Cairo University
FCI

Theory of Computation

Push Down Automata (PDA)

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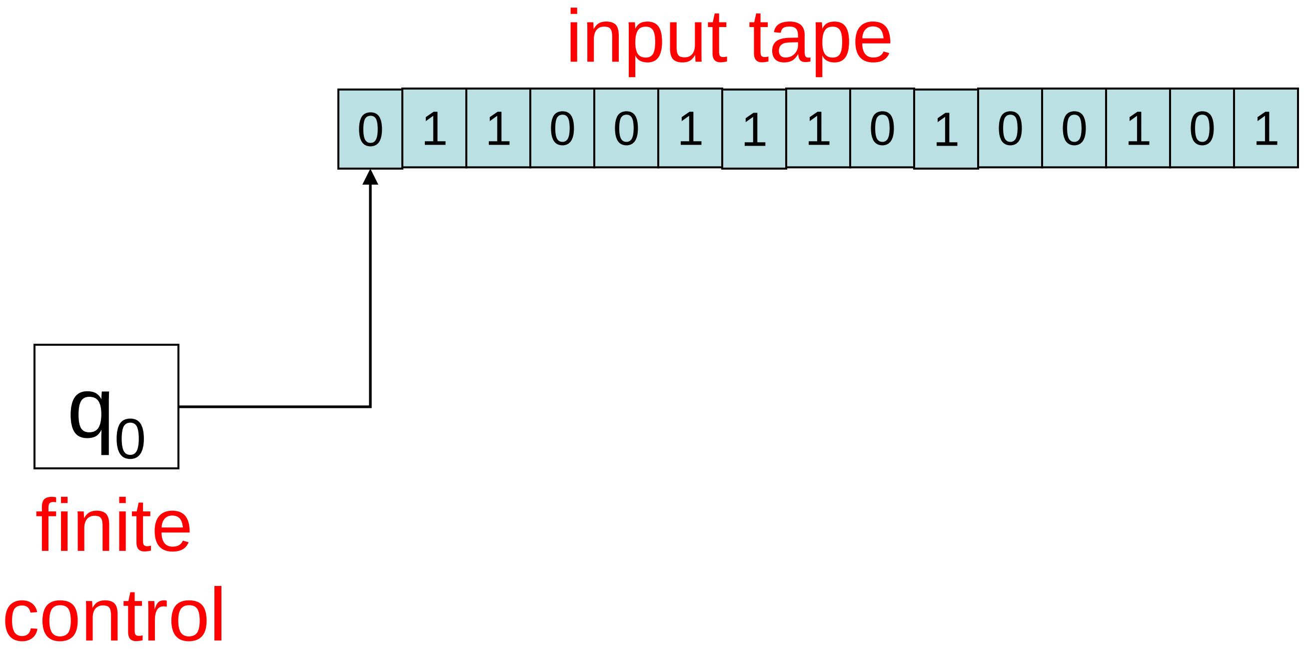
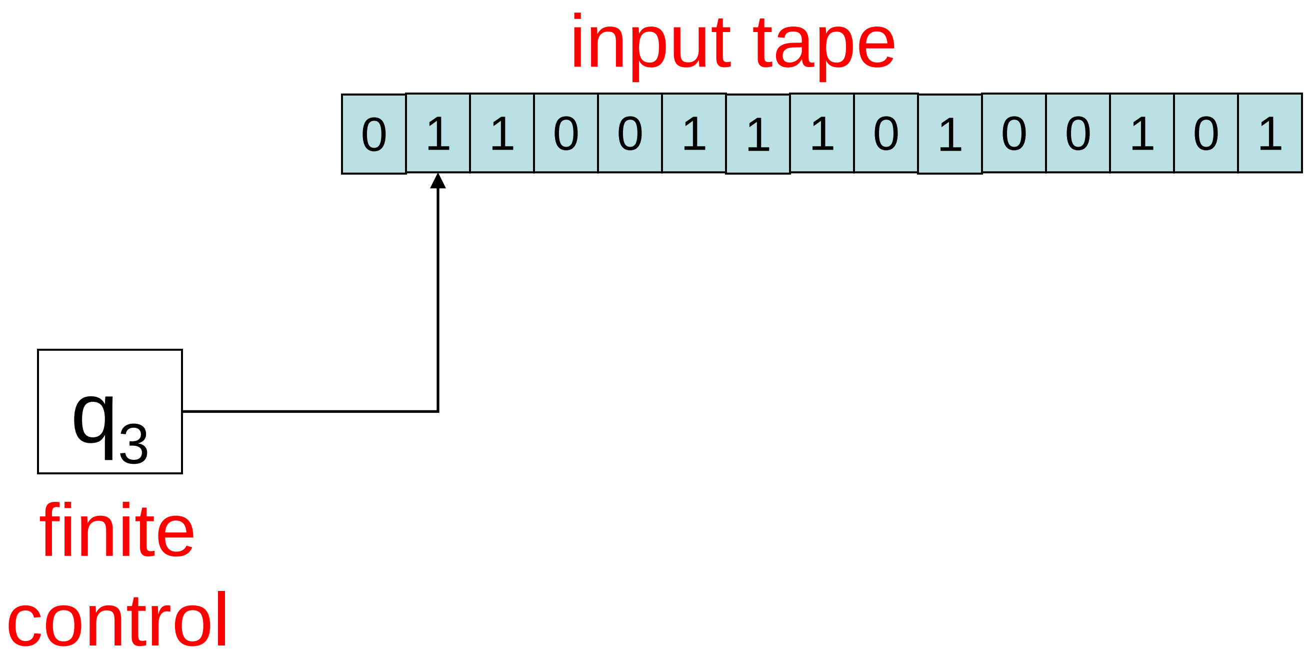
Some of this lecture slides are taken from previous course of prof . Dr.Abdulhussein Mohsin Abdullah

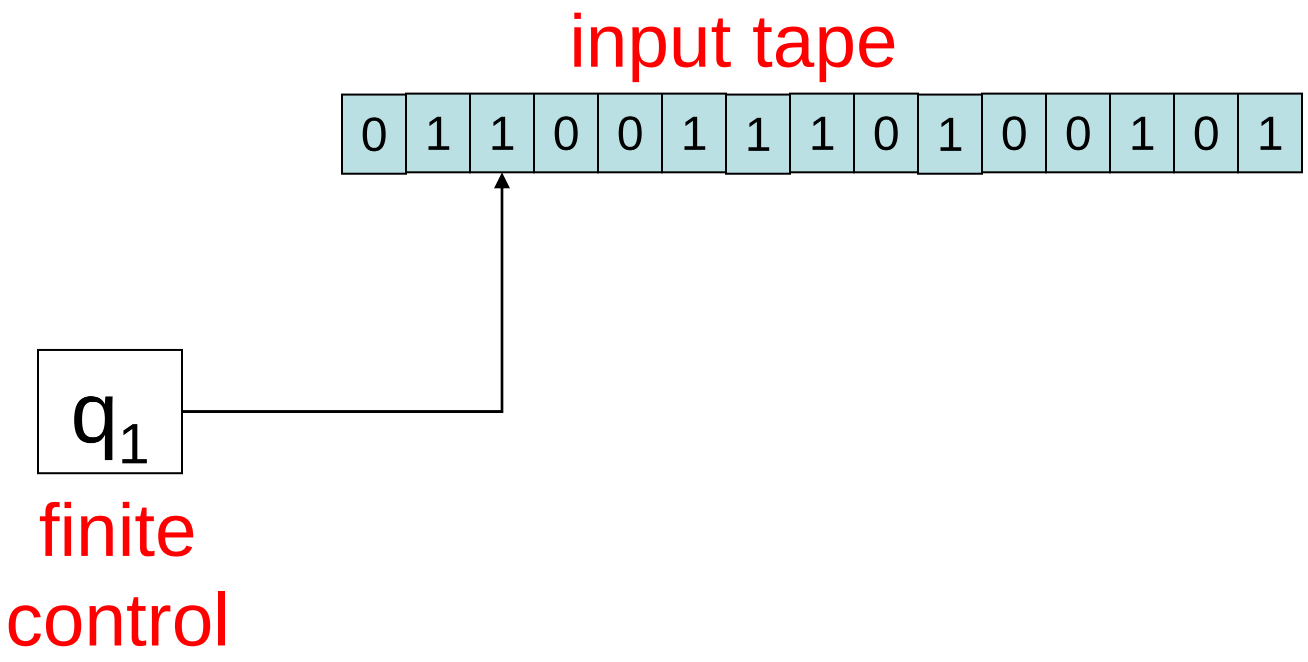
Machine view of FA



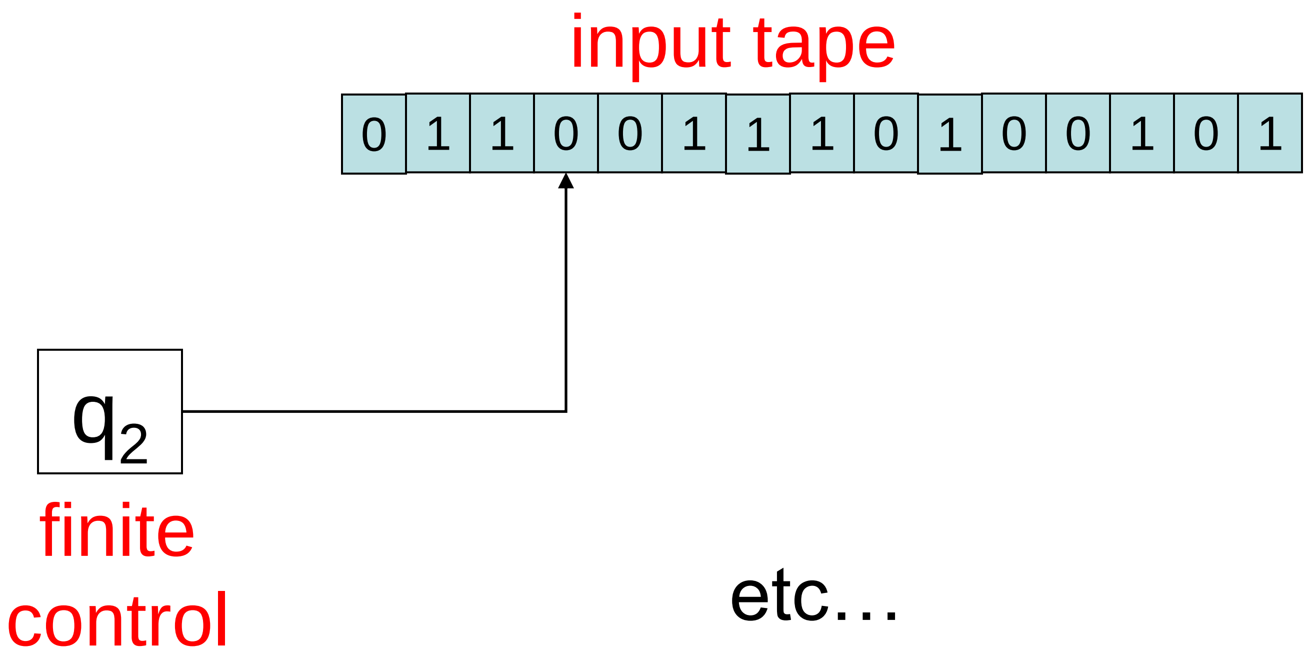
Machine view of FA



Machine view of FA



Machine view of FA



etc…

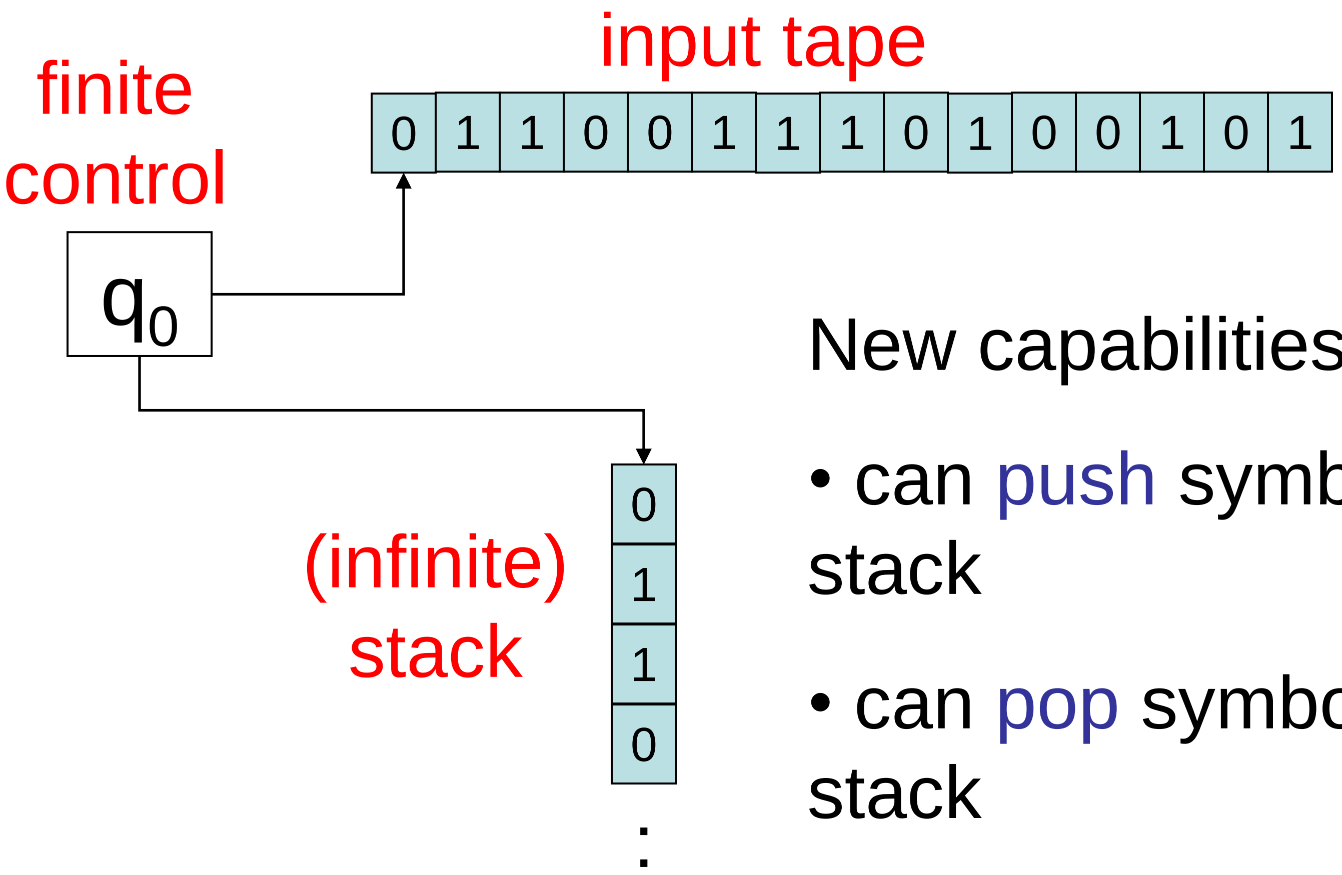
A more powerful machine

- limitation of FA related to fact that they can only "remember" a bounded amount of information

- What is the simplest alteration that adds unbounded "memory" to our machine?

- Should be able to recognize, e.g., $\{0^n1^n: n \geq 0\}$

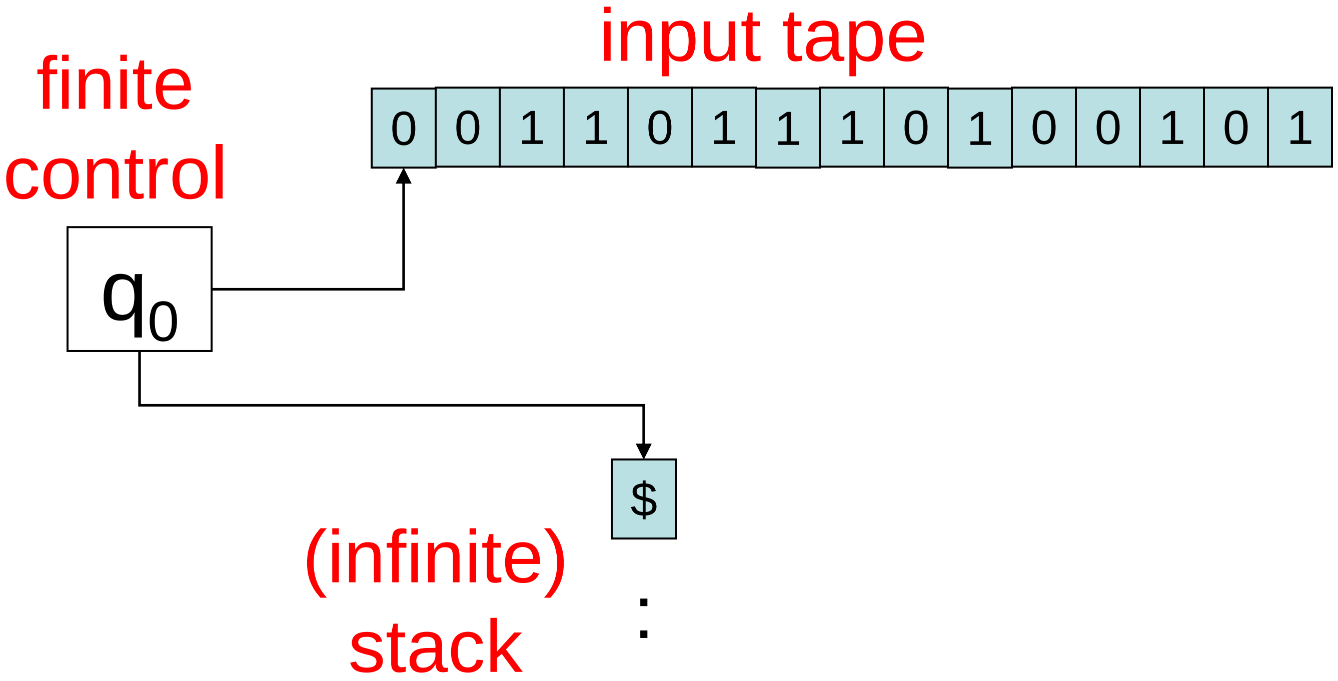
Pushdown Automata



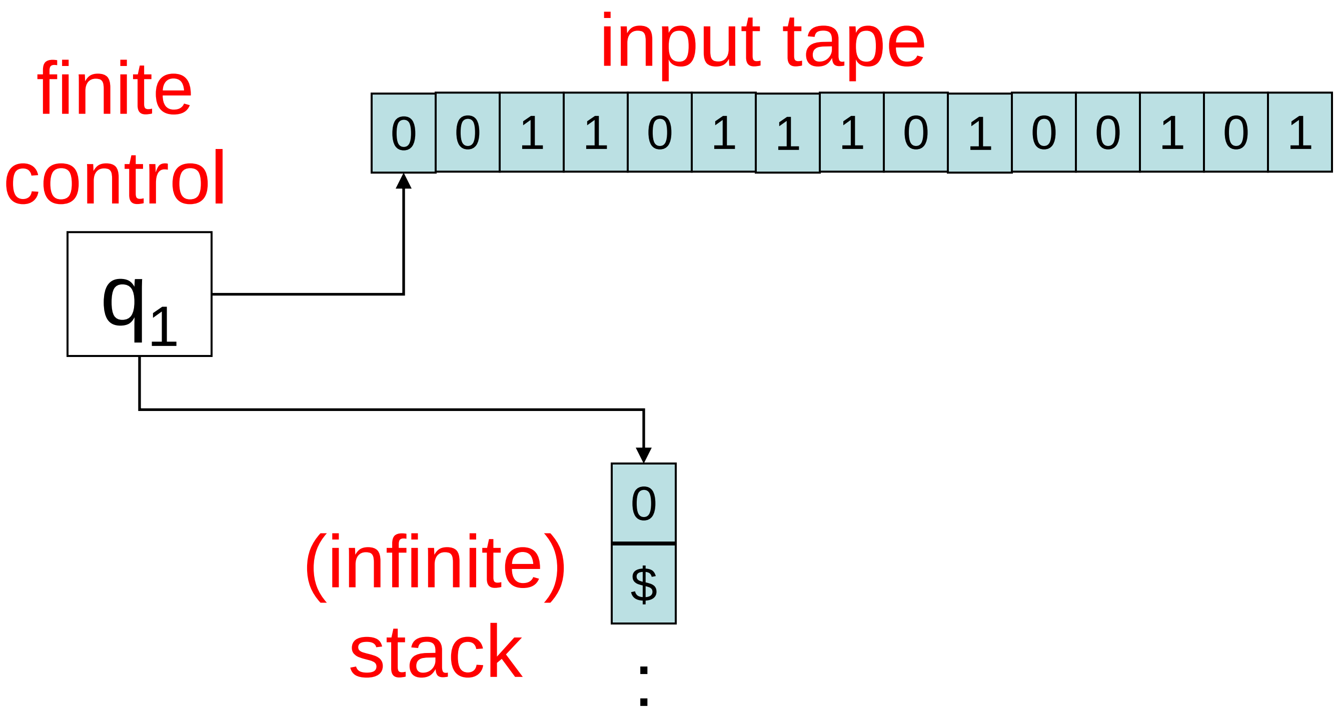
New capabilities:

- can push symbol onto stack
- can pop symbol off of stack

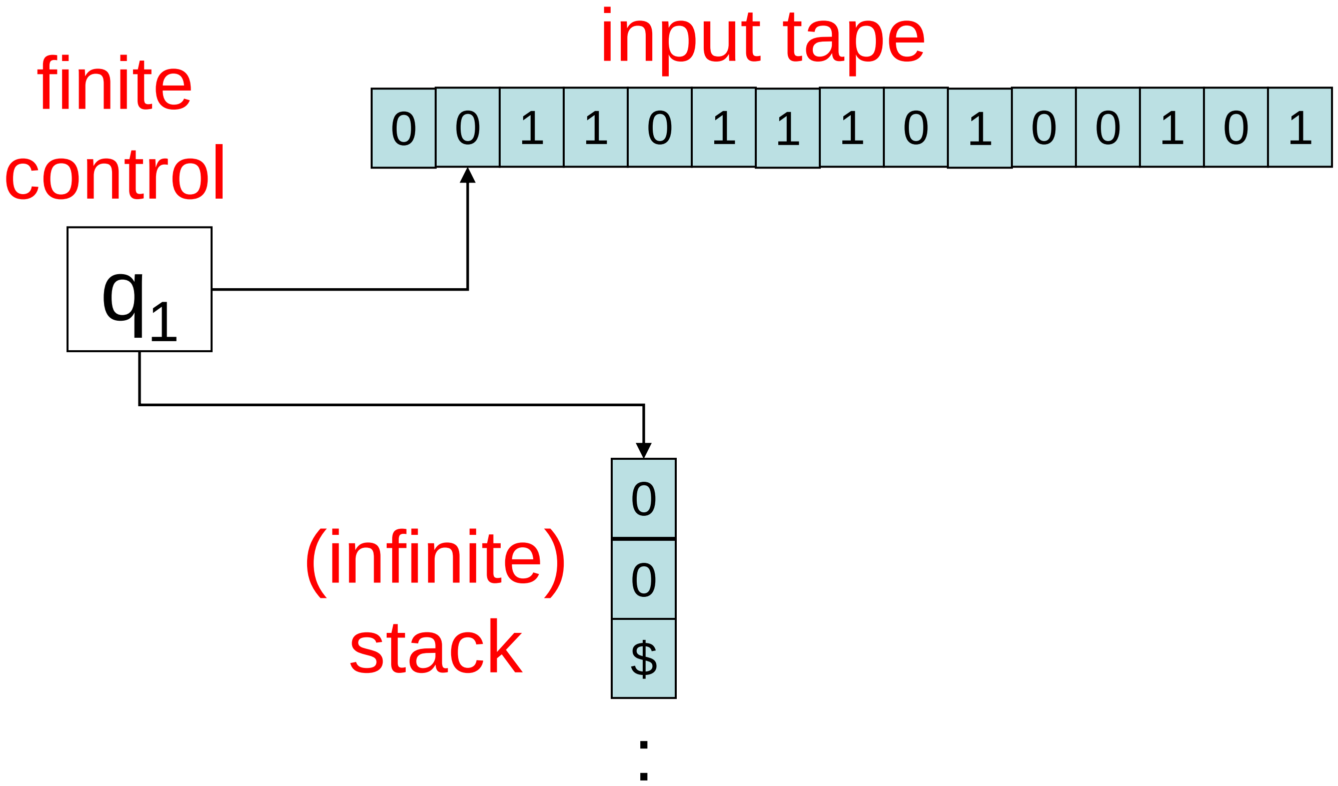
Pushdown Automata



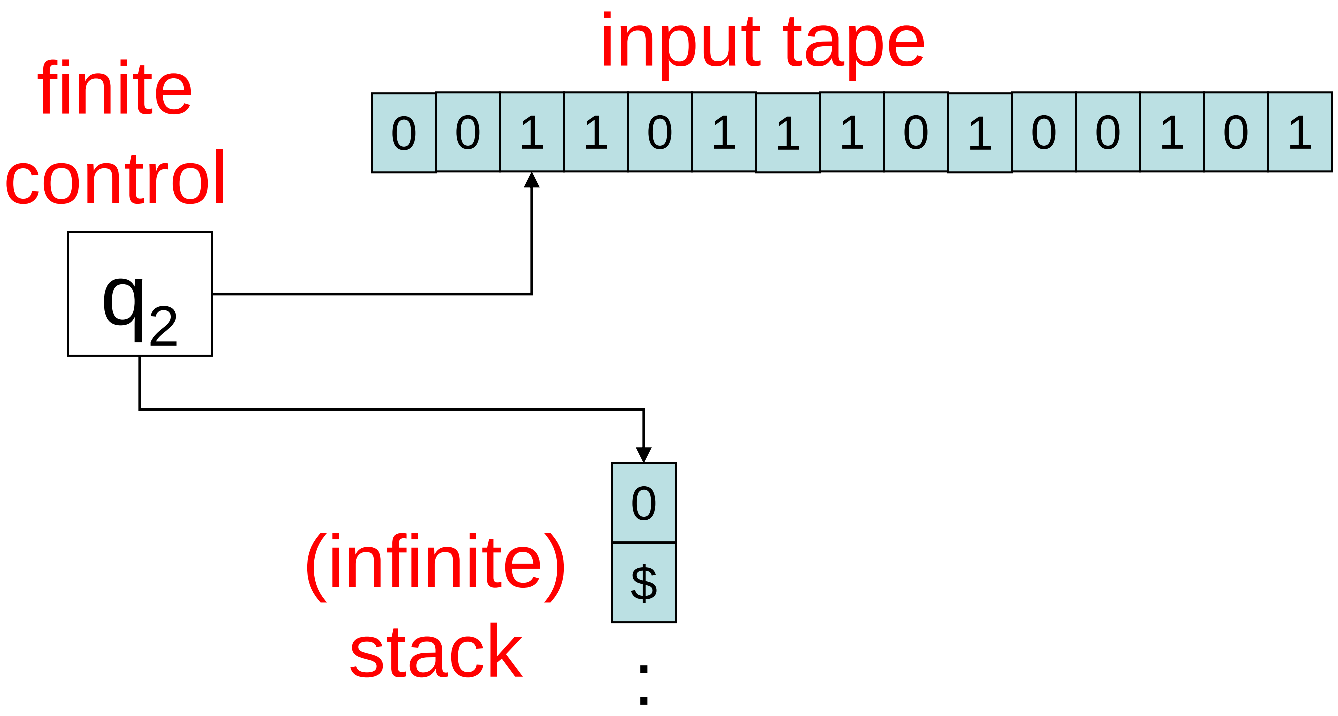
Pushdown Automata



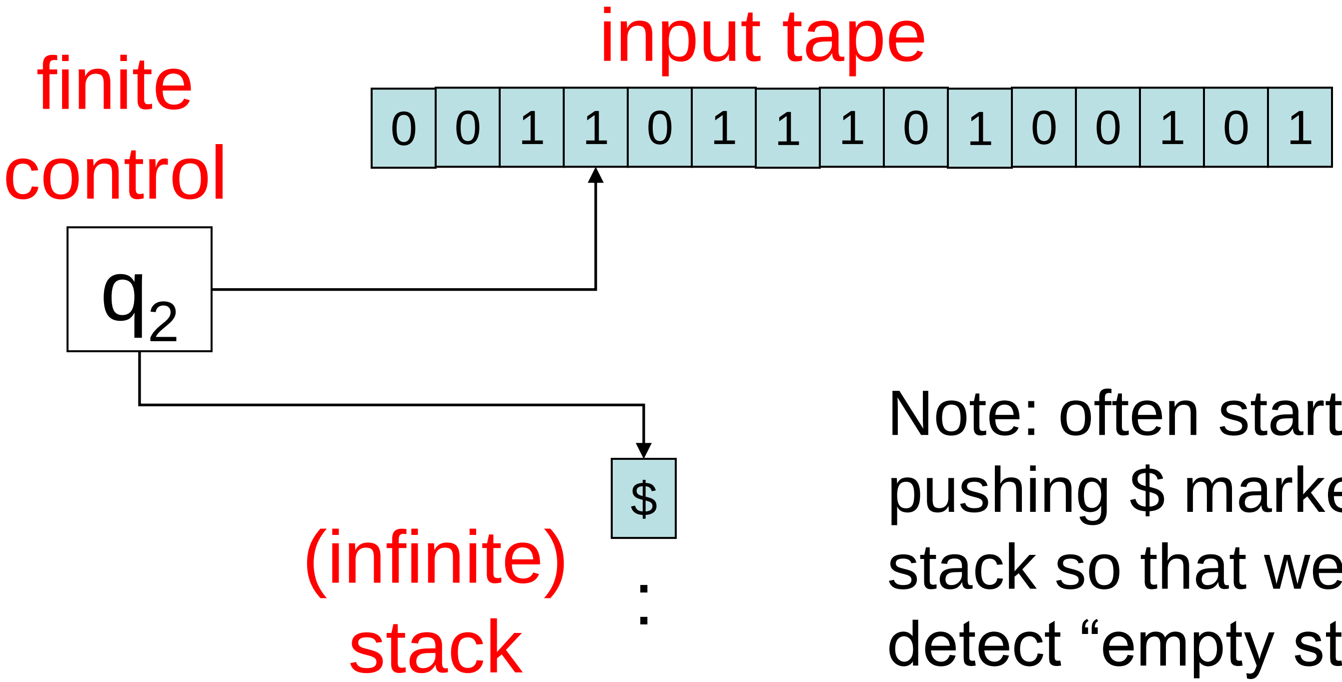
Pushdown Automata



Pushdown Automata



Pushdown Automata



Note: often start by pushing $ marker onto stack so that we can detect “empty stack”

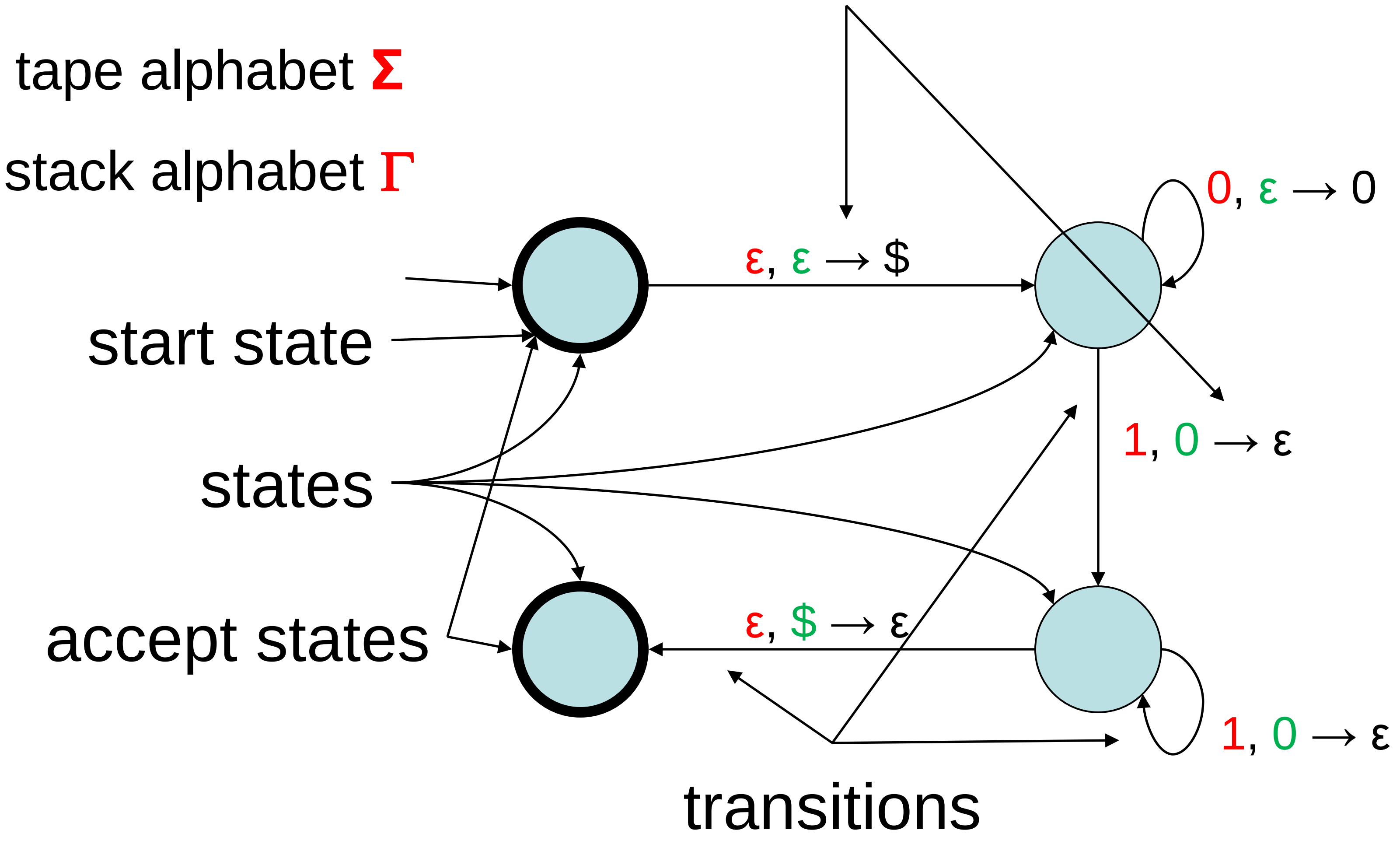
Pushdown Automata (PDA)

- Two ways to describe PDA
  - diagram
  - formal definition

PDA diagram

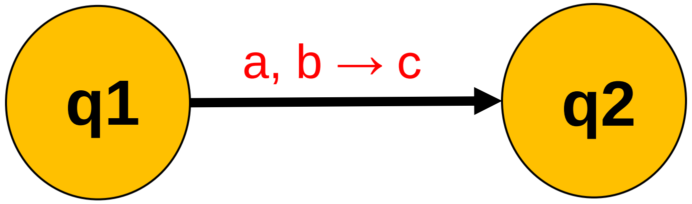
transition label:

(**tape symbol read**, stack symbol popped → stack symbol pushed)



PDA operation

- Taking a transition labeled:



  - $a \in (\Sigma \cup \{\varepsilon\})$
  - $b,c \in (\Gamma \cup \{\varepsilon\})$

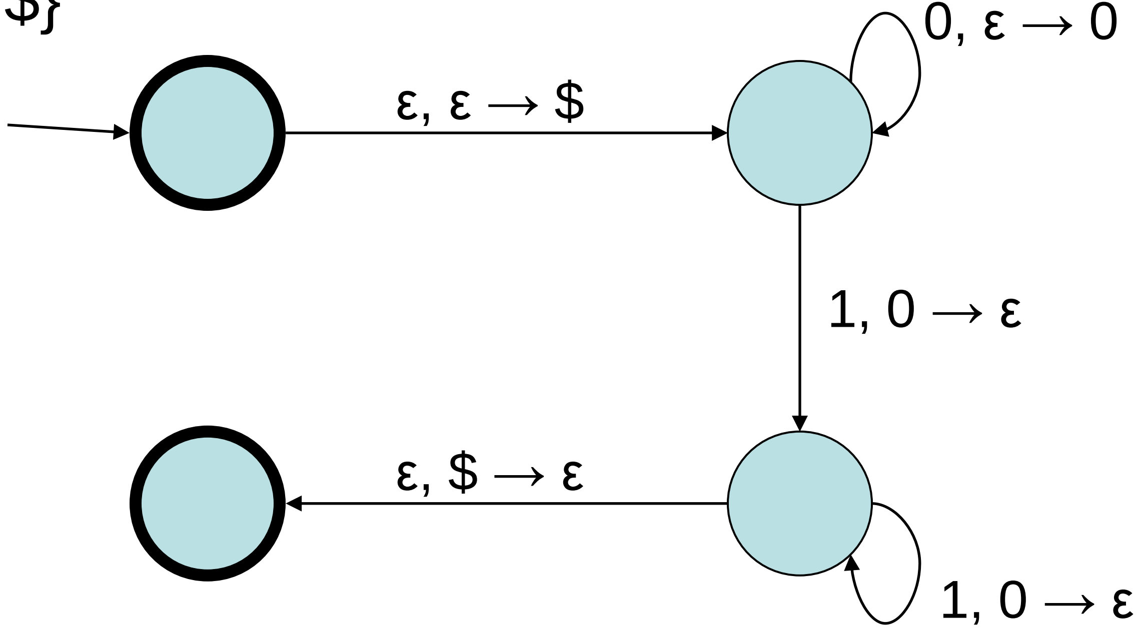
If the input symbol is **a** **and**
the top stack symbol is **b** **then**
q_1 to q_2, pop **b**, push **c**, advance read head

read a from tape, or don't read from tape if $a = \varepsilon$
pop b from stack, or don't pop from stack if $b = \varepsilon$
push c onto stack, or don't push onto stack if $c = \varepsilon$

Example PDA

$\Sigma = \{0, 1\}$

$\Gamma = \{0, 1, \$\}$

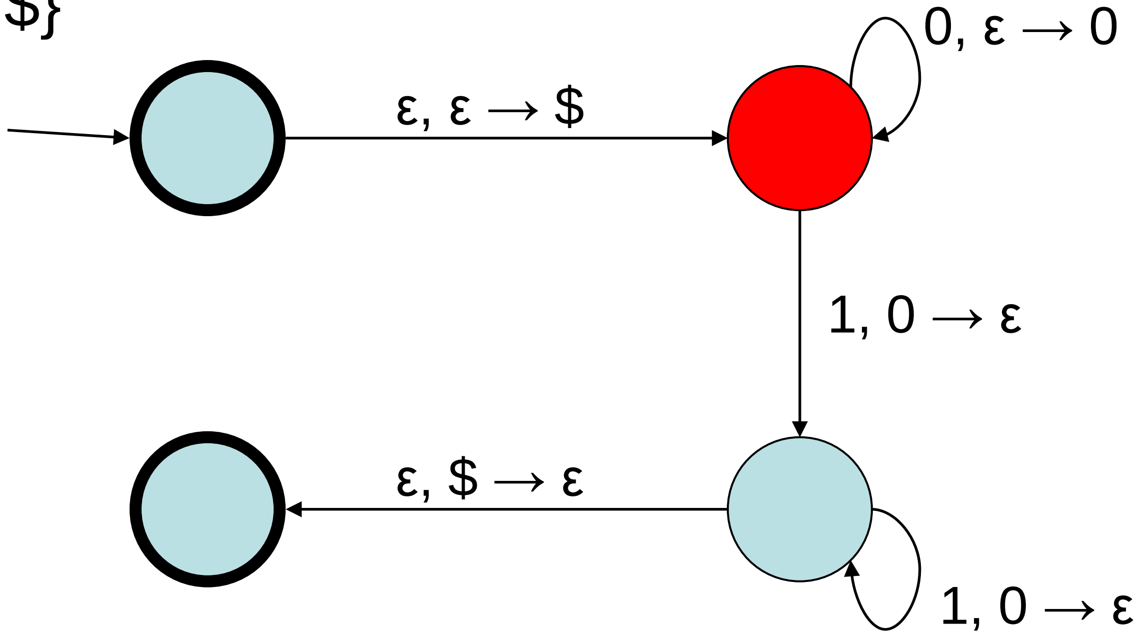


- tape: 0 0 1 1 Stack contents: $

Example PDA

$\Sigma = \{0, 1\}$

$\Gamma = \{0, 1, \$\}$



- tape: 0 0 1 1 Stack contents: 0 $

Example PDA

$\Sigma = \{0, 1\}$

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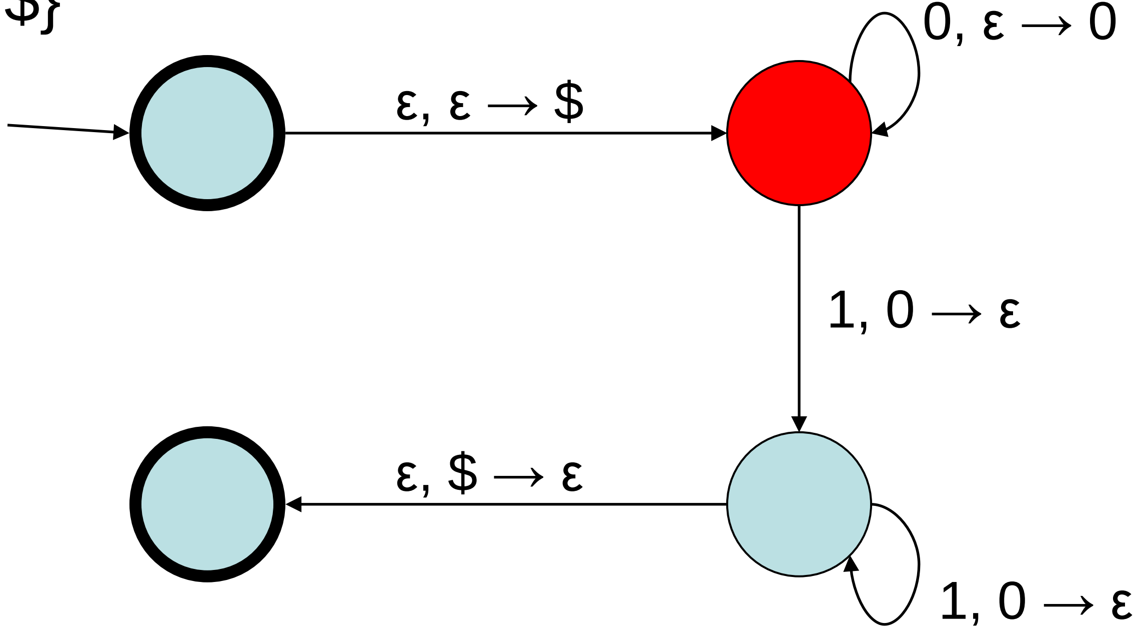

- tape: 0 0 1 1 Stack contents: 0 0 $

Example PDA

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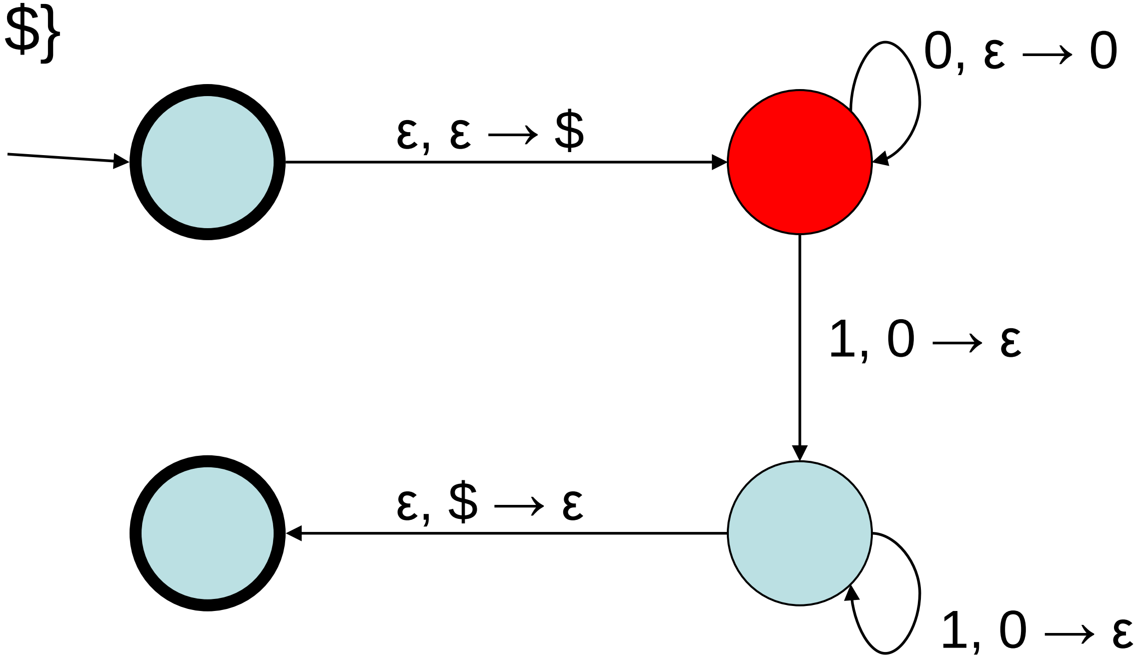

- tape: 0 0 1 1 Stack contents: 0 0 $

Example PDA

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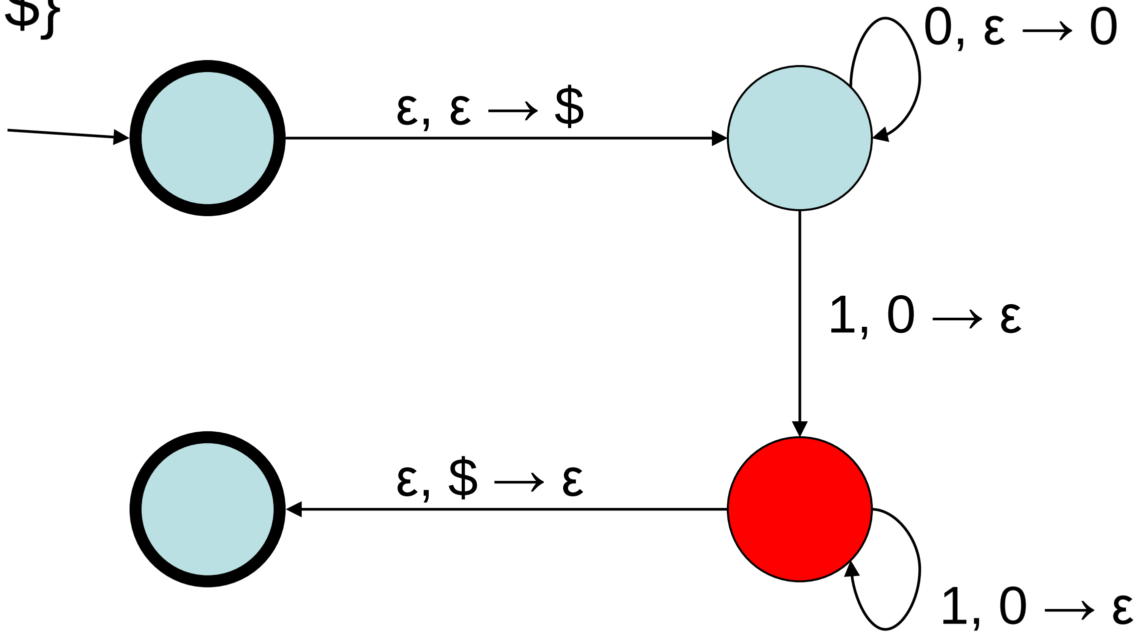

- tape: 0 0 1 1 Stack contents: 0 $

Example PDA

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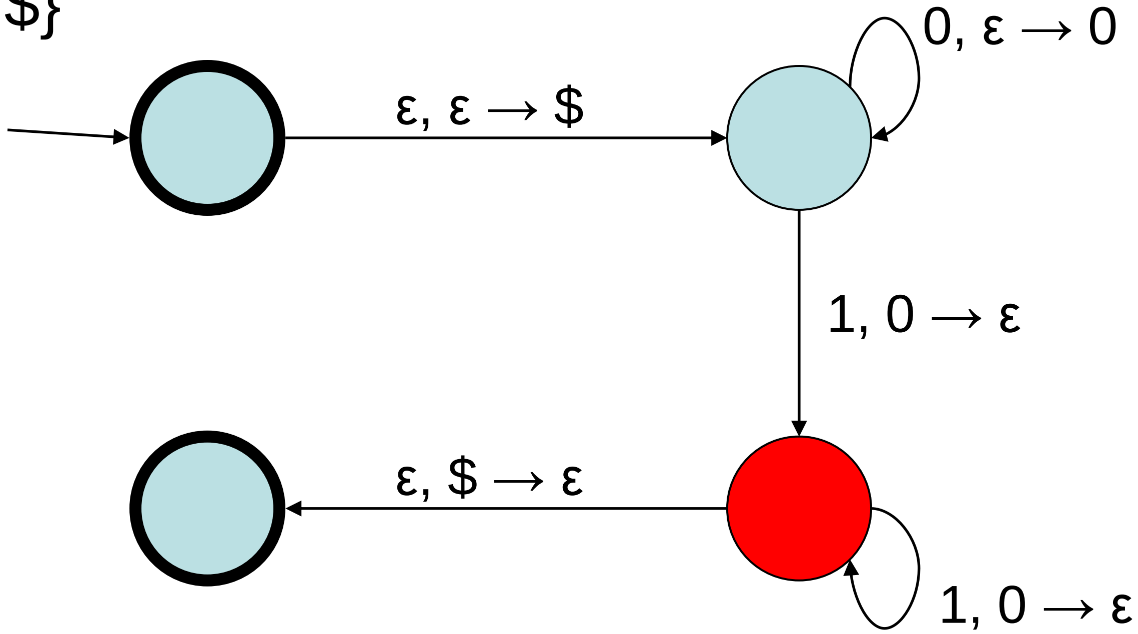
$\Gamma = \{0, 1, \$\}$



- tape: 0 0 1 1 accepted

Stack contents: $

Example PDA

$\Sigma = \{0, 1\}$

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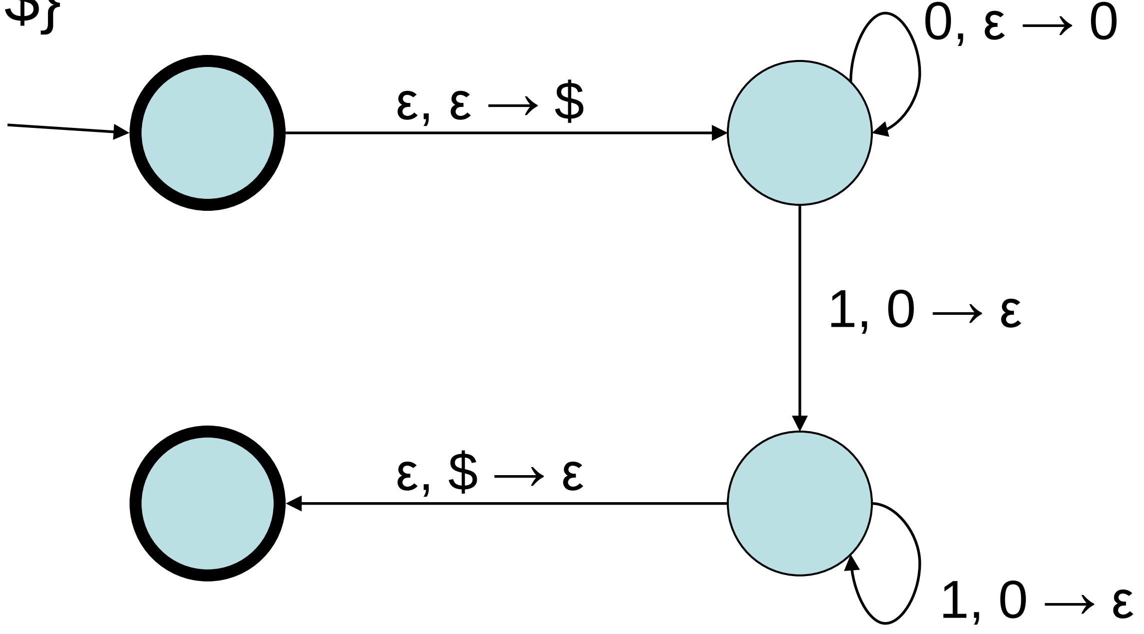

- tape: 0 0 1   Stack contents: $

Example PDA

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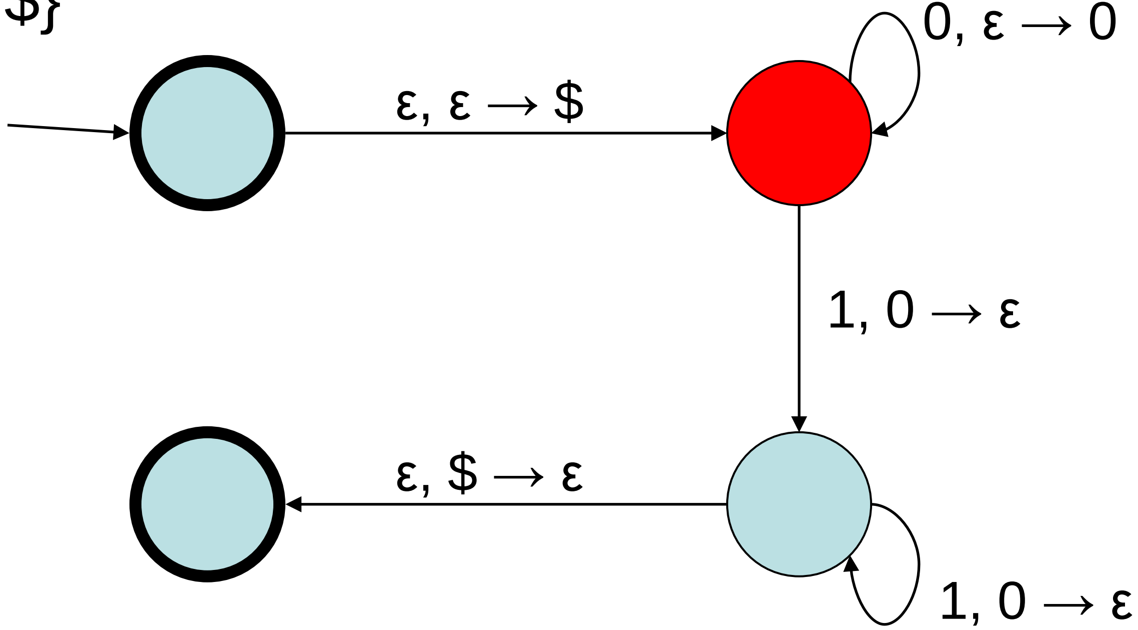

- tape: 0 0 1 Stack contents: 0 $

Example PDA

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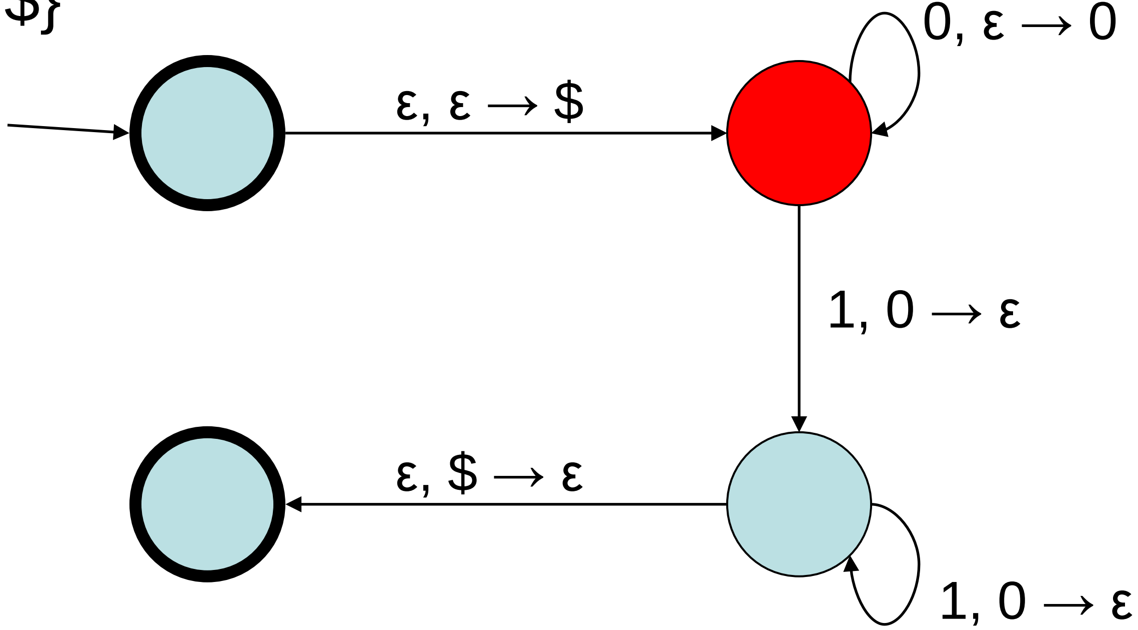


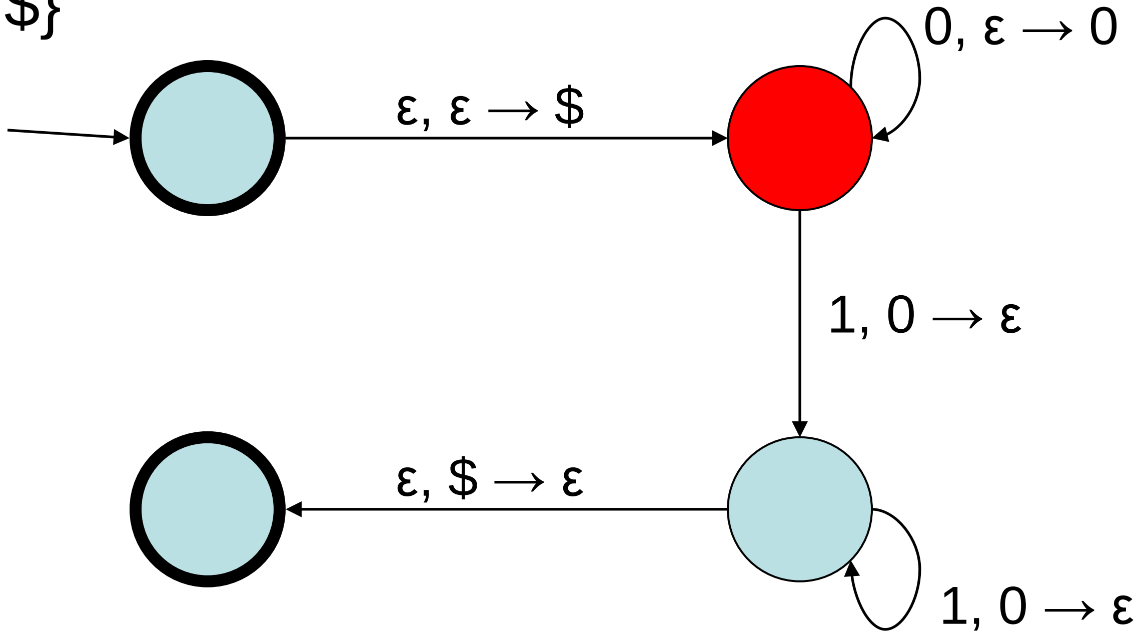
- tape: 0 0 1

Stack contents: 0 0 $

Example PDA

$\Sigma = \{0, 1\}$

$\Gamma = \{0, 1, \$\}$



- tape: 0 0 1 Stack contents: 0 0 $

Example PDA

$\Sigma = \{0, 1\}$

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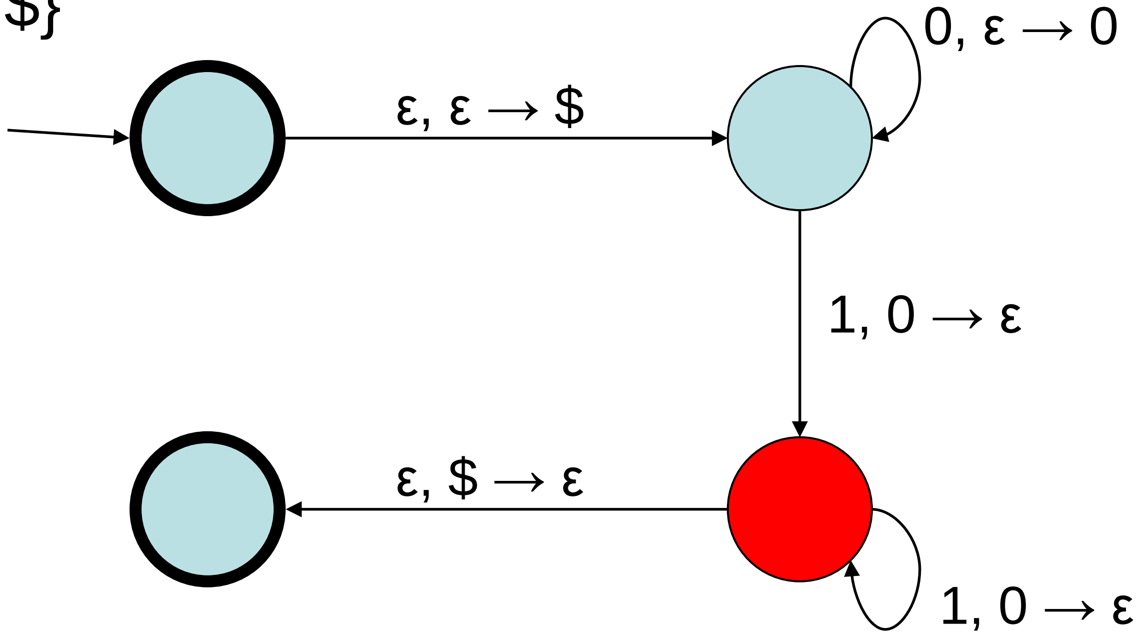


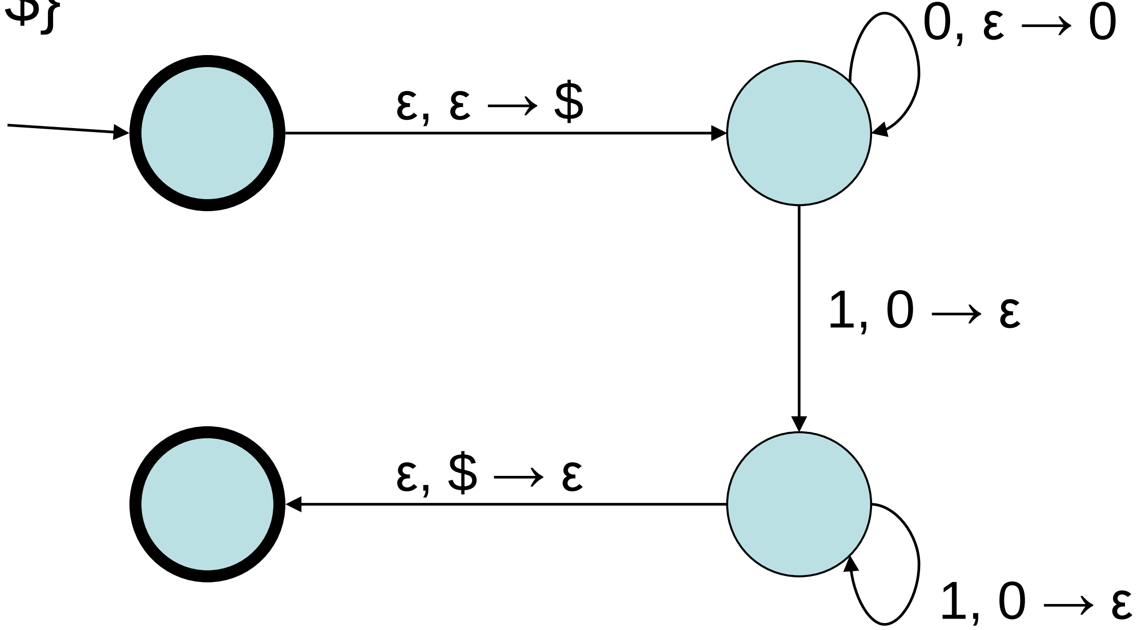
- tape: 0 0 1 Stack contents: 0 $

  not accepted

Example PDA

$\Sigma = \{0, 1\}$

$\Gamma = \{0, 1, \$\}$



- What language does this PDA accept?

Formal definition of PDA

- A PDA is a 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$ where:
  - Q is a finite set called the states
  - Σ is a finite set called the tape alphabet
  - Γ is a finite set called the stack alphabet
  - $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\}) \rightarrow \wp(Q \times (\Gamma \cup \{\varepsilon\}))$ is a function called the transition function
  - q_0 is an element of Q called the start state
  - F is a subset of Q called the accept states

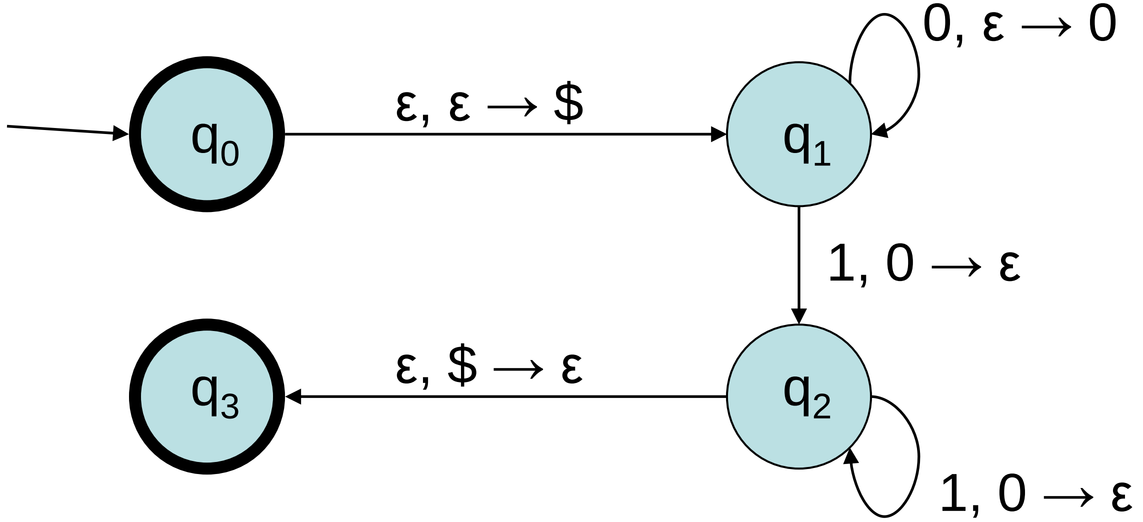
Formal definition of PDA

- PDA $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ accepts string $w \in \Sigma^*$ if w can be written as

$$w_1 w_2 w_3 \ldots w_m \in (\Sigma \cup \{\varepsilon\})^*, \text{ and}$$

- there exist states $r_0, r_1, r_2, \ldots, r_m$, and
- there exist strings $s_0, s_1, \ldots, s_m$ in $(\Gamma \cup \{\varepsilon\})^*$
  - $r_0 = q_0$ and $s_0 = \varepsilon$
  - $(r_{i+1}, b) \in \delta(r_i, w_{i+1}, a)$, where $s_i = at$, $s_{i+1} = bt$ for some $t \in \Gamma^*$
  - $r_m \in F$

Example of formal definition



- $Q = \{q_0, q_1, q_2, q_3\}$
- $\Sigma = \{0,1\}$
- $\Gamma = \{0, 1, \$\}$
- $F = \{q_0, q_3\}$

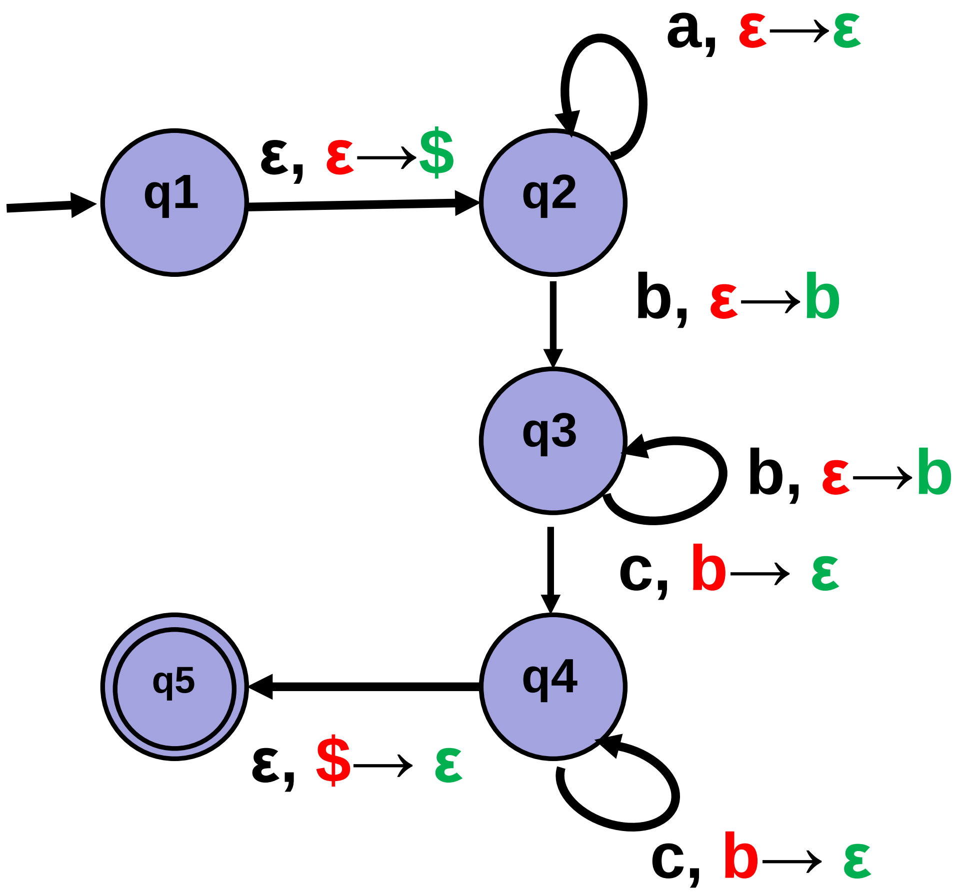
- $\delta(q_0, \varepsilon, \varepsilon) = \{(q_1, \$)\}$
- $\delta(q_1, 0, \varepsilon) = \{(q_1, 0)\}$
- $\delta(q_1, 1, 0) = \{(q_2, \varepsilon)\}$
- $\delta(q_2, 1, 0) = \{(q_2, \varepsilon)\}$
- $\delta(q_2, \varepsilon, \$) = \{(q_3, \varepsilon)\}$

other values of

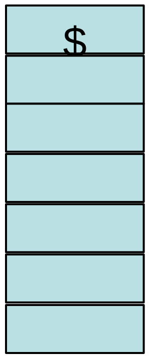
$\delta(\bullet, \bullet, \bullet)$ equal $\{\}$

Example

$L = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } j = k\}$



abc
aabbbbcccc
aaabbbbbccccc



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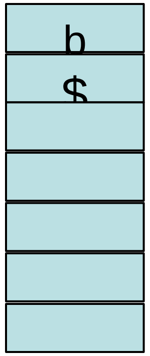

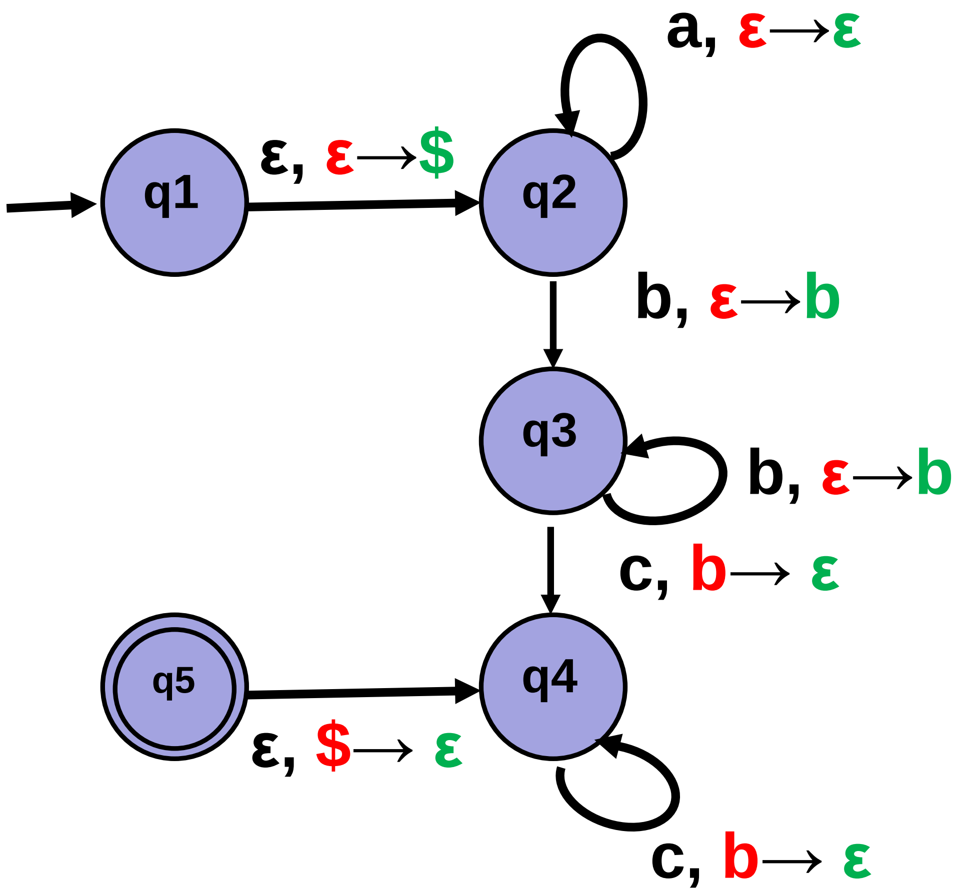

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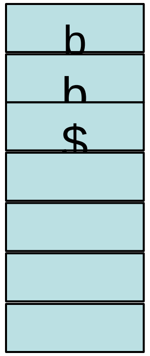

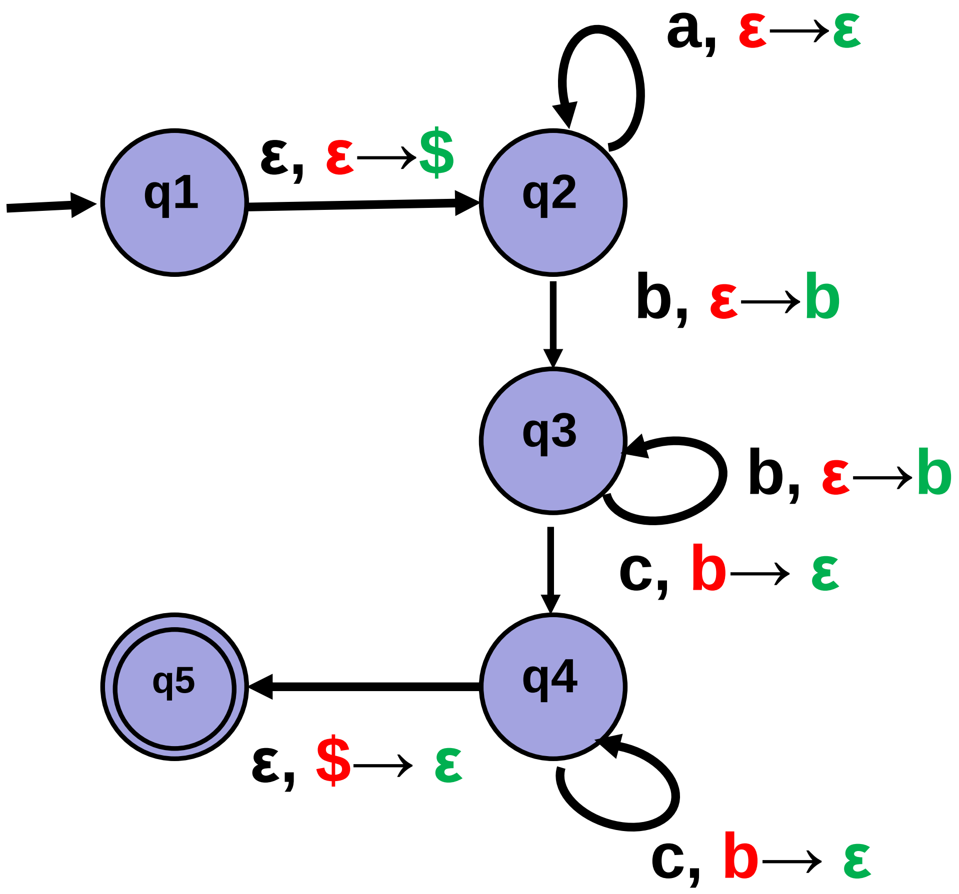

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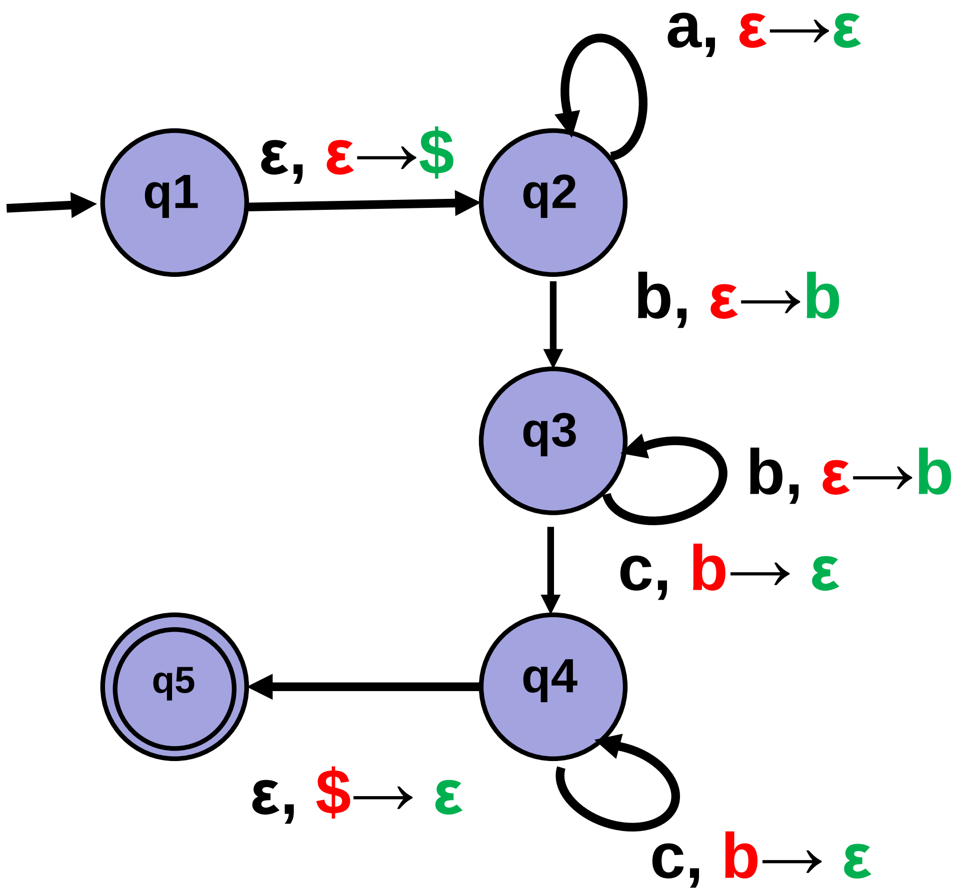

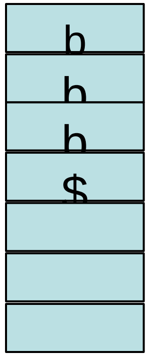

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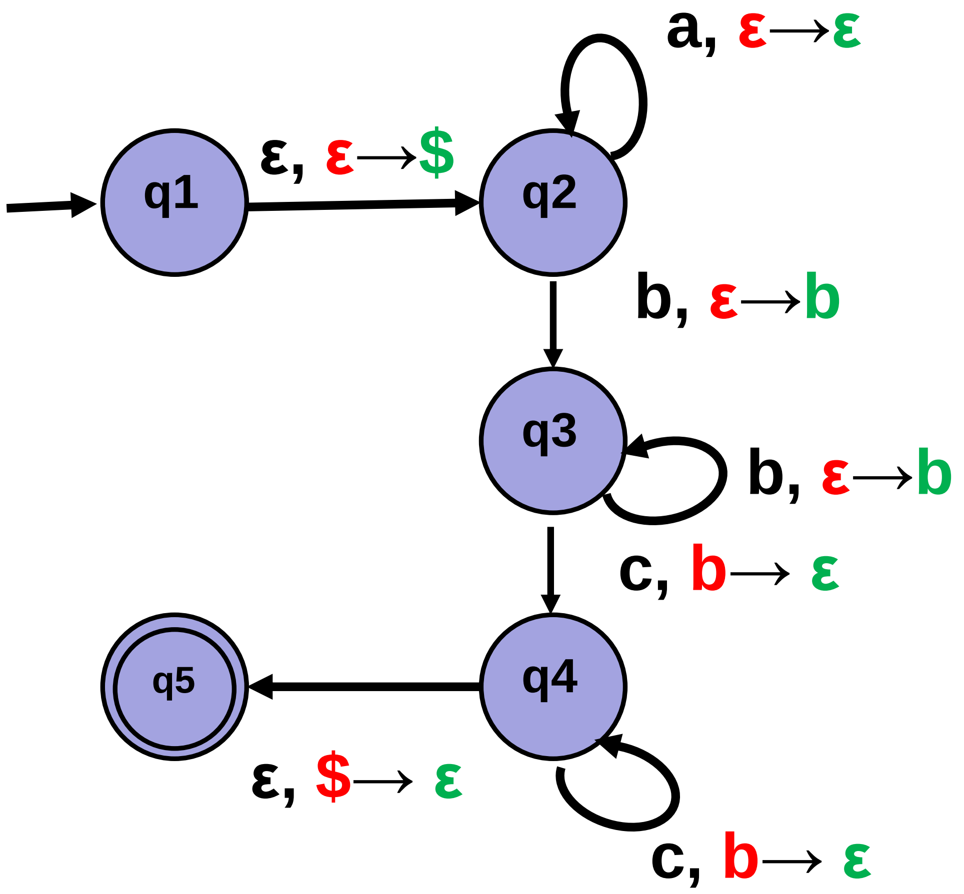

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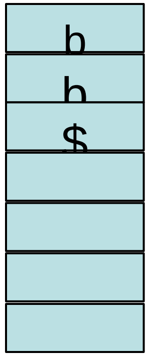

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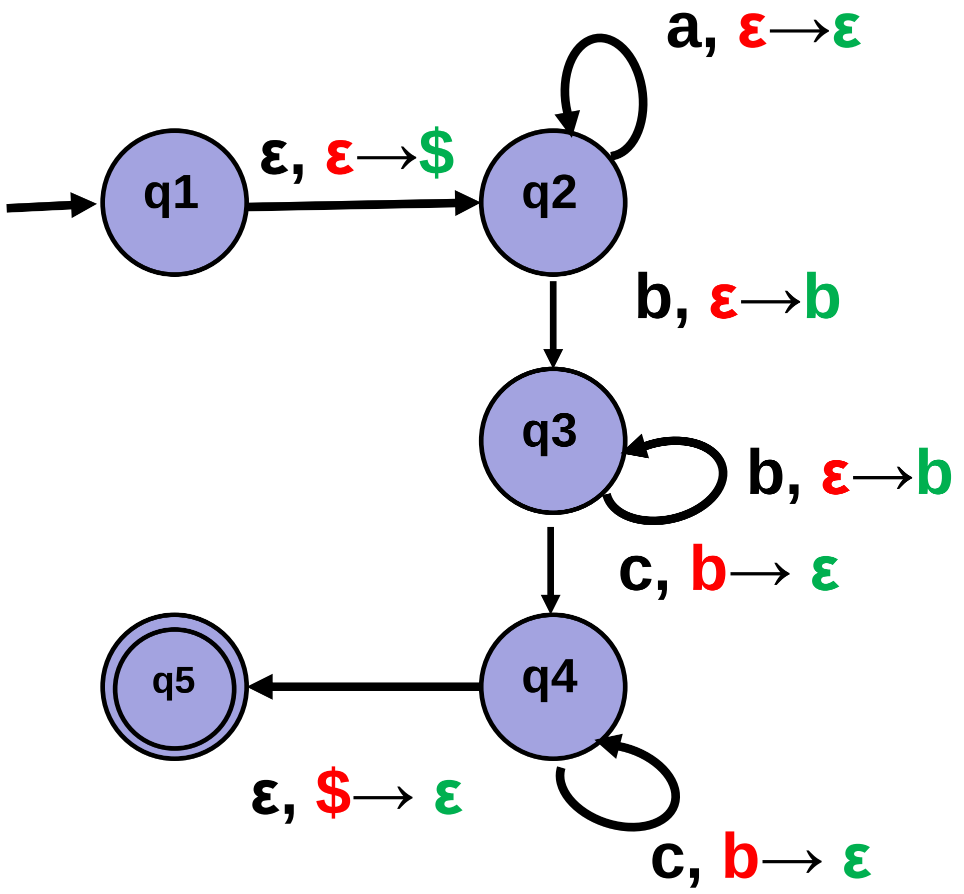

abc
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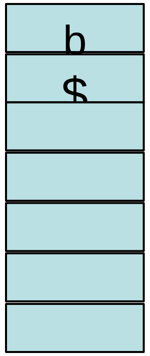
aaabbbbbccccc



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