Typed Embedding of a Relational Language in OCaml

Dmitrii Kosarev, Dmitrii Boulytchev

Saint-Petersburg State University
JetBrains Research

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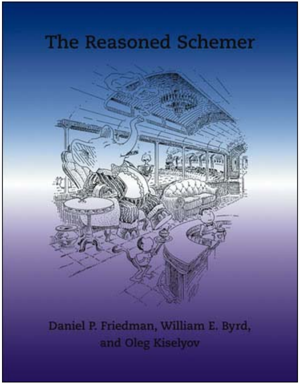
Relational Programming in miniKanren

From programs as *functions* to programs as *relations*:

$$f : X \to Y \rightsquigarrow f^o \subseteq X \times Y$$



- Daniel P. Friedman, William Byrd and Oleg Kiselyov. *The Reasoned Schemer*, The MIT Press, Cambridge, MA, 2005
- A DSL for Scheme/Racket with rather simple minimal implementation
- A family of languages (μKanren, α-Kanren, cKanren etc.)
- Implemented as DSL for a wide range of host languages (including OCaml, Haskell, Scala etc.)

An Example: Relational List Append

$\texttt{append}\colon \alpha\,\texttt{list} \to \alpha\,\texttt{list} \to \alpha\,\texttt{list}$

$\texttt{append}^o \subseteq \alpha\,\texttt{list} \times \alpha\,\texttt{list} \times \alpha\,\texttt{list}$

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append^o ⊆ α list × α list × α list

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```
(define (appendᵒ xs ys xys)
   (conde
      [(≡ '() xs) (≡ ys xys)]
      [(fresh (h t tys)
         (≡ `(,h . ,t) xs)
         (≡ `(,h . ,tys) xys)
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Refinement of answers	$\texttt{refine} : \sigma \to X \to T$

Unification and refinement are virtually the main things to implement

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  Might be a good solution (lightweight, efficient), if type safety is justified

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- the safety of unification/refinement implementation has to be justified separately;
- states must not escape their scope (otherwise the coherence between variable types and terms, stored in states, can be lost).

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$$\rightsquigarrow$$

the refinement is type-safe, if a variable is refined in a state, which is an inheritor of the state that variable was created in.

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The framework guarantees, that variables are refined only in correct states.

Datatype-Generic Pattern

Injecting a user-type into logic domain and projecting the logical results back:

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type $\texttt{tree}^o$ = (($\texttt{int}^o$, ltree) $\texttt{tree}_f)^o$

Datatype-Generic Pattern

Injecting a user-type into logic domain and projecting the logical results back:

$$\begin{array}{rcl} \uparrow_t & : & t \to t^o \\ \downarrow_t & : & t^o \to t \end{array}$$

Can be done systematically using generic programming:

- "$\uparrow_\forall$", "$\downarrow_\forall$" are polymorphic shallow injection/projection;
- for the deep case, make the type a functor and use *fmap*.

```
type tree = Leaf of int | Node of tree * tree
⇝
type ('int, 'tree) tree_f = Leaf of 'int | Node of 'tree * 'tree
type tree  = (int, tree) tree_f
type tree^o = ((int^o, ltree) tree_f)^o

let rec (↑tree) t = ↑∀ ( fmap_tree_f  (↑∀) (↑tree) t)
```

Datatype-Generic Pattern

Injecting a user-type into logic domain and projecting the logical results back:

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type tree = Leaf of int | Node of tree * tree
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type tree  = (int, tree) tree_f
type tree^o = ((int^o, ltree) tree_f)^o

let rec (↑tree) t = ↑∀ ( fmap_tree_f (↑∀) (↑tree) t)
let rec (↓tree) l = fmap_tree_f (↓∀) (↓tree) (↓∀ l)
```

Example

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Current Implementation

- Repository: `https://github.com/dboulytchev/OCanren`
- Implements μKanren + disequality constraints
- Passes most of the orignal tests
- Outperforms μKanren on long queries