Linear Combination and Span- Theoretical Definitions

In general terms, the simple definition of a **linear combination** is a multiplication of a scalar to a variable and addition of those terms.

For example:

If x, y and z are variables,

and a_1, a_2 and a_3 are scalars,

the following equations will be a linear combination:

$$v = a_1x + a_2y + a_3z$$

Equation 6

Let's now put it into the Linear Algebra context.

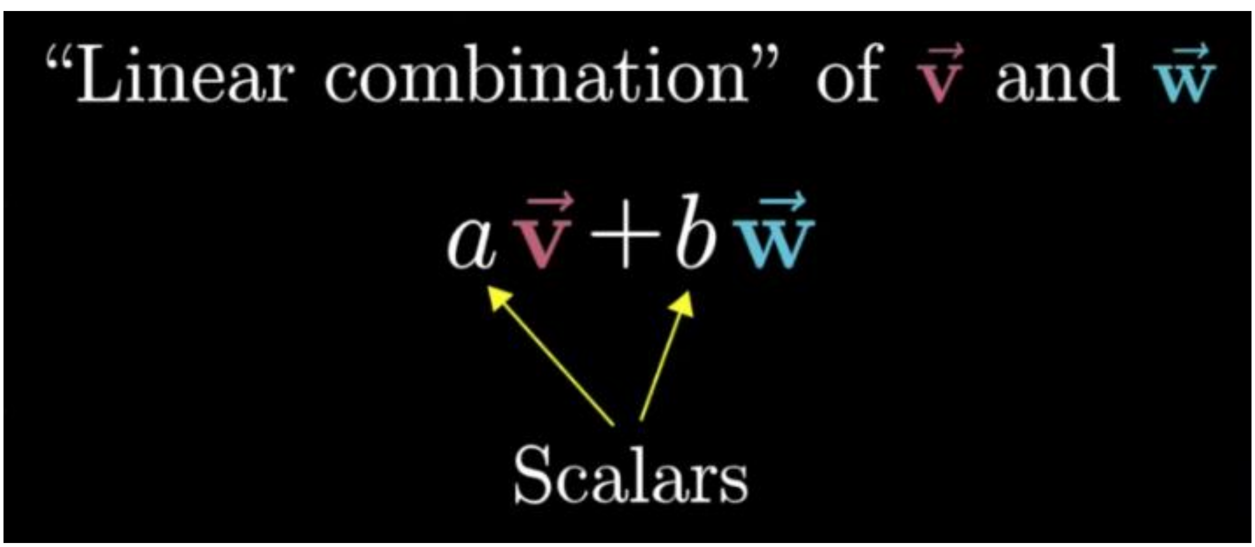
Our variables will now be vectors: $\vec{x}$, $\vec{y}$ and $\vec{z}$ are variables.

The scalars can remain the same: a_1, a_2 and a_3.

A linear combination of a scalar by a vector will be a new vector:

$$\vec{v} = a_1\vec{x} + a_2\vec{y} + a_3\vec{z}$$

Equation 7



A linear combination can be of a single addition, or (as shown in equations 4 above) of any number of additions.

The general notation of a vector by a scalar linear combination will be:

$$\sum_1^n a_i \, v_i$$

Equation 8

What is the **Span**?

The "span" of $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ is the set of all their linear combinations.

If $\vec{v_1}, \vec{v_2}, \ldots.., \vec{v_n} \in \mathbb{R}$ then

the **Span** of those vectors (sometimes also referred to as the Linear Span) is the set of all possible linear combinations of those vectors.

Mathematically, the span of the set of vectors $\vec{v}_1, \vec{v}_1, \ldots, \vec{v}_n$ is written as:

$$Sp(\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n)$$

For example:

The three following vectors:

$\vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{v_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ span any vector in $\mathbb{R}^3$

To prove that, we will take a random vector $\vec{v} = \begin{bmatrix} p \\ q \\ t \end{bmatrix}$

and show that it can be generated as a linear combination of of vectors $\vec{v}_1, \vec{v}_2, and \ \vec{v}_3$.

In a quick observation we can see that:

$$\vec{v} = p \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + q \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$