

Sheet 3 – Section 1

K-Map Summary:

- The K-Map was designed to ensure a more reliable way for function optimization, by utilizing the Identity property in digital logic $(A+A')=1$ this can be achieved by applying the adjacency property in K-Map.
- Firstly we have to group the implicants in the map to represent the simplified function either in SOP or POS form, the implicants will be ones in case of SOP and zeros in case of POS.

- ☐ No diagonals.
- ☐ Only power of 2 number of cells in each group.
- ☐ Groups should be as large as possible.
- ☐ Every one must be in at least one group.
- ☐ Overlapping allowed.
- ☐ Wrap around allowed.
- ☐ Fewest number of groups possible.

Q-1: a

$$F(A,B) = \sum_m (1,2),$$

Get the Maxterms form $\Rightarrow F(A,B) = \pi_M (0,3)$

1- Simplified function in SOP form

| | | B | |
|---|---|---------|---------|
| | | 0 | 1 |
| A | 0 | m0 0 | m1 1 |
| | 1 | m2 1 | m3 0 |

$$F(A,B) = A'B + AB'$$

2- Simplified function in POS form

| | | B | |
|---|---|---------|---------|
| | | 0 | 1 |
| A | 0 | m0 0 | m1 0 |
| | 1 | m2 0 | m3 0 |

$$F(A,B) = (A+B) \cdot (A'+B')$$

Q-1: h

$$F(A,B,C) = \pi_M(0,1,5,6),$$

Get the Maxterms form $\Rightarrow F(A,B) = \sum_m(2,3,4,7)$

1- Simplified function in POS form

| | | BC | | | |
|---|---|------|------|----|------|
| | | 00 | 01 | 11 | 10 |
| A | 0 | 0 m0 | 0 m1 | m3 | m2 |
| | 1 | m4 | 0 m5 | m7 | 0 m6 |

$$F(A,B) = (A+B).(B+C').(A'+B'+C)$$

2- Simplified function in SOP form

| | | BC | | | |
|---|---|------|----|------|------|
| | | 00 | 01 | 11 | 10 |
| A | 0 | m0 | m1 | 1 m3 | 1 m2 |
| | 1 | 1 m4 | m5 | 1 m7 | m6 |

$$F(A,B) = A'B + BC + AB'C'$$

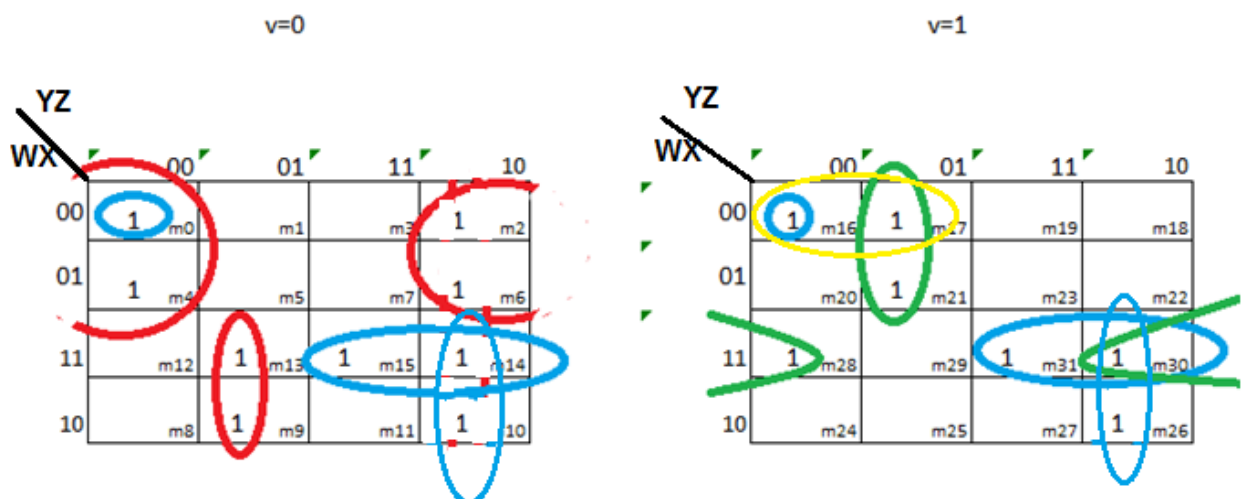
Q-1: ii

$$F(V,W,X,Y,Z) = \sum_m(0,2,4,6,9,10,13,14,15,16,17,21,26,28,30,31)$$

Get the Maxterms form =>

$$F(V,W,X,Y,Z) = \pi_M(1,3,5,7,8,11,12,18,19,20,22,23,24,25,27,29)$$

1- Simplified function in SOP form



Note: the red font is used to describe the implicant of the first table, the green font is used to describe the implicants of the second table and the blue font is used to describe the intersected implicants across the two tables and the yellow font is used to describe another optional combination.

$$F(V,W,X,Y,Z) = V'W'Z' + V'WY'Z + WXY + VW'Y'Z + VWXZ' + WYZ + W'X'Y'Z' \text{ or } VW'X'Y'$$

v=0

| | | YZ | | | |
|----|----|----|-----|-----|-----|
| | | 00 | 01 | 11 | 10 |
| WX | 00 | m0 | 0 | 0 | m2 |
| | 01 | m4 | 0 | 0 | m6 |
| | 11 | 0 | m12 | m13 | m14 |
| | 10 | 0 | m8 | m9 | m10 |

v=1

| | | YZ | | | |
|----|----|-----|-----|-----|-----|
| | | 00 | 01 | 11 | 10 |
| WX | 00 | m16 | m17 | 0 | m18 |
| | 01 | 0 | m20 | 0 | m22 |
| | 11 | m28 | 0 | m29 | m30 |
| | 10 | 0 | m24 | 0 | m26 |

2- Simplified function in POS form

Note: the red font is used to describe the implicant of the first table, the green font is used to describe the implicants of the second table, the blue font is used to describe the intersected implicants across the two tables and the yellow font is used to describe another optional combination.

$$F(V,W,X,Y,Z) = (V+W+Z') \cdot (V+W'+Y+Z) \cdot (X+Y'+Z') \cdot (V'+W+Y') \cdot (V'+W+X'+Z) \cdot (V'+W'+Y+Z') \cdot (W'+X+Y+Z) \text{ or } (V'+W'+X+Y)$$

Q-1: oo

$$F(A,B,C) = \sum_m(4,5) + d(0,6,7)$$

Get the Maxterms form =>

$$F(A,B,C) = \pi_M(1,2,3) + d(0,6,7)$$

Note: The Don't care cells are cells with either values of 1 or 0 depending on we want to use them for example if it will help us to simplify in the SOP form so we will consider the don't care cell as one and vice versa, take also in consideration that we don't have to use all the Don't care cells.

1- Simplified function in SOP form

| BC | | | | | |
|----|---|------|------|------|------|
| A | | 00 | 01 | 11 | 10 |
| | 0 | X m0 | m1 | m3 | m2 |
| | 1 | 1 m4 | 1 m5 | X m7 | X m6 |

$$F(A,B,C) = A$$

2- Simplified function in POS form

| | | BC | | | |
|---|---|-----------------|-----------------|-----------------|-----------------|
| A | | 00 | 01 | 11 | 10 |
| | 0 | X _{m0} | 0 _{m1} | 0 _{m3} | 0 _{m2} |
| | 1 | | | X _{m7} | X _{m6} |

Note: some of the POS outputs are intersected with the Don't care outputs, in this case we consider the cells to be filled with the Don't care values (X).

$$F(A,B,C) = A$$

Q-1: rr

$$F(A,B,C,D) = ACD + BD' + B'C + B'C' + B'D + C'D' + d(A'BC'D)$$

Note: We don't have to get the truth table then get the Minterms & Maxterms from it, we can shortly fill in the cells of K-Map with ones that achieve each corresponding term in the upper expression.

| CD \ AB | | CD | | | |
|---------|-------|-------|-------|-------|----|
| | | 00 | 01 | 11 | 10 |
| 00 | 1 m0 | 1 m1 | 1 m3 | 1 m2 | |
| 01 | 1 m4 | X m5 | 0 m7 | 1 m6 | |
| 11 | 1 m12 | 0 m13 | 1 m15 | 1 m14 | |
| 10 | 1 m8 | 1 m9 | 1 m11 | 1 m10 | |

Note: the green font is used to describe the SOP implicants, while the blue font is used to describe the POS implicants.

1- Simplified function in SOP form

$$F(A,B,C,D) = B' + D' + AC$$

2- Simplified function in POS form

$$F(A,B,C,D) = (A+B'+D')(B'+C+D')$$

Q-2

$F(A,B,C,D) = A'C + AC'D$, where $A=C=1$ can never occur

Note: We don't have to get the truth table then get the Minterms & Maxterms from it, we can shortly fill in the cells of K-Map with ones that achieve each corresponding term in the upper expression.

Also take in consideration to fill all the cells in the K-Map by **X (do not care)** when $A=C=1$.

| AB \ CD | | | | |
|---------|-----|-------|-------|-------|
| | 00 | 01 | 11 | 10 |
| 00 | m0 | m1 | 1 m3 | 1 m2 |
| 01 | m4 | m5 | 1 m7 | 1 m6 |
| 11 | m12 | 1 m13 | X m15 | X m14 |
| 10 | m8 | 1 m9 | X m11 | X m10 |

1- Simplified function in SOP form

$$F(A,B,C,D) = C + AD$$

Q -3: p (ORs, AND and NOTs)

$$F(W,X,Y,Z) = WX' + Y'Z' + W'YZ'$$

Note: We don't have to get the truth table then get the Minterms & Maxterms from it, we can shortly fill in the cells of K-Map with ones that achieve each corresponding term in the upper expression.

Also take in consideration that we shall present the output in the POS form to match the drawing criteria for the logical diagram using ORs, AND and NOTs, so we will group the implicants of zeros.

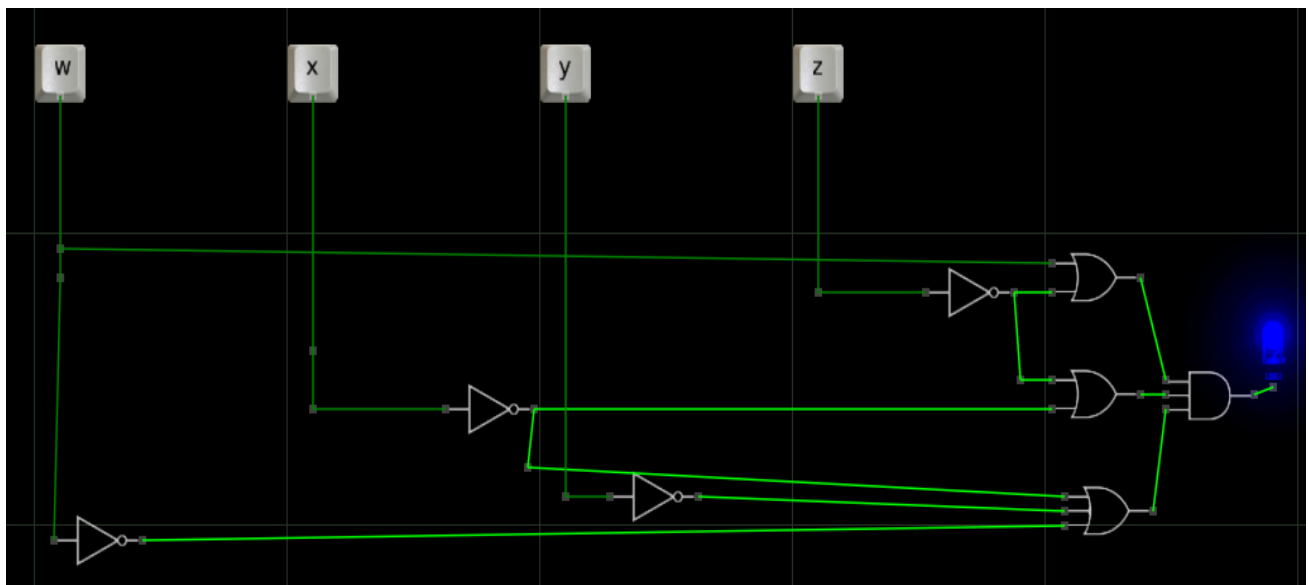
A 4x4 Karnaugh Map for the function F(W,X,Y,Z). The vertical axis is labeled WX (00, 01, 11, 10) and the horizontal axis is labeled YZ (00, 01, 11, 10). The map contains 1s at m0, m2, m4, m6, m8, m10, m12, and m14. There are 0s at m1, m3, m5, m7, m9, m11, m13, and m15. Three green loops are drawn around the 0s: a vertical loop for m1, m5, m9, m13; a horizontal loop for m5, m7, m13, m15; and a horizontal loop for m13, m15, m9, m11. A red diagonal line is drawn from the top-left corner (m0) to the bottom-right corner (m15).

| WX \ YZ | 00 | 01 | 11 | 10 |
|---------|-------|-------|-------|-------|
| 00 | 1 m0 | 0 m1 | 0 m3 | 1 m2 |
| 01 | 1 m4 | 0 m5 | 0 m7 | 1 m6 |
| 11 | 1 m12 | 0 m13 | 0 m15 | 0 m14 |
| 10 | 1 m8 | 1 m9 | 1 m11 | 1 m10 |

1- Simplified function in POS form

$$F(W,X,Y,Z) = (W + Z')(X' + Z')(W' + X' + Y')$$

2- Logical Diagram using the specified gates on the simplified function



Q -3: t (ANDs, OR and NOTs)

$$F(A,B,C,D) = (A' + C).(A' + C').(A + B + C + D')$$

Note: We don't have to get the truth table then get the Minterms & Maxterms from it, we can shortly fill in the cells of K-Map with zeros that achieve each corresponding term in the upper expression.

Also take in consideration that we shall present the output in the SOP form to match the drawing criteria for the logical diagram using ANDs, OR and NOTs, so we will group the implicants of ones.

CD

AB

| | | | | |
|----|-------|-------|-------|-------|
| | 00 | 01 | 11 | 10 |
| 00 | 1 m0 | 0 m1 | 1 m3 | 1 m2 |
| 01 | 1 m4 | 1 m5 | 1 m7 | 1 m6 |
| 11 | 0 m12 | 0 m13 | 0 m15 | 0 m14 |
| 10 | 0 m8 | 0 m9 | 0 m11 | 0 m10 |

1- Simplified function in SOP form

$$F(A,B,C,D) = A'B + A'C + A'D'$$

2- Logical Diagram using the specified gates on the simplified function

