Modeling the 2010 Gulf of Mexico Oil Spill

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1 Abstract

The Deepwater Horizon Oil Spill was a catastrophic oil spill that occurred in 2010 resulting in the spilling of millions of barrels of oil spreading throughout the Gulf of Mexico. A decade later, scientists still do not truly know the entire breadth of the spill and what it affected. Using different concepts from calculus and estimated data gathered by the EPA, we were able to analyze and interpret as accurately as possible just how far out the oil diffused through the water. The method of finding the volume was achieved through using Riemann Sums and the Interference of Light formula created by prominent mathematicians that have contributed to the sciences immensely. The oil spill was also looked at from a view of social justice and how it affected local communities. The four million barrels of oil that spilled considerably impacted the marine life in the Gulf as well as the livelihoods of those who depended on the Gulf. A societal and mathematical perspective taken on the Deepwater Horizon Oil Spill presents the deep environmental injustice that transpired in 2010 and grants us with the opportunity to prevent such a tragedy from happening again.

2 Key Words

- Bernhard Riemann
- Calculus
- Deepwater Horizon
- Derivative
- Function
- Gottfried Wilhelm Leibniz

- Integral
- Interference of light
- Isaac Newton
- Riemann Sum
- Simpson's Rule
- Social Justice
- Thomas Young
- Young's Double Slit Experiment

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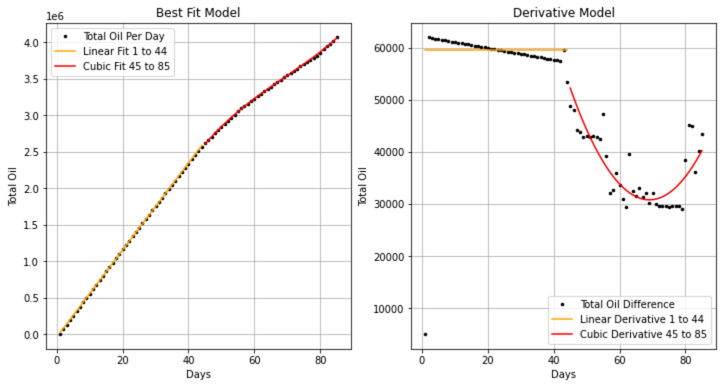

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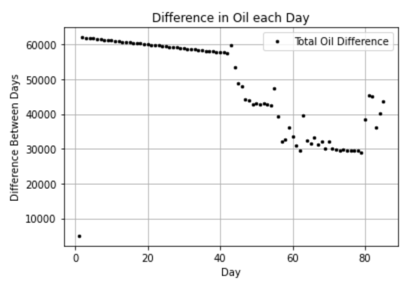

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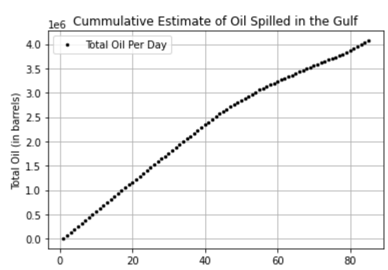

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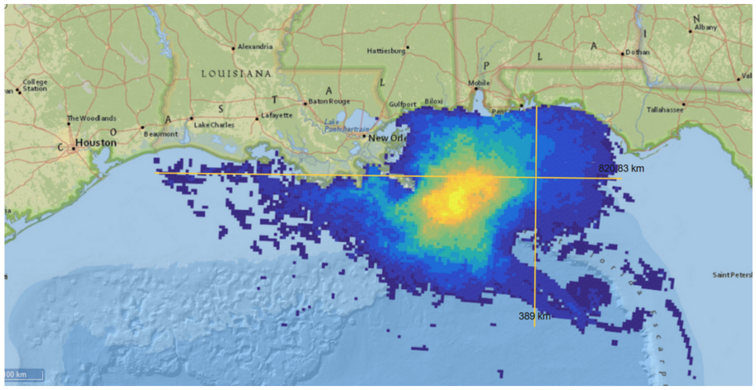

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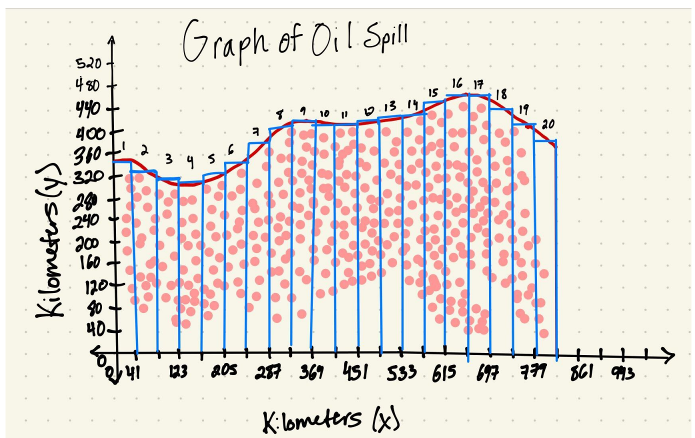

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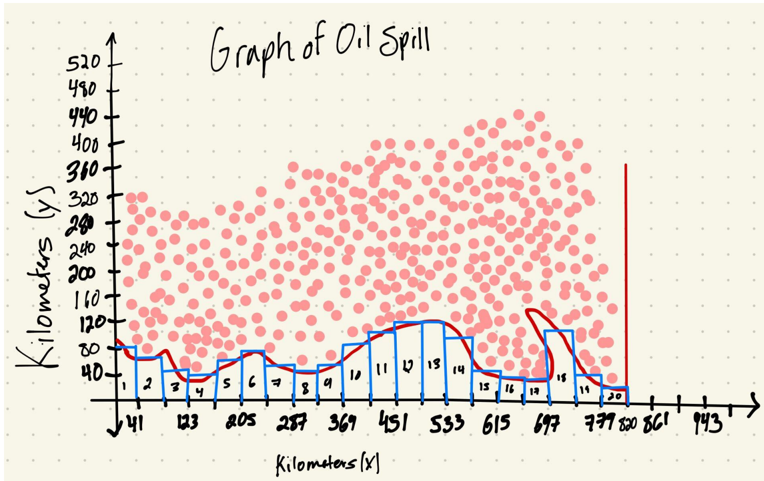

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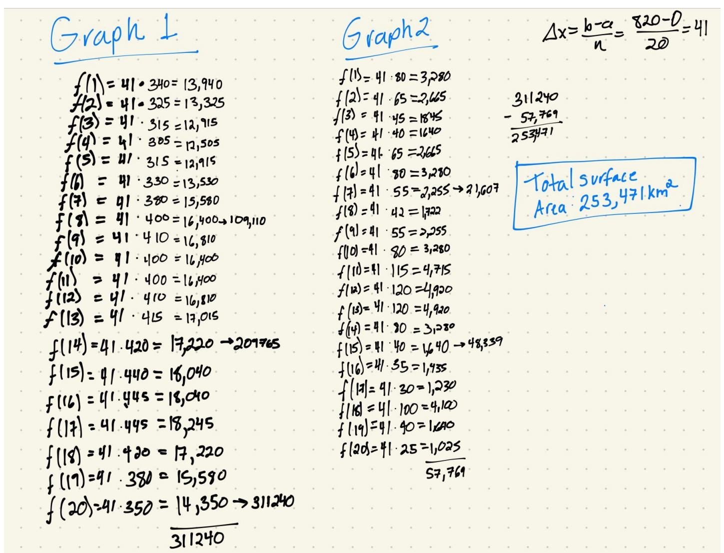

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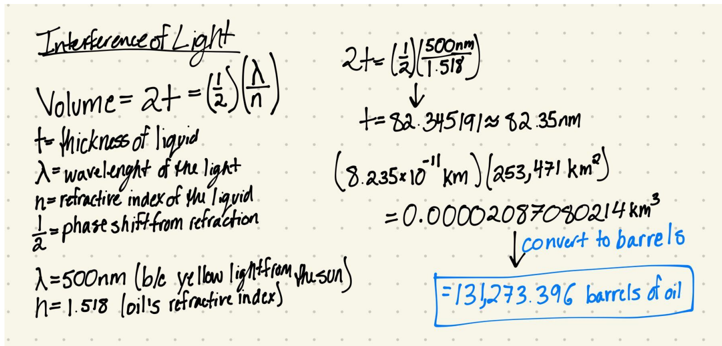

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Cumulative oil (in barrels) that entered the Gulf based on the flow-rate estimates, adjusting for oil recovered by BP

Table 1: Cumulative oil (in barrels) that entered the Gulf

Day	Total Oil	Day	Total Oil	Day	Total Oil	Day	Total Oil
1	5000	23	1343013	45	2616605	67	3459022
2	67000	24	1402583	46	2664639	68	3491212
3	128888	25	1461950	47	2708787	69	3521418
4	190663	26	1521250	48	2752609	70	3553606
5	252325	27	1580438	49	2795460	71	3583557
6	313875	28	1639513	50	2838563	72	3613208
7	375313	29	1698475	51	2881351	73	3642791
8	436638	30	1757325	52	2924493	74	3672451
9	497850	31	1816063	53	2967304	75	3701926
10	558950	32	1874688	54	3009741	76	3731460
11	619938	33	1933200	55	3056988	77	3761015
12	680813	34	1991600	56	3096295	78	3790587
13	741575	35	2049888	57	3128371	79	3819604
14	802225	36	2108063	58	3161029	80	3858052
15	862763	37	2166125	59	3197035	81	3903303
16	923188	38	2224075	60	3230630	82	3948324
17	983500	39	2281913	61	3261537	83	3984423
18	1043700	40	2339638	62	3291002	84	4024581
19	1103788	41	2397250	63	3330535	85	4068111
20	1163763	42	2454750	64	3363039	-	-
21	1223625	43	2514372	65	3394567	-	-
22	1283375	44	2567787	66	3427724	-	-

5 Introduction

In the spring of 2010, the Deepwater Horizon, an oil rig in the Gulf of Mexico, exploded and sank causing one of the largest oil spills in history. Transocean was the company operating the rig and it was leased by BP oil company. The rig was situated over an oil well that was about fifteen-hundred meters below sea level. On April 20^{th}, a swell of natural gas rushed through a recently installed concrete core that was installed to seal the well. The natural gas traveled up to the rig's platform where it killed almost a dozen workers and injured several more. On April 22^{nd}, the rig sank which broke the machine that was responsible for balancing out the upward force of the gas and oil. Without its opposing force, oil began seeping into the gulf. The U.S. government estimated that approximately sixty-thousand barrels of oil were discharged per day. The device on the rig, designed to close the oil channel, malfunctioned. A pipe on the device had bent due to the pressure of the gas and oil rising from below. Several efforts were made to contain the leak until BP turned to a machine called the Lower Marine Riser Package (LMRP). The device was placed over the original broken machinery, allowing some oil to continue to leak out, but collecting approximately fifteen-thousand barrels per day into its tank. Another system was added to the LMRP that increased the collection rate to twenty-five-thousand barrels per day. In July, a more permanent cap was placed on the oil seal. By this point, almost five-million barrels had leaked into the gulf with only about sixteen percent being collected. It is important to conduct research on this oil spill to understand the effects the oil leak has caused to the marine life living in the Gulf of Mexico.

The research that we plan to carry out is the cleaning up and data collection as a result of the spill. There were different strategies employed to clean up the spill, each with different amounts of success. There also is a large amount of data that was collected to see the total cost and the impact that such a spill had on the environment. This information allows us to analyze and understand what exactly occurred with this event and the lasting effect that it has caused. The importance of knowing this information allows us to know the warning signs and ways to prevent such an event from ever occurring again. And if it does, then use the fastest methods for cleaning up spills and keeping damage to the environment at a minimum. Therefore, we hypothesize that the 2010 BP oil spill had very deep effects across the Gulf of Mexico, especially involving how far it spread and the marine life is affected. By collecting data based on this hypothesis, we will be able to see the true magnitude the spill had on the Gulf and the environment.

Before this project, we had minimal knowledge of the Deep Horizon oil spill. We only knew that there was an accident, in which an oil machine broke and released tons of gallons of oil in the oceans. Scientists were having difficulties in removing the oil because it was hard to control and spread all over the ocean. However, we had no knowledge of its severity. We didn't know how it happened, what it cost, and never thought of a way to resolve the problem.

In this research, we shall be focusing on cleaning up the oil and collecting data to see the progress. We will have to use calculus in order to do that. By doing this project, we will realize the use of calculus in real-life problems. We will be able to collect data, match those with the calculated one, and solve problems. We can also prove the theories of calculus by getting an accurate result. This study will build our knowledge and accuracy on this topic and will make us understand various issues at a time.

6 Historical Events, Bio of Mathematicians, etc.

6.1 Young's Light Experiments

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One of the problems that we have discussed before was cleaning up the oil and seeing how far it has spread. To see how much oil needs to be cleaned up, we need to find the volume of the oil released by the spill. We know from chemistry that oil and water won't mix because of the polarity of water molecules and the nonpolarity of oil molecules. However, while we need to see the entire breadth of the spread, oil does have a thickness when floating on top of the water. This means we have to use a concept in physics known as the interference of light. Initially proposed by Isaac Newton, Newton stated that light is made up of tiny particles. It was later on where a scientist named Thomas Young determined the wave nature of light with his experiment called Young's Double Slit Experiment in 1800. Later experiments with this discovery lead to the interference of light, which can be explained with lights being made up of waves (Heavens and Ditchburn, 1991).

Using the properties of interference of light, we can calculate the thickness of oil by using fluorescence and seeing how the light bounces off the oil and then bounces off the water. When the light passes through the oil to get to the water, it then bounces back through the oil again and back to the air. This is known as the interference of light and why oil sometimes looks like it has rainbow circles on the surface. Thus, we can calculate the thickness of the oil by measuring the extra distance a wave of light has to travel when passing through the oil and back again. The extra distance is equal to twice the thickness of the oil because the wave of light has to go through the oil, hit the water, and then bounce back through the oil again. Because the way light shifts and assuming normal sunlight (wavelength of $500nm$) the equation we need to solve this problem is $2t = (\frac{1}{2})(\frac{\lambda}{n})$ where t represents the thickness of the oil, n represents the refractive index of the oil, and λ represents the wavelength of passing through the oil. The $\frac{1}{2}$ is necessary because a phase shift occurs at the surface of the oil that causes the wavelength to be half its original size (Heavens and Ditchburn).

We can obtain the thickness of the oil using Thomas Young's experiments, but now we need to find the area. Multiplying the thickness of the oil above the water by the area of the spread will give us the volume of all the oil released. To calculate the volume, we will use the Reimann sum to find the area of the oil spill. Reimann sums are named after nineteenth-century German mathematician Bernhard Riemann. They are an approximation of an integral through a finite sum along a certain interval. The primary use of the Riemann sum is to find the area under the curve of a line. The sum is found by dividing the region into shapes that together form a region that is as close as to the original shape as possible, i.e. rectangles, trapezoids. The area of these separate shapes is calculated and you end up with the area for the whole region when you add the smaller areas of the separate shapes together (C. H. Edwards, 1979).

6.2 Leibniz's Role in Calculus

Calculus today has become a branch of mathematics that is taught worldwide and is standard in education. It focuses on subjects like integration and tangents. Although the baselines were laid out by ancient mathematicians in Greece, such as Antiphon the Sophist, and the Eleatic philosopher Zeno (Howard Eves, Page 380), there are two figures that are prominently seen as the fathers of calculus. Sir Isaac Newton and Gottfried Wilhelm Leibniz are these figures. Sir Isaac Newton, being a Lucasian Professor at Cambridge University and the plague occurring during the 1660's (C. H. Edwards, Page 190-191), focused seriously on the study of mathematics, beginning with Euclid's Elements and Descartes' Geometrie. He soon turned his focus onto major contributions to mathematics and calculus during this time to be published later on in his life. Gottfried Wilhelm Leibniz mastered mathematics at a young age, eventually developing the general concepts of mathematics in use today and influencing many mathematicians, like Georgle Boole (Howard Eves, Page 403) in the future.

Leibniz's work has become the foundation of modern calculus, with his notations and concepts is what is currently used today. Early rules and foundations of Calculus, which a student learns early in a beginning course of calculus, were derived by Leibniz (Howard Eves, Page 405).

On October 29, 1675, Leibniz used the first modern integral sign (Howard Eves, Page 404). A few weeks later, he started writing differentials and derivatives in the form used today, with dx as the finite interval and dy being the proportion (Howard Eves, 405). Leibniz came up with $f(g(x))g'(x)dx = f(u)du$ (C. H. Edwards, Page 233).The substitution of $u = g(x), du = g'(x)$ makes the formula above seem inevitable.

"This amounts to the invariance of the differential form $f(u)du$ with respect to arbitrary changes of variable - one of Leibniz' most important discoveries." He was also the first to create two ideas that are essential to calculus, "the transformation of integrals by means of substitution, and the reduction of quadrature problems to inverse tangent problems, the latter being problems in which a curve is to be determined from a knowledge of its tangent line (C. H. Edwards, Page 244)".

These advanced discoveries were supported by the fundamentals Leibniz first came up with during the revelation about determining a tangent of a curve he claimed to have after viewing Pascal's triangle (Boyer, 1959). This revelation was that the tangent depended on the differences in the x and y axes and that this value was inversely related to the sum of infinitely thin rectangles for infinite values on the x axis. In today's terms, we know this "difference" to be the derivative, and the ”sum” to be the indefinite integral. Not only did he come up with the substitution rule for indefinite integrals, Leibniz also outlined basic rules for derivatives. After much indecisive thinking, he represented the power rule, product rule, and quotient rule correctly using his notation. Leibniz's product and quotient rules are as follows:

$$d(xy) = xdy + ydx$$

and

$$d\left(\frac{x}{y}\right) = \frac{ydx - xdy}{y^2}.$$

From this, Leibniz extended these to all integral powers of a variable, the power rule, with the “difference”, derivative, of x^n being $nx^{n-1}dx$ (Boyer, 1959).

Likewise, Leibniz demonstrated the inverse of x^n, its summation, as the integral of x^n being $\frac{x^{n+1}}{n+1}$ (Boyer, 1959). Finally, Leibniz also noted how multiple derivatives can be taken on a function, and the final derivative of a function would end up being 0, $x^2 \Rightarrow 2x \Rightarrow 2 \Rightarrow 0$ (D. E. Smith, 1959). These fundamental rules are all seen in modern calculus, albeit with proper arithmetic and analytical proofs.

With everything Leibniz has contributed to the development of calculus, he is easily one of the most important mathematicians who ever lived. The impact of calculus on our society is immense, as it played a key role in many technological discoveries that might not have happened otherwise. Leibniz's discoveries and work on integrals and the fundamental theorem of calculus will forever be remembered and used for centuries afterward.

7 Social Justice Aspects of Our Problem

7.1 How did the oil spill occur?

The oil spill occurred when a rush of natural gas traveled up an oil rig situated over an oil well in the Gulf of Mexico.

7.2 Cleaning up the spill

7.2.1 When did the different cleanup strategies begin?

Cleanup began before the seal was repaired. The spill began in April and cleanup efforts began in May, before the seal was closed in July.

7.2.2 What was involved in the strategies?

The first thing that had to be done was containing the spreading oil. This was done with the use of containment booms, floating devices that are temporary floating barriers meant to hold in oil during a spill. With "...BP and the Coast Guard [deploying] some 2.7 million feet of boom..." to contain the spreading oil (Grier, 2010). After such measures were taken to prevent the further spread of the oil, people sought to disperse the oil. An oil dispersant called Corexit was used where it has fast-acting properties to prevent further damage to the shoreline (Butler, 2011). This method was not effective and has been debated to have made the oil much more damaging to the environment than the oil itself. The most effective methods were burning the oil, filtering it offshore, and collecting it for later processing. Other methods include skimming from the surface, natural dispersion, and evaporation and dissolving.

7.2.3 How long did each strategy last?

After two months in, around 75% of the oil had remained in the environment even while using all of the various methods (Kiger, 2012). Much work was put in to halt the spread of the oil and hasten its removal from the environment. All of these methods had various amounts of success in their time in use. There are no specific numbers on how long each method was used but on April 15, 2014, BP had released a statement that the cleanup was essentially complete but lots of work remained (Robertson, 2014).

7.2.4 For each strategy, what was the outcome? Explain whether it helped and if so, by how much.

Cleanup strategies: Chemical dispersants were sprayed on the surface of the gulf to pull apart the oil particles from the water. These chemicals reduced the size of the oil particles to make them able to be degraded by bacteria. The benefit of this strategy was that the chemicals could be dispersed throughout a large area from above. Once the chemicals were applied, the oil became less toxic. The disadvantage of this strategy was that, over time, the chemicals may lose their ability to reduce the size of the oil particles as the only remains in the gulf. The chemicals themselves, are also toxic and have negative impacts on the environment.

Underwater dispersants were similar to the dispersants sprayed on the surface. By injecting the chemicals into the oil while underwater, the oil particles will break apart before they reach the surface. This strategy is less effective at extreme depths because of the lack of vehicles with the ability to transfer the chemicals.

Manual and mechanical cleanups are used in almost every oil spill. Workers were placed on the coast with shovels and gloves to collect any oil that reaches the shore. Heavy machinery was used in areas where the beaches are heavily soaked in oil and debris. These strategies are time-consuming but manual cleanup is preferred. Minimal training is required to perform this cleanup and a large number of workers can be hired. Heavy machinery is also quick to clean up large areas, but the machines may cause damage to the areas they are ridding of oil.

7.3 Data for the spill

7.3.1 Who is collecting data on the spill?

The U.S. EPA Office of Solid Waste and Emergency Response collected the information on the BP oil spill (U.S. Environmental Protection Agency, 2016). All of the information collected was compiled together for record-keeping.

7.3.2 How is the amount of spill estimated?

A total of 4.9 million barrels of oil were estimated to be spilled at this disaster, with around 12,000 to 19,000 barrels spilling each day as estimated by the U.S. Government. It has been stated that there were no proven techniques for estimating the flow of oil at the broken wellheads. Several factors were preventing this, such as an unbounded source of oil and limited timeline to work with as well as several different leak points Visual estimation in combination of mass balances, while unreliable, had to be used to estimate, with several changes in the estimated amount of oil spilled per day (United States Coast Guard, 2011).

7.3.3 Why is an accurate assessment useful?

An accurate assessment of the amount of oil spilled is useful because it allows us to determine the actual amount of oil spilled. With this information, different parties can determine the true impact of the ocean effect. Others are also able to gather resources and plan according to the proper size of the spill. In the early days of the spill, low estimates were given which led to a lack of proper resources to handle the spill (United States Coast Guard, 2011).

7.4 What were the impacts of the spill on the coastal region?

7.4.1 How were people affected?

By June 2010, many workers cleaning up the spill, as well as several gulf residents, reported smelling strong odors as well as experiencing nausea, headaches, and irritation in their ears, eyes, and throat. A study eight years after the spill demonstrates that those people involved in the oil spill cleanup operations experience persistent alterations or worsening of their hematological, hepatic, pulmonary, and cardiac functions. Also, these subjects experienced prolonged or worsening illness symptoms even 7 years after their exposure to the oil spill. (D'Andrea and Reddy, 2018)

7.4.2 How was the environment affected?

The oil spill caused the death of thousands of marine animals and polluted habitats. Not concerning the anthropocentric impacts of the spill, we see that the spill harmed or killed around 25,900 marine mammals, including sea turtles and dolphins, as well as approximately 82,000 birds of 102 species (NOAA, 2011).

7.4.3 How was the economy affected?

The cleanup required an abundance of workers to be hired to clean the beaches and polluted areas. The fishing industry was negatively impacted due to the decrease in what could be caught and sold.

7.4.4 What responsibilities did the corporations involved incur?

BP faced the responsibility of addressing the damage done to human life, wildlife, and the economy. BP spent four billion dollars to contain and clean up the oil spill and another four-to-five billion dollars in penalties for those impacted.

7.4.5 How did states use their settlements from the spill? Were there behaviors that led to more effective use of funds?

Alabama, Florida, Louisiana, Mississippi, and Texas received around 20.8 billion dollars in a settlement under the Clean Water Act and Oil Pollution Act. A statement was released by Interior Secretary Sally Jewell saying, "Today's settlement is a significant step in restoring the natural resources that were impacted by the Deepwater Horizon oil spill and a breakthrough for building back the resilience of this region. The Trustees will continue to work with people along the coast to ensure they have every opportunity to be engaged in these meaningful recovery and restoration efforts that will generate jobs, improve water quality, support our tribal responsibilities and result in an improved wildlife habitat for migratory birds and hundreds of vulnerable species" (Office of Public Affairs, 2016). Trustees developed a plan to determine how the money would be spent. Restoration and conservation of habitats would get 4.7 billion dollars. Restoration of water quality would get 410 million dollars. Replenishment and protection of coastal living resources would get 1.8 billion dollars. Enhancement of recreational opportunities would receive 420 million dollars. And finally, the monitoring, adaptive management, and oversight of these actions would receive 1.5 billion dollars. Further funds were used to fund studies and many organizations, businesses, and private citizens lawsuit money to get money back from the damage caused to them (U.S. Department of Commerce, 2017).

7.5 Social Justice

7.5.1 What is it?

Social justice is justice under the conditions of equal rights, equal opportunity, and equal treatment. Within the framework of social justice issues there exists an issue known as environmental justice. Just as social injustices against one's equal rights occur, environmental injustices can also occur. Environmental injustice is the "disproportionate exposure of certain populations to environmental hazards" (Maantay, 2002).

7.5.2 How does it relate to the oil spill?

Social and environmental injustices are vastly interwoven into the aftermath of oil spills, especially one as large as the Deepwater Horizon Oil Spill in 2010. Not only was the spill an environmental disaster, but the spill also didn't affect everyone equally either. With an accident of such scale located near a populated region, there was bound to be a certain portion of the population that was unequally affected which is usually those on the lower end of the income spectrum. Because the oil spill polluted the water and disturbance of marine wildlife, the Gulf's seafood industry received the most adverse effect of the spill. African Americans and Southeast Asians who depended on the Gulf for their jobs were now at a loss for income.

The government had also approved dumping debris that resulted from the spill near landfills near neighborhoods that were predominately black, Latinx, and Asian. The combination of environmental injustices and research done by Robert Bullard produced an even further social injustice onto the affected communities: environmental racism.

A study conducted in 1990 by Robert Bullard showed that black neighborhoods were consistently chosen for waste facilities. Locally Unwanted Land Uses, or LULUs, are facilities that you most likely don't want to be living near and are often time pollutive like waste incinerators, landfills, power plants, etc. Bullard conducted many studies that showed LULUs correlations to low-income communities and people of color. He also ended up correlating this concept to the BP oil spill. "The affected communities have health statistics that are among the poorest in the nation and had previously been battered and disrupted by natural disasters" (Hansel et al., 2017). The loss of jobs, landfills (LULUs), environmental racism, and health issues that followed the 2010 Gulf oil spill shows that a mass amount of social injustices have occurred because of the spill.

8 Nomenclature/Data Description

Function - It is a set of ordered pairs $[(x, y)]$, where the domain, or x, has a second element of y, represented by $f(x)$, usually by an equation. An example would be $f(x) = 3x$.

Derivative - The derivative of a function is the slope of a line that is tangent to the function at a specific point. Typically represented by dx next to a function.

Integral - An integral is a way to apply a function to a variety of different topics, such as finding an area, an infinite number of times. It looks like

$$\int_a^b f(x)\,dx$$

Here the $\int$ represents we are doing an integral. $f(x)$ represents the function that we are using. dx represents that we are going to have to use the derivative of the function for the integral. The a and b represent the function being continuous from these two points on the x-axis.

Riemann's Sum - A Riemann's sum is a sum of n products. It looks like this:

$$S_n = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + ... + f(x_n)\Delta x +$$

or

$$S_n = \sum_{i=1}^{n} f(x_i)\Delta x$$

Δx is an equation where $\Delta x = \frac{b-a}{n}$ where b and a are a sub interval. This is what will be used to determine the area of the spilled oil.

Interference of Light - Interference of light This is represented by the formula

$$V = \int_a^b f(x)(\Delta x)(t)$$

This, in combination with Riemann's sums, will determine the full amount of oil spilled and the area that the oil has spilled into.

Data Used: From Maps and GIS software provided by the Office of Response and Restoration, The largest width-wise area affected would be around 510 miles long, which is 820.765 km. We will round it to be 820 km. The largest length-wise area affected would be around 242 miles, which is about 389 km (Office of Response and Restoration). The thickness of oil is 89 nm thick. In terms of kilometers, it would be $8.9 * 10^{-11}$ km thick. With these figures, we will be able to use them in conjunction with Riemann's sums and the interference of light formulas.

9 Problem Formulation

The project our team was assigned was modeling the 2010 Gulf of Mexico oil spill. We will first provide background information on the topic at hand: the Gulf of Mexico oil spill. We will describe how the oil spill occurred, the cleaning strategies that were used on the spill, we will obtain data on the spill and speak upon its accuracy, as well as talk about the impact of the spill on the coastal region is affected.

To determine the impact of the oil spill, we have to use the thickness of the oil and Riemann's sums. We start with the Riemann's sums. This method will enable us to get the general area of the oil spill. To aid in this, we will code in python a program to enable us to be able to use N to a greater number than we can calculate feasibly by hand, i.e. 100 or 1000, to get as accurate as possible to the real area. With this number we can multiply it with the thickness of the oil, found using the interference of light formula, to calculate the volume of the oil spilled. Putting the thickness together with the Riemann sum and applying Leibniz's integral notation, we can provide a formula for the volume with

$$V = \int_a^b f(x)(\Delta x)(t)$$

where x is a point along with the interval $[a, b]$, $f(x)$ is the height of the rectangles, (Δx) is the width of the rectangles, and t is the thickness of the oil. Finding this integral will allow us to calculate the full extent of the oil spilled in the Gulf of Mexico.

10 Mathematical/Problem Formulation

From the GIS system provided by the National Oceanic and Atmospheric Administration, we were able to determine the farthest width to be around 820 km and the furthest height to be 389 km. Using this information, we can create a Riemann's sum by slicing up Figure 4 into increments of 41 for 20 different sections. With this information, we are able to determine the volume of surface-level oil that was spilled. Each section has its own height and would be determined by the work found in Figure 5 and 6.

(Note: These graphs are visualizations and not drawn to scale. All data was gathered from the GIS system)

Using data from the GIS system, we drew a baseline on the system to act as the x-axis, and a vertical line to act as the y-axis. With this information, we determined the surface area of the section by finding the Riemann's sum of the graphs in Figure 5 and 6 and then subtracting them to each other to determine the surface area. The midpoint of each section was use to get as accurate an answer. This can be seen in Figure 7.

After this Riemann's sum is complete, we can take the interference of light of oil to determine the thickness of oil and then determine the km^3 or the volume of the oil on the surface. From there we can convert the volume to barrels of oil to get our result. The work can be found in Figure 8.

The final answer to our question is 131,273.396 barrels of oil spilled on the surface.

11 Results And Discussion

With the math fully done, now we can explain what it means. Only getting the surface amount of oil, 131,273 barrels of oil, really puts into perspective just how much oil was spilled during this incident. 131,273 is such a small fraction of the 4 million barrels that BP managed to spill and yet it is still a massive amount of oil to spill in the ocean. A perspective like this can really let people know just how devastating this event was to the ocean life and people it affected. Such a massive amount of oil was spilled in the course of three months that the effects are still felt to this day. People's lives are still in ruin from their loss of jobs and the health effects suffered from the oil. The animal and plant life in the region was ravaged and required intense care and cleaning jobs that were still not sufficient enough. An event like this was horrible and hopefully an event like this will never happen again.

12 Code

The following code is a function that can calculate the Riemann sum of an array of points depending on a certain $[a, b]$ and n value. Since we had to deal with two functions from the NOAA GIS data, we take the Riemann sum of both and subtract the below one $g(x)$ from $f(x)$, $f(x) - g(x)$. In the code, this is represented as `TotalSum = sumFx - sumGx`. The arrays `points1` and `points2` can be inputted directed into the code or asked from the user with additional code.

```
def riemannSum(a, b, n, points):
    sum = 0
    deltaX =(b-a)/n

    for i in range(n):
        sum += points[i][1] * deltaX

    return sum

# points1 = []
# points2 = []
sumFx = riemannSum(0, 820, 20, points1)
sumGx = riemannSum(0, 820, 20, points2)

TotalSum = sumFx - sumGx

print(TotalSum)
```

Algorithm 1: Riemann Sum Code

The next algorithm allowed us to find the equation that best fit the data from Table 1 and Figure 3 and produce the plots in Figure 1. The array `df1` was an array that contained all the data from Table 1, and thus let us manipulate, analyze, and plot the data. Using numpy's extensive data analytic functions, we used `numpy.polyfit` to create polynomial equations of varying degrees. Taking the resulting equation, we then applied `numpy.polyval` to get the range of the polynomial based on a certain amount of x values. Based on Figure 2, we determined that Figure 3 was in fact a piecewise function therefore two equations were made: linear and cubic. `numpy.linspace` allowed us to take the intervals we determined the data changed on, $[1, 44]$ and $[45, 85]$, and create points between each x value. In this instance, `x_1_44` and `x_45_85` both used `numpy.linspace` to interpolate 100 points between each integer from 1 to 44 and from 45 to 85. This was then able to be applied to `numpy.polyval` to produce y values of `linearFormula_1_44` and `cubicFormula_45_85` with 100 points between each point as well. This produced a piecewise equation that fit the graph as best as we could:

$$f(x) = \begin{cases} 59639.03x - 37726.59 & 1 \leq x \leq 44 \\ 12.40x^3 - 2568.12x^2 + 208080.14x - 2686701.45 & x \geq 45 \end{cases}$$

The code is as follows:

```python
import numpy as np

x_1_44 = np.linspace(1,44,440)
linearFormula_1_44=np.polyfit(df1["Days"][0:44],df1["Total Oil"][0:44],1)
linear_y_1_44 = np.polyval(linearFormula_1_44, x_1_44)

x_45_85 = np.linspace(45,85, 410)
cubicFormula_45_85 = np.polyfit(df1["Days"][44:],df1["Total Oil"][44:],3)
cubic_y_45_85 = np.polyval(cubicFormula_45_85, x_45_85)
```

Algorithm 2: Best-Fit Equation

13 Feedback and Difficulties

Abbey, Alison: At the beginning of this project, I felt very overwhelmed. I was unclear on the instructions for the final project and was not yet comfortable speaking up to my group members. One of my biggest difficulties was communication. I had never used Latex before and never had read code before. It was a lot harder for me to understand some of the aspects of the project because I didn't fully understand the direction the project was going.

I found it interesting to research the event and the impacts it had over time. It was a helpful way to see numbers representing such a social issue and an economical issue. I feel I could have communicated my questions more when I was unclear. I found this project to be more difficult than beneficial to me.

Fisherman, Mylo: When attempting to put our project into LaTeX I ran into a few difficulties. The only background I have in writing LaTeX in Overleaf was a 1 credit course called Mathematics Orientation Seminar. I have not done anything Overleaf since that class which I took in the Fall of 2019 so I was a little rusty, to say the least.

I was fairly familiar with the basics going into starting this project. I was able to use a template that I was familiar with to start the project and use sections and subsections to order the project in a cohesive and organized way. Putting in the information that was just written words was also easy. It was when it came to putting in some of the math I ran into some trouble. I never learned in my course how to put summations, delta, integrals, tables, or pictures into LaTeX so, I was left to figure out how to do it. I learned from my tutors last year that if I can not figure out how to code something to Google it so that is what

I did. I was able to easily find the information I need in Google, however, it still took me some trial and error and fiddling around in the LaTeX document in order to figure things out. Additionally, towards the end, when most of our project was coded, I was trying to clear up the few error messages that were popping up. One error was for the pictures and table the document wanted me to use 'th!' instead of 'th!' as 'th!' was too strict of a command for the LaTeX document. Another error that took me a while to figure out was when I tried to code "g'(x)" I kept getting a right apostrophe error because I did not realize there was a code for a prime other than the apostrophe so I fixed it after I found that out. The rest of the errors were fairly easy to fix to the point at which I can not recall what they were.

Muller, Karl: Going into this research paper, I had some previous experience writing a research paper, examining data, and creating a poster for that research paper. Therefore, I was semi-confident going into this paper yet it ended up being a lot more intensive and time consuming than I expected. I learned python in my first semester, Fall of 2019, and had only fiddled slightly with it since then so I had to readjust to python and all of its libraries. The most difficult part was probably using matplotlib and numpy to analyze all the data from the oil spill. I had never used matplotlib and had only dabbled in numpy. Learning and dealing with all the constant errors I would get when trying to create a plot of a dataset wasn't always the easiest to fix or find out what was wrong. However, once I had spent a considerable amount of time editing two different plots, I was able to get the hang of it and make a lot more. I had never heard of or used Jupyter Notebook either but it turned out to be really helpful and interactive as I was able to learn it quickly. Applying what we learned in calculus to python and the plots turned out to be more fun and interesting than I thought, especially because the derivative graph made sense alongside its antiderivative. I didn't think I was going to see a good example of calculus in the real world like that.

There were definitely some errors, especially since I extrapolated data and tried to estimate lines of best fit. While these were based on observations of the trend of the data, you can never get a truly accurate answer. However, using python's numpy allowed me to get as close as I think I could. For example, even though the trend of the data showed a lowering of the change in oil between each day, the cubic line of best fit predicted a million more barrels would have spilt into the Gulf in 18 days, August 1st, when it had taken 30 days for the previous million to accumulate. The data itself also wasn't exact as they were estimates of how much was spilling each day. We don't know if the oil leaked from multiple locations under the water or there is leakage that is not accounted for. There can be unknown factors in any data set. When you record data, you can never get a full record of that data either. On day 1, our data says the equivalent of 5000 barrels of oil spilled from the oil rig into the Gulf. Was it exactly 5000.0000 barrels? Most likely no. Our data above and in many other studies are approximations, even when using the most advanced technology to measure something. There can always be an extra decimal place that is missed or isn't measured. 5.5 can be a lot different than 5 in a data set. Thus, errors are very common in research, no matter how small, and shows that real life data isn't always 100% reliable. So our study was also liable to those data errors. However, we were still able to analyze the data we had fairly well.

Muller, Kurt: This was an interesting project to work on. It definitely is the biggest one that I have worked on currently. Using previous work on research papers, the paper part was not too difficult. I was able to do most of it with basic research and learning a few new topics. The poster and presentation of the project also were interesting to work into the project to show others the work that we have done. I got the hang of everything that needed to be done fairly quickly and was able to get through most of the project without too much difficulty.

The difficulties that I had came about when creating and working on the calculus problem of the project. Our initial assumptions of how the data was going to be were false and did not support the Riemann's sum that we initially planned to do. Wanting to stay with Riemann's sums as it was a topic everyone was familiar with. As the deadline closed in, a panic ensued as we did not realize this until then. It was a scrambling trying to salvage a bit of work that was done already before realizing we had to go a different route. With the new way of working, we were able to find out the volume of oil spilled without having to redo much of the previous work. There is also the difficulty of some group members not communicating or doing big parts of the project. Essentially Karl, Mylo, and I had to do most of the leg work in the project.

Ritee, Zahin: This project was very helpful and interesting to work on. I never worked on this type of project before so it was a bit difficult for me. I was very worried about this project but with the help of my peer members, it seemed okay. I didn't know coding at all so I couldn't work at that. I also didn't know that there could be such a clear example of calculus in real life. This project really made my idea wrong.

There were errors and difficulties in doing this project. The calculus problems were hard and difficult but with the help of my peers it worked out pretty well. When creating the poster, we also weren't sure what to put on first. After talking with the professor, it was clear. This project can really help students to get clear ideas and examples of calculus in real life.

Zgombick, Jonah: If I'm being completely honest, I felt lost for almost the entirety of this project. Aside from the beginning, I couldn't get a sense of direction on what to do next. There was a definite lack of communication for the duration of this, with sometimes up to a week without anyone saying anything in the Slack channel. Sometimes I would ask what we had to next or what I could work on and I'd often get no response. This ended up with Kurt, Karl, and Mylo doing most of the work on the project. Constant updates are key in a group project like this, especially during COVID, and it felt like people just did things without saying them, making it hard to know what was and wasn't done yet.

However, I definitely could have communicated more myself, I should have followed up on my questions much more frequently than I did. I also should not have let my confusion/lack of direction on the project stop me from working on it. Overall I did not enjoy this project at all, though I'll admit much of that was on me since I didn't put as much time into this class as I did with my other ones.

14 Conclusion

The oil spill was a devastating event. Starting off small, there was an opportunity to end it right there. There was an insufficient amount of supplies and work put behind the effort resolving the matter because their estimation of the amount of oil spilled was way off from the actual number. Panic ensued as the problem spiraled out of control, taking almost three months of work to get the spill under control. By that time, it was far too late and the damage had been done. Permanent damage was dealt to the ocean life and people who relied on the ocean for their living. Animal's habitats were destroyed and the fishing industry died in the area. Many lost their jobs and their homes. Many different forms of life felt the impact from the spill. Such a massive amount of oil was spilled, totaling to 4 million barrels worth of oil. An event like this should never have happened in the first place. Preventative measures and proper guidelines for handling a situation like this have been drawn up in the case that an event like this should occur again. Hopefully proper measures are taken to prevent an oil spill of this magnitude from ever happening again.

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