Fuzzy Logic Introduction

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Outline

- Fuzzy Logic
  - What is Fuzzy Logic?
  - Applications
- Fuzzy Sets
  - Examples
- Operations of Fuzzy Sets
- Fuzzy Rules
- Fuzzy Controls(Architecture)
  - Components/Steps
  - Examples

WHAT IS FUZZY LOGIC?

- Boolean logic uses sharp distinctions. It forces us to draw lines between members of a class and non- members. For instance, we may say, someone is tall because his height is 181 cm. If we drew a line at 180 cm, we would find that a person, who is 179 cm, is short. Is he really a short man or we have just drawn an arbitrary line in the sand?

- Assign a possibility that a man 181 cm tall is really tall to be a value of 0.86. This work led to an inexact reasoning technique often called **possibility theory**.

- **Possibility theory** is modified to a formal system of mathematical logic and introduced a new concept for applying natural language terms.

WHAT IS FUZZY LOGIC?

- Logic that extended the range of truth values to all real numbers in the interval between 0 and 1. It used a number in this interval **to represent the possibility that a given statement was true or false**.

- **Fuzzy logic,** Unlike two-valued Boolean logic, is multi-valued. It deals with degrees of membership and degrees of truth.

Fuzzy Logic Properties

- Fuzzy logic can handle the concept of partial truth - truth values between “completely true” and “completely false”
- Fuzzy logic reflects how people think. It attempts to model our sense of words, our decision making and our common sense. As a result, it is leading to new, more human, intelligent systems.
- Is a **form of knowledge representation** suitable for notions that cannot be defined precisely.
- Accepts **noisy**, **imprecise** input.
- Is based on the idea that **all things admit of degrees**. Temperature, height, speed, distance, beauty – all come on a sliding scale.

Sample Applications

For washing machines, Fuzzy Logic control is almost becoming a standard feature

fuzzy controllers to load-weight, fabric-mix, and dirt sensors and automatically set the wash cycle for the best use of power, water, and detergent.

GE WPRB9110WH Top Load Washer

Haier ESL-T21 Top Load Washer

[LG WD14121 Front Load Washer]

[Miele WT945 Front Load All-in-One Washer / Dryer]

[AEG LL1610 Front Load Washer]

[Zanussi ZWF1430W Front Load Washer]

Others: Samsung, Toshiba, National, Matsushita, etc.

Sample Applications

Station subway system is controlled by a fuzzy computer (Seiji Yasunobu and Soji Miyamoto of Hitachi)

Nissan – fuzzy automatic transmission, fuzzy anti-skid braking system

CSK, Hitachi – Hand-writing Recognition

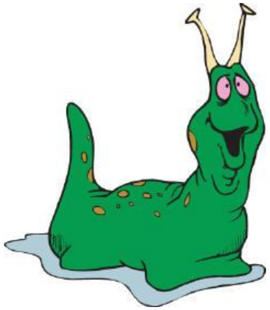
Sony - Hand-printed character recognition

Ricoh, Hitachi – Voice recognition

NASA has studied fuzzy control for automated space docking: simulations show that a fuzzy control system can greatly reduce fuel consumption

The Canon camera's fuzzy control system uses 12 inputs: 6 to obtain the current clarity data provided by the CCD and 6 to measure the rate of change of lens movement. The output is the position of the lens. The fuzzy control system uses 13 rules and requires 1.1 kilobytes of memory.

TRADITIONAL REPRESENTATION OF LOGIC



Slow

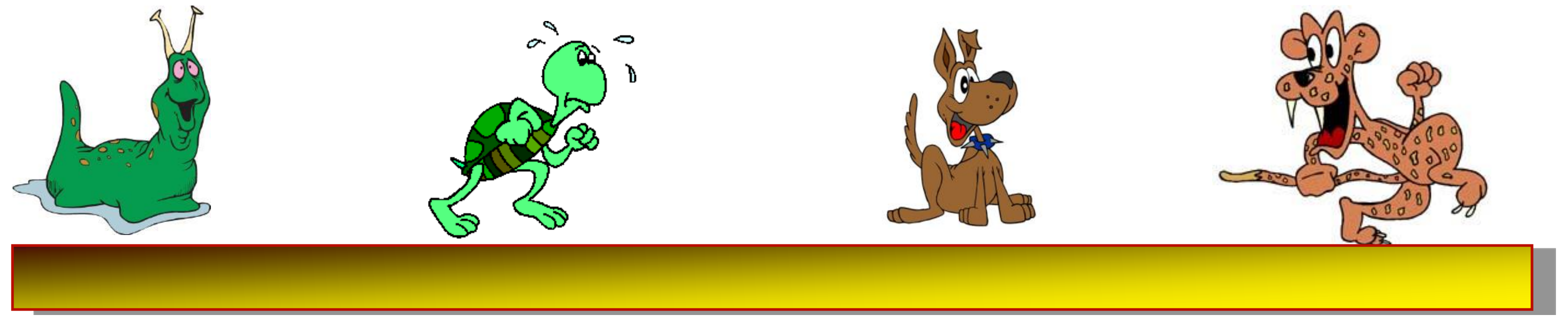
Speed = 0

Fast

Speed = 1

```
bool speed;
// get the speed
if ( speed == 0)
{
        // speed is slow
}
else
{
        // speed is fast
}
```

FUZZY LOGIC REPRESENTATION



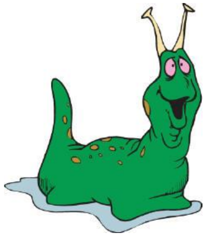
Slowest Slow Fast Fastest

```
float speed;
// get the speed
if ((speed >= 0.0)&&(speed < 0.25))
{
    //  speed is slowest
}
else if ((speed >= 0.25)&&(speed < 0.5))
{
    //  speed is slow
}
else if ((speed >= 0.5)&&(speed < 0.75))
{
    //  speed is fast
}
else // speed >= 0.75 && speed < 1.0
{
    //  speed is fastest
}
```

FUZZY LOGIC REPRESENTATION

- Some problems must be represented in terms of fuzzy sets.

- What are Fuzzy Sets?



Slowest

[0.0 – 0.25]

Slow

[0.25 – 0.50]

Fast

[0.50 – 0.75]

Fastest

[0.75 – 1.00]

Fuzzy Sets

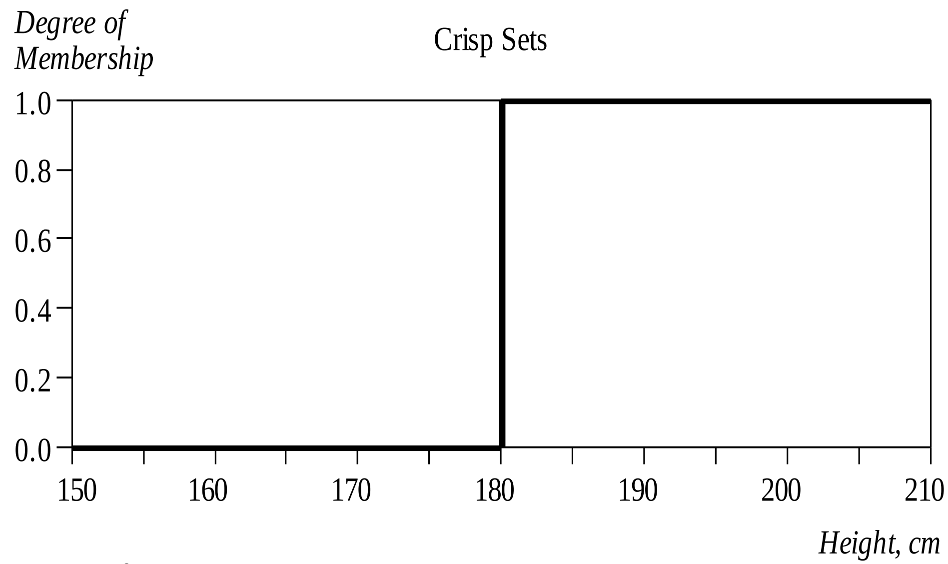
- Fuzzy Sets can represent the degree to which a quality/degree is possessed.
- Fuzzy Sets (Simple Fuzzy Variables) have values in the range of [0…1]
- The classical example in fuzzy sets is tall men. The elements of the fuzzy set "tall men" are all men, but their degrees of membership depend on their height.

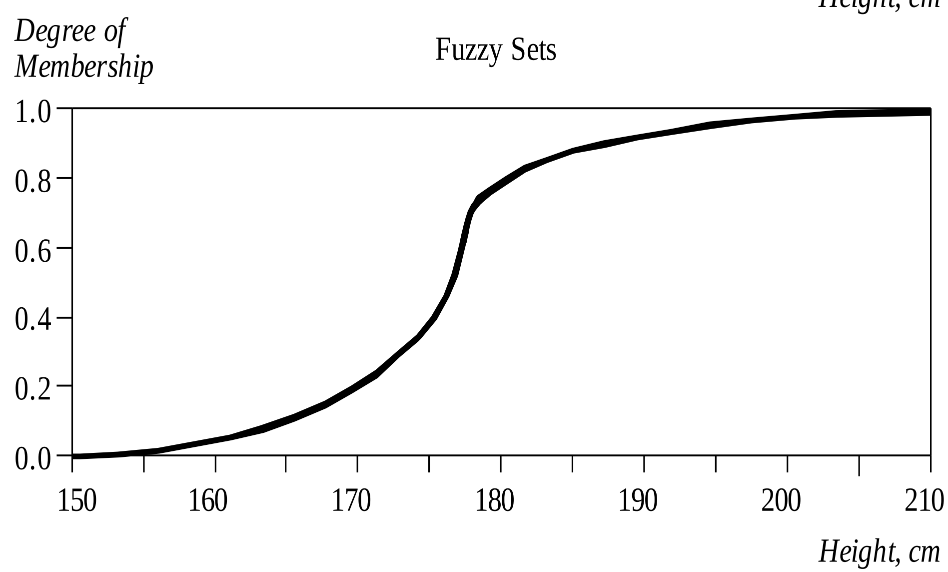
Name	Height, cm	Degree of Membership	
		Crisp	*Fuzzy*
Chris	208	1	1.00
Mark	205	1	1.00
John	198	1	0.98
Tom	181	1	0.82
David	179	0	0.78
Mike	172	0	0.24
Bob	167	0	0.15
Steven	158	0	0.06
Bill	155	0	0.01
Peter	152	0	0.00

Fuzzy Sets

- The x-axis represents the **universe of discourse** – the range of all possible values applicable to a chosen variable. In tall example, the variable is the man height. According to this representation, the universe of men’s heights consists of all tall men.

- The y-axis represents the **membership value** of the fuzzy set. In tall example, the fuzzy set of “tall men” maps height values into corresponding membership values.





Fuzzy Sets

- Let X be the universe of discourse and its elements be denoted as x. In the classical set theory, **crisp set** A of X is defined as function $f_A(x)$ called the characteristic function of A

$f_A(x)$: X -> {0, 1}, where

$$f_A(x) = \begin{cases} 1, \text{if } x \in A \\ 0, \text{if } x \notin A \end{cases}$$

- In the fuzzy theory, fuzzy set A of universe X is defined by function $m_A(x)$ called the **membership function** of set A

$m_A(x)$: X -> [0, 1], where

$m_A(x) = 1$ if x is **totally** in A;
$m_A(x) = 0$ if x is **not** in A;
$0 < m_A(x) < 1$ if x is **partly** in A.

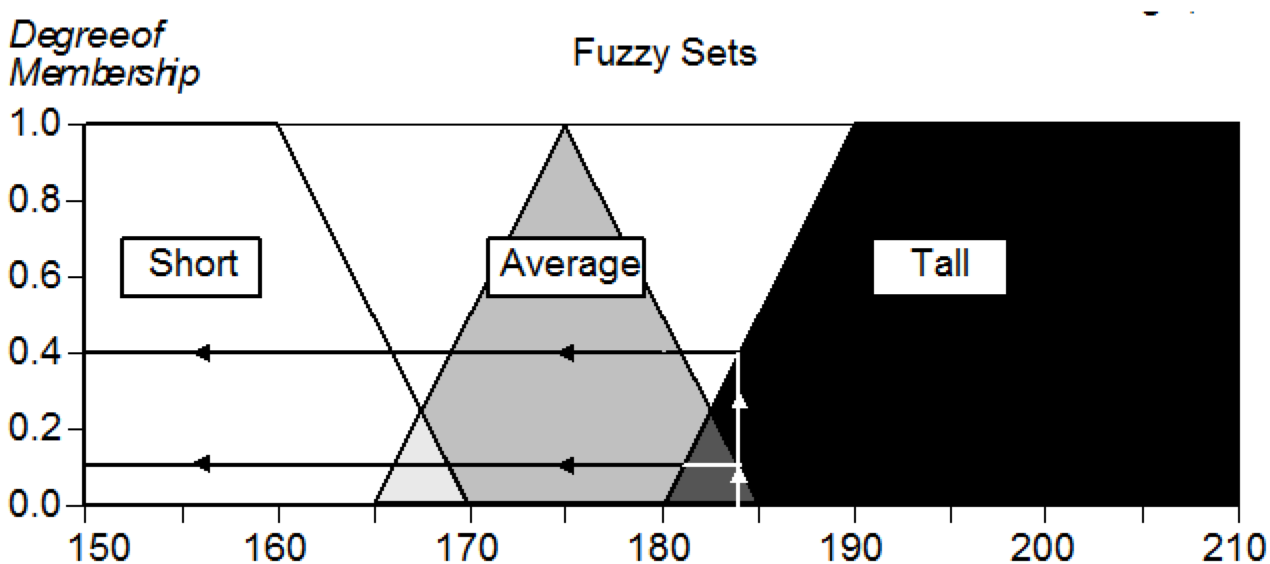
Fuzzy Linguistic Variables

- At the root of fuzzy set theory lies the idea of linguistic variables.

- A linguistic variable is a **fuzzy variable**. For example, the statement “John is tall” implies that the **linguistic variable John** takes the **linguistic value tall**.

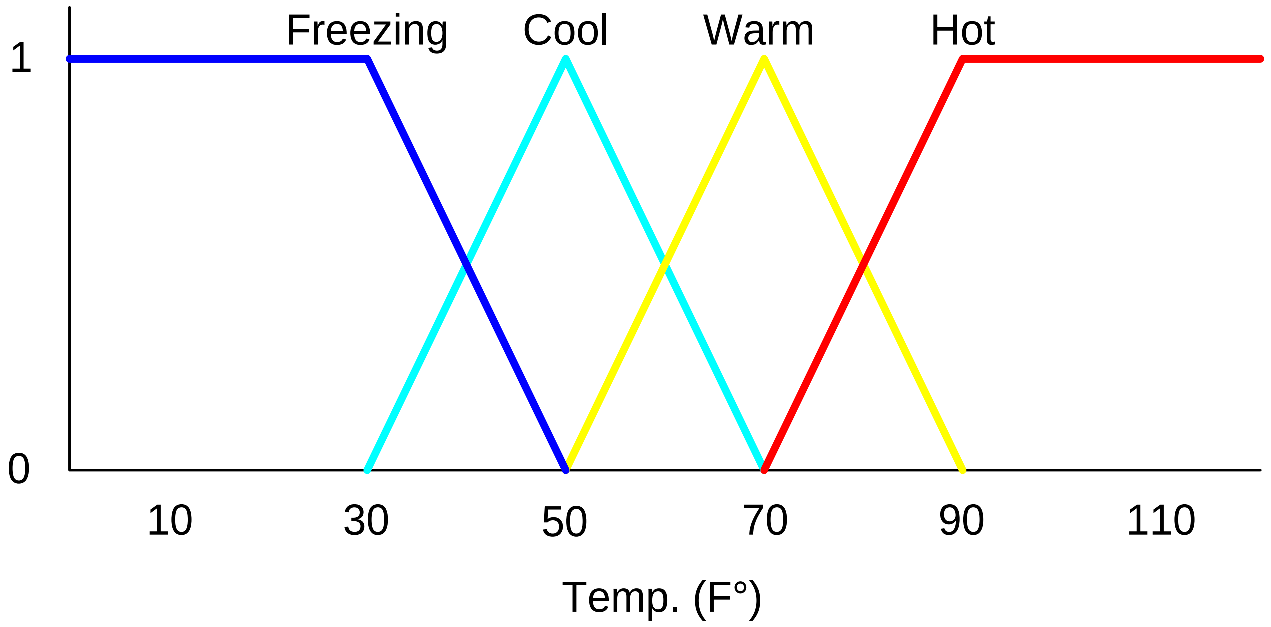
Example 1

- men’s height: **{short, average, tall}**
- Question: How tall is man with 184 cm?
- Answer: a man who is 184 cm tall is a member of the **average** set with a **degree of membership** of **0.1**, and at the same time, he is also a member of the **tall** set with a degree of **0.4**.



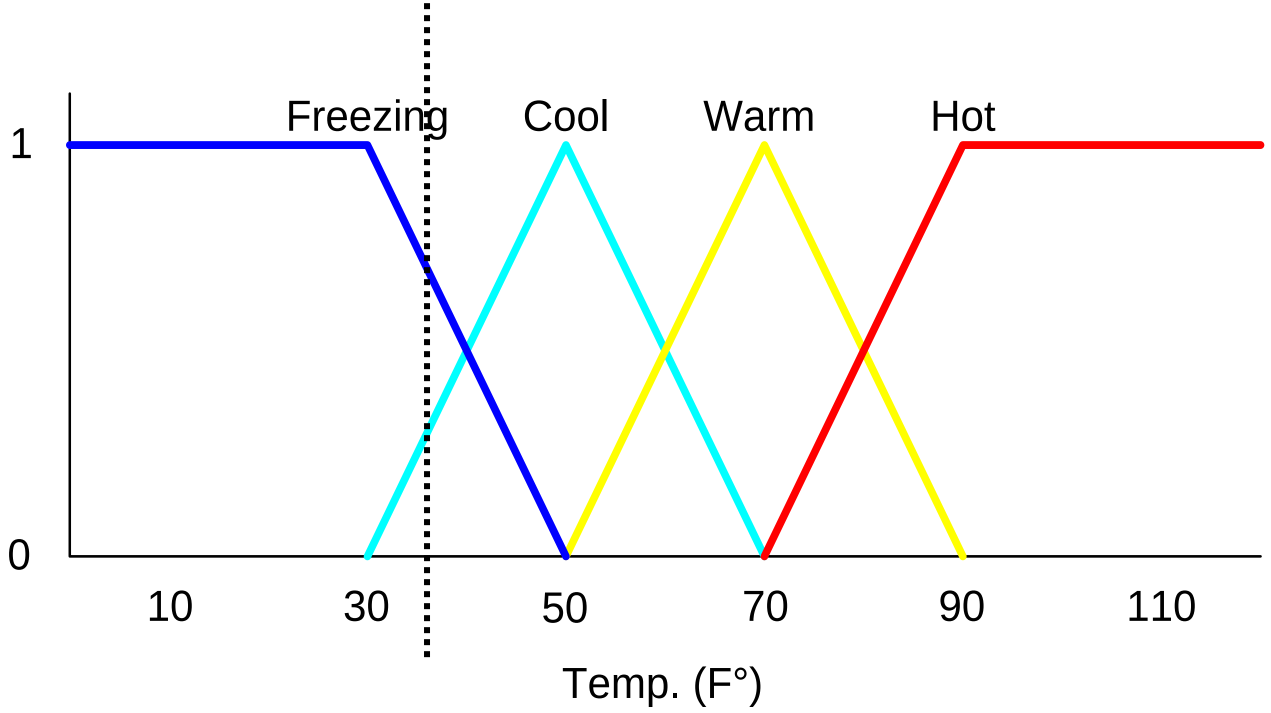
Membership Functions

- Temp: {Freezing, Cool, Warm, Hot}
- Degree of Truth or "Membership"



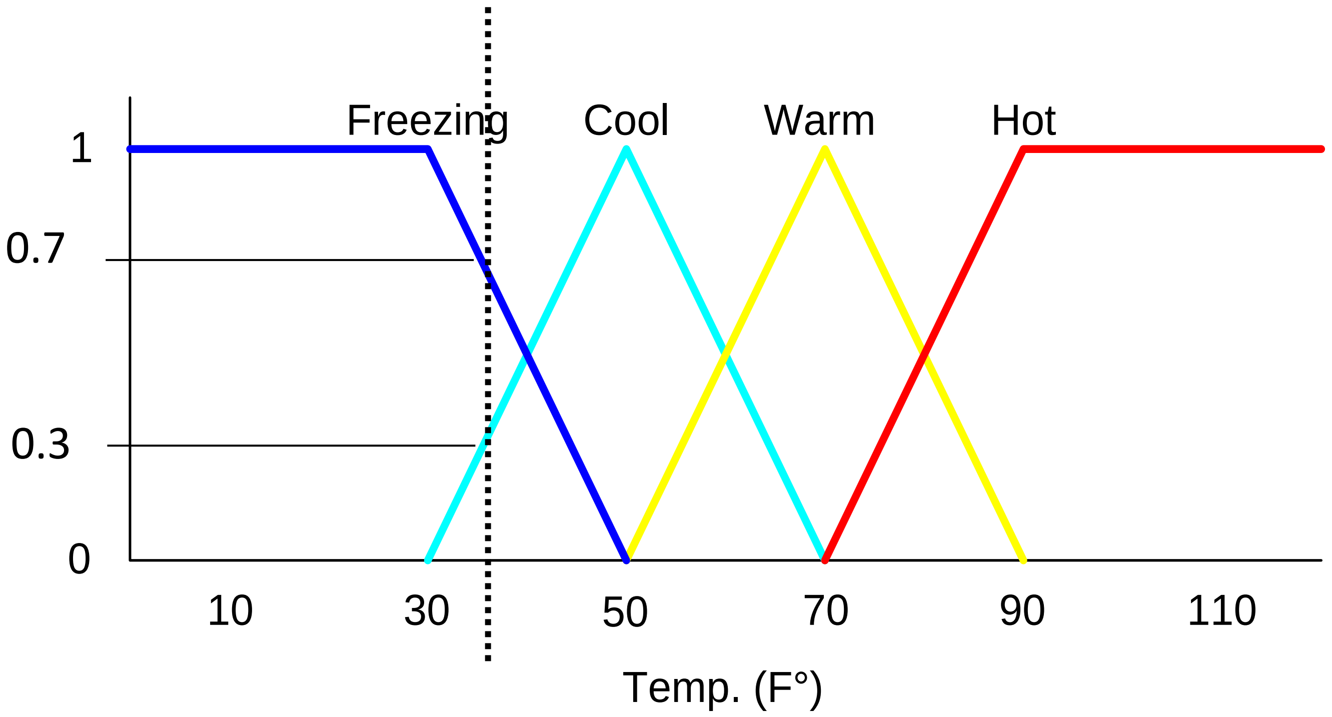
Membership Functions

- How cool is 36 F° ?



Membership Functions

- How cool is 36 F° ?
- It is 30% Cool and 70% Freezing



Membership Functions

- **Shape of membership function is usually triangle** (but it could also be trapezoidal or other shape)
- **Height usually normalized to 1 (so in total: [0..1])**
- **Width of the base of function depends on number of functions**
- **Center points of functions evaluate to 1**
- **Overlap is usually 50% at base of function**

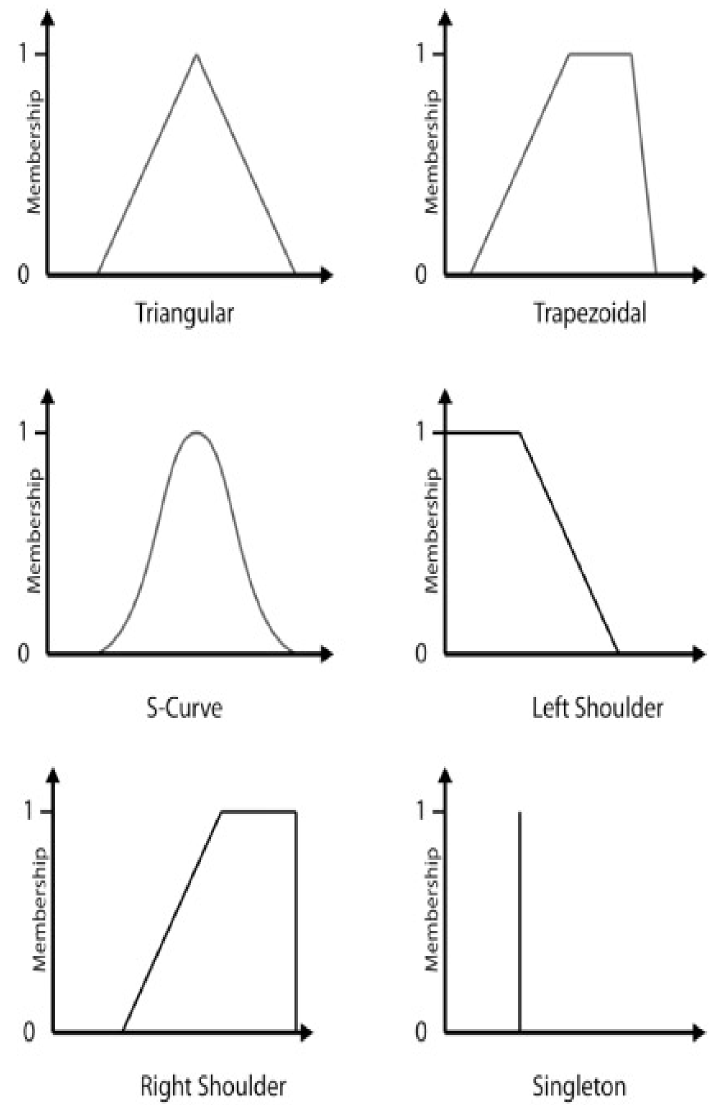
Well known Membership Functions

<u>For triangular fuzzy sets:</u>

A fuzzy set can be defined by 3 points {a, b, c} on the X-axis, where membership of a and c are zeros, while membership of b is one.

<u>For trapezoidal fuzzy sets:</u>

A fuzzy set can be defined by 4 points {a, b, c, d} on the X-axis, where membership of a and d are zeros, while membership of b and c is one.



Examples

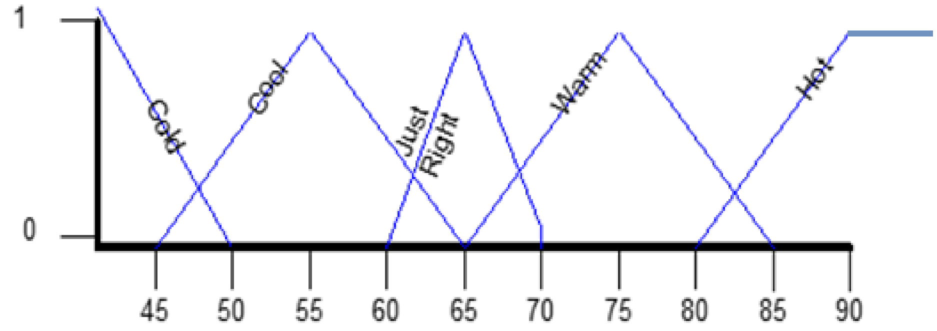
Air Temperature

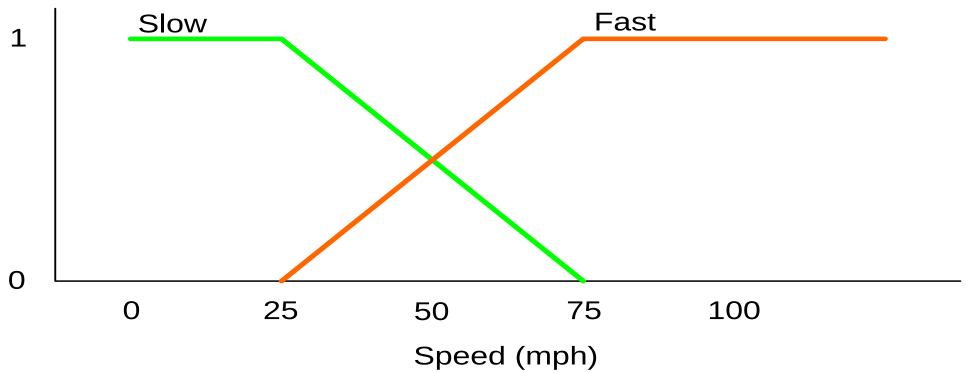
- Set cold {0, 0, 50}
- Set cool {45, 55, 65}
- Set just right {60, 65, 70}
- Set warm {65, 75, 85}
- Set hot {80, 90, ∞, ∞}

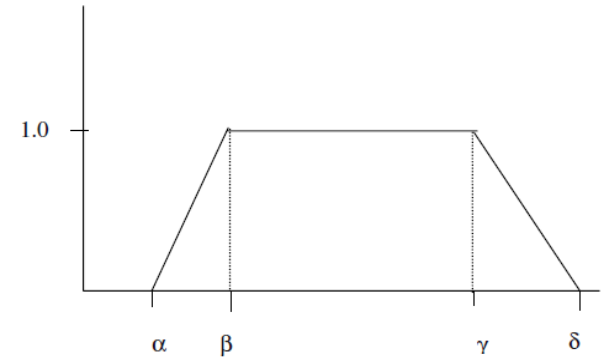
Speed

- Set slow {0, 0, 25,75}
- Set fast {25, 75,∞, ∞}

Set {α, β, γ, δ}

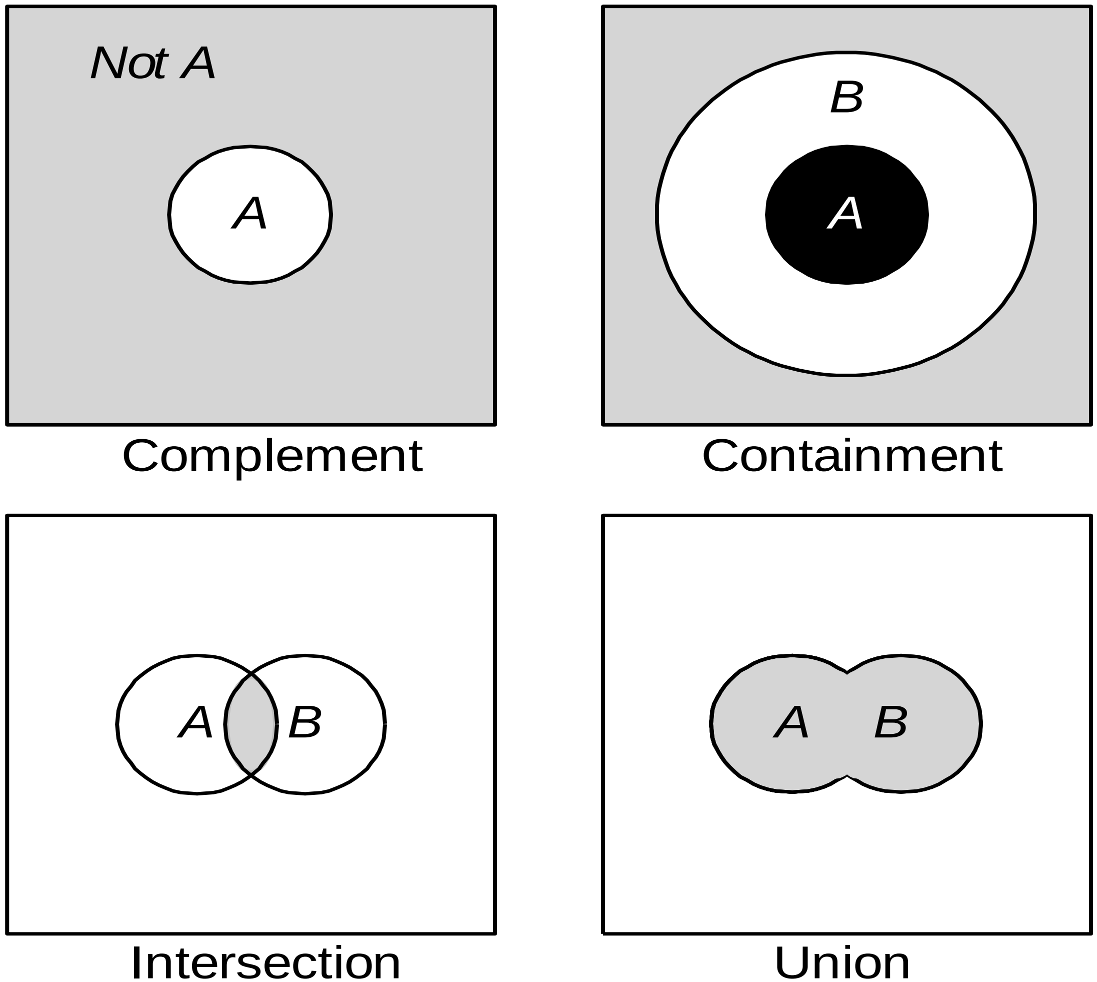






Operations of Fuzzy Sets

- Traditional set operations:

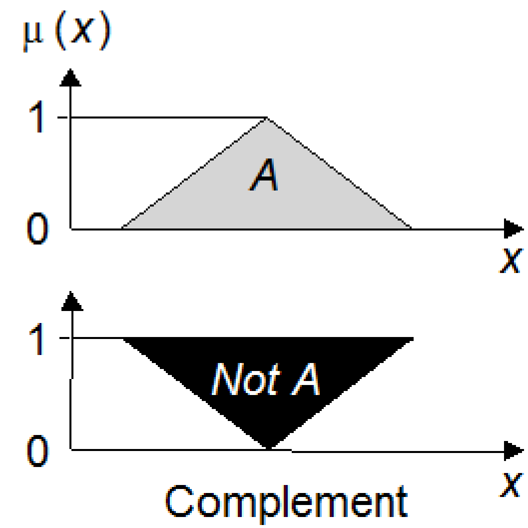


Operations of Fuzzy Sets

Complement

- Crisp Sets: **Who** does not belong to the set?
- Fuzzy Sets: **How much** do elements not belong to the set?
- Example: if we have the set of tall men, its complement is the set of NOT tall men.
- If A is the fuzzy set, its complement $\neg A$ can be found as follows:

$$\mu_{\neg A}(x) = 1 - \mu_A(x)$$



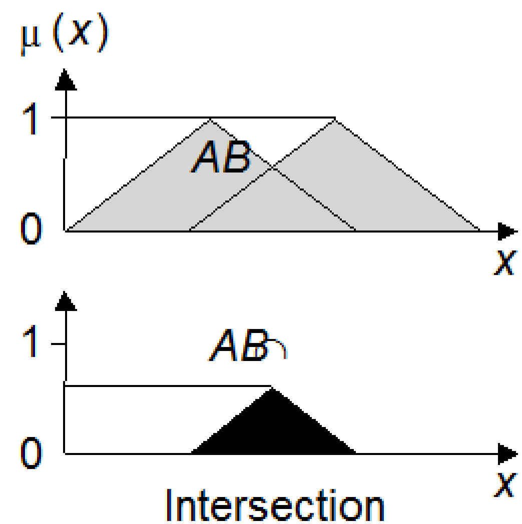
Operations of Fuzzy Sets

Intersection

- Crisp Sets: Which element belongs to both sets?
- Fuzzy Sets: How much of the element is in both sets? an element may partly belong to both sets with different memberships.
- Fuzzy intersection is the lower membership in both sets of each element.
- The fuzzy intersection of two fuzzy sets A and B on universe of discourse X:

$$\mu_{A \cap B}(x) = \min [\mu_A (x), \mu_B (x)] = \mu_A (x) \cap \mu_B(x),$$

where $x \in X$



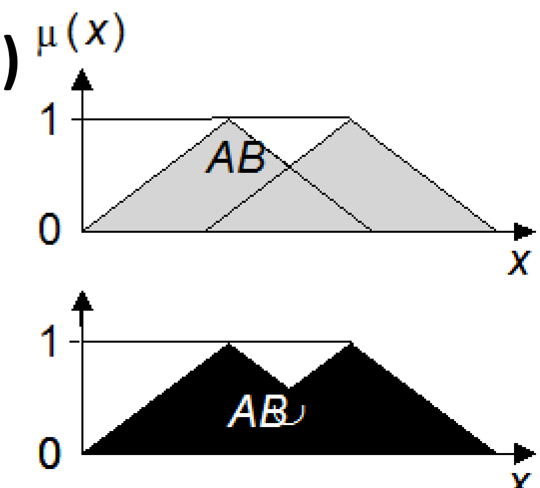
Operations of Fuzzy Sets

Union

- Crisp Sets: Which element belongs to either set?
- Fuzzy Sets: How much of the element is in either set?
- Fuzzy Union is the reverse of the intersection, so the union is the largest membership value of the element in either set.
- The fuzzy operation for forming the union of two fuzzy sets A and B on universe X can be given as:

$$\boldsymbol{\mu_{A \cup B}(x) = \max\,[\mu_A(x), \mu_B(x)] = \mu_A(x) \cup \mu_B(x)}$$

where $x \in X$



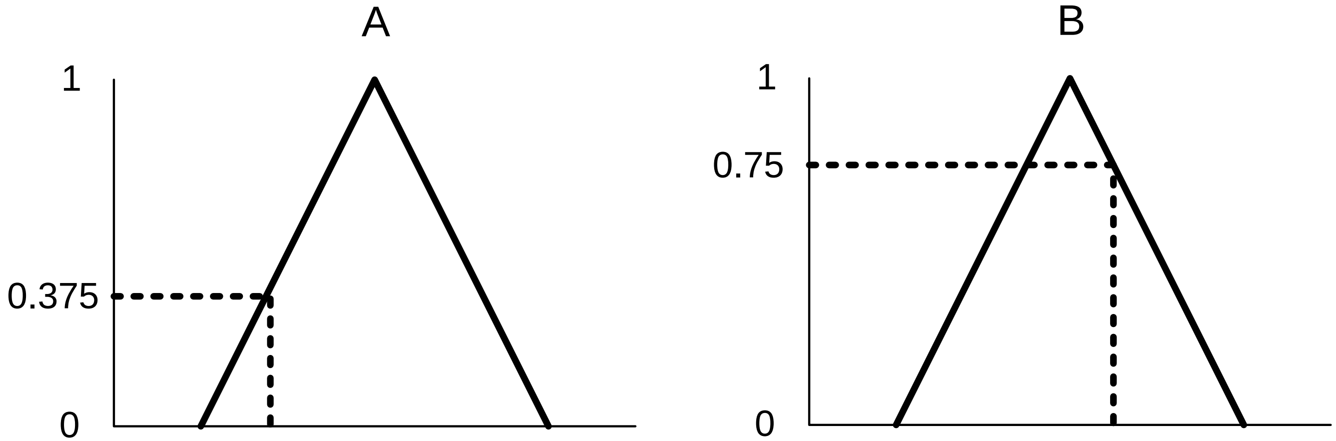
Union

Fuzzy Logic

- How do we use fuzzy membership functions in predicate logic?
- Fuzzy logic Connectives:
  - Fuzzy Conjunction(Intersection), $\wedge$
  - Fuzzy Disjunction(union), $\vee$
- Operate on degrees of membership in fuzzy sets

Fuzzy Disjunction

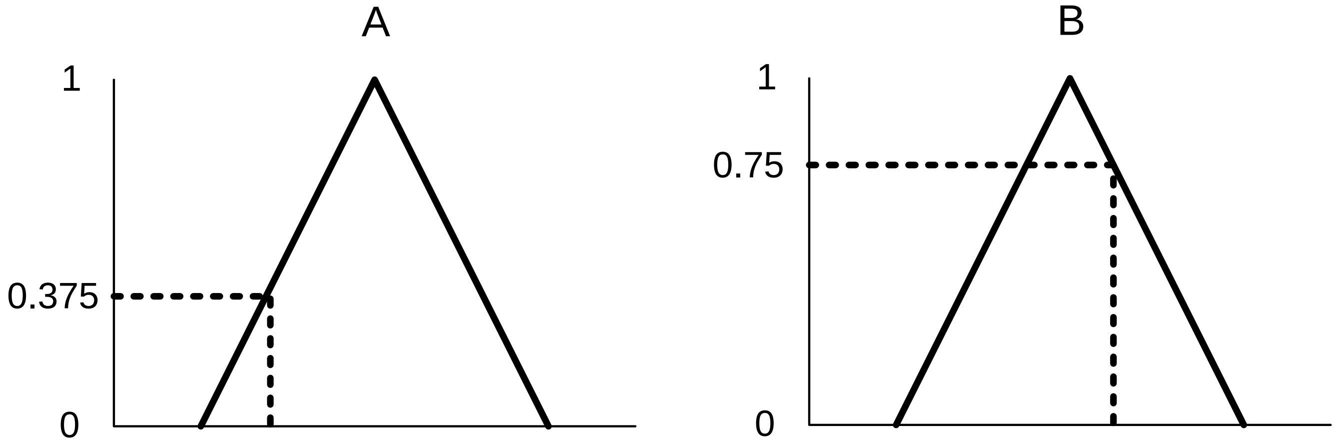
- $A \vee B \triangleq \max(A, B)$
- $A \vee B = C$ "Quality C is the disjunction of Quality A and B"



$$(A \vee B = C) \Rightarrow (C = 0.75)$$

Fuzzy Conjunction

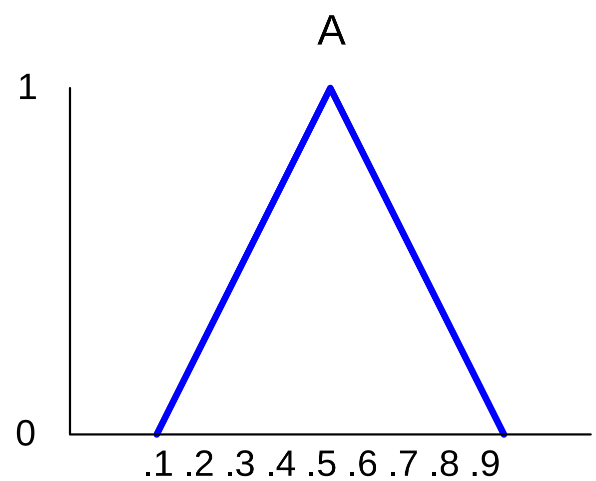
- $A \wedge B \triangleq \min(A, B)$
- $A \wedge B = C$ "Quality C is the conjunction of Quality A and B"

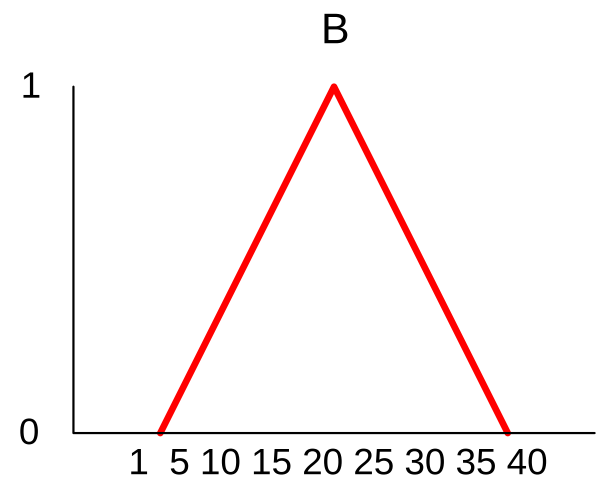


$$(A \wedge B = C) \Rightarrow (C = 0.375)$$

Example: Fuzzy Conjunction

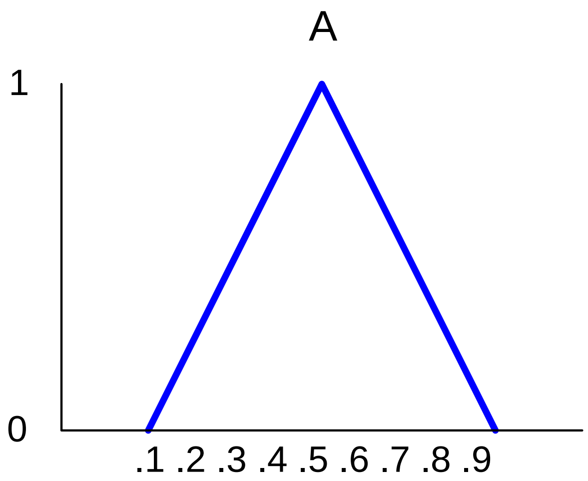
Calculate A∧B given that A is .4 and B is 20

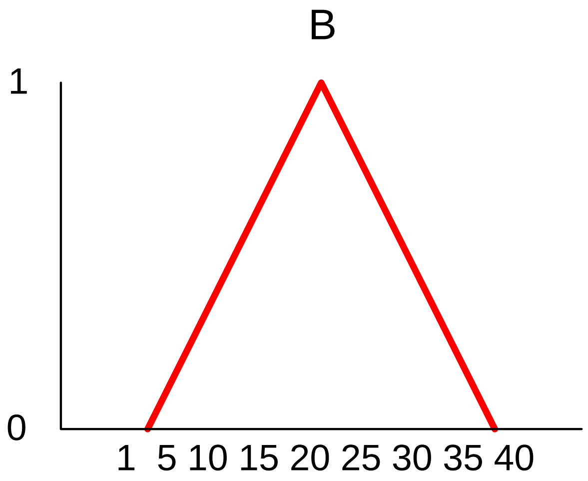




Example: Fuzzy Conjunction

Calculate $A \wedge B$ given that A is .4 and B is 20

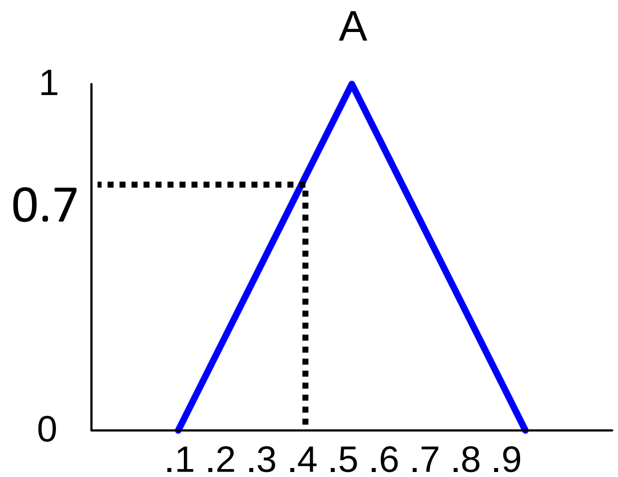


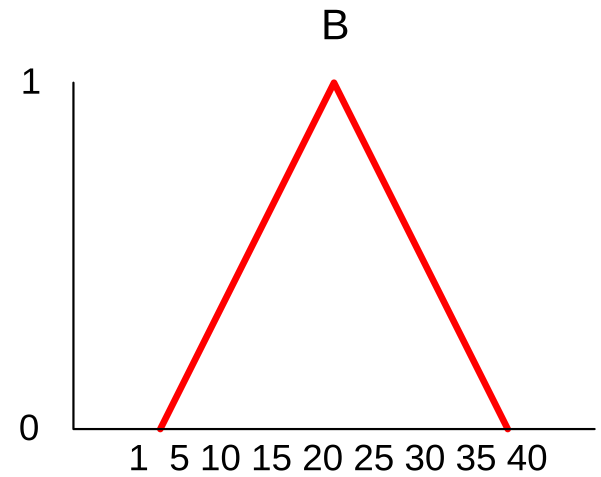


Determine degrees of membership:

Example: Fuzzy Conjunction

Calculate A∧B given that A is .4 and B is 20



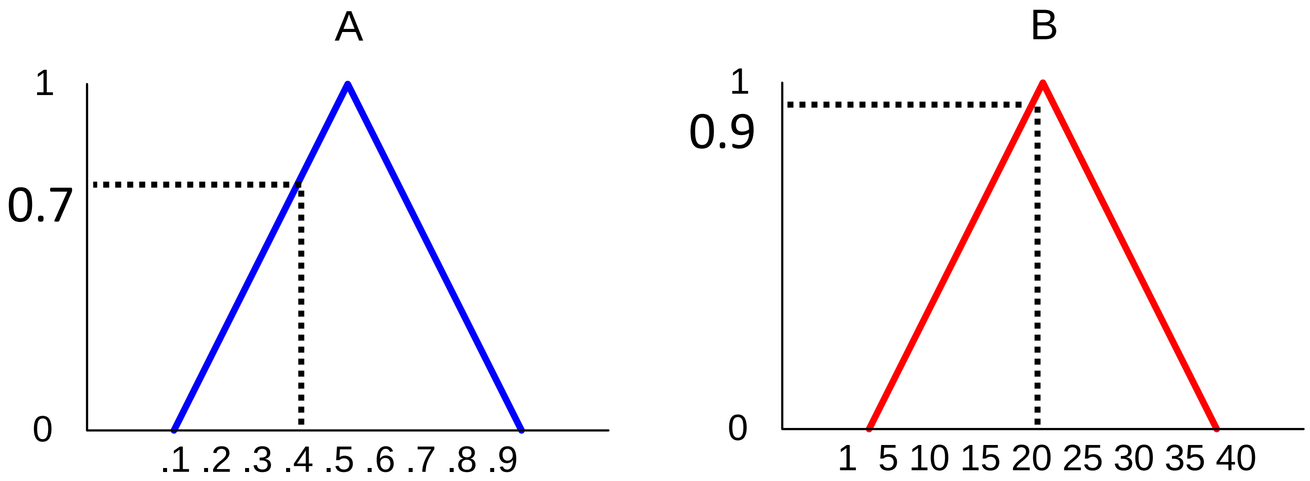


Determine degrees of membership:

A = 0.7

Example: Fuzzy Conjunction

Calculate $A \wedge B$ given that A is .4 and B is 20

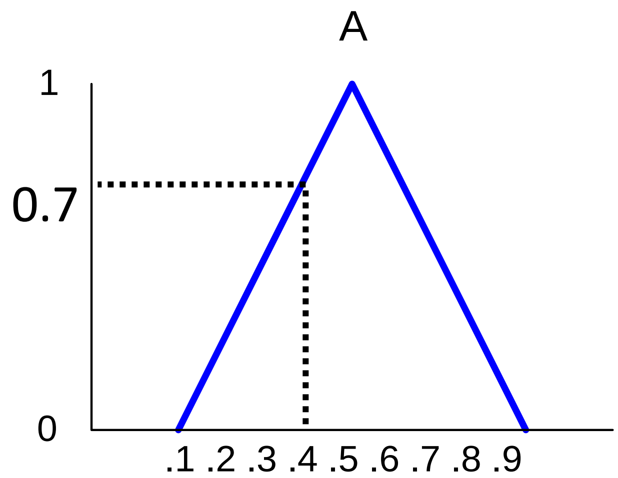


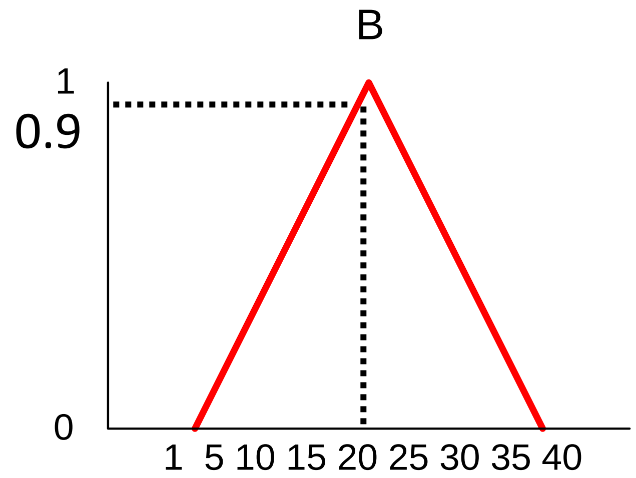
Determine degrees of membership:

A = 0.7   B = 0.9

Example: Fuzzy Conjunction

Calculate A∧B given that A is .4 and B is 20





Determine degrees of membership:

A = 0.7   B = 0.9

Apply Fuzzy AND

A∧B = min(A, B) = 0.7

Fuzzy Rules

- A fuzzy rule can be defined as a conditional statement in the form:
  IF x is A
  THEN y is B

- Where x and y are **linguistic variables**; and A and B are linguistic values determined by fuzzy sets on the universe of discourses X and Y, respectively.

- If **antecedents** (x) is true to some degree of membership, then **consequent** (y) is also true to that same degree.

- EX:
  IF **speed** is **slow**
  THEN **stopping_distance** is **short**

Fuzzy Rules

- A fuzzy rule can have multiple antecedents, for example:

IF **project_duration** is **long**
AND **project_staffing** is **large**
AND **project_funding** is **inadequate**
THEN **risk** is **high**

- The consequent of a fuzzy rule can also include multiple parts, for instance:

IF **temperature** is **hot**
THEN **hot_water** is **reduced**;
cold_water is **increased**

Fuzzy Rules

- If it's Sunny and Warm, drive Fast

  $Sunny(Cover) \wedge Warm(Temp) \Rightarrow Fast(Speed)$

- If it's Cloudy and Cool, drive Slow

  $Cloudy(Cover) \wedge Cool(Temp) \Rightarrow Slow(Speed)$

- Driving Speed is the combination of output of these rules...

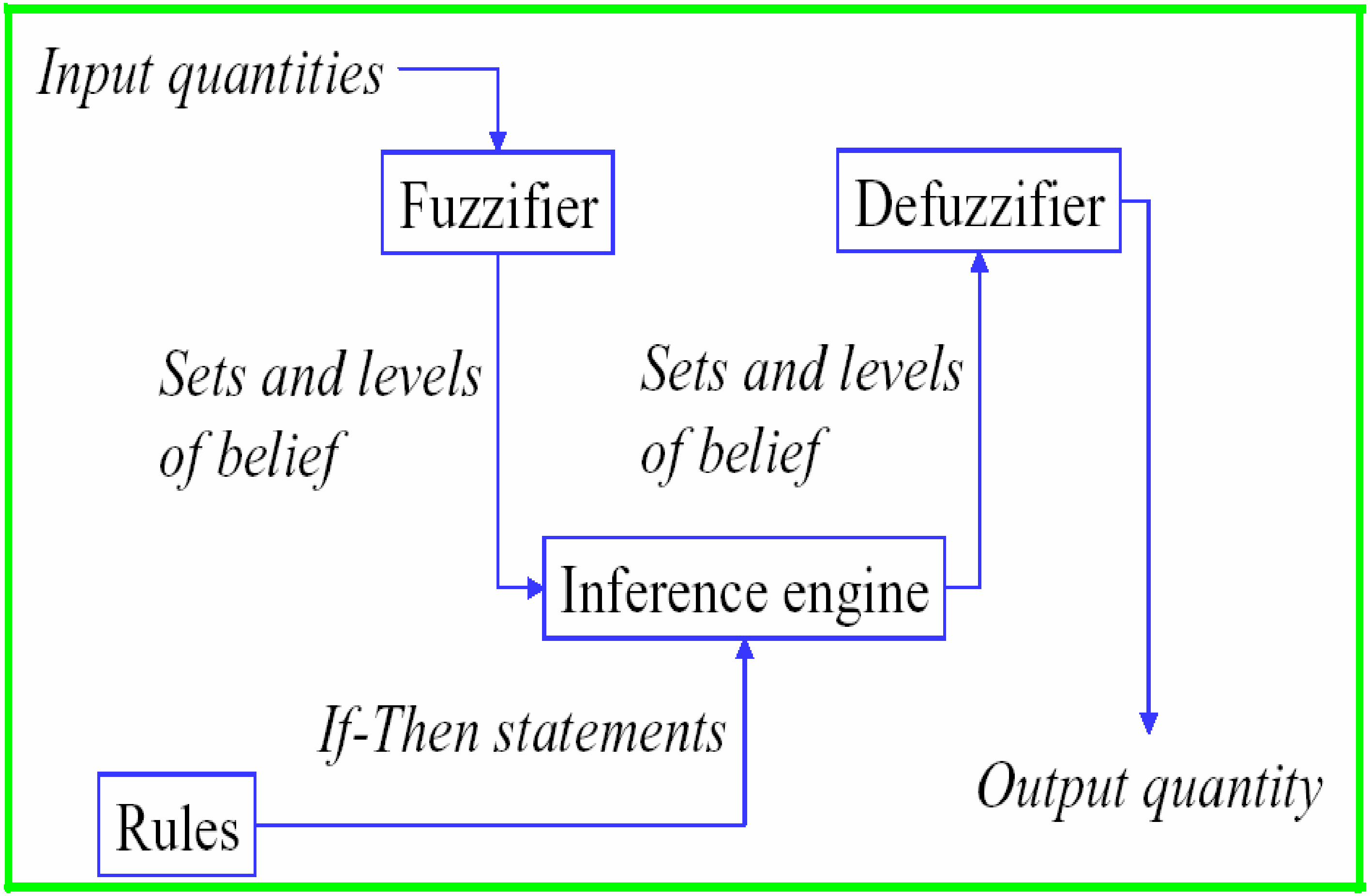
Fuzzy Control

- Fuzzy Control combines the use of fuzzy linguistic variables with fuzzy logic
- Example: Speed Control
- How fast am I going to drive today?
- It depends on the weather.

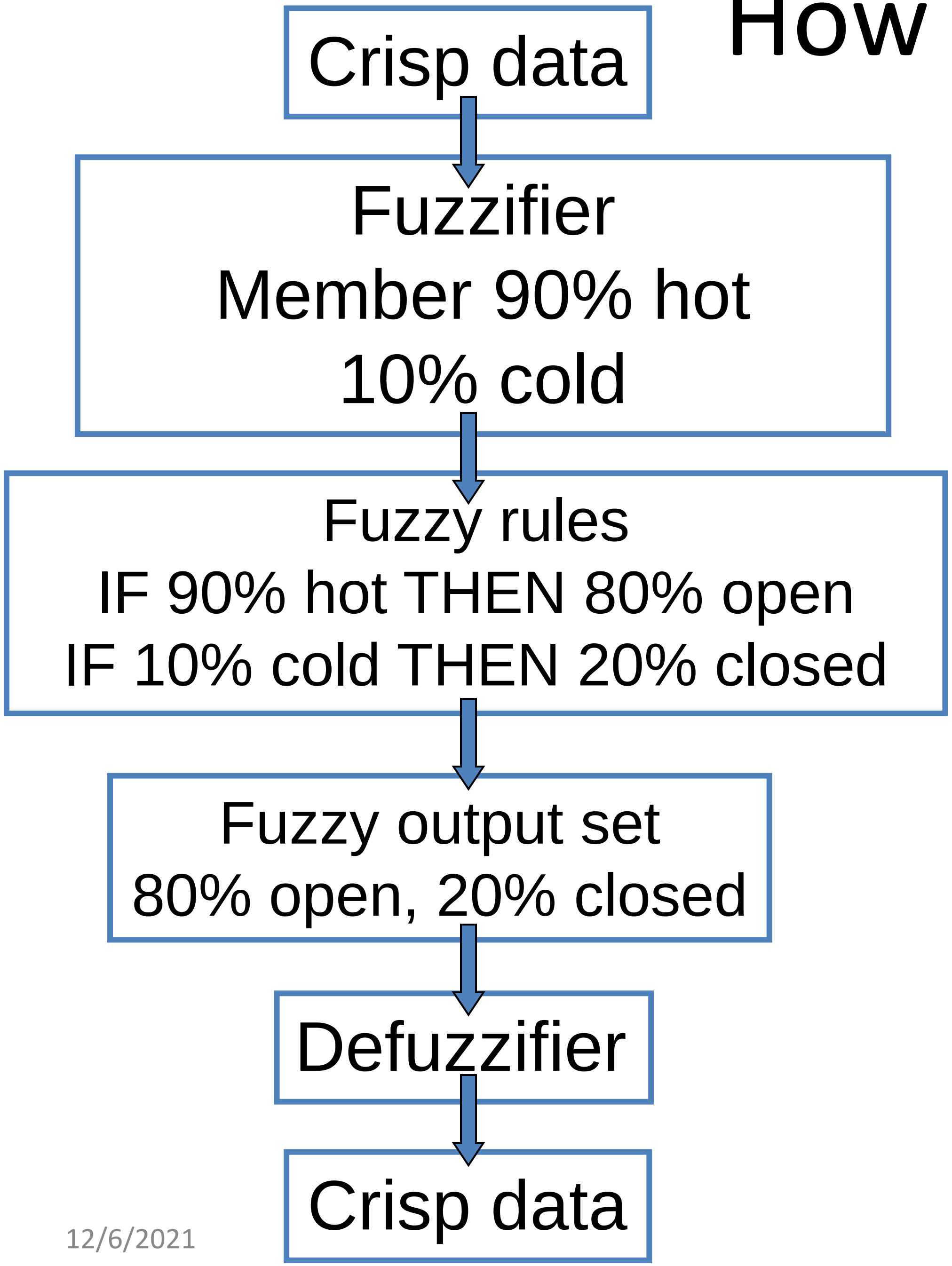
What can we do now?

- What should be done is summarized into *three main stages*
  - Fuzzification
    - *Membership functions* used to *graphically* describe a situation
  - Evaluation of Rules (Inference)
    - Application of the *fuzzy logic rules*
  - De-fuzzification
    - Obtaining the crisp results

Fuzzy Control Components



How it work?



Inputs converted to degrees of membership of fuzzy sets.

Fuzzy rules applied to get new sets of members.

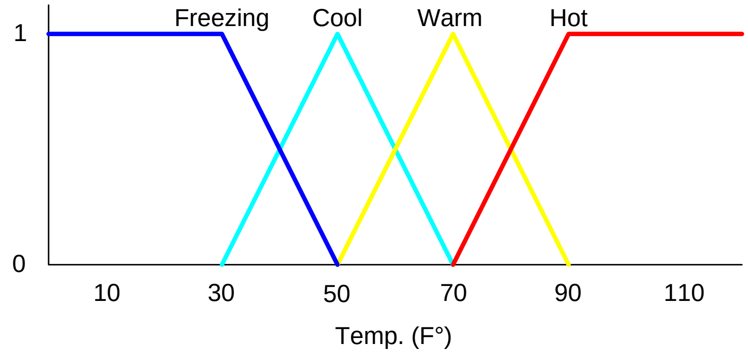
These sets are then converted back to real numbers.

Steps

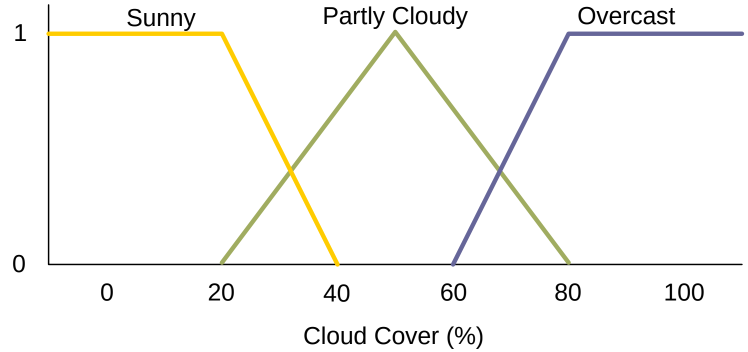
1. Determining a set of fuzzy rules
2. Fuzzifying the inputs using the input membership functions.
3. Combining the fuzzified inputs according to the fuzzy rules to establish a rule strength,
4. Finding the consequence of the rule by combining the rule strength and the output membership function.
5. Combining the consequences to get an output distribution.
6. Defuzzifying the output distribution.

Example: Speed Calculation

- Inputs: Temperature, Cloud Cover
- Temp: {Freezing, Cool, Warm, Hot}

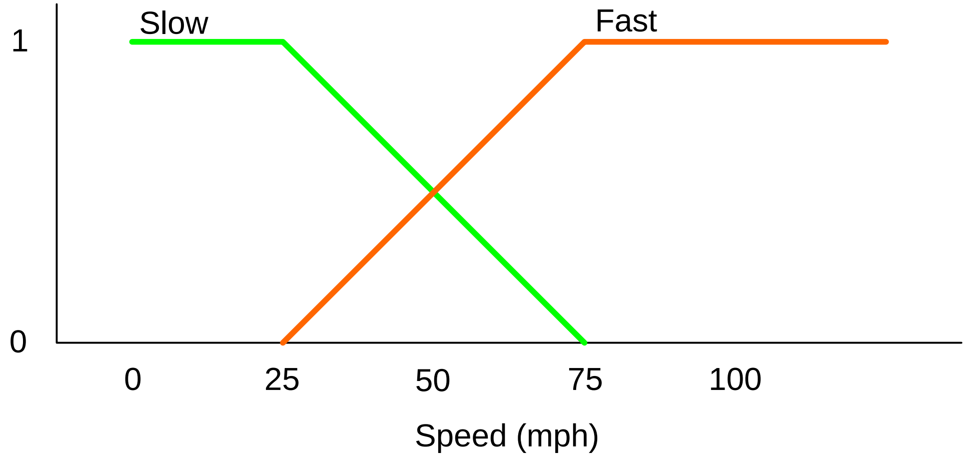


- Cover: {Sunny, Partly, Overcast}



Example: Speed Calculation

- Output: Speed
- Speed: {Slow, Fast}

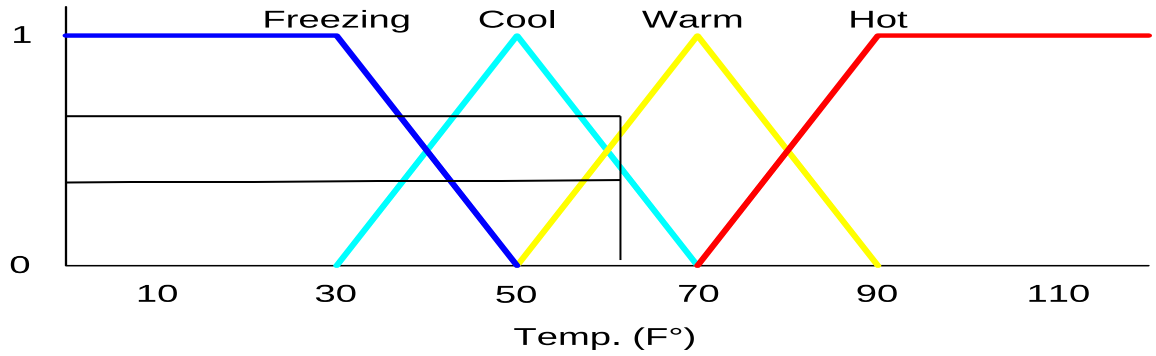


Example: Speed Calculation

- How fast will I go if it is
  - 65 F°
  - 25 % Cloud Cover ?

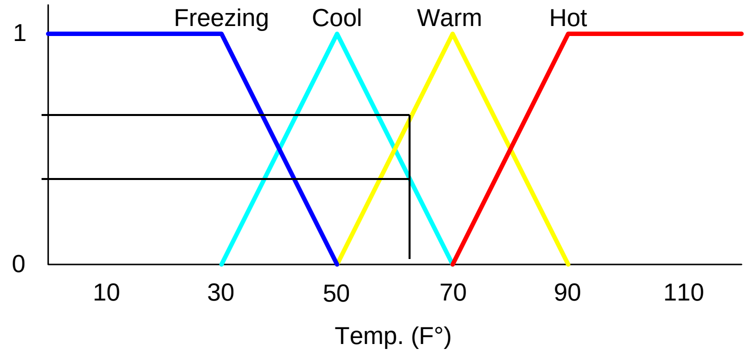
Fuzzification:
Calculate Input Membership Levels

- 65 F° $\Rightarrow$ Cool = 0.4, Warm= 0.7

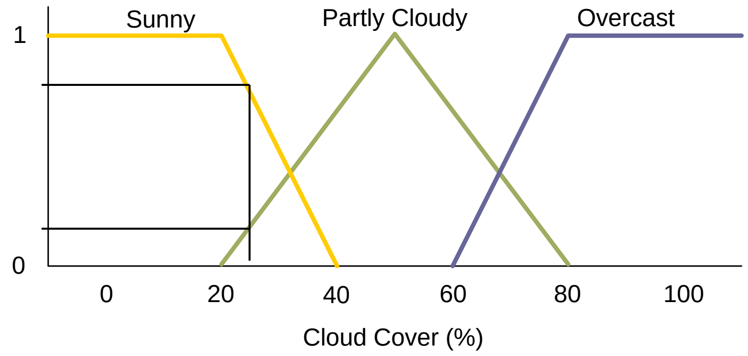


Fuzzification: Calculate Input Membership Levels

- 65 F° ⇒ Cool = 0.4, Warm= 0.7



- 25% Cover ⇒Sunny = 0.8, Cloudy = 0.2



...Calculating...

- If it's Sunny and Warm, drive Fast

$$\text{Sunny(Cover)} \wedge \text{Warm(Temp)} \Rightarrow \text{Fast(Speed)}$$

$$0.8 \wedge 0.7 = 0.7$$

$$\Rightarrow \textbf{Fast = 0.7}$$

- If it's Cloudy and Cool, drive Slow

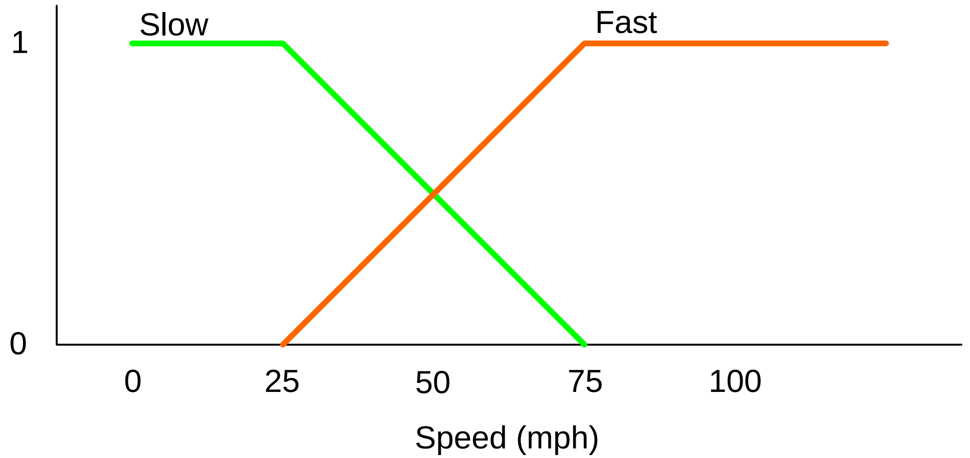
$$\text{Cloudy(Cover)} \wedge \text{Cool(Temp)} \Rightarrow \text{Slow(Speed)}$$

$$0.2 \wedge 0.4 = 0.2$$

$$\Rightarrow \textbf{Slow = 0.2}$$

Defuzzification: Constructing the Output

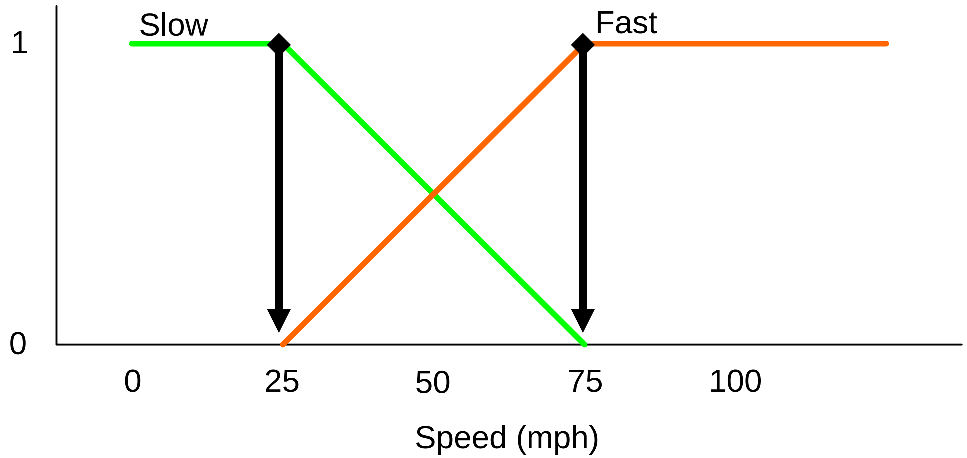
- Speed is 20% Slow and 70% Fast



- Find centroids: Location where membership is 100%

Defuzzification: Constructing the Output

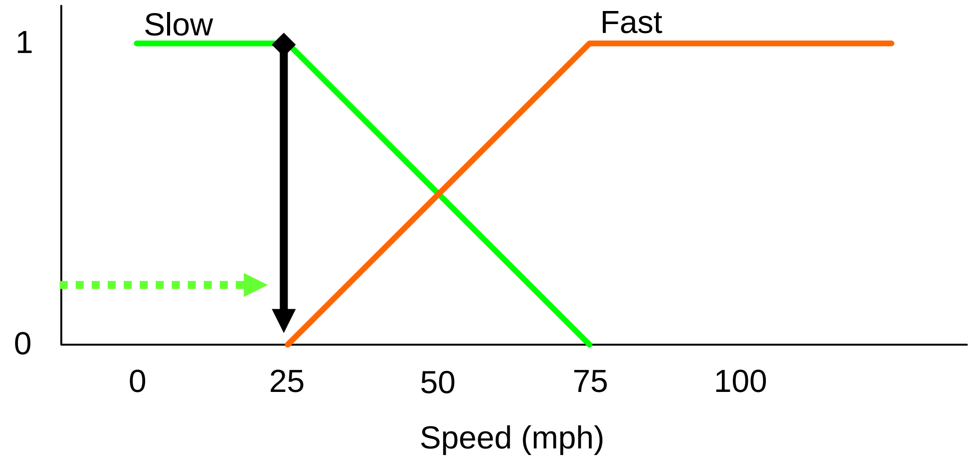
- Speed is 20% Slow and 70% Fast



- Find centroids: Location where membership is 100%

Defuzzification: Constructing the Output

- Speed is 20% Slow and 70% Fast



- Speed = weighted average mean

  = (0.2*25+...

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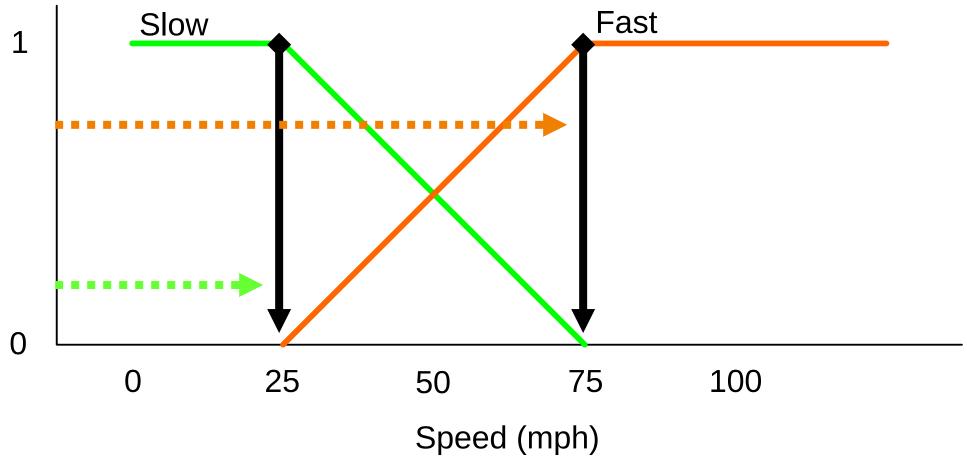


- Speed = weighted mean
  = (0.2*25+0.7*75)/(0.2+0.7)
  = 63.8 mph

Fuzzy Air Conditioner: Rule Base

Air Temperature

- Set cold {50, 0, 0}
- Set cool {65, 55, 45}
- Set just right {70, 65, 60}
- Set warm {85, 75, 65}
- Set hot {∞, 90, 80}

Fan Speed

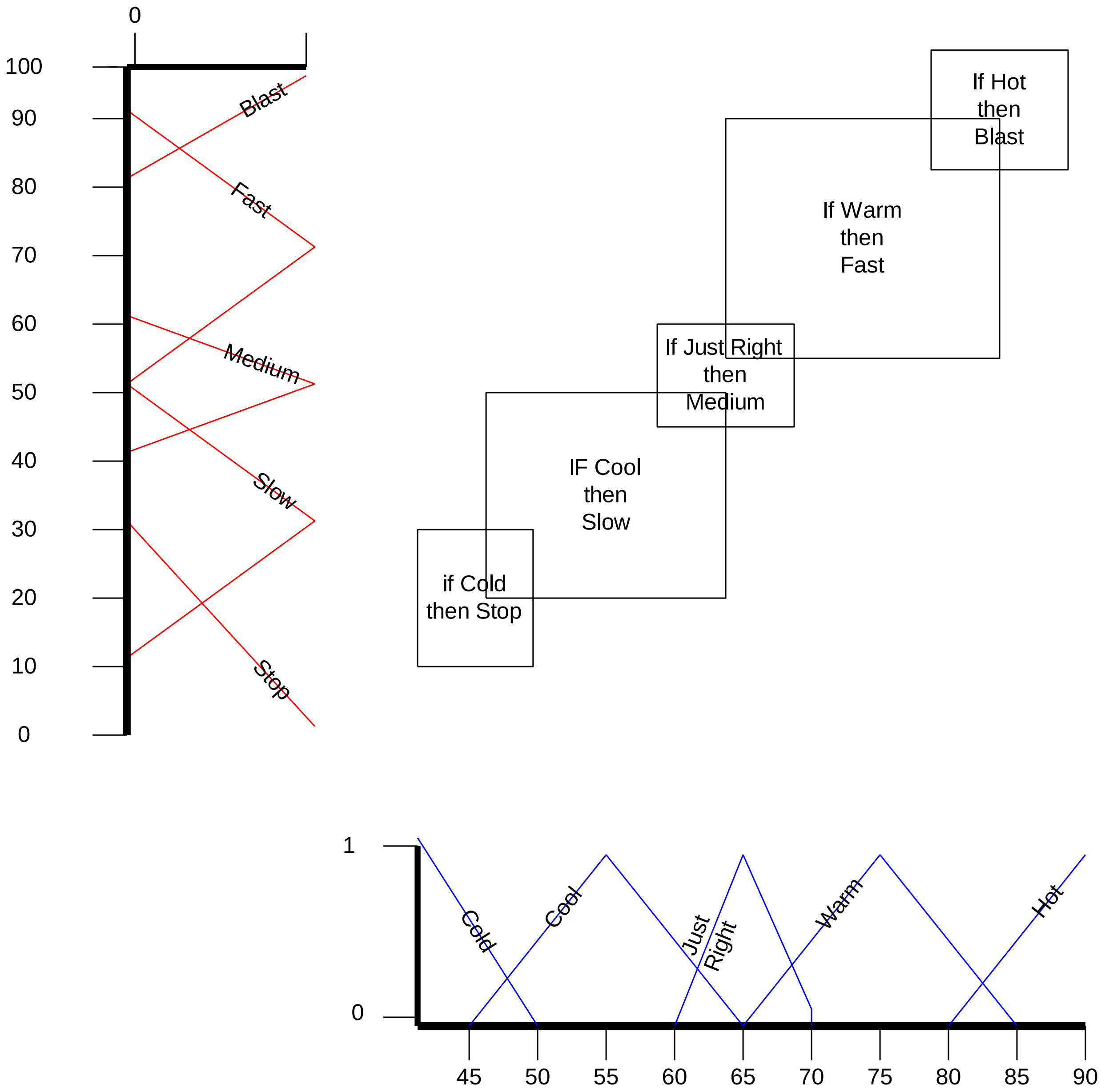
- Set stop {0, 0, 0}
- Set slow {50, 30, 10}
- Set medium {60, 50, 40}
- Set fast {90, 70, 50}
- Set blast {∞, 100, 80}

Fuzzy Air Conditioner: Rules

Air Conditioning Controller Example:

- IF Cold then Stop
- If Cool then Slow
- If just right then Medium
- If Warm then Fast
- IF Hot then Blast

Fuzzy Air Conditioner



Hybrid with other techniques

- Complicated systems may require several iterations to find a set of rules resulting in a stable system.
- It’s often helpful to get other AI techniques to generate the membership functions – e.g. Neural Nets and Genetic Algorithms.
- Combining Neural Networks with fuzzy logic reduces time to establish rules by analyzing clusters of data.