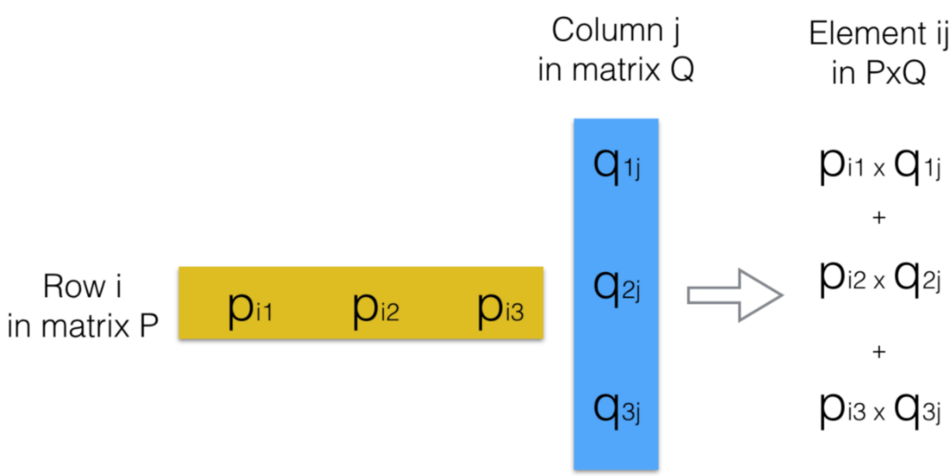
Matrix Multiplication - General

So far we only concentrated on square matrix multiplications. However, we are not limited to square matrices alone.

Lets look again at our image that represents the multiplication $P \times Q$:

Remember, the order here does matter, as matrix multiplication is not commutative



If you look closely at the image that represents the multiplication of $P \times Q$, you will see that the number columns in P equals the number of rows in Q. (In the case of our example, it was 3).

How many rows do we have in matrix P? How many columns do we have in matrix Q? That will deretmine the dimensions of the resulting multiplication.

In other words, if P is a matrix with dimensions $t \times m$ and Q is a matrix with dimensions $m \times v$ then:

- $P \times Q$ is possible as the dimensions match. (P has m columns and Q has m rows).
- $P \times Q$ will be a matrix of $t \times v$. (t rows and v columns).

Lets look at an example:

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \end{bmatrix}$$

$$Q = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \end{bmatrix}$$

We would like to calculate $P \times Q$.

Our first step is to see if the calculation is possible. We will do that by looking at the dimensions of the matrices.

Matrix P has 3 columns and matrix Q has 3 rows. Perfect! multiplication is possible.

We expect to have a final result $P \times Q$ with the dimensions 2×4 (2 rows and 4 columns).

$P \times Q$ =

$$\begin{bmatrix} p11q11 + p12q21 + p13q31 & p11q12 + p12q22 + p13q32 & p11q13 + p12q23 + p13q33 & p11q14 + p12q24 + p13q34 \\ p21q11 + p22q21 + p23q31 & p21q12 + p22q22 + p23q32 & p21q13 + p22q23 + p23q33 & p21q14 + p22q24 + p23q34 \end{bmatrix}$$

Equation 16