Combinational Circuits Design

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Logic Circuits

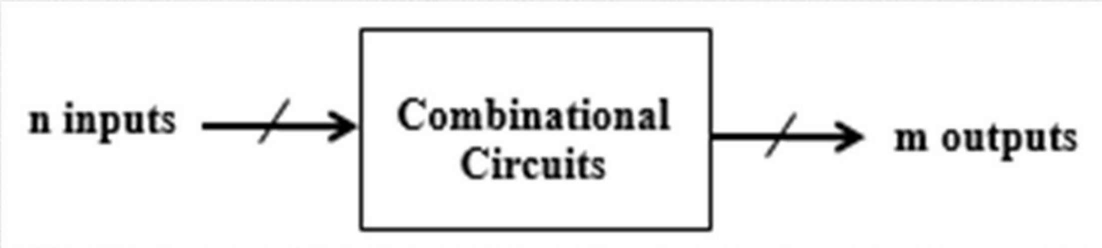
Two types of logic circuits exist:

- **Combinational circuits.**
- **Sequential circuits.**

Combinational Circuits

- Combinational circuits:

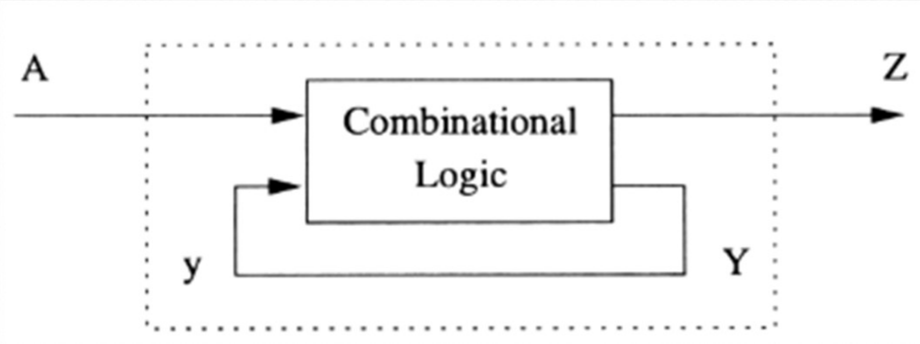
  A combinational circuit is a logical circuit, where the output depends on the present value of the input signals without regards to previous inputs.



Sequential Circuits

- **Sequential circuits.**

  A sequential circuit is a logical circuit, where the output depends on the present value of the input signal as well as the sequence of past inputs.



Combinational Circuits

It consists of:

- n binary inputs (1 or more).
- m binary outputs (1 or more) produced by the internal combinational circuit.
- Logic gates building the combinational circuit.

Combinational Circuits Design Procedure

1. Specification
   - Write a specification for the circuit if one is not already available.
   - Specify (number and label) the inputs and outputs.
2. Formulation
   - Derive a truth table or initial Boolean equations that define the required relationships between the inputs and outputs.
   - Note that non-existing inputs values are considered as don't care in outputs.

Combinational Circuits Design Procedure

- Apply hierarchical design if appropriate.

3. Optimization

- Apply optimization (Boolean Algebra, K-Map, Tabular Method … etc) to get the function of the outputs in terms of the inputs.
- Draw a logic diagram.

4. Verification (Optional)

- Verify the correctness of the final design manually or using simulation.

Combinational Circuits Design (Example 1)

<u>Example 1:</u>

Design a combinational circuit with three inputs and one output. The output is equal to 1 when the decimal value of the input is less than 3. Otherwise the output is 0.

1. Specification
   - The specification for the circuit already exists.
   - 3 inputs A,B,C and 1 output Y.

Combinational Circuits Design
(Example 1)

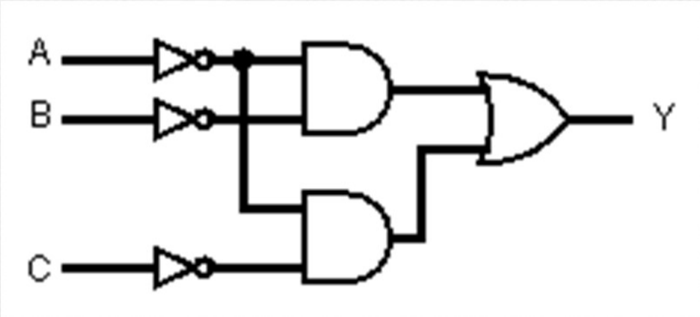
2.

Decimal Value	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

3.

A \ BC	00	01	11	10
0	1	1		1
1				

$$Y = \overline{A}\,\overline{B} + \overline{A}\,\overline{C}$$



Combinational Circuits Design (Example 2)

Example 2:

Design a combinational circuit with three inputs and a number of outputs, the output is a number that equal to the double of the inputs.

1. Specification
   - The specification for the circuit already exists.
   - 3 inputs A,B,C and 4 outputs W,X,Y,Z.

Combinational Circuits Design

(Example 2)

2.

Decimal Value	A	B	C	W	X	Y	Z
0	0	0	0	0	0	0	0
1	0	0	1	0	0	1	0
2	0	1	0	0	1	0	0
3	0	1	1	0	1	1	0
4	1	0	0	1	0	0	0
5	1	0	1	1	0	1	0
6	1	1	0	1	1	0	0
7	1	1	1	1	1	1	0

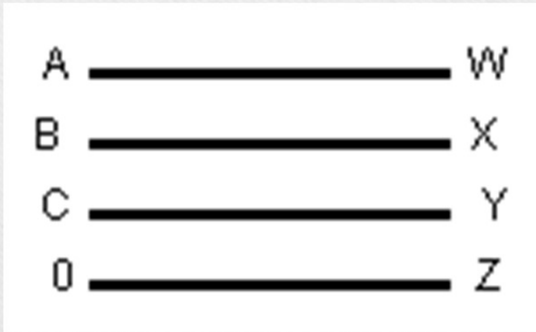
3. Without using any simplification method. It is clear that:

W=A

X=B

Y=C

Z=0



Combinational Circuits Design
(Example 3)

<u>Example 3:</u>

Design a combinational circuit that subtract 5 from a 4-bit binary number (input cannot take a value less than 5).

1. **Specification**
   - **The specification for the circuit already exists.**
   - **4 inputs A,B,C,D and 4 outputs W,X,Y,Z.**

Combinational Circuits Design

(Example 3)

2.

Inputs				Outputs			
A	B	C	D	W	X	Y	Z
0	0	0	0	X	X	X	X
0	0	0	1	X	X	X	X
0	0	1	0	X	X	X	X
0	0	1	1	X	X	X	X
0	1	0	0	X	X	X	X
0	1	0	1	0	0	0	0
0	1	1	0	0	0	0	1
0	1	1	1	0	0	1	0
1	0	0	0	0	0	1	1
1	0	0	1	0	1	0	0
1	0	1	0	0	1	0	1
1	0	1	1	0	1	1	0
1	1	0	0	0	1	1	1
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	1
1	1	1	1	1	0	1	0

3.

AB \ CD	00	01	11	10
00	X	X	X	X
01	X			
11		1	1	1
10				

W= ABD + ABC

Combinational Circuits Design

(Example 3)

2.

Inputs				Outputs			
A	B	C	D	W	X	Y	Z
0	0	0	0	X	X	X	X
0	0	0	1	X	X	X	X
0	0	1	0	X	X	X	X
0	0	1	1	X	X	X	X
0	1	0	0	X	X	X	X
0	1	0	1	0	0	0	0
0	1	1	0	0	0	0	1
0	1	1	1	0	0	1	0
1	0	0	0	0	0	1	1
1	0	0	1	0	1	0	0
1	0	1	0	0	1	0	1
1	0	1	1	0	1	1	0
1	1	0	0	0	1	1	1
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	1
1	1	1	1	1	0	1	0

3.

AB \ CD	00	01	11	10
00	X	X	X	X
01	X			
11	1			
10		1	1	1

$$X = B\bar{C}\,\bar{D} + \bar{B}D + \bar{B}C$$

Combinational Circuits Design

(Example 3)

2.

Inputs				Outputs			
A	B	C	D	W	X	Y	Z
0	0	0	0	X	X	X	X
0	0	0	1	X	X	X	X
0	0	1	0	X	X	X	X
0	0	1	1	X	X	X	X
0	1	0	0	X	X	X	X
0	1	0	1	0	0	0	0
0	1	1	0	0	0	0	1
0	1	1	1	0	0	1	0
1	0	0	0	0	0	1	1
1	0	0	1	0	1	0	0
1	0	1	0	0	1	0	1
1	0	1	1	0	1	1	0
1	1	0	0	0	1	1	1
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	1
1	1	1	1	1	0	1	0

3.

AB \ CD	00	01	11	10
00	X	X	X	X
01	X		1	
11	1		1	
10	1		1	

$$Y = \bar{C}\,\bar{D} + CD$$

Combinational Circuits Design

(Example 3)

2.

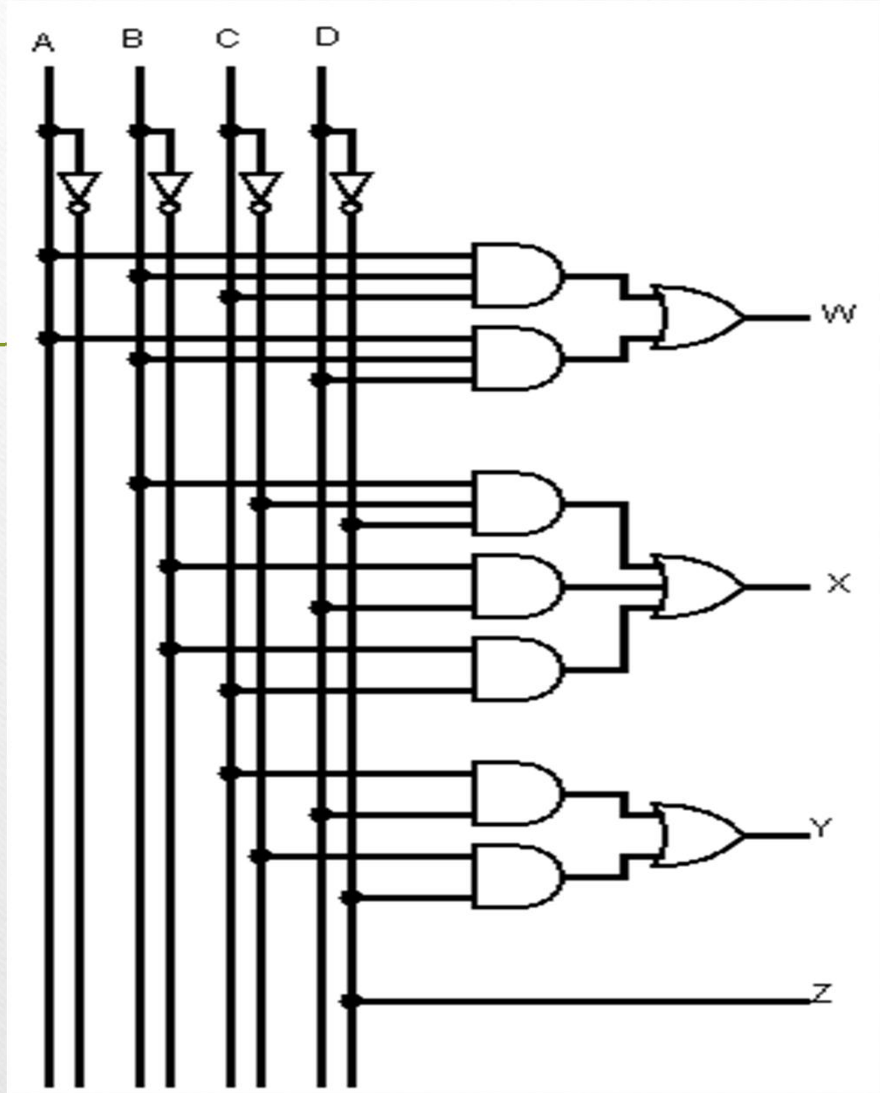
Inputs				Outputs			
A	B	C	D	W	X	Y	Z
0	0	0	0	X	X	X	X
0	0	0	1	X	X	X	X
0	0	1	0	X	X	X	X
0	0	1	1	X	X	X	X
0	1	0	0	X	X	X	X
0	1	0	1	0	0	0	0
0	1	1	0	0	0	0	1
0	1	1	1	0	0	1	0
1	0	0	0	0	0	1	1
1	0	0	1	0	1	0	0
1	0	1	0	0	1	0	1
1	0	1	1	0	1	1	0
1	1	0	0	0	1	1	1
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	1
1	1	1	1	1	0	1	0

3.

AB \ CD	00	01	11	10
00	X	X	X	X
01	X			1
11	1			1
10	1			1

$Z = \overline{D}$

Combinational Circuits Design
(Example 3)



Combinational Circuits Design
(Example 4)

<u>Example 4:</u>

Design a combinational circuit that converts 4-bit gray code to a binary number, using 2-input XOR gates only

4 inputs A,B,C,D and 4 outputs W,X,Y,Z.

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Combinational Circuits Design
(Example 4)

CD \ AB	00	01	11	10
00				
01				
11	1	1	1	1
10	1	1	1	1

W= A

Inputs				Outputs			
A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	1	0	0	1	0
0	0	1	0	0	0	1	1
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	1
0	1	0	1	0	1	1	0
0	1	0	0	0	1	1	1
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	1
1	1	1	1	1	0	1	0
1	1	1	0	1	0	1	1
1	0	1	0	1	1	0	0
1	0	1	1	1	1	0	1
1	0	0	1	1	1	1	0
1	0	0	0	1	1	1	1

Combinational Circuits Design
(Example 4)

AB \ CD	00	01	11	10
00				
01	1	1	1	1
11				
10	1	1	1	1

$$X = A\bar{B} + \bar{A}B$$
$$= A \oplus B$$

Inputs				Outputs			
A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	1	0	0	1	0
0	0	1	0	0	0	1	1
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	1
0	1	0	1	0	1	1	0
0	1	0	0	0	1	1	1
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	1
1	1	1	1	1	0	1	0
1	1	1	0	1	0	1	1
1	0	1	0	1	1	0	0
1	0	1	1	1	1	0	1
1	0	0	1	1	1	1	0
1	0	0	0	1	1	1	1

Combinational Circuits Design

(Example 4)

AB \ CD	00	01	11	10
00			1	1
01	1	1		
11			1	1
10	1	1		

$$Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + ABC + A\bar{B}\bar{C}$$

$$= \bar{A}(\bar{B}C + B\bar{C}) + A(BC + \bar{B}\bar{C})$$

$$= A \oplus B \oplus C = X \oplus C$$

Inputs				Outputs			
A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	1	0	0	1	0
0	0	1	0	0	0	1	1
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	1
0	1	0	1	0	1	1	0
0	1	0	0	0	1	1	1
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	1
1	1	1	1	1	0	1	0
1	1	1	0	1	0	1	1
1	0	1	0	1	1	0	0
1	0	1	1	1	1	0	1
1	0	0	1	1	1	1	0
1	0	0	0	1	1	1	1

Combinational Circuits Design

(Example 4)

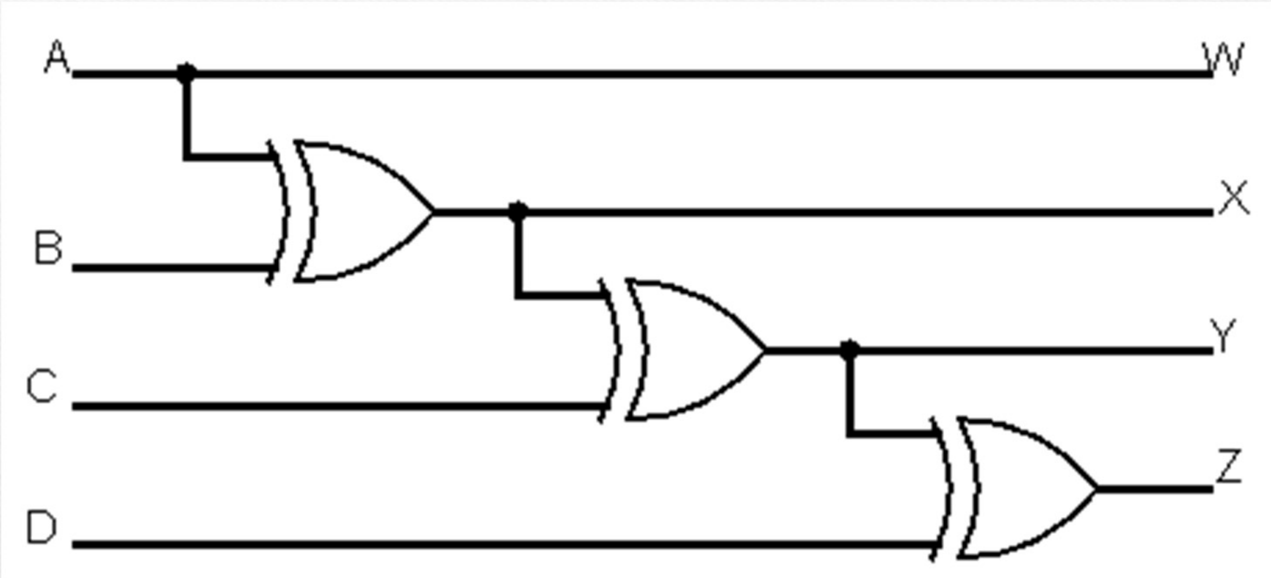
CD \ AB	00	01	11	10
00		1		1
01	1		1	
11		1		1
10	1		1	

$$Z = A \oplus B \oplus C \oplus D$$

$$= Y \oplus D$$

Inputs				Outputs			
A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	1	0	0	1	0
0	0	1	0	0	0	1	1
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	1
0	1	0	1	0	1	1	0
0	1	0	0	0	1	1	1
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	1
1	1	1	1	1	0	1	0
1	1	1	0	1	0	1	1
1	0	1	0	1	1	0	0
1	0	1	1	1	1	0	1
1	0	0	1	1	1	1	0
1	0	0	0	1	1	1	1

Combinational Circuits Design
(Example 4)



Combinational Circuits Design
(Example 5)

<u>Example 5:</u>

Design a combinational circuit that detects error in the representation of a decimal digit in BCD.

Input is a BCD number so 4-bit input ABCD.

1 output F.

Combinational Circuits Design

(Example 5)

- Optimize a function for the output in terms of inputs.
- Then draw the logic circuit.

Inputs				Output
A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Combinational Circuits Design

(Example 6)

<u>Example 6:</u>

A logic circuit has four inputs (I_3, I_2, I_1, I_0) and two outputs (O_1, O_0) At least one of the inputs is always asserted high. If a given input line has a logic 1 applied to it, the output signals will encode its index in binary. If two or more inputs are at logic 1, the output will be set according to which input has the highest index ($I_0 < I_1 < I_2 < I_3$). Design a combinational circuit that satisfies these specification

4 inputs I_3, I_2, I_1, I_0 and 2 outputs O_1, O_0

Combinational Circuits Design (Example 6)

- Optimize functions for the outputs in terms of inputs.
- Then draw the logic circuit.

Inputs				Outputs	
I3	I2	I1	I0	O1	O0
0	0	0	0	X	X
0	0	0	1	0	0
0	0	1	0	0	1
0	0	1	1	0	1
0	1	0	0	1	0
0	1	0	1	1	0
0	1	1	0	1	0
0	1	1	1	1	0
1	0	0	0	1	1
1	0	0	1	1	1
1	0	1	0	1	1
1	0	1	1	1	1
1	1	0	0	1	1
1	1	0	1	1	1
1	1	1	0	1	1
1	1	1	1	1	1

Combinational Circuits Design
(Example 7)

Example 7:

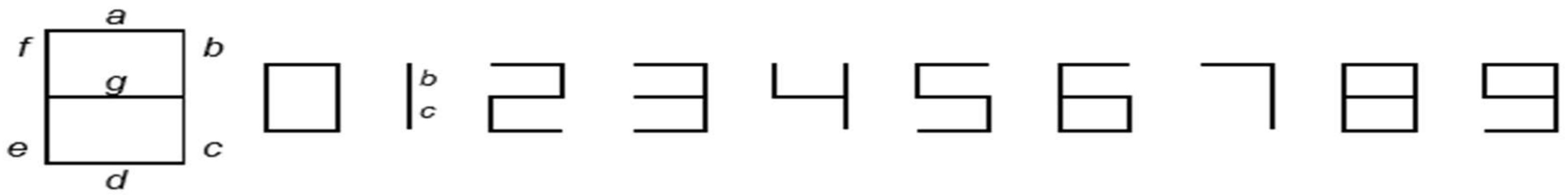
A BCD-to-seven-segment decoder is a combinational circuit that converts a decimal digit in BCD to an appropriate code for the selection of segments in a display indicator used for displaying the decimal digit in a familiar form. The seven outputs of the decoder (*a,b,c,d,e,f,g*) select the corresponding segments in the display as shown in the figure. The numeric designation chosen to represent the decimal digit as shown. Design the BCD-to-seven-segment decoder.

Combinational Circuits Design

(Example 7)

Four inputs w,x,y,z

Seven outputs *a,b,c,d,e,f,g*


a
f
b
g
e
c
d
b
c

Combinational Circuits Design
(Example 7)

- Optimize functions for the outputs in terms of inputs.
- Then draw the logic circuit.

Inputs				Outputs						
w	x	y	z	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	0
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	x	x	x	x	x	x	x
1	0	1	1	x	x	x	x	x	x	x
1	1	0	0	x	x	x	x	x	x	x
1	1	0	1	x	x	x	x	x	x	x
1	1	1	0	x	x	x	x	x	x	x
1	1	1	1	x	x	x	x	x	x	x

Combinational Circuits Design

Exercise 1:

Design a combinational circuit that will recognize the occurrence of 3 consecutive 1s in an eight-bit-parallel message.

Combinational Circuits Design

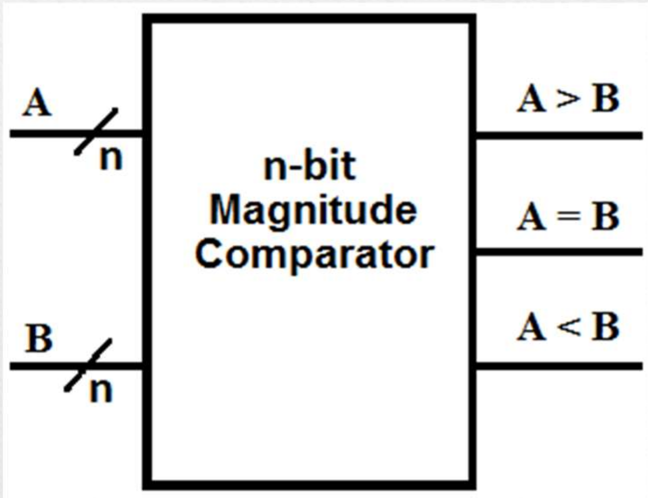
<u>**Exercise 2:**</u>

Design a combinational circuit that convert a BCD number to 84-2-1 code.

n-bit Magnitude Comparator

A magnitude comparator is a logic circuit that takes two binary numbers as inputs, and determines whether one number is greater than, less than or equal to the other number.

It is not logical to use the ordinary method to build this circuit.



4-bit Magnitude Comparator

<u>**Example 8:**</u>

Design a 4 bit comparator.

4-bit Magnitude Comparator

Having two 4-bit numbers; ($A_3A_2A_1A_0$) and ($B_3B_2B_1B_0$).

Three outputs are used:

EQ= 1 if $A_3A_2A_1A_0=B_3B_2B_1B_0$

GT= 1 if $A_3A_2A_1A_0>B_3B_2B_1B_0$

LT= 1 if $A_3A_2A_1A_0<B_3B_2B_1B_0$

<u>Define</u>

$X_i=A_iB_i+\overline{A_i}\,\overline{B_i} = \overline{A_i \oplus B_i}$

It is clear that $X_i=1$ if $A_i=B_i$

4-bit Magnitude Comparator

Design the equal output (EQ):

EQ=1 if $A_3=B_3$ & $A_2=B_2$ & $A_1=B_1$ & $A_0=B_0$

i.e. if $X_3=1$ & $X_2=1$ & $X_1=1$ & $X_0=1$

i.e. if $X_3X_2X_1X_0=1$

So EQ= $X_3X_2X_1X_0$

4-bit Magnitude Comparator

<u>**Design the greater output (GT):**</u>

GT=1 if $A_3>B_3$ (i.e. $A_3=1$&$B_3=0$) $\rightarrow$ $A_3\overline{B_3}$

if $A_3=B_3$ & $A_2>B_2$ (i.e. $X_3=1$ & $A_2=1$&$B_2=0$)

$\rightarrow$ $X_3A_2\overline{B_2}$

if $A_3=B_3$ & $A_2=B_2$ & $A_1>B_1$

(i.e. $X_3=1$ & $X_2=1$ & $A_1=1$&$B_1=0$)

$\rightarrow$ $X_3X_2A_1\overline{B_1}$

4-bit Magnitude Comparator

if $A_3=B_3$ & $A_2=B_2$ & $A_1=B_1$ & $A_0>B_0$

(i.e. $X_3=1$ & $X_2=1$ & $X_1=1$ & $A_0=1$&$B_0=0$)

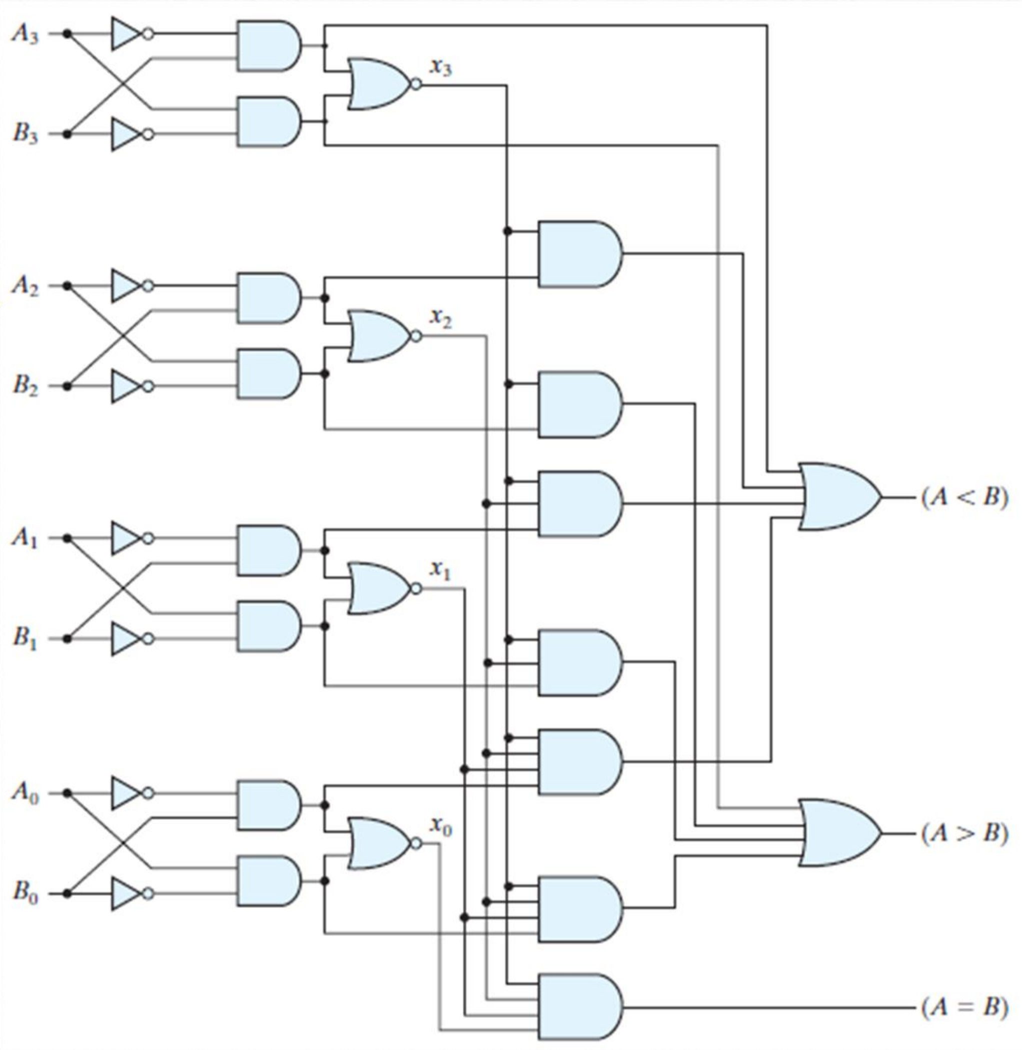
$\rightarrow \quad X_3X_2X_1A_0\overline{B_0}$

So $GT = A_3\overline{B_3}+X_3A_2\overline{B_2}+X_3X_2A_1\overline{B_1} + X_3X_2X_1A_0\overline{B_0}$

<u>Design of the less than output (LT):</u> In a similar way

$$LT = \overline{A_3}B_3+X_3\overline{A_2}B_2+X_3X_2\overline{A_1}B_1 + X_3X_2X_1\overline{A_0}B_0$$

4-bit Magnitude Comparator



4-bit Magnitude Comparator

Of course we can obtain a simpler design, as any two outputs can leads to the third one.

For example:

If $A \ngtr B$ & $A \nless B \rightarrow A = B$

i.e.

If GT=0 & LT=0 → EQ=1

so $EQ = \overline{GT + LT}$