Multiplication of Square Matrices

When multiplying 2 matrices we need to consider the dimensions of each, as multiplication is not possible if the dimensions are not aligned appropriately.

The easiest multiplication to consider is that of two **square matrices** of the same dimensions $n \times n$.

A square matrix is a matrix that has the same number of rows and columns.

The matrix below is a square matrix of $m \times m$ It has mmm rows and m columns.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & .. & a_{1m} \\ a_{21} & a_{22} & a_{23} & .. & a_{2m} \\ a_{31} & a_{32} & a_{33} & .. & a_{3m} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & ...... & a_{mm} \end{bmatrix}$$

Equation 14

Matrix A in *Equation 14* can be multiplied with other square matrices of the same dimension , $m \times m$.

The result will be a new square matrix of the same dimensions.

The easiest way to demonstrate the actual multiplication is with an example:

Let P and Q be two square matrices of 3×3.

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

$$Q = \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix}$$

To multiply PPP by QQQ we need to do the following:

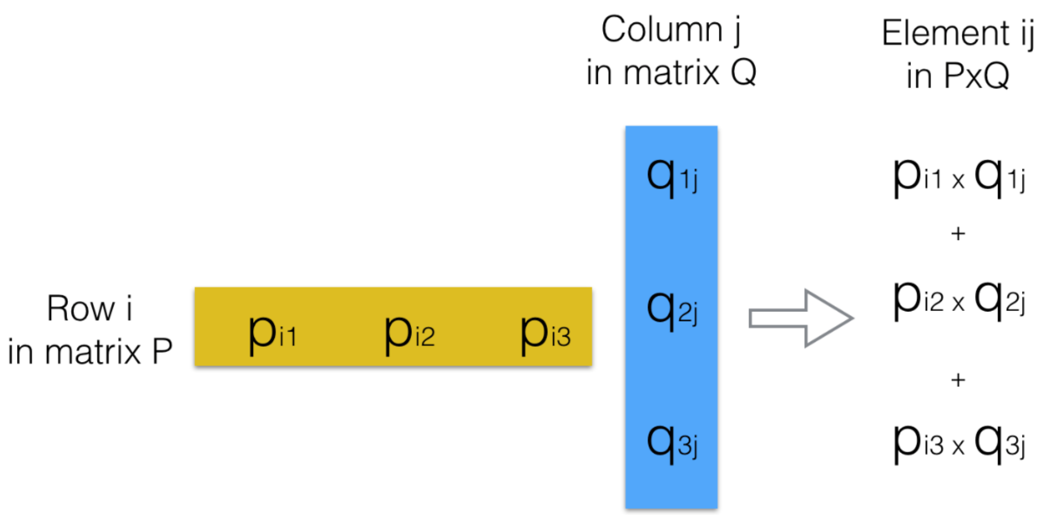
$$P \times Q = \begin{bmatrix} p_{11}q_{11} + p_{12}q_{21} + p_{13}q_{31} & p_{11}q_{12} + p_{12}q_{22} + p_{13}q_{32} & p_{11}q_{13} + p_{12}q_{23} + p_{13}q_{33} \\ p_{21}q_{11} + p_{22}q_{21} + p_{23}q_{31} & p_{21}q_{12} + p_{22}q_{22} + p_{23}q_{32} & p_{21}q_{13} + p_{22}q_{23} + p_{23}q_{33} \\ p_{31}q_{11} + p_{32}q_{21} + p_{33}q_{31} & p_{31}q_{12} + p_{32}q_{22} + p_{33}q_{32} & p_{31}q_{13} + p_{32}q_{23} + p_{33}q_{33} \end{bmatrix}$$

Equation 15

Notice the pattern of finding each element in the multiplication result.

Each element ij in $Q \times P$ is a result of multiplying all elements in row i of matrix P with the corresponding j elements in column j of matrix Q.

See the picture below for an illustration :



if A is an $n \times m$ matrix and B is an $m \times p$ matrix, their matrix product AB is an $n \times p$ matrix, in which the m entries across a row of A are multiplied with the m entries down a column of B and summed to produce an entry of AB. When two linear transformations are represented by matrices, then the matrix product represents the composition of the two transformations.