POKHARA UNIVERSITY

Level: Bachelor | Semester – Spring | Year: 2020
Program: BE | Full Marks: 70
Course: Engineering Mathematics IV | Pass Marks: 31.5
Time: **2** hrs.

Candidates are required to answer in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

Group - A: (5×10=50)

Q. 1 Define differentiability of the complex function. How is it related to the analyticity of the function. What is the harmonic function and its conjugate. Is v = arg(z) = $\tan^{-1}\left(\frac{y}{x}\right)$ harmonic? If yes, find its harmonic conjugate function 2+2+1+5

Q. N. 2 What type of transformation is conformal mapping? Name the four types of elementary conformal mappings. Find the image of triangular region of the Z-plane bounded by the lines x=0, y=0 and $\sqrt{3}x + y = 1$ under the transformation $W = e^{i\pi/3}.z$. Also sketch the image. 2+2+6

Q. N. 3 What is the difference between the Cauchy integral formula and Cauchy residue theorem? . Can you verify Cauchy's integral theorem for the function $f(z) = z$ taking C to be the circle $|z| = 2$? 2+2+6

Evaluate the integral $\int_C \frac{2z^2 - z - 3}{(z-2)^3} dz$ where C is the circle given by $|z| = 3$.

OR

Is Maclaurian's series a special part of Taylor's series? Does every function have Taylor series development? Explain your answer with an example. Represent the function $f(z) = \frac{4z+3}{z(z-3)(z+2)}$ in Laurent series i). Within $|z| = 1$ ii). In the annular region between $|z| = 2$ and $|z| = 3$ iii). Exterior to $|z| = 3$.

Q. N. 4 What is the difference between Fourier integral and Fourier transform? Find the Fourier transform of $f(x) = \begin{cases} e^{-2x} & for\ x > 0 \\ 0 & for\ x < 0 \end{cases}$. Can we determine the Fourier transform of a function which is not continuous? Discuss. 2+6+2

Q. N. 5 a) How do you define first shifting property of z-transform? Can you use this property to find $Z(\sinh at \cos bt)$? 1+4

b) What id difference equation? Obtain the solution of the difference equation $X_{k+2} + 6X_{k+1} + 9X_k = 2^k$, given $X_0 = X_1 = 0$ using z-transform. [5] 1+4

Group - B: (1×20=20)

Q. N. 6 What are the assumptions used to determine the one dimensional wave equation in an elastic string. Derive the one dimensional wave equation. The Laplacian of u in Cartesian form is given by $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$. Transform it into the polar form. Solve the equation: $u_{tt} + u_{ss} = 0$ by separating the variables. 3+6+7+4