Typed embedding of relational programming language

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Reverse execution.

3rd example (1)

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| [] → ys
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We are going to emulate *reverse execution* using *relational programming*.

```
function: (xs,ys) -> result
relation: (xs, ys, result)
```

3rd example (2)

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let rec appendo xs ys rez =

$((\texttt{xs} \equiv [\,]) \wedge (\texttt{ys} \equiv \texttt{rez}))$

$\vee$

$((\texttt{xs} \equiv \texttt{h::tl}) \wedge (\texttt{appendo tl ys tmp}) \wedge (\texttt{rez} \equiv \texttt{h::tmp}))$

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Going to rewrite as ‘mathy‘ formula:

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let rec appendo xs ys rez =
  ((xs ≡ []) ∧ (ys ≡ rez))
∨
  ∃ h,tl,tmp:
    ((xs≡ h::tl) ∧ (appendo tl ys tmp) ∧ (rez≡ h::tmp))
```

```
let rec appendo xs ys rez =
  ((xs === llist_nil) &&& (ys===rez))
  |||
  Fresh.three (fun h tl tmp →
    (xs === h % tl) &&& (appendo tl ys tmp) &&& (rez === x % tmp)
  )
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Let's try to execute it.

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# ...give me q such that (appendo [1;2] [3;4] q)
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# ...give me q such that (appendo q [3;4] [1;2;3;4])
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# ...give me (q,r) such that (appendo q r [1;2])
([], [1;2])
([1], [2])
([1;2], [])
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let rec appendo xs ys rez =
  ((xs === llist_nil) &&& (ys===rez))
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     (xs === h % tl) &&& (appendo tl ys tmp) &&& (rez === x % tmp)
  )
```

```
# ...give me (q,r) such that (appendo q r [1;2])
([], [1;2])
([1], [2])
([1;2], [])

# ...give me (r,q) such that (appendo r q q)
([], _.0)
```

Relational programming

- Relational programming $\equiv$ miniKanren
- `http://minikanren.org/`
- A family of many languages: $alpha$-Kanren, μKanren, constrained Kanren, etc.
- Implemented in many languages: various LISPS, OCaml, Haskell, F#, Scala, etc.

Contributions of this work

- 2nd implementation of miniKanren in OCaml (more type-safe than first one: `https://github.com/lightyang/minikanren-ocaml`.
- Less verbose programming than other approaches
  - No need for ad-hoc conversions to `term` type.
- No typeclasses used.
- We are reusing OCaml specific features in *unification* (algorithm behind $\equiv$).

```
let foo ints strings =
  Fresh.two (fun h tl →
    (ints === h % tl) &&& (h === inj 5) &&&
    Fresh.two (fun h' tl' →
      (strings === h % tl') &&& (tl' === (inj "asdf") % llist_nil)
     ))
```

```
File "regression/test000.ml", line 37, characters 50-51:
Error: This expression has type bytes llist logic but
       an expression was expected of type int llist logic
       Type bytes is not compatible with type int
```

About implementations with `term` type

- Special type for relation-related evaluations.
- ```
  type term = Var of int
            | ConstString of string
            | ConstInt of int
            | ListNil
            | ListCons of term * term
            | Atom of string
            | etc...
  ```
- Special convertion functions for every type `t`:

  ```
  val inject_t:  t → term
  val project_t: term → t
  ```
- And `project` is partial by two reasons:
  - Value of type `term` can contain free logic variables. ”
  - The value `ListCons(ConstInt, ListCons(ConstString "a"),ListNil))` cannot be converted neither to OCaml `int list` nor to `string list`.
  ```