Cairo University
Faculty of Computers & Artificial Intelligence
Theory of Computations

Lab 11

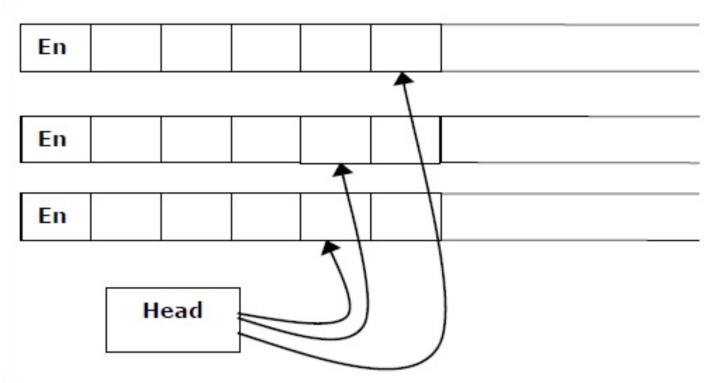
Multi-tape Turing Machine

Multi-tape Turing Machines have multiple tapes where each tape is accessed with a separate head. Each head can move independently of the other heads. Initially the input is on tape 1 and others are blank. At first, the first tape is occupied by the input and the other tapes are kept blank. Next, the machine reads consecutive symbols under its heads and the TM prints a symbol on each tape and moves its heads.

On a single move, depending on the state of the finite control and symbol read from each tape, the machine can perform following operations:

1. Change the state to a new state,
2. Write a new symbol at the current head position on each tape, and,
3. Each head can move right or left or remain stationary on each tape independently.

In each step (transitions) TM reads symbols scanned by all heads, depending on head position and current state, next, Each head writes, moves R (right) or L (left), and control-unit enter into new state. Actions of R-W heads are independent of each other.



A Multi-tape Turing machine can be formally described as a 6-tuple (Q, X, B, δ, q_0, F) where −

Q is a finite set of states

X is the tape alphabet

B is the blank symbol

δ is a relation on states and symbols where

$\delta: Q \times X^k \rightarrow Q \times (X \times \{\text{Left\_shift, Right\_shift, No\_shift}\})^k$

where there is k number of tapes

q_0 is the initial state

F is the set of final states

Note − Every Multi-tape Turing machine has an equivalent single-tape Turing machine.

Example 1

Example 20.3 *Construct a Two-tape TM to recognize language* $L = \{a^n b^n \mid n \geq 0\}$

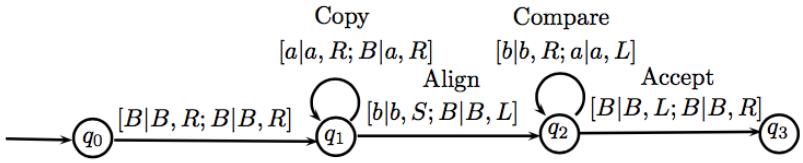


Figure 20.4: Recognition of language $L = \{a^n b^n \mid n \geq 0\}$ using a two-tape TM

With the original string w on tape 1, the transitions (instructions) performed by the ™ are shown in Fig. 20.4, and major steps are as follows:

1. Copy: Copies the string of a's on tape-1 to tape-2.
2. Align: Moves head 1 to begin of b's on tape-1, and head-2 on last a on tape-2.
3. Compare: Compares number of b's tape-1 with number of a's on tape-2 by moving heads in opposite directions.

Example 2

Example 20.4 *Two-tape TM to recognize language* $L = \{ww^R \mid w \in \{a, b\}^*\}$.

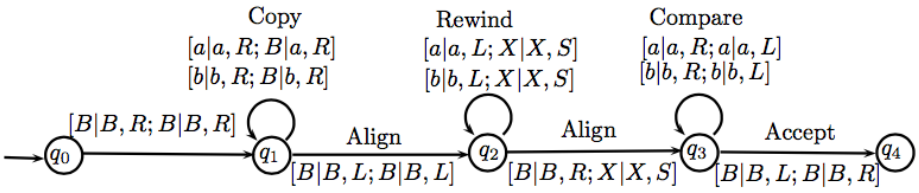


Figure 20.5: Two tape Turing machine to recognize $L = \{ww^R \mid w \in \{a, b\}^*\}$

original string w is on tape 1, perform the following steps:
1. Copy string w on tape 2,
2. Move head 1 to extreme left (rewind),
3. Aligns both heads on opposite, i.e, head 1 to extreme left, head 2 to extreme right, 4. Compare tape 1 and 2, by moving heads in opposite directions.