Typed Embedding of a Relational Language in OCaml

Dmitrii Kosarev, Dmitrii Boulytchev

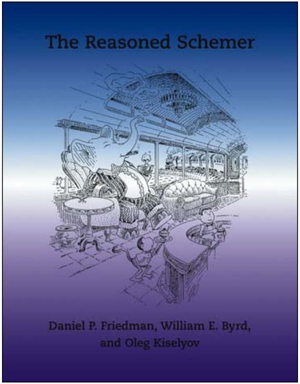
Saint-Petersburg State University
JetBrains Research

ML Family Workshop
September 22, 2016
Nara, Japan

Relational Programming in miniKanren

From programs as *functions* to programs as *relations*:

$$f : X \to Y \rightsquigarrow f^o \subseteq X \times Y$$



- Daniel P. Friedman, William Byrd and Oleg Kiselyov. *The Reasoned Schemer*, The MIT Press, Cambridge, MA, 2005
- A DSL for Scheme/Racket with rather simple minimal implementation
- A family of languages (μKanren, α-Kanren, cKanren etc.)
- Implemented as DSL for a wide range of host languages (including OCaml, Haskell, Scala etc.)

An Example: Relational List Append

$\texttt{append}\colon \alpha\,\texttt{list} \to \alpha\,\texttt{list} \to \alpha\,\texttt{list}$ $\texttt{append}^o \subseteq \alpha\,\texttt{list} \times \alpha\,\texttt{list} \times \alpha\,\texttt{list}$

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`append: α list → α list → α list`

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`appendᵒ ⊆ α list × α list × α list`

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```
(define (appendᵒ xs ys xys)
   (conde
      [(≡ '() xs) (≡ ys xys)]
      [(fresh (h t tys)
         (≡ `(,h . ,t) xs)
         (≡ `(,h . ,tys) xys)
         (appendᵒ t ys tys))]))
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Refinement of answers	$\texttt{refine} : \sigma \to X \to T$

Unification and refinement are virtually the main things to implement

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  Might be a good solution (lightweight, efficient), if type safety is justified

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- the safety of unification/refinement implementation has to be justified separately;
- states must not escape their scope (otherwise the coherence between variable types and terms, stored in states, can be lost).

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the refinement is type-safe, if a variable is refined in a state, which is an inheritor of the state that variable was created in.

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Here:

- `run` — the only way to run goals;
- $\bar{n}$ — a *numeral*, describing the number of fresh variables, available for running the goal `g`; numerals can be manufactured *quantum satis* using the successor function, which is provided as well;
- $q_1, q_2 \ldots q_n$ — these fresh variables;
- $a_1, a_2 \ldots a_n$ — the streams of *refined* answers for the variables $q_1, q_2 \ldots q_n$ respectively;
- `h` — a *handler*, which can make use of refined answers.

The framework guarantees, that variables are refined only in correct states.

Datatype-Generic Pattern

Injecting a user-type into logic domain and projecting the logical results back:

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\begin{array}{rcl}
\uparrow_t & : & t \to t^o \\
\downarrow_t & : & t^o \to t
\end{array}
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$\rightsquigarrow$

$\mathbf{type}\ (\mathtt{'int},\ \mathtt{'tree})\ \mathtt{tree}_f = \mathtt{Leaf}\ \mathbf{of}\ \mathtt{'int} \mid \mathtt{Node}\ \mathbf{of}\ \mathtt{'tree} * \mathtt{'tree}$

$\mathbf{type}\ \mathtt{tree} = (\mathtt{int},\ \mathtt{tree})\ \mathtt{tree}_f$

$\mathbf{type}\ \mathtt{tree}^o = ((\mathtt{int}^o,\ \mathtt{tree}^o)\ \mathtt{tree}_f)^o$

$\mathbf{let\ rec}\ (\uparrow_{\mathtt{tree}})\ \mathtt{t} = \uparrow_\forall\ (\ \boxed{\mathit{fmap}_{\mathtt{tree}_f}}\ (\uparrow_\forall)\ (\uparrow_{\mathtt{tree}})\ \mathtt{t})$

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type ('int, 'tree) tree_f = Leaf of 'int | Node of 'tree * 'tree
type tree   = (int, tree) tree_f
type tree^o = ((int^o, tree^o) tree_f)^o

let rec (↑tree) t = ↑∀ ( fmap_tree_f (↑∀) (↑tree) t)
let rec (↓tree) l = fmap_tree_f (↓∀) (↓tree) (↓∀ l)
```

Example

.

Current Implementation

- Repository: `https://github.com/dboulytchev/OCanren`
- Implements μKanren + disequality constraints
- Passes most of the orignal tests
- Outperforms μKanren on long queries