Boolean Function Simplification Using Karnaugh Map

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Karnaugh Map

Karnaugh Map (K-map) is a diagram made up of squares, with each square representing one minterm/maxterm of the function that is to be minimized. Since any Boolean function can be expressed as a sum of minterms (or a product of maxterms), it follows that a Boolean function is recognized graphically in the map from the area enclosed by those squares whose minterms/maxterms are included in the function.

Karnaugh map is used for Boolean function simplification.

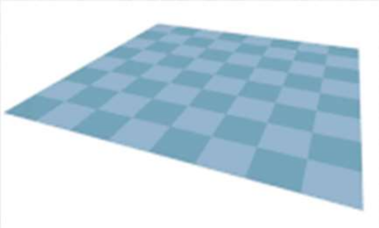
Karnaugh Map

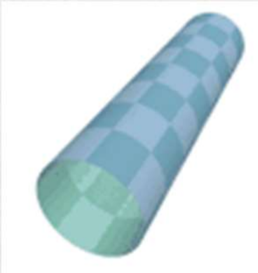
The minterms/maxterms are transferred from a truth table onto a two-dimensional grid where, in Karnaugh maps, the cells are ordered in Gray code.

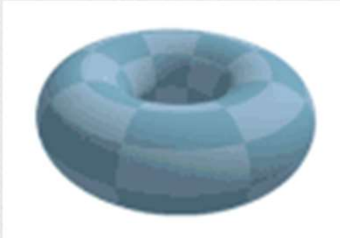
Simplification is obtained by optimal groups of 1s or 0s are identified, which represent the terms of a canonical form of the logic in the original truth table. These terms can be used to write a minimal Boolean expression representing the required logic.

Karnaugh Map

The grid is toroidally connected, which means that rectangular groups can wrap across the edges. Cells on the extreme right are actually 'adjacent' to those on the far left, in the sense that the corresponding input values only differ by one bit; similarly, so are those at the very top and those at the bottom.







Karnaugh Map

Combine the cells (squares) in the toroidal grid into one or more group, each of 2^n, where n=0,1,2,3 … etc.

It is better to have larger groups (i.e. it is better to maximize the number of grouped cells), as it will leads to less variables in a term.

A cell may be included in more than one group.

Do not add new groups if all cells are included in others.

Karnaugh Map

<u>In SoP</u> Groups are composed of 1's

A term is composed of the product of the variable(s) whom value is not changed in the group.

Unchanged variable with value=1 is used as it is. But if the value of the unchanged variable=0, its complement is used.

Then the sum of the terms are used.

For example: $F(A,B,C,D)=\overline{B}\,\overline{C}+CD+\overline{A}D$

Karnaugh Map

<u>In PoS,</u> Groups are composed of 0's

A term is composed of the sum of the variable(s) whom value is not changed in the group.

Unchanged variable with value=0 is used as it is. But if the value of the unchanged variable=1, its complement is used.

Then the product of the terms are used.

For example: $F(W,X,Y,Z) = (W + X + \overline{Y} + \overline{Z})(\overline{X} + \overline{Y} + Z)(\overline{W} + Y + \overline{Z})$

2-Variable K-map

Var1 \ Var2	0	1
0	00 0	01 1
1	10 2	11 3

In each square:

Black values represents the binary values.
Red values represents the decimal values.

2-Variable K-map (Example 1)

Example 1:

Using k-map, optimize:

$F(A,B)= \sum_m (1,3)$

As SoP form.

A \ B	0	1
0		1
1		1

F(A,B)=B

3-Variable K-map

Var1 \ Var2 Var3	00	01	11	10
0	000 0	001 1	011 3	010 2
1	100 4	101 5	111 7	110 6

In each square:

Black values represents the binary values.
Red values represents the decimal values.

3-Variable K-map (Example 2)

<u>Example 2:</u>

Using k-map, optimize:

F(A,B,C)= $\sum_m$ (0,1,4) as PoS form.

As we need F as PoS, so we need to use maxterms.

F(A,B,C)= $\prod_M$ (2,3,5,6,7)

A \ BC	00	01	11	10
0			0	0
1		0	0	0

$$F(A,B,C) = \overline{B}(\overline{A} + \overline{C})$$

3-Variable K-map

Exercise:

Using k-map, optimize: $F(X,Y,Z)= \sum_m (0,1,3,5,6)$ as PoS form.

3-Variable K-map

<u>Exercise:</u>

Using k-map, optimize the following function as SoP form.

$$F(A,B,C) = \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + A\overline{B}C + AB\overline{C} + ABC$$

3-Variable K-map

Exercise:

Using k-map, optimize the following function as PoP form.

$$F(A,B,C) = \overline{A}\,\overline{B} + \overline{A}B\overline{C}$$

4-Variable K-map

Var1 Var2 \ Var3 Var4	00	01	11	10
00	0000 **0**	0001 **1**	0011 **3**	0010 **2**
01	0100 **4**	0101 **5**	0111 **7**	0110 **6**
11	1100 **12**	1101 **13**	1111 **15**	1110 **14**
10	1000 **8**	1001 **9**	1011 **11**	1010 **10**

In each square:

Black values represents the binary values.
Red values represents the decimal values.

4-Variable K-map (Example 3)

<u>Example 3:</u>

Using k-map, optimize:

F(A,B,C,D) =

ΠM (1,2,5,6,9,11,13,14,15)

as PoS.

AB \ CD	00	01	11	10
00		0		0
01		0		0
11		0	0	0
10		0	0	

$$\mathbf{F(A,B,C,D) = (C+\bar{D})(\bar{A}+\bar{D})(A+\bar{C}+D)}$$

$$\mathbf{(\bar{A}+\bar{B}+\bar{C})}$$

or

$$\mathbf{(\bar{B}+\bar{C}+D)}$$

4-Variable K-map (Example 4)

Example 4:

Using k-map, optimize F in an SoP form:

$$\mathbf{F(A, B, C, D) = \overline{A}\,\overline{C}\,\overline{D} + \overline{A}D + \overline{B}C + CD + A\overline{B}\,\overline{D}}$$

We need to find minterms or maxterms. We can use truth table. But an easier way can be used is by adding 1's (in case of SoP) or 0's (in case of PoS) in the square(s) that met the value of the variables. Of course do not add two 1's (or 0's) in the same square.

4-Variable K-map (Example 4)

- For $\bar{A}\bar{B}\bar{C}$ add 1's in the squares where A=0, B=0, C=0 (2)
- For $\bar{A}D$ add 1's in the squares where A=0, D=1 (4)
- For $\bar{B}C$ add 1's in the squares where B=0, C=1 (4)
- For CD add 1's in the squares where C=1, D=1 (4)
- For $A\bar{B}\bar{D}$ add 1's in the squares where A=1, B=0, D=0 (2)

4-Variable K-map (Example 4)

$$F(A, B, C, D) = \bar{A}\bar{C}\bar{D} + \bar{A}D + \bar{B}C + CD + A\bar{B}\bar{D}$$

AB \ CD	00	01	11	10
00	1	1	1 ~~1~~ ~~1~~	1
01	1	1	1 ~~1~~	
11			1	
10	1		1 ~~1~~	1 ~~1~~

$\Rightarrow$

AB \ CD	00	01	11	10
00	1	1	1	1
01	1	1	1	
11			1	
10	1		1	1

$$F(A, B, C, D) = \bar{A}\bar{C} + CD + \bar{B}\bar{D}$$

4-Variable K-map (Example 4)

$$F(A,B,C,D) = \bar{A}\bar{C}\bar{D} + \bar{A}D + \bar{B}C + CD + A\bar{B}\bar{D}$$

AB \ CD	00	01	11	10
00	1	1	1	1
01	1	1	1	
11			1	
10	1		1	1

$\Rightarrow$ **Note that we can obtain the minterms and maxterms directly from the k-map.**

F(A,B,C,D) =

$=\sum_m (0,1,2,3,4,5,7,8,10,11,15)$

$=\prod_M (6,9,12,13,14)$

4-Variable K-map

<u>Exercise:</u>

Using k-map, optimize F as SoP.

$F(W,X,Y,Z)=\Pi_M(0,1,2,4,7,8,9,10,12,15)$

4-Variable K-map

Exercise:

Using k-map, optimize F as SoP & PoS

$$F(A,B,C,D) = B\overline{C} + \overline{A}B + BC\overline{D} + \overline{A}\,\overline{B}D + A\overline{B}\,\overline{C}D$$

5-Variable K-map

Var1=0

Var2 Var3 \ Var4 Var5	00	01	11	10
00	00000 0	00001 1	00011 3	00010 2
01	00100 4	00101 5	00111 7	00110 6
11	01100 12	01101 13	01111 15	01110 14
10	01000 8	01001 9	01011 11	01010 10

Var1=1

Var2 Var3 \ Var4 Var5	00	01	11	10
00	10000 16	10001 17	10011 19	10010 18
01	10100 20	10101 21	10111 23	10110 22
11	11100 28	11101 29	11111 31	11110 30
10	11000 24	11001 25	11011 27	11010 26

In each square: Black values represents the binary values.
Red values represents the decimal values.

5-Variable K-map (Example 5)

Example 5:

Using k-map, optimize:

$$F(V,W,X,Y,Z) = \Pi_M(0,4,7,8,12,15,16,20,23,24,28,29,30,31)$$

As PoS form.

5-Variable K-map (Example 5)

V=0

WX \ YZ	00	01	11	10
00	0			
01	0		0	
11	0		0	
10	0			

V=1

WX \ YZ	00	01	11	10
00	0			
01	0		0	
11	0	0	0	0
10	0			

$$F(V, W, X, Y, Z) = (Y + Z)(\overline{X} + \overline{Y} + \overline{Z})(\overline{V} + \overline{W} + \overline{Z})$$

5-Variable K-map

<u>**Exercise:**</u>

Using k-map, optimize F as SoP.

$$F(V,W,X,Y,Z) = \sum m\,(0,2,4,6,9,10,12,13,14,15,16,17,21,25,26,28,29,30,31)$$

$$F(V,W,X,Y,Z) = \sum m\,(0,2,4,6,9,10,12,13,14,15,16,17,21,25,26,28,29,30,31)$$

5-Variable K-map

<u>**Exercise:**</u>

Using k-map, optimize F as SoP.

$$F(A,B,C,D,E) = (A+\overline{B}+\overline{C})(\overline{A}+D+\overline{E})(\overline{C}+\overline{D}+E)(B+\overline{C})$$

Then get its minterms & maxterms.

$$F(A,B,C,D,E) = (A + \overline{B} + \overline{C})(\overline{A} + D + \overline{E})(\overline{C} + \overline{D} + E)(B + \overline{C})$$

don’t care conditions

A don't-care term for a function is an input-sequence (a series of bits) for which the function output does not matter. An input that is known never to occur is a can't-happen term. Both these types of conditions are treated the same way in logic design and may be referred to collectively as don't-care conditions.

An “x” is added on the cell of don’t care conditions. The cell may be used iff it will lead to more simplified function.

don’t care conditions (Example 6)

<u>Example 6:</u>

Using k-map, optimize:

F(A,B,C,D)=

$\sum_m(0,2,3,6,7,8,10,11) + d(5,14,15)$

As PoS form.

$F(A,B,C,D)=\Pi_M(1,4,9,12,13)$

$+ d(5,14,15)$

AB \ CD	00	01	11	10
00		0		
01	0	x		
11	0	0	x	x
10		0		

$$F(A,B,C,D) = (\bar{B} + C)(C + \bar{D})$$

don’t care conditions

<u>**Exercise:**</u>

Compare the simplified SoP function (using k-map) of:

$$F(A,B,C,D)=\sum{}_m(0,2,3,6,7,8,10,11)$$

$$F(A,B,C,D)=\sum{}_m(0,2,3,6,7,8,10,11) + d(5,14,15)$$

$$F(A,B,C,D)=\sum{}_{m}(0,2,3,6,7,8,10,11) + d(5,14,15)$$

don’t care conditions

<u>Exercise:</u>

Using k-map, optimize F(A,B,C) =$\sum_m$ (4,5)+ d(0,6,7) as SoP.

don’t care conditions

<u>Exercise:</u>

Using k-map, optimize F as SoP

$$F(A,B,C,D,E) = ABE(\overline{C}D + \overline{D}) + \overline{A}(\overline{C}E + \overline{B}D)$$

$$+d(5,12,13,14,15,17,22,23,31)$$

Also get the function’ minterms & maxterms.

$$F(A,B,C,D,E) = ABE(\overline{C}D + \overline{D}) + \overline{A}(\overline{C}E + \overline{B}D)$$

$$\mathbf{+d(5,12,13,14,15,17,22,23,31)}$$