

## Linear Boundaries

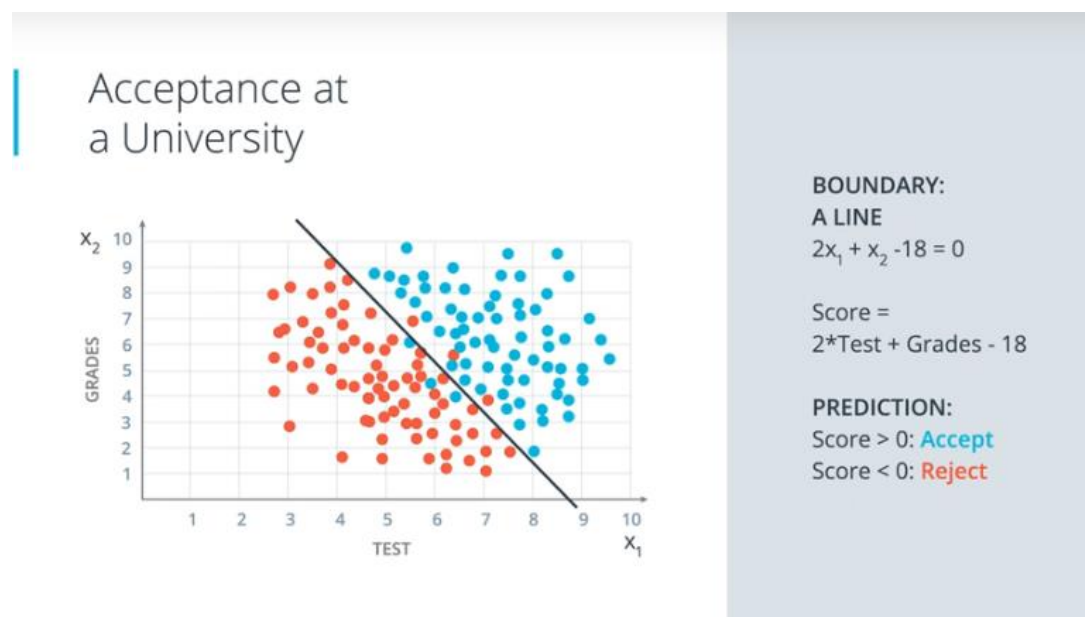
We're going to label the horizontal axis corresponding to the test by the variable  $x_1$ , and the vertical axis corresponding to the grades by the variable  $x_2$ .

So this boundary line that separates the blue and the red points is going to have a linear equation.

The one drawn has equation  $2x_1 + x_2 - 18 = 0$ .

What does this mean? This means that our method for accepting or rejecting students simply says the following: take this equation as our score, the score is  $2x_{\text{test}} + \text{grades} - 18$ .

Now when the student comes in, we check their score. If their score is a positive number, then we accept the student and if the score is a negative number then we reject the student. This is called a prediction. We can say by convention that if the score is 0, we'll accept a student although this won't matter much at the end.



And that's it. That linear equation is our model.

In the more general case, our boundary will be an equation of the following  $w_1x_1 + w_2x_2 + b = 0$ .

We'll abbreviate this equation in vector notation as  $w \cdot x + b = 0$ , where  $w$  is the vector  $(w_1, w_2)$  and  $x$  is the vector  $(x_1, x_2)$ .

And we simply take the product of the two vectors.

We'll refer to  $x$  as the input, to  $w$  as the weights and  $b$  as the bias.

we'll denote a label as  $Y$  and the label is what we're trying to predict.

So if the student gets accepted, namely the point is blue, then the label is  $Y+1$ .

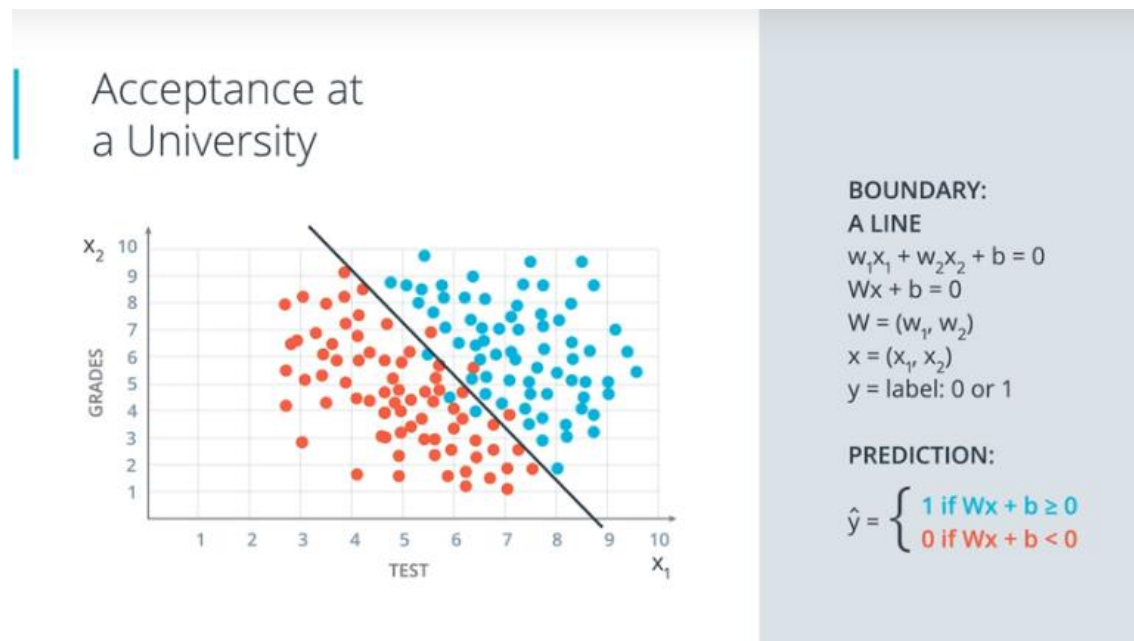
And if the student gets rejected, namely the point is red and then the label is  $Y=0$ .

Thus, each point is in the form  $x_1, x_2, Y$  or  $Y$  is 1 for the blue points and 0 for the red points.

And finally, our prediction is going to be called  $\hat{Y}$  and it will be what the algorithm predicts that the label will be.

In this case,  $\hat{Y}$  is one of the algorithm predicts that the student gets accepted, which means the point lies over the line. And,  $\hat{Y}$  is 0 if the algorithm predicts that this didn't get rejected, which means the point is under the line.

In math terms, this means that the prediction  $\hat{Y}$  is 1 if  $wx+b$  is greater than or equal to zero and 0 if  $wx+b$  is less than 0.



So, to summarize, the points above the line have  $\hat{Y}=1$  and the points below the line have  $\hat{Y}=0$ .

And, the blue points have  $Y=1$  and the red points have  $Y=0$ . And, the goal of the algorithm is to have  $\hat{Y}$  resembling  $Y$  as closely as possible, which is exactly equivalent to finding the boundary line that keeps most of the blue points above it and most of the red points below it.

Now that you know the equation for the line ( $2x_1 + x_2 - 18=0$ ), and similarly the "score" ( $2x_1 + x_2 - 18$ ), what is the score of the student who got 7 in the test and 6 for grades?

Answer: 2