Boolean Function Simplification Using Tabular Method

Digital Design. M. Morris Mano

Tabular Method

The tabular method (also known as the Quine-McCluskey) method is useful when minimizing functions having a large number of variables.

This method can be programmed (in fact programs have been developed employing this algorithm).

The method reduces a function to a set of prime implicants, from which as many variables are eliminated as possible.

Tabular Method (Rules)

Rules of tabular method:

1. **Represent the function as a sum of minterms or product of maxterms.**
2. **Get the terms into 0's and 1's**
3. **Divide the terms in the function into groups depending upon the number of 1s they have. This form the first table.**

Tabular Method (Rules)

Rules of tabular method (cont):

4. **For each two successive groups, the terms of the first group are successively matched with those in the next adjacent higher order group to look for any possible matching and consequent reduction.**
   - **The terms are considered matched if all literals except for one match.**

Tabular Method (Rules)

Rules of tabular method (cont):

- The pairs of matched terms are replaced with a single term where the position of the unmatched literals is replaced with a dash (—).
- These new terms (formed as a result of the matching process) are placed in a second table.
- Mark used terms
- Unused term(s) is(are) considered as prime implicants.

Tabular Method (Rules)

Rules of tabular method (cont):

5. **Repeat number 4 for the second table, to form a third table, … etc., until the terms become irreducible any further. Remove repeated terms.**

   Note that while comparing the terms for a match, it is important that the dash (—) is also treated like any other literal, that is, the dash signs also need to match.

Tabular Method (Rules)

Rules of tabular method (cont):

6. **An optimum selection of prime implicants to account for all the original terms leads to the terms for the minimized expression.**

Note that don't care terms are considered for matching, but they do not have to be accounted for prime implicants if they are not used for match.

Tabular Method (Example 1)

Example 1:

Using Tabular (Quine-McCluskey) method to simply the function $F(A,B,C)=\sum m(1,3,5,6)+d(0,7)$

1- $F(A,B,C)=\sum m(1,3,5,6)+d(0,7)$

2- Function represented as 1's and 0's:

1(001), 3(011), 5(101), 6(110)

Don't cares are 0(000), 7(111)

Tabular Method (Example 1)

3- Groups are formed:

000

001

011 101 110

111

First table is formed

Terms		Group
In decimal	In 0's and 1's	
0	000	1st group
1	001	2nd group
3	011	3rd group
5	101	
6	110	
7	111	4th group

Tabular Method (Example 1)

4- Match terms:

Use first table to form the second one.

Terms		Mark/prime implicant
In decimal	In 0's and 1's	
0	000	√
1	001	√
3	011	√
5	101	√
6	110	√
7	111	√

New Terms		Groups
In decimal	In 0's and 1's	
0,1	0 0 -	1st group
1,3	0 - 1	2nd group
1,5	- 0 1	
3,7	- 1 1	
5,7	1 - 1	3rd group
6,7	1 1 -	

Tabular Method (Example 1)

5- Match terms:

Use second table to form the third one.

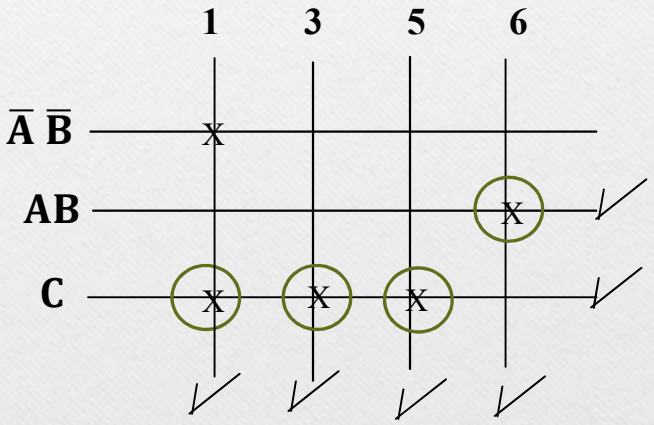
New Terms		Mark/prime implicant
In decimal	In 0's and 1's	
0,1	0 0 -	$\bar{A}\,\bar{B}$
1,3	0 - 1	√
1,5	- 0 1	√
3,7	- 1 1	√
5,7	1 - 1	√
6,7	1 1 -	AB

New Terms		Mark/prime implicant
In decimal	In 0's and 1's	
1,3,5,7	- - 1	C
~~1,3,5,7~~	~~- - 1~~	

As only one terms remains, so end of the matching is reached (terms are irreducible any more).

Tabular Method (Example 1)

6- Selection of optimum prime implicants



$$F(A, B, C) = AB + C$$

Tabular Method (Example 2)

Example 2:

Using Tabular (Quine-McCluskey) method to simply the function as SoP:

$$F(A, B, C, D) = \overline{B}\overline{D} + \overline{A}\overline{B}D + \overline{A}BD + ABD + AB\overline{C}\overline{D}$$

First get the minterms (for example using k-map)

$F(A,B,C,D)=\sum m(0,1,2,3,5,7,8,10,12,13,15)$

$=\sum m(0000,0001,0010,0011,0101,0111,1000,1010,1100,1101,1111)$

Tabular Method

(Example 2)

First table

0	0000	√
1	0001	√
2	0010	√
8	1000	√
3	0011	√
5	0101	√
10	1010	√
12	1100	√
7	0111	√
13	1101	√
15	1111	√

Second table

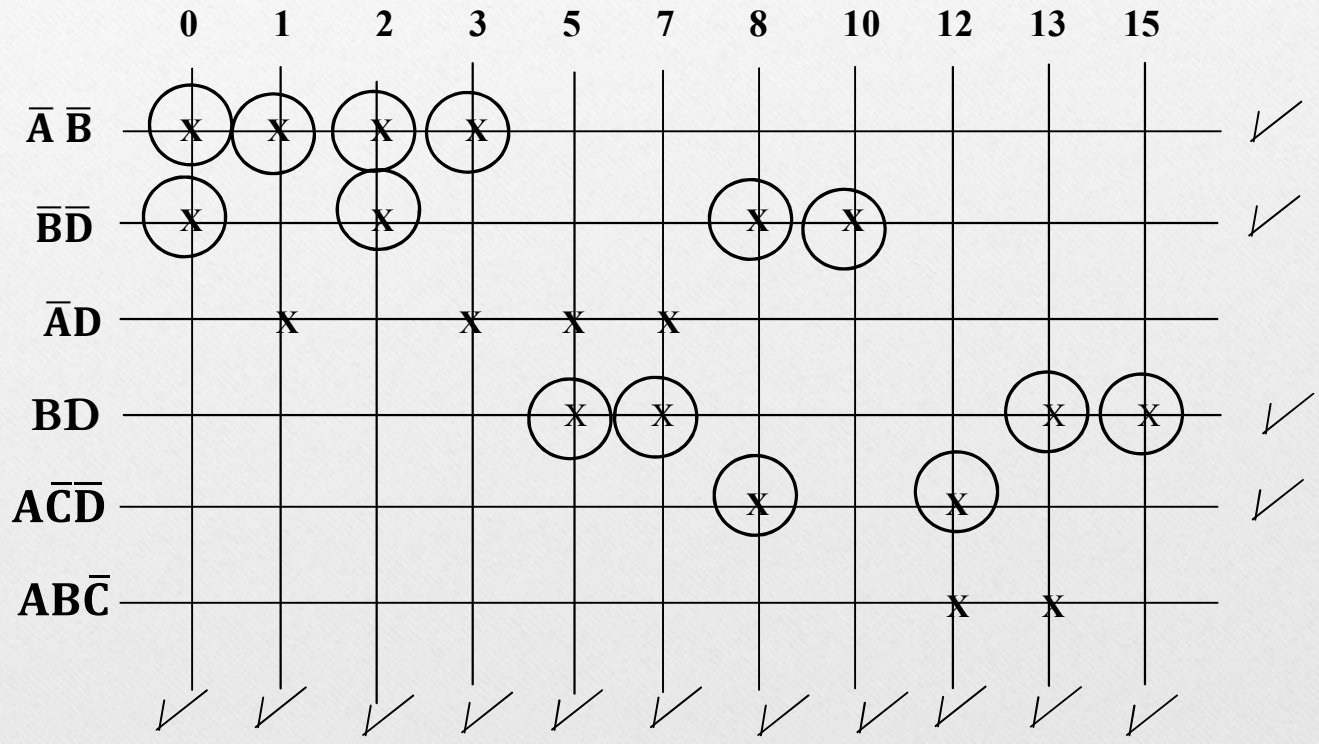
0,1	0	0	0	-	√
0,2	0	0	-	0	√
0,8	-	0	0	0	√
1,3	0	0	-	1	√
1,5	0	-	0	1	√
2,3	0	0	1	-	√
2,10	-	-	1	0	√
8,10	1	0	-	0	√
8,12	1	-	0	0	$A\bar{C}\bar{D}$
3.7	0	-	1	1	√
5,7	0	1	-	1	√
5,13	-	1	0	1	√
12,13	1	1	0	-	$AB\bar{C}$
7,15	-	1	1	1	√
13,15	1	1	-	1	√

Tabular Method (Example 2)

Third table

0,1,2,3	0 0 - -	$\bar{A}\bar{B}$
~~0,2,1,3~~	~~0 0 - -~~	
0,2,8,10	- 0 - 0	$\bar{B}\bar{D}$
~~0,8,2,10~~	~~- 0 - 0~~	
1,3,5,7	0 - - 1	$\bar{A}D$
~~1,5,3,7~~	~~0 - - 1~~	
5,7,13,15	- 1 - 1	BD
~~5,13,7,15~~	~~- 1 - 1~~	

Tabular Method (Example 2)



$$F(A, B, C, D) = \bar{A}\bar{B} + \bar{B}\bar{D} + BD + A\bar{C}\bar{D}$$

Tabular Method

Exercise:

Using tabular method, optimize F as SoP.

$$F(A,B,C,D) = B\overline{C}\,\overline{D} + \overline{A}\,\overline{B}D + A\overline{C}D + \overline{A}BC + \overline{A}B\overline{C}D$$
$$+ d(A\overline{B}\overline{C}\,\overline{D} + ABC + A\overline{B}C\overline{D})$$

$$F(A,B,C,D)=\sum m(1,3,4,5,6,7,9,12,13)+d(8,10,14,15)$$

	1,3	0	0	-	1			7,15	-	1	1	1								
	1,5	0	-	0	1			13,15	1	1	-	1								
	1,9	-	0	0	1			14,15	1	1	1	-								
	4,5	0	1	0	-															
	4,6	0	1	-	0															
	4,12	-	1	0	0															
	8,9	1	0	0	-															
	8,10	1	0	-	0															
	8,12	1	-	0	0															
	3,7	0	-	1	1															
	5,7	0	1	-	1															
	5,13	-	1	0	1															
	6,7	0	1	1	-															
	6,14	-	1	1	0															
	9,13	1	-	0	1															
	10,14	1	-	1	0															
	12,13	1	1	0	-															
	12,14	1	1	-	0															

$F(A,B,C,D)=\sum m(1,3,4,5,6,7,9,12,13)+d(8,10,14,15)$

1,3,5,7	0	-	-	1
1,5,9,13	-	-	0	1
4,5,6,7	0	1	-	-
4,5,12,13	-	1	0	-
4,6,12,14	-	1	-	0
8,9,12,13	1	-	0	-
8,10,12,14	1	-	-	0
5,7,13,15	-	1	-	1
6,7,14,15	-	1	1	-
12,13,14,15	1	1	-	-

Tabular Method

How to use Tabular method to simply the function into PoS:

1. **Get the maxterms as forms of variables Mi, then transfer it into terms of 1's and 0's**
   - **Note that, if terms are in variables, used variables as it is take 0, while used complement of variables take 1's.**
2. **Simply it using the tabular method as usual, but prime implicants is formed as maxterms Mi (in variables).**
3. **Then the function is written as PoS.**

Tabular Method (Example 3)

Example 3:

How to use Tabular (Quine-McCluskey) method to simply the function:

$$F(A,B,C,D) = (\overline{A}+\overline{B}+\overline{C}+\overline{D})(\overline{A}+\overline{B}+\overline{C}+D)$$
$$(\overline{A}+\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+\overline{D})\ (A+\overline{B}+C+\overline{D})$$

First get the maxterms.

$F(A,B,C,D)=\Pi_M(5,7,13,14,15) =\Pi_M(0101,0111,1101,1110,1111)$

Tabular Method (Example 3)

First table

5	0101	√
7	0111	√
13	1101	√
14	1110	√
15	1111	√

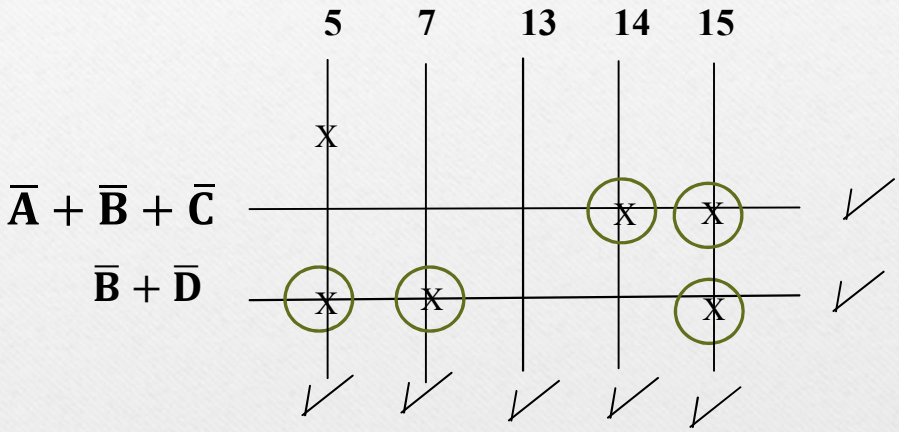
Second table

5,7	0	1	-	1	√
5,13	-	1	0	1	√
7,15	-	1	1	1	√
13,15	1	1	-	1	√
14.15	1	1	1	-	$\overline{A} + \overline{B} + \overline{C}$

Third table

5,7,13.15	-	1	-	1	$\overline{B} + \overline{D}$
~~5,7,13.15~~	-	~~1~~	-	~~1~~	

Tabular Method (Example 3)



$$F(A, B, C, D) = (\overline{A} + \overline{B} + \overline{C})(\overline{B} + \overline{D})$$

Tabular Method

Exercise:

Using tabular method, optimize F as PoS.

$$F(A,B,C,D) = B\overline{C}\,\overline{D} + \overline{A}\,\overline{B}D + A\overline{C}D + \overline{A}BC + \overline{A}B\overline{C}D$$
$$+ d(A\overline{B}\overline{C}\,\overline{D} + ABC + A\overline{B}C\overline{D})$$