

# Statistical Machine Learning

7주차

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**1. What is Ensemble?**

**2. Ensemble Methods**

**3. Ensemble Models**

# 1. What is Ensemble Learning?

# Ensemble

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Wisdom of the crowd



# Ensemble

## Ensemble learning

- 다수의 기본 분류 모델( base classifier, weak classifier)의 예측 결과를 종합하여, 정확한 예측 성능을 얻도록 하는 방법론

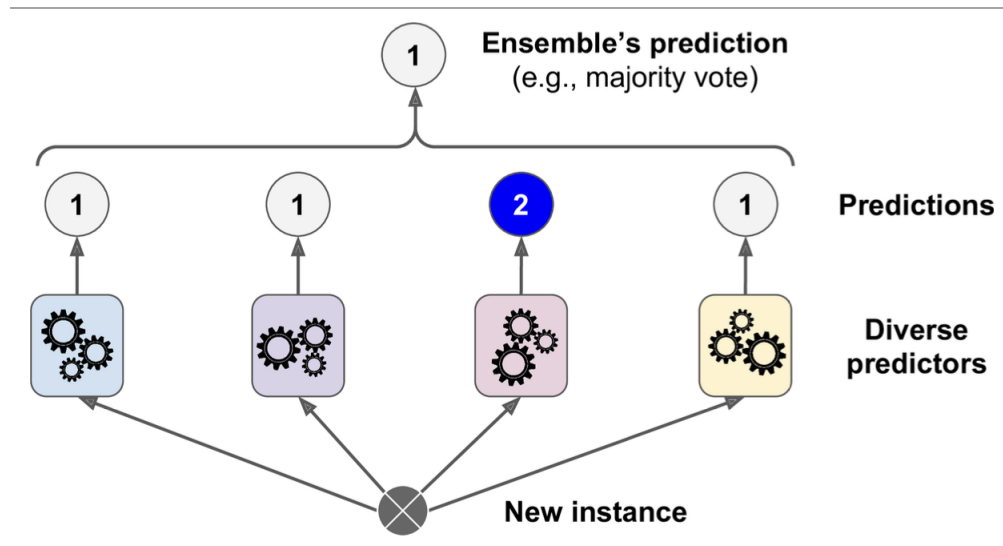
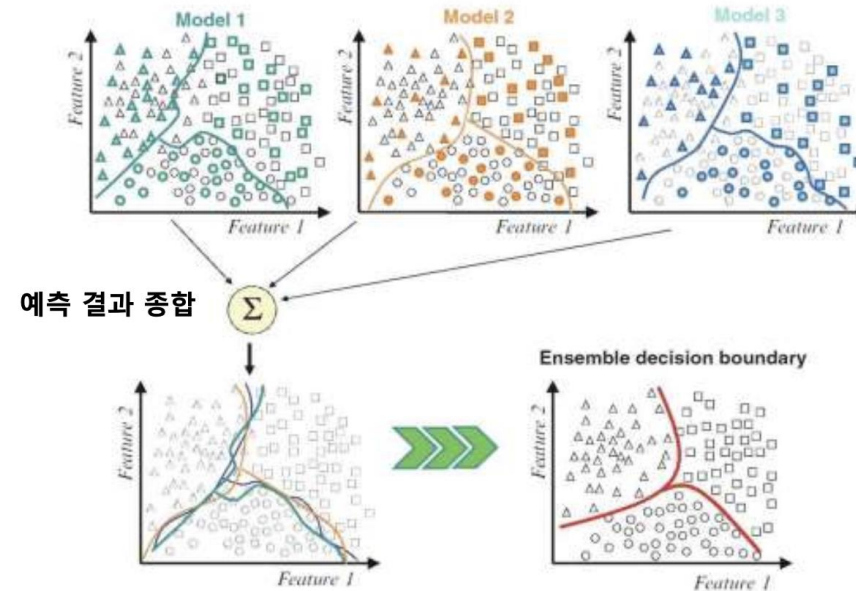


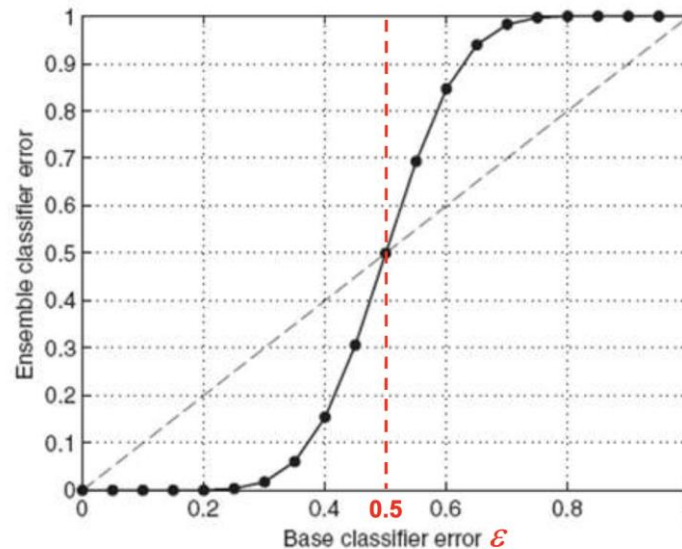
Figure 7-2. Hard voting classifier predictions



# Ensemble

## Example

- 25 base classifiers
- Error rate  $\varepsilon = 0.35$
- Each independent
- Ensemble classifier : Majority vote

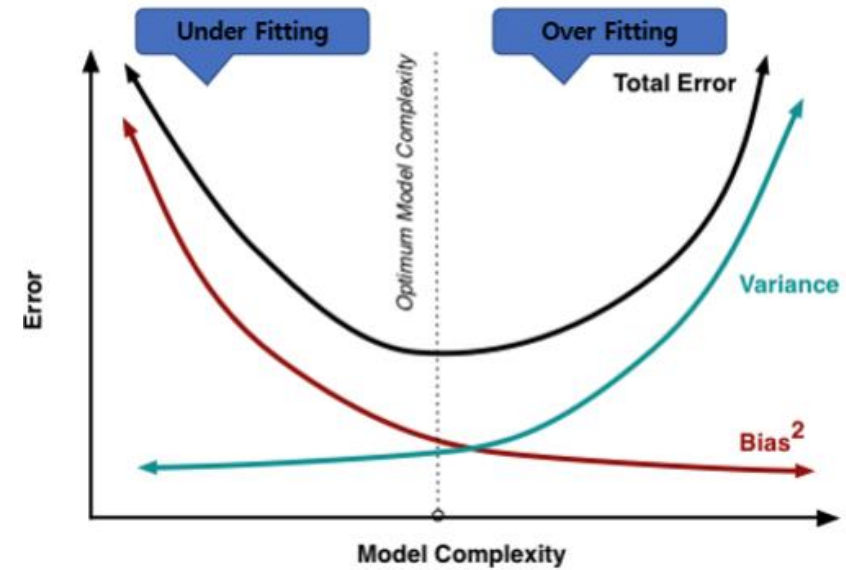
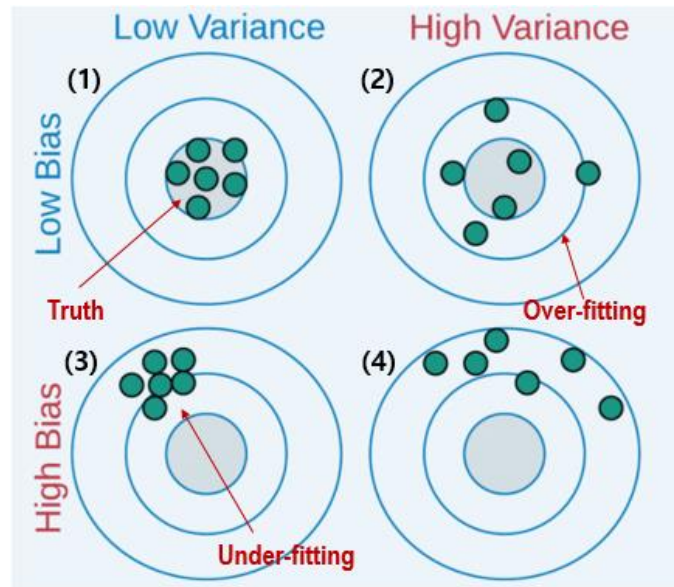


$$e_{ensemble} \sim \text{Binomial}(25, 0.35)$$

$$P(\text{incorrect classifier} \geq 13) = e_{ensemble} = \sum_{i=13}^{25} \binom{25}{i} \varepsilon^i (1 - \varepsilon)^{25-i} = 0.06$$

Probability that the ensemble classifier makes wrong prediction

# Bias and Variance



$$Y = f(x) + e \quad (Y: \text{predictions}, x: \text{covariates})$$

$$\text{Error}(x) = E[(Y - f'(x))^2] = E[((f(x) - f'(x)) + e)^2]$$

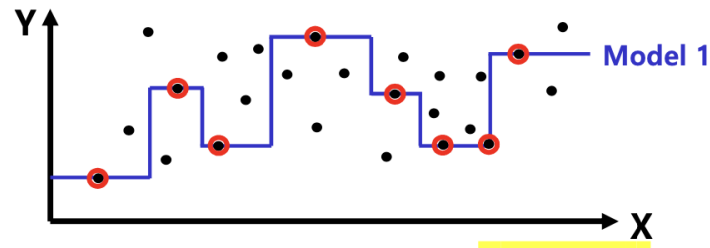
$$= \underbrace{(E[f'(x) - f(x)])^2}_{\text{Bias}} + \underbrace{E[(f'(x) - E[f'(x)])^2]}_{\text{Variance}} + \sigma_e^2$$

$$\text{Error}(x) = \text{Bias}^2 + \text{Variance} + \text{Irreducible Error}$$

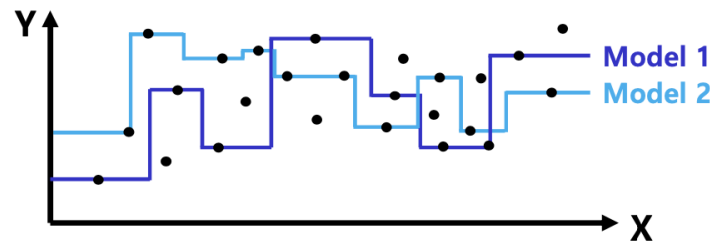
# Ensemble

## Ensemble Learning

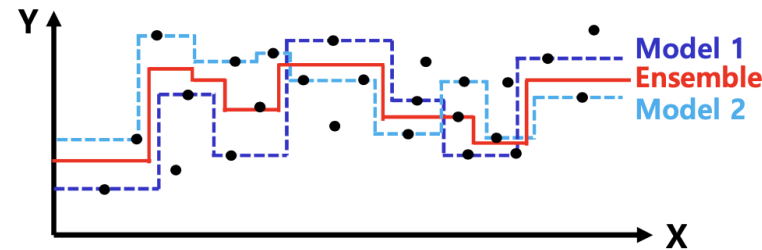
- Reduce Learning error
- Reduce Bias
- Reduce Variance



Sample 데이터에 대해 잘 예측 "Low bias"



Model 1과 Model 2 예측 값이 서로 다름 "High variance"



Model 1과 Model 2 예측 값의 평균 사용: Ensemble

"Low variance"



# Ensemble

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## Voting

Hard Voting  
Soft Voting  
Weighted Voting

## Stacking

Meta level Learning  
Blending

## Bagging

Bootstrap + Aggregating

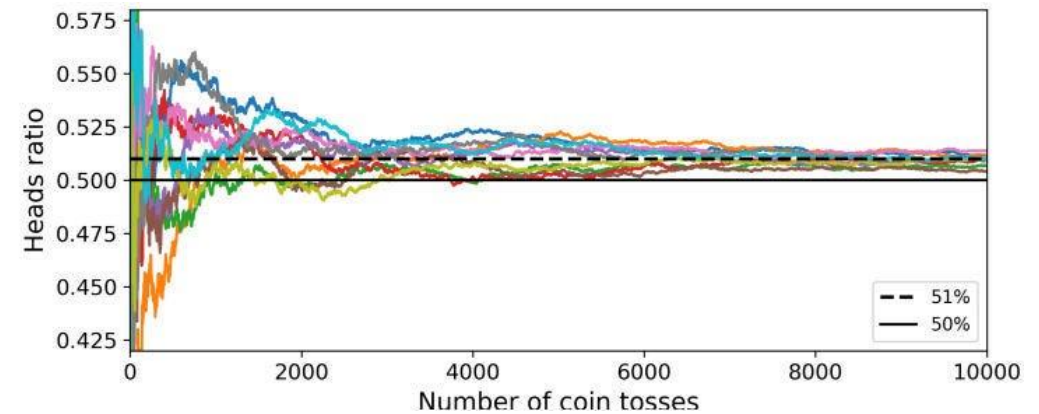
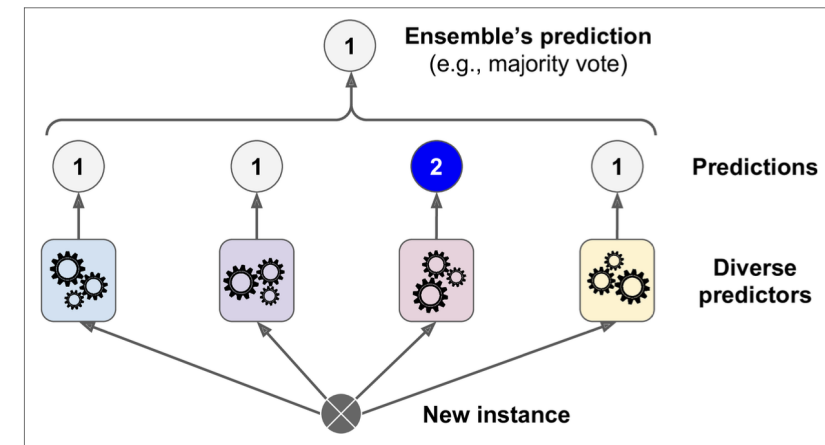
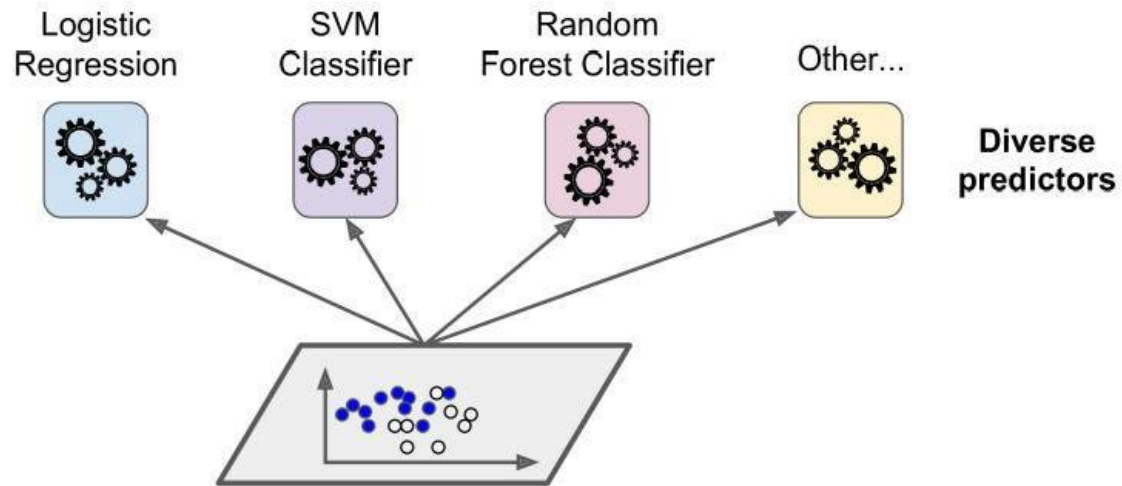
## Boosting

Error learner

## 2. Ensemble Methods

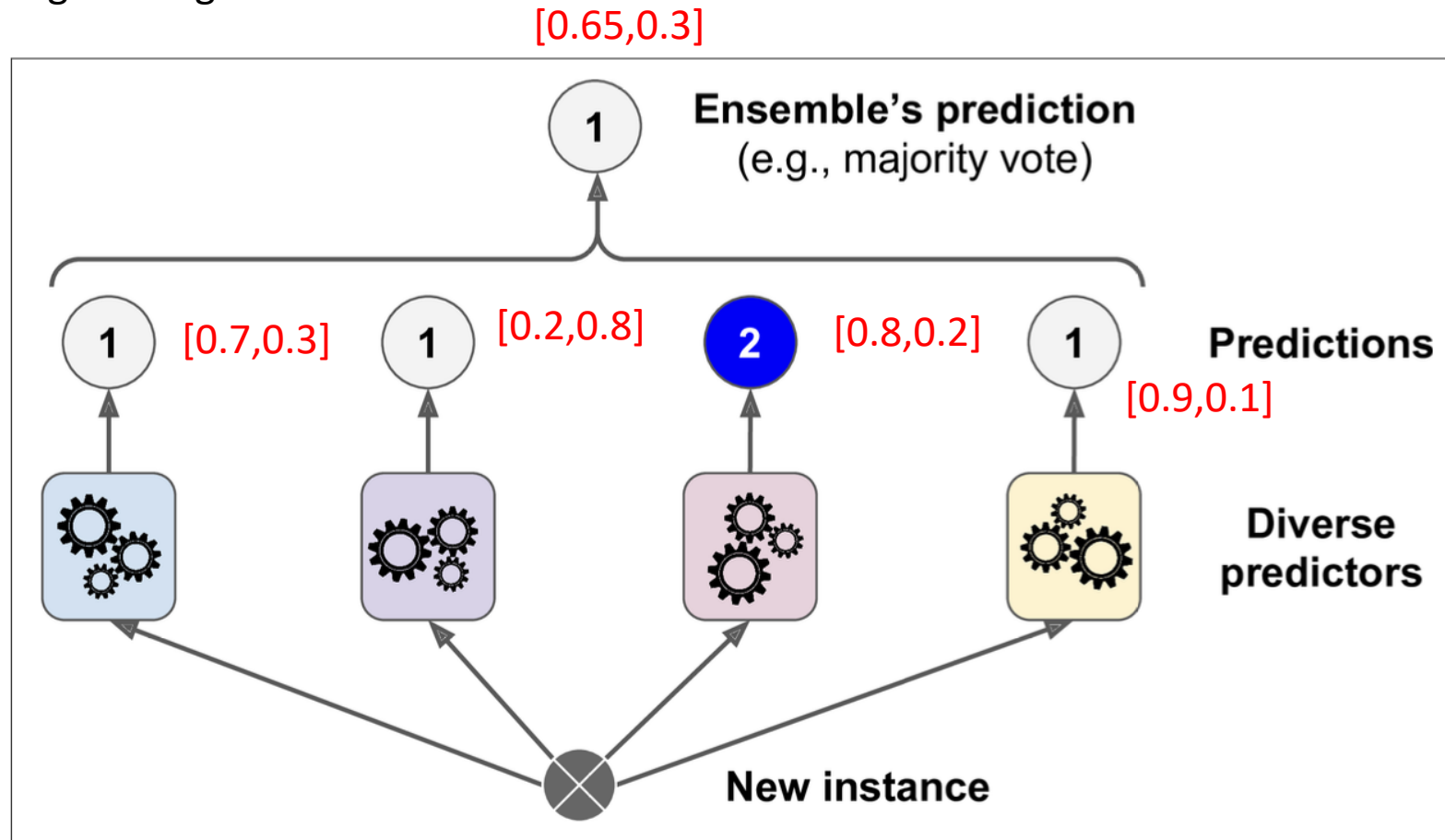
# Voting

## Hard Voting : Majority Voting



# Voting

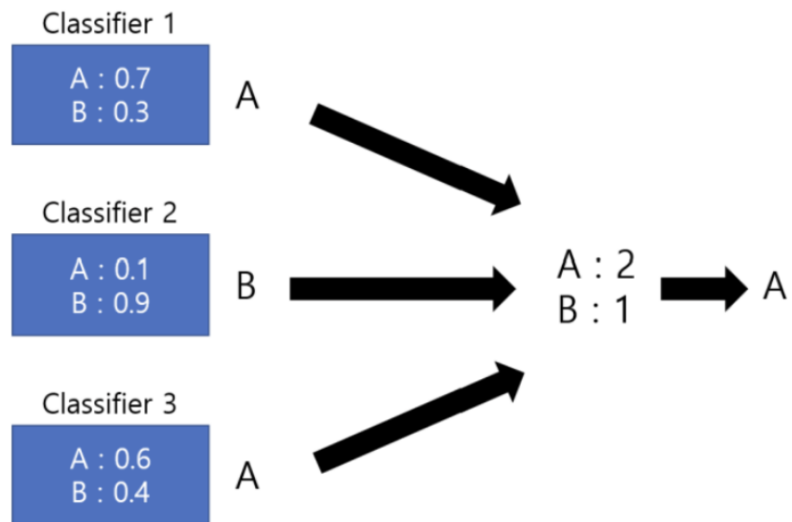
**Soft Voting** : Average Voting



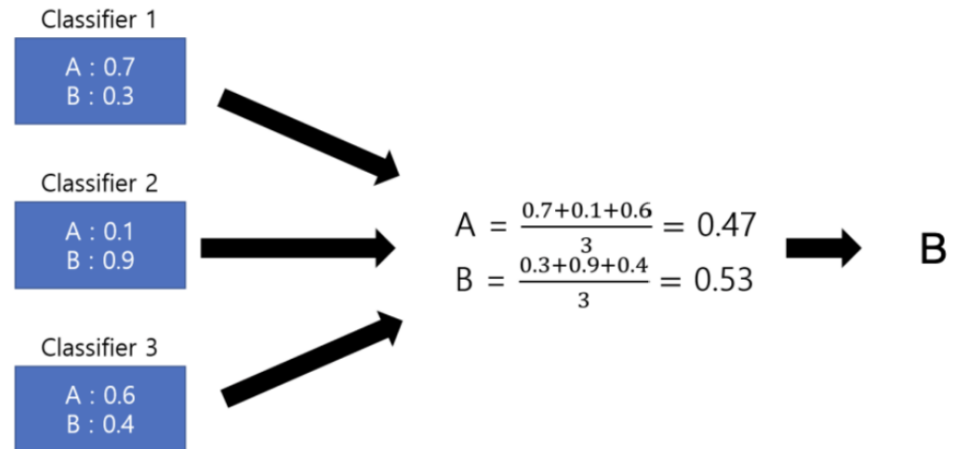
# Voting

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## Hard Voting



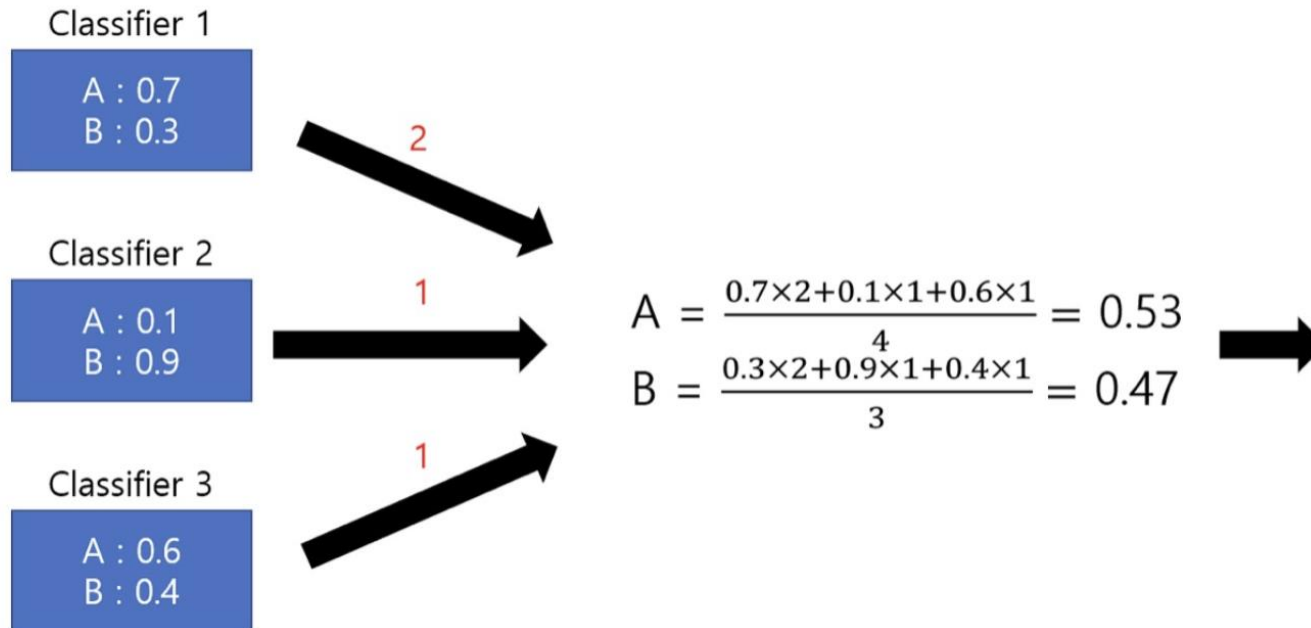
## Soft Voting



# Voting

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## Soft + Weighted



# Bagging

**Bagging** = Bootstrap + Aggregating(Average)

**Bootstrap** : sampling with Replacement → Variance 개선

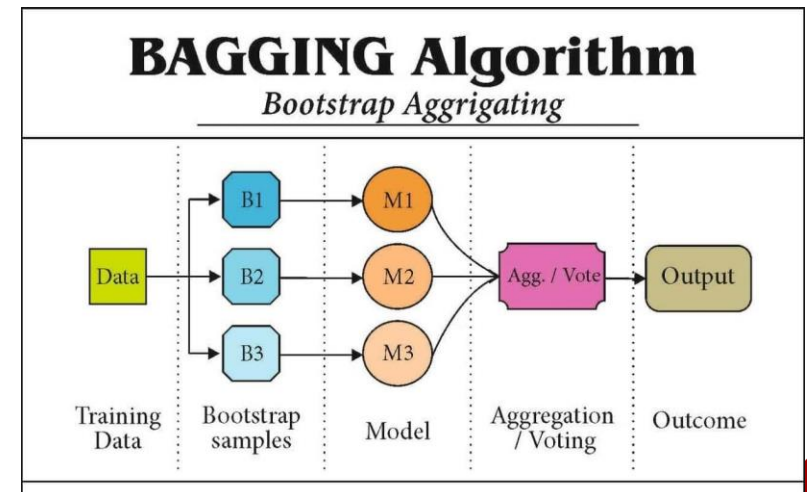
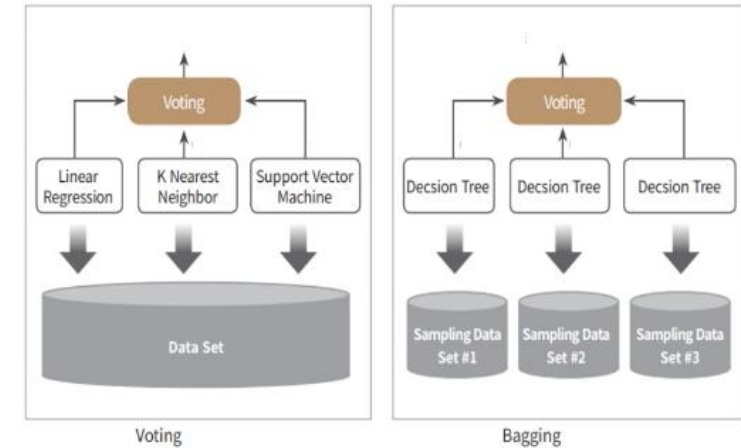
Probability that one sample is not chosen by bootstrap  
(N records, N sample size)

$$= \left(1 - \frac{1}{N}\right)^N$$

If N is large enough, then  $\lim_{N \rightarrow \infty} \left(1 - \frac{1}{N}\right)^N = e^{-1} = 0.3678$

**36.7%** of original train dataset

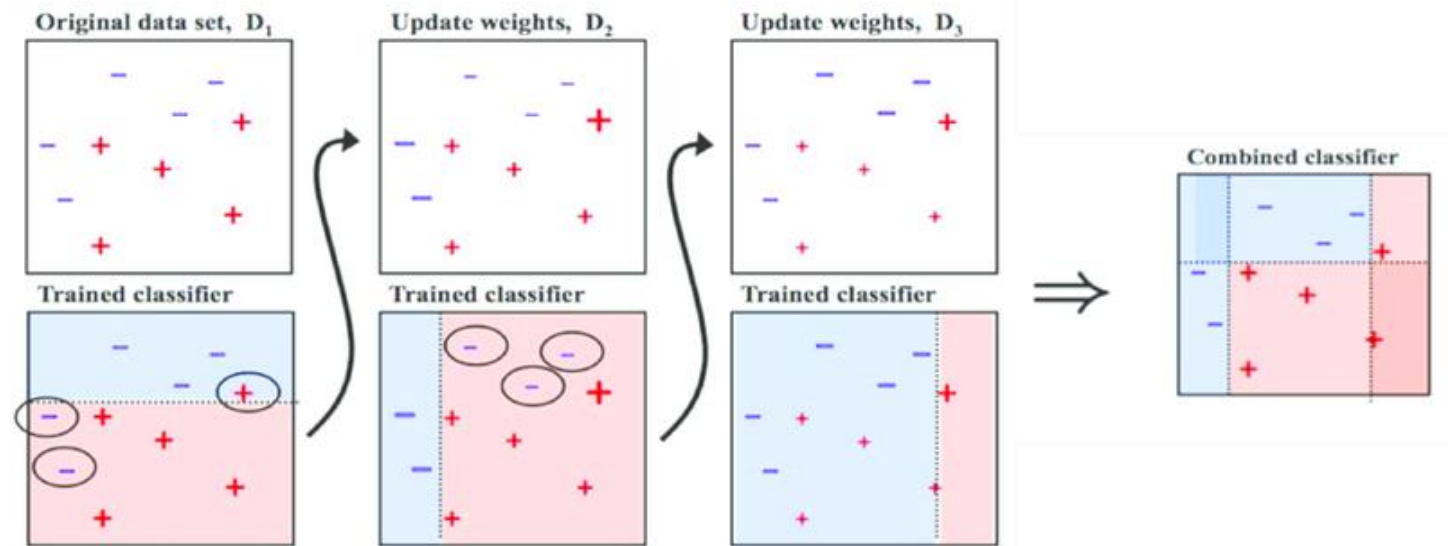
**Aggregating** : Majority Voting, Weighting, Soft voting



# Boosting

## Boosting

- 오분류된 샘플에 더 많은 가중치 부여 -> 오답을 다시 학습
  - 예측이 틀린 데이터가 다시 뽑힐 가중치가 높아진다.
  - 이전 모델이 잘못 예측한 부분을 집중적으로 학습
- Bias 개선

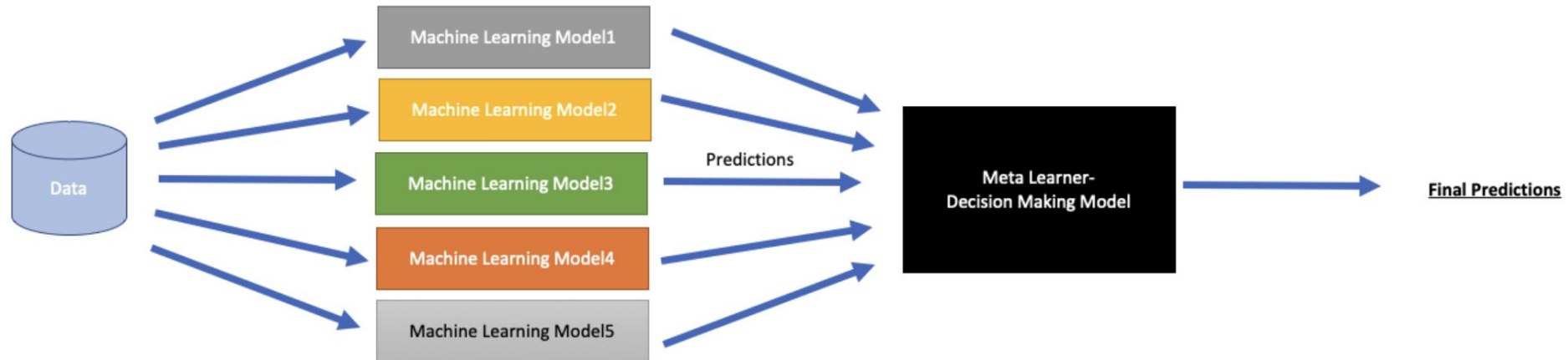




# Stacking

## Stacking Generalization

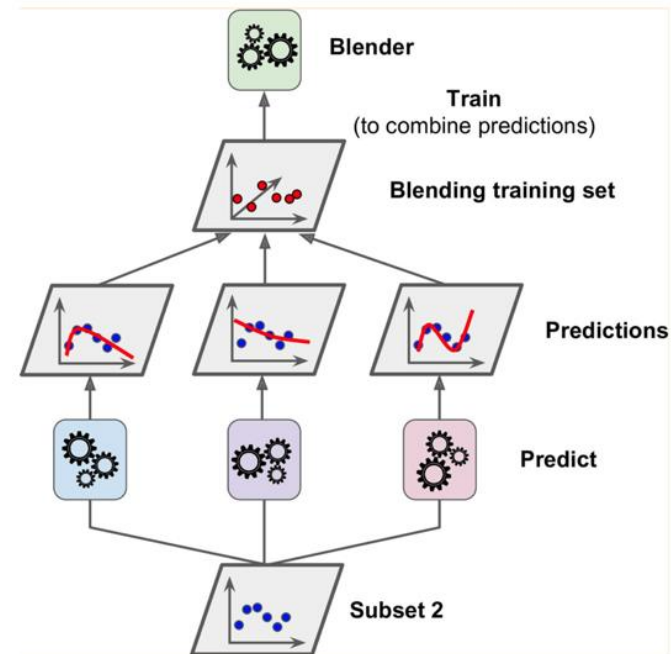
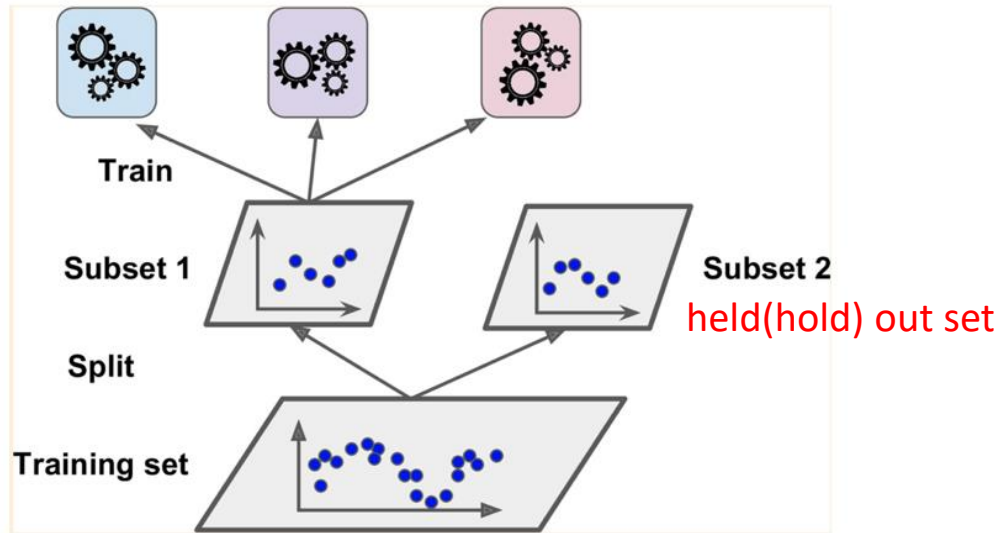
- Meta-learning model
- 여러 가지 모델들의 예측값을 최종 모델의 학습 데이터로 사용
- K-fold cv
- Step 0 : 각 weak model에 k-fold cv를 적용하여 예측 데이터를 형성
- Step 1 : step 0에서 만든 예측 데이터를 stack.하여 meta-model을 train 및 예측



# Blending

## Blending Generalization

- Meta-learning model
- 개별 모델의 예측값을 다시 input으로 사용
- Use hold-out set



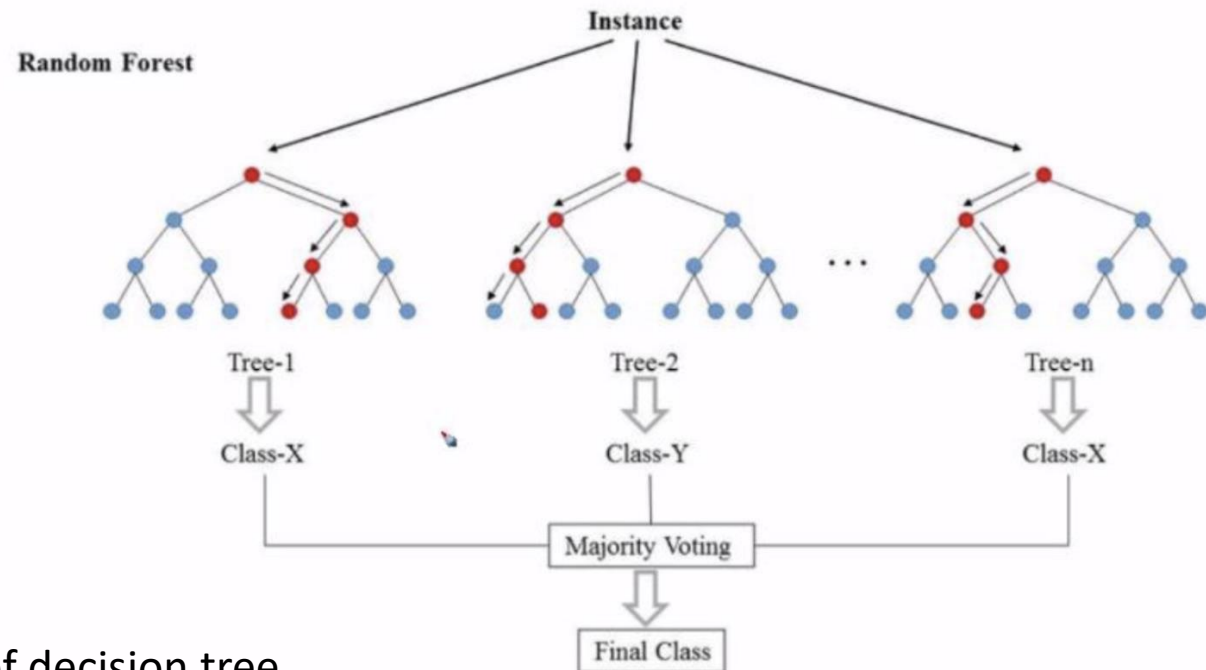
### 3. Ensemble Models

# RandomForest

Feature Bagging → RandomForest

## RandomForest Decision Tree Generation

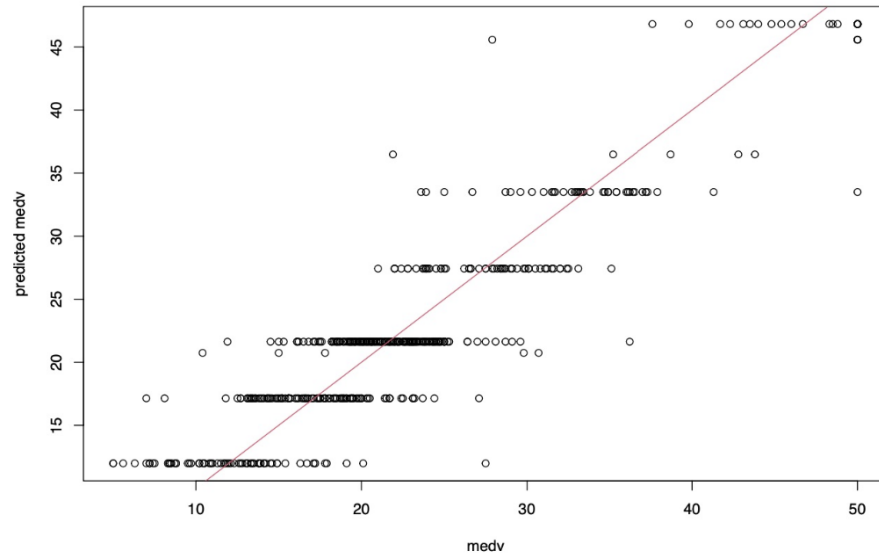
- **Forest-RI(random input)**  
randomly select  $F$  features  
to split each node of decision tree
- **Forest-RC(randomly combined)**  
 $F$  randomly combined new features  
( $F$  linear combination)
- **Randomly select**  
one of the  $F$  best splits at each node of decision tree



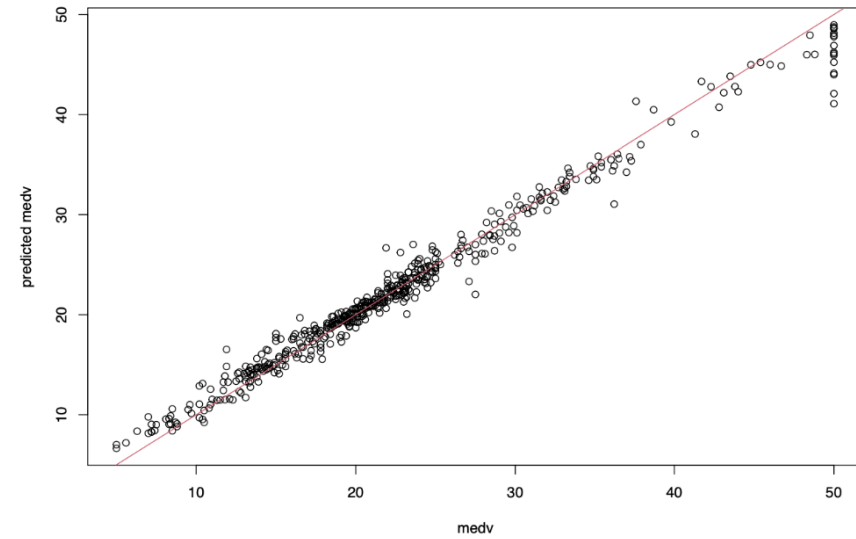
# RandomForest

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## Single Tree



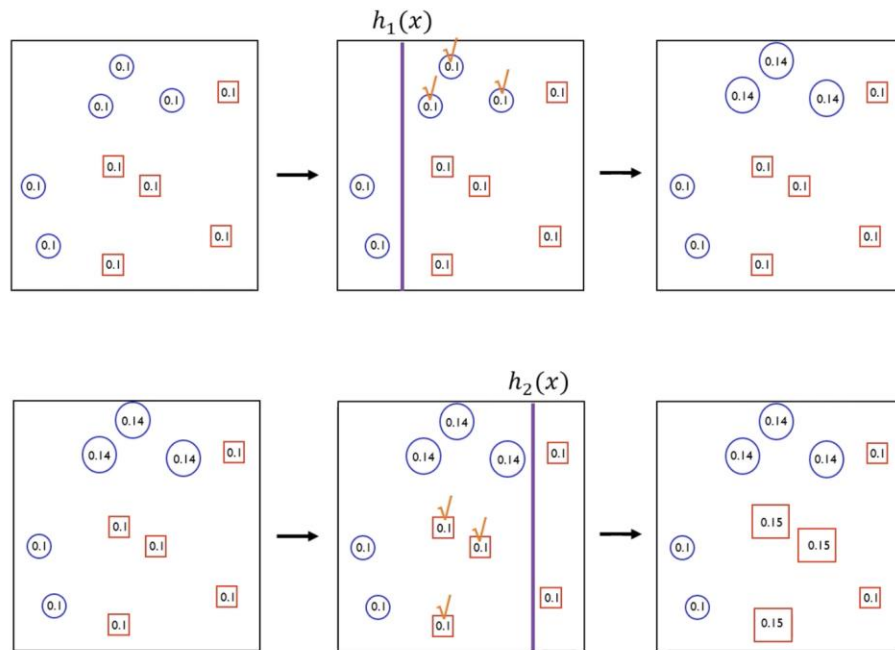
## Random Forest



# Adaboost

**Adaboost** : Adaptive + Boosting

- **Adaptive** : 이전 모델이 잘못 분류한 데이터의 가중치를 adaptive하게 변경
- **Boosting** : 이전 모델이 잘못 분류한 데이터들을 중심으로 학습

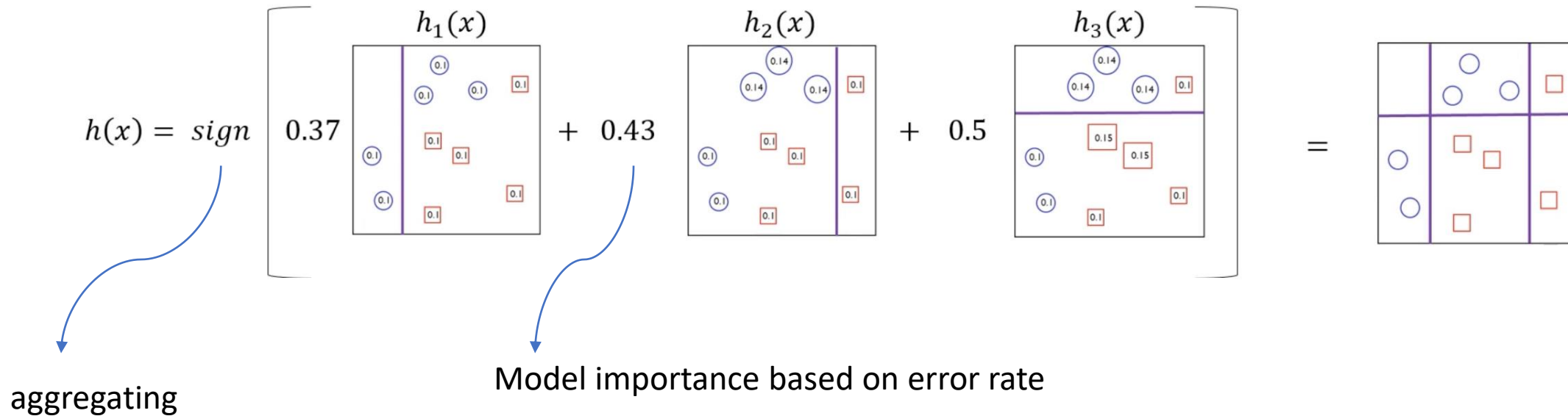


정분류 sample : 그대로  
오분류 sample : 가중치 ↑

# Adaboost

$$W_i = \frac{1}{n} \quad L_j = \frac{\sum_{i=1}^n W_i I(y_i \neq h_i(x))}{\sum_{i=1}^n W_i} \cdot \alpha_j = \log\left(\frac{1 - L_j}{L_j}\right)$$

$$h(x) = \text{sign} \left[ \sum_{i=1}^{m=3} \alpha_j h_j(x) \right]$$

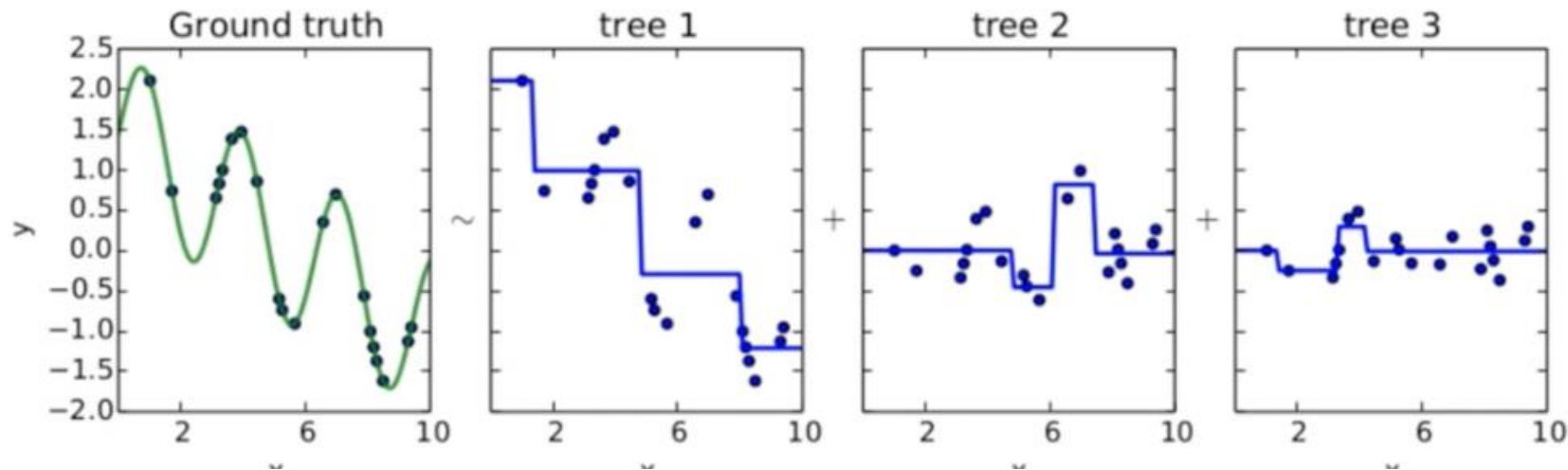


# Gradient Boosting(GBM)

- Gradient boosting = Boosting with gradient decent
- Tree 1을 통해 Y를 예측하고 residua로 tree2 다시 학습
- 점차 residual(실제값과 예측값의 차이) 작아짐
- Gradient boosting model = tree1 + tree2 + tree3

$$\text{loss function} : (y, f(x)) = \frac{1}{2} (y - f(x))^2$$

$$\text{negative gradient} : \frac{\partial (y, f(x))}{\partial f(x)} = - \frac{\partial \left[ \frac{1}{2} (y - f(x))^2 \right]}{\partial f(x)} = -(f(x) - y) = y - f(x)$$





# Ensemble models

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- RandomForest
- ExtraTrees
- Adaboost
- GradientBoost
  - XGBoost
  - LightGBM
  - CatBoost

수고하셨습니다!

해당 세션자료는 KUBIG Github에서 보실 수 있습니다!