Typed Embedding of a Relational Language in OCaml

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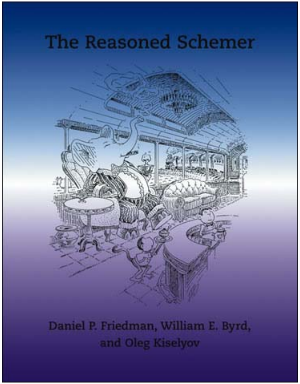
Relational Programming in miniKanren

From programs as *functions* to programs as *relations*:

$$f : X \to Y \rightsquigarrow f^o \subseteq X \times Y$$



- Daniel P. Friedman, William Byrd and Oleg Kiselyov. *The Reasoned Schemer*, The MIT Press, Cambridge, MA, 2005
- A DSL for Scheme/Racket with rather simple minimal implementation
- A family of languages (μKanren, α-Kanren, cKanren etc.)
- Implemented as DSL for a wide range of host languages (including OCaml, Haskell, Scala etc.)

Relational List Append (OCaml/OCanren/miniKanren)

$\texttt{append}\colon \alpha\,\texttt{list} \to \alpha\,\texttt{list} \to \alpha\,\texttt{list}$

$\texttt{append}^o \subseteq \alpha\,\texttt{list} \times \alpha\,\texttt{list} \times \alpha\,\texttt{list}$

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let rec append xs ys =
  match xs with
  | []    → ys
  | h::tl → h :: (append tl ys)
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append^o ⊆ α list × α list × α list

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In the original miniKanren:

```
(define (appendᵒ xs ys xys)
   (conde
      [(≡ '() xs) (≡ ys xys)]
      [(fresh (h t tys)
         (≡ `(,h . ,t) xs)
         (≡ `(,h . ,tys) xys)
         (appendᵒ t ys tys))]))
```

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Refinement of answers	`refine`: $\sigma \to X \to T$

Unification and refinement are virtually the main things to implement

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  Might be a good solution (lightweight, efficient), if type safety is justified

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- the safety of unification/refinement implementation has to be justified separately;
- states must not escape their scope (otherwise the coherence between variable types and terms, stored in states, can be lost).

Properties of Polymorphic Unification

It can be shown, that for our concrete implementation:

- variables in a substitution are always associated with the terms of the same type;
- all variables preserve their types, assigned by the compiler;
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the refinement is type-safe, if a variable is refined in a state, which is an inheritor of the state that variable was created in.

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Here:

- `run` — the only way to run goals;
- $\bar{n}$ — a *numeral*, describing the number of fresh variables, available for running the goal `g`; numerals can be manufactured *quantum satis* using the successor function, which is provided as well;
- $q_1, q_2 \ldots q_n$ — these fresh variables;
- $a_1, a_2 \ldots a_n$ — the streams of *refined* answers for the variables $q_1, q_2 \ldots q_n$ respectively;
- `h` — a *handler*, which can make use of refined answers.

The framework guarantees, that variables are refined only in correct states.

Datatype-Generic Pattern

Injecting a user-type into logic domain and projecting the logical results back:

$$
\begin{array}{lcl}
\uparrow_t & : & t \to t^o \\
\downarrow_t & : & t^o \to t
\end{array}
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type tree  = (int, tree) tree_f
type tree^o = ((int^o, tree^o) tree_f)^o
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```
let rec (↑tree) t = ↑∀ ( fmap_tree_f (↑∀) (↑tree) t)
```

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$\rightsquigarrow$

$\textbf{type}\ (\mathtt{'int},\ \mathtt{'tree})\ \mathtt{tree}_f = \mathtt{Leaf}\ \textbf{of}\ \mathtt{'int}\ |\ \mathtt{Node}\ \textbf{of}\ \mathtt{'tree} * \mathtt{'tree}$
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Example

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Current Implementation

- Repository: `https://github.com/dboulytchev/OCanren`
- Implements μKanren + disequality constraints
- Passes most of the original tests
- Outperforms μKanren on long queries