Quick Reference for Fairness Measures

Fairness Metrics

Metrics by Category

Category	Metric	Definition	Weakness	References
Group Fairness	**Demographic Parity**	A model has **Demographic Parity** if the predicted positive rates (selection rates) are approximately the same for all protected attribute groups. $$\frac{P(\hat{y}=1 \vert unprivileged)}{P(\hat{y}=1 \vert privileged)}$$	Historical biases present in the data are not addressed and may still bias the model.	Zafar *et al* (2017)
	Equalized Odds	Odds are equalized if $P(+)$ is approximately the same for all protected attribute groups. **Equal Opportunity** is a special case of equalized odds specifying that $$P(+\vert y=1)$$ is approximately the same across groups.	Historical biases present in the data are not addressed and may still bias the model.	Hardt *et al* (2016)
	Predictive Parity	This parity exists where the Positive Predictive Value and Negative Predictive Value are each approximately the same for all protected attribute groups.	Historical biases present in the data are not addressed and may still bias the model.	Zafar *et al* (2017)
Similarity-Based Measures	**Individual Fairness**	Individual fairness exists if "similar" individuals (ignoring the protected attribute) are likely to have similar predictions.	The appropriate metric for similarity may be ambiguous.	Dwork (2012), Zemel (2013), Kim *et al* (2018)
	Unawareness	A model is unaware if the protected attribute is not used.	Removal of a protected attribute may be ineffectual due to the presence of proxy features highly correlated with the protected attribute.	Zemel *et al* (2013), Barocas and Selbst (2016)
Causal Reasoning	**Counterfactual Fairness** *	Counterfactual fairness exists where counterfactual replacement of the protected attribute does not significantly alter predictive performance. This counterfactual change must be propogated to correlated variables.	It may be intractable to develop a counterfactual model for some problems.	Russell *et al* (2017)

Statistical Definitions of Group Fairness

Metric	Statistical Criteria	Definition	Description
Demographic Parity	Statistical Independence	$R \perp\!\!\!\perp G$	sensitive attributes (A) are statistically independent of the prediction result (R)

Metric	Statistical Criteria	Definition	Description
Equalized Odds	Statistical Separation	$R \perp\!\!\!\perp A \mid Y$	sensitive attributes (A) are statistically independent of the prediction result (R) given the ground truth (Y)
Predictive Parity	Statistical Sufficiency	$Y \perp\!\!\!\perp A \mid R$	sensitive attributes (A) are statistically independent of the ground truth (Y) given the prediction (R)

From: Verma & Rubin, 2018

Fairness Measures

Name	Definition	Description
Demographic Parity	$P(\hat{y} \mid G = u) = P(\hat{y} \mid G = p)$	Predictions must be st independent from the Subjects in all groups probability of being as positive class.
Conditional Statistical Parity	$P(\hat{y} = 1 \mid L = l, G = u) = P(\hat{y} = 1 \mid L = l, G = p)$	Subjects in all groups probability of being as positive class conditio factors (L)
False positive error rate (FPR) balance	$P(\hat{y} = 1 \mid Y = 0, G = u) = P(\hat{y} = 1 \mid Y = 0, G = p)$	Equal probabilities for negative class to have predictions. Mathematically equiva P(d=0\lvert{Y=0,G=m} =0,G=f})
False negative error rate (FNR) balance	$P(\hat{y} = 0 \mid Y = 1, G = u) = P(\hat{y} = 0 \mid Y = 1, G = p)$	Equal probabilities for positive class to have predictions. Mathematically equiva $P(d = 1 \mid Y = 1, G = m) =$.
Equalized Odds	$P(\hat{y} = 1 \mid Y = c, G = u) = P(\hat{y} = 1 \mid Y = c, G = p), c \in 0, 1$.	Equal TPR and equal F
Predictive Parity	$P(Y = 1 \mid \hat{y} = 1, G = u) = P(Y = 1 \mid \hat{y} = 1, G = p)$	All groups have equal that a subject with a p actually belongs to the Mathematically equiva Discovery Rate (FDR): $P(Y = 0 \mid d = 1, G = m) =$
Conditional use accuracy equality	$(P(Y = 1 \mid \hat{y} = 1, G = u) = P(Y = 1 \mid \hat{y} = 1, G = p)) \wedge (P(Y = 0 \mid \hat{y} = 0, G = u) = P(Y = 0 \mid \hat{y} = 0, G = p))$	
Overall Accuracy Equity	$P(\hat{y} = Y, G = m) = P(\hat{y} = Y, G = p)$	

Name	Definition	Description
Treatment Equality	$FNu/FPu = FNp/FPp$	Groups have equal rati Negative Rates to Fals
Calibration	$P(Y = 1 \mid S = s, G = u) = P(Y = 1 \mid S = s, G = p)$	For a predicted probab groups should have eq belonging to the positi
Well-calibration	$P(Y = 1 \mid S = s, G = u) = P(Y = 1 \mid S = s, G = p) = s$	For a predicted probab groups should have eq belonging to the positi probability is equal to
Balance for positive class	$E(S \mid Y = 1, G = u) = E(S \mid Y = 1, G = p)$	Subjects in the positiv groups have equal ave probability score S
Balance for negative class	$E(S \mid Y = 0, G = u) = E(S \mid Y = 0, G = p)$	Subjects in the negativ groups have equal ave probability score S
Causal discrimination	$(X_p = X_u \wedge G_p != G_u) \rightarrow \hat{y}_u = \hat{y}_p$	Same classification pr subjects with the exac
Fairness through unawareness	$X_i = X_j \rightarrow \hat{y}_i = \hat{y}_j$	No sensitive attributes in the decision-making
Fairness through awareness (Individual Fairness)	for a set of applicants V , a distance metric between applicants k : V Å~V → R, a mapping from a set of applicants to probability distributions over outcomes M : V → δA, and a distance D metric between distribution of outputs, fairness is achieved iff $D(M(x), M(y)) \leq k(x, y)$.	Similar individuals (as distance metric) shoul classification
Counterfactual fairness	A causal graph is counterfactually fair if the predicted outcome d in the graph does not depend on a descendant of the protected attribute G.	

Interpretations of Common Measures

Group Measure Type	Examples	"Fair" Range
Statistical Ratio	Disparate Impact Ratio, Equalized Odds Ratio	0.8 <= "Fair" <= 1.2
Statistical Difference	Equalized Odds Difference, Predictive Parity Difference	-0.1 <= "Fair" <= 0.1

Metric	Measure	Equation	Interpretation
	Selection Rate	$\sum_{i=0}^{N}(\hat{y}_i)/N$	-
Group Fairness Measures	Demographic (Statistical) Parity Difference	$P(\hat{y} = 1 \mid unprivileged) - P(\hat{y} = 1 \mid privileged)$	(-) favors privileged group (+) favors unprivileged group

Metric	Measure	Equation	Interpretation
	Disparate Impact Ratio (Demographic Parity Ratio)	$\frac{P(\hat{y}=1 \mid unprivileged)}{P(\hat{y}=1 \mid privileged)} = \frac{selection\_rate(\hat{y}_{unprivileged})}{selection\_rate(\hat{y}_{privileged})}$	< 1 favors privileged group > 1 favors unprivileged group
	Positive Rate Difference	$precision(\hat{y}_{unprivileged}) - precision(\hat{y}_{unprivileged})$	(-) favors privileged group (+) favors unprivileged group
	Average Odds Difference	$\frac{(FPR_{unprivileged} - FPR_{privileged}) + (TPR_{unprivileged} - TPR_{privileged})}{2}$	(-) favors privileged group (+) favors unprivileged group
	Average Odds Error	$\frac{\lvert FPR_{unprivileged} - FPR_{privileged}\rvert + \lvert TPR_{unprivileged} - TPR_{privileged}\rvert}{2}$	(-) favors privileged group (+) favors unprivileged group
	Equal Opportunity Difference	$recall(\hat{y}_{unprivileged}) - recall(\hat{y}_{privileged})$	(-) favors privileged group (+) favors unprivileged group
	Equalized Odds Difference	$max((FPR_{unprivileged} - FPR_{privileged}), (TPR_{unprivileged} - TPR_{privileged}))$	(-) favors privileged group (+) favors unprivileged group
	Equalized Odds Ratio	$min(\frac{FPR_{smaller}}{FPR_{larger}}, \frac{TPR_{smaller}}{TPR_{larger}})$	< 1 favors privileged group > 1 favors unprivileged group
Individual Fairness Measures	Consistency Score	$1 - \frac{1}{n \cdot N_{n_neighbors}} * \sum_{i=1}^{n} \lvert\hat{y}_i - \sum_{j \in \mathrm{N}_{neighbors}(x_i)} \hat{y}_j\rvert$	1 is consistent 0 is inconsistent
	Generalized Entropy Index	$GE = \mathrm{E}(\alpha) = \begin{cases} \frac{1}{n\alpha(\alpha-1)} \sum_{i=1}^{n} [(\frac{b_i}{\mu})^{\alpha} - 1], & \alpha \neq 0, 1 \\ \frac{1}{n} \sum_{i=1}^{n} \frac{b_i}{\mu} \ln \frac{b_i}{\mu}, & \alpha = 1 \\ -\frac{1}{n} \sum_{i=1}^{n} \ln \frac{b_i}{\mu}, & \alpha = 0 \end{cases}$	-
	Generalized Entropy Error	$GE(\hat{y}_i - y_i + 1)$	-
	Between-Group Generalized Entropy Error	$GE([N_{unprivileged} * mean(Error_{unprivileged}), N_{privileged} * mean(Error_{privileged})])$	0 is fair (+) is unfair

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