Typed Embedding of a Relational Language in OCaml

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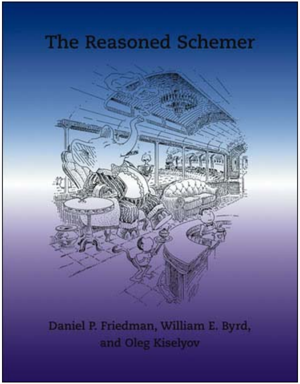
Relational Programming in miniKanren

From programs as *functions* to programs as *relations*:

$$f : X \to Y \rightsquigarrow f^o \subseteq X \times Y$$



- Daniel P. Friedman, William Byrd and Oleg Kiselyov. *The Reasoned Schemer*, The MIT Press, Cambridge, MA, 2005
- A DSL for Scheme/Racket with rather simple minimal implementation
- A family of languages (μKanren, α-Kanren, cKanren etc.)
- Implemented as DSL for a wide range of host languages (including OCaml, Haskell, Scala etc.)

An Example: Relational List Append

$\texttt{append}: \alpha\,\texttt{list} \to \alpha\,\texttt{list} \to \alpha\,\texttt{list}$

$\texttt{append}^o \subseteq \alpha\,\texttt{list} \times \alpha\,\texttt{list} \times \alpha\,\texttt{list}$

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$\texttt{append}: \alpha\,\texttt{list} \rightarrow \alpha\,\texttt{list} \rightarrow \alpha\,\texttt{list}$

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let rec append xs ys
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let rec append xs ys =
  match xs with
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append: α list $\rightarrow$ α list $\rightarrow$ α list

append^o $\subseteq$ α list $\times$ α list $\times$ α list

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let rec append xs ys =
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```

```
(define (appendᵒ xs ys xys)
  (conde
    [(≡ '() xs) (≡ ys xys)]
    [(fresh (h t tys)
       (≡ `(,h . ,t) xs)
       (≡ `(,h . ,tys) xys)
       (appendᵒ t ys tys))]))
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Unification is virtually the main thing to implement

Dealing with Typed Terms

- Non-solution:
  - implement unification for a fixed term representation;
  - convert user-type data to- and from that universal representation.

Polymorphic Unification

- Types are erased in substitution
- Have to be reconstructed properly during refinement
- States must not escape the scope

Datatype-Generic Pattern

Injecting a user-type into logic domain and projecting the logical results back:

$$\begin{array}{rcl} \uparrow_t & : & t \to t^o \\ \downarrow_t & : & t^o \to t \end{array}$$

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- "$\uparrow_\forall$", "$\downarrow_\forall$" are polymorphic shallow injection/projection;
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type tree = Leaf of int | Node of tree * tree
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type ('int, 'tree) tree_f = Leaf of 'int | Node of 'tree * 'tree
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type tree = (int, tree) $\texttt{tree}_f$
type $\texttt{tree}^o$ = (($\texttt{int}^o$, ltree) $\texttt{tree}_f$)^o

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$\textbf{type}\ (\texttt{'int}, \texttt{'tree})\ \texttt{tree}_f = \texttt{Leaf}\ \textbf{of}\ \texttt{'int} \mid \texttt{Node}\ \textbf{of}\ \texttt{'tree} * \texttt{'tree}$

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$\textbf{let rec}\ (\uparrow_{\texttt{tree}})\ \texttt{t} = \uparrow_\forall\ (\ \boxed{\mathit{fmap}_{\texttt{tree}_f}}\ (\uparrow_\forall)\ (\uparrow_{\texttt{tree}})\ \texttt{t})$

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Capturing the States

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Example

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Current Implementation

- Repository: `https://github.com/dboulytchev/OCanren`
- Implements μKanren + disequality constraints
- Passes most of the orignal tests
- Outperforms μKanren on long queries