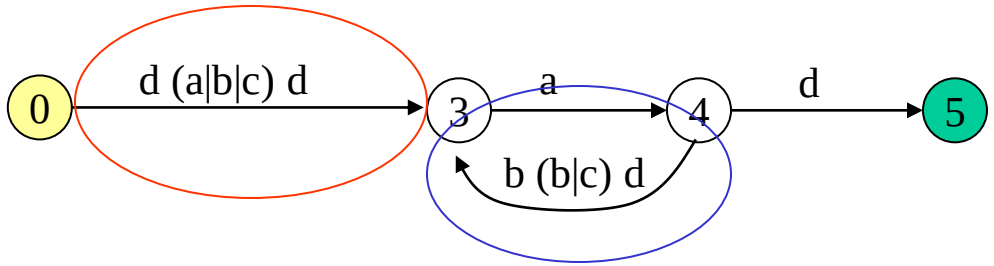
parallel edges become alternation



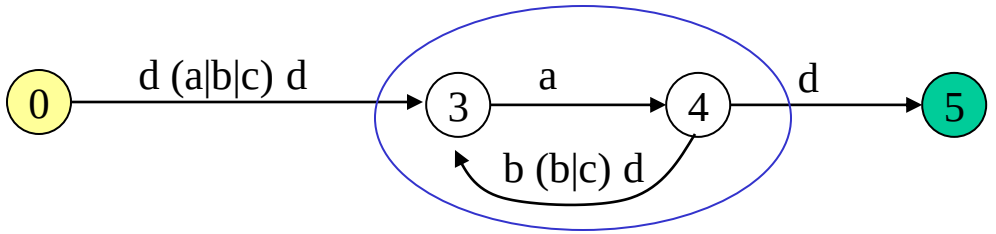
Example



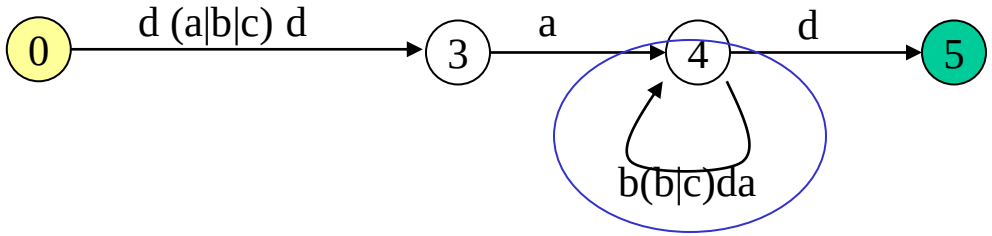
serial edges become concatenation



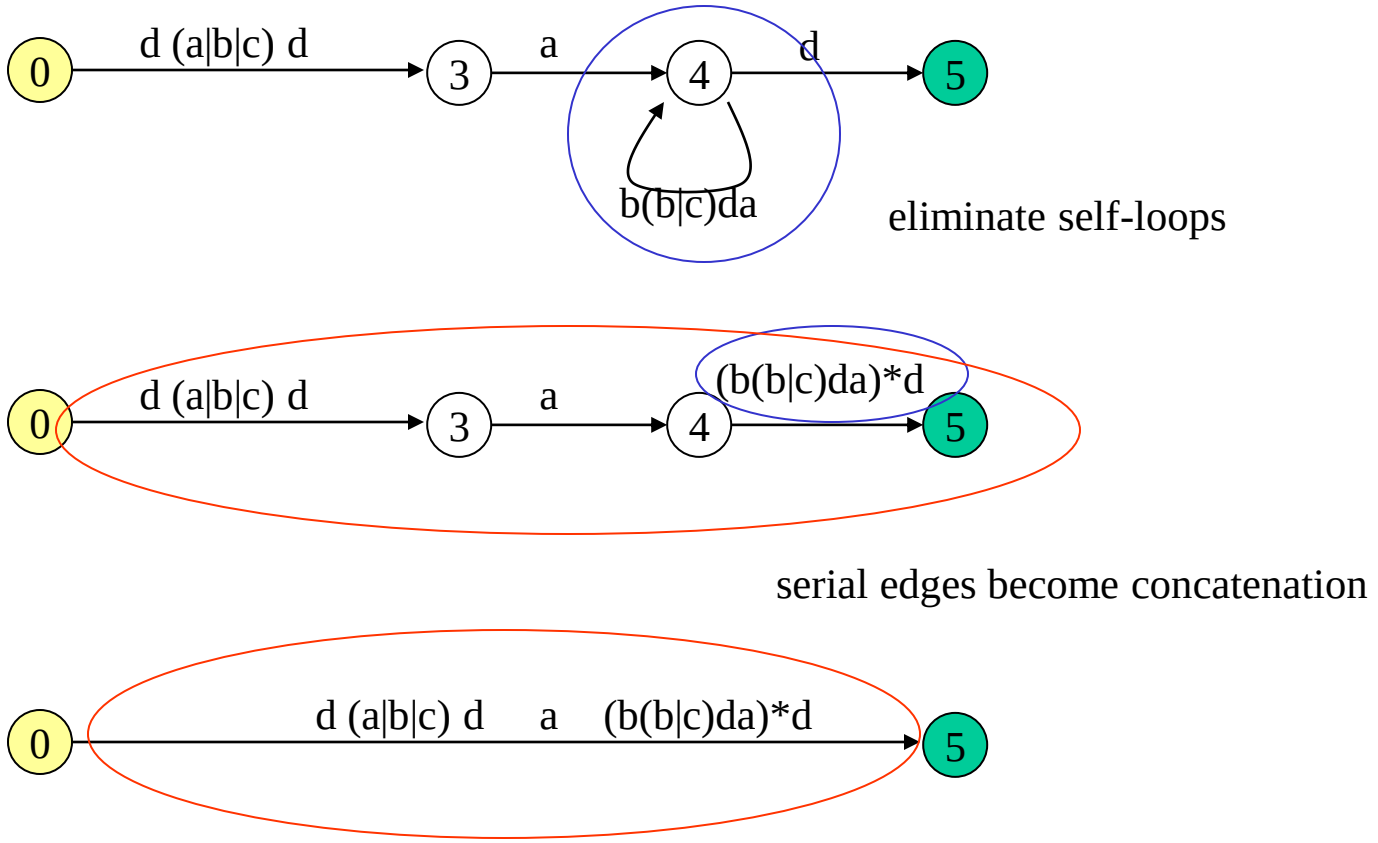
Example



Find paths that can be “shortened”



Example


0
d (a|b|c) d
3
a
4
d
5
b(b|c)da
eliminate self-loops
0
d (a|b|c) d
3
a
4
(b(b|c)da)*d
5
serial edges become concatenation
0
d (a|b|c) d
a
(b(b|c)da)*d
5

Describing Regular Languages

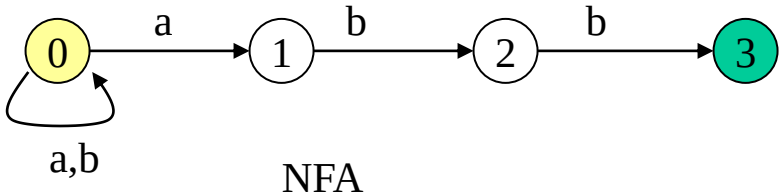
- Generate ***all*** strings in the language
- Generate ***only*** strings in the language

Try the following:

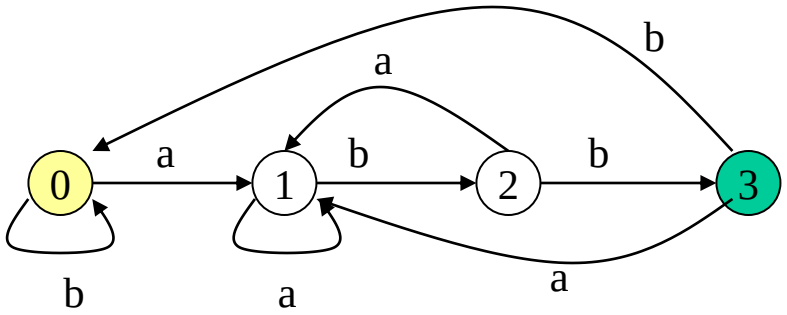
- Strings of {*a*,*b*} that end with '*abb*'
- Strings of {*a*,*b*} that don't end with '*abb*'
- Strings of {*a*,*b*} where every *a* is followed by at least one *b*

Strings of (a|b)* that end in abb

re: (a|b)*abb



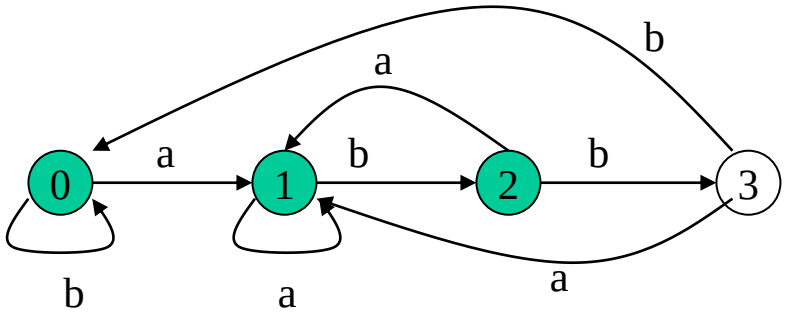
NFA



DFA

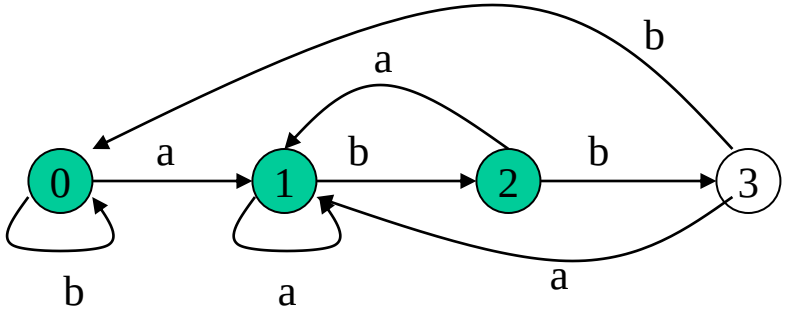
Strings of (a|b)* that don't end in abb

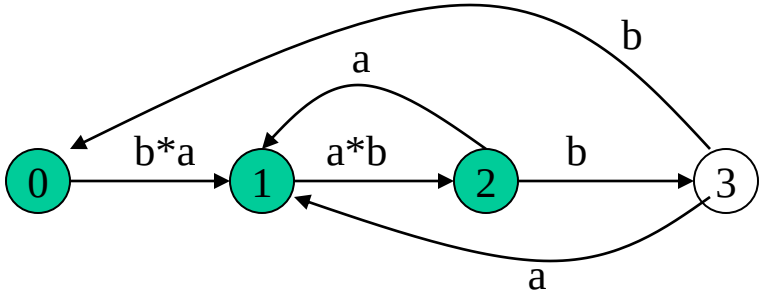
re: ??

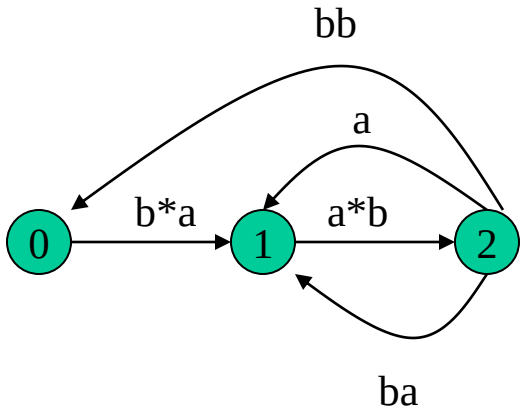


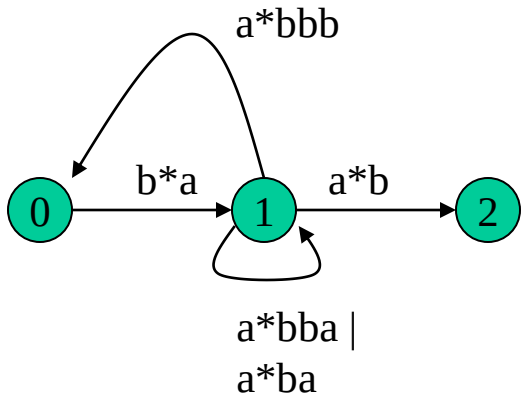
DFA/NFA

Strings of (a|b)* that don't end in abb









Suggestions for writing NFA/DFA/RE

- Typically, one of these formalisms is more natural for the problem. Start with that and convert if necessary.
- In NFA/DFAs, each state typically captures some partial solution
- Be sure that you include all relevant edges (ask – does every state have an outgoing transition for all alphabet symbols?)

Non-Regular Languages

Not all languages are regular”

- The language *ww* where *w*=(a|b)*

Non-regular languages cannot be described using REs, NFAs and DFAs.