Digital Signal Analysis and Processing

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Introduction to DSP

DT Signal and Systems

Basic Operations on Signals

Basic Signal Models

System Properties and LTI Systems

Convolution and Properties

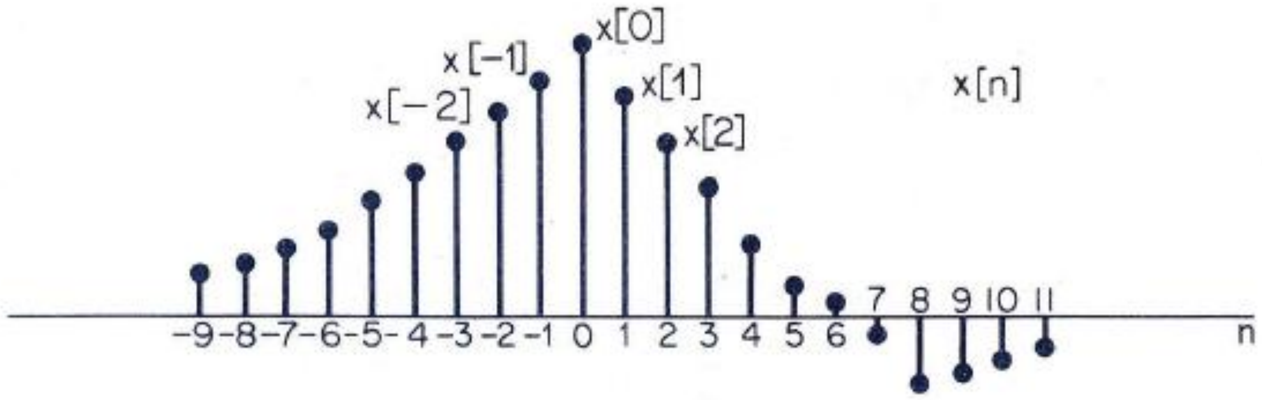
Difference Equations

DT Fourier Series and Transform

DT Signals

- Discrete Time (DT) Signals:

  - Independent variable (time) is discrete
  - Signal is defined only for discrete values of time
  - Time instants need not be equidistant but usually are considered to be for computational simplicity.
  - Represented by *x[n]* or *x[nT]*



DT Signal Representation

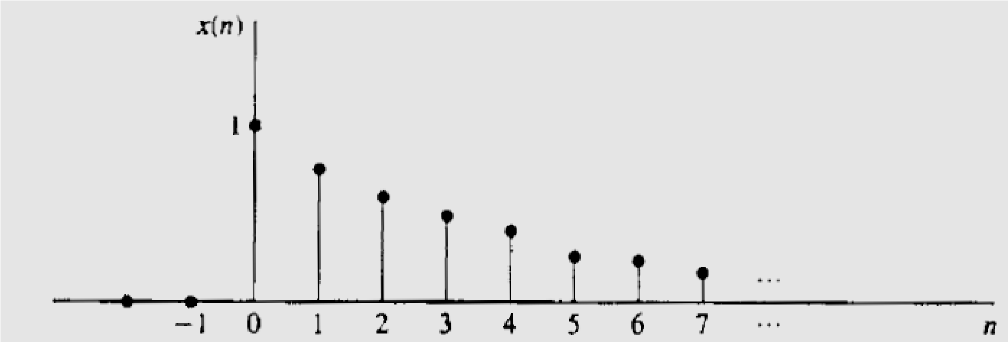
- Sequential representation:

  - by a sequence of numbers
  - e.g., $x[n] = \{1, 0.8, 0.64, 0.512, \ldots\ldots\ldots\}$

- Functional representation

  - by a function of n
  - e.g., $$x[n] = \begin{cases} 0.8^n & \text{for} \quad n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- Graphical representation



DT Signal Origin

- DT signals may arise in two ways:

  - Sampling CT signals

    - taking values of CT signal at equal intervals.
    - e.g., $x[n]$ shown earlier can be obtained by sampling

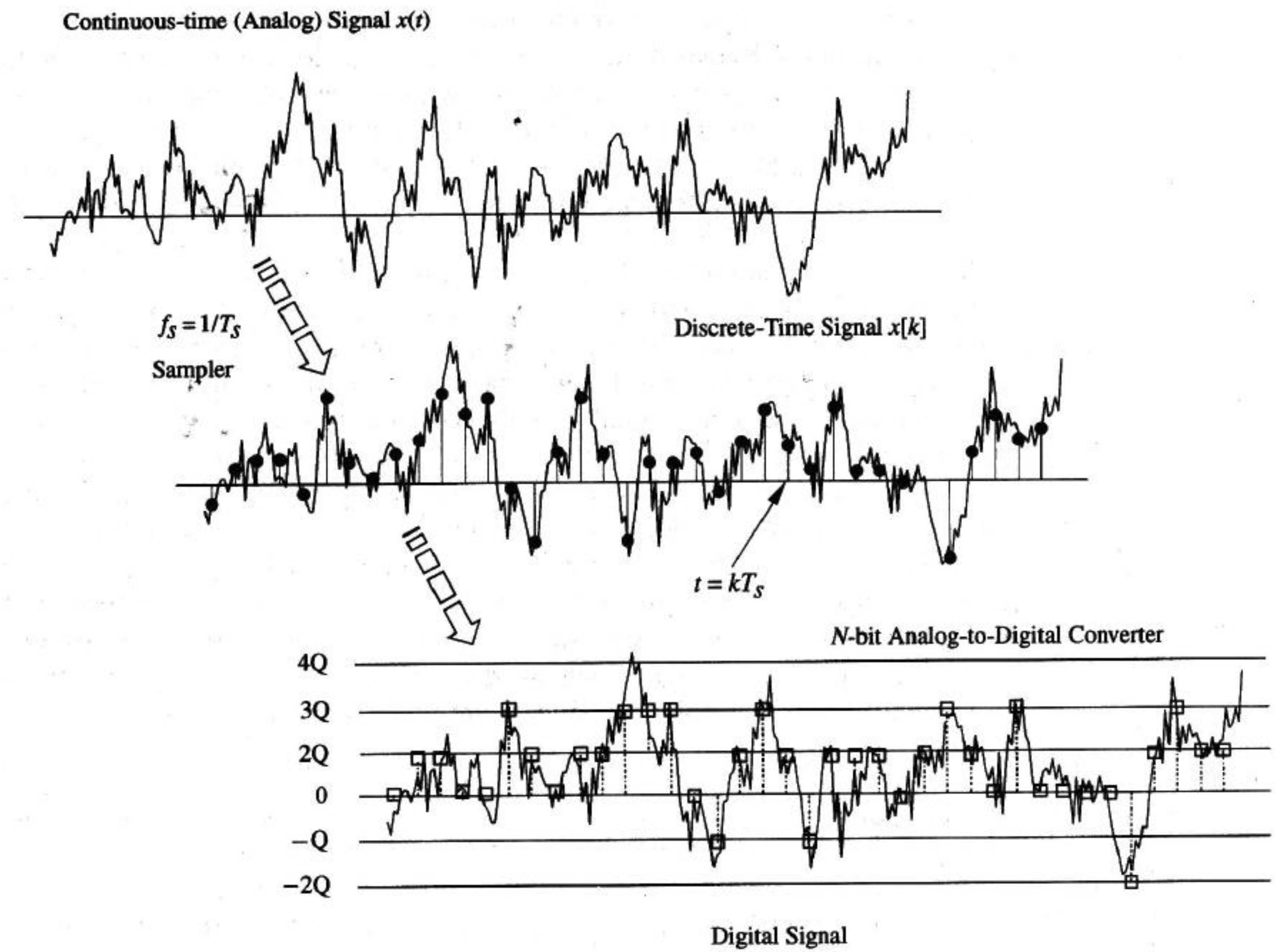
    $$x(t) = \begin{cases} 0.8^t & for \;\; t \geq 0 \\ 0 & otherwise \end{cases}$$ every one second

    - measurement of temperature of a room every one hour

  - Accumulating variables over a time

    - these signals are inherently discrete
    - e.g. value of gold every day, amount of rainfall every day

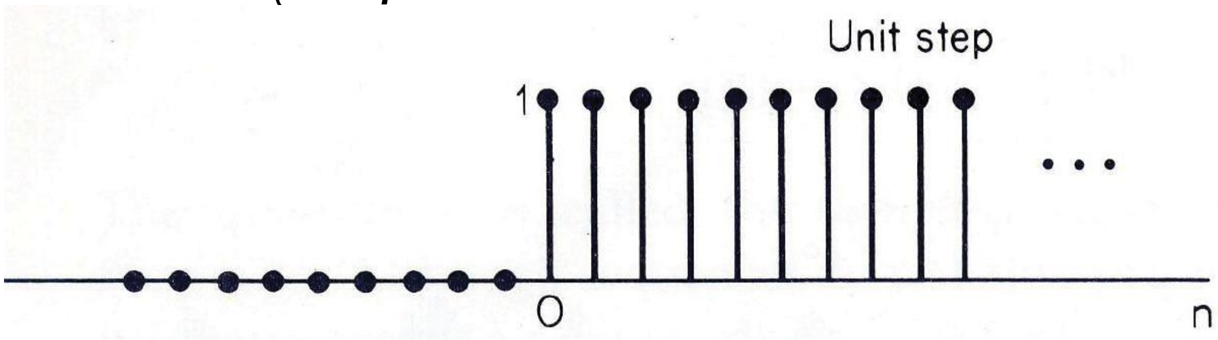
Analog/CT/DT /Digital



DT Unit Step Signal

- DT unit step is defined as:

$$u[n] = \begin{cases} 1 & for \quad n \geq 0 \\ 0 & for \quad n < 0 \end{cases}$$



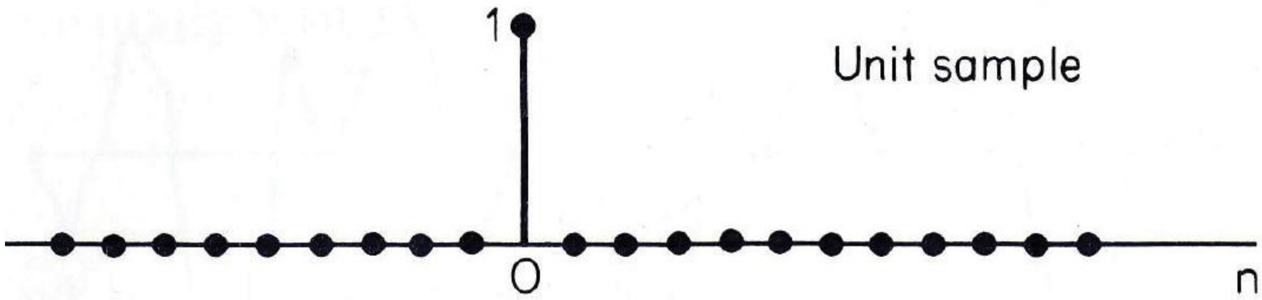
- used to model phenomenon that change value in steps.
- also used to model causal signals

$$Ku[n] = \begin{cases} K & for \quad n \geq 0 \\ 0 & for \quad n < 0 \end{cases}$$

DT Unit Impulse Signal

- DT unit impulse (also unit sample) signal is defined as:

$$\delta[n] = \begin{cases} 1 & for \quad n = 0 \\ 0 & for \quad n \neq 0 \end{cases}$$



- used to model phenomenon that appear only for short time
- also used to model sampling of signals

$$K\delta[n] = \begin{cases} K & for \quad n = 0 \\ 0 & for \quad n \neq 0 \end{cases}$$

- Any DT signal may be expressed in terms of impulses:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

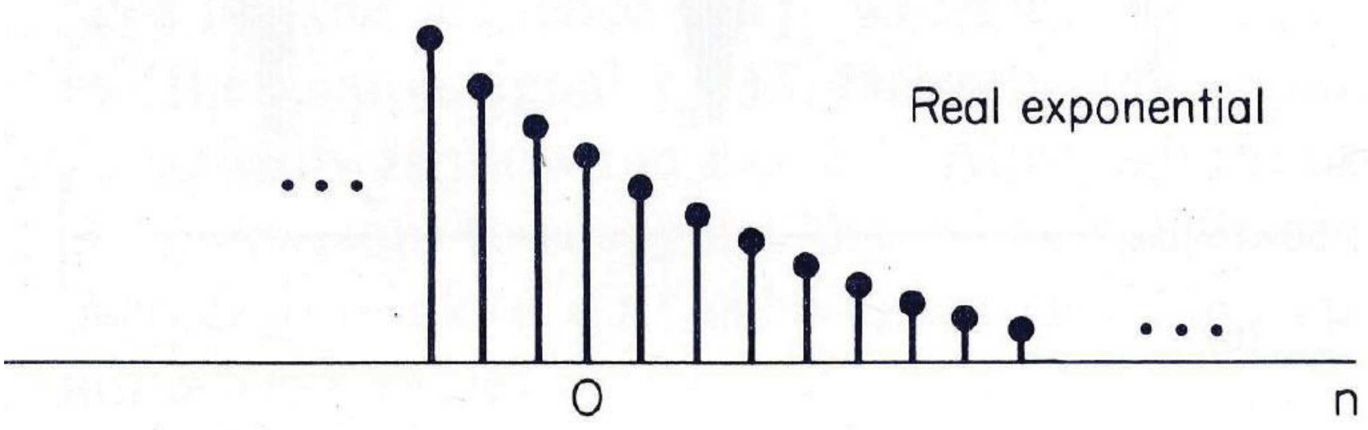
DT Exponential Signal

- DT exponential signal is defined as:

$$x[n] = A\alpha^n$$

- may be decaying or growing
(growing if $|\alpha| > 1$ while decaying if $|\alpha| < 1$)
(sign of values alternates each sample if α is negative)

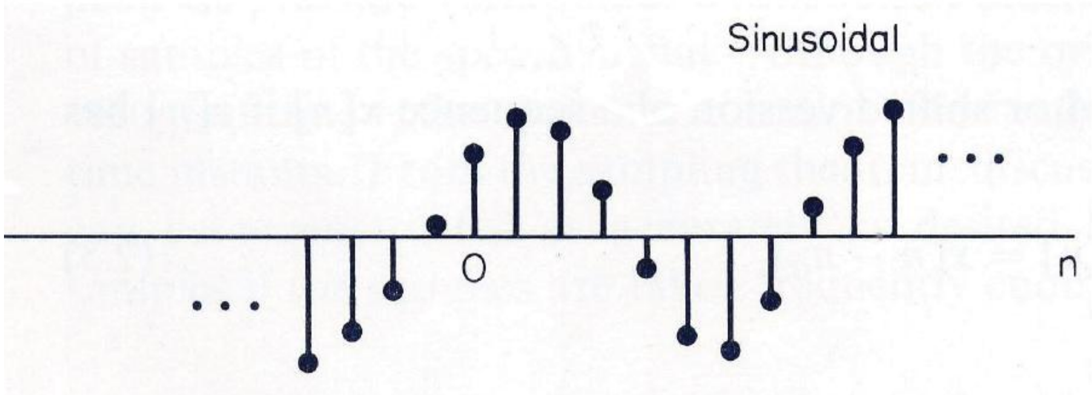
- may be real or complex
(real only when both A and α are real)



Sinusoidal/Complex Exponential

- DT sinusoidal signal is defined as:

$$x[n] = A\cos(\omega_0 n + \phi)$$



- Complex exponential has the form: $x[n] = A\alpha^n$

$$\alpha = |\alpha| e^{j\omega_\circ} \qquad A = |A| e^{j\phi}$$

$$x[n] = |A||\alpha|^n e^{j(\omega_\circ n + \phi)}$$

$$x[n] = |A||\alpha|\cos(\omega_0 n + \phi) + j|A||\alpha|\sin(\omega_0 n + \phi)$$

DT Complex Exponential Signal

- Particularly interesting case of DT complex exponential signal is:

$$x[n] = e^{j\omega_0 n}$$
$$= \cos(\omega_0 n) + j\sin(\omega_0 n)$$

- It seems to be periodic
- However, it is periodic only in particular condition.

$$x[n+N] = e^{j\omega_0 (n+N)} = e^{j\omega_0 n} e^{j\omega_0 N}$$

- So $x[n] = e^{j\omega_0 n}$ is periodic only when $e^{j\omega_0 N} = 1$

- This happens only when

$$\omega_0 N = k2\pi, \ where \ k \ is \ an \, \text{int}\, eger$$

$$or, \omega_0 = k\frac{2\pi}{N}$$

Frequency In DT Signals

- For a DT complex exponential signal $x[n] = e^{j\omega_0 n}$

$$e^{j(2\pi+\omega_0)n} = e^{j\omega_0 n}e^{j2\pi n} = e^{j\omega_0 n}$$

(Since n is always an integer)

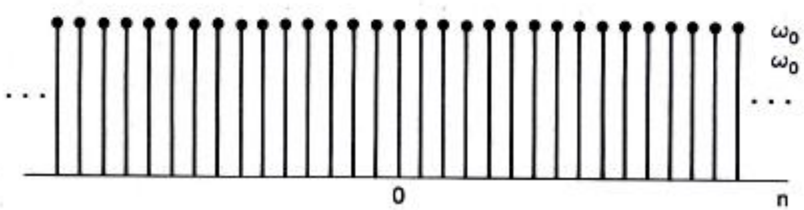
-This means, DT signals with frequencies differing by 2π are same

-Thus, in DT signals, unique range of frequencies is only 2π (either 0 to 2π or – π to π)

-Frequency value of π corresponds to highest frequency while 0 or π corresponds to lowest frequency.

- This is in contrast to CT signals where a signal is unique for all unique value of frequency and the possible range of frequency is -∞ to ∞.

Frequency in DT Signals



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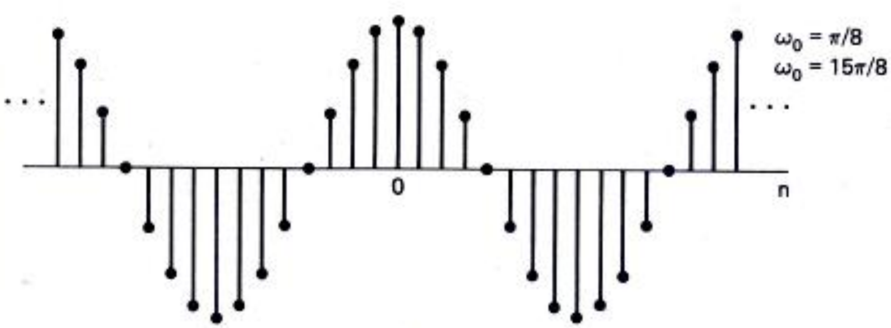

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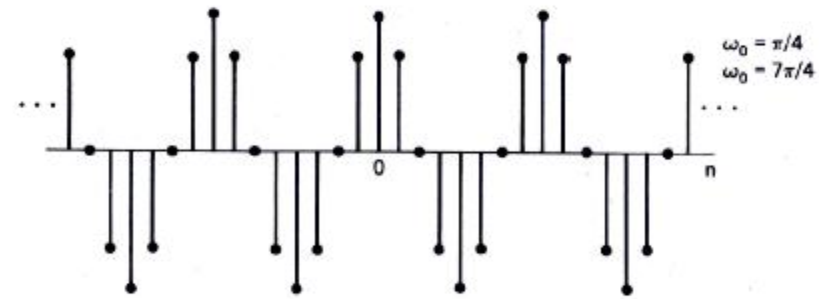

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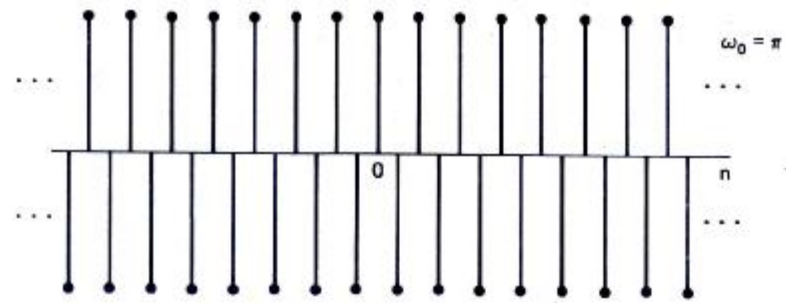


(d)

Periodic DT Signals

- Periodic DT signals repeat themselves after every fixed samples.

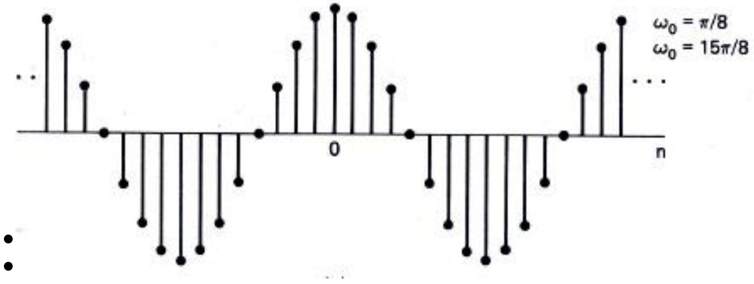
- Mathematically, for a periodic signal $x[n]$,

$$x[n+N] = x[n] \qquad\qquad for\ \ all\ \ \ n$$

where, smallest value of N for which above equation is true is called fundamental period.

- Fundamental frequency then is:

$$f = \frac{1}{N}$$



- Fundamental radian frequency is:

$$\omega = \frac{2\pi}{N}$$

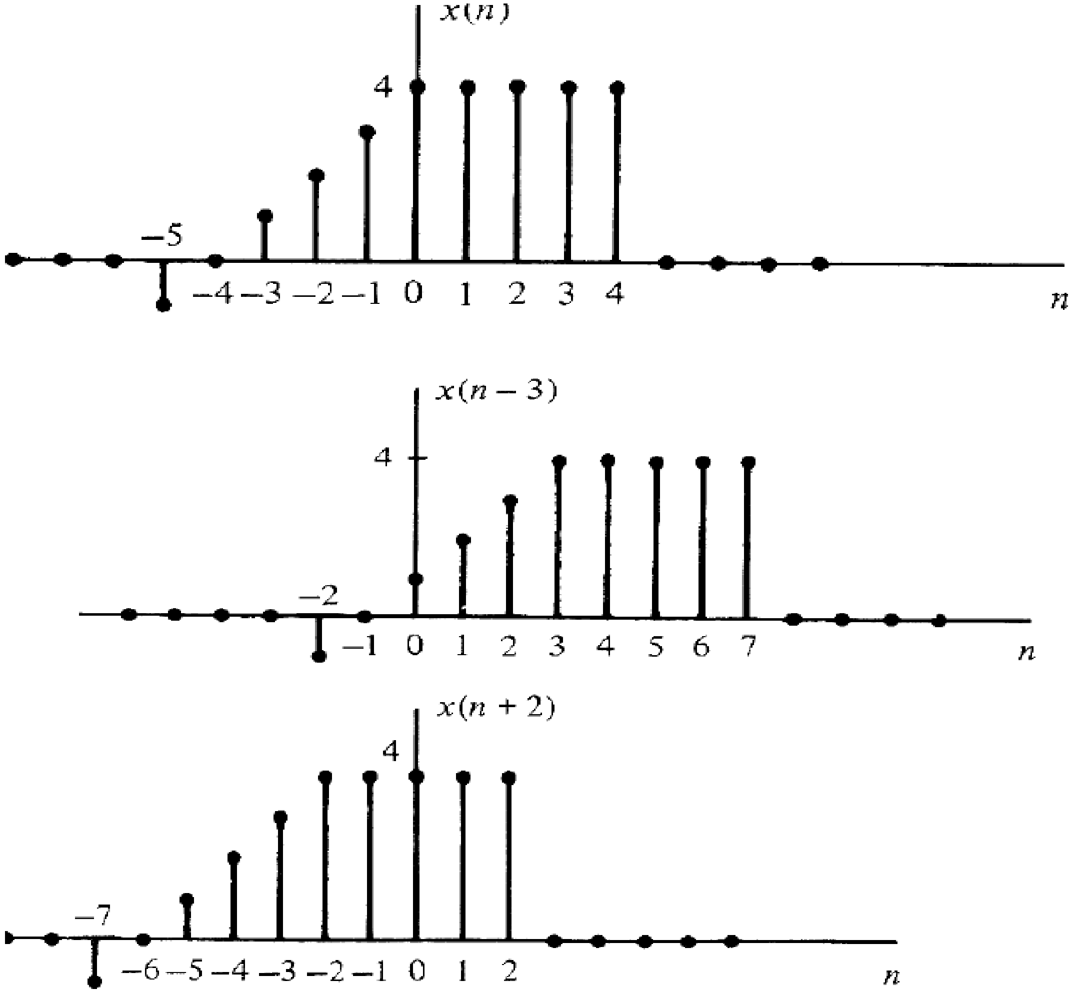
Power and Energy Signals

For a DT signal $x[n]$,

- Instantaneous power is: $P = |x[n]|^2$
- Total signal energy then is: $E_T = \sum_{n=-\infty}^{\infty} |x[n]|^2$
- Average power is: $P_{avg} = \frac{1}{N} \sum_{n=-N/2}^{N/2} |x[n]|^2$
- Any DT signal that has finite total energy is called *energy signal.* Such signals have zero average power. Aperiodic finite duration signals are energy signals.
- Any DT signal that has finite average power is called *power signal.* Such signals have infinite total energy. Periodic signals are power signals.
- Some signals are *neither energy nor power signals*.

Time Shifting

- Shifting the signal on time axis.
- Replacing *n* by *n-k* where *k* is shifting constant.

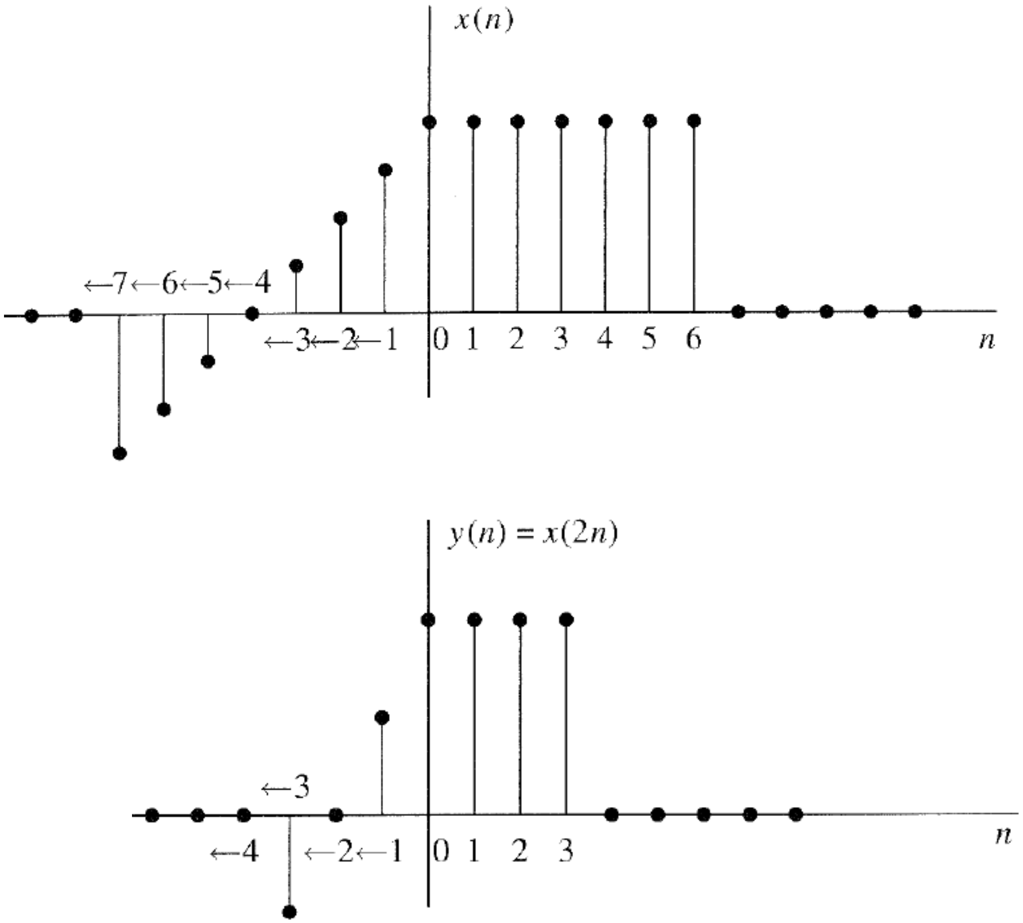


If k is positive, shifted to right. So, signal is delayed

If k is negative, shifted to left. So, signal is advanced

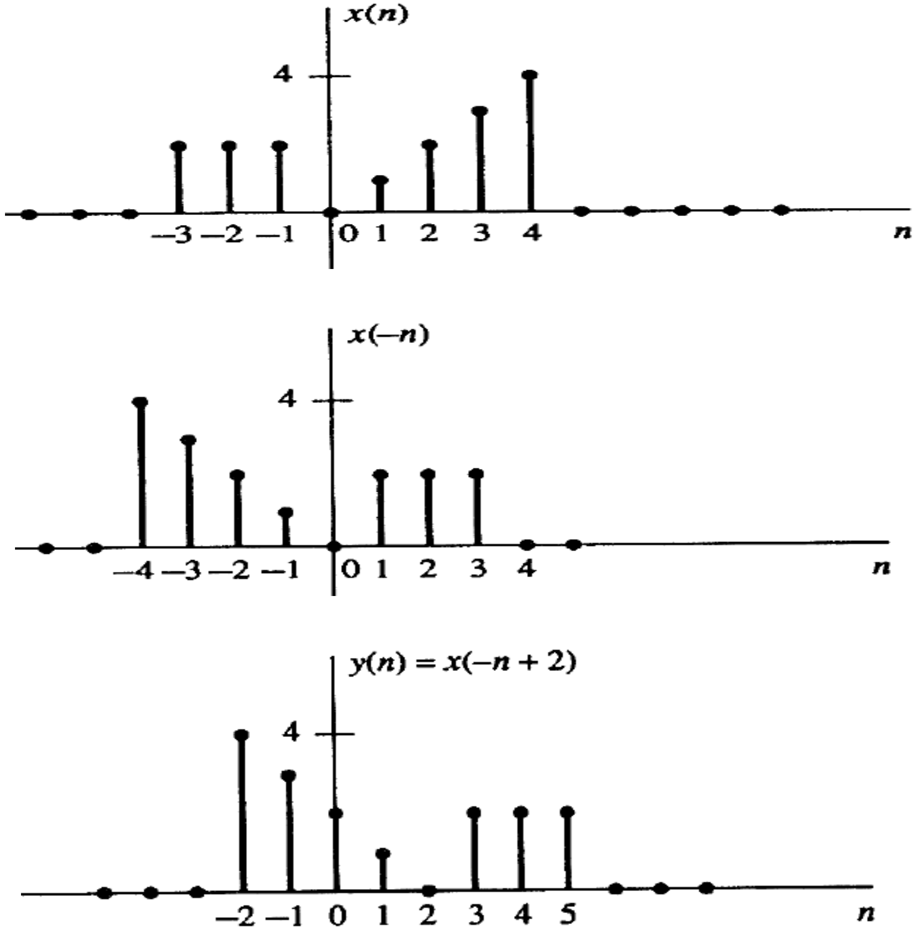
Time Scaling

- Changing the duration of signal. Compression
- Replacing *n* by *kn*. *k* is integer.



Time Inversion

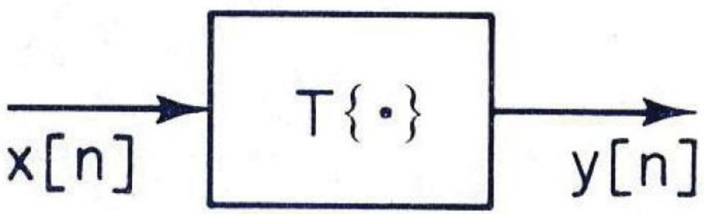
- Folding or reflection signal about time origin.
- Replacing *n* by –*n*.



DT Systems

- A DT system is defined as a transformation or operator that maps an input sequence with values $x[n]$ into an output sequence with values $y[n]$.

$$y[n] = T\{x[n]\}$$



- e.g., $y[n] = 2x[n]$ *Amplifier*

$$y[n] = \sum_{k=-\infty}^{n} x[k] \quad \textit{Accumulator}$$

$$y[n] = \frac{1}{2}\{x[n] + x[n-1]\} \quad \textit{moving average system}$$

Memoryless Systems

- A DT system is called a memoryless system if, the output $y[n]$ at every value of n depends only on the input $x[n]$ at the same value of n.

- So, output depends on present input values but not on past and future input values.

- Also called static systems

e.g., $$y[n] = 2x[n]$$

Causality

- A DT system is called a causal system if, the output $y[n]$ at any value of n_0 depends only on the input $x[n]$ at the values of $n<n_0$.

- So, output depends on present and past input values but not on future input values.

- Thus is nonanticipative.

  e.g.,

$$y[n] = 2x[n] + 5x^2[n-1]$$

Linearity

- A DT system is called a linear system if, it follows the properties of homogeneity and superposition.

- For a linear system, if

$$x_1[n] \rightarrow y_1[n]$$

$$and \;\; x_2[n] \rightarrow y_2[n]$$

then, for constants A and B,

$$x_3[n] = A\,x_1[n] + B\,x_2[n] \rightarrow y_3[n] = A\,y_1[n] + B\,y_2[n]$$

Check for linearity:

Time Invariance

- If the properties of the system does not change with time, the system is called time invariant system.

- Specifically, a system is called a time invariant system if, a time shift or delay of the input causes a corresponding shift in the output.

- Mathematically, for a time invariant system,

$$\text{if,}\quad x[n] \rightarrow y[n]$$

$$\text{then,}\quad x[n-n_0] \rightarrow y[n-n_0]$$

Check: $y[n] = x[Mn]$ (time variant)

$y[n] = x[n] - x[n-1]$ (time invariant)

Stability

- A system is stable in the bounded-input bounded-output (BIBO) sense if and only if every bounded input sequence produces a bounded output sequence.

- Mathematically, for a stable system,

  if, $x[n] \rightarrow y[n]$

  then for all $|x[n]| \leq B_x < \infty,$ for all n

  $|y[n]| \leq B_y < \infty$ for all n

- Check:

Linear Time Invariant Systems

- Any system which follows both linearity and time invariance.

- Many practical systems are LTI systems

- LTI systems are represented by impulse responses and difference equations.

LTI Systems & Impulse Response

- Impulse response of a system is the output of the system when the input is an unit impulse response.

- Usually represented as $h[n]$.

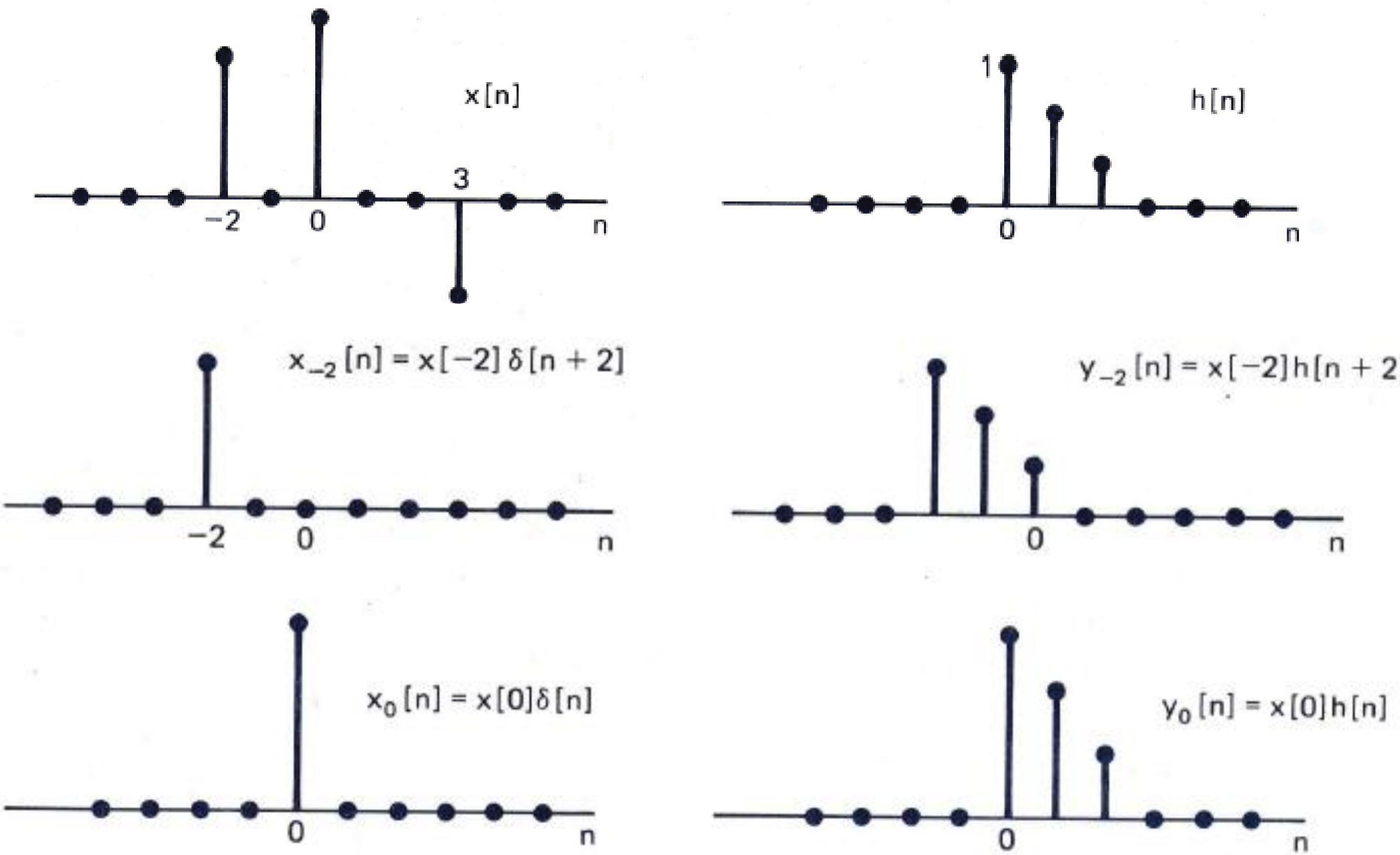
- For any LTI system with input $x[n]$ and impulse response $h[n]$, the output signal $y[n]$ is given by:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

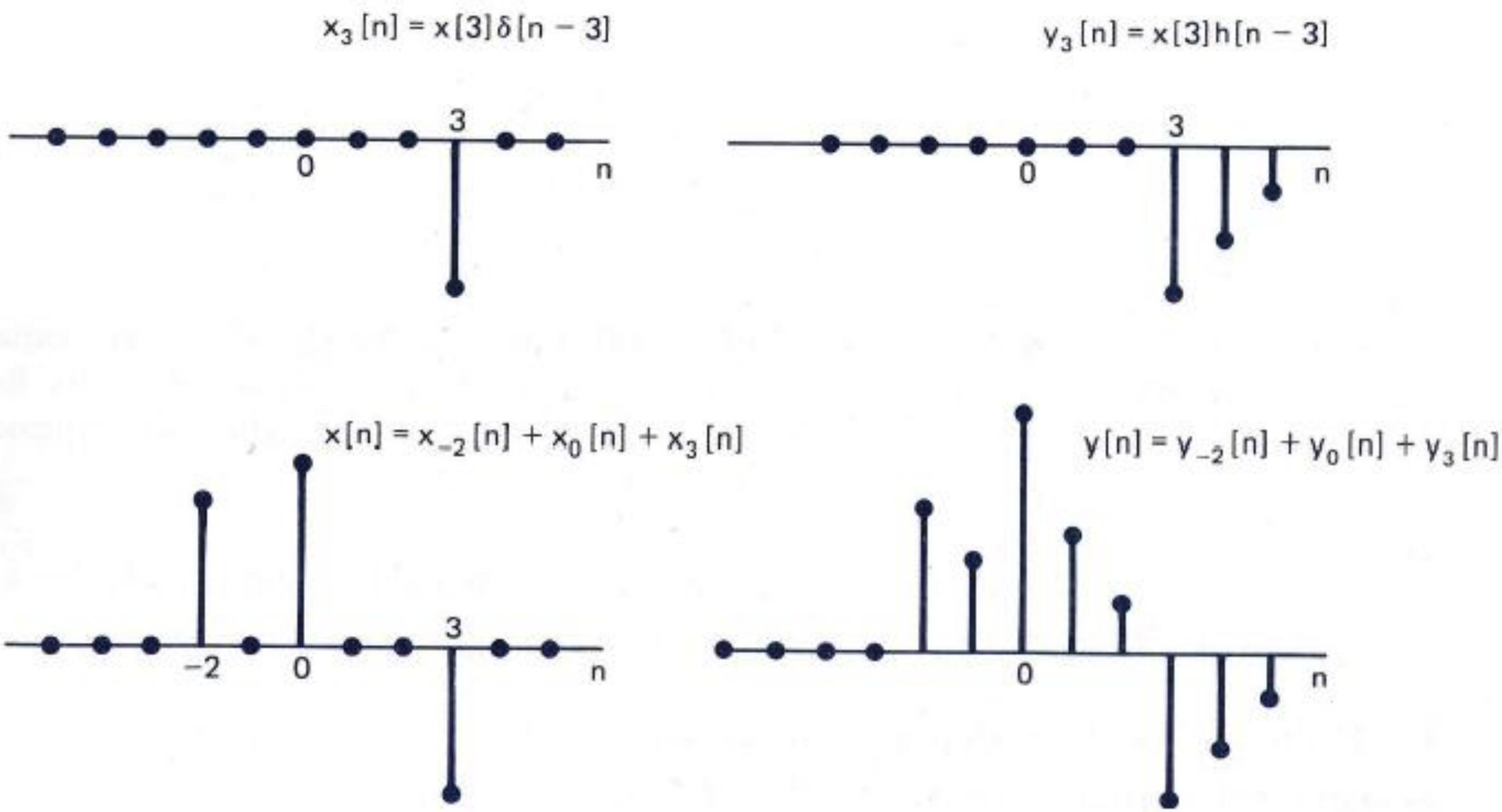
Convolution Sum

$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Convolution: Understanding



Convolution

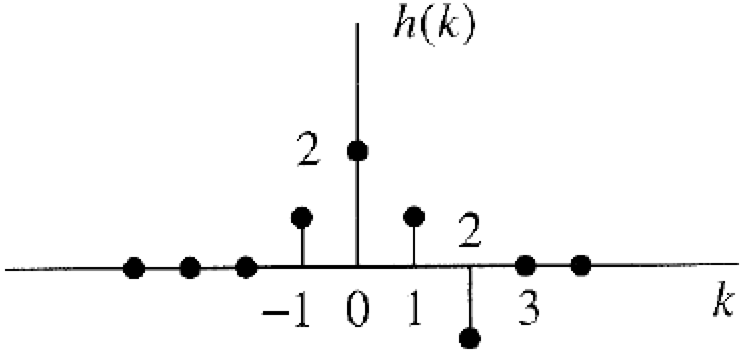


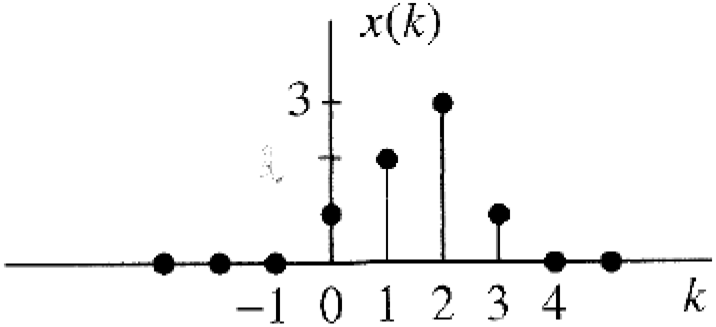
Fold and Shift Interpretation: Graphical Convolution

1. *Folding*. Fold $h(k)$ about $k = 0$ to obtain $h(-k)$.
2. *Shifting*. Shift $h(-k)$ by n_0 to the right (left) if n_0 is positive (negative), to obtain $h(n_0 - k)$.
3. *Multiplication*. Multiply $x(k)$ by $h(n_0 - k)$ to obtain the product sequence $v_{n_0}(k) \equiv x(k)h(n_0 - k)$.
4. *Summation*. Sum all the values of the product sequence $v_{n_0}(k)$ to obtain the value of the output at time $n = n_0$.

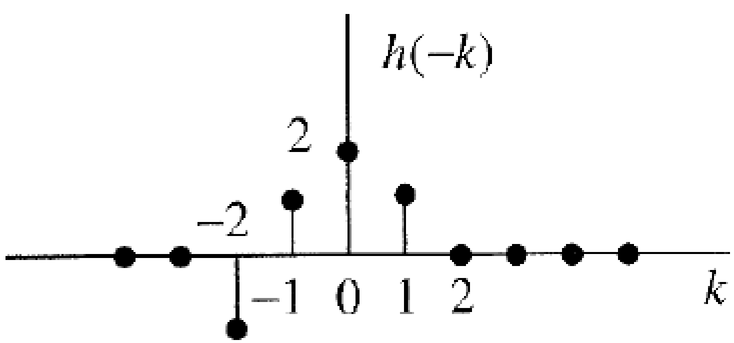
Fold and Shift Interpretation: Graphical Convolution

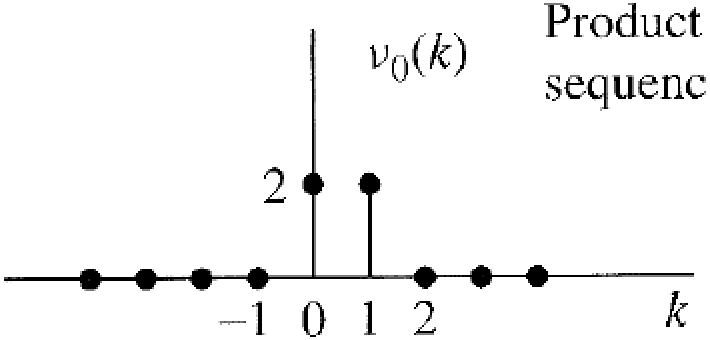
$$h(n) = \{1, \underset{\uparrow}{2}, 1, -1\} \qquad x(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$$





$$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k) \qquad v_0(k) \equiv x(k)h(-k)$$



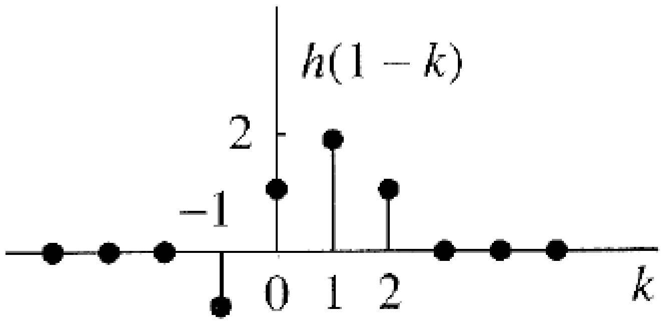


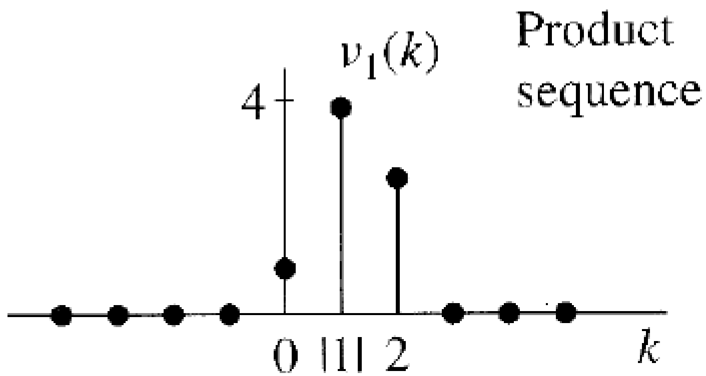
Product sequence

$$y[0] = 2 + 2 = 4$$

Fold and Shift Interpretation: Graphical Convolution

$$y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k) \qquad v_1(k) = x(k)h(1-k)$$





$$y[1] = 1 + 4 + 3 = 8$$

Similarly we can compute:

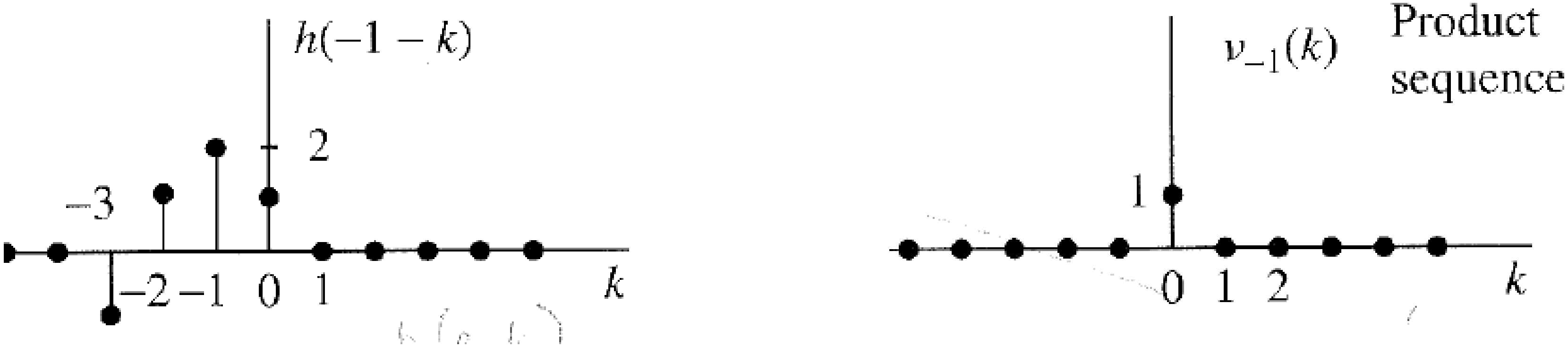
$$y[2] = 8 \qquad y[3] = 3 \qquad y[4] = -2 \qquad y[5] = -1$$

$$y[n] = 0 \quad for \quad n > 5$$

Fold and Shift Interpretation: Graphical Convolution

What about negative values of n?

$$y(-1) = \sum_{k=-\infty}^{\infty} x(k)h(-1-k)$$



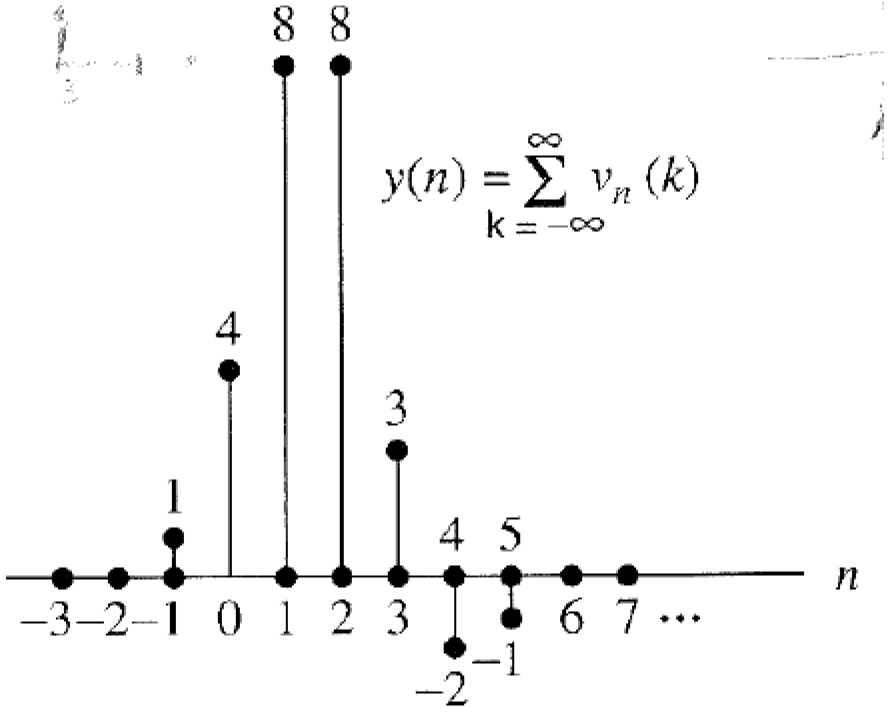
$$y[-1] = 1$$

Similarly we can compute: $y[-2] = 0$ $\quad y[-3] = 0$ $\quad y[-4] = 0$

$$y[n] = 0 \quad \mathit{for} \quad n < -1$$

Fold and Shift Interpretation: Graphical Convolution

Finally,



$$y(n) = \{\ldots, 0, 0, 1, \underset{\uparrow}{4}, 8, 8, 3, -2, -1, 0, 0, \ldots\}$$

Properties of LTI System

- Convolution operation follows several properties which impose some important characteristics to the LTI systems.

-

Commutative Property

$$\boxed{x[n]*h[n]=h[n]*x[n]}$$

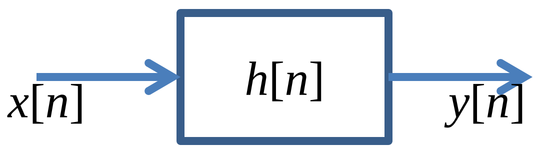
- Proof:

$$x[n]*h[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]$$

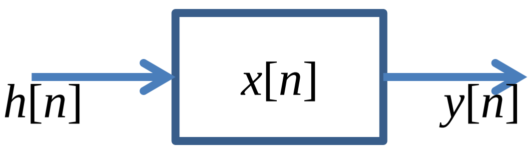
Let n-k=p

$$=\sum_{p=\infty}^{-\infty}x[n-p]h[p]=\sum_{p=-\infty}^{\infty}h[p]x[n-p]$$

$$=h[n]*x[n]$$



Is same to



Interchanging the role of input and impulse response does not change the output of the system.

Distributive over Addition

$$\boxed{x[n]*\{h_1[n]+h_2[n]\}=\{x[n]*h_1[n]\}+\{x[n]*h_2[n]\}}$$

- Proof: Let, $h_1[n]+h_2[n]=h_3[n]$

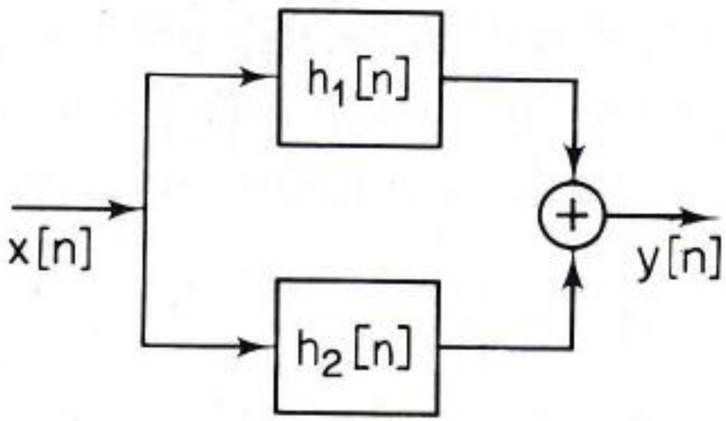
$$x[n]*\{h_1[n]+h_2[n]\}=\sum_{k=-\infty}^{\infty}x[k]h_3[n-k]$$

$$=\sum_{k=-\infty}^{\infty}x[k]\{h_1[n-k]+h_2[n-k]\}$$

$$=\sum_{k=-\infty}^{\infty}x[k]h_1[n-k]+\sum_{k=-\infty}^{\infty}x[k]h_2[n-k]$$

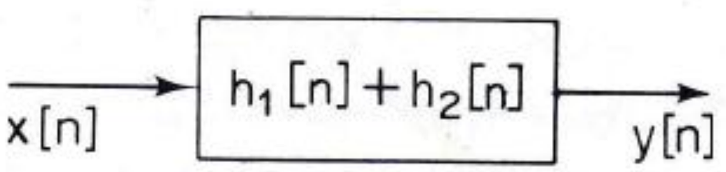
$$=\{x[n]*h_1[n]\}+\{x[n]*h_2[n]\}$$

Parallel connection



$$y[n] = \{x[n] * h_1[n]\} + \{x[n] * h_2[n]\}$$

By application of distributive property, above system is equivalent to:



$$y[n] = x[n] * \{h_1[n] + h_2[n]\}$$

When two or more LTI systems are connected in parallel, it is equivalent to a single system whose impulse response is equal to the sum of individual responses.

Associative

$$\boxed{x[n]*\{h_1[n]*h_2[n]\}=\{x[n]*h_1[n]\}*h_2[n]}$$

- Proof: Let, $h_1[n]+h_2[n]=h_3[n]$

$$x[n]*\{h_1[n]*h_2[n]\}=\sum_{k=-\infty}^{\infty}x[k]h_3[n-k]$$

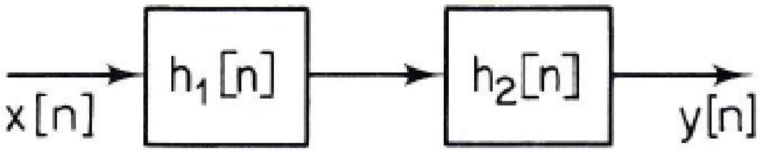
$$=\sum_{k=-\infty}^{\infty}x[k]\sum_{p=-\infty}^{\infty}h_2[p]h_1[n-k-p]$$

$$=\sum_{p=-\infty}^{\infty}h_2[p]\sum_{k=-\infty}^{\infty}x[k]h_1[n-p-k]$$

$$=\sum_{p=-\infty}^{\infty}h_2[p]\{x[n-p]*h_1[n-p]\}=\sum_{p=-\infty}^{\infty}h_2[p]r[n-p]$$

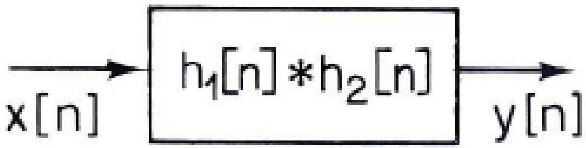
$$=h_2[n]*r[n]=h_2[n]*\{x[n]*h_1[n]\}=\{x[n]*h_1[n]\}*h_2[n]$$

Series connection



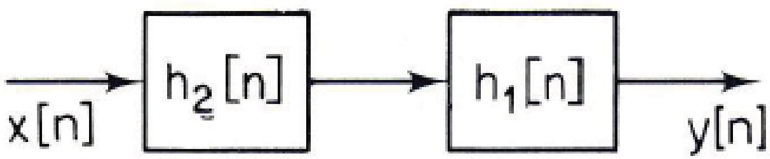
$$y[n] = \{x[n] * h_1[n]\} * h_2[n]$$

By application of associative property, above system is equivalent to:



$$y[n] = x[n] * \{h_1[n] * h_2[n]\}$$

Which is again equal to :



$$y[n] = \{x[n] * h_2[n]\} * h_1[n]$$

When two or more LTI systems are connected in cascade,it is equivalent to a single system whose impulse response is equal to the convolution of individual responses.

Also, order of cascading does not change net I/O relationship.

Stability and Causality

- Stability: A DT LTI system is stable if, its impulse response is absolutely sumable.

$$\sum_{k=-\infty}^{\infty} \left| h[k] \right| \leq M < \infty$$

- Causality: A DT LTI system is causal if, its impulse response is causal.

$$h[n] = 0 \qquad for \;\; all \;\; n < 0$$

- Memory: A DT LTI system is memory less if

$$h[n] = 0 \qquad for \;\; all \;\; n \neq 0$$

Some Common Systems and Impulse Response

- Finite Delay:

$$y[n] = x[n-n_0] \quad \Longrightarrow \quad h[n] = \delta[n-n_0]$$

- Moving average system:

$$y[n] = \frac{1}{N}\sum_{k=0}^{N-1} x[n-k] \quad \Longrightarrow \quad h[n] = \frac{1}{N}\sum_{k=0}^{N-1} \delta[n-k]$$

$$= \begin{cases} 1/N & for\ \ 0 \le n \le N-1 \\ 0 & otherwise \end{cases}$$

- Accumulator:

$$y[n] = \sum_{k=-\infty}^{n} x[k] \quad \Longrightarrow \quad h[n] = \sum_{k=-\infty}^{n} \delta[k] = \begin{cases} 1 & for\ \ n \ge 0 \\ 0 & otherwise \end{cases}$$

$$= u[n]$$

Some Common Systems and Impulse Response

- Forward difference:

$$y[n] = x[n+1] - x[n] \quad \Longrightarrow \quad h[n] = \delta[n+1] - \delta[n]$$

- Backward difference:

$$y[n] = x[n] - x[n-1] \quad \Longrightarrow \quad h[n] = \delta[n] - \delta[n-1]$$

Difference Equations

- A wide variety of systems that form an important subclass of LTI systems are described by difference equations.

- For such systems, their input and output are related by linear constant coefficient difference equation of order N.

- A general Nth order linear constant coefficient difference equation has the form:

$$\boxed{\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]}$$

- e.g.,:

Difference Equations

- Difference equations can be solved using classic methods based on forced response and transient response.

- It can also be solved recursively.

Frequency Analysis of Signals

- In addition to time domain representation, signal and systems can also be represented and analyzed in frequency domain.

- In many situations and applications, such representations are very useful and vital.

- Several transformation tools are used to obtain such frequency domain representations.

- Fourier analysis tools including Fourier Series and Transforms are one of the most important such tool.

Fourier Analysis of DT Signals

- Discrete time signals may be represented in terms of sinusoids or complex exponentials.

- Such representation is called Fourier representations.

- Periodic signals are represented by *Fourier Series* while aperiodic signals are represented by *Fourier Transforms*.

- Importance of Fourier representations lie in the fact that a wide range of practical signals have Fourier representations. Also, complex exponentials are the *eigen functions* for LTI systems.

Fourier Analysis of DT Signals

- For a DT LTI system with impulse response h[n], the output for a complex exponential input is.

$$y[n]=\sum_{k=-\infty}^{\infty} h[k]e^{j\omega(n-k)}$$

$$=e^{j\omega n}\left(\sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}\right)$$

H

$$H(e^{j\omega})=\sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}$$

$$y[n]=H(e^{j\omega})e^{j\omega n}$$

- Thus if any signal can be expressed in terns of combination of complex exponentials, output can be expressed as combination of eigen value times the complex exponentials.

DT Fourier Series

- A periodic DT signal can be expressed as weighed linear combination of a complex exponential and its harmonics.
- The resulting series is called Fourier series representation

- For a periodic signal $x[n]$ with fundamental period N and frequency ω_0,

$$x[n]=\sum_{k=\langle N\rangle} a_k e^{jk\omega_0 n}=\sum_{k=\langle N\rangle} a_k e^{jk\frac{2\pi}{N}n}$$

Where Fourier Coefficients a_k are given as:

$$a_k=\frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk\frac{2\pi}{N}n} \qquad k=0,1,2\ldots N-1$$

DT Fourier Series -Properties

- DTFS is a finite series. DTFS coefficients are periodic.

Let us consider $x[n]$ and $y[n]$ be periodic signals with period N such that,

$$x[n] \xleftrightarrow{DTFS} a_k$$

$$y[n] \xleftrightarrow{DTFS} b_k$$

- Linearity:

For constants A and B,

$$\boxed{A\,x[n] + B\,y[n] \xleftrightarrow{DTFS} A a_k + B b_k}$$

DT Fourier Series -Properties

- Time shifting:

Shifting the signal in time results in change of phase in spectrum.

$$\boxed{x[n-n_0] \xleftrightarrow{DTFS} a_k e^{-jk\omega_0 n_0}}$$

- Frequency shifting:

$$\boxed{e^{jM\omega_0 n} x[n] \xleftrightarrow{DTFS} a_{k-M}}$$

- Time Reversal:

$$\boxed{x[-n] \xleftrightarrow{DTFS} a_{-k}}$$

- Conjugation:

$$\boxed{x^*[n] \xleftrightarrow{DTFS} a^*_{-k}}$$

DT Fourier Series - Properties

- Periodic Convolution:

$$\sum_{l=\langle N \rangle} x[l]\, y[n-l] \xleftrightarrow{DTFS} N\, a_k b_k$$

- Multiplication:

$$x[n]\, y[n] \xleftrightarrow{DTFS} \sum_{l=\langle N \rangle} a_l b_{k-l}$$

- First Difference:

$$x[n] - x[n-1] \xleftrightarrow{DTFS} \left(1 - e^{-jk\omega_0}\right) a_k$$

- Running Sum:

$$\sum_{k=-\infty}^{\infty} x[k] \xleftrightarrow{DTFS} \frac{1}{\left(1 - e^{-jk\omega_0}\right)} a_k$$

DT Fourier Transform

- DTFT is used to represent aperiodic signals in terms of sinusoids or complex exponentials.

For a finite duration signal $x[n]$,

$$\boxed{x[n] = \frac{1}{2\pi}\int_{2\pi} X\left(e^{j\omega}\right)e^{j\omega n}\,d\omega}$$

Where, $X\left(e^{j\omega}\right)$ is called the Fourier Transform of $x[n]$ and is given as:

$$\boxed{X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}}$$

DTFT is a continuous function of ω and periodic in ω with period 2π.

Fourier Transform-Properties

Let us consider $x[n]$ and $y[n]$ be the finite duration signals such that,

$$x[n] \xleftrightarrow{DTFT} X(e^{j\omega})$$

$$y[n] \xleftrightarrow{DTFT} Y(e^{j\omega})$$

- Linearity:

  For constants A and B,

$$\boxed{A\,x[n] + B\,y[n] \xleftrightarrow{DTFT} A\,X(e^{j\omega}) + B\,Y(e^{j\omega})}$$

- Time shifting:

  Shifting the signal in time results in change of phase in spectrum

$$\boxed{x[n-n_0] \xleftrightarrow{DTFT} e^{-j\omega n_0} X(e^{j\omega})}$$

Fourier Transform-Properties

- Frequency Shifting:

$$\boxed{e^{j\omega_0 n}x[n] \xleftrightarrow{DTFT} X(e^{j(\omega-\omega_0)})}$$

- Conjugation:

$$\boxed{x^*[n] \xleftrightarrow{DTFT} X^*(e^{-j\omega})}$$

- Differencing and Accumulation:

$$\boxed{x[n]-x[n-1] \xleftrightarrow{DTFT} (1-e^{-j\omega})X(e^{j\omega})}$$

$$\boxed{\sum_{k=-\infty}^{n} x[k] \xleftrightarrow{DTFT} \frac{1}{(1-e^{-j\omega})}X(e^{j\omega})+\pi X(e^{j0})\sum_{k=-\infty}^{\infty}\delta(\omega-2\pi k)}$$

Fourier Transform-Properties

- Time Reversal:

$$\boxed{x[-n] \xleftrightarrow{DTFT} X(e^{-j\omega})}$$

- Differentiation in frequency:

$$\boxed{n\,x[n] \xleftrightarrow{DTFT} j\frac{d\,X(e^{j\omega})}{d\omega}}$$

- Convolution:

$$\boxed{x[n] * y[n] \xleftrightarrow{DTFT} X(e^{j\omega})Y(e^{j\omega})}$$

$$\boxed{x[n]\,y[n] \xleftrightarrow{DTFT} \frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta}$$

Frequency Response of systems

- As already known, complex exponentials are eigen functions of LTI systems. For LTI systems, output is given by convolution of input and impulse response.

$$y[n] = x[n] * h[n]$$

- Using Fourier Transform in both sides,

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

- $H(e^{j\omega})$ The Fourier Transform of $h[n]$ which is also equal to the ratio of $Y(e^{j\omega})$ to $X(e^{j\omega})$ is called frequency response.

Frequency Response of systems

- Frequency response is used to represent LTI systems in frequency domain.

- $\left|H(e^{j\omega})\right|$ is called the magnitude response while $\angle H(e^{j\omega})$ is called the phase response.

- For a system described by the difference equation,

$$\boxed{\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]}$$

The frequency response is given as: $H(e^{j\omega}) = \dfrac{\sum_{k=0}^{M} b_k e^{-j\omega k}}{\sum_{k=0}^{N} a_k e^{-j\omega k}}$