Cairo University
Faculty of Computers & Artificial Intelligence
Theory of Computations



## Lab#6

### Description

- 1- Construct DFA for a given language problem.
- 2- Construct a NFA for a given language problem.
- 3- Convert NFA to DFA problem.

TA's will revise topic and let Students solve the questions

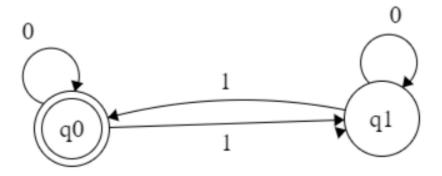
### **Problem 1**

 Construct a DFA recognizing the language {x | the number of 1's is divisible by 2, and 0'sby 3} over an alphabet ∑={0,1}

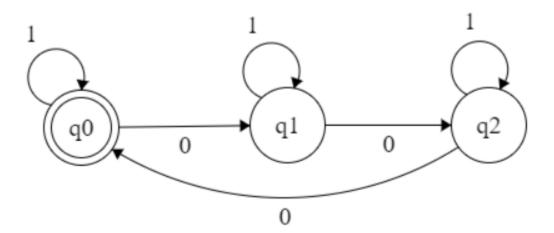
The given language L={ x | the number of 1's is divisible by 2, and 0's by 3} over an alphabet  $\Sigma$ ={0,1}.

The language is divided into two parts, first we need to find the number of 1's divisible by 2 and second find out the number of 0's divisible by 3, finally combine the two parts to generate a result.

Step 1 – DFA for the first part, number of 1's divisible by 2.



Step 2 – DFA for the second part, number of 0's divisible by 3.

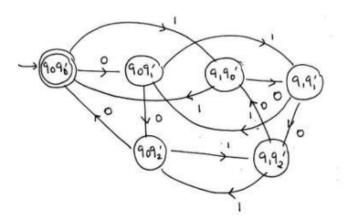


Step 3 – The final DFA is: DFA first part X DFA second part.

-	-	-
States	0	1
{q0q0'}	{q0q1'}	{q1q0'}
{q0q1'}	{q0q2'}	{q1q1'}
{q0q2'}	{q0q0'}	{q1q2'}
{q1q0'}	{q1q1'}	{q0q0'}
{q1q1'}	{q1q2'}	{q0q1'}
{q1q2'}	{q1q0'}	{q0q2'}

# Transition diagram

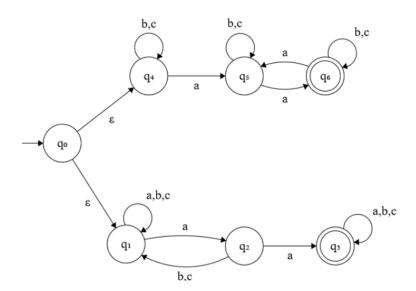
The transition diagram for the DFA is as follows –



# Problem 2

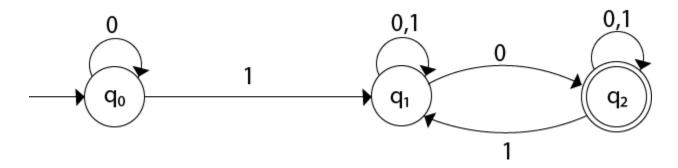
2. Construct an NFA to recognize the following language, where  $\Sigma = \{a, b, c\}$  L2 =  $\{w : w \text{ contains an even number of a's or contains the pattern 'aa'}\}$ 

#### Answer:



## **Problem 3**

Convert the given NFA to DFA.



Solution: For the given transition diagram we will first construct the transition table.

State	0	1
→q0	q0	q1
q1	{q1, q2}	q1
*q2	q2	{q1, q2}

Now we will obtain  $\delta'$  transition for state q0.

- 1.  $\delta'([q0], 0) = [q0]$
- 2.  $\delta'([q0], 1) = [q1]$

The  $\delta'$  transition for state q1 is obtained as:

- 1.  $\delta'([q1], 0) = [q1, q2]$  (new state generated)
- 2.  $\delta'([q1], 1) = [q1]$

Now we will obtain  $\delta^{\prime}$  transition on [q1, q2].

1. 
$$\delta'([q1, q2], 0) = \delta(q1, 0) \cup \delta(q2, 0)$$

2. 
$$= \{q1, q2\} \cup \{q2\}$$
  
3.  $= [q1, q2]$ 

4. 
$$\delta'([q1, q2], 1) = \delta(q1, 1) \cup \delta(q2, 1)$$

5. 
$$= \{q1\} \cup \{q1, q2\}$$

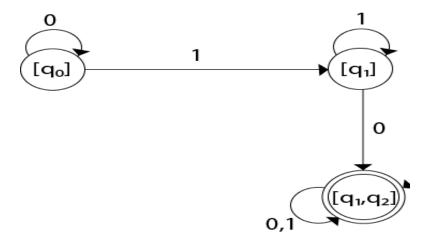
6. 
$$= \{q1, q2\}$$

7. 
$$= [q1, q2]$$

The state [q1, q2] is the final state as well because it contains a final state q2. The transition table for the constructed DFA will be:

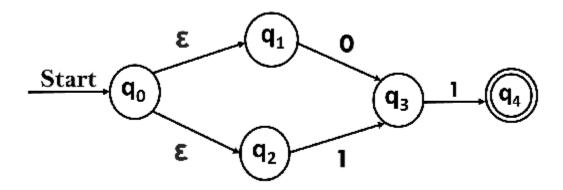
State	0	1
→[q0]	[q0]	[q1]
[q1]	[q1, q2]	[q1]
*[q1, q2]	[q1, q2]	[q1, q2]

The Transition diagram will be:



## **Problem 4**

Convert the NFA with  $\epsilon$  into its equivalent DFA.



#### Solution:

Let us obtain  $\epsilon$ -closure of each state.

- 1.  $\epsilon$ -closure {q0} = {q0, q1, q2}
- 2.  $\epsilon$ -closure  $\{q1\} = \{q1\}$
- 3.  $\epsilon$ -closure  $\{q2\} = \{q2\}$
- 4.  $\epsilon$ -closure {q3} = {q3}
- 5.  $\epsilon$ -closure  $\{q4\} = \{q4\}$

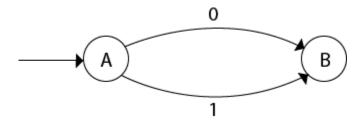
Now, let  $\epsilon$ -closure  $\{q0\}$  =  $\{q0, q1, q2\}$  be state A.

Hence

$$\begin{split} \delta'(A,0) &= \epsilon\text{-closure} \, \{\delta((q0,\,q1,\,q2),\,0) \,\} \\ &= \epsilon\text{-closure} \, \{\delta(q0,\,0) \, \cup \, \delta(q1,\,0) \, \cup \, \delta(q2,\,0) \,\} \\ &= \epsilon\text{-closure} \, \{q3\} \\ &= \{q3\} \qquad \qquad \textbf{call it as state B}. \end{split}$$

$$\begin{split} \delta'(A,1) &= \epsilon\text{-closure} \left\{ \delta((q0,q1,q2),1) \right\} \\ &= \epsilon\text{-closure} \left\{ \delta((q0,1) \cup \delta(q1,1) \cup \delta(q2,1) \right\} \\ &= \epsilon\text{-closure} \left\{ q3 \right\} \\ &= \left\{ q3 \right\} = B. \end{split}$$

The partial DFA will be



Now,

$$\delta'(B, 0) = \epsilon\text{-closure } \{\delta(q3, 0)\}$$

$$= \varphi$$

$$\delta'(B, 1) = \epsilon\text{-closure } \{\delta(q3, 1)\}$$

$$= \epsilon\text{-closure } \{q4\}$$

$$= \{q4\}$$
i.e. state C

For state C:

1. 
$$\delta'(C, 0) = \epsilon$$
-closure  $\{\delta(q4, 0)\}$ 

3. 
$$\delta'(C, 1) = \epsilon$$
-closure  $\{\delta(q4, 1)\}$ 

## The DFA will be,

