Cairo University
Faculty of Computers & Artificial Intelligence
Logic Design

Sheet 3

Tabular & Universal Gates

1. oo) Simplify the following function into its POS form

$$\mathbf{f(A,B,C) = \sum_m(4,5) + d(0,6,7)}$$

First, we get the maxterms of the function

$$f(A,B,C) = \Pi_m(1,2,3) + d(0,6,7)$$

Then, we get the terms into 0's and 1's and divide the terms in the function into groups depending upon the number of 1s they have. This forms the first table.

(0)	0	0	0
(1)	0	0	1
(2)	0	1	0
(3)	0	1	1
(6)	1	1	0
(7)	1	1	1

For each two successive groups, the terms of the first group are successively matched with those in the next adjacent higher order group. The terms are considered matched if all literals except for one match. The pairs of matched terms are replaced with a single term where the position of the unmatched literals is replaced with a dash (—). These new terms are placed in a second table and we mark used terms

(0)	0	0	0	✓
(1)	0	0	1	✓
(2)	0	1	0	✓
(3)	0	1	1	✓
(6)	1	1	0	✓
(7)	1	1	1	✓

(0,1)	0	0	–
(0,2)	0	–	0
(1,3)	0	–	1
(2,3)	0	1	–
(2,6)	–	1	0
(3,7)	–	1	1
(6,7)	1	1	–

We repeat for the second table, to form a third table, ... etc., until the terms become irreducible any further. Then we remove repeated terms.

$$
\begin{array}{cccccc}
(0) & 0 & 0 & 0 & \checkmark \\
\hline
(1) & 0 & 0 & 1 & \checkmark \\
(2) & 0 & 1 & 0 & \checkmark \\
\hline
(3) & 0 & 1 & 1 & \checkmark \\
(6) & 1 & 1 & 0 & \checkmark \\
\hline
(7) & 1 & 1 & 1 & \checkmark
\end{array}
\qquad
\begin{array}{ccccc}
(0,1) & 0 & 0 & - & \checkmark \\
(0,2) & 0 & - & 0 & \checkmark \\
\hline
(1,3) & 0 & - & 1 & \checkmark \\
(2,3) & 0 & 1 & - & \checkmark \\
(2,6) & - & 1 & 0 & \checkmark \\
\hline
(3,7) & - & 1 & 1 & \checkmark \\
(6,7) & 1 & 1 & - & \checkmark
\end{array}
\qquad
\begin{array}{ccccc}
(0,1,2,3) & 0 & - & - & \\
(0,2,1,3) & 0 & - & - & \times \\
\hline
(2,3,6,7) & - & 1 & - & \\
(2,6,3,7) & - & 1 & - & \times
\end{array}
$$

Unused terms are considered as prime implicants. As the question requires simplifying the function into POS, we will convert the prime implicants to maxterms.

$$
\begin{array}{cccccc}
(0) & 0 & 0 & 0 & \checkmark \\
\hline
(1) & 0 & 0 & 1 & \checkmark \\
(2) & 0 & 1 & 0 & \checkmark \\
\hline
(3) & 0 & 1 & 1 & \checkmark \\
(6) & 1 & 1 & 0 & \checkmark \\
\hline
(7) & 1 & 1 & 1 & \checkmark
\end{array}
\qquad
\begin{array}{ccccc}
(0,1) & 0 & 0 & - & \checkmark \\
(0,2) & 0 & - & 0 & \checkmark \\
\hline
(1,3) & 0 & - & 1 & \checkmark \\
(2,3) & 0 & 1 & - & \checkmark \\
(2,6) & - & 1 & 0 & \checkmark \\
\hline
(3,7) & - & 1 & 1 & \checkmark \\
(6,7) & 1 & 1 & - & \checkmark
\end{array}
\qquad
\begin{array}{ccccc}
(0,1,2,3) & 0 & - & - & A \\
(0,2,1,3) & 0 & - & - & \times \\
\hline
(2,3,6,7) & - & 1 & - & \overline{B} \\
(2,6,3,7) & - & 1 & - & \times
\end{array}
$$

Next we will select optimally the prime implicants that account for all the original terms.

$$
\begin{array}{cccc}
 & 1 & 2 & 3 \\
\hline
A & \circledast & \circledast & \circledast \\
\overline{B} & & \star & \star
\end{array}
$$

Because A covers all the maxterms of the function we can discard $\overline{B}$. The final function will be

$$f(A,B,C) = A$$

1. uu) Simplify the following function into its SOP form

$$\mathbf{f(A,B,C,D)=\Pi_M(1,3,5,9,12,14)+d(4,11,15)}$$

First, we get the minterms of the function

$$f(A,B,C)=\sum_m(0,2,6,7,8,10,13)+d(4,11,15)$$

We will create the tables using the same method used in the previous question. However, instead of translating the binary into maxterms, we will translate it into minterms

(0)	0	0	0	0	✓
(2)	0	0	1	0	✓
(4)	0	1	0	0	✓
(8)	1	0	0	0	✓
(6)	0	1	1	0	✓
(10)	1	0	1	0	✓
(7)	0	1	1	1	✓
(11)	1	0	1	1	✓
(13)	1	1	0	1	✓
(15)	1	1	1	1	✓

(0, 2)	0	0	–	0	✓
(0, 4)	0	–	0	0	✓
(0, 8)	–	0	0	0	✓
(2, 6)	0	–	1	0	✓
(2, 10)	–	0	1	0	✓
(4, 6)	0	1	–	0	✓
(8, 10)	1	0	–	0	✓
(6, 7)	0	1	1	–	$\overline{A}BC$
(10, 11)	1	0	1	–	$A\overline{B}C$
(7, 15)	–	1	1	1	BCD
(11, 15)	1	–	1	1	ACD
(13, 15)	1	1	–	1	ABD

(0, 2, 4, 6)	0	–	–	0	$\overline{A}\,\overline{D}$
(0, 2, 8, 10)	–	0	–	0	$\overline{B}\,\overline{D}$
(0, 4, 2, 6)	0	–	–	0	×
(0, 8, 2, 10)	–	0	–	0	×

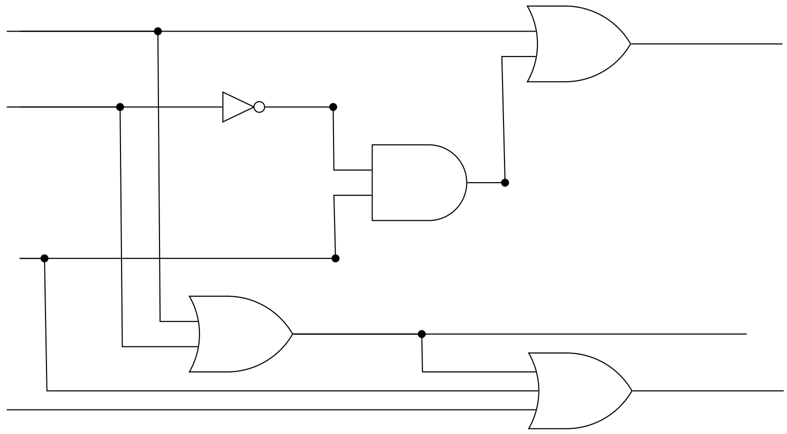
Next we will select optimally the prime implicants that account for all the original terms.

	0	2	6	7	8	10	13
$\overline{A}\,\overline{D}$	⋆	⋆	⋆				
$\overline{B}\,\overline{D}$	⊛	⊛			⊛	⊛	
BCD				⋆			
ACD							
ABD							⊛
$\overline{A}BC$			⊛	⊛			
$A\overline{B}C$						⋆	

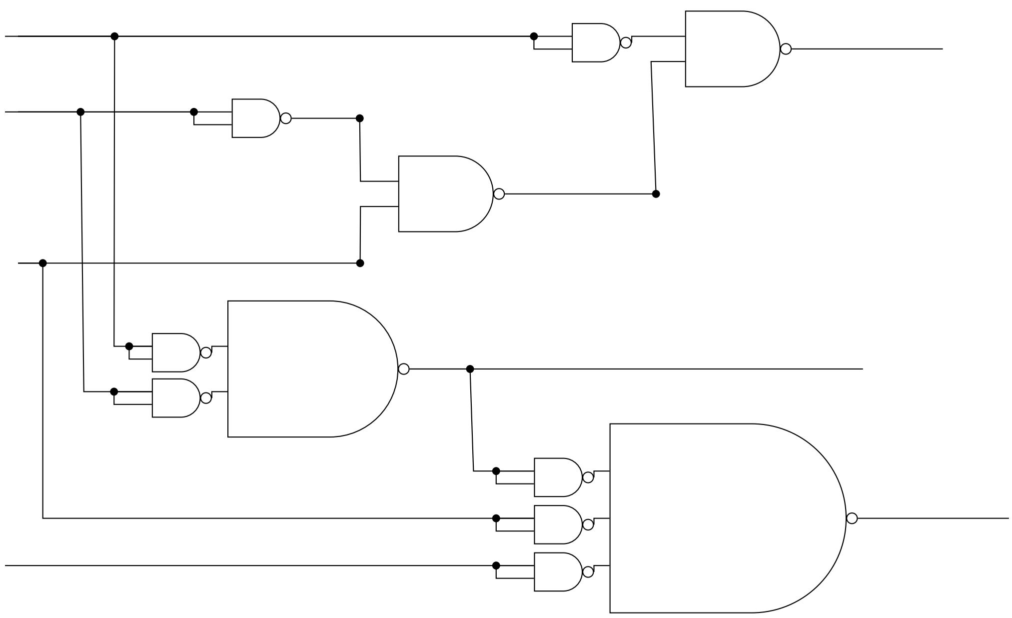
The final function will be

$$f(A,B,C)=\overline{B}\,\overline{D}+ABD+\overline{A}BC$$

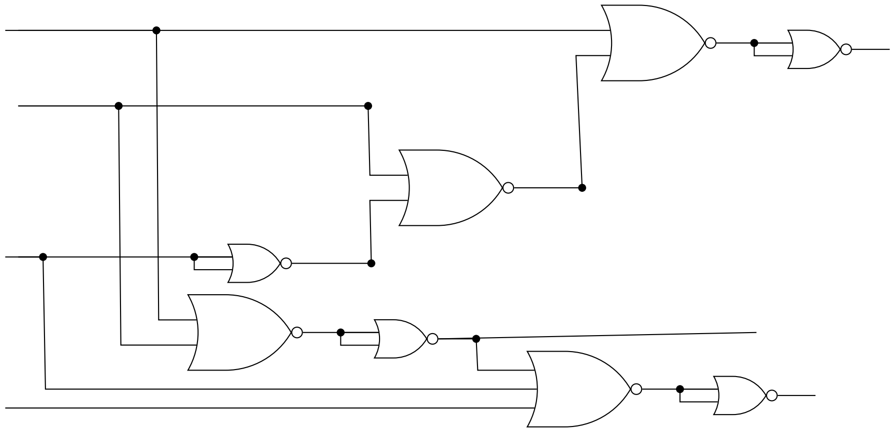
4. j) Draw the logic diagram of the following Boolean functions using: NANDs only NORs only



NANDS



NORS



3. n) Simplify the following function into its SOP and POS forms using K-map then draw the logic diagram using NANDs only and NORs only

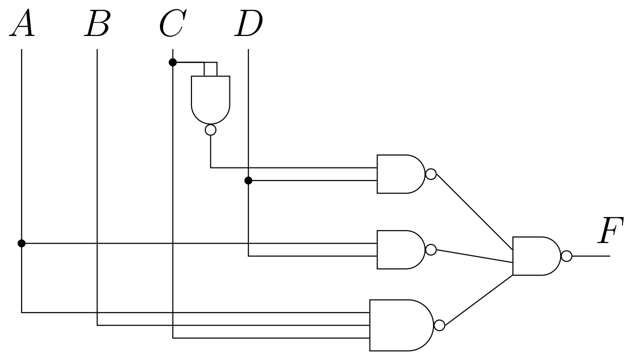
$$\mathbf{F(A,B,C,D) = AD + \overline{A}\,\overline{C}D + ABC\overline{D}}$$

AB \ CD	00	01	11	10
00		1		
01		1		
11		1	1	1
10		1	1	

AB \ CD	00	01	11	10
00		1		
01		1		
11		1	1	1
10		1	1	

AB \ CD	00	01	11	10
00		1		
01		1		
11		1	1	1
10		1	1	

SOP: $F(A,B,C,D) = \overline{C}D + AD + ABC$



AB \ CD	00	01	11	10
00	0		0	0
01	0		0	0
11	0			
10	0			0

AB \ CD	00	01	11	10
00	0		0	0
01	0		0	0
11	0			
10	0			0

AB \ CD	00	01	11	10
00	0		0	0
01	0		0	0
11	0			
10	0			0

POS: $F(A,B,C,D) = (C+D)(A+\overline{C})(B+D)$