

## Midterm

29<sup>th</sup> of March 2015

### Question1

1. (1 mark) Prove that  $(P \vee Q) \wedge (R \vee P) \wedge (\neg Q \vee \neg R \vee P) \equiv P$

**Solution (it is OK if students prove by using the truth table):**

$$(P \vee Q) \wedge (R \vee P) \wedge (\neg Q \vee \neg R \vee P) \equiv$$

$$[\neg Q \wedge (P \vee Q) \wedge (R \vee P)] \vee [\neg R \wedge (P \vee Q) \wedge (R \vee P)] \vee [P \wedge (P \vee Q) \wedge (R \vee P)] \equiv$$
$$P \vee P \vee P \equiv P$$

2. (1 mark) Given the premises  $(\exists x)P(x)$  and  $(\forall x)[P(x) \rightarrow Q(x)]$  give a series of steps concluding that  $(\exists x)Q(x)$ .

#### Solution

- 1)  $(\exists x)P(x)$  (premise)
  - 2)  $P(a)$  for some  $a$  from the domain (from 1 using Existential Specification – **skolem**  
i. **constant**)
  - 3)  $(\forall x)[P(x) \rightarrow Q(x)]$  (premise)
  - 4)  $P(a) \rightarrow Q(a)$  for some  $a$  from the domain (from 3 using Universal Specification)
  - 5)  $Q(a)$  for some  $a$  from the domain (from 2 and 4 using Modus Ponens)
  - 6)  $(\exists x)Q(x)$  (from 5 using Existential Generalization).
3. In the domain of people, consider the following sentences:
- Mona is a Professor
  - Mohamed is the Dean.
  - Deans are professors.
  - All professors consider the dean a friend or don't know him.
  - Everyone is a friend of someone.
  - People only criticize people that are not their friends.
  - Mona criticized Mohamed.
- a) (1 marks) Translate these sentences into Predicate Logic.
  - b) (1 mark) Transform the above Predicate Logic sentences to CNF.
  - c) (1 mark) Prove by resolution, the following statement:  
Mona is not friend of Mohamed
  - d) (1 mark) Prove, by using contradiction, the following statement:  
Mona is not friend of Mohamed.

**Solution:**

a) FOPL:

1. Professor(Mona).
2. Dean(Mohamed).
3.  $\forall x (\text{Dean}(x) \rightarrow \text{Professor}(x))$
4.  $\forall x \forall y \text{Professor}(x) \wedge \text{Dean}(y) \rightarrow \text{Friend}(x, y) \vee \neg \text{Know}(x, y)$
5.  $\forall x \exists y \text{Friend}(x, y)$
6.  $\forall x \forall y \text{Criticize}(x, y) \rightarrow \neg \text{Friend}(x, y)$
7. Criticize(Mona, Mohamed)

b) Transform to CNF:

1. Professor(Mona).
2. Dean(Mohamed).
3.  $\neg \text{Dean}(x) \vee \text{Professor}(x)$
4.  $\neg \text{Professor}(x) \vee \neg \text{Dean}(y) \vee \text{Friend}(x, y) \vee \neg \text{Know}(x, y)$
5.  $\neg \text{Criticize}(x, y) \vee \neg \text{Friend}(x, y)$
6. Criticize(Mona, Mohamed)

c) The proof by resolution:

5+6= 7 { y/Mohamed, x/Mona}:  $\neg \text{Friend}(\text{Mona}, \text{Mohamd})$

d) The proof by contradiction:

**Assume: 7. Friend(Mona, Mohamed).**

5+7= 8 {y/Mohamed, x/Mona}:  $\neg \text{Criticize}(\text{Mona}, \text{Mohamed})$

6+8 = 9: nil (contradiction)

$\diamond \neg \text{Friend}(\text{Mona}, \text{Mohamed})$

## Question2

(1 marks)  $X=[b,d,f]$ .

(1 marks) It should be like that:

$\max(X,Y,X) :- X \geq Y, !.$

$\max(X,Y,Y).$

## Question3

class precedence list:

Mohamed –Director-manager -sales –technical-permanent- employee 1/2 marks

Mohamed salary = 10000 L.E 1/2 marks

class precedence list

Ahmed- consultant – temporary –manager- sales-employee 1/2 marks

Ahmed salary =  $200 \times 15 = 30,00$  1/2 marks