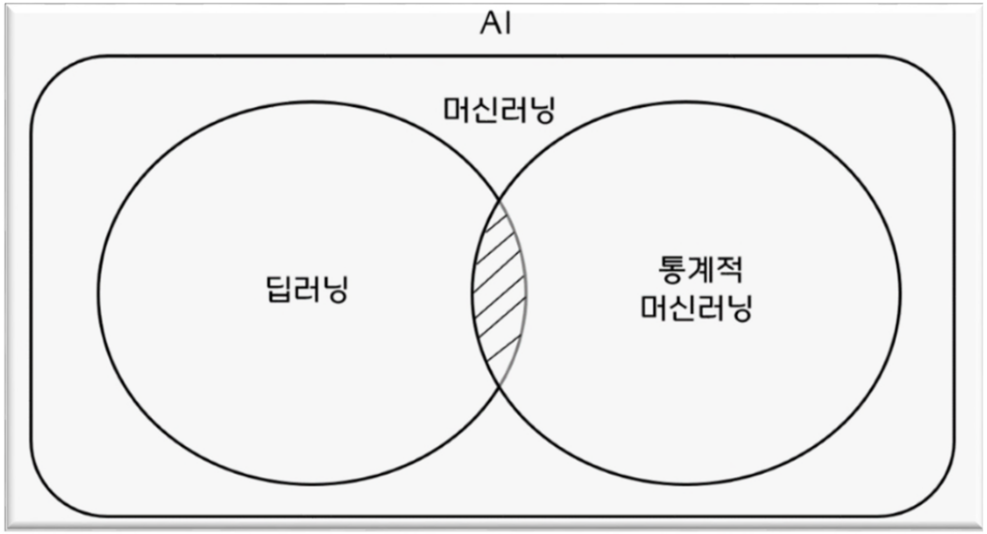
Statistical Machine Learning

1주차

담당 : 11기 명재성

KU-BIG

Statistical Machine Learning



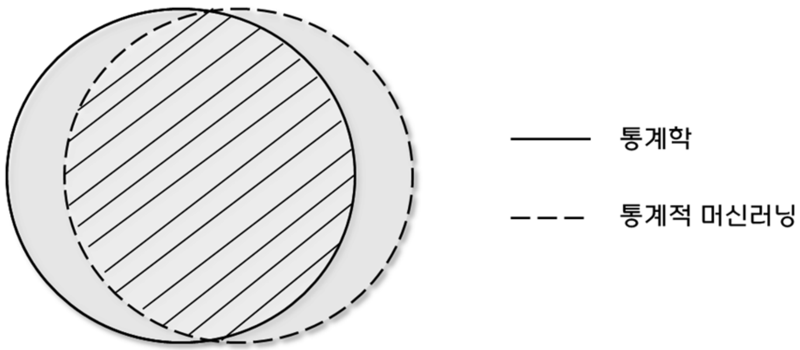
Statistical Machine Learning

전통적인 통계학

- 규칙의 통계적 추론에 중점 (전문적인 통계적, 수학적 지식)
- 자료의 특성(다변량, 시계열, 범주형 등)에 따라 분석.

통계적 머신러닝

- 규칙의 일반화에 중점
- 목적변수의 관측여부에 따라 지도학습, 비지도학습으로 분석



BigData

- Volume
- Velocity
- Variety

구분	통계적 머신러닝	딥러닝
데이터 크기	중/소 크기	빅데이터
분석자료 형태	2차원 텐서	2차원 텐서이상
강점을 갖는 자료	정형화된 자료	비정형자료
특성변수	특성변수를 만들어야 함	특성변수가 만들어짐
특성변수의 정규화 및 표준화	선택	필요
모형	매우 많음	기본적으로 3 개의 모형
최적화	일반적으로 전체 데이터 사용	배치데이터
해석여부	해석이 쉬움 (단, SVM과 boosting 제외)	어렵거나 불가능
하드웨어	중급	고성능(GPU 요구)
실행요구시간	최대 시간 단위	최대 주단위 시간

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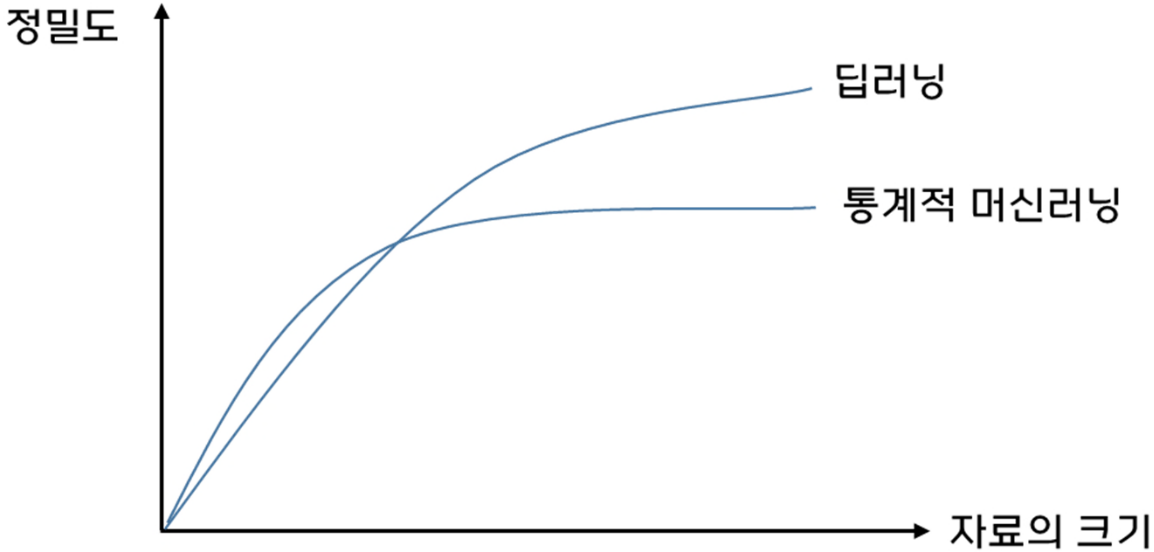
Statistical Machine Learning and Deep Learning



Statistical Machine Learning Type

- 지도학습(supervised learning)
  비지도학습(unsupervised learning)
  강화학습(Reinforcement learning)

- 배치학습(Batch learning)
  온라인학습(Online learning)

- 사례기반(Instance-based learning)
  모형기반(Model-based learning)

Linear Regression

- Linearity?

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_p X_{pi} + \epsilon_i$$

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i} + \epsilon_i$$

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \cdots + \beta_p X_i^p + \epsilon_i$$

Linear Model

- Linearity? $\longrightarrow$ Linear Model

$$Y_i \overset{ind}{\sim} (\mu_i(\mathbf{X}_i),\ \sigma^2) \qquad \text{where} \quad E[Y_i] = \mu_i(\mathbf{X}_i)$$

$$\mu_i(\mathbf{X}_i) = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_p X_{pi} = \boldsymbol{\beta}^T \mathbf{X}_i$$

$$\boldsymbol{\mu}(\mathbf{X}) = \mathbf{X}\ \boldsymbol{\beta}$$

Regression

- Least Square Estimator

$$\sum \epsilon_i^2 = \sum (Y_i - \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_p X_{pi})^2$$

$$\frac{\partial}{\partial \beta_0} \sum (Y_i - \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_p X_{pi})^2 \overset{set}{=} 0$$

$$\frac{\partial}{\partial \beta_1} \sum (Y_i - \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_p X_{pi})^2 \overset{set}{=} 0$$

$$\vdots$$

$$\frac{\partial}{\partial \beta_p} \sum (Y_i - \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_p X_{pi})^2 \overset{set}{=} 0$$

Regression

- Least Square Estimator

```
> summary(model.a<-lm(exp~income+ factor(Region)))

Call:
lm(formula = exp ~ income + factor(Region))

Residuals:
    Min      1Q  Median      3Q     Max 
-77.624 -26.431  -8.821  19.391 174.548 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept)     21.94531   60.05982   0.365   0.7165    
income           0.05337    0.01169   4.566 3.84e-05 ***
factor(Region)2  1.21498   20.02606   0.061   0.9519    
factor(Region)3 -0.44452   20.91222  -0.021   0.9831    
factor(Region)4 49.92487   19.78310   2.524   0.0152 *  
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

Regression

- Risk Function

$$R(\theta, T(X)) = E[L(\tau(\theta), T(X))]$$

- Loss Function

$$L[\tau(\theta), T(X)] = \sum(Y_i - \hat{Y}_i)^2$$

$$= \sum|Y_i - \hat{Y}_i|$$

Regression

- Error term?
  - Mean 0
  - Identical, Independent
  - Normal?

Regression

- Likelihood function

Definition (Likelihood)

For $X_1, \cdots, X_n \overset{iid}{\sim} f_X(x;\theta)$, where θ denotes a parameter of interest. The likelihood function is

$$L(\theta;\mathbf{X}) = L(\theta; X_1, \cdots, X_n) = \prod_{i=1}^{n} f_X(X_i;\theta)$$

Regression

- Maximum Likelihood Estimator

Definition (Maximum likelihood estimator, MLE)

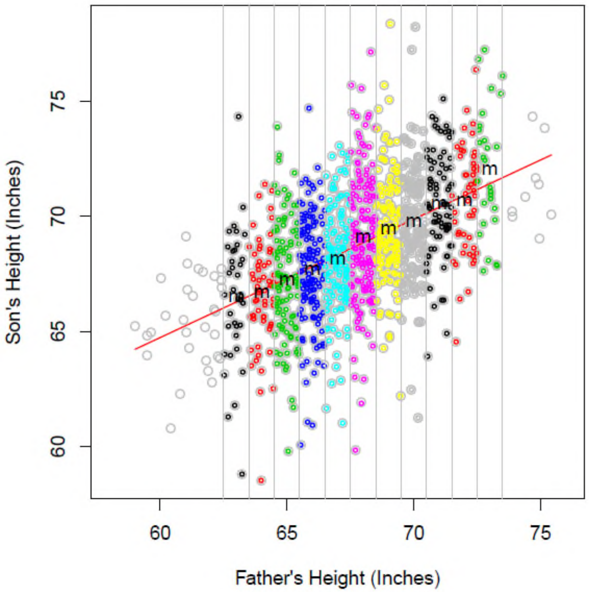
For $X_1, \cdots, X_n \overset{iid}{\sim} f_X(x;\theta)$, the MLE of θ is

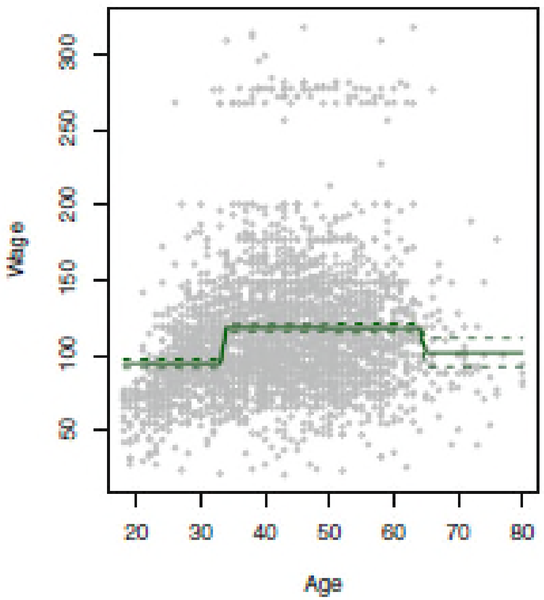
$$\hat{\theta}_{MLE} = \underset{\theta}{\operatorname{argmax}}\, L(\theta;\mathbf{x}).$$

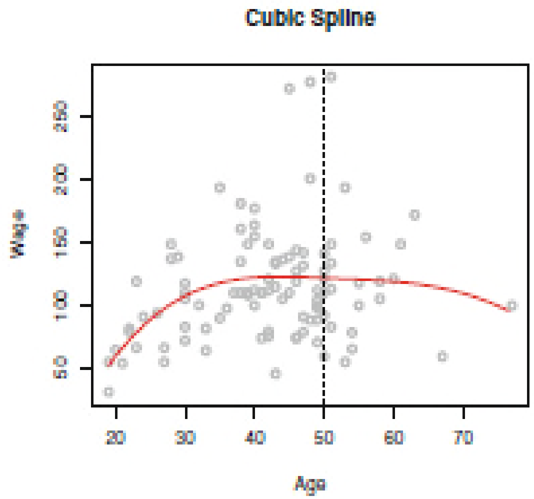
which is equivalent to maximize the logarithm of $L(\theta;\mathbf{x})$ which we call the log-likelihood

$$\ell(\theta;\mathbf{x}) = \log L(\theta;\mathbf{x}).$$

Other Regression..?







Logistic Regression

$$\log\left(\frac{\pi(\mathbf{X})}{1-\pi(\mathbf{X})}\right) = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_p X_{pi}$$

$$\mathrm{P}(Y = 1|\mathbf{X}) = \pi(\mathbf{X}) = \frac{e^{\beta_0+\beta_1 X_{1i}+\beta_2 X_{2i}+\cdots+\beta_p X_{pi}}}{1+e^{\beta_0+\beta_1 X_{1i}+\beta_2 X_{2i}+\cdots+\beta_p X_{pi}}}$$

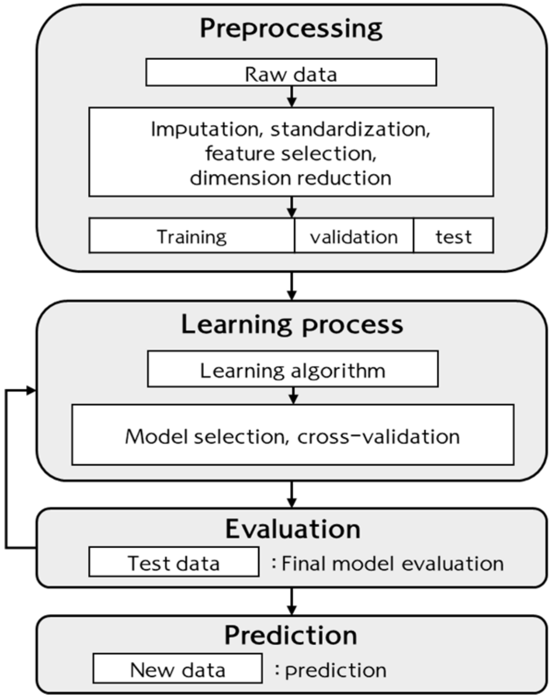
$$= \frac{e^{\boldsymbol{\beta}^T\mathbf{X}}}{1+e^{\boldsymbol{\beta}^T\mathbf{X}}}$$

Generalized Linear Model

	Normal	Poisson	Binomial	Gamma	Inv Gaussian
Notation	$N(\mu, \sigma^2)$	$P(\mu)$	$B(n, \pi)/n$	$G(\mu, v)$	$IG(\mu, \sigma^2)$
Support	$(-\infty, \infty)$	$\{0, 1, \cdots\}$	$\{0, \cdots, n\}/n$	$(0, \infty)$	$(0, \infty)$
$a(\phi)$	$\phi = \sigma^2$	1	$1/m$	v^{-1}	σ^2
$b(\theta)$	$\theta^2/2$	e^θ	$\log(1 + e^\theta)$	$-\log(-\theta)$	$-(-2\theta)^{1/2}$
$b'(\theta) = E(Y)$	θ	e^θ	$\frac{e^\theta}{1+e^\theta}$	$-1/\theta$	$(-2\theta)^{-1/2}$
$(b')^{-1}(\mu) = g(\mu)$	μ	$\log(\mu)$	$\log \frac{\mu}{1-\mu}$	μ^{-1}	μ^{-2}
$b''(\theta)$	1	μ	$\mu(1-\mu)$	μ^2	μ^3

Table: Summary of some popular GLM models.

Summary


Preprocessing
Raw data
Imputation, standardization,
feature selection,
dimension reduction
Training
validation
test
Learning process
Learning algorithm
Model selection, cross-validation
Evaluation
Test data
: Final model evaluation
Prediction
New data
: prediction

reference

자료

19-2 STAT424 통계적 머신러닝 - 박유성 교수님

교재

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