Part I

1 **Operations on matrices :** + , . ,scaler mult.

2 **Types :** commute, anti-commute, inverse of, symmetric, skew-symmetric, orthogonal,

3 **Properties :**

- $A + B = B + A$,
- $A + (B + C) = (A + B) + C$,
- $\lambda (A + B) = \lambda A + \lambda B$, where λ is a scalar ,
- $A(B + C) = AB + AC$,
- $(A + B)C = AC + BC$,
- $A(BC) = (AB) C$,
- $(AB)^{-1} = B^{-1}A^{-1}$
- $(A^T)^T = A$ and $(\lambda A)^T = \lambda A^T$
- $(A + B)^T = A^T + B^T$
- $(AB)^T = B^T A^T$
-

- $A + A^T$ must be symmetric
- $A - A^T$ must be skew-symmetric

Prove that :

4- Row Operations

5- Determinants

Properties

$|A^T| = |A|$

$|AB| = |A||B|$

The determinant of any orthogonal matrix is either +1 or –1.

How to find an inverse for a 3x3 matrix?

6- vectors

7- System of Linear Equations

$$A x = b$$

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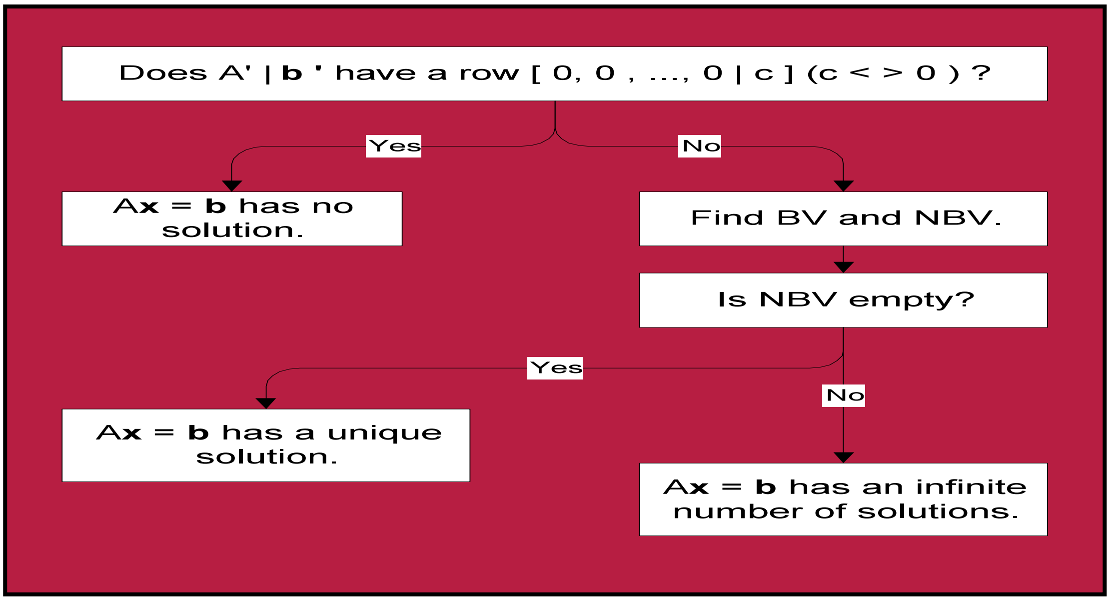
1- Gaussian-Jordan Elimination Method

2- matrix inverses Method

3- Cramer's Rule

$$\begin{aligned} x_1 \quad + x_2 \quad + x_3 &= 6 \\ x_1 \quad + 2x_2 \quad + x_3 &= 7 \\ 2x_1 \quad + 3x_2 \quad + 2x_3 &= 14 \end{aligned}$$



8- A Vector Space

- Definition of a vector space
- A Linear combination
- Linear Independence and Dependence of Vectors
- A set $B=\{b_1,b_2,\ldots,b_k\}$ of vectors is a **basis** for a vector space V
- The Rank of a Matrix
- A method of determining whether a set of vectors $V = \{v_1, v_2, \ldots, v_m\}$ is linearly dependent
- Linear Transformations
  - ✓ The standard (associated) matrix AT for the Linear Transformation T
  - ✓ Let B denote the reduced echelon form of A.

    1- If m > n, then T cannot be onto.

    2- If m < n, then T cannot be one-to-one.

    3- T is onto if and only if B has a pivot in every row.

    4- T is one-to-one if and only if B has a pivot in every column.
  - ✓ Tables 1,2, ..

9- **Eigenvalues and Eigenvectors:** $\mathbf{Ax} = \lambda\, \mathbf{x}$, *A is a square matrix*

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$\det(A - \lambda I) = 0 \Rightarrow \lambda_i$ (real or complex)

at λ_i : find $x(\lambda_i) = x_i$ [eigen value] from $(A - \lambda_i I)x_i = 0$

Part II

1- Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$$\mathcal{L}\left\{e^{at}\right\} = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$$

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$$\mathcal{L}^{-1}\left\{\frac{-2s+6}{s^2+4}\right\}.$$

2- TRANSFORMS OF DERIVATIVES

IVP $\frac{dy}{dt} + 3y = 13\sin 2t, \quad y(0) = 6.$

3- Step functions

$$h(t) = \begin{cases} 2 & , 0 \le t < 4 \\ 5 & , 4 \le t < 7 \\ -1 & , 7 \le t < 9 \\ 1 & , \quad t \ge 9 \end{cases} == 2 + 3u_4 - 6u_7 + 2u_9$$

$$\boxed{\text{i.e., } \mathcal{L}\{u_a(t)\} = \frac{e^{-as}}{s}, \quad (s > 0).}$$

4- First and Second Shifting Theorem

If $\mathcal{L}\{f(t)\} = F(s)$, then $\mathcal{L}\{e^{at} f(t)\} = F(s - a)$.

$\mathcal{L}\{f(t-a)\, u_a(t)\} = e^{-as} F(s)$,

$\mathcal{L}\{t u_2(t)\} = ?$

5- FOURIER SERIES

$$f(x) \cong S(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{n\pi x}{T}\right) + b_n\sin\left(\frac{n\pi x}{T}\right)\right]$$

$$a_n = \frac{1}{T}\int_{-T}^{T} f(x)\cos\left(\frac{n\pi x}{T}\right)dx \qquad b_n = \frac{1}{T}\int_{-T}^{T} f(x)\sin\left(\frac{n\pi x}{T}\right)dx$$