Boolean Functions

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Boolean Functions

- A Boolean Function is described by an algebraic expression that consists of binary variables, the constants 0 and 1 and the logic operation symbols.
- For a given value of binary variables, the function can be equal to either 1 or 0.
- Example: $F(A, B, C) = A\overline{B} + BC + C$

  F is a boolean function that depends on three variables inputs A,B and C. A,B,C and F can only take 0 or 1.

Truth table

- A truth is a presentation of a Boolean function.
- For n input variables the number of rows = 2^n
- The binary combinations for the truth table are obtained from the binary numbers by counting from 0 to 2^n-1

Truth table

- **Example:** $F(A, B, C) = A\bar{B} + BC + C$

Decimal Value	A	B	C	$\bar{B}$	$A\bar{B}$	BC	F
0							
1							
2							
3							
4							
5							
6							
7							

Truth table

- Example: $F(A, B, C) = A\overline{B} + BC + C$

Decimal Value	A	B	C	$\overline{B}$	$A\overline{B}$	BC	F
0	0	0	0	1	0	0	0
1	0	0	1	1	0	0	1
2	0	1	0	0	0	0	0
3	0	1	1	0	0	1	1
4	1	0	0	1	1	0	1
5	1	0	1	1	1	0	1
6	1	1	0	0	0	0	0
7	1	1	1	0	0	1	1

Logic Circuit

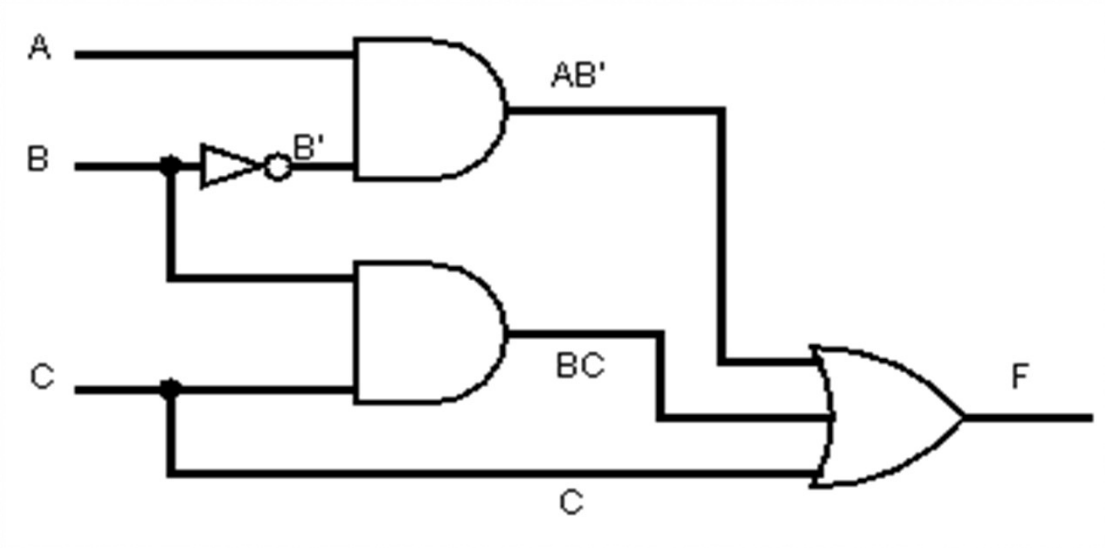
- A logic circuit is defined as a collection of coincidence and mixing gates.
- Voltage-operated logic circuits respond to two separate voltage levels that represent a binary variable equal to logic 1 or logic 0.
- A digital logic circuit is defined as the one in which voltages are assumed to have a finite number of distinct value. Types of digital logic circuits are combinational logic circuits and sequential logic circuits.

Logic circuit

- Example: $F(A, B, C) = A\overline{B} + BC + C$

Logic circuit

- Example: $F(A, B, C) = A\overline{B} + BC+C$



Minterms and Maxterms

Minterms

- A minterm is a product (AND) of all variables in the function, in direct or complemented form.
- A minterm has the property that it is equal to 1 on exactly one row of the truth table, i.e. it gives a true value (i.e., 1) for just one combination of the input variables.
- Minterms are called products because they are the logical AND of a set of variables.

Minterms and Maxterms

Minterms

- **A minterm is assigned an index (mi) based on a conventional binary encoding of the complementation pattern of the variables (where the variables in all the minterms are written in the same order, usually alphabetical). This convention assigns the value 1 to the direct form and 0 to the complemented form.**

Minterms and Maxterms

Maxterms

- A maxterm is a sum (OR) of all variables in the function, in direct or complemented form.
- A maxterm has the property that it is equal to 0 on exactly one row of the truth table, i.e. it gives a false value (i.e., 0) for just one combination of the input variables.
- Maxterms are called sums because they are the logical OR of a set of variables.

Minterms and Maxterms

Maxterms

- **A maxterm is assigned an index (Mi) based on a conventional binary encoding of the complementation pattern of the variables (where the variables in all the maxterms are written in the same order, usually alphabetical). The maxterm convention assigns the value 0 to the direct form and 1 to the complemented form.**

Minterms and Maxterms

			Minterms		Maxterms	
x	y	z	Term	Designation	Term	Designation
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

Minterms and Maxterms

Minterms (Example)

Example:

$$F(A, B, C) = A\bar{B} + BC + C$$

Minterms are

m_1, m_3, m_4, m_5, m_7

$\bar{A}\bar{B}C, \bar{A}BC, A\bar{B}\bar{C}, A\bar{B}C, ABC$

001, 011, 100, 101, 111

Decimal Value	A	B	C	F	
0	0	0	0	0	
1	0	0	1	1	m_1
2	0	1	0	0	
3	0	1	1	1	m_3
4	1	0	0	1	m_4
5	1	0	1	1	m_5
6	1	1	0	0	
7	1	1	1	1	m_7

Minterms and Maxterms

Maxterms (Example)

Example:

$$F(A, B, C) = A\overline{B} + BC + C$$

Maxterms are

$$M_0, M_2, M_6$$

$$A + B + C, A + \overline{B} + C, \overline{A} + \overline{B} + C$$

000, 010, 110

Decimal Value	A	B	C	F	
0	0	0	0	0	M_0
1	0	0	1	1	
2	0	1	0	0	M_2
3	0	1	1	1	
4	1	0	0	1	
5	1	0	1	1	
6	1	1	0	0	M_6
7	1	1	1	1	

Canonical Forms

Two dual canonical forms of any Boolean function are used:

- The term "Sum of Products" or "SoP" is used for the canonical form that is a disjunction (OR) of minterms (sum of minterms).
- The term "Product of Sums" or "PoS" is used for the canonical form that is a conjunction (AND) of maxterms (product of maxterms).

Canonical Forms

SoP (Example)

Example:

$$F(A,B,C) = A\bar{B} + BC + C$$

$$F(A,B,C) = m_1 + m_3 + m_4 + m_5 + m_7$$

$$F(A,B,C) = \sum m(1,3,4,5,7)$$

$$F(A,B,C) = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC$$

Canonical Forms

PoS (Example)

Example:

$$F(A,B,C) = A\overline{B} + BC + C$$

$$F(A,B,C) = M_0 M_2 M_6$$

$$F(A,B,C) = \Pi_M(0,2,6)$$

$$F(A,B,C) = (A + B + C)(A + \overline{B} + C)(\overline{A} + \overline{B} + C)$$

Boolean Functions Representation

- Boolean function.
- Truth table.
- Logic circuit.
- Sum of minterms.
- Product of maxterms.
- Sum of Products.
- Product of Sum.

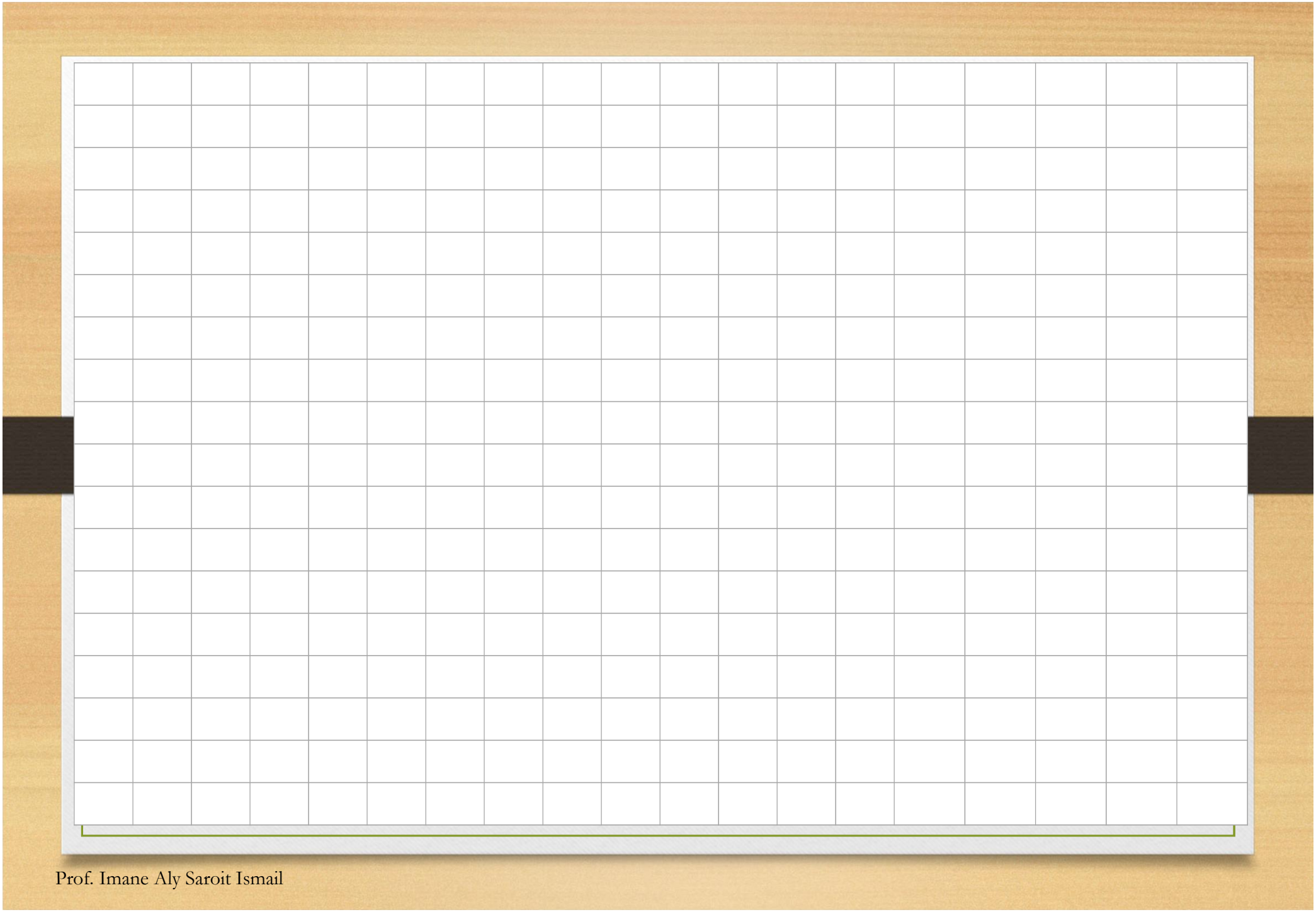
Boolean Functions Representation

Exercise:

For the Boolean function F given in the truth table:

- List the minterms of F.
- List the maxterms of F.
- List the minterms of $\bar{F}$.
- List the maxterms of $\bar{F}$.
- Obtain the Boolean function of F as SoP.
- Obtain the Boolean function of $\bar{F}$ as PoS.
- Draw the logic diagram of F.

W	X	Y	Z	F
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1



Boolean Functions Representation

Exercise:

For the Boolean function F do the following:

$$F(W,X,Y,Z) = W(X+Y+Z) + X\bar{Y}Z$$

- Obtain the truth table
- Express the function in sum of minterms.
- Express the function in product of maxterms.
- Draw its logic diagram

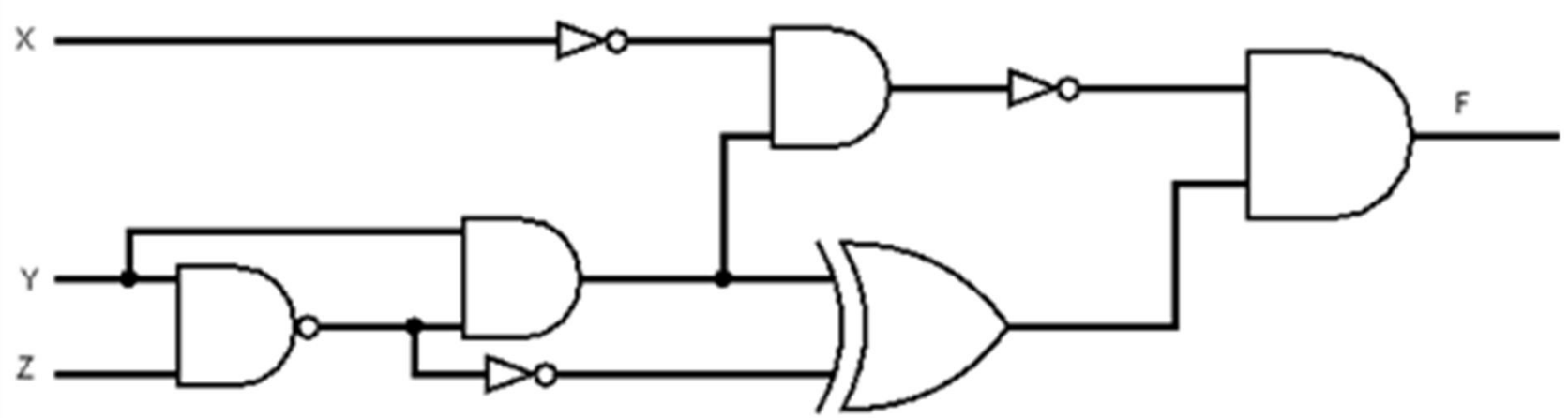
$$F(W,X,Y,Z) = W(X+Y+Z) + X\bar{Y}Z$$

Boolean Functions Representation

Exercise:

For the following logic circuit, find:

- The Boolean function of F.
- List the minterms of F.
- List the maxterms of F.
- List the minterms of $\bar{F}$.
- List the maxterms of $\bar{F}$.
- Obtain the Boolean function of F (as SoP and PoS).
- Draw the Circuit using only one And, no of ORs & Inverter(s).
- Using truth table; prove that F is equivalent to Y(X+Z).


X
Y
Z
F