Chapter 4

Beyond Classical Search

CS361 Artificial Intelligence
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Spring 2023

(This is the instructor's notes, and the student must read the textbook for complete material.)

Chapter Outline

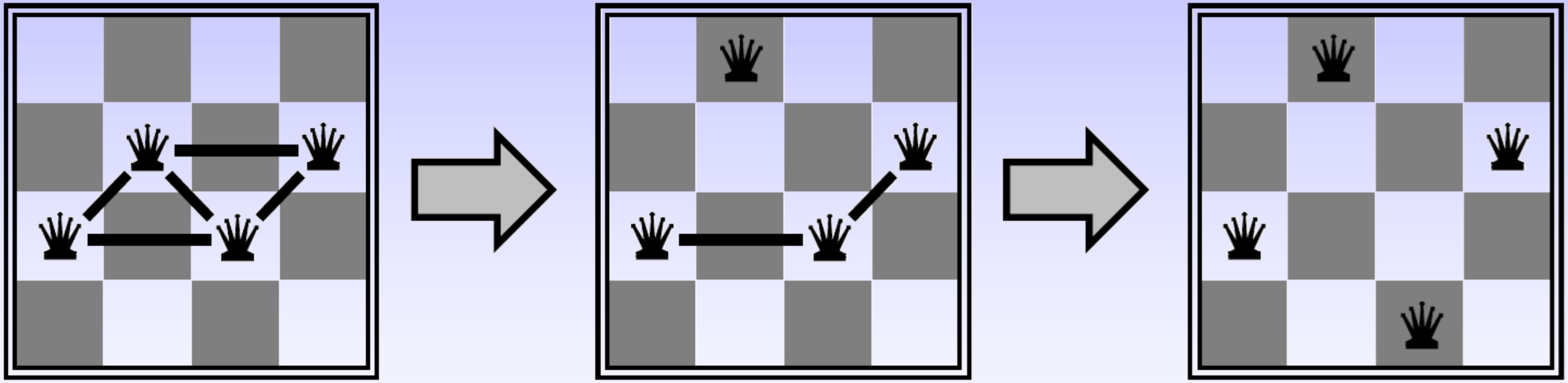
- Local Search Algorithms
  - Hill-climbing search
  - Simulated annealing search
  - Local beam search
  - Genetic algorithms (briefly)
- Searching with Nondeterministic Actions
- Searching with No Observation
- Searching with Partial Observations

Local Search Algorithms

- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution.
  - State space is set of "complete" configurations of a problem.
  - Target is to find a configuration that satisfy constraints.
  - For example, in *n*-queens problem, what matters is the final queens configuration, not the order in which they are added.
- In such cases, we can use **local search algorithms**.
  - Keep a single "current" state and try to improve it.
- Useful for solving pure optimization problems, in which the aim is to find the best state according to an objective function.

Example: *n*-queens

- Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal.
  – Move a queen to reduce number of conflicts

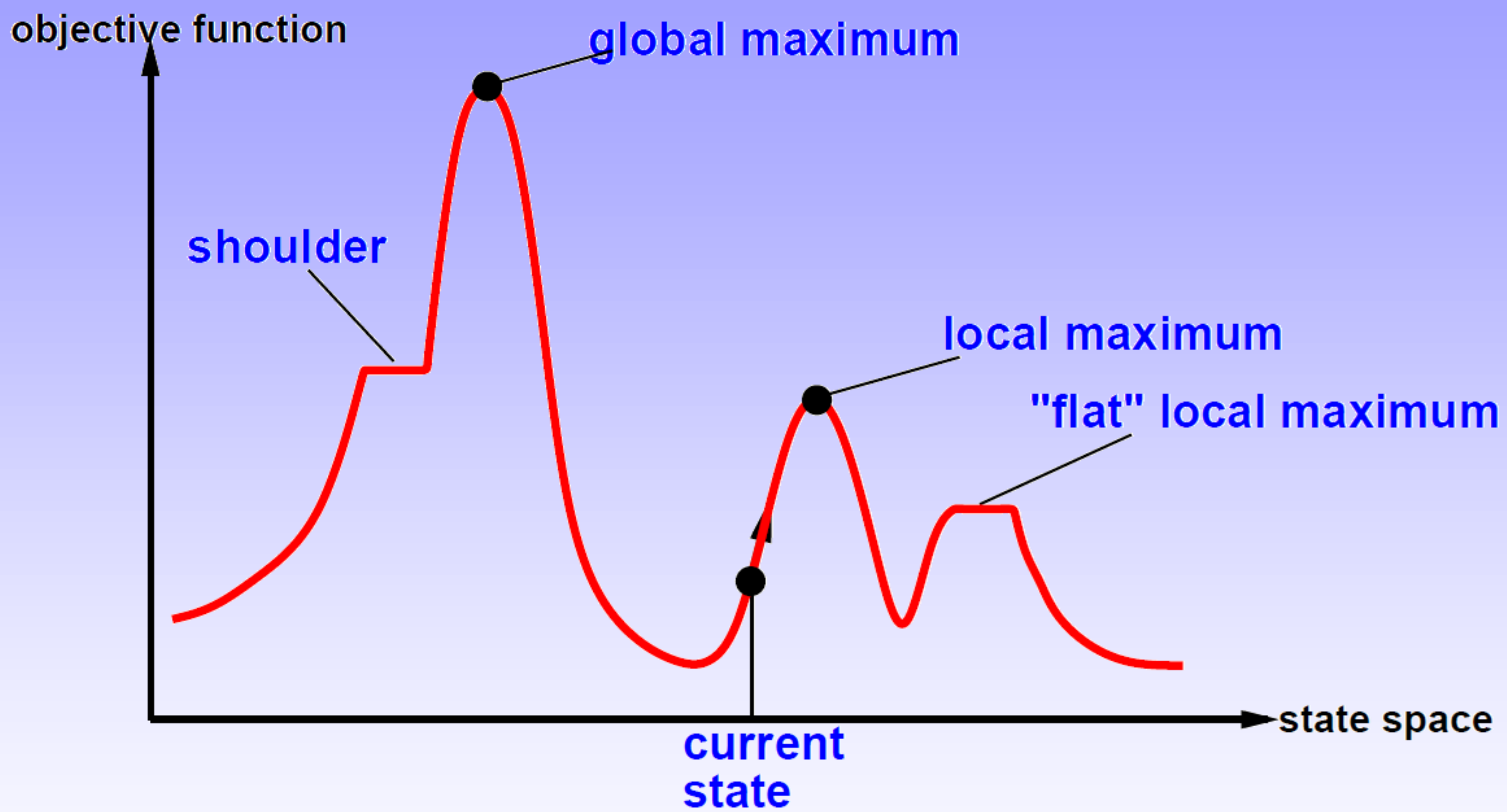


Initial state … Improve it … using local transformations

Local Search Algorithms

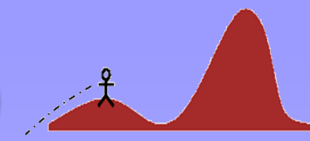
- To understand local search, it is helpful to consider the **state-space landscape** that contains the following:
  - "location" defined by the state
  - "elevation" defined by a heuristic cost or objective function
    - » If elevation corresponds to cost, then the aim is to find the lowest valley—***a global minimum***.
    - » If elevation corresponds to an objective function, then the aim is to find the highest peak—***a global maximum***.
- Local search algorithms explore this landscape.
  - A complete local search algorithm always finds a goal if one exists; an optimal algorithm always finds a global minimum/maximum.

State-space Landscape



- Problem: depending on initial state, can get stuck in local maxima.

Hill-climbing Search (HCS)

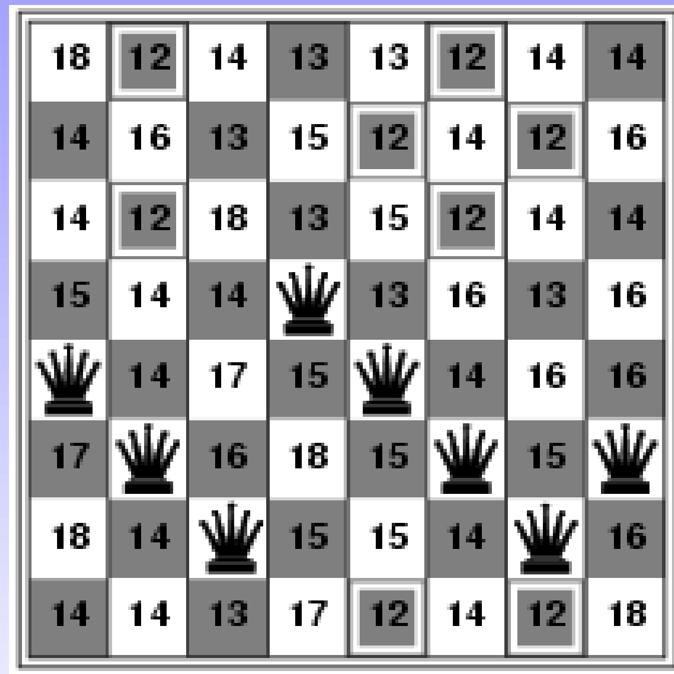


- "Like climbing Everest in thick fog with forgetfulness"

```
function HILL-CLIMBING(problem) returns a state that is a local maximum

  current ← MAKE-NODE(problem.INITIAL-STATE)
  loop do
     neighbor ← a highest-valued successor of current
     if neighbor.VALUE ≤ current.VALUE then return current.STATE
     current ← neighbor
```

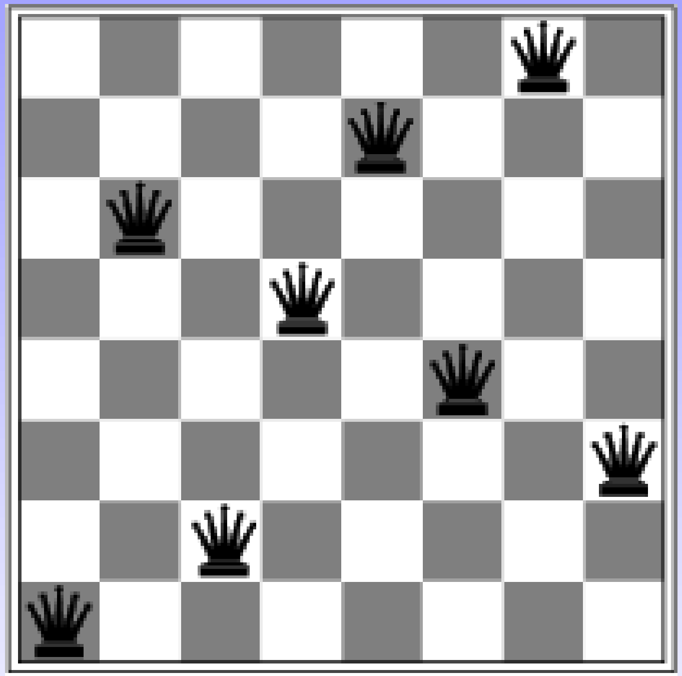
- Loop that continually moves in the direction of increasing value and terminates when it reaches a "peak" where no neighbor has a higher value.
  - Does not maintain a search tree
  - So, the current node need only record the state and the value of the objective function.

Hill-climbing Search: 8-queens Problem



- Heuristic cost function *h* = number of queen-pairs that are attacking each other, either directly or indirectly.
- *h* = 17 for the above state and best next is *h* = 12

Hill-climbing Search: 8-queens Problem

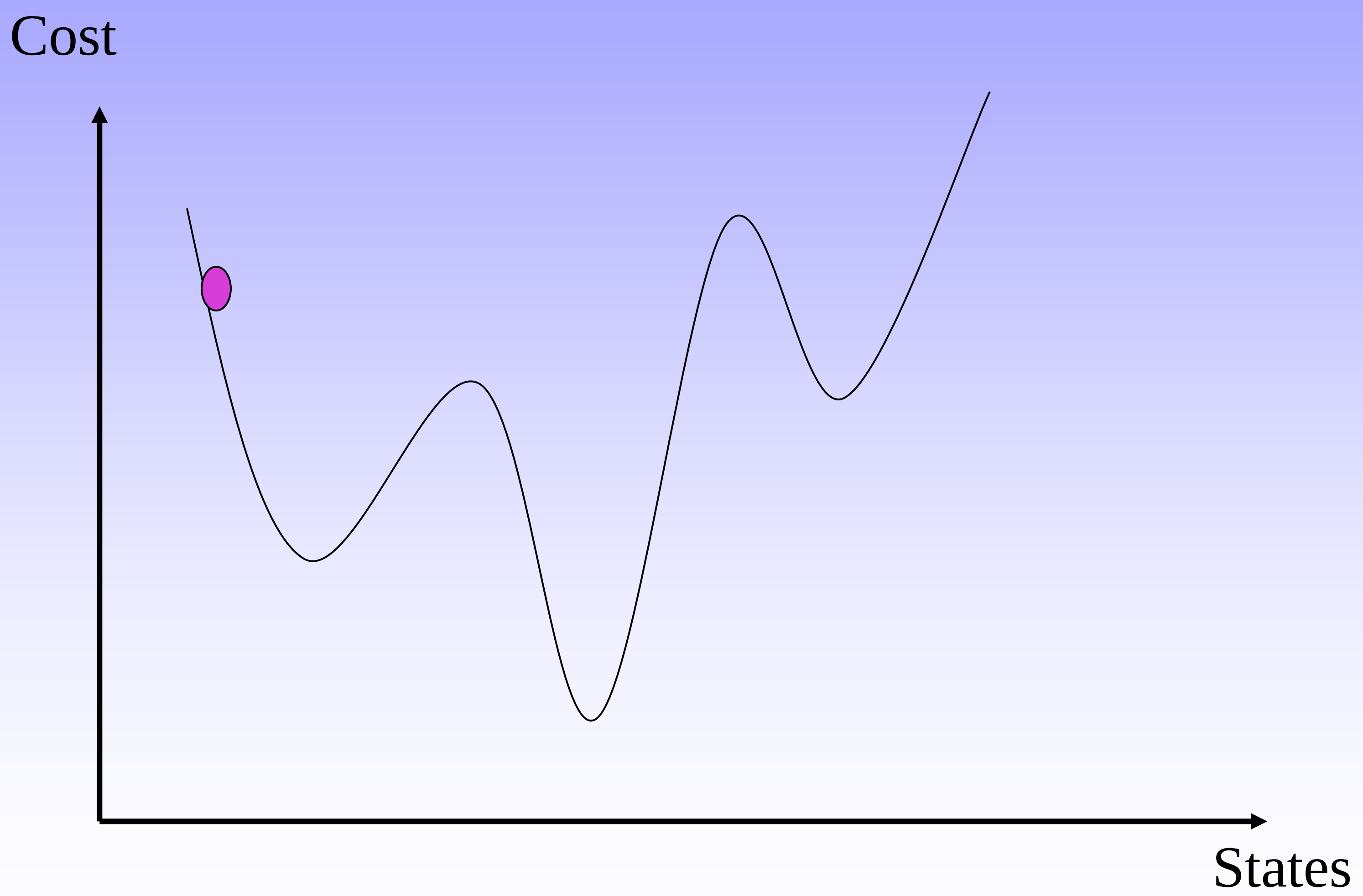


- A local minimum state with h = 1 but every successor has a higher cost.
- Therefore, hill climbing is sometimes called **greedy local search**.

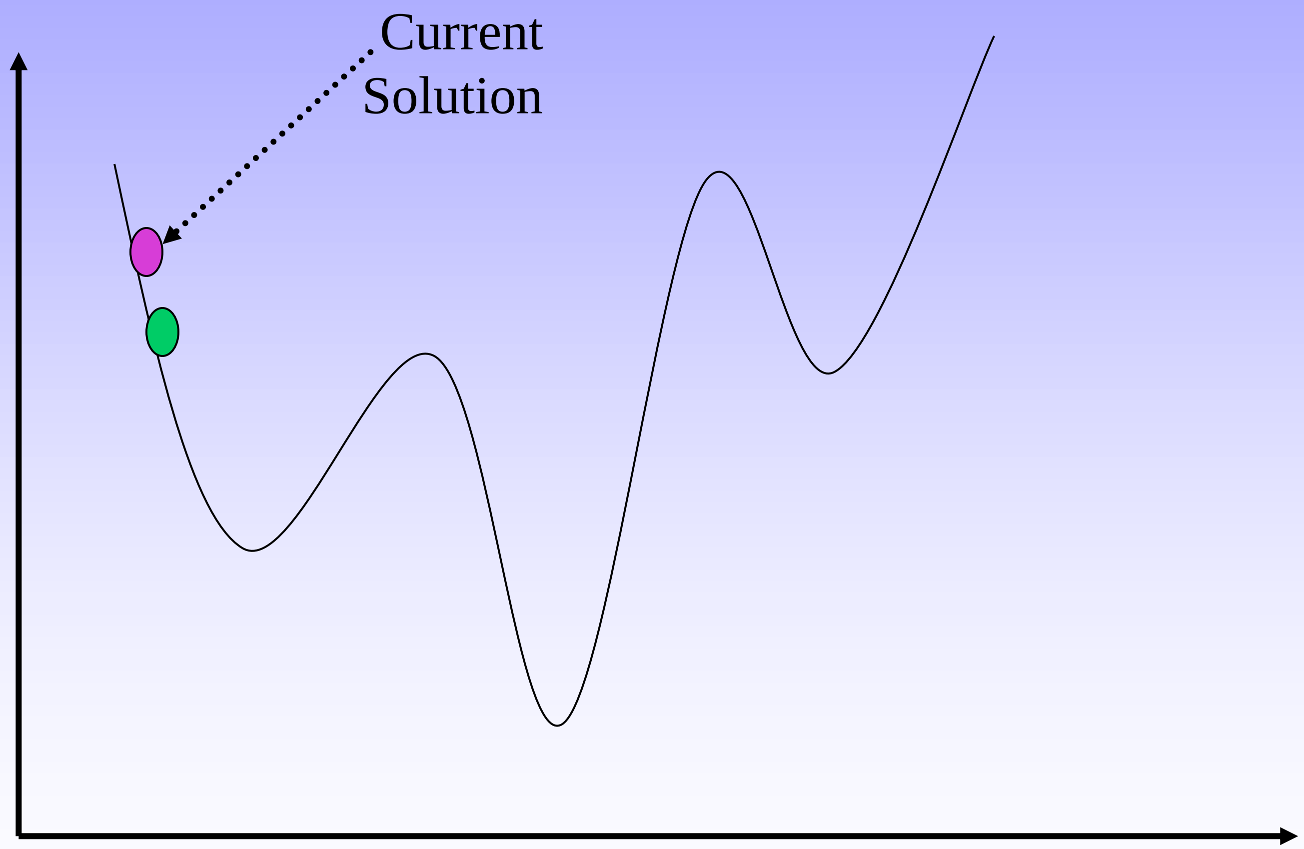
Hill-climbing Search

- HCS often makes rapid progress toward a solution because it is usually quite easy to improve a bad state.
- But, HCS frequently gets stuck when reaches a point at which no progress is being made for the following reasons:
  - **Local maxima**: a local maximum is a peak that is higher than each of its neighboring states but lower than the global maximum.
  - **Ridges**: result in a sequence of local maxima that is very difficult for greedy algorithms to navigate.
  - **Plateaux**: can be a flat local maximum with no uphill exit exists, or shoulder, from which progress is possible.

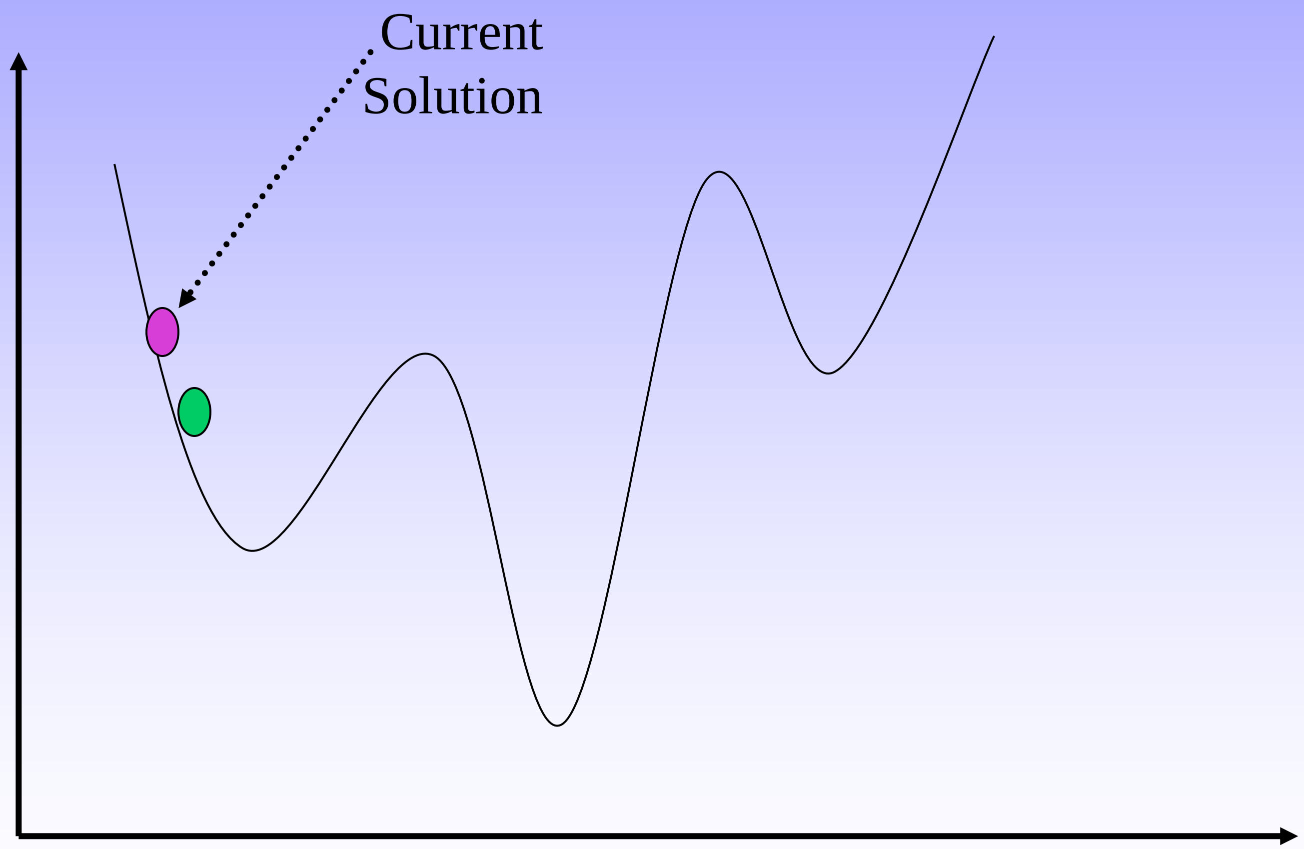
Hill-climbing Search



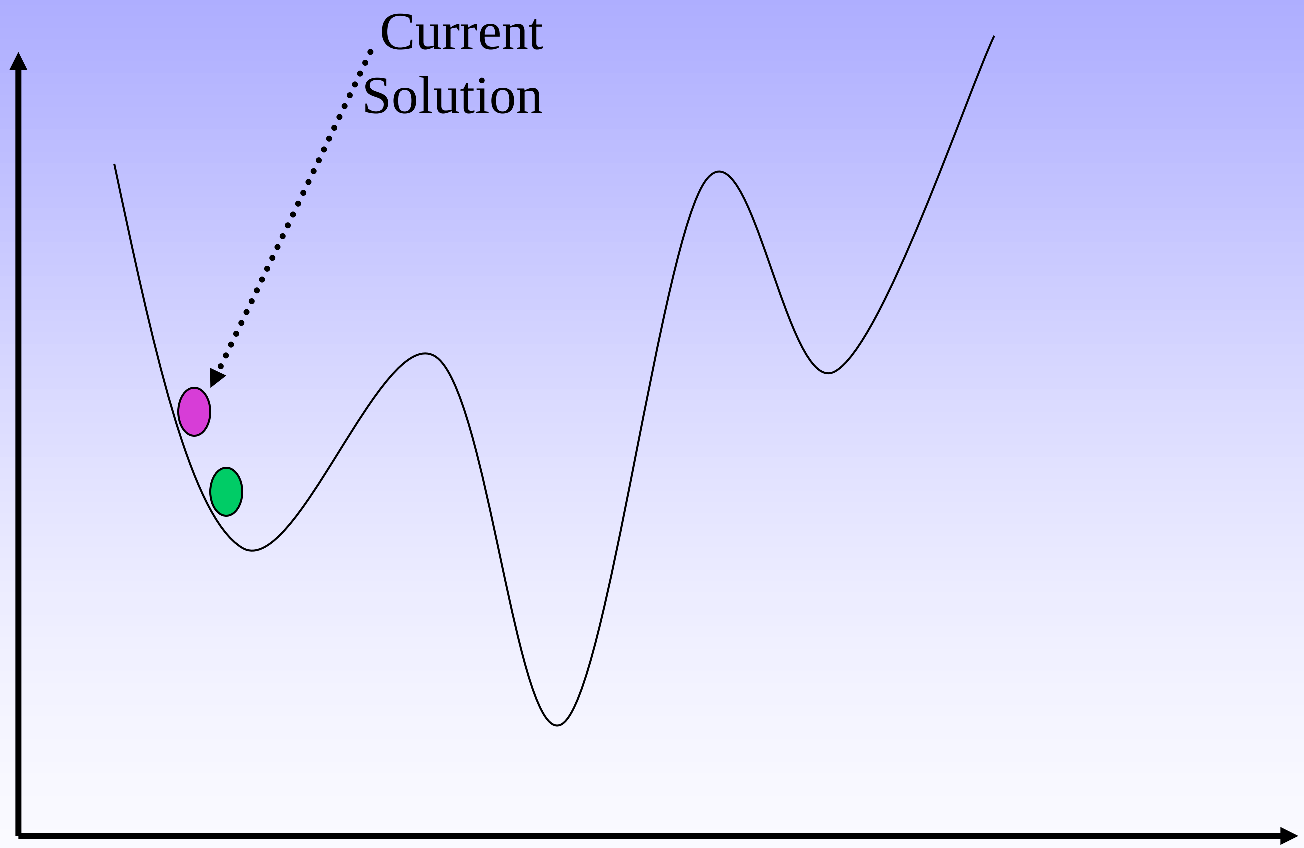
Hill-climbing Search



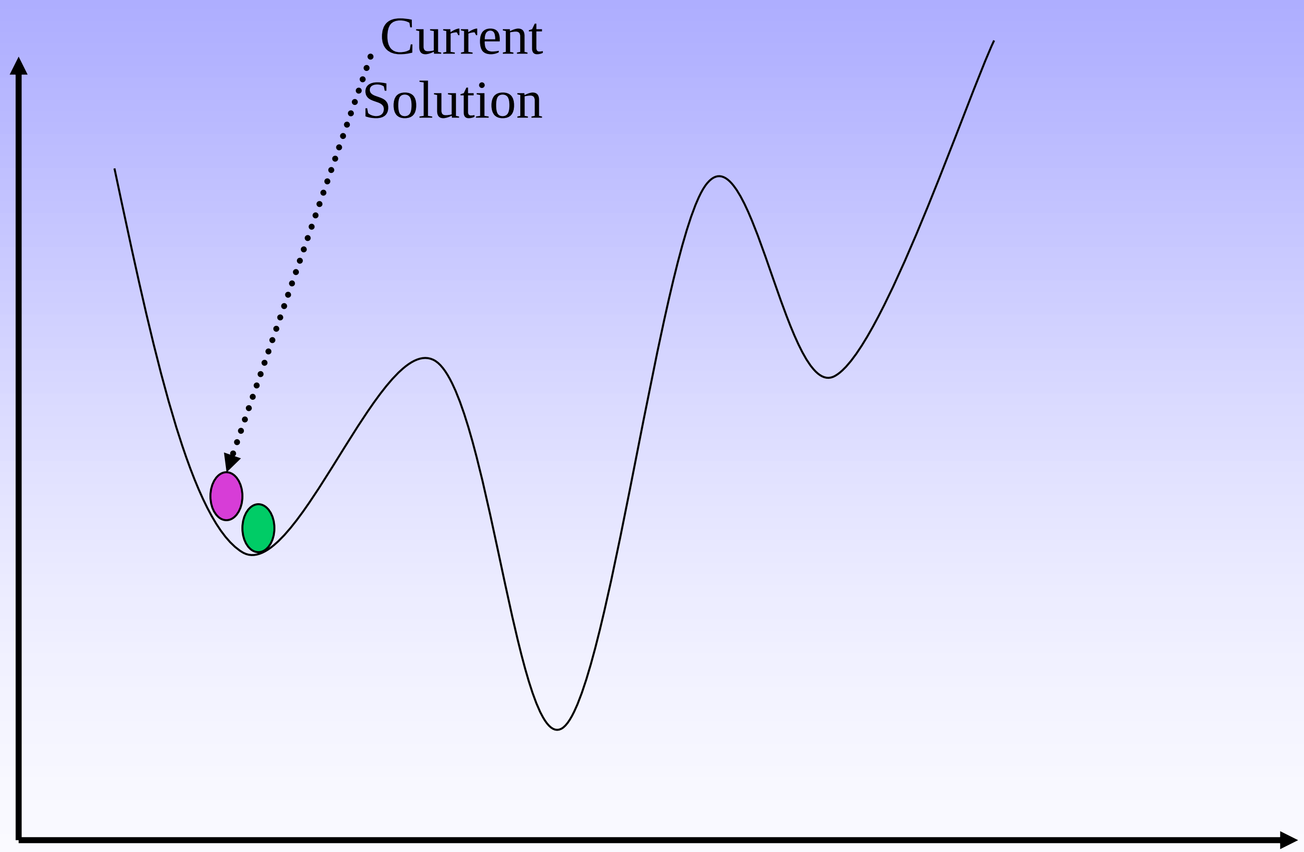
Hill-climbing Search



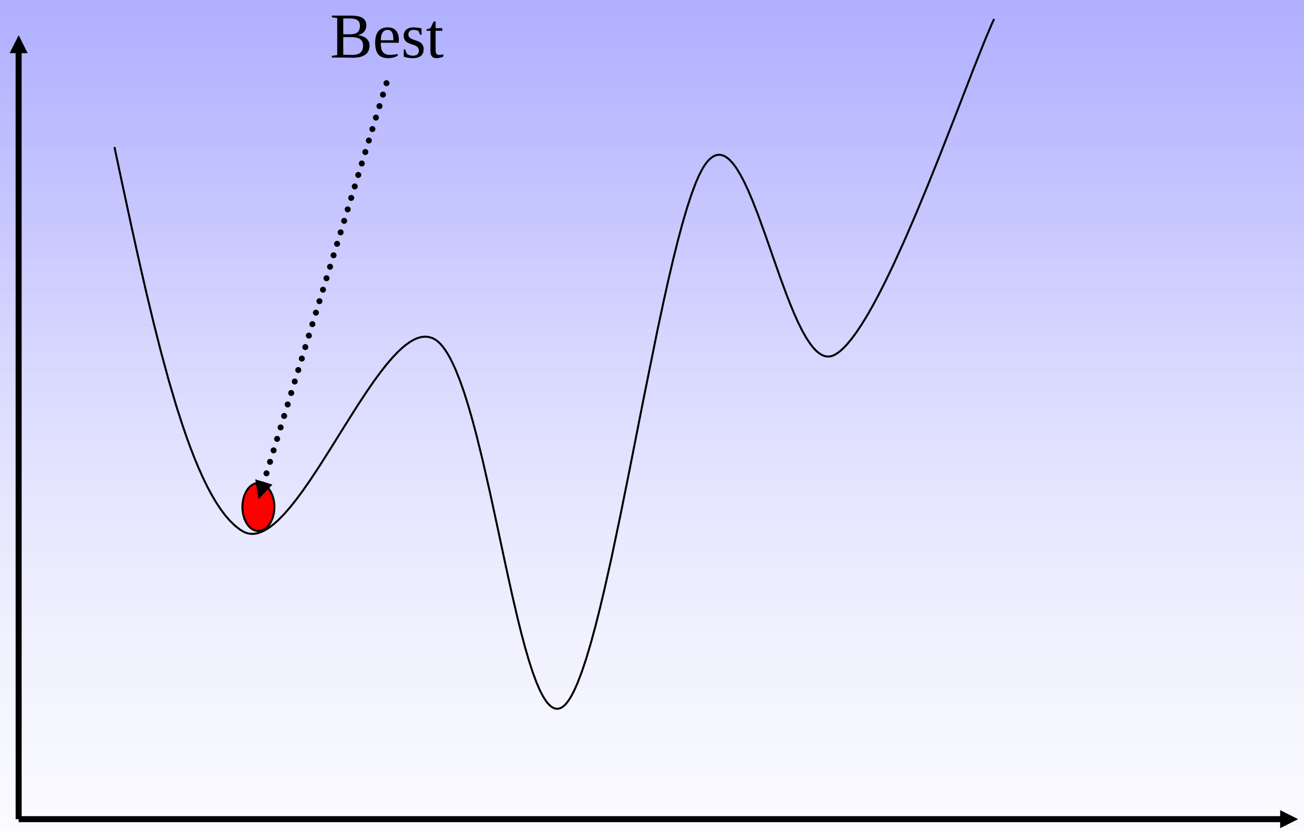
Hill-climbing Search



Hill-climbing Search



Hill-climbing Search



Simulated Annealing Search

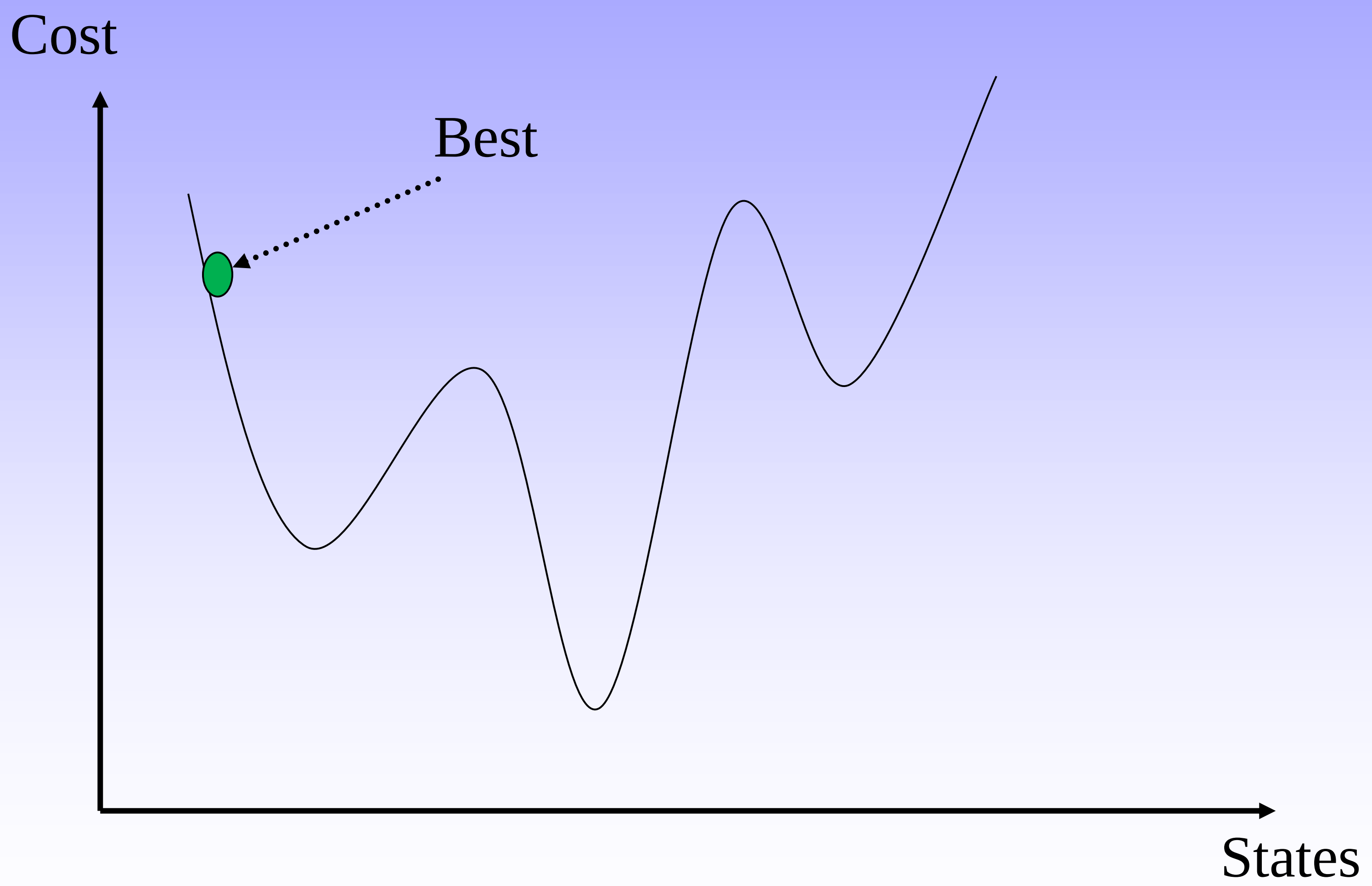
- HCS algorithm is incomplete and can get stuck on a local maximum because it *never* makes downhill moves toward states with lower value (or higher cost).
- Key idea: escape local maxima by allowing some "bad" moves but **gradually decrease** their size and frequency.
  - Instead of picking the best move, it picks a *random* move.
  - Take some downhill steps to escape the local maximum.
  - If the move improves the situation, it is always accepted.
  - Else, it accepts the move with some probability less than 1.
- Physical analogy with annealing process to harden metals:
  - Heating them to a high temp. and then gradually cooling them
  - The heuristic value is the energy, E
  - Temperature parameter, T, controls speed of convergence.

Simulated Annealing Search

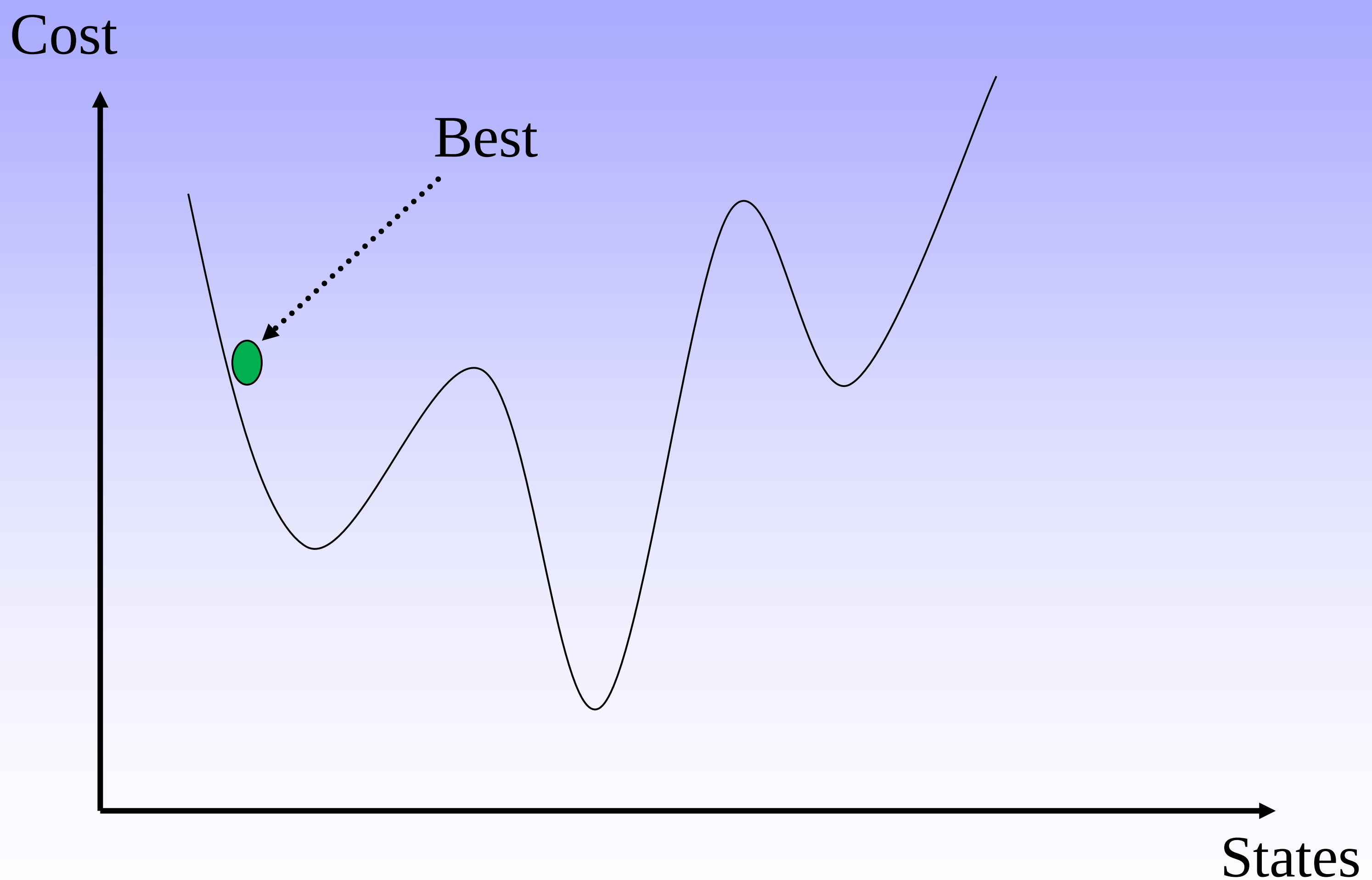
```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
  inputs: problem, a problem
          schedule, a mapping from time to "temperature"

  current ← MAKE-NODE(problem.INITIAL-STATE)
  for t = 1 to ∞ do
      T ← schedule(t)
      if T = 0 then return current
      next ← a randomly selected successor of current
      ΔE ← next.VALUE – current.VALUE
      if ΔE > 0 then current ← next
      else current ← next only with probability e^(ΔE/T)
```

- Can prove: If T decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1.
- Widely used in VLSI layout, airline scheduling, etc.

Simulated Annealing Search



Simulated Annealing Search



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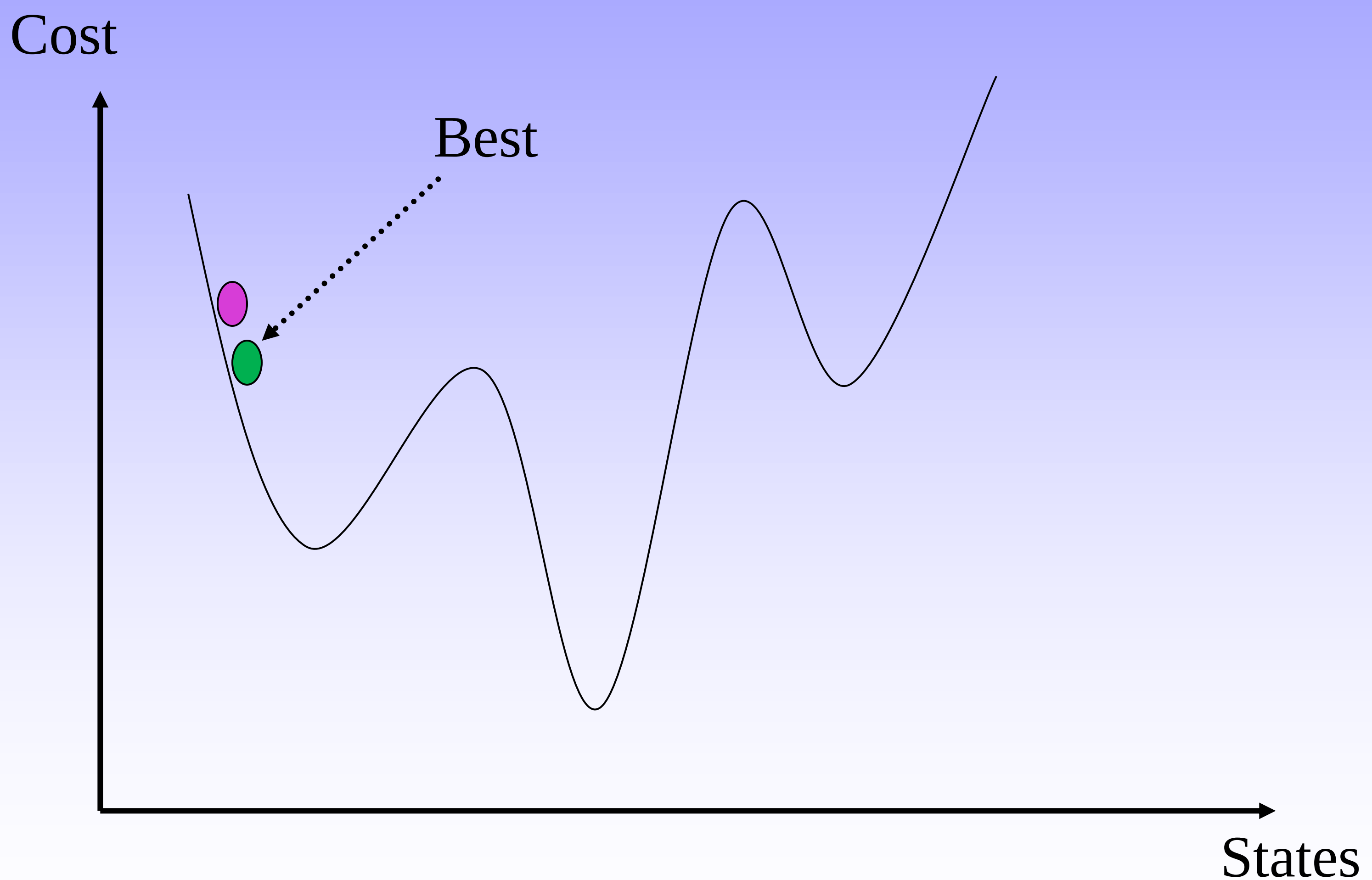

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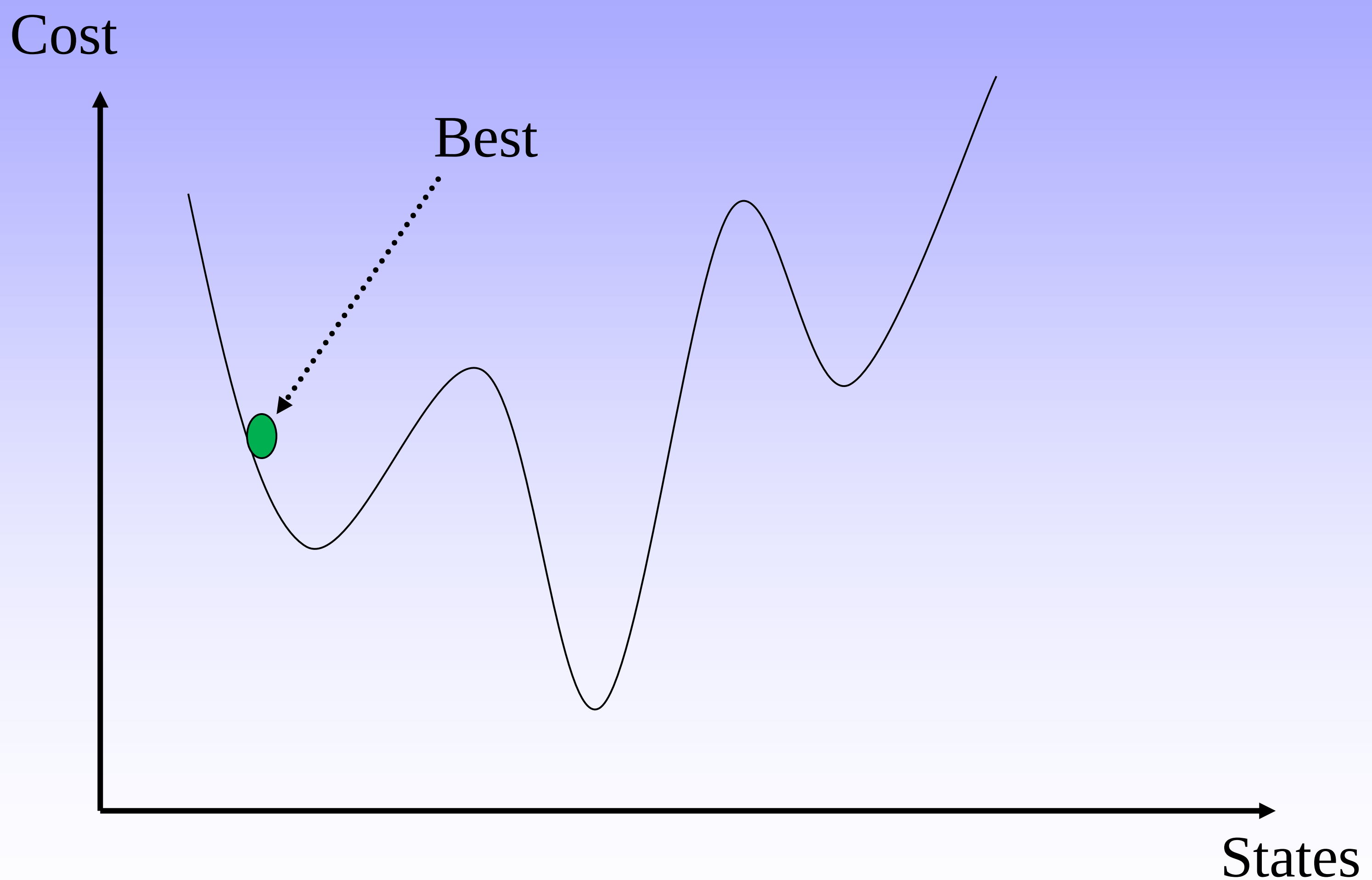

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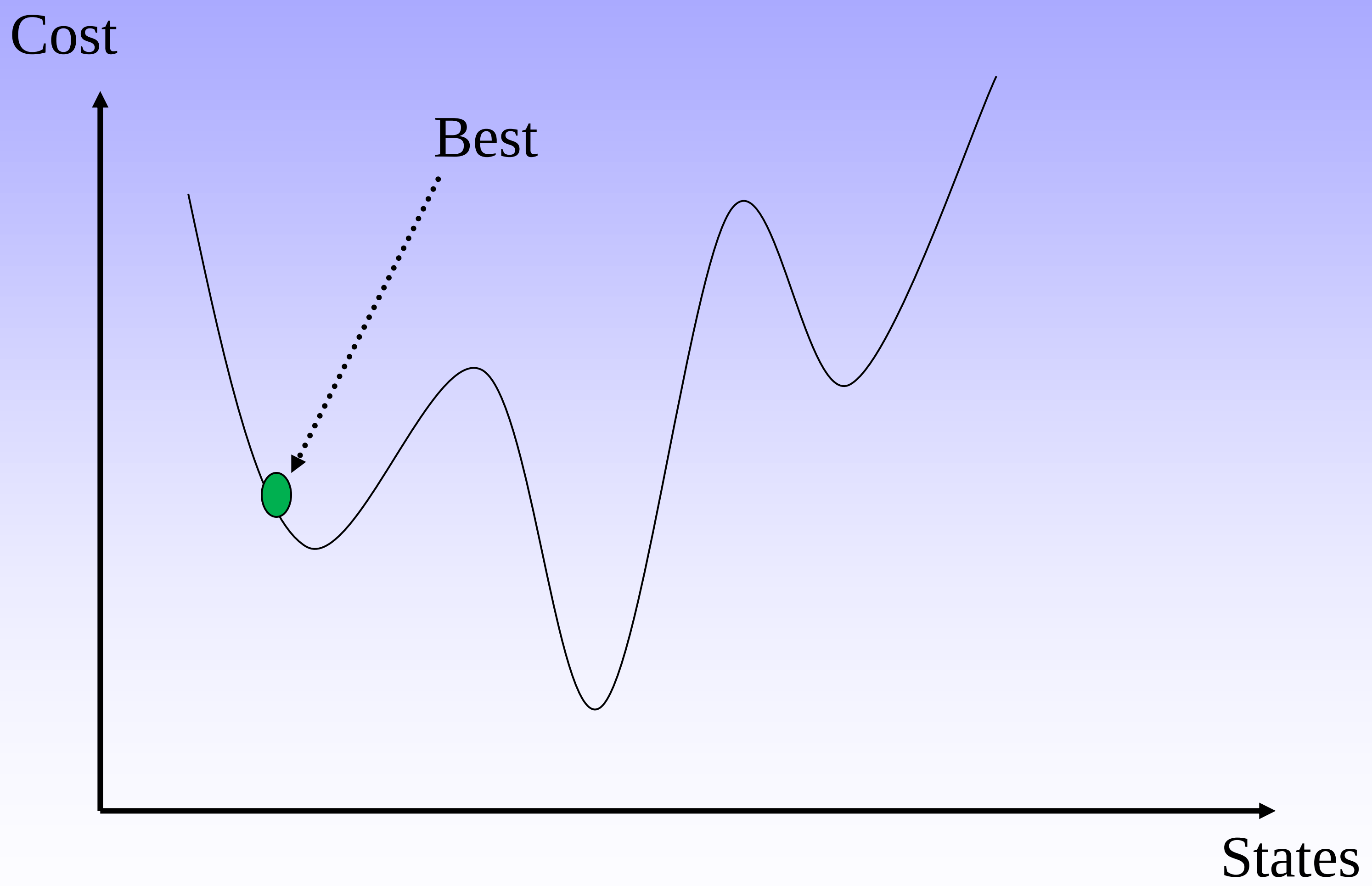

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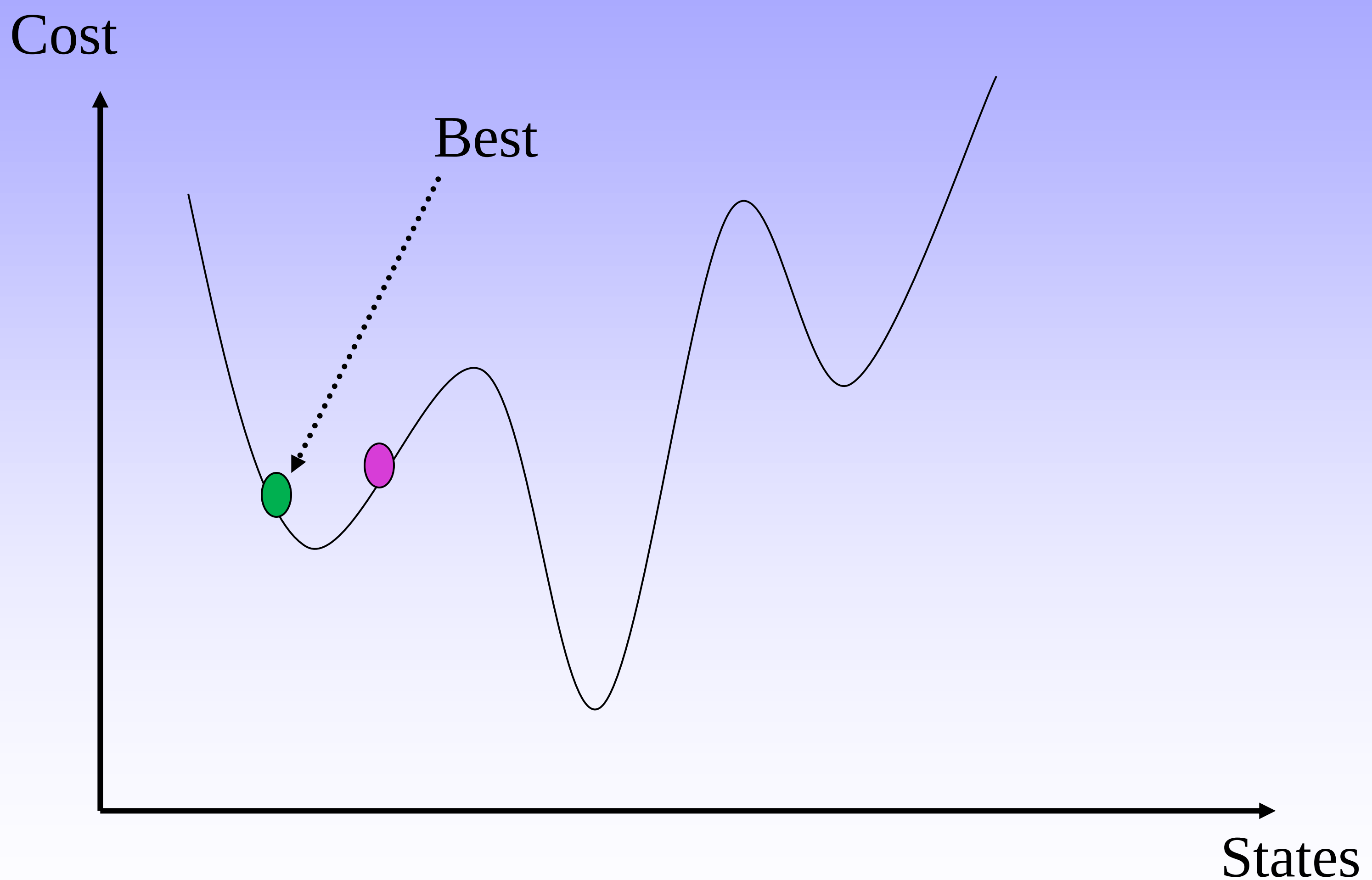

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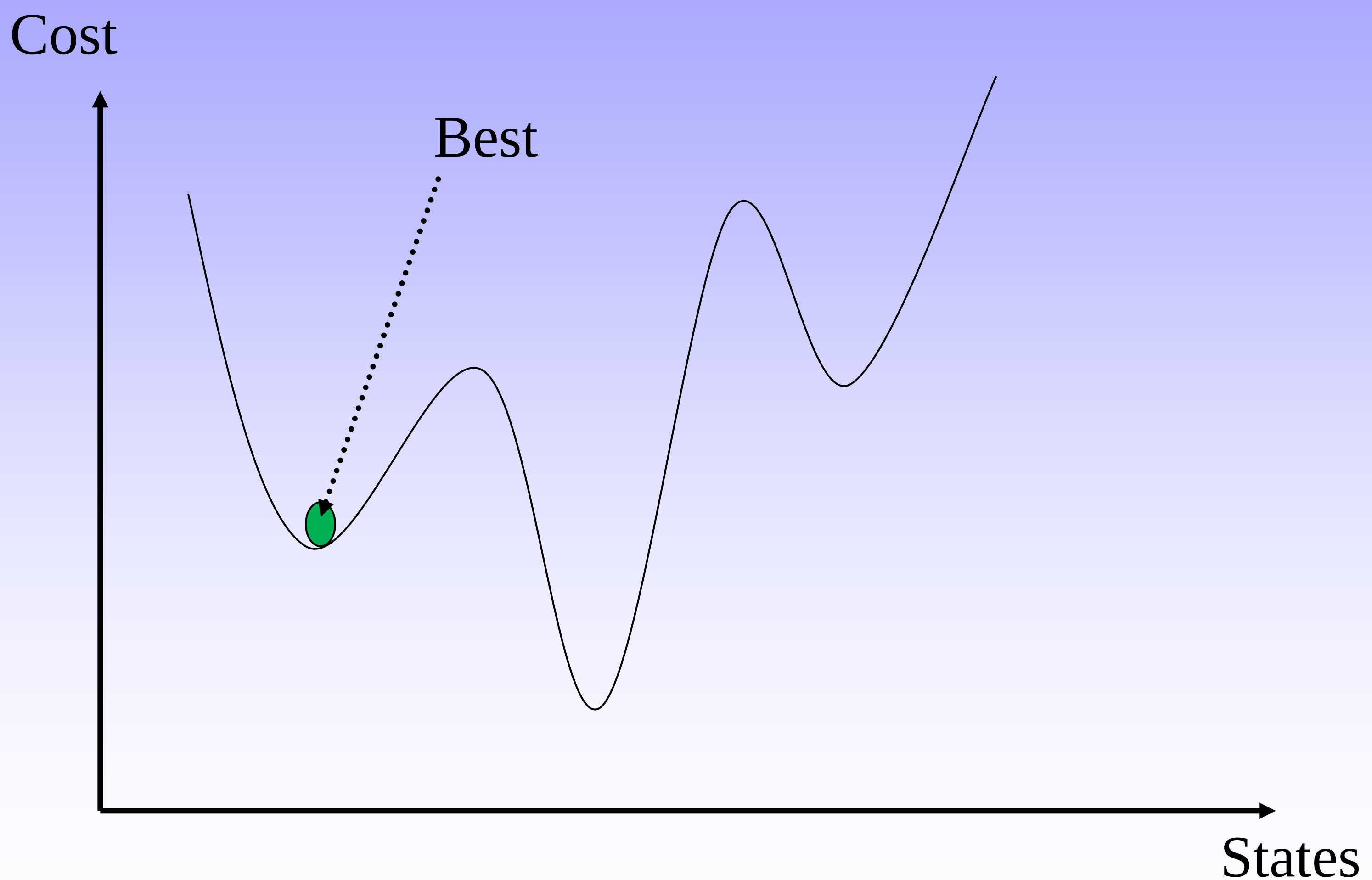

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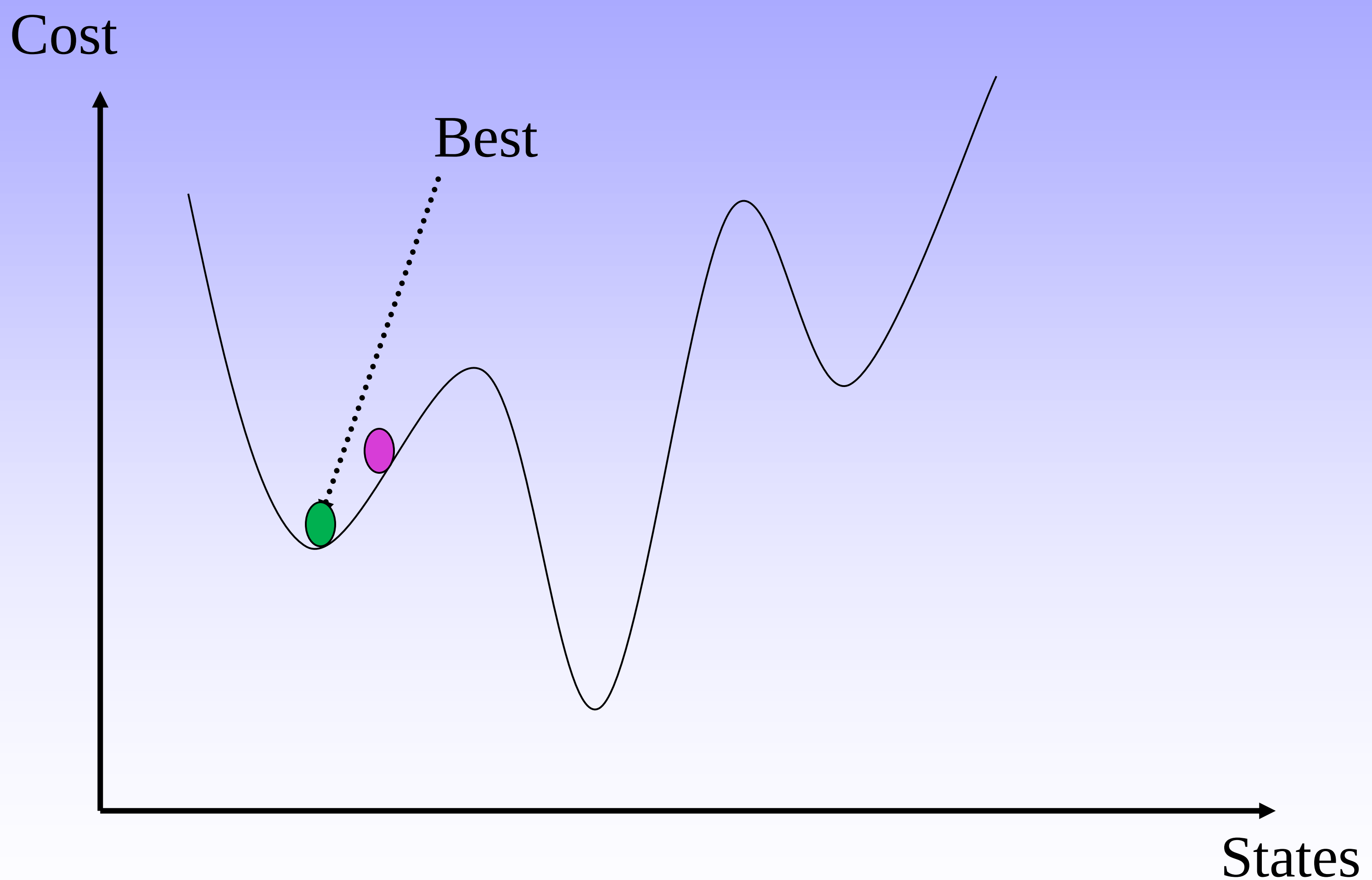

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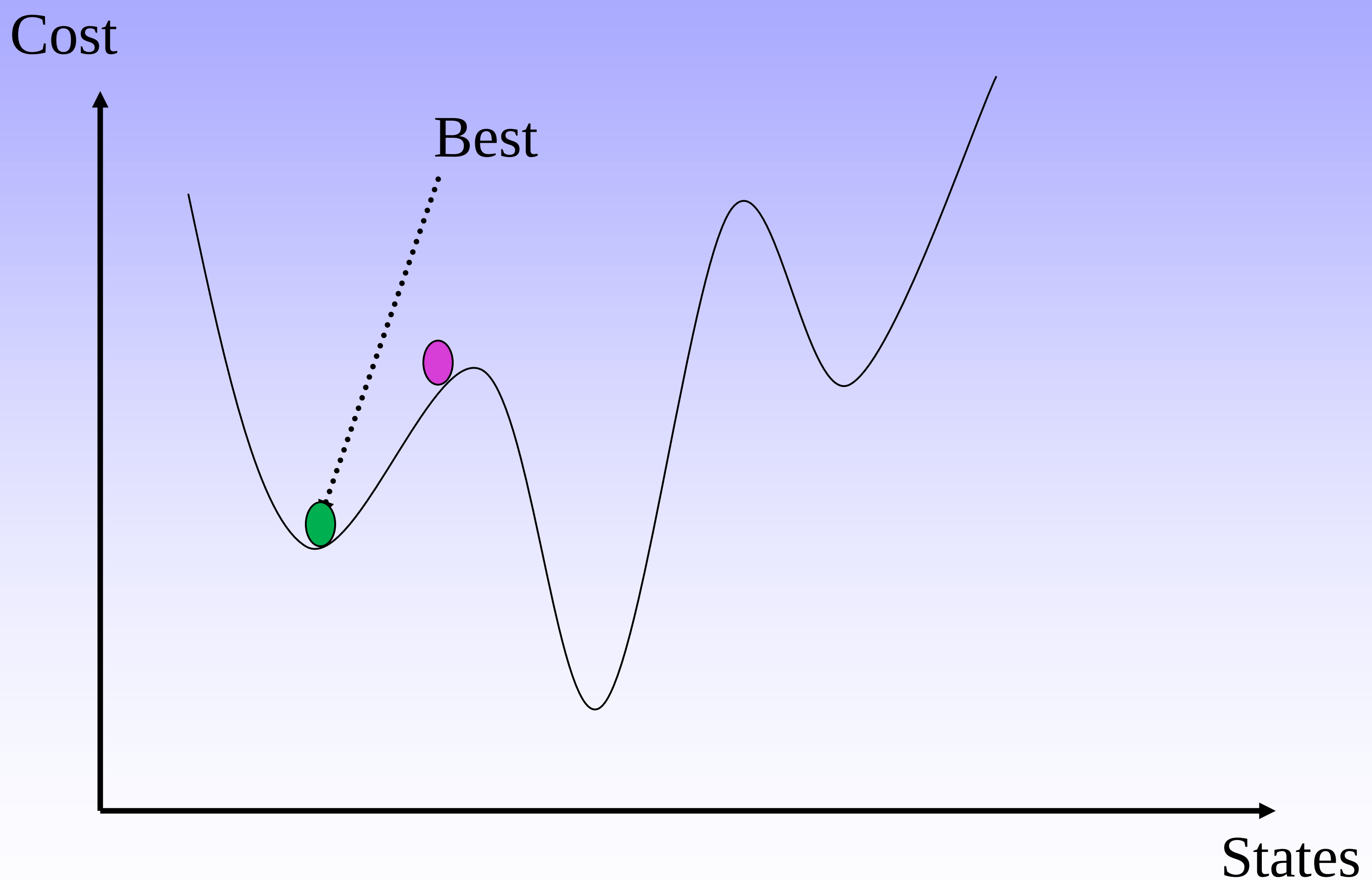

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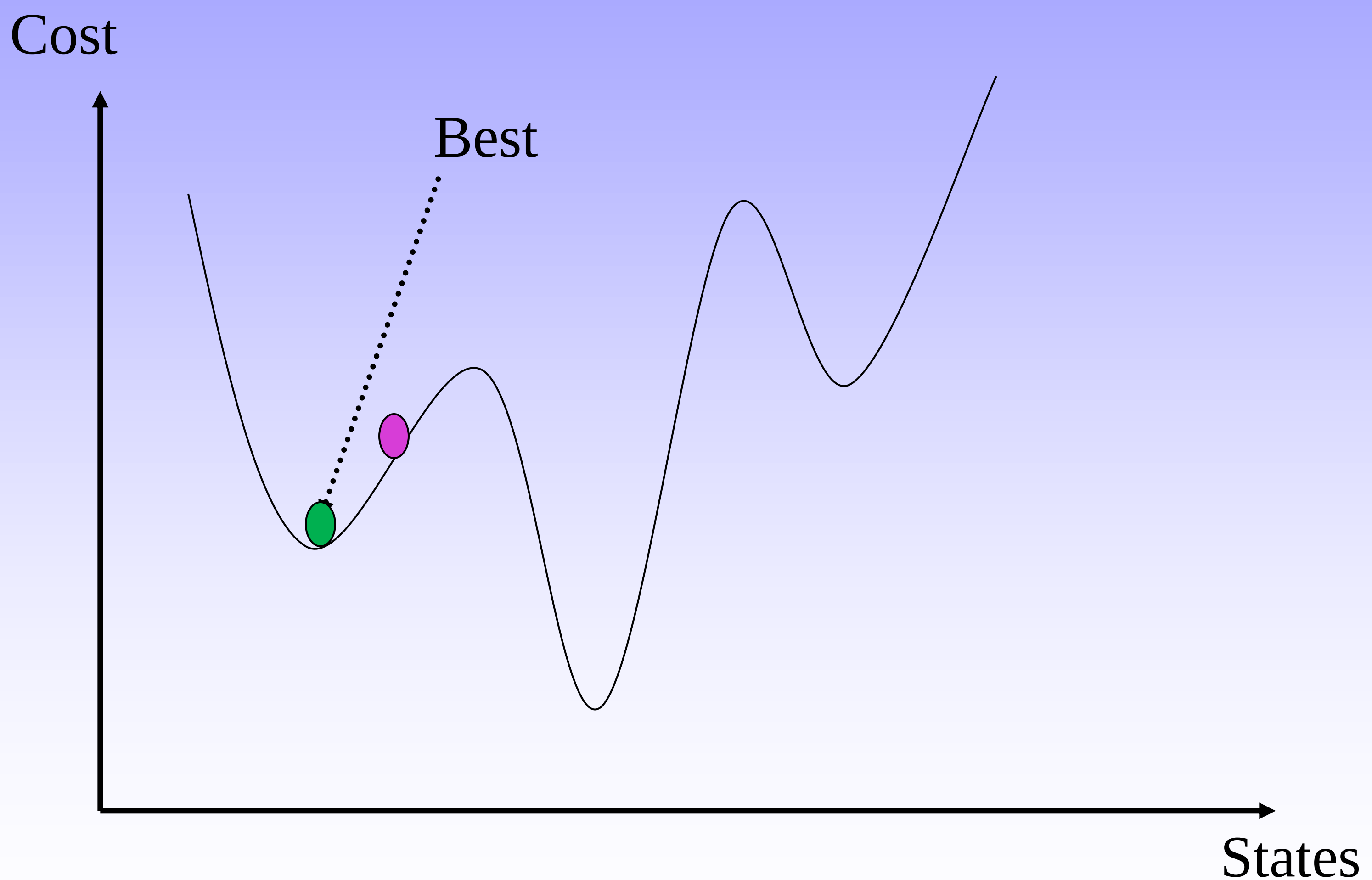

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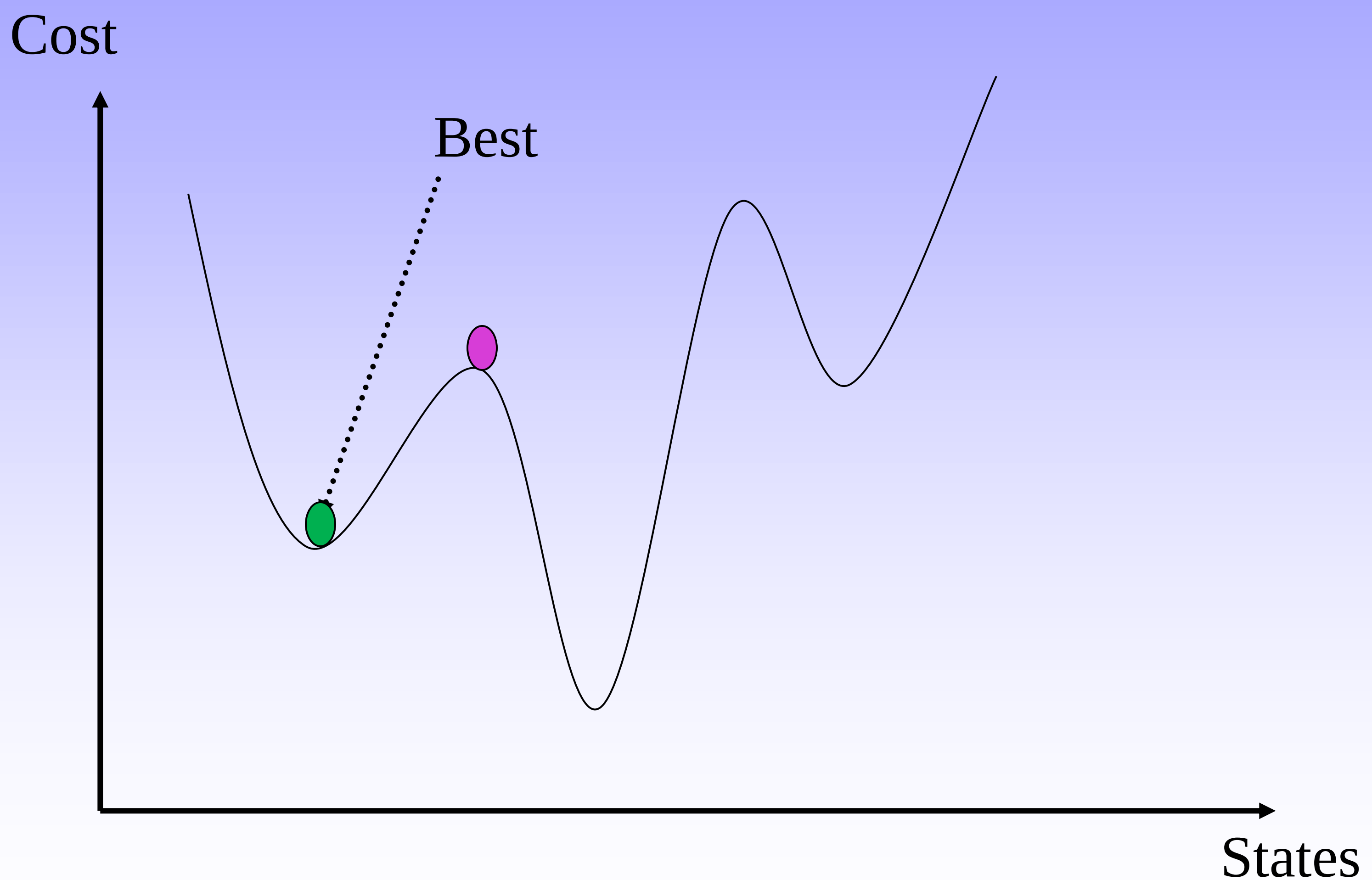

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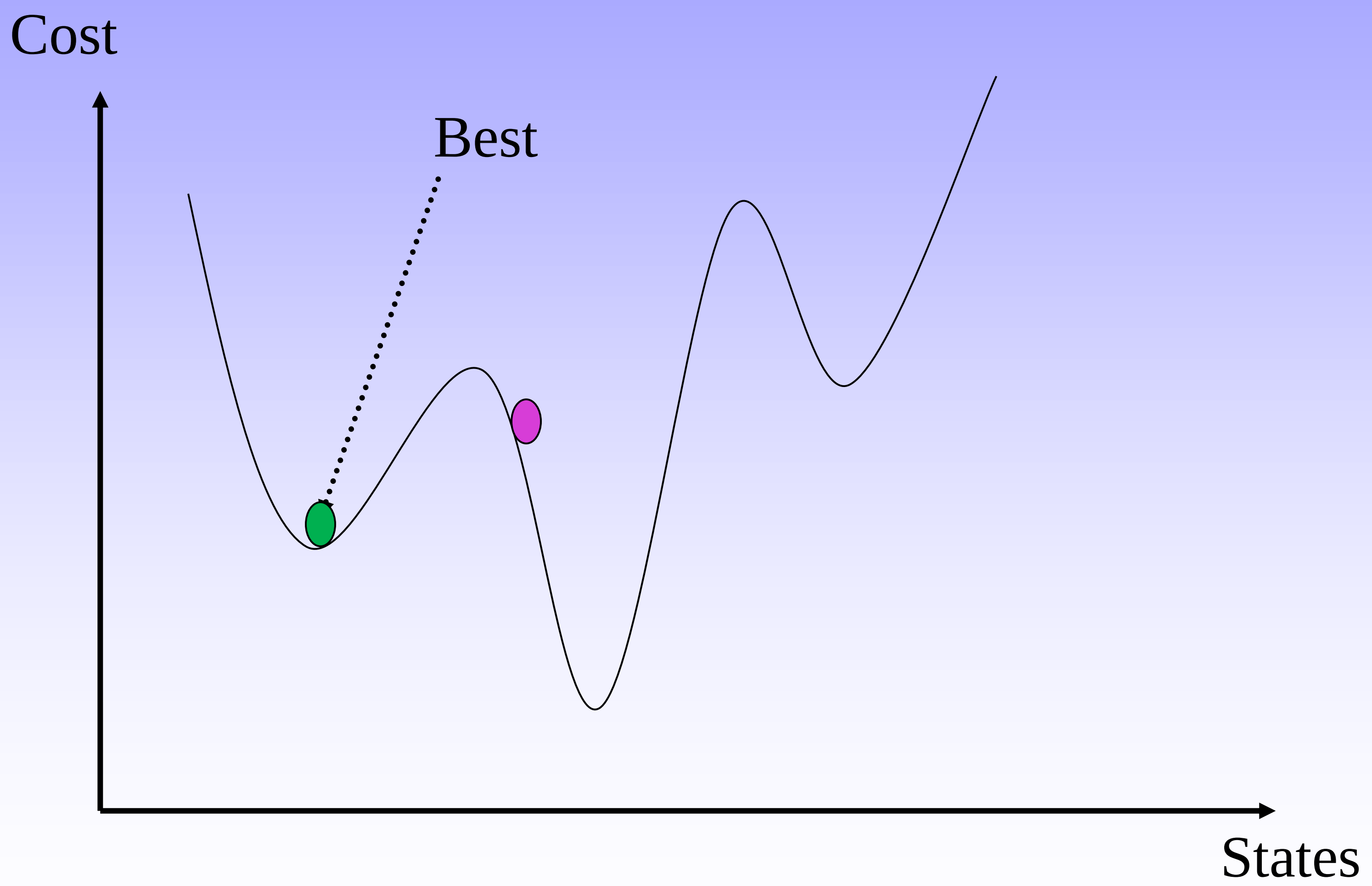

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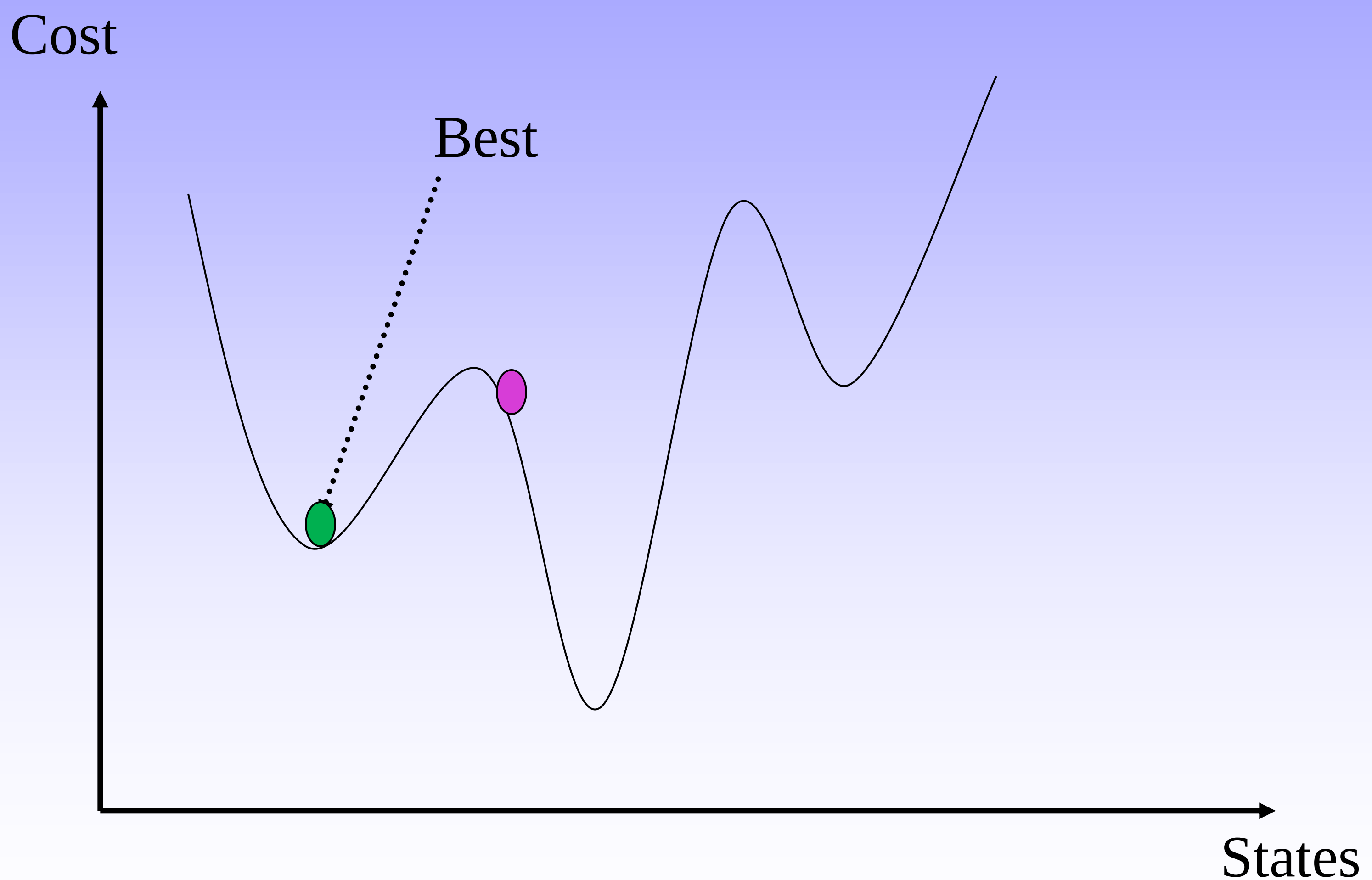

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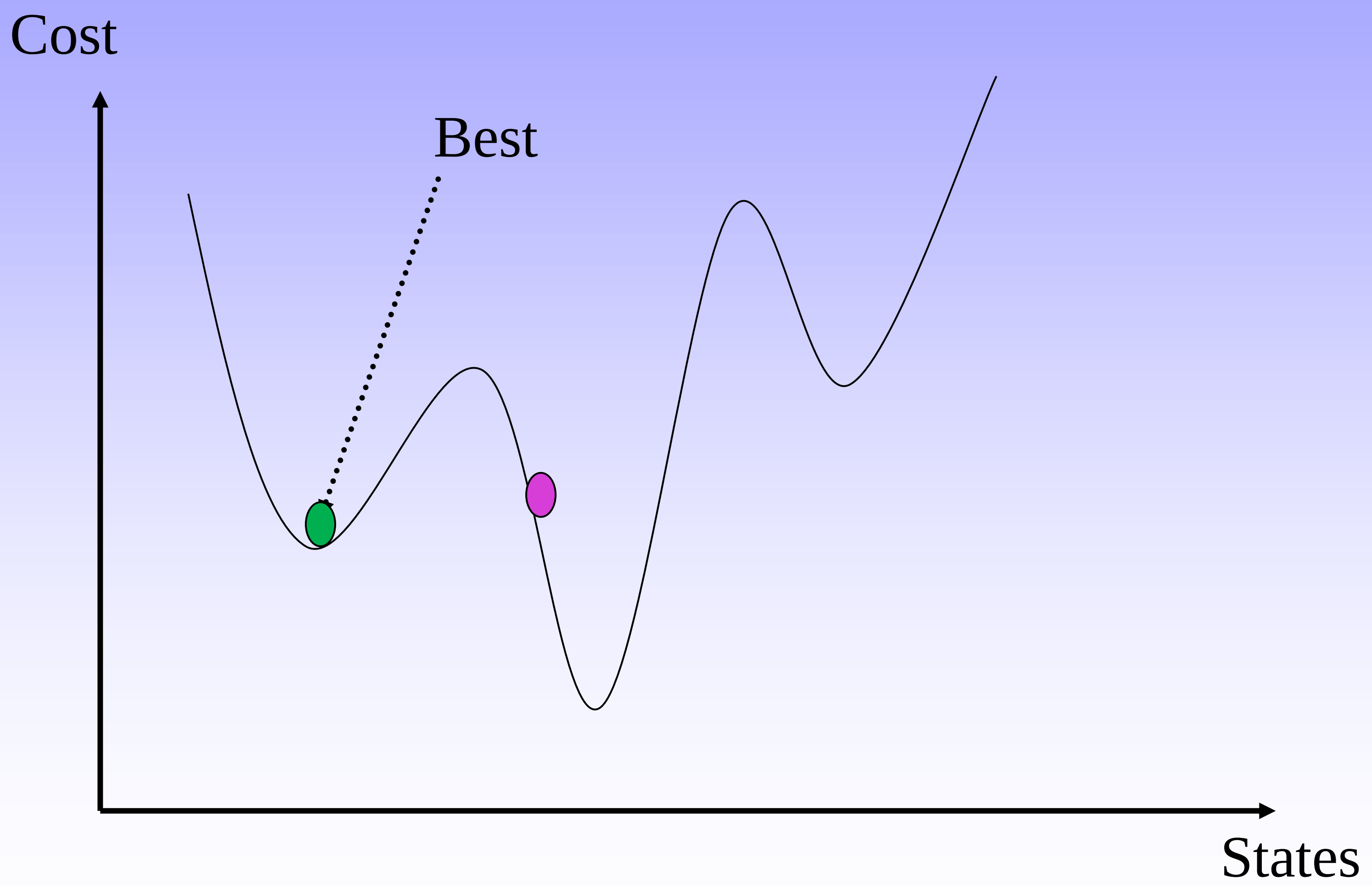

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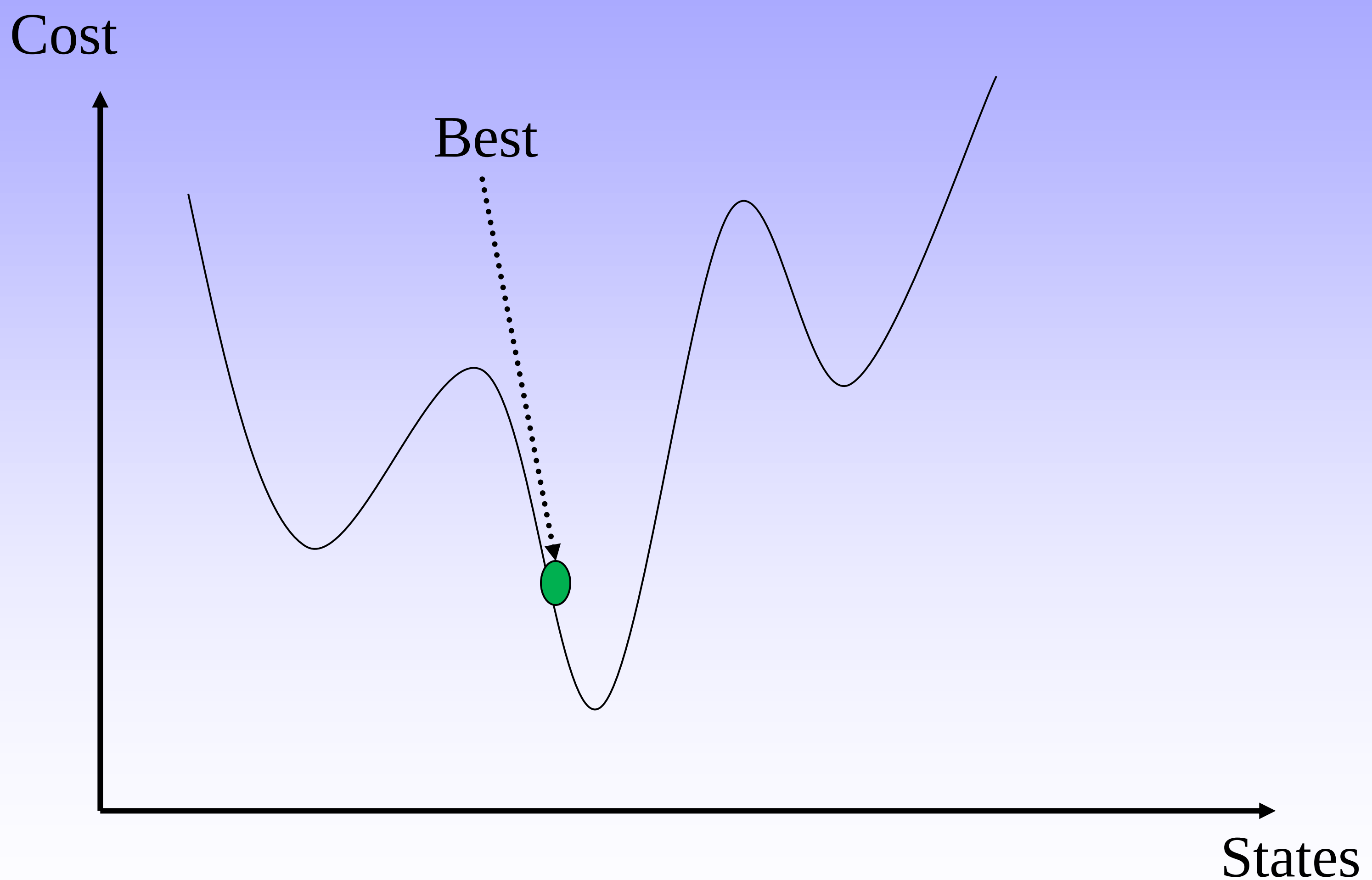

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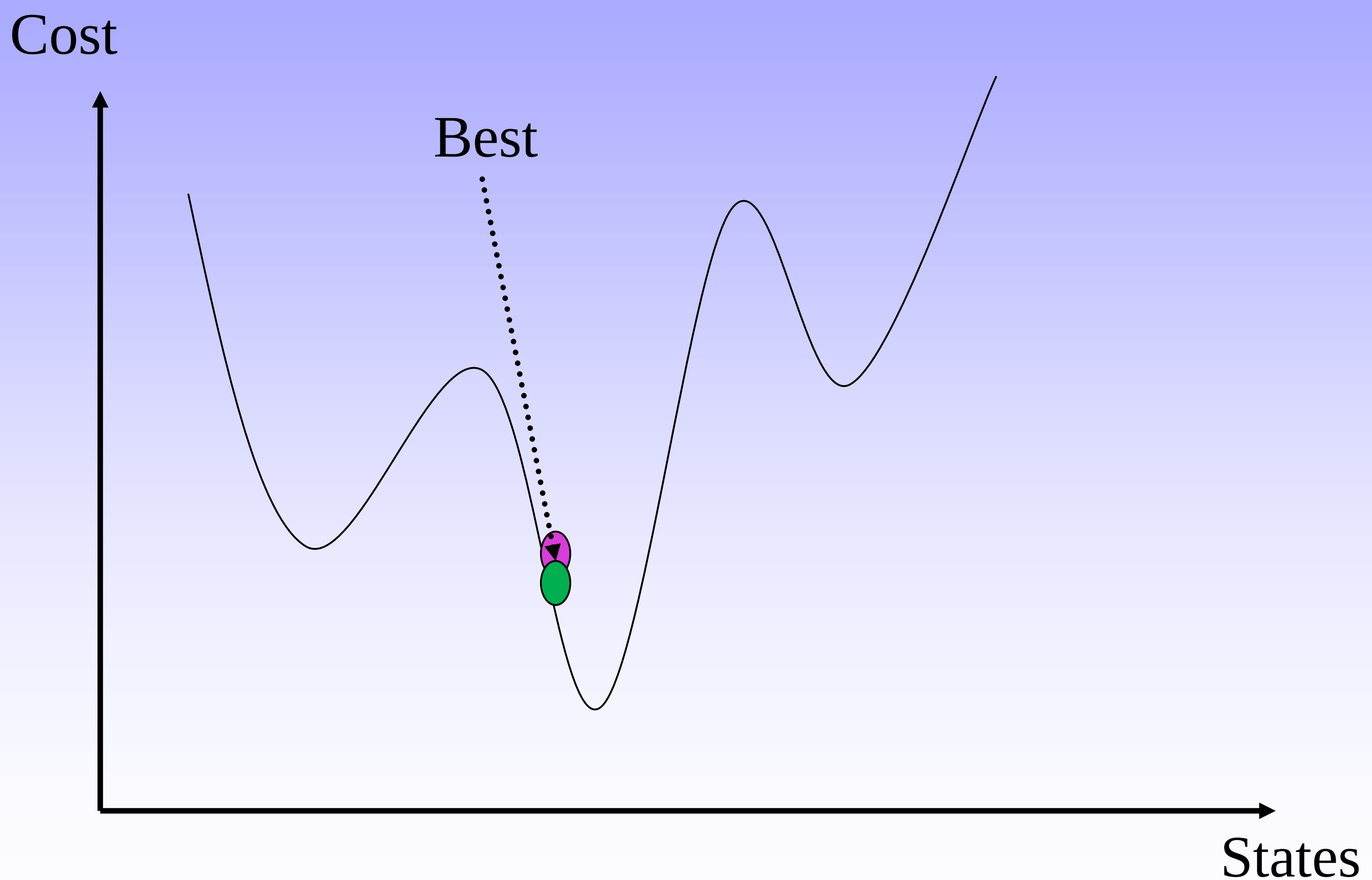

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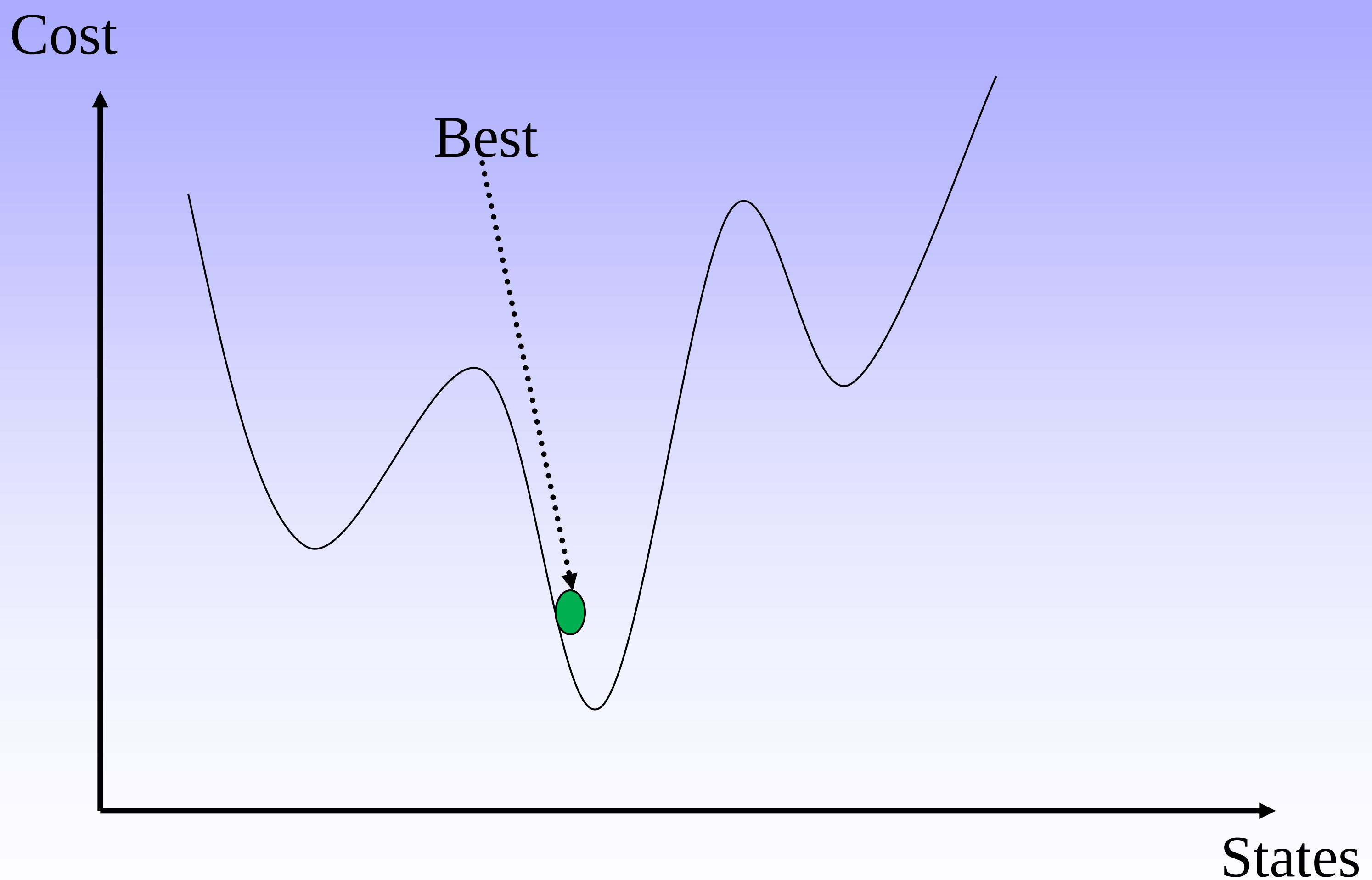

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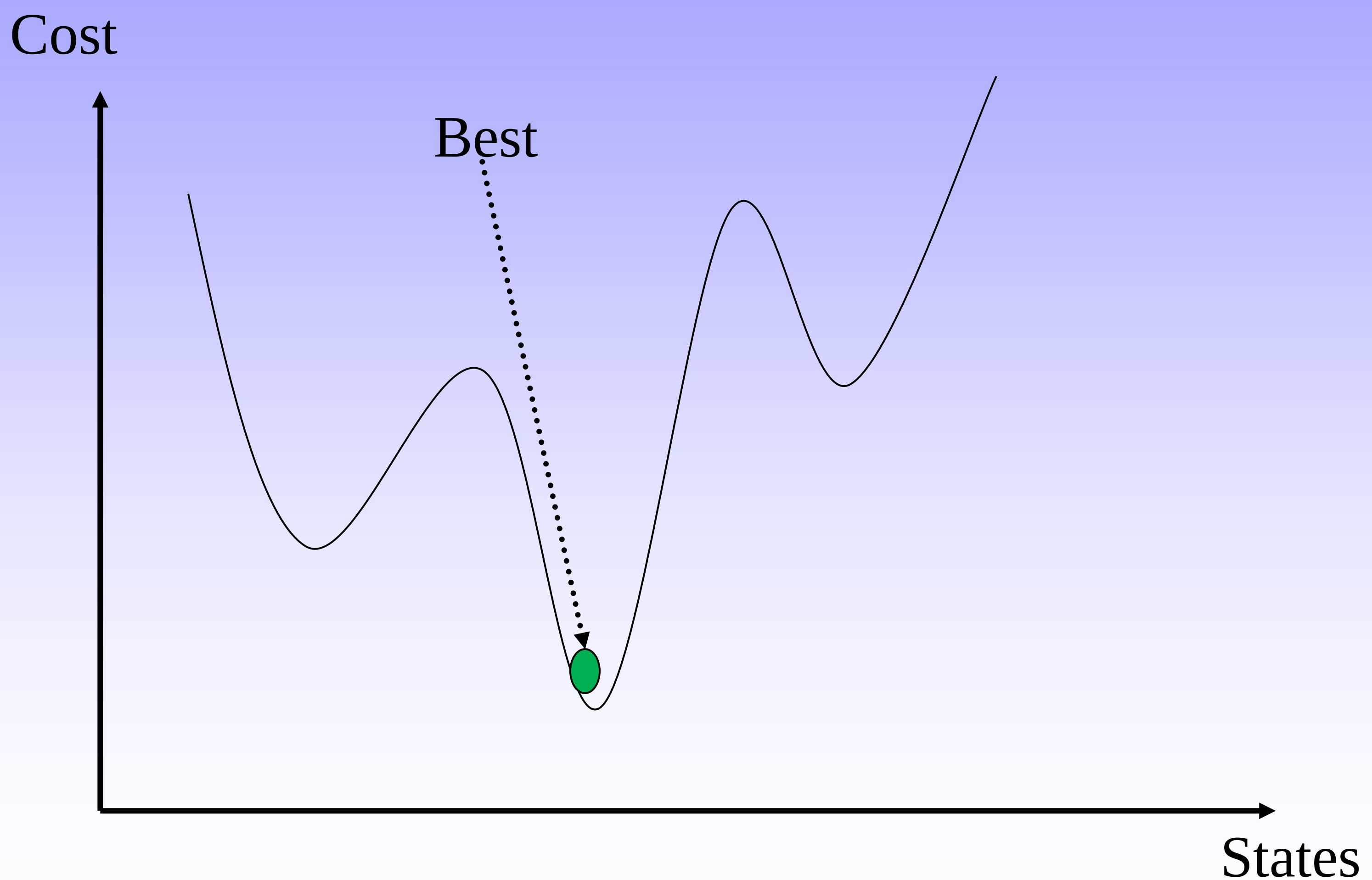

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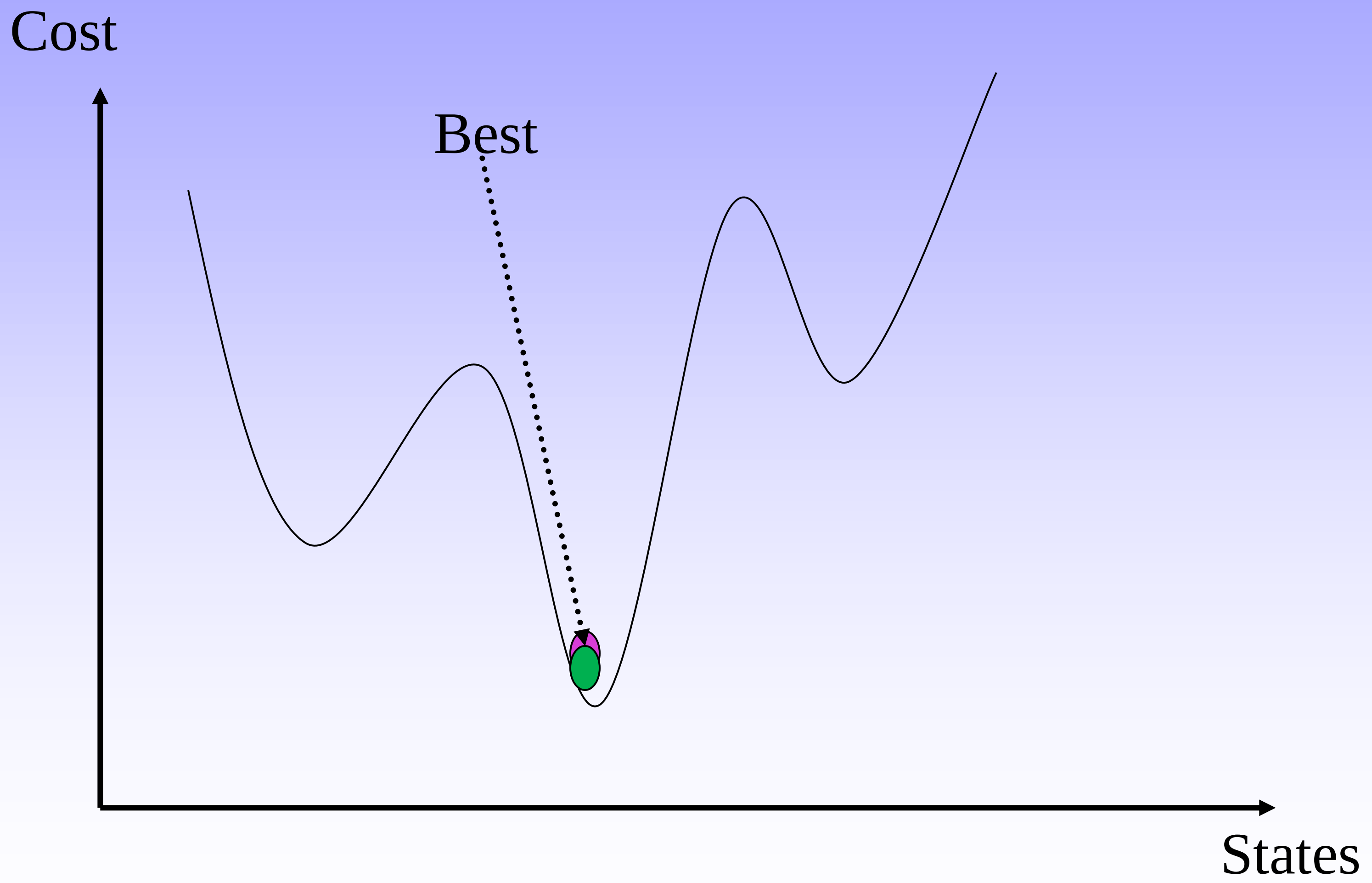

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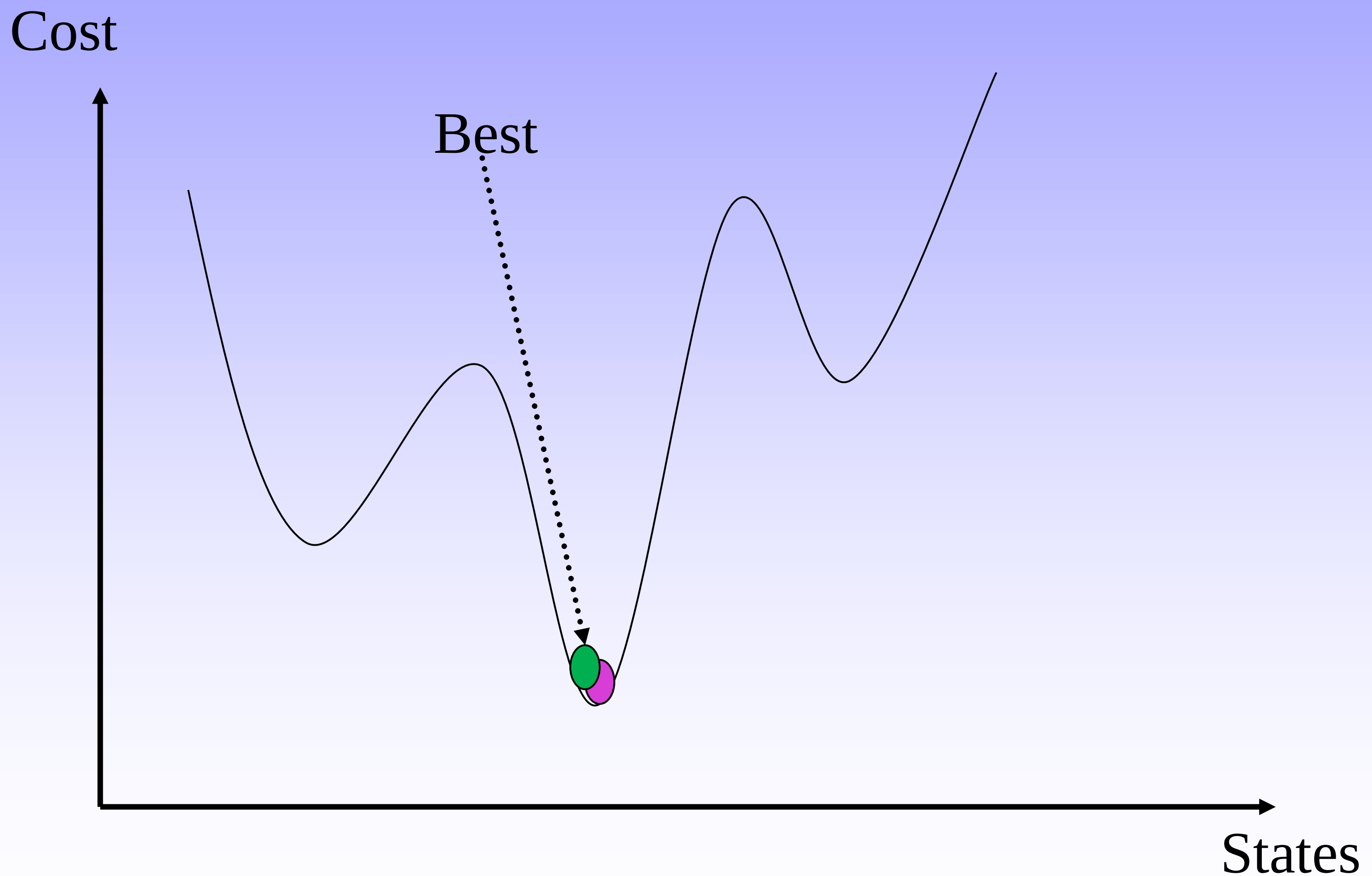

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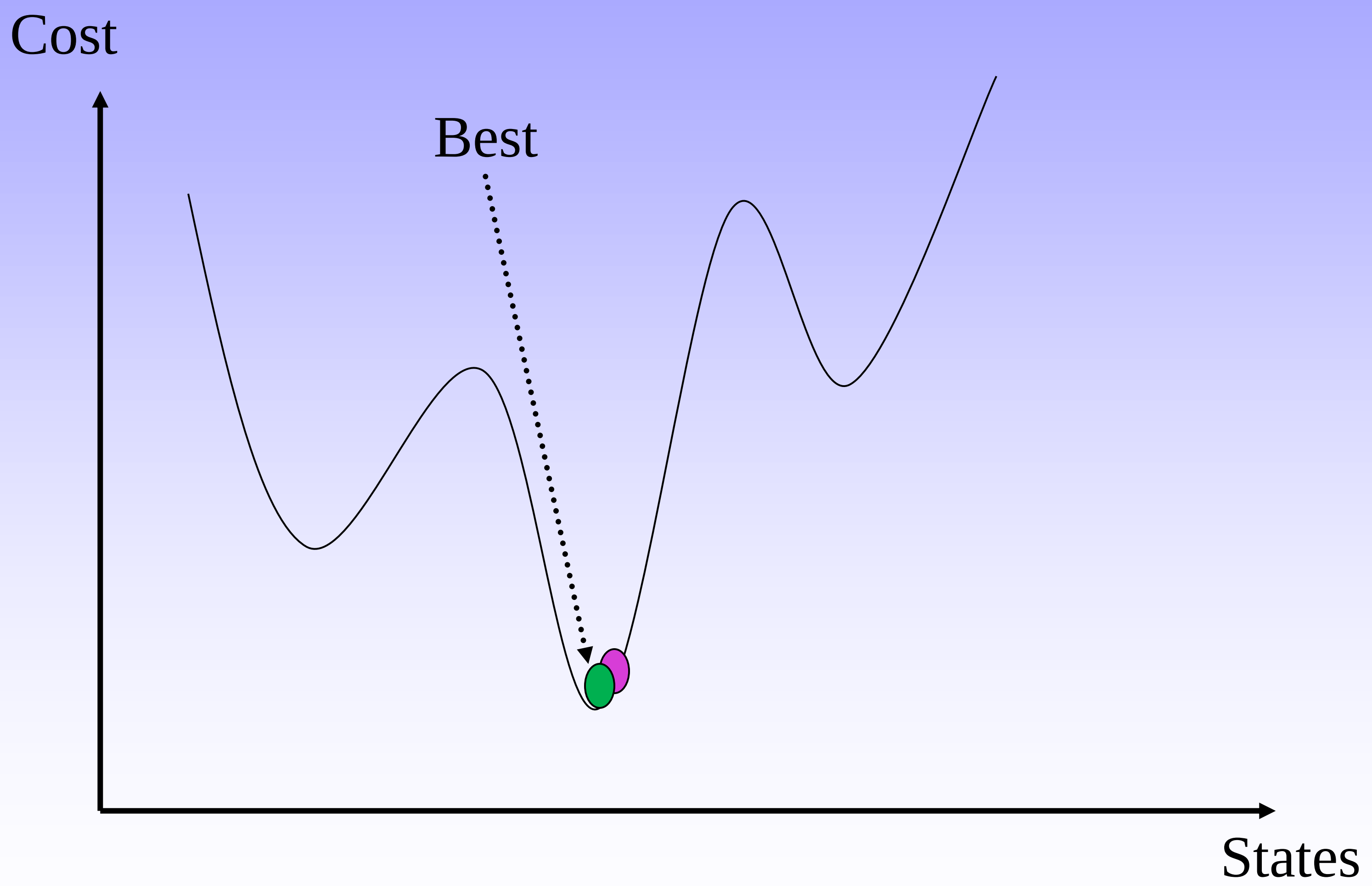

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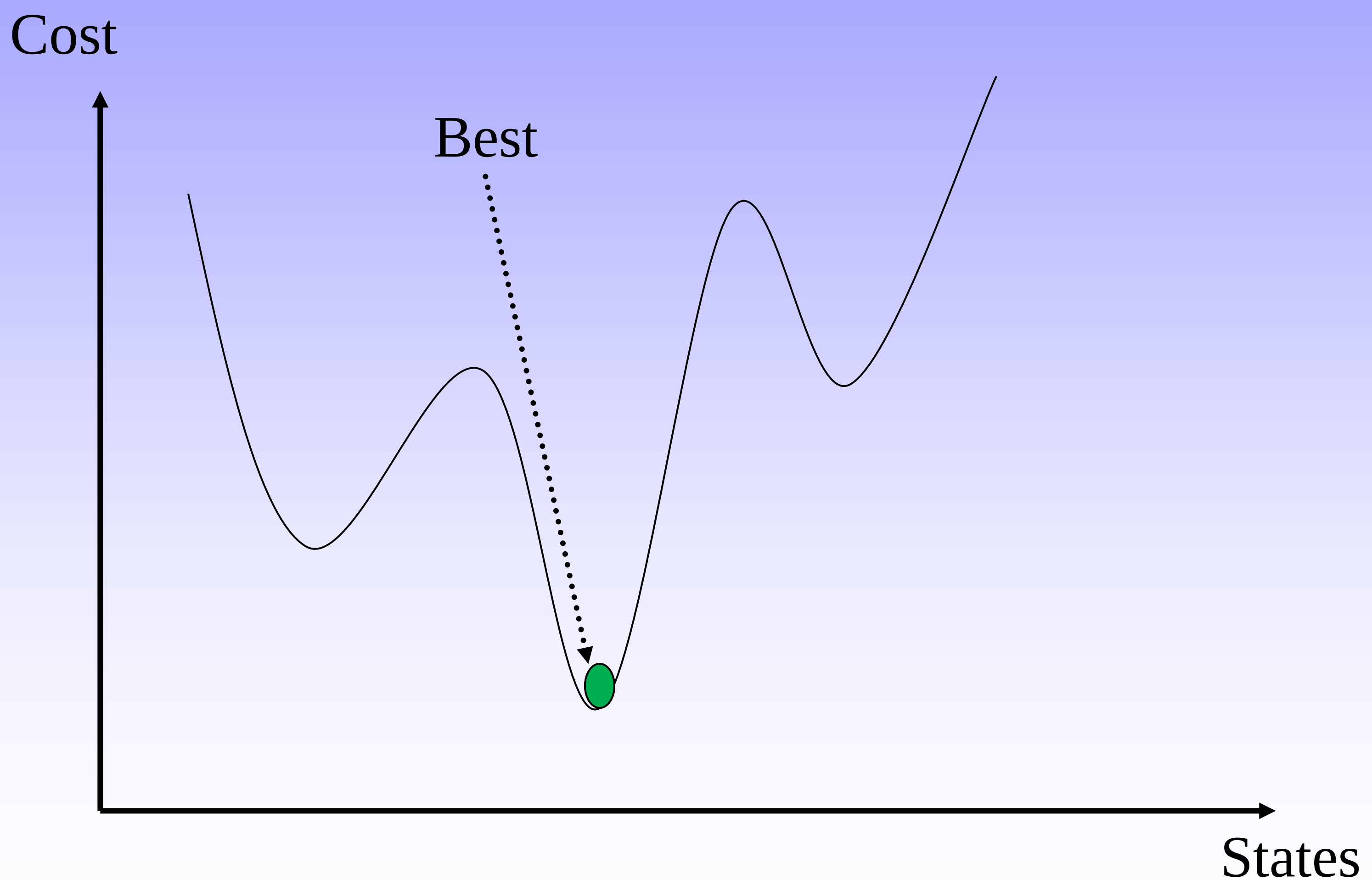

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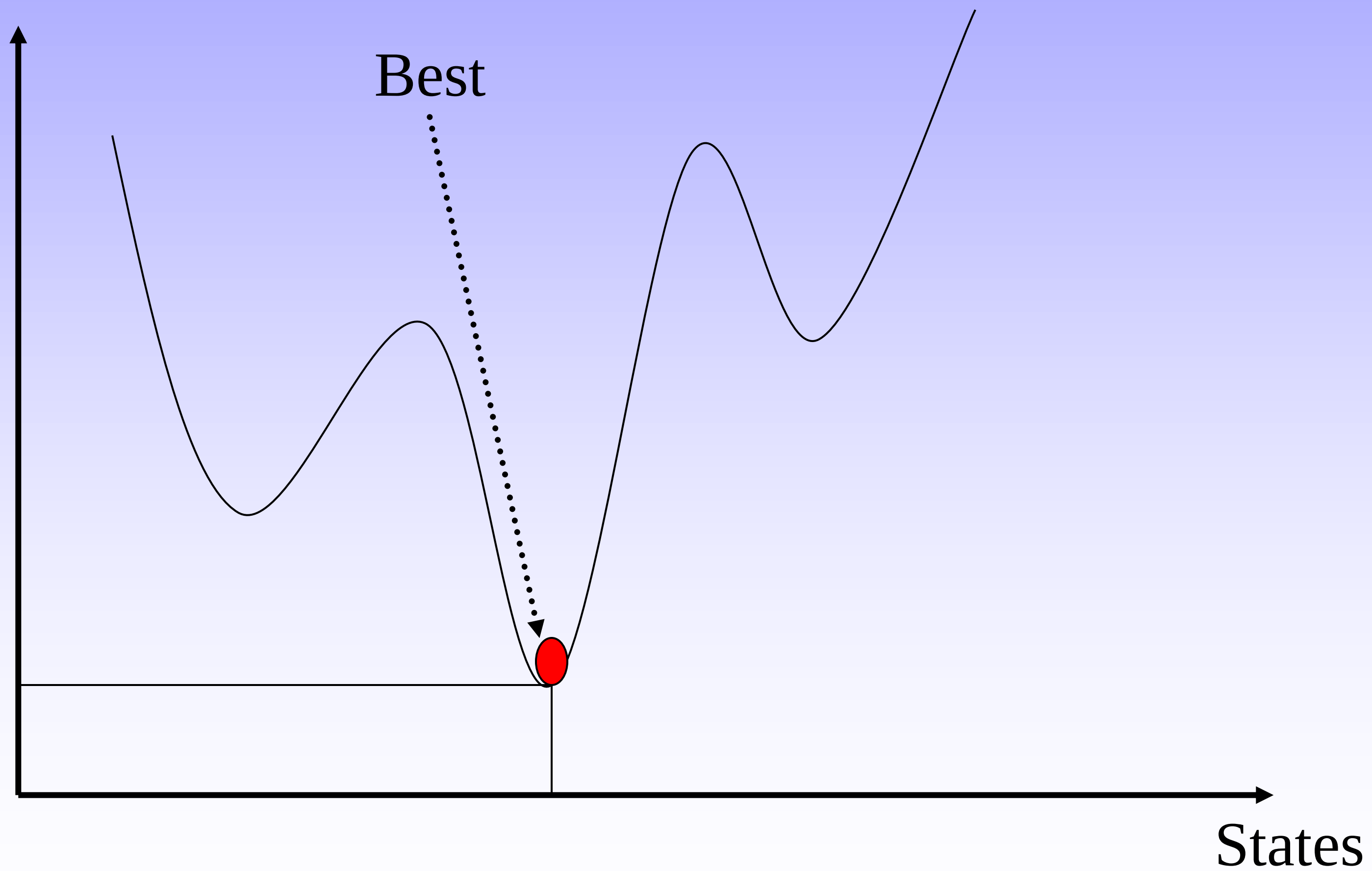


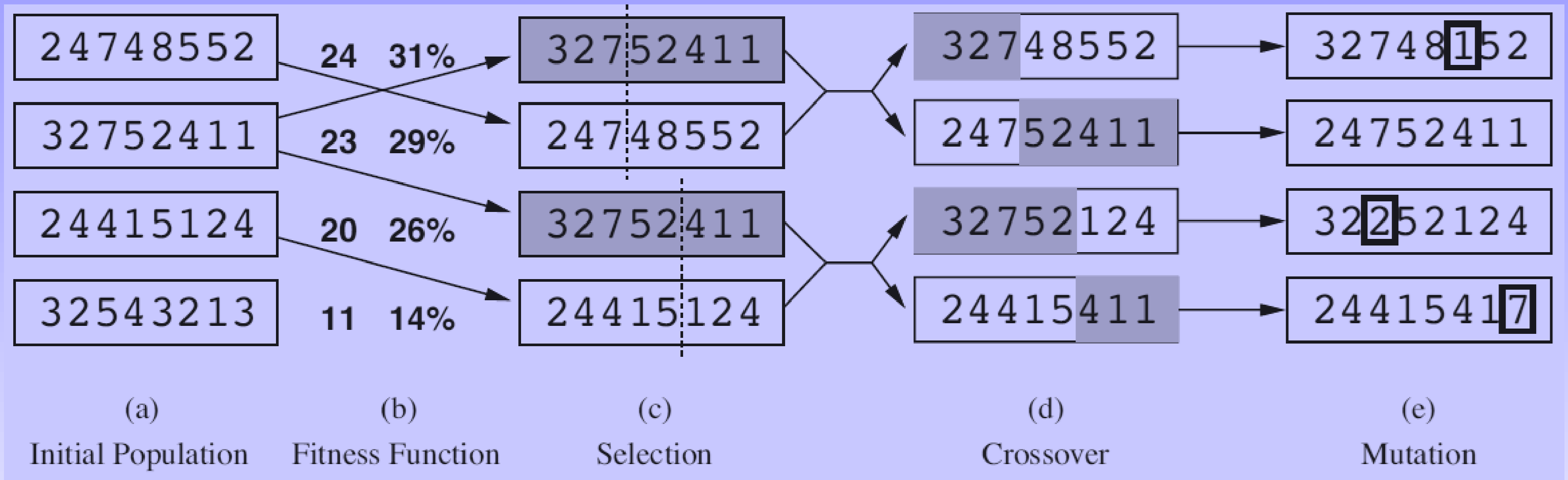
Local Beam Search (LBS)

- Idea: keep track of *k* states rather than just one
- Start with *k* randomly generated states:
  - At each iteration, all the successors of all *k* states are generated.
  - If any one is a goal state, stop; else select the *k* best successors from the complete list and repeat.
- Useful information can be passed among the parallel search threads.
  - LBS quickly abandons unfruitful searches and moves its resources to where the most progress is being made.
- Problem: LBS can suffer from a lack of diversity among the *k* states, making the search an expensive version of hill climbing.

Genetic Algorithms (GAs)

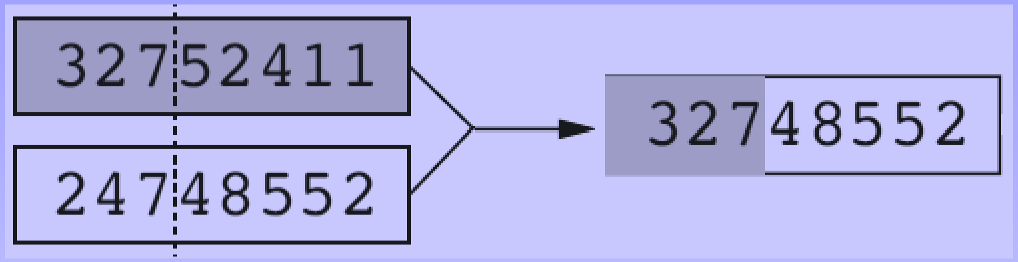
- A successor state is generated by combining two parent states rather than by modifying a single state.
- Like beam searches, start with *k* randomly generated states (called **population**).
  - Each state, or **individual**, is represented as a string over a finite alphabet (most commonly, a string of 0s and 1s).
  - Each state is rated by the objective function (or **fitness function** in GA terminology).
    - » A fitness function should return higher values for better states.
    - » The probability of selecting a state for reproducing is directly proportional to the fitness score.
- Produce the next generation of states by selection, crossover, and mutation.

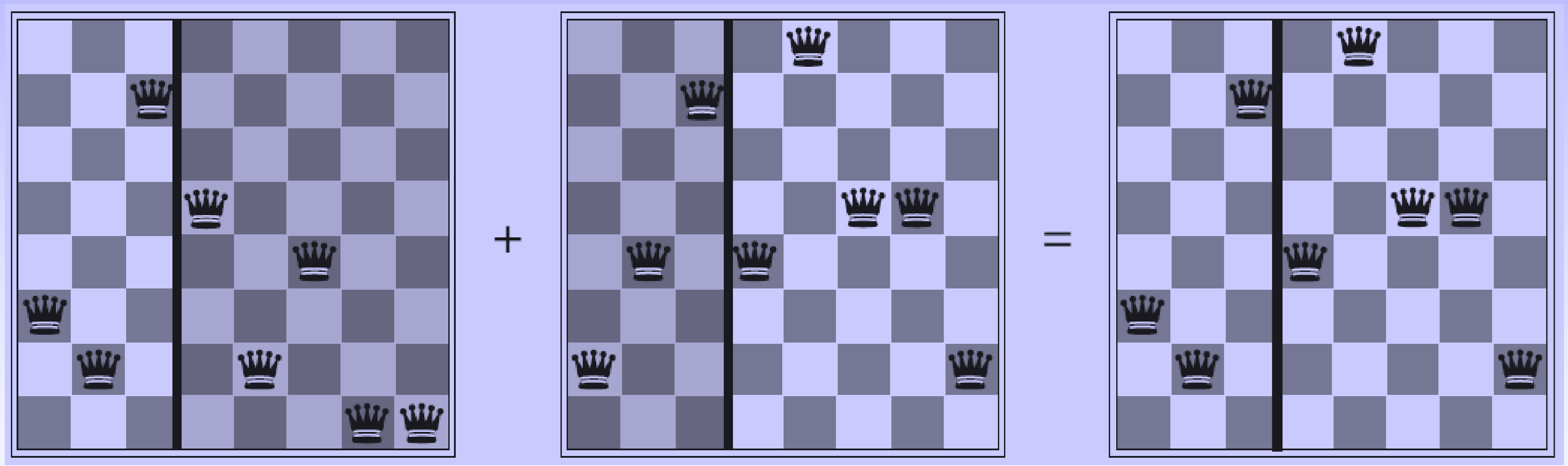
Genetic Algorithms: 8-queens Problem



- <u>Fitness function</u>: number of non-attacking pairs of queens:
  - Min rate = 0, and Max rate = (8 × 7)/2 = 28
  - 24/(24+23+20+11) = 31%
  - 23/(24+23+20+11) = 29% etc.

Genetic Algorithms: 8-queens Problem





- The shaded columns are lost in the crossover step and the unshaded columns are retained.

Genetic Algorithms

- Two pairs are selected at random for reproduction, in accordance with the probabilities.
  - There are many variants of this **selection** rule.
  - The method of *culling*, in which all individuals below a given threshold are discarded, can be shown to converge faster than the random version.
- For each pair to be mated, a **crossover** point is chosen randomly from the positions in the string.
  - The offspring themselves are created by crossing over the parent strings at the crossover point.
- Finally, each location is subject to random **mutation** with a small independent probability.
  - In 8-queens problem, this corresponds to choosing a queen at random and moving it to a random square in its column.

Genetic Algorithms

```
function GENETIC-ALGORITHM(population, FITNESS-FN) returns an individual
  inputs: population, a set of individuals
          FITNESS-FN, a function that measures the fitness of an individual

  repeat
     new_population ← empty set
     for i = 1 to SIZE(population) do
        x ← RANDOM-SELECTION(population, FITNESS-FN)
        y ← RANDOM-SELECTION(population, FITNESS-FN)
        child ← REPRODUCE(x, y)
        if (small random probability) then child ← MUTATE(child)
        add child to new_population
     population ← new_population
  until some individual is fit enough, or enough time has elapsed
  return the best individual in population, according to FITNESS-FN
```

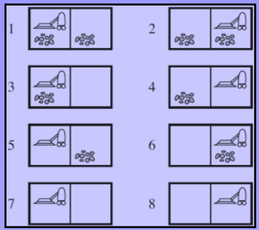
```
function REPRODUCE(x, y) returns an individual
  inputs: x, y, parent individuals

  n ← LENGTH(x); c ← random number from 1 to n
  return APPEND(SUBSTRING(x, 1, c), SUBSTRING(y, c + 1, n))
```

Searching with Nondeterministic Actions

- In case of fully observable and deterministic environment, the agent knows the effects of each action and its percepts provide no new information.
  - Therefore, the agent can calculate exactly which state results from any sequence of actions.
- But, when the environment is nondeterministic, percepts tell the agent which of the possible outcomes of its actions has actually occurred.
  - So, the future percepts cannot be determined in advance and the agent's future actions will depend on those future percepts.
  - Thus, a problem solution is not a sequence but a **contingency plan** that specifies what to do depending on what percepts are received.

Erratic Vacuum World



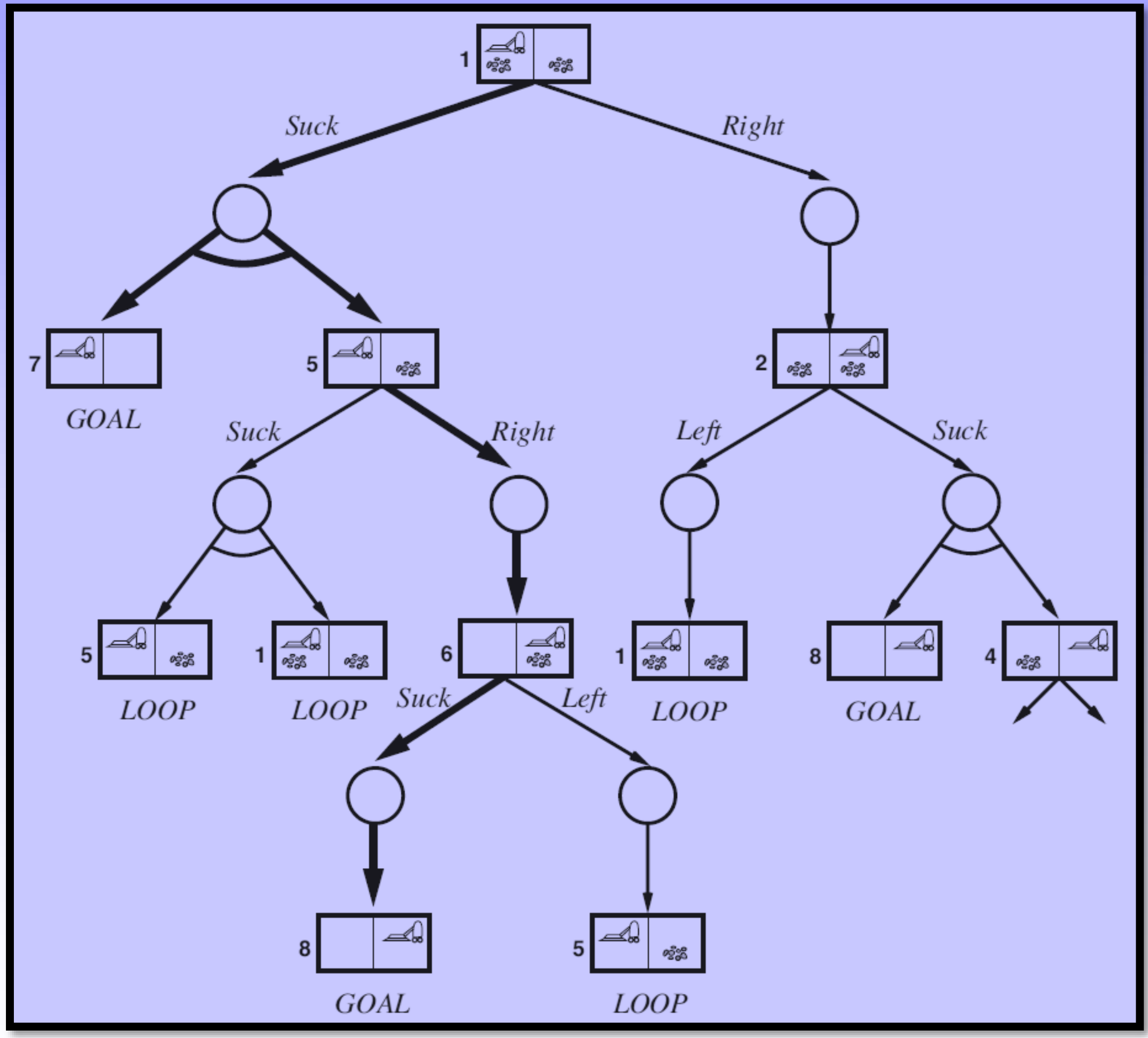
- It is a powerful but erratic vacuum cleaner, such that the *Suck* action works as follows:
  - When applied to a dirty square the action cleans the square and sometimes cleans up dirt in an adjacent square, too.
  - When applied to a clean square the action sometimes deposits dirt on the carpet.
- In this case, we use a RESULTS function that returns a *set* of possible outcome States:
  - The *Suck* action in state 1 leads to a state in the set {5, 7}
- Need also to generalize the notion of a problem **solution**:
  - If we start in state 1, we need a contingency plan such as the following:

    [*Suck*, **if** State = 5 **then** [*Right*, *Suck*] **else** [ ]] .

Searching with Nondeterministic Actions

- Consequently, solutions for nondeterministic problems can contain nested **if–then–else** (or case) statements.
  - Means that the solutions are ***trees*** rather than sequences.
  - This allows the selection of actions based on contingencies arising during execution.
- In the search tree of a deterministic environment, the only branching is introduced by the agent's own choices in each state – we call these nodes **OR nodes**.
- Otherwise, in a nondeterministic environment, branching is also presented by the *environment's* choice of outcome for each action – we call these nodes **AND nodes**.
  - These two kinds of nodes alternate, leading to an **AND–OR tree**.

Erratic Vacuum World: AND–OR Tree



Searching with Nondeterministic Actions

- A solution for an AND–OR search problem is a subtree that:

  (1) has a goal node at every leaf

  (2) specifies one action at each of its OR nodes

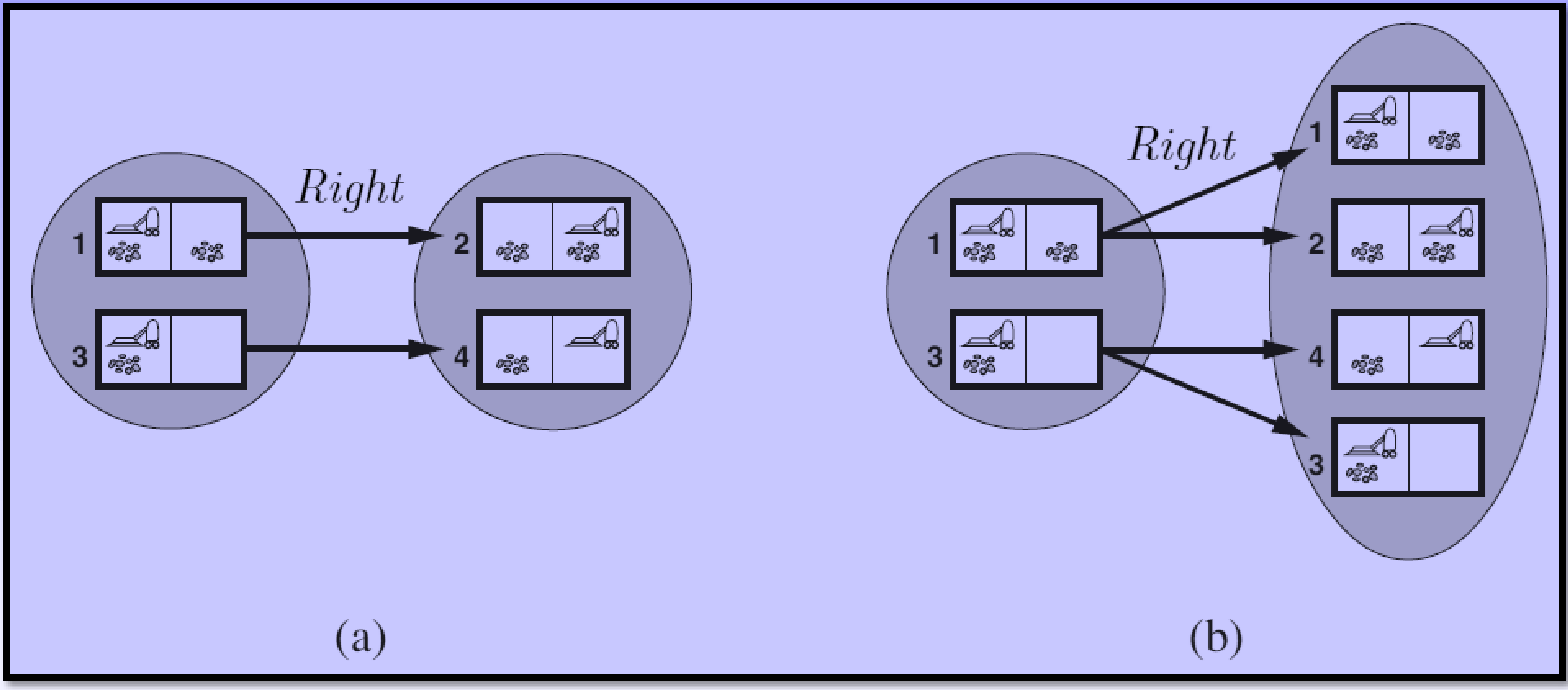
  (3) includes every outcome branch at each of its AND nodes

- AND–OR trees can be explored by depth-first, breadth-first or best-first methods.
  - The concept of a heuristic function must be modified to estimate cost of a contingent solution rather than a sequence.
  - But the notion of admissibility carries over and there is an analog of the A^* algorithm for finding optimal solutions.
- This type of interleaving of search and execution is also useful for exploration problems and for game playing.

Searching with No Observation

- When the agent's percepts provide *no information at all*, we have what is called a **sensorless** problem.
  - Sensorless agents can be useful, primarily because they *don't* rely on sensors working properly.
  - Many ingenious methods have been developed for orienting parts correctly from an unknown initial position.
- To solve sensorless problems, we search in the space of **belief states** rather than physical states.
  - A belief state represents the agent's current belief about the possible physical states it might be in, given the sequence of actions and percepts up to that point.
- In belief-state space, the problem is fully observable and the solution (if any) is always a sequence of actions.

Sensorless Vacuum World



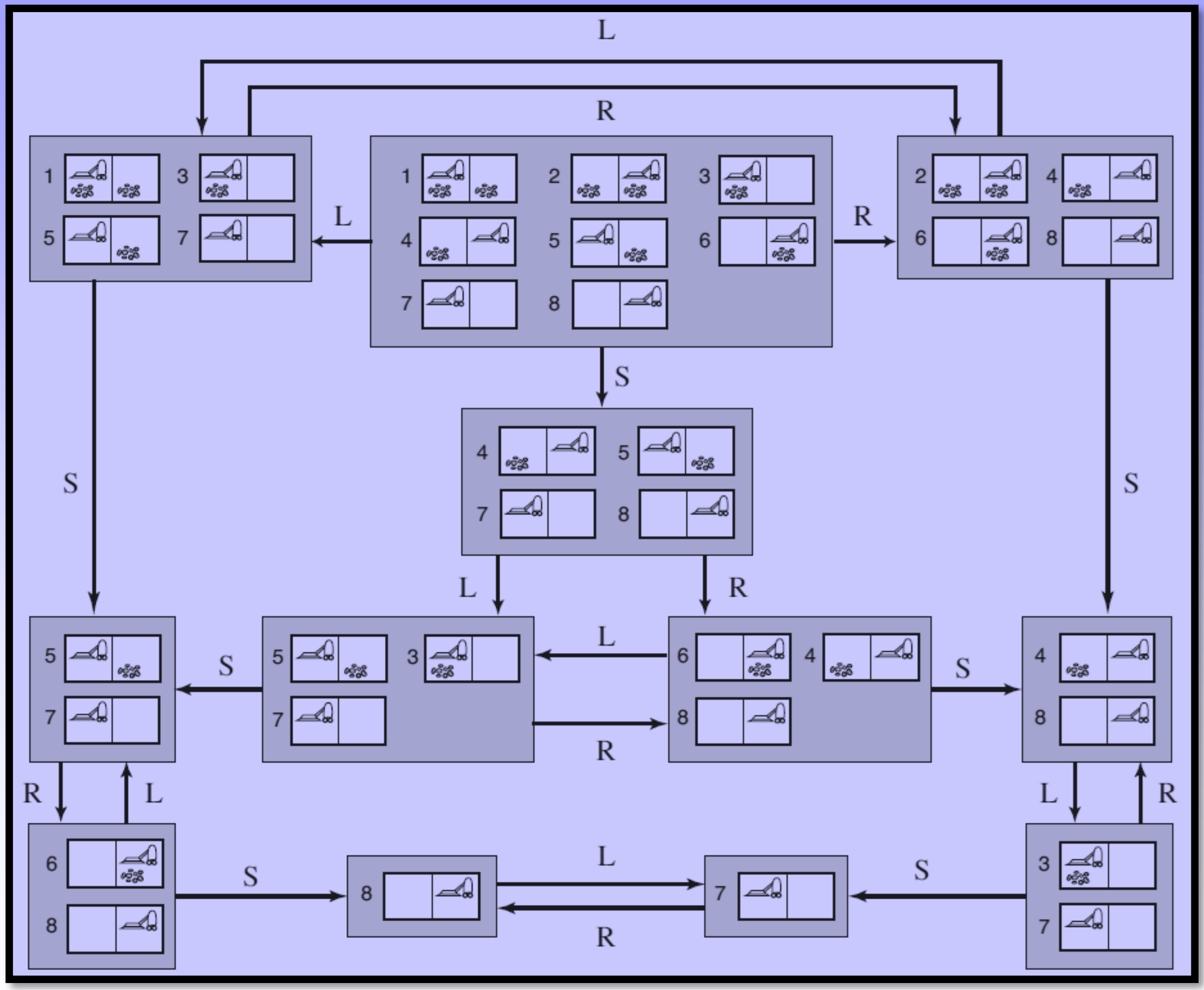
(a) Predicting the next belief state with a deterministic action, *Right*.

(b) Prediction for the same belief state and action in slippery vacuum.

Searching with No Observation

- The entire belief-state space contains every possible set of physical states.
  - If a problem has N states, then the sensorless problem has up to 2^N states, although many may be unreachable from the initial state.
- Typically, the initial state of a problem is the set of all physical states of that problem.
- The agent doesn't know which physical state in the belief state is the right one.
  - It might get to any of the physical states resulting from applying a certain action to one of the states in the belief state.
- A belief state satisfies the goal only if *all* the physical states in it satisfy the problem GOAL-TEST.

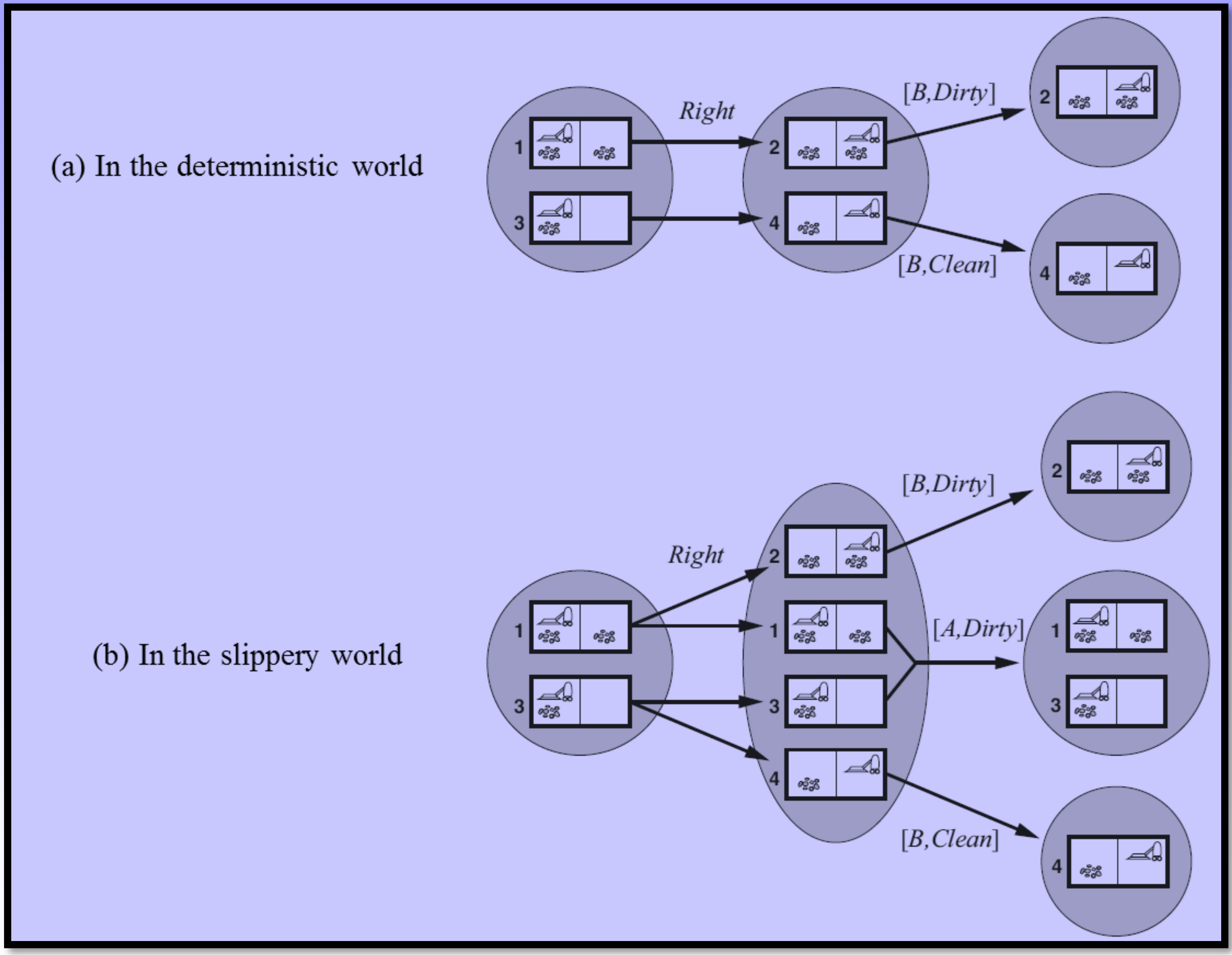
Sensorless Vacuum: Belief-state Space



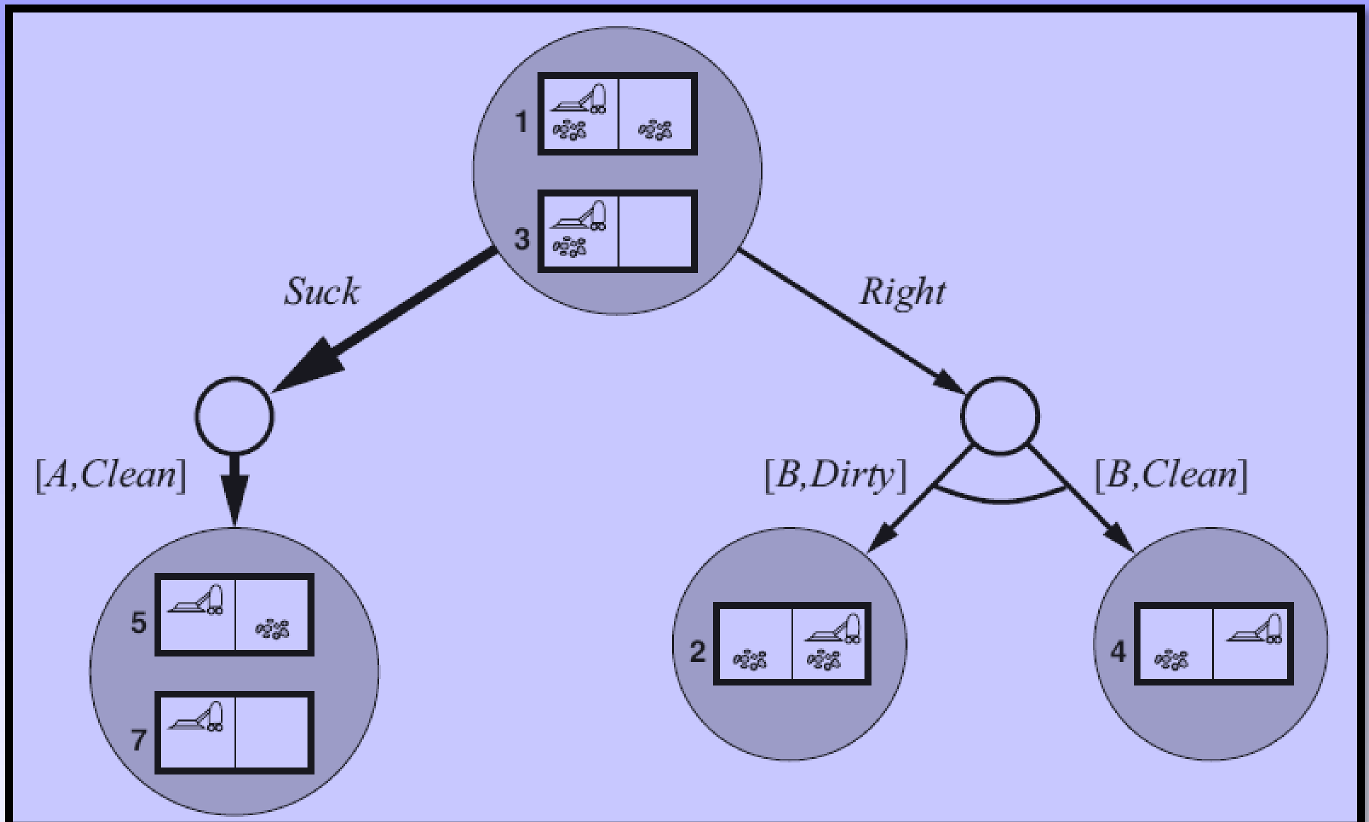
Searching with Partial Observations

- When observations are partial, the agent's percepts do not suffice to pin down the exact state.
  - It is the case that several states could have produced any given percept.
- The belief-state space of a problem is constructed from the underlying physical problem just as for sensorless problem, but the transition model is more complicated.
  - Partial observations can only help reduce uncertainty compared to the sensorless case.
- Example: Local-sensing vacuum world
  - The agent has a position sensor and a local dirt sensor.
  - But it has no sensor capable of detecting dirt in other squares.

Local-sensing Vacuum World



Local-sensing Vacuum: AND–OR Tree



Assuming an initial percept [*A*, *Dirty*] then the solution is the conditional plan as follow:

[*Suck*, *Right*, **if** *Dirty* **then** *Suck* **else** [ ]] .