Contents


```
% ID - 202183504, Name - Choudhary Kailash
```

Original Code Link: GitHub_Code

```
clear all
close all
```

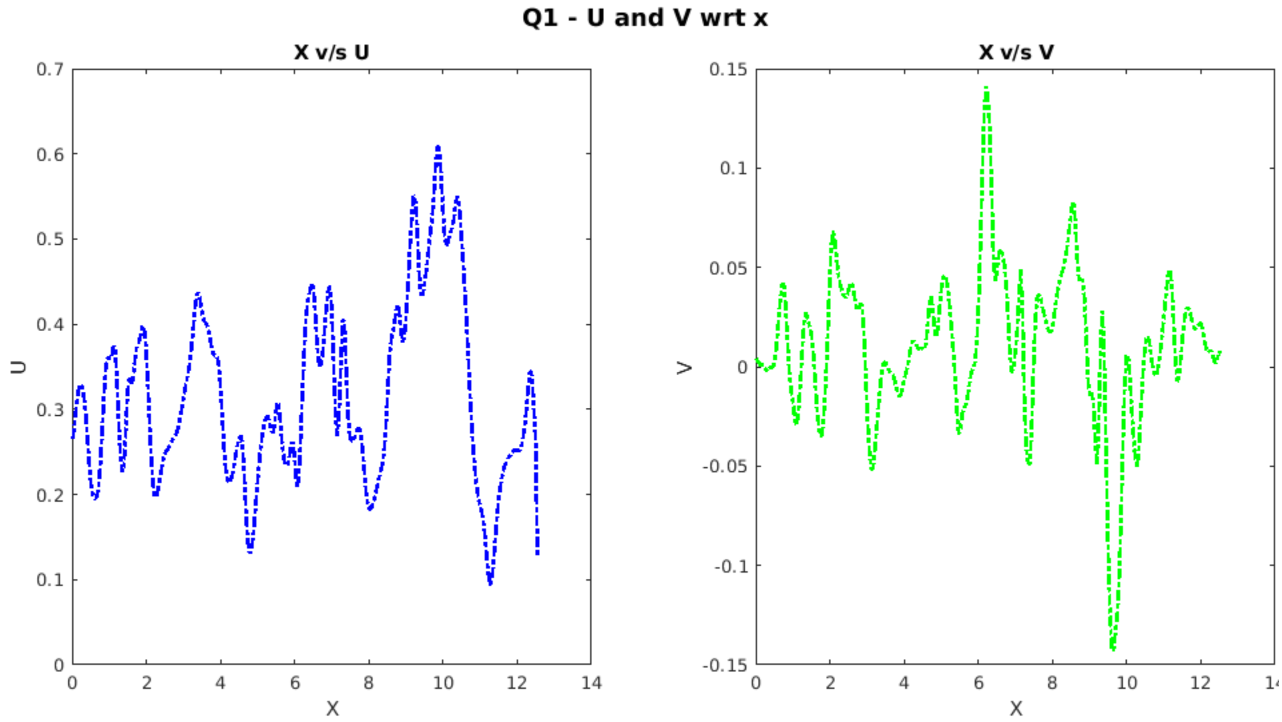
Reading Data

```
clc
data_path = "/media/ksm/Kailash_3/Kailash/Course_work/Turbulence/Project/dataset"
addpath(data_path);


startRow = 2;
formatSpec = '%13f%13f%f%[^\n\r]';

n_files = dir(fullfile(data_path, '*.dat'));

data = {};
for i=1:numel(n_files)

    filename = strcat(sprintf('INS_Vel_%06d.dat', i));
    fileID = fopen(filename,'r');
    dataArray = textscan(fileID, formatSpec, 'Delimiter', '', 'WhiteSpace', '', 'TextType', 'string', 'HeaderLines' ,startRow-1, 'EndOfLine', '\r\n');

    data{i} = table(dataArray{1:end-1}, 'VariableNames', {'X','U','V'});
end
clear fileID dataArray startRow i formatSpec;
```

```
data_path =

    "/media/ksm/Kailash_3/Kailash/Course_work/Turbulence/Project/dataset"
```

Q1 - Plot the signal of U and V wrt x

```
figure(1)
data1 = data{1};
subplot(1,2,1);
plot(data1.X, data1.U, '-.b','LineWidth',2,'MarkerSize',1);
xlabel('X');
ylabel('U');
title('X v/s U');
set(gca, 'FontSize', 10);

subplot(1,2,2);
plot(data1.X, data1.V, '-.g','LineWidth',2,'MarkerSize',1);
xlabel('X');
ylabel('V');
title('X v/s V')
sgtitle('Q1 - U and V wrt x', 'FontWeight', 'Bold');
set(gca, 'FontSize', 10);
set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12,6])
```

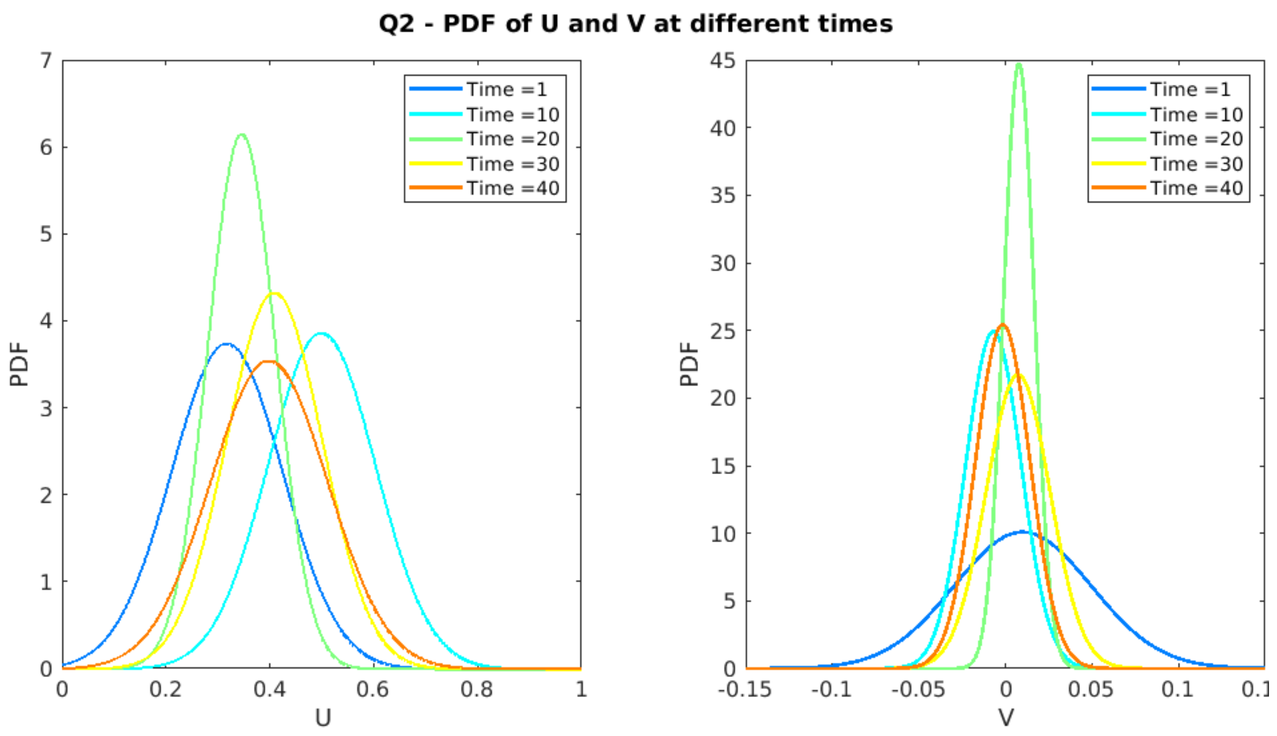


Q2 - Plot the probability density function (PDF) of U and V. Show that the PDF satisfies the equation (3.16) in the textbook.

```
% As all velocity data will satisfy eq 3.16. So to have denced data set,
% lets combine all data

clc
U = [];
V = [];
X = data{1}.X;


for i=1:numel(data)
    U = [U, data{i}.U];
    V = [V, data{i}.V];
end

% So, basically, U_a has all the u-velocity data. Each coloum will have
% data corresponding to one value of time. And each row will have data
% corresponding to one value of possition. Same for V_a




lgd = {};
lgd_V = {};
times = [1, 10, 20, 30, 40];


CM = jet(length(times));

figure(2)

for i=1:numel(times)
    U_i = U(:,times(i));
    V_i = V(:,times(i));


    U_mean = mean(U_i);
    U_skew = skewness(U_i);
    U_std = std(U_i);

    V_mean = mean(V_i);
    V_skew = skewness(V_i);
    V_std = std(V_i);

    % pd = fitdist(U_i, 'Normal'); %pdf(pd, U_range);
    U_range = 0:0.001:1;
    U_pdf = pdf('Normal',U_range, mean(U_i),std(U_i));

    area_U = trapz(U_range, U_pdf);
    ax1 = subplot(1,2,1);
    fig1 = plot(U_range, U_pdf, 'color', CM(i, :), 'LineWidth', 2);
    hold on
    lgd{i} = strcat('Time = ', num2str(times(i)));
```

```
    %pd = fitdist(V_i, 'Normal');  %pdf(pd, V_range);
    V_range = -0.15:0.001:0.15;
    V_pdf = pdf('Normal',V_range, mean(V_i),std(V_i));
    area_V = trapz(V_range, V_pdf);
    ax2 = subplot(1,2,2);
    plot(V_range, V_pdf, 'color', CM(i, :), 'LineWidth', 2)
    hold on
    disp(['Time = ', num2str(times(i))]);
    disp(['Area unter the U-PDF = ', num2str(area_U)]);
    disp(['Area unter the V-PDF = ', num2str(area_V)]);
    disp('-------------------------------')



end
sgtitle('Q2 - PDF of U and V at different times', 'FontWeight', 'Bold');
legend(ax1, lgd);
legend(ax2, lgd);

set(ax1, 'FontSize', 12);
set(ax2, 'FontSize', 12);

xlabel(ax1, 'U');
xlabel(ax2, 'V');

ylabel(ax1, 'PDF');
ylabel(ax2, 'PDF');
set(gcf, 'Units', 'Inches', 'Position',...
    [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12 6])

% name = 'Part2.png';
% print('-dpng','-r600',name);
%
```

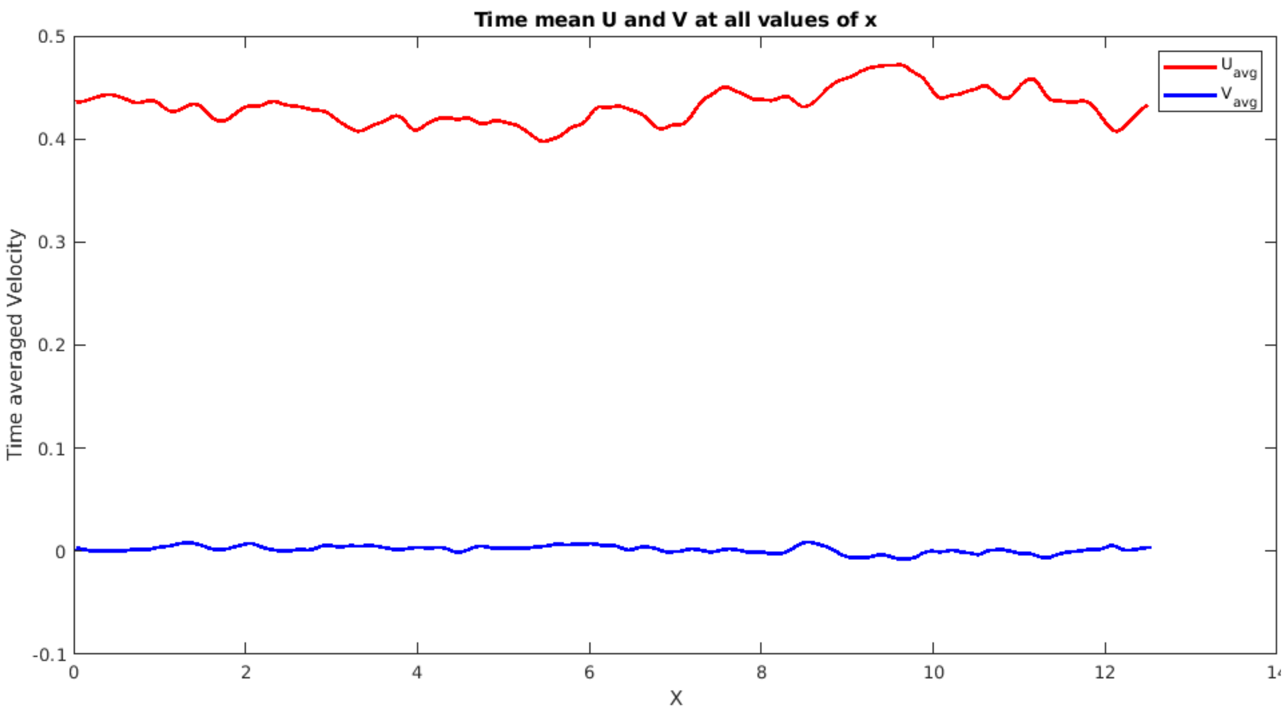
```
Time = 1
Area unter the U-PDF = 0.99849
Area unter the V-PDF = 0.99977
-------------------------------
Time = 10
Area unter the U-PDF = 1
Area unter the V-PDF = 1
-------------------------------
Time = 20
Area unter the U-PDF = 1
Area unter the V-PDF = 1
-------------------------------
Time = 30
Area unter the U-PDF = 0.99999
Area unter the V-PDF = 1
-------------------------------
Time = 40
Area unter the U-PDF = 0.99979
Area unter the V-PDF = 1
-------------------------------
```



Q3 - Show that U and V are statistically homogeneous in the x direction.

```
% Statistically homogeneous in x direction means if we replace x with x+X,
% the statistics (mean) should not change. So, lets take mean of U and V
% for all time intances (row wise), so we will get time mean velocity of
% each position

% Dear Sir, the last point (X = 12.54) U-mean deviating largely from the mean
% data, so I am ignoring that data in plot(last row of the data set)



U_mean = mean(U, 2); % time mean
V_mean = mean(V, 2);


% Mean absolute percentage error

abs_err_U = 100 * (1/length(U_mean)) * sum(abs((U_mean - mean(U_mean))./U_mean));

% Absolute value of V is very low, so the absolute error in V coming high
abs_err_V = 100 * (1/length(V_mean)) * sum(abs((V_mean - mean(V_mean))./V_mean));



figure
plot(X(1:end-1), U_mean(1:end-1), 'r', 'LineWidth', 2);
hold on
plot(X, V_mean, 'b', 'LineWidth', 2);
title('Time mean U and V at all values of x')
xlabel('X');
ylabel('Time averaged Velocity');
legend('U_{avg}', 'V_{avg}');

set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12, 6])
% name = 'Part3.png';
% print('-dpng','-r600',name);
disp('The mean U and V are almost constant with positions, So, they are homogeneous with x');
```

```
The mean U and V are almost constant with positions, So, they are homogeneous with x
```



Q- 4 Compute the ensemble averages of U and V

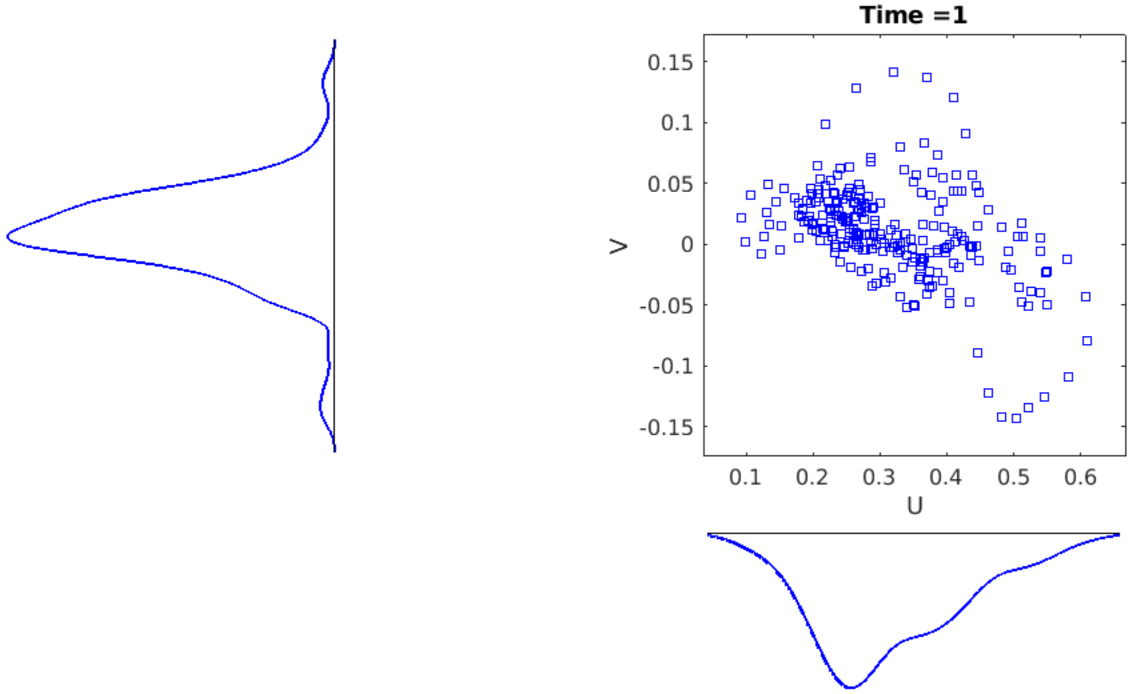
```
% ensemble averaging is just average over N-repetitions, So, bassically at
% each position, is is time average.
% In code it is just average of each row (Same as last question)

% Morever, ensemble average itself a random variable and average of this
% random veriable will be <U>

% Mean of ensemble average random variable

avg_ens_U = mean(U_mean); % <U>
avg_ens_V = mean(V_mean); % <V>
```

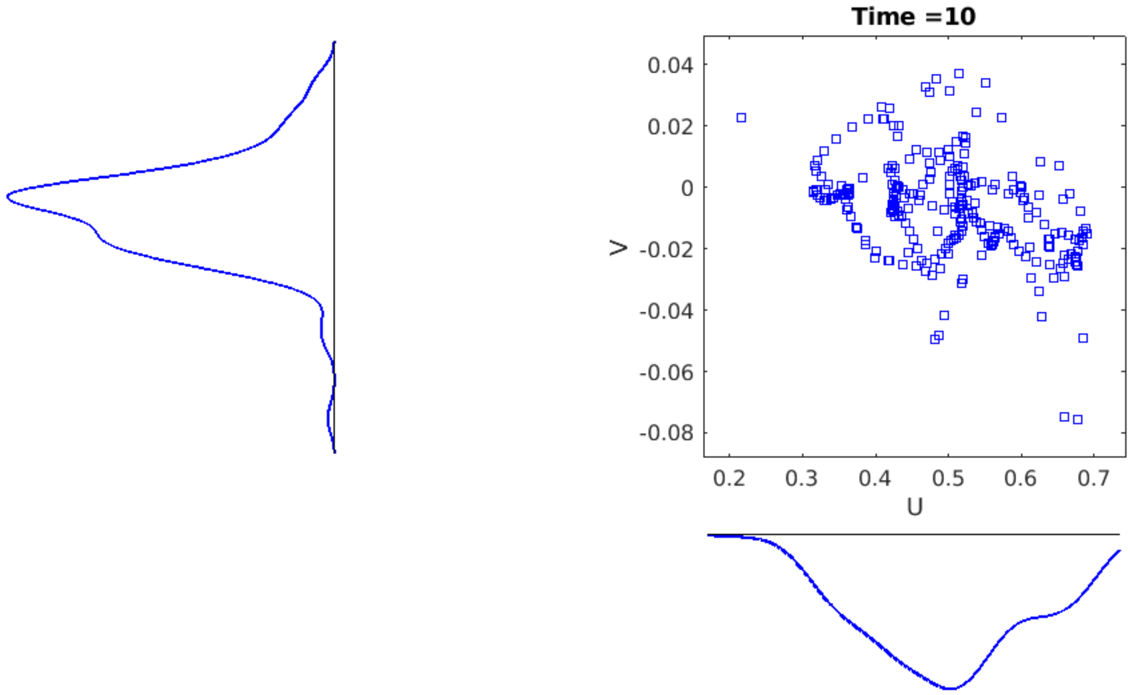
```
disp('ensempble average already plotted in Q3');
disp(['<U> = ', num2str(avg_ens_U)]);
disp(['<V> = ', num2str(avg_ens_V)]);
```

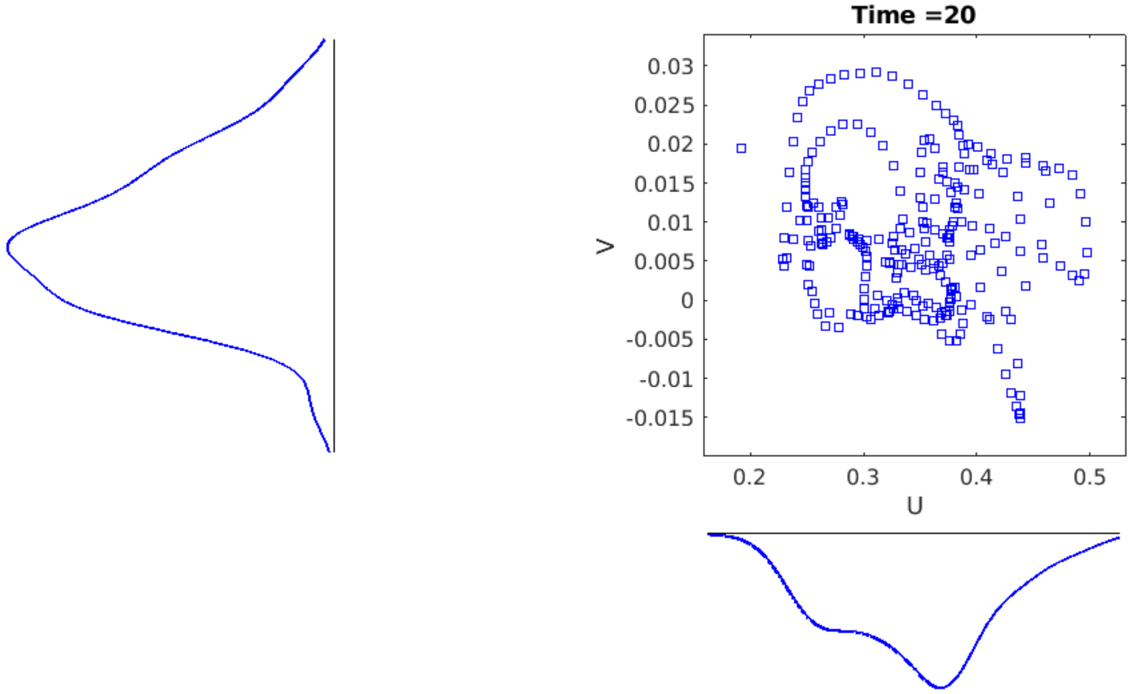
```
ensempble average already plotted in Q3
<U> = 0.43098
<V> = 0.0017193
```

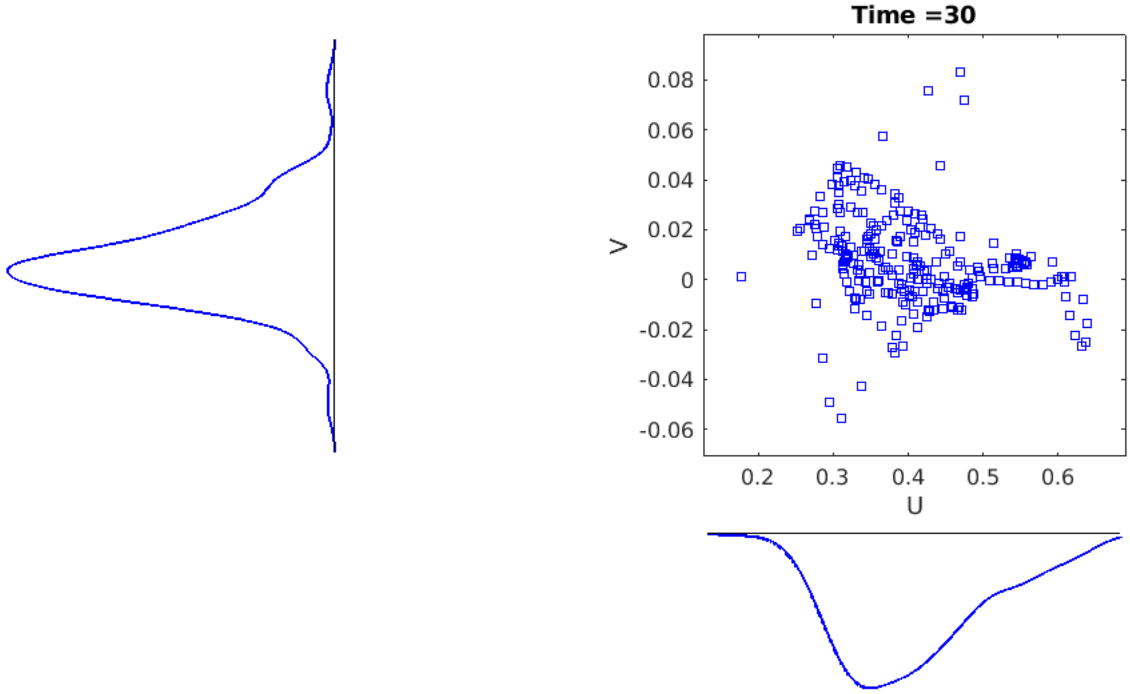
Q - 5 Plot the scatter plot of U and V.

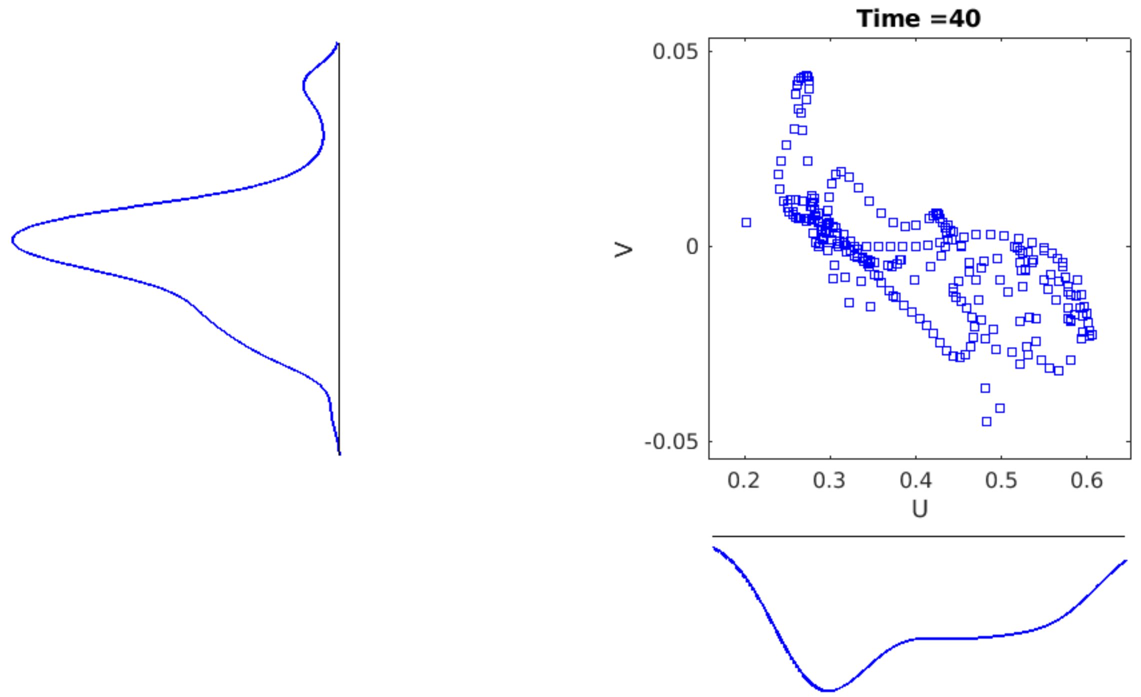
```
lgd = {};
times = [1, 10, 20, 30, 40];


CM = jet(length(times));




for i=1:numel(times)
    U_i = U(:,times(i));
    V_i = V(:,times(i));


    figure
    scatterhist(U_i, V_i, 'Kernel','on','Location','SouthWest',...
    'Direction','out','Color','b','LineWidth',2,'Marker','s','MarkerSize',5);%, 'MarkerEdgeColor','red', 'MarkerFaceColor',[1 .6 .6])

    title(strcat('Time = ', num2str(times(i))));
    set(gca, 'FontSize', 12);
    axis('square');
    xlabel('U');
    ylabel('V');
    set(gcf, 'Units', 'Inches', 'Position',...
        [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12 6]);

end
```


Time =1
V
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
U
0.1
0.2
0.3
0.4
0.5
0.6


Time =10
V
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
U
0.2
0.3
0.4
0.5
0.6
0.7


Time =20
V
0.03
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
0.2
0.3
0.4
0.5
U

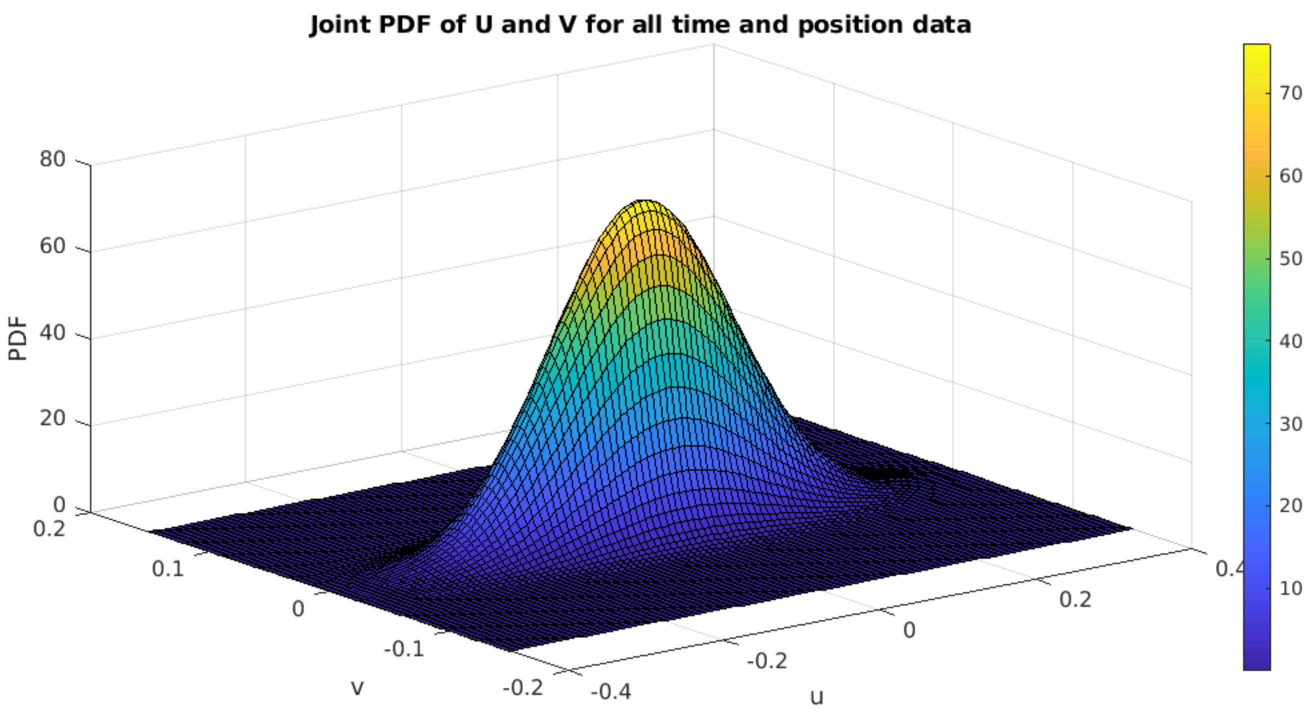

Time =30
V
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
0.2
0.3
0.4
0.5
0.6
U

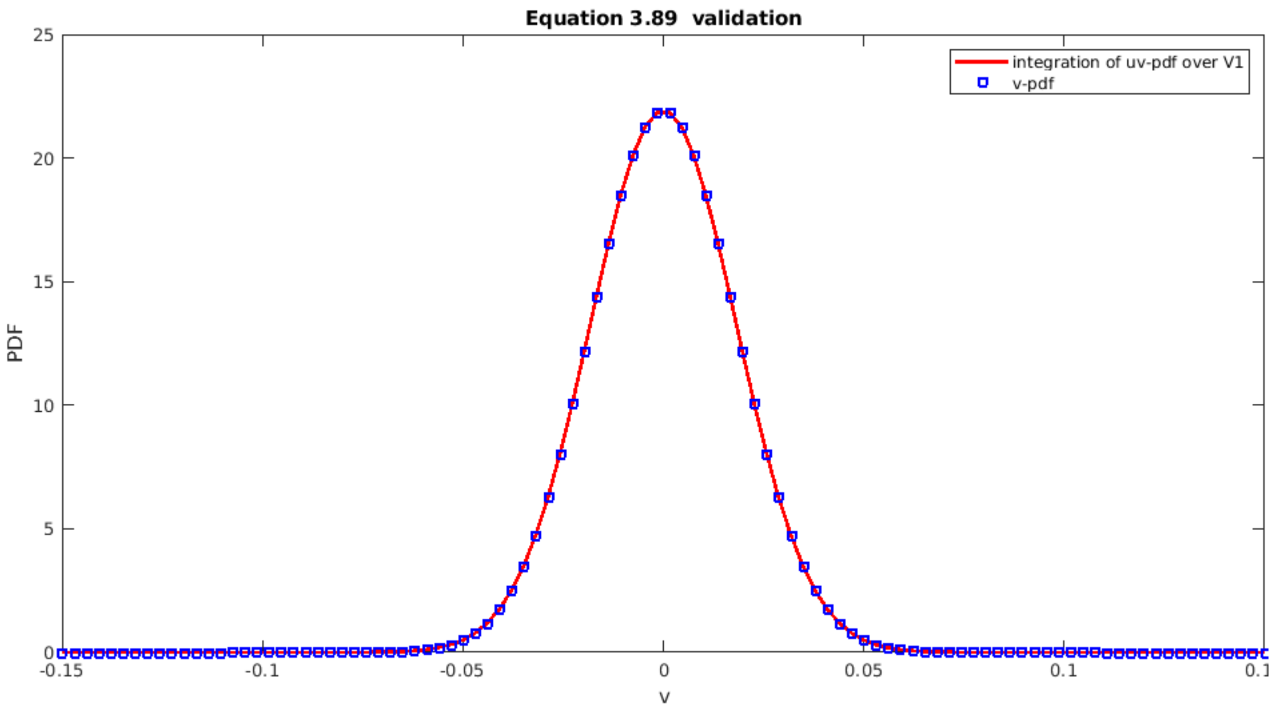


Q6 Plot the joint probability density function (JPDF) of u and v, where u and v are velocity fluctuations in the streamwise and wall-normal directions, respectively. Show that theJPDF satisfies the equation (3.89) and (3.90) in the textbook

```
% Perturbation is just u = U - <U>. Value of <U> can be calculated by
% taking avgrage of ensemble average. (Eq. 3.109)


u = U - avg_ens_U;
v = V - avg_ens_V;


u_range = linspace(-0.4, 0.4, 100);
v_range = linspace(-0.15, 0.15, 100);

u_pdf = pdf('Normal', u_range, mean2(u), std2(u));
v_pdf = pdf('Normal', v_range, mean2(v), std2(v));



uv_pdf = v_pdf'*u_pdf;
[u_r, v_r] = meshgrid(u_range, v_range);


figure
surf(u_r, v_r, uv_pdf);

colormap('parula');
cb = colorbar('eastoutside');
set(gca, 'FontSize', 12);
xlabel('u');
ylabel('v');
zlabel('PDF');
set(gcf, 'Units', 'Inches', 'Position',...
    [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12 6])
title('Joint PDF of U and V for all time and position data');



% eq 3.89
f2_v2 = trapz(u_range, uv_pdf');
figure
plot(v_range, f2_v2, '-r', 'LineWidth', 2);
hold on
plot(v_range, v_pdf, 'sb', 'LineWidth', 2);


xlabel('v');
ylabel('PDF');
legend('integration of uv-pdf over V1', 'v-pdf');

set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12, 6])
title('Equation 3.89  validation');
disp('Plots of both side of equation 3.89 are overlapping');

area_uv_pdf = trapz(u_range, trapz(v_range, uv_pdf));
disp(['Equation 3.90, area under the uv_pdf = ', num2str(area_uv_pdf)]);
```

```
Plots of both side of equation 3.89 are overlapping
Equation 3.90, area under the uv_pdf = 0.99952
```





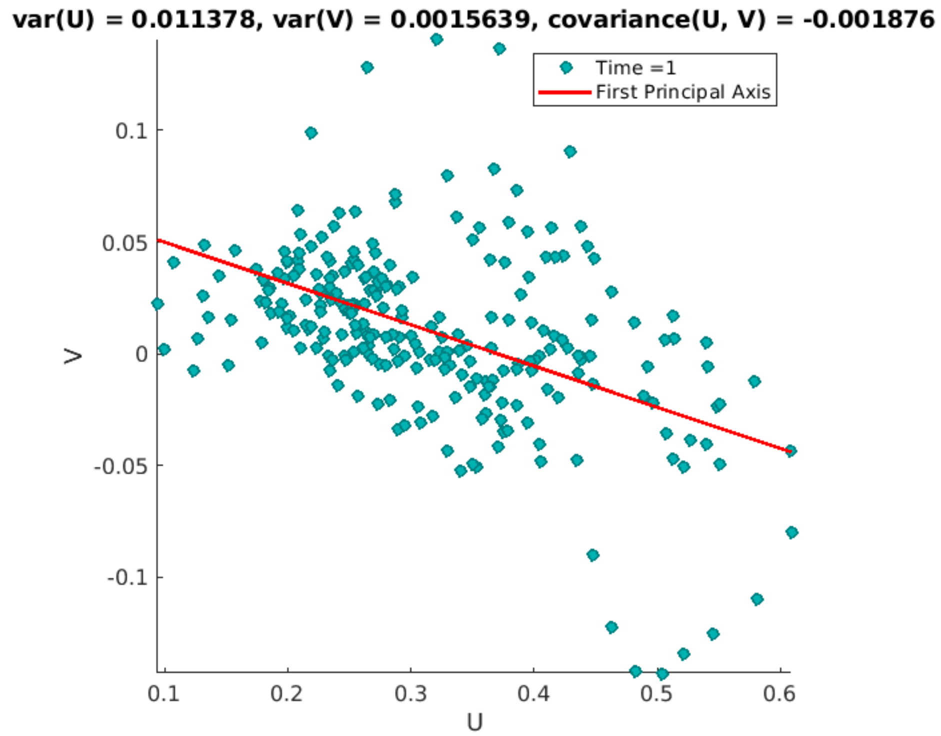
Q7 - Compute the variance of U and V and the covariance of U and V

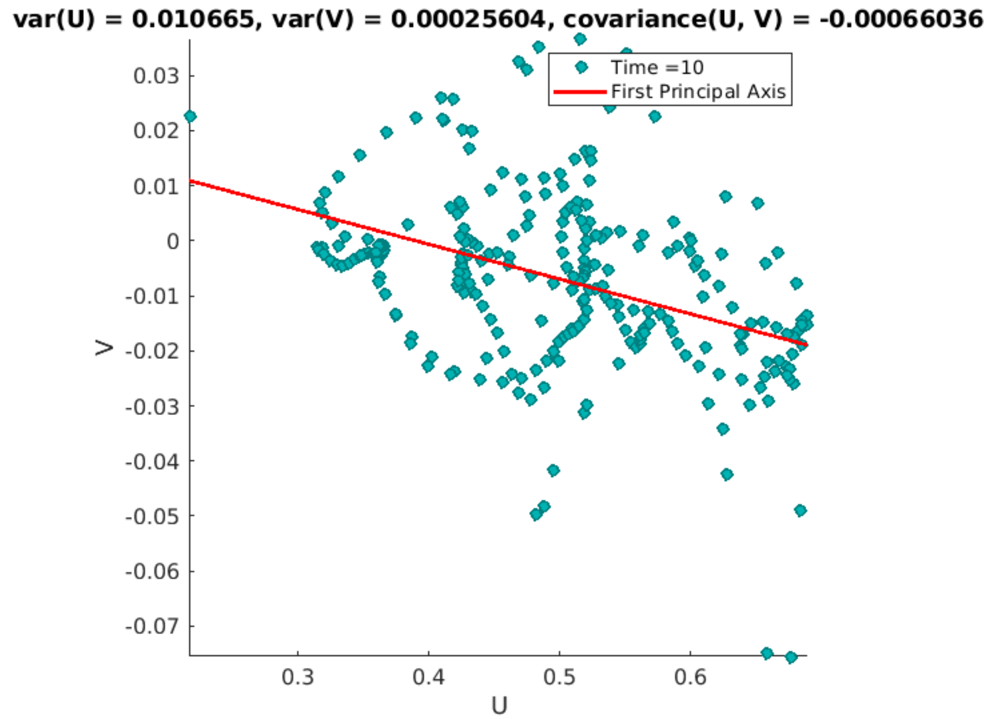
```
% variance and covariance have step to substracet the mean <U> from the
% data set. So basically variance or covariance of U and V wil be same as
% variance or covariance of u and v.




lgd = {};
times = [1, 10, 20, 30, 40];


CM = jet(length(times));
```

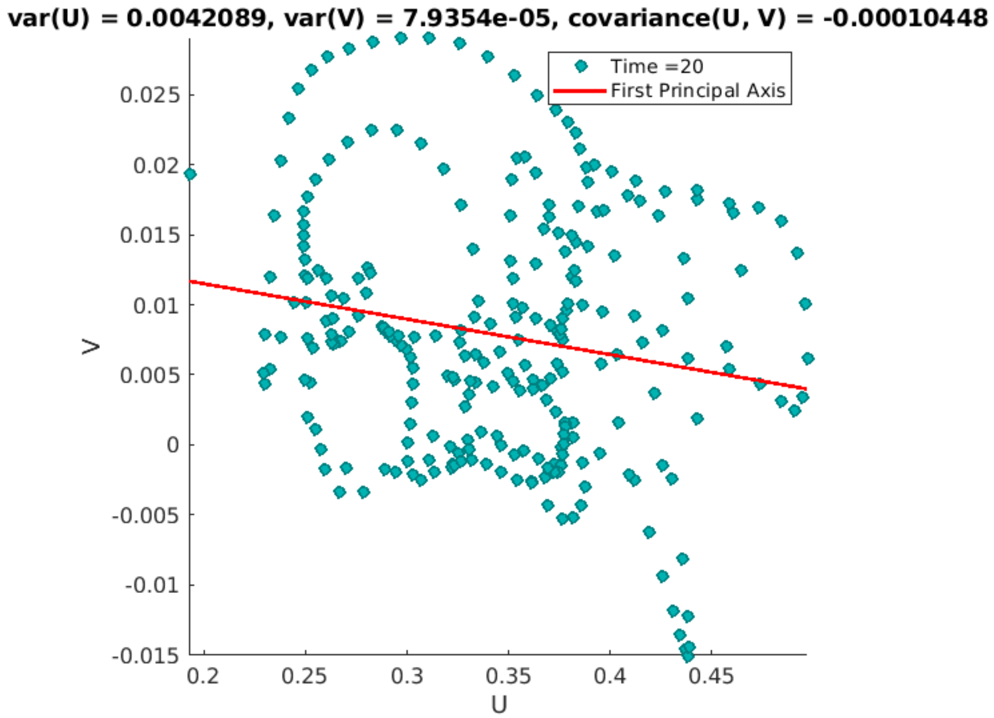
```
for i=1:numel(times)
    U_i = U(:,times(i));
    V_i = V(:,times(i));



    disp(['Time = ', num2str(times(i))]);
    % Covariance
    cov_UV = cov(U_i, V_i)

    %Principle axis - vec- eigenvector, val - eign values matrix

    [vec, val] = eig(cov_UV);
    prin_axis = [vec(:,1) * val(1,1), vec(:,2) * val(2,2)]
    sz = 40;
    figure
    scatter(U_i, V_i, sz,'MarkerEdgeColor',[0 .5 .5],...
              'MarkerFaceColor',[0 .7 .7],...
              'LineWidth',1.5);
    hold on
    %plotv(vec, '-', 'LineWidth', 2)
    %set(v1, 'LineWidth', 2)

    m1 = vec(2,1)/vec(1,1);
    b1 = mean(V_i) - m1 * mean(U_i);

    m2 = vec(2,2)/vec(1,2);
    b2 = mean(V_i) - m2 * mean(U_i);

    axisx = [min(U_i), max(U_i)];
    axis1y = m1 * axisx + b1;
    axis2y = m2 * axisx + b2;


    if val(1,1)>=val(2,2)
        plot(axisx, axis1y, '-r', 'Linewidth', 2);
    else
        plot(axisx, axis2y, '-r', 'Linewidth', 2);
    end
    title(['var(U) = ', num2str(var(U_i)), ', var(V) = ', num2str(var(V_i)), ', covariance(U, V) = ', num2str(cov_UV(2,1))]);
    xlim(axisx);ylim([min(V_i),max(V_i)]);
    %plot


    legend(strcat('Time = ', num2str(times(i))), 'First Principal Axis');
    set(gca, 'FontSize', 12);
    axis('square');
    xlabel('U');
    ylabel('V');
    set(gcf, 'Units', 'Inches', 'Position',...
        [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12 6])
    disp('-----------------------------------');
end
```

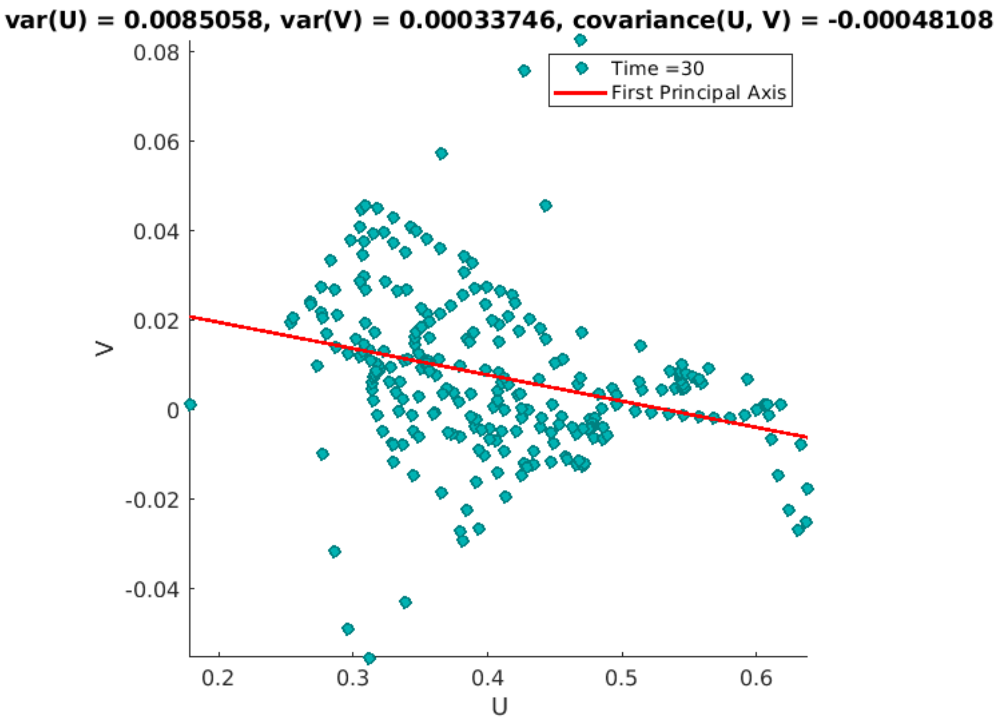
```
Time = 1

cov_UV =

    0.0114   -0.0019
   -0.0019    0.0016


prin_axis =

   -0.0002   -0.0115
   -0.0012    0.0021

-----------------------------------
Time = 10

cov_UV =

    0.0107   -0.0007
   -0.0007    0.0003


prin_axis =

   -0.0000   -0.0107
   -0.0002    0.0007

-----------------------------------
Time = 20

cov_UV =

    0.0042   -0.0001
   -0.0001    0.0001


prin_axis =
```

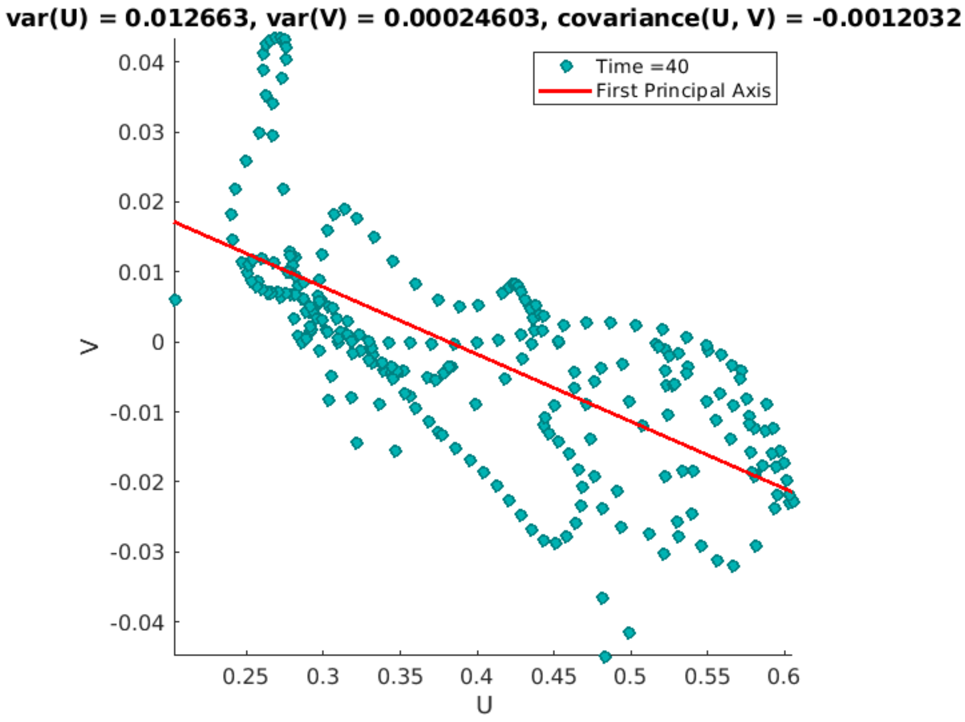
```
   -0.0000   -0.0042
   -0.0001    0.0001

----------------------------------
Time = 30

cov_UV =

    0.0085   -0.0005
   -0.0005    0.0003


prin_axis =

   -0.0000   -0.0085
   -0.0003    0.0005

----------------------------------
Time = 40

cov_UV =

    0.0127   -0.0012
   -0.0012    0.0002


prin_axis =

   -0.0000   -0.0127
   -0.0001    0.0012

----------------------------------
```


var(U) = 0.011378, var(V) = 0.0015639, covariance(U, V) = -0.001876
Time =1
First Principal Axis
0.1
0.05
0
-0.05
-0.1
V
0.1
0.2
0.3
0.4
0.5
0.6
U


var(U) = 0.010665, var(V) = 0.00025604, covariance(U, V) = -0.00066036
Time =10
First Principal Axis
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
V
0.3
0.4
0.5
0.6
U


var(U) = 0.0042089, var(V) = 7.9354e-05, covariance(U, V) = -0.00010448
Time =20
First Principal Axis
V
U
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
0.2
0.25
0.3
0.35
0.4
0.45

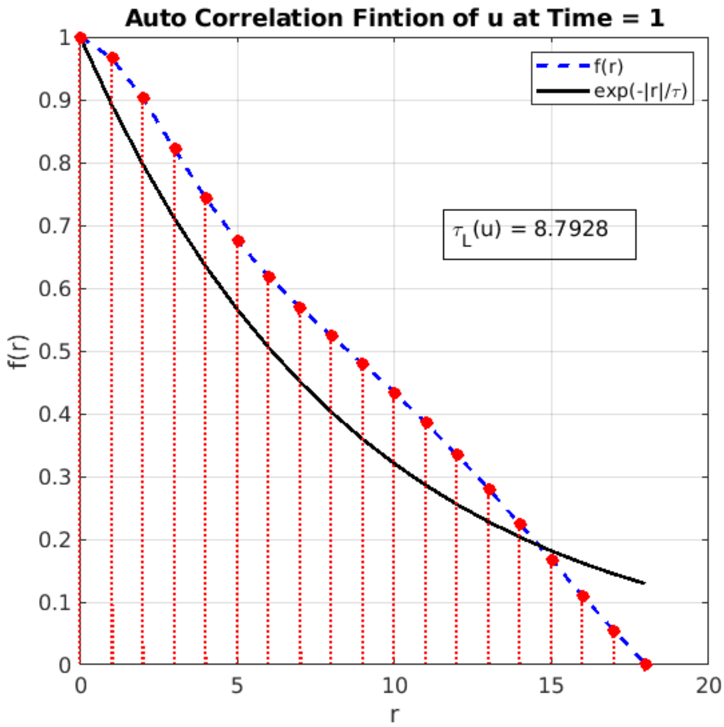

var(U) = 0.0085058, var(V) = 0.00033746, covariance(U, V) = -0.00048108
Time =30
First Principal Axis
V
U
0.08
0.06
0.04
0.02
0
-0.02
-0.04
0.2
0.3
0.4
0.5
0.6

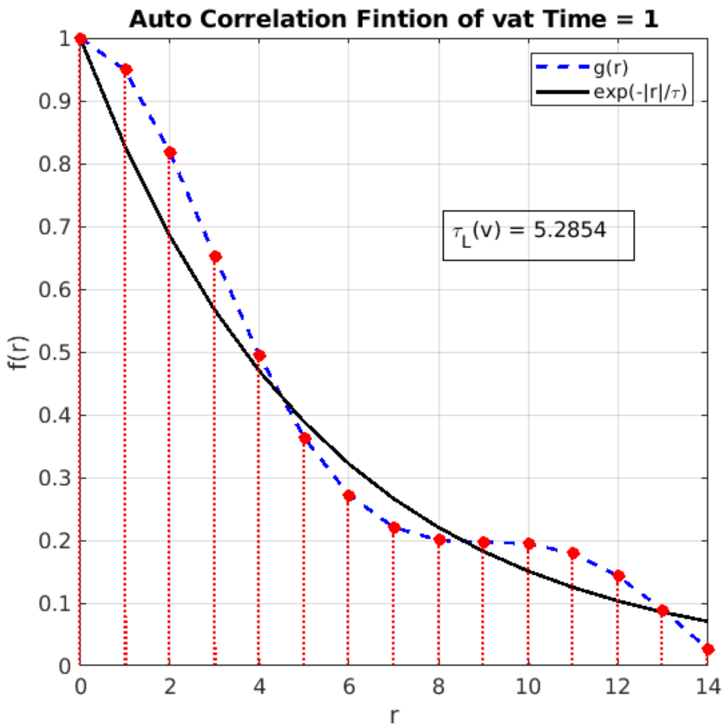


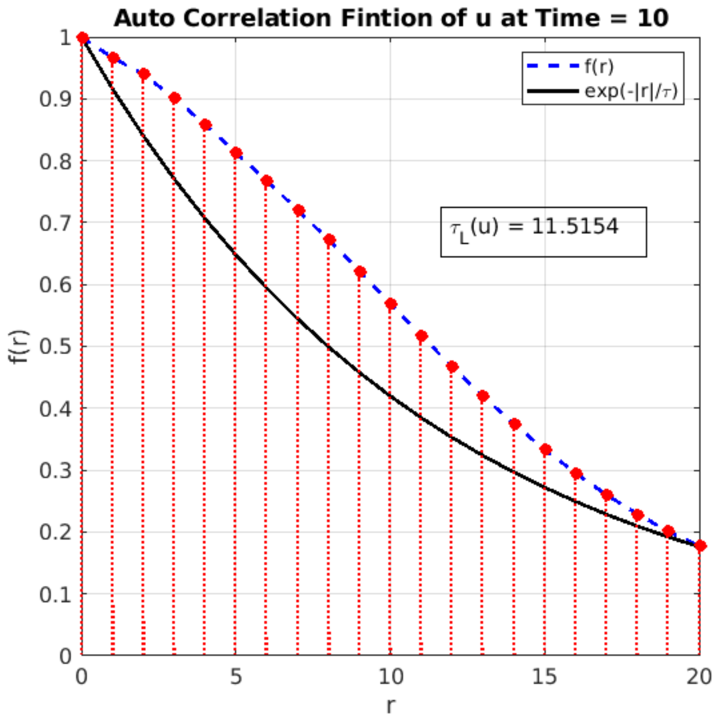
Q 8 - Plot the autocorrelation functions f(r/h) and g(r/h) and compute the corresponding integral length scales

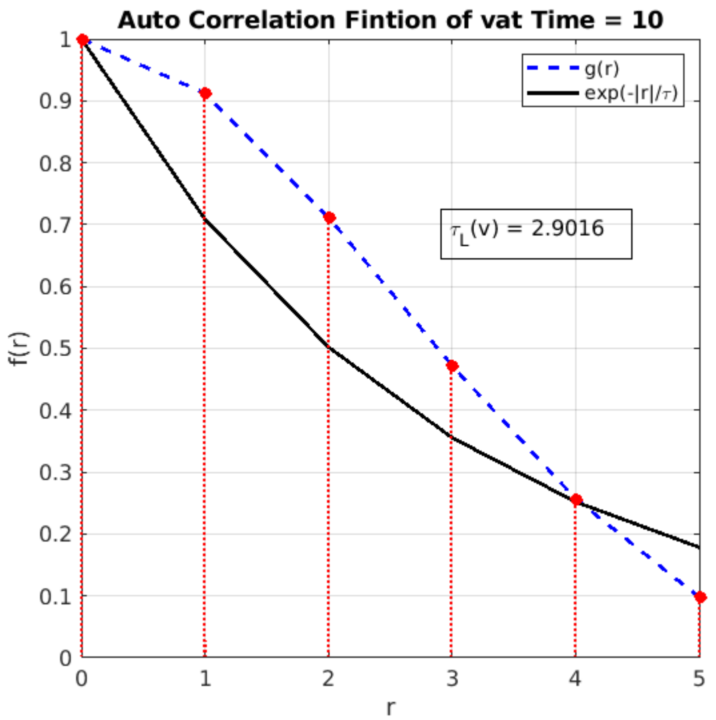
```
lgd = {};
times = [1, 10, 20, 30, 40];


CM = jet(length(times));

clc


for i=1:numel(times)
    u_i = u(:,times(i));
    v_i = v(:,times(i));


    % auto correlation function (Matlab funtion "autocorr" returns the normalized auto-covariance so no need to devide by variance)
    [fr, r_fr] = autocorr(u_i);
    [gr, r_gr]= autocorr(v_i);


    % To find out the lenth scale, we need to consider the auto-covariance
    % function only up to positive values (1 to 0).

    % first negative value finder
    if any(fr<0)
        fr_1st_neg = find(~(fr>=0), 1, 'first');
        fr = fr(1:fr_1st_neg-1);
        r_fr = r_fr(1:fr_1st_neg-1); % to start from zero
    end

    if any(gr<0)
        gr_1st_neg = find(~(gr>=0), 1, 'first');
        gr = gr(1:gr_1st_neg-1);
        r_gr = r_gr(1:gr_1st_neg-1); % to start from zero
    end




    % integral length scales
    tl_u = trapz(r_fr, fr);
    tl_v = trapz(r_gr, gr);


    % exp(-|r|/tl) plot
    f_exp = exp(-r_fr/tl_u);
    g_exp = exp(-r_gr/tl_v);


    figure
    plot(r_fr, fr, '--b', 'Linewidth', 2)
    hold on
    plot(r_fr, f_exp, '-k', 'linewidth', 2);
    stem(r_fr, fr, ':r', 'filled', 'linewidth', 1.5)
```

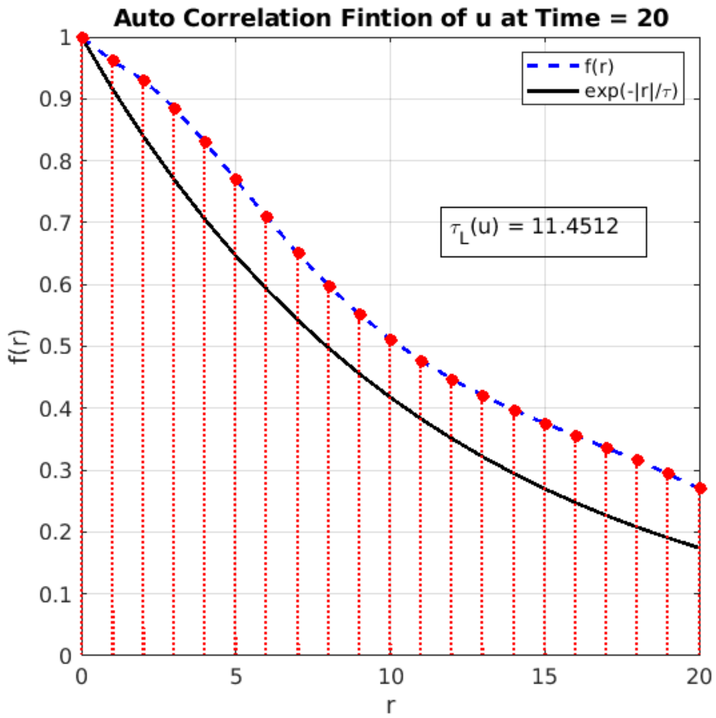
```
    dim = [0.55 0.4 0.3 0.3];
    str = {['\tau_L(u) = ', num2str(tl_u)]};
    annotation('textbox',dim,'String',str,'FitBoxToText','on', 'Fontsize', 12);
    legend('f(r)', 'exp(-|r|/\tau)');
    grid on
    title(['Auto Correlation Fintion of u at Time = ', num2str(times(i))])
    set(gca, 'FontSize', 12);
    axis('square');
    xlabel('r');
    ylabel('f(r)');
    set(gcf, 'Units', 'Inches', 'Position',...
        [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12 6])


    figure
    plot(r_gr, gr, '--b', 'Linewidth', 2)
    hold on
    plot(r_gr, g_exp, '-k', 'linewidth', 2);
    stem(r_gr, gr, ':r', 'filled', 'linewidth', 1.5)


    dim = [0.55 0.4 0.3 0.3];
    str = {['\tau_L(v) = ', num2str(tl_v)]};
    annotation('textbox',dim,'String',str,'FitBoxToText','on', 'Fontsize', 12);

    legend('g(r)', 'exp(-|r|/\tau)');
    grid on
    title(['Auto Correlation Fintion of vat Time = ', num2str(times(i))])
    set(gca, 'FontSize', 12);
    axis('square');
    xlabel('r');
    ylabel('f(r)');
    set(gcf, 'Units', 'Inches', 'Position',...
        [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12 6])

end
```

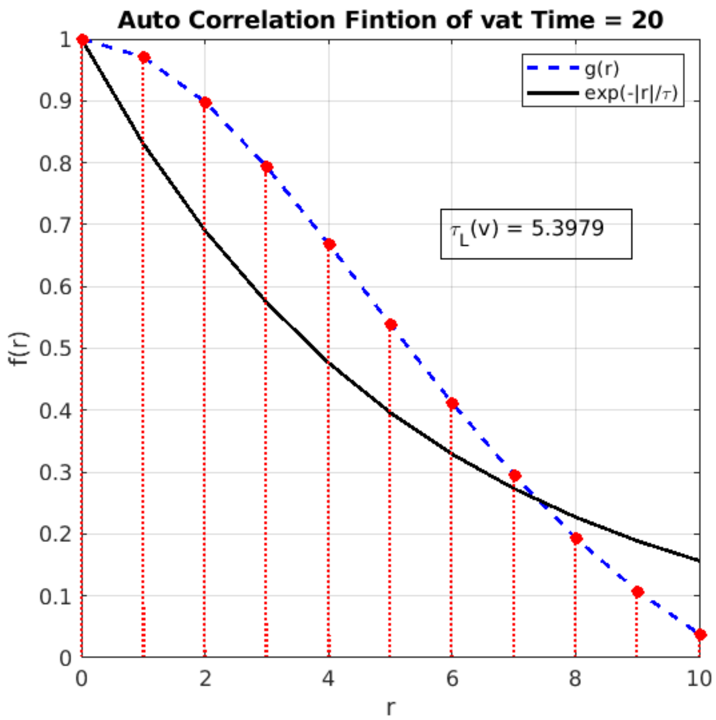

Auto Correlation Fintion of u at Time = 1
f(r)
exp(-|r|/τ)
τL(u) = 8.7928
f(r)
r
0 5 10 15 20
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

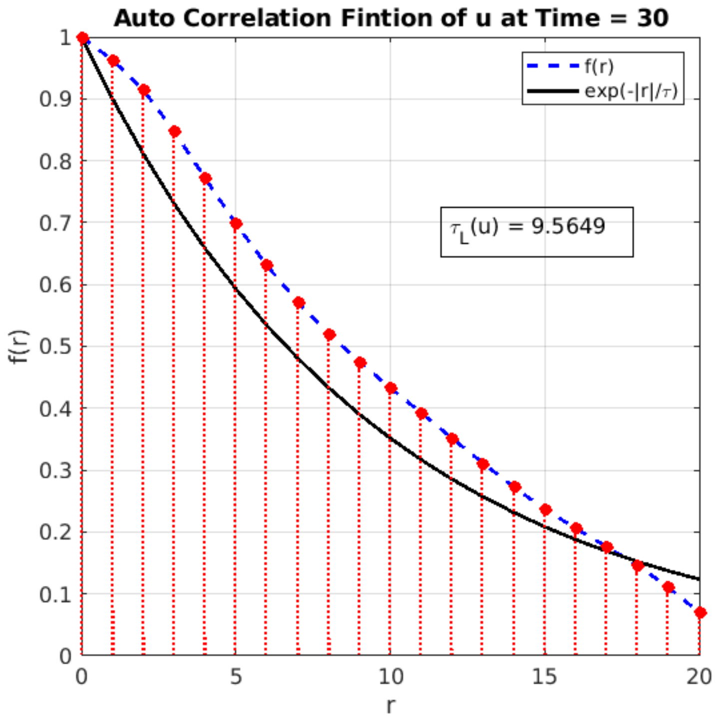

Auto Correlation Fintion of vat Time = 1
g(r)
exp(-|r|/τ)
τL(v) = 5.2854
f(r)
r
0 2 4 6 8 10 12 14
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

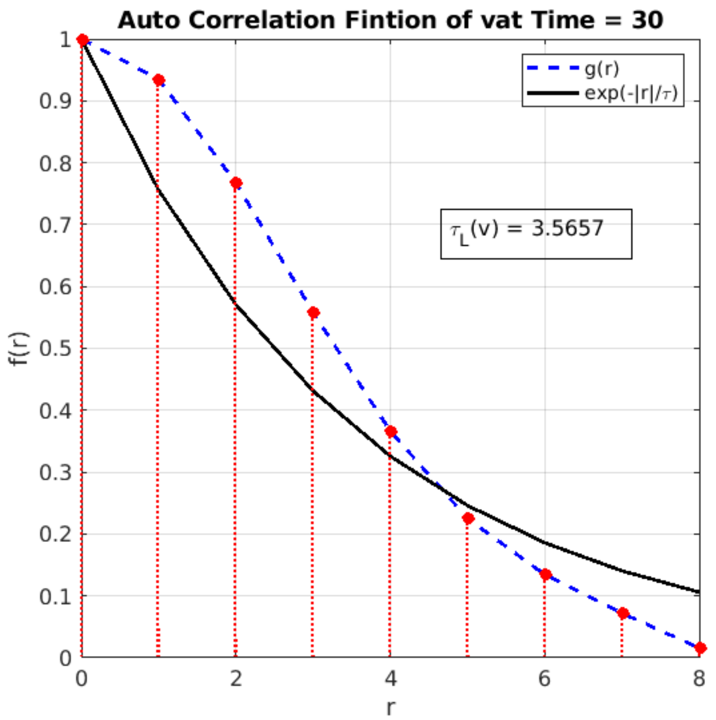

Auto Correlation Fintion of u at Time = 10
f(r)
exp(-|r|/τ)
τ_L(u) = 11.5154
f(r)
r

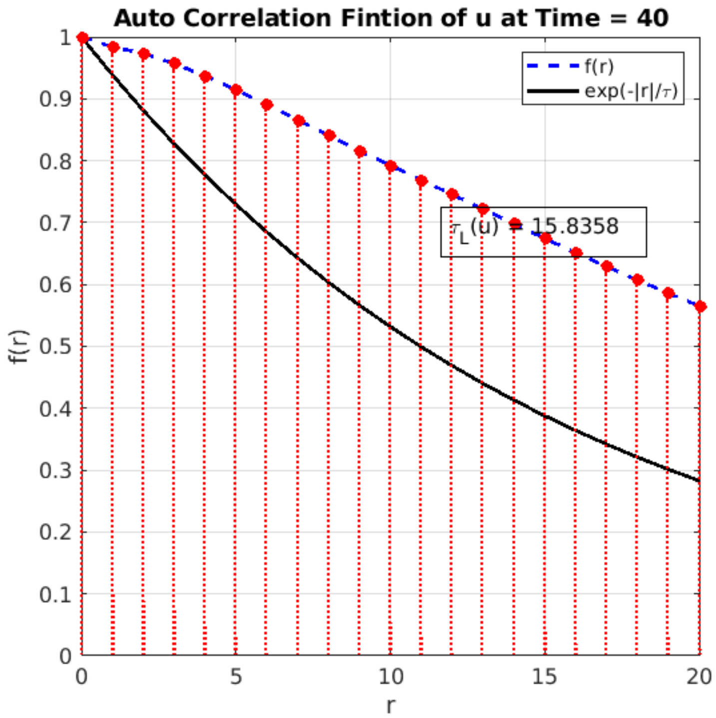

Auto Correlation Fintion of vat Time = 10
g(r)
exp(-|r|/τ)
τ_L(v) = 2.9016
f(r)
r

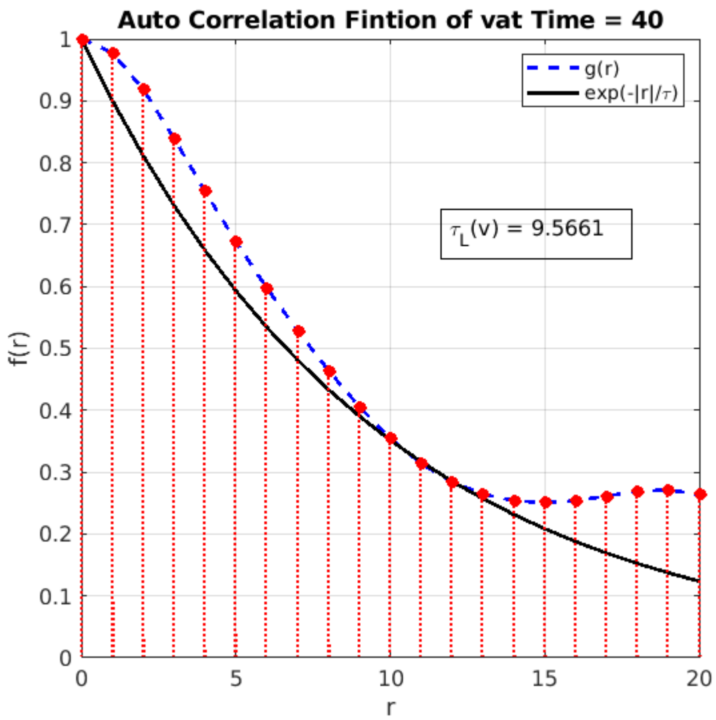

Auto Correlation Fintion of u at Time = 20
f(r)
exp(-|r|/τ)
τL(u) = 11.4512
f(r)
r


Auto Correlation Fintion of vat Time = 20
g(r)
exp(-|r|/τ)
τL(v) = 5.3979
f(r)
r


Auto Correlation Fintion of u at Time = 30
f(r)
exp(-|r|/τ)
τL(u) = 9.5649
f(r)
r
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20


Auto Correlation Fintion of vat Time = 30
g(r)
exp(-|r|/τ)
τL(v) = 3.5657
f(r)
r
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
2
4
6
8





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