

Q1.

(i)

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix} = 1(-1+2) - 2(-3-0) + 1(3-0) \\ = 1 + 6 + 3 = 10 \neq 0 \text{ non-singular}$$

(ii)

$$\begin{bmatrix} 1 & 2 & -1 \\ -3 & 4 & 5 \\ -4 & 2 & 6 \end{bmatrix} = 1(24-10) - 2(-18+20) - (-6+15) \\ = 14 - 4 - 10 = 0 \text{ Singular}$$

Q2.

(i)

$$A = \begin{bmatrix} 2 & 6 & 7 \\ 6 & -2 & 3 \\ 7 & 3 & 0 \end{bmatrix}, A^T = \begin{bmatrix} 2 & 6 & 7 \\ 6 & -2 & 3 \\ 7 & 3 & 0 \end{bmatrix}$$

$$A = A^T \rightarrow A \text{ is symmetric}$$

(ii)

$$A = \begin{bmatrix} 0 & 3 & -5 \\ -3 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}, A^T = \begin{bmatrix} 0 & -3 & 5 \\ 3 & 0 & -6 \\ -5 & 6 & 0 \end{bmatrix}$$

$$A = -A^T \rightarrow A \text{ is Skew symmetric}$$

Q3.

(ii)

$$\begin{bmatrix} 1 & 2 & -1 \\ -3 & 4 & k \\ -4 & 2 & 6 \end{bmatrix} = 0$$

$$= 1(24-2k) - 2(-18+4k) - (-6+16) = 0$$

$$= 24 - 2k + 36 - 8k - 10 = 0$$

$$= 50 - 10k = 0 \rightarrow k = 5$$

iii)

$$\begin{bmatrix} 1 & 1 & -2 \\ 3 & -1 & 1 \\ k & 3 & -6 \end{bmatrix} = 0$$

$$1(6-3) - 1(-18-k) - 2(6-k) = 0$$

$$3 + 18 + k - 12 + 2k = 0$$

$$9 + 3k = 0 \Rightarrow -3k = 9 \Rightarrow k = -3$$

Q4.

ii)

$$\begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

$$|A| = 0[-6+6] + 2[-2+3] - 3[-2+3] = -1$$

$$a_{11} = \begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix} = 0$$

$$a_{12} = \begin{bmatrix} 1 & 3 \\ -1 & -2 \end{bmatrix} = 1$$

$$a_{13} = \begin{bmatrix} 1 & 3 \\ -1 & -2 \end{bmatrix} = 1$$

$$a_{21} = \begin{bmatrix} -2 & -3 \\ -2 & -2 \end{bmatrix} = -2$$

$$a_{22} = \begin{bmatrix} 0 & -3 \\ -1 & -2 \end{bmatrix} = -3$$

$$a_{23} = \begin{bmatrix} 0 & -2 \\ -1 & -2 \end{bmatrix} = -2$$

$$a_{31} = \begin{bmatrix} -2 & -3 \\ 3 & 3 \end{bmatrix} = 3$$

$$a_{32} = \begin{bmatrix} 0 & -3 \\ 1 & 3 \end{bmatrix} = 3$$

$$a_{33} = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix} = 2$$

$$A^{-1} = - \begin{bmatrix} 0 & 1 & 1 \\ -2 & -3 & -2 \\ 3 & 3 & 2 \end{bmatrix}$$

Q₅

1- $x + y = 3$

$2x + y = 4$

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & 1 & 4 \end{array} \right] \rightarrow -2R_1 + R_2$$

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -1 & -2 \end{array} \right]$$

$y = 2$, $x = 1 \rightarrow$ unique solution

2- $2x + 4y = 2$

$x + 2y = 1$

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 1 & 2 & 1 \end{array} \right] \rightarrow R_1 - R_2$$

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 0 & 0 \end{array} \right]$$

Let $y = t$, $x = 1 - 2t \rightarrow$ infinite solution

3- $x + 2y = 1$

$x + 2y = 5$

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 1 & 2 & 5 \end{array} \right] \rightarrow -R_1 + R_2$$

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 0 & 4 \end{array} \right]$$

no solution

4- $2x + y + 2z = 0$

$x + y + 3z = 1$

$3x + 2y + 5z = 3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 2 & 1 & 2 & 0 \\ 3 & 2 & 5 & 3 \end{array} \right] \begin{array}{l} \rightarrow -2R_1 + R_2 \\ \rightarrow -3R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 0 & -1 & -4 & -2 \\ 0 & -1 & -4 & 0 \end{array} \right] \rightarrow -R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 0 & -1 & -4 & -2 \\ 0 & 0 & 0 & 2 \end{array} \right]$$

\rightarrow no solution

5- $x + 2y + 2z = 1$

$2x + y + 3z = 1$

$3x + 3y + 5z = 2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 2 & 1 & 3 & 1 \\ 3 & 3 & 5 & 2 \end{array} \right] \begin{array}{l} \rightarrow -2R_1 + R_2 \\ \rightarrow -3R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & -3 & -1 & -1 \\ 0 & -3 & -1 & -1 \end{array} \right] \rightarrow R_2 - R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Let $z = t$, $y = \frac{1}{3}(1-t)$, $x = \frac{1}{3}(1-4t)$
 \rightarrow infinite solution