Image Processing and Pattern Recognition (IPPR)

Lecture 3

Image Enhancement in Spatial Domain Cont…

Basanta Joshi, PhD

Asst. Prof., Depart of Electronics and Computer Engineering

Program Coordinator, MSc in Information and Communication Engineering

Member, Laboratory for ICT Research and Development (LICT)

Member, Research Management Cell (RMC)

Institute of Engineering

basanta@ioe.edu.np

http://www.basantajoshi.com.np

https://scholar.google.com/citations?user=iocLiGcAAAAJ
https://www.researchgate.net/profile/Basanta_Joshi2

Image Histograms

The histogram of an image shows us the distribution of grey levels in the image

Massively useful in image processing, especially in segmentation

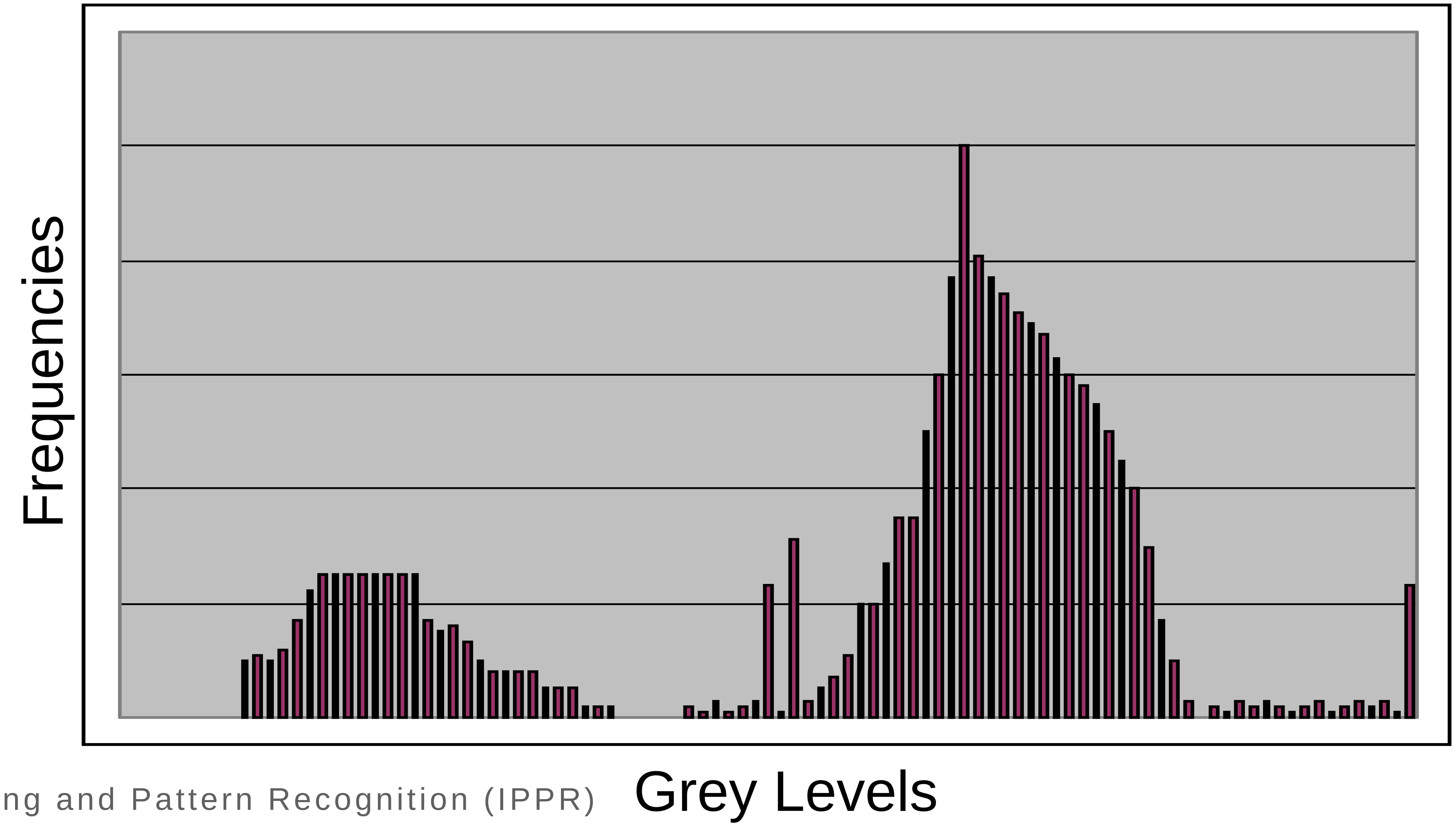


Image Histograms

- The histogram of a digital image, f, (with intensities [0,L-1]) is a discrete function

$h(r_k) = n_k$

- Where r_k is the kth intensity value and n_k is the number of

pixels in f with intensity r_k

- Normalizing the histogram is common practice – Divide the components by the total number of pixels in the image – Assuming an MxN image, this yields

$p(r_k) = n_k/MN$ for $k=0,1,2,....,L-1$ – $p(r_k)$ is, basically, an estimate of the probability of occurrence of intensity level r_k in an image
$\Sigma\, p(r_k) = 1$

Uses for Histogram Processing

- Image enhancements
- Image statistics
- Image compression
- Image segmentation
- Simple to calculate in software
- Economic hardware implementations
  - – Popular tool in real-time image processing
- A plot of this function for all values of k provides a global description of the appearance of the image (gives useful information for contrast enhancement)
- Histograms commonly viewed in plots as

$h(r_k) = n_k$ versus r_k

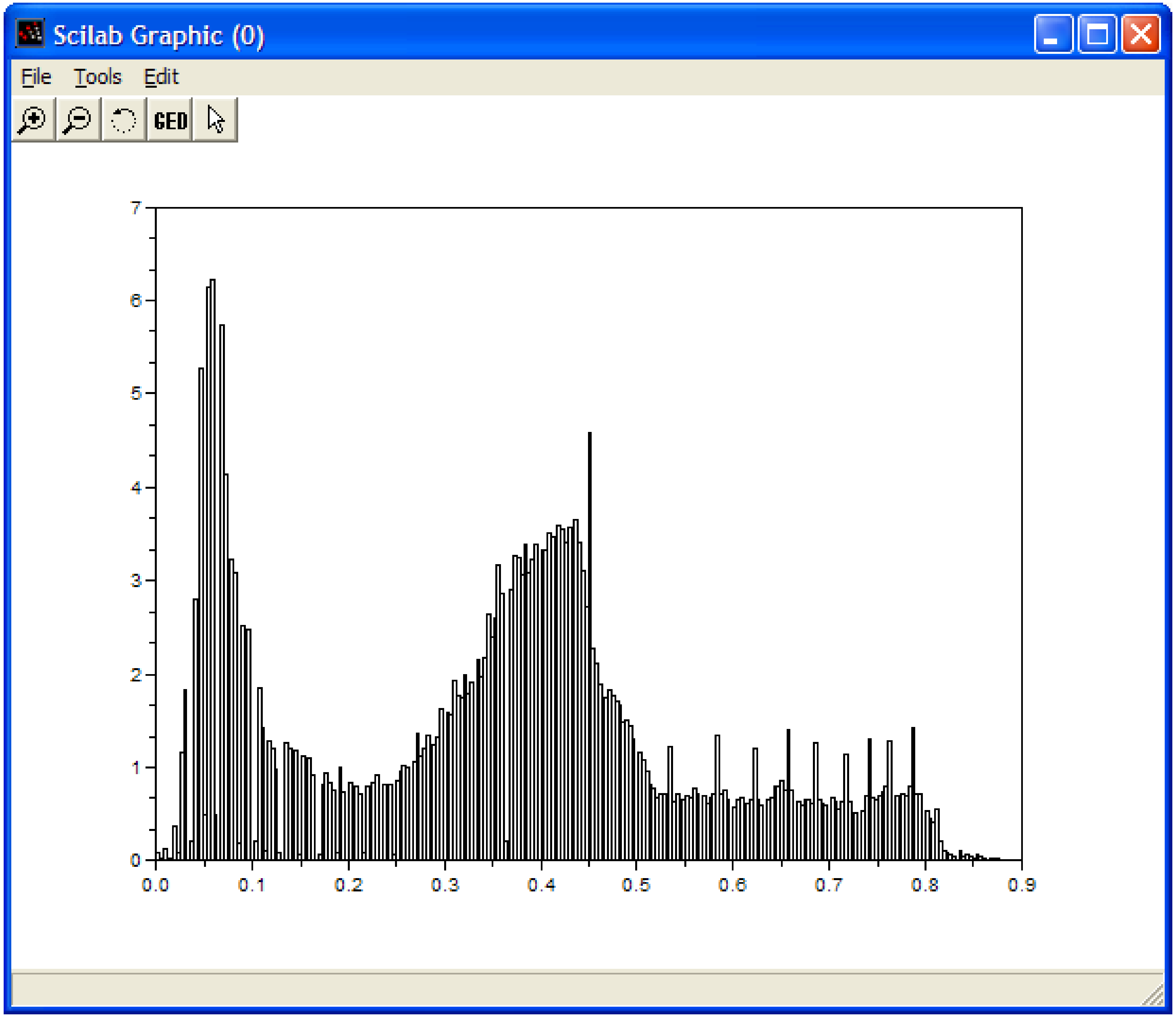
$p(r_k) = n_k / MN$ versus r_k

Histogram Examples

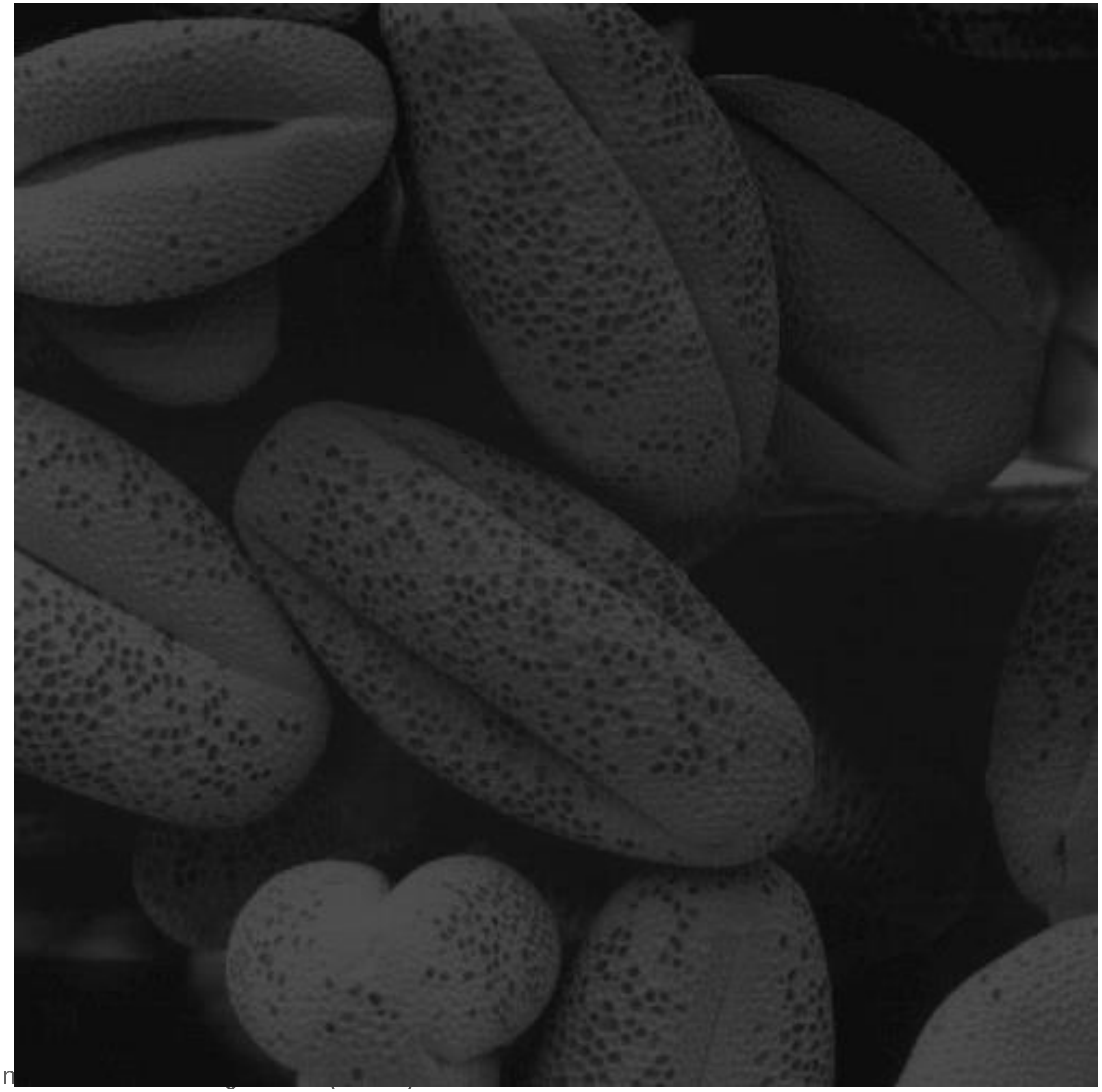
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Histogram Examples (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

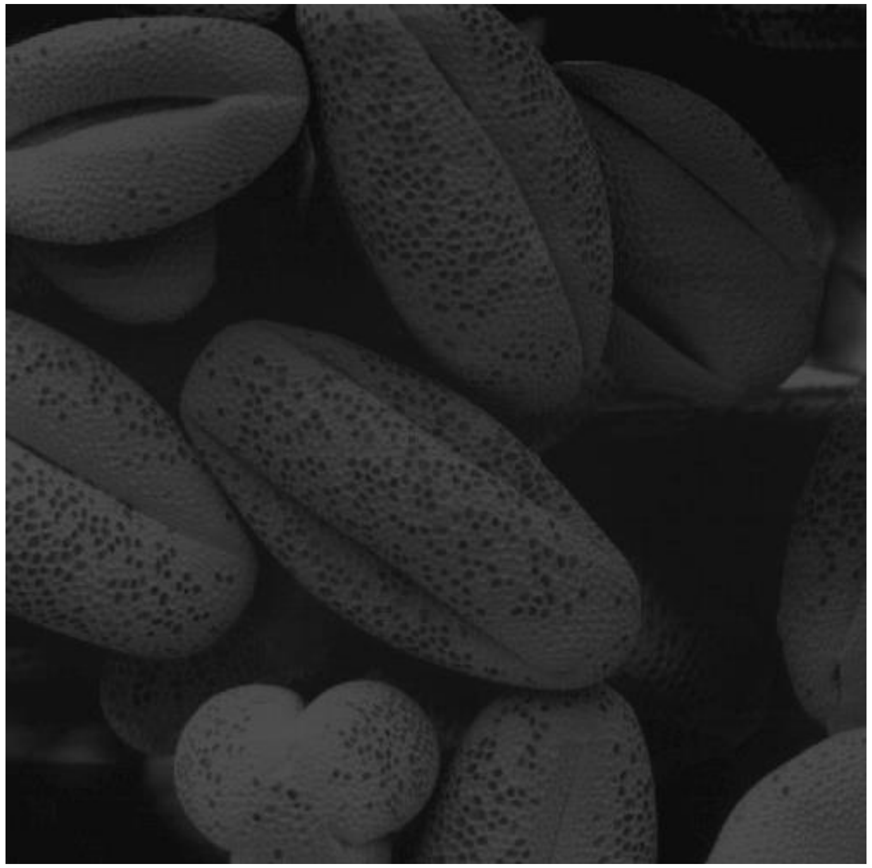


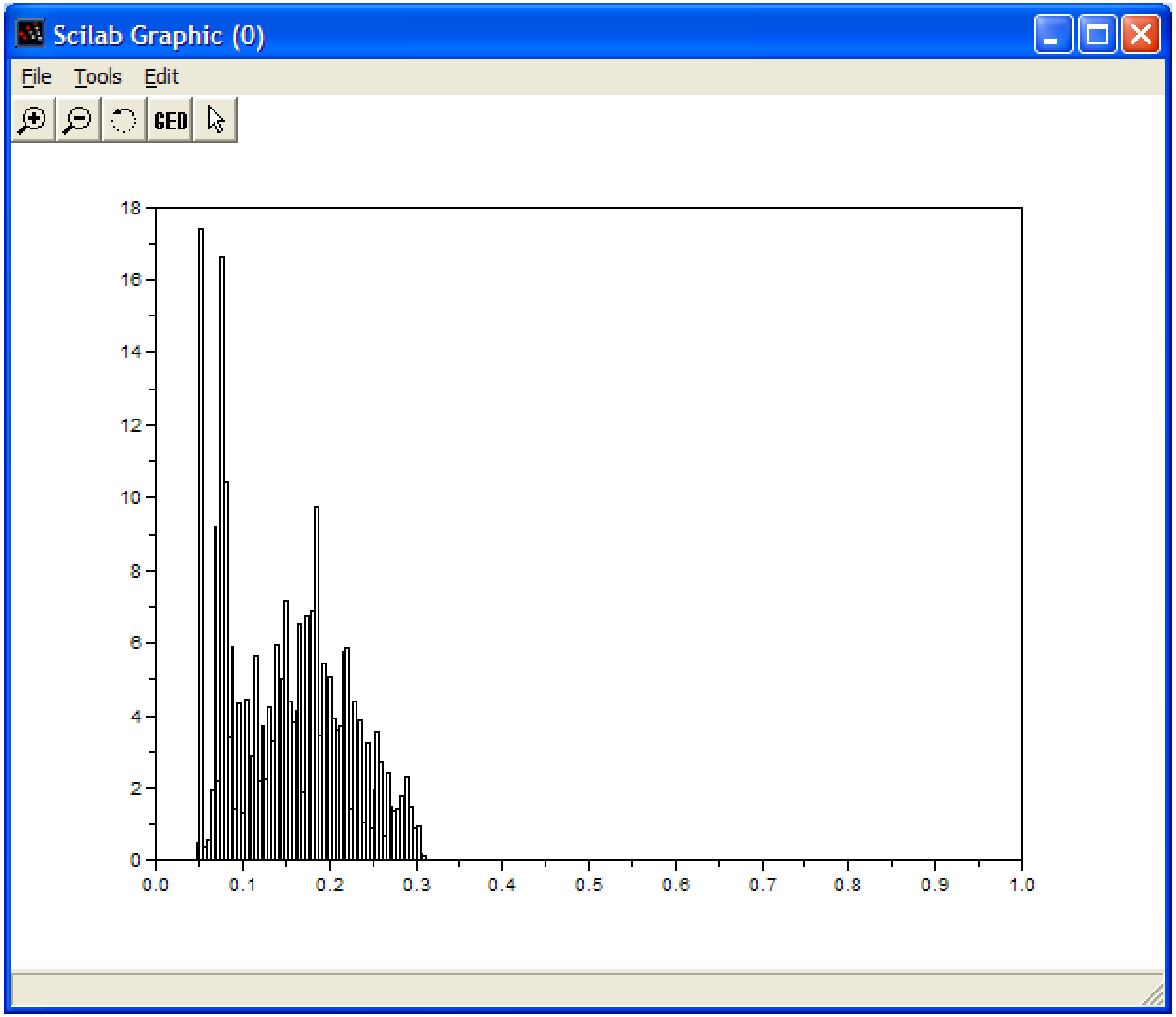
Histogram Examples (cont…)



Histogram Examples (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)



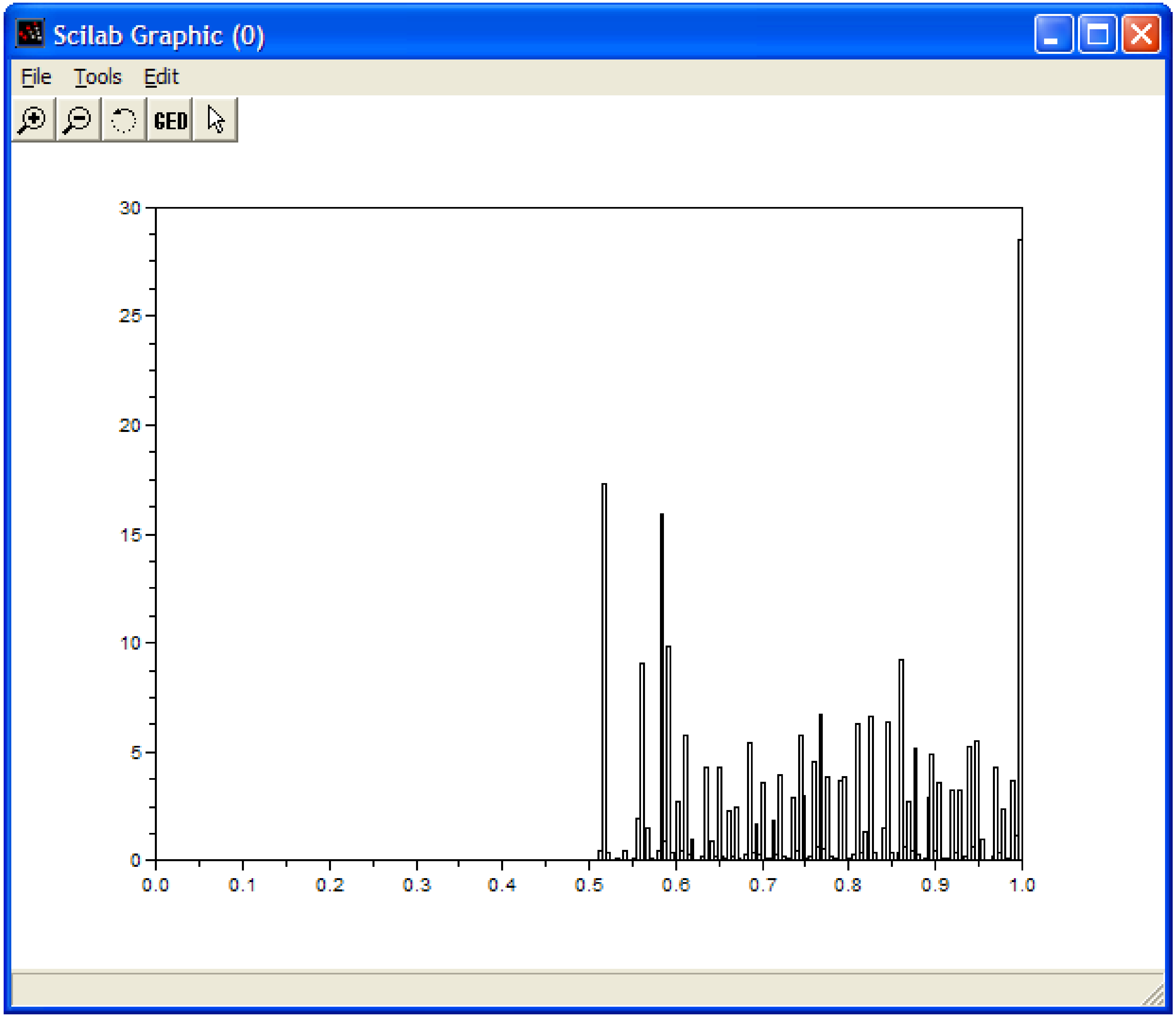


Histogram Examples (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Histogram Examples (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

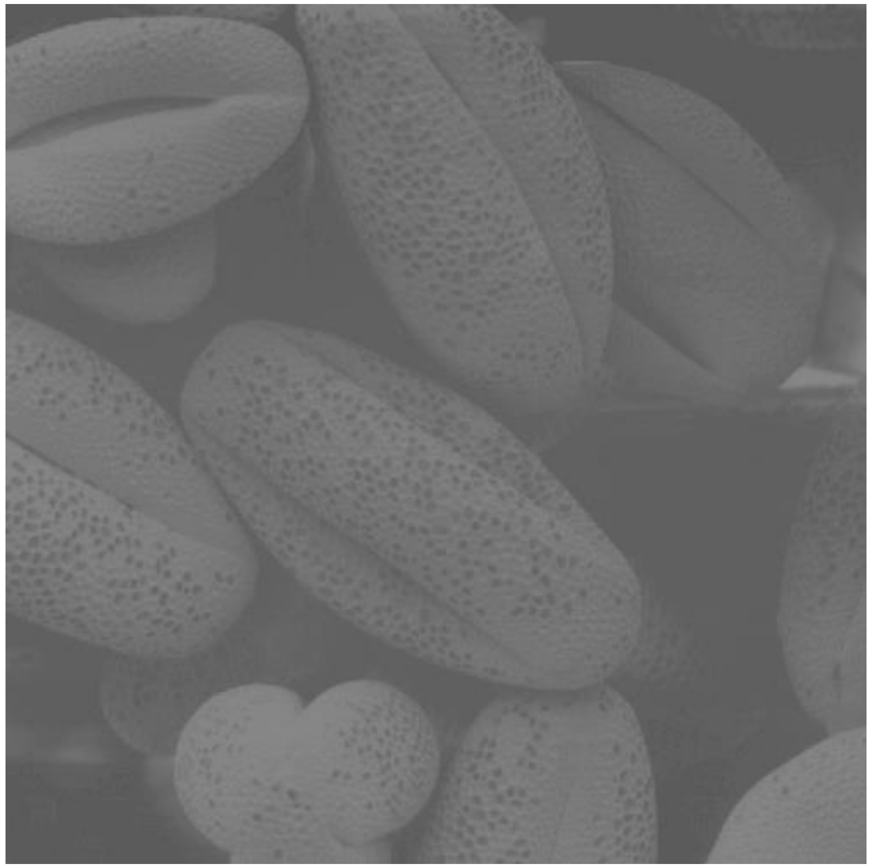


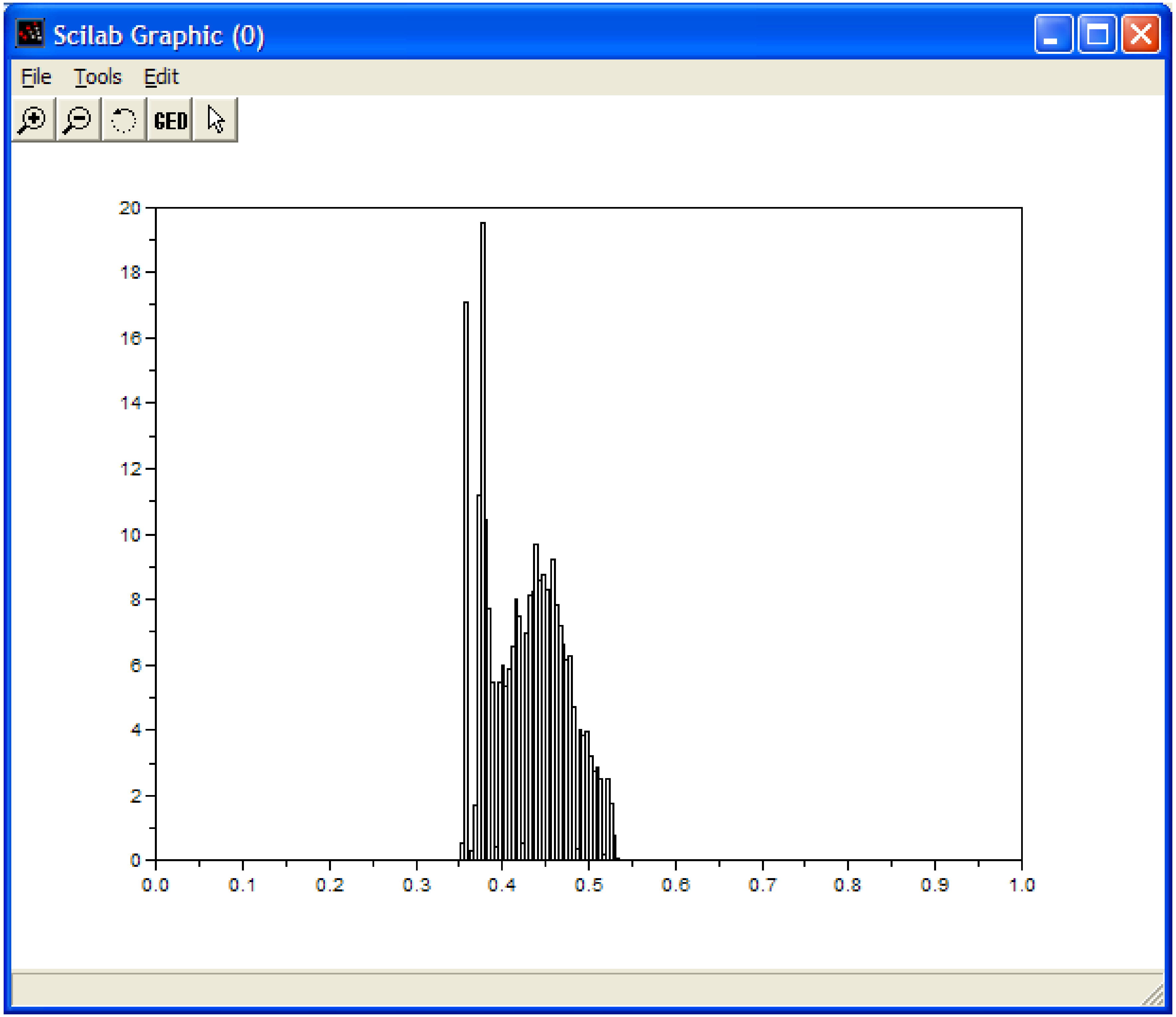
Histogram Examples (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Histogram Examples (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

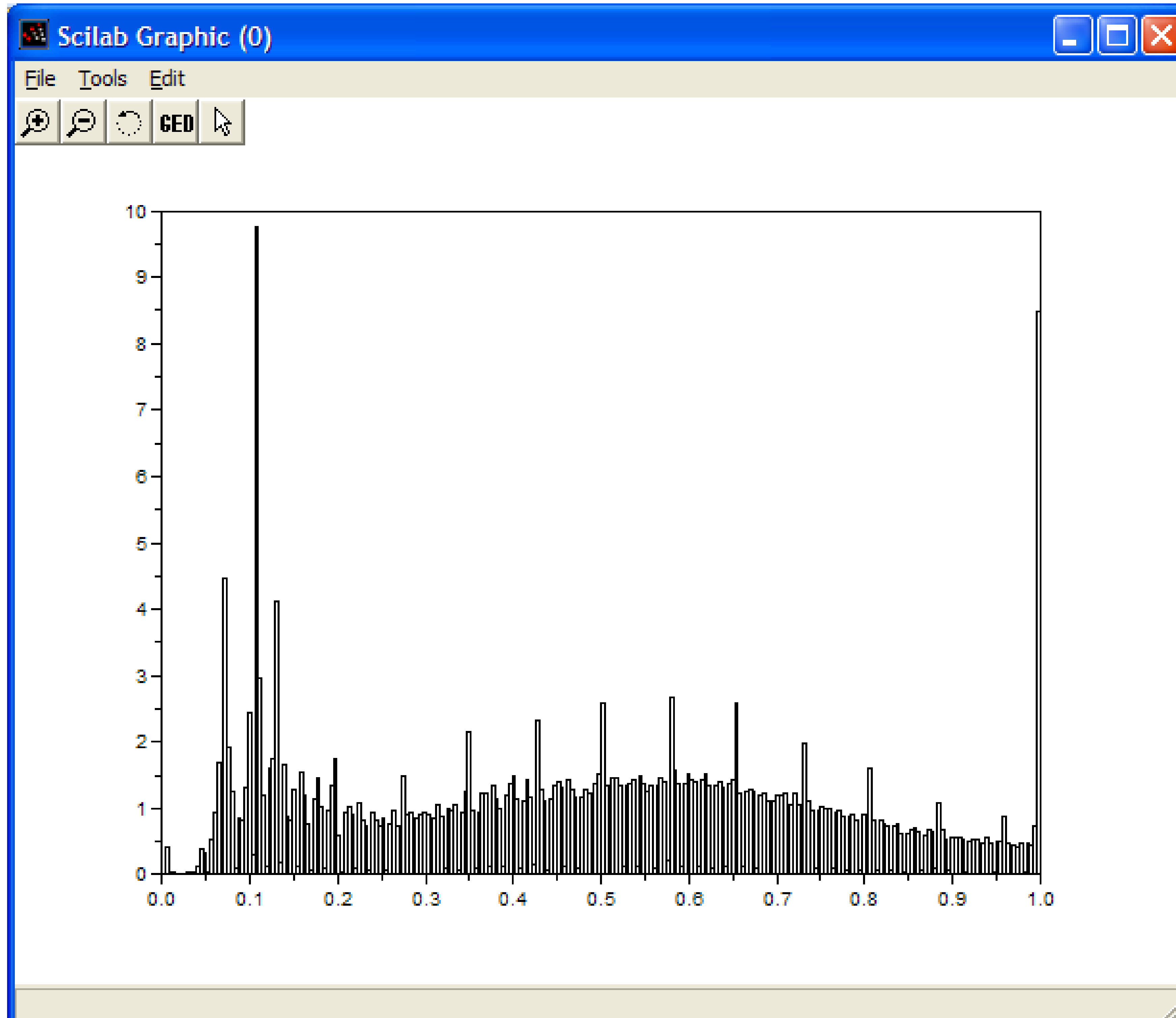




Histogram Examples (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Histogram Examples (cont…)



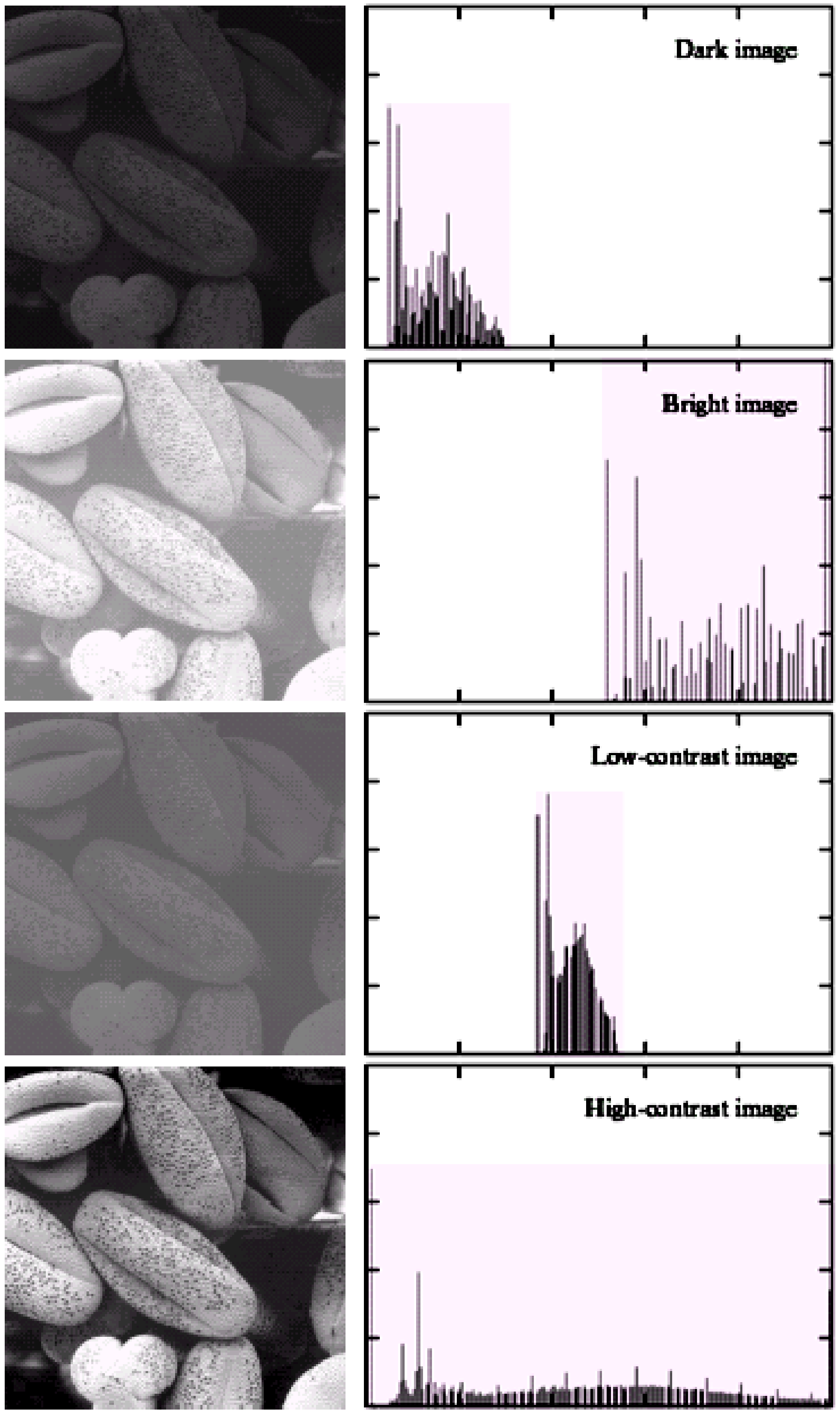
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Histogram Examples (cont…)

A selection of images and their histograms

Notice the relationships between the images and their histograms

Note that the high contrast image has the most evenly spaced histogram

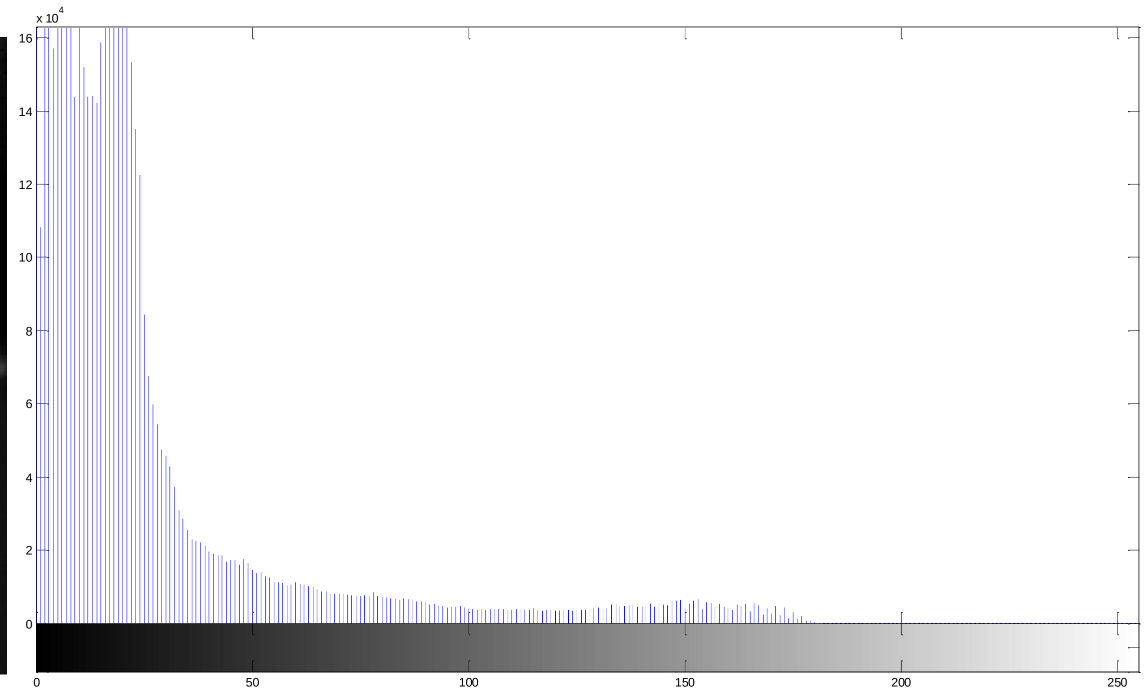


Images taken from Gonzalez & Woods, Digital Image Processing (2002)

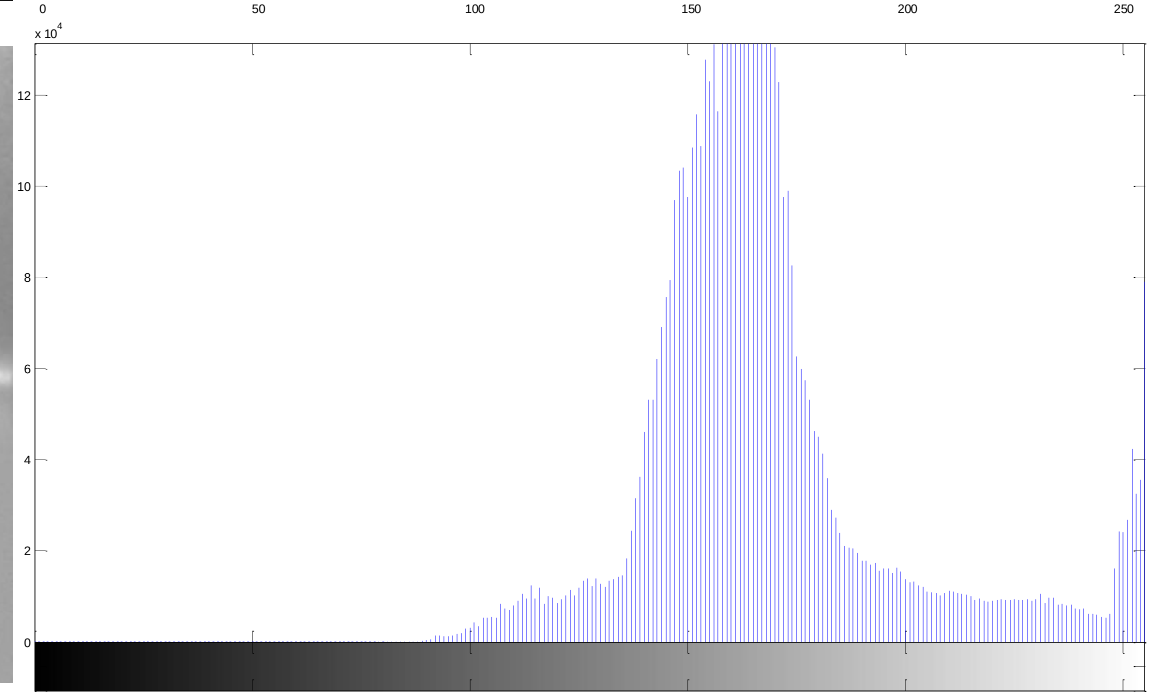
Contrast Stretching

Contrast Stretching: improves the contrast in an image by stretching the range of intensity values to span a desired range of values.

Histogram of 4 basic grey-level characteristics

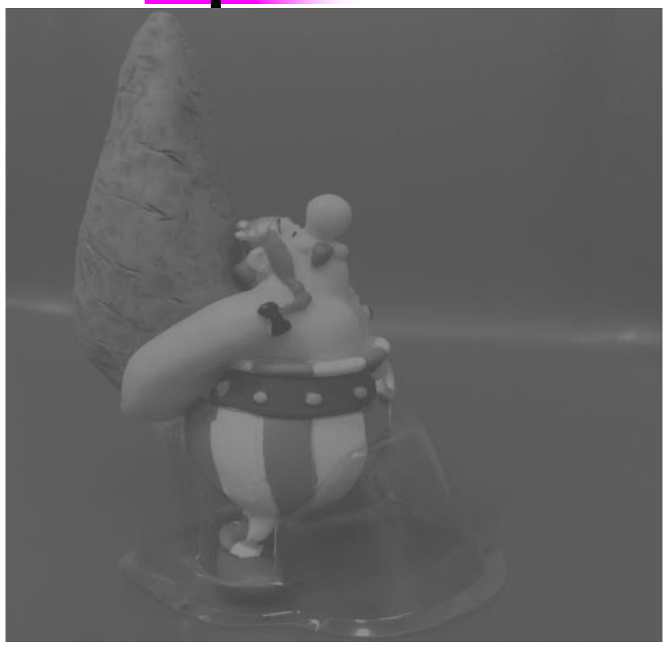


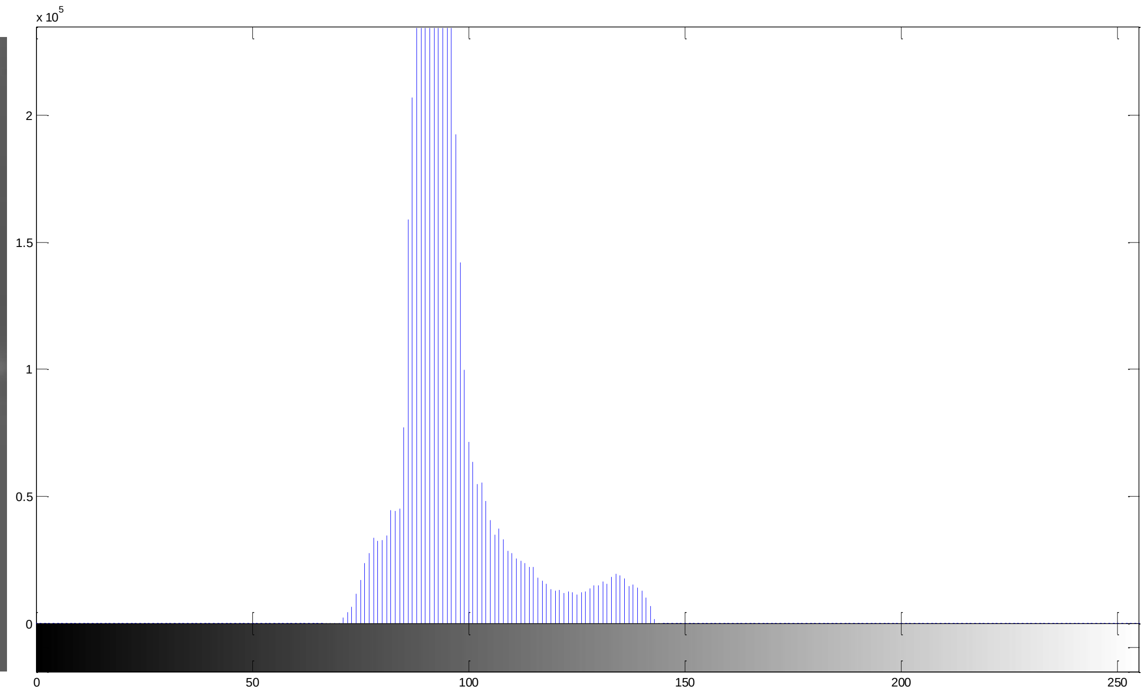
Dark image



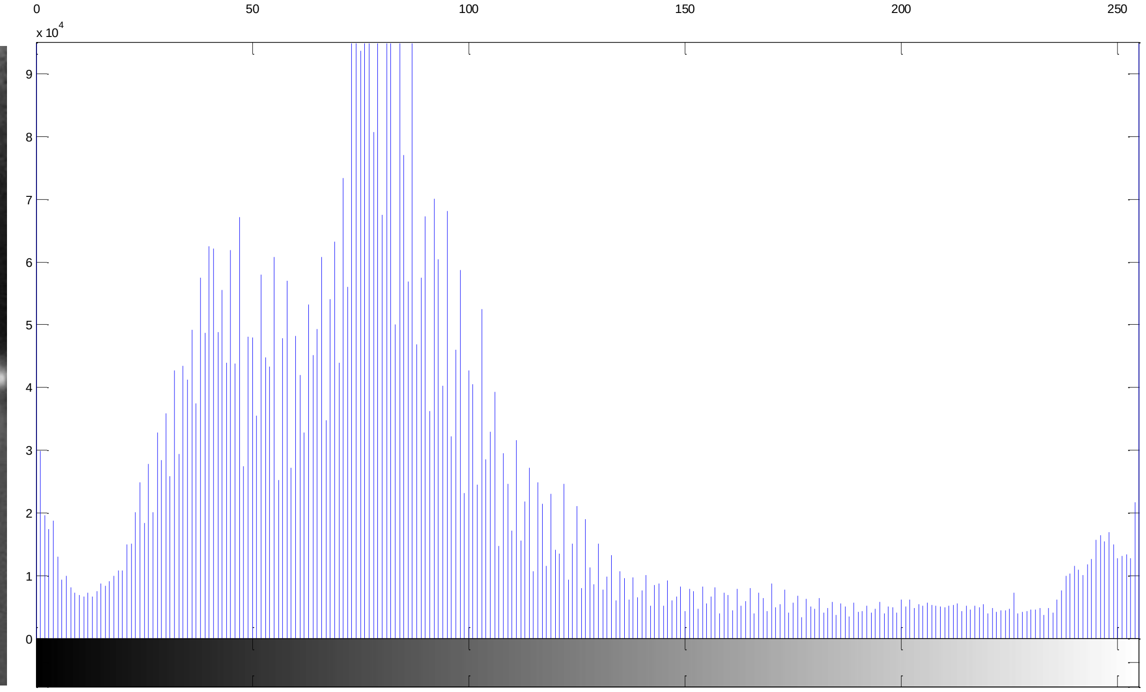
Bright image

Histogram of 4 basic grey-level characteristics





Low contrast image



High contrast image

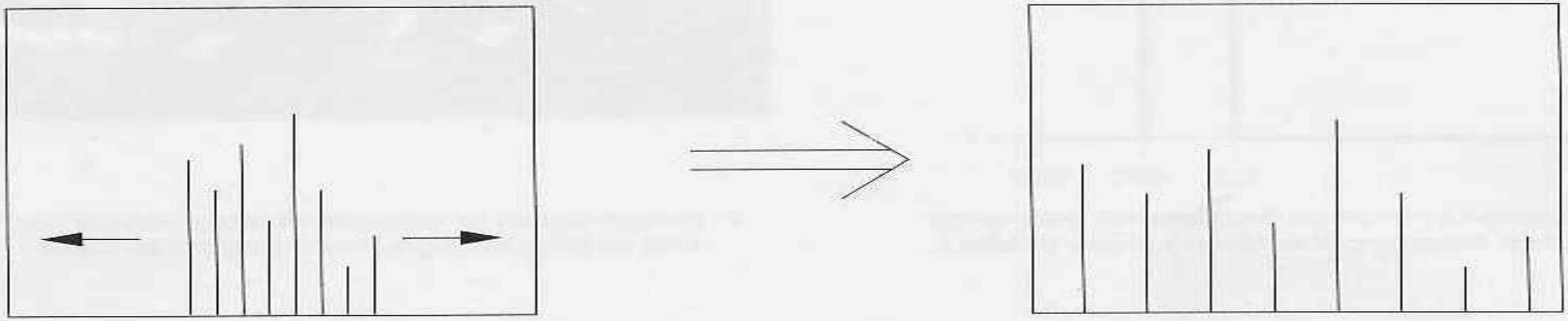
Contrast Stretching

We can fix images that have poor contrast by applying a pretty simple contrast specification

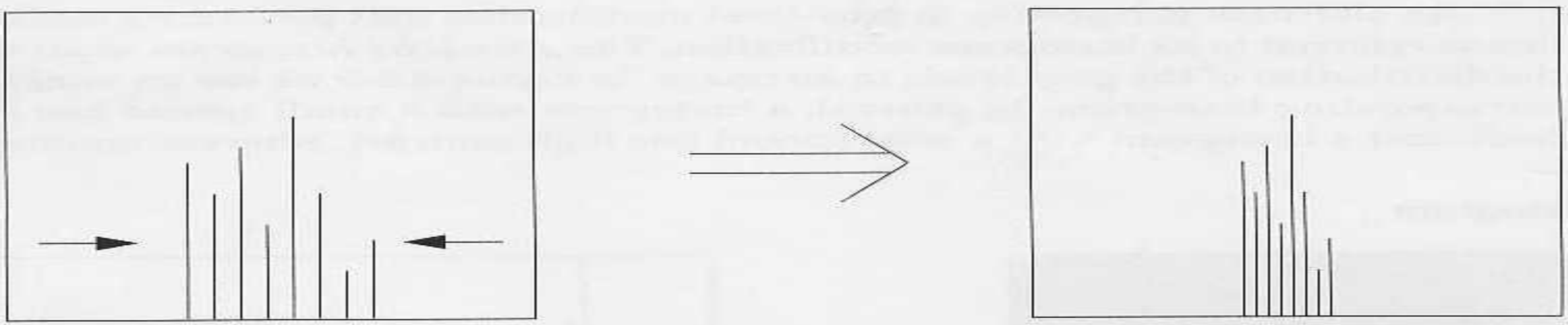
The interesting part is how do we decide on this transformation function?

Histogram Modifications

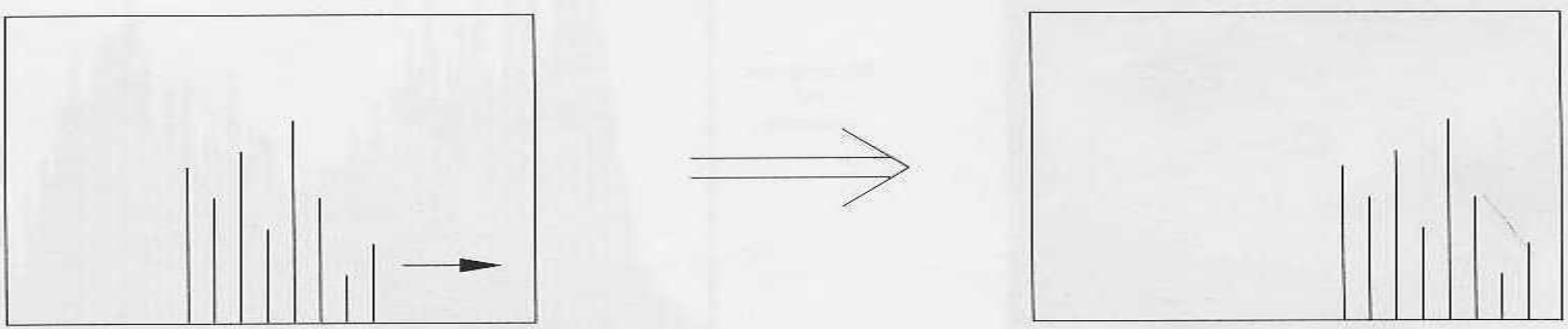
- **The gray level histogram of an image is the distribution of the gray level in an image.**
- **The histogram can be modified by mapping functions, which will stretch, shrink (compress), or slide the histogram.**
- **Next Figure illustrates a graphical representation of histogram stretch, shrink and slide.**



a. Histogram stretch.



b. Histogram shrink.



c. Histogram slide.

Mapping function

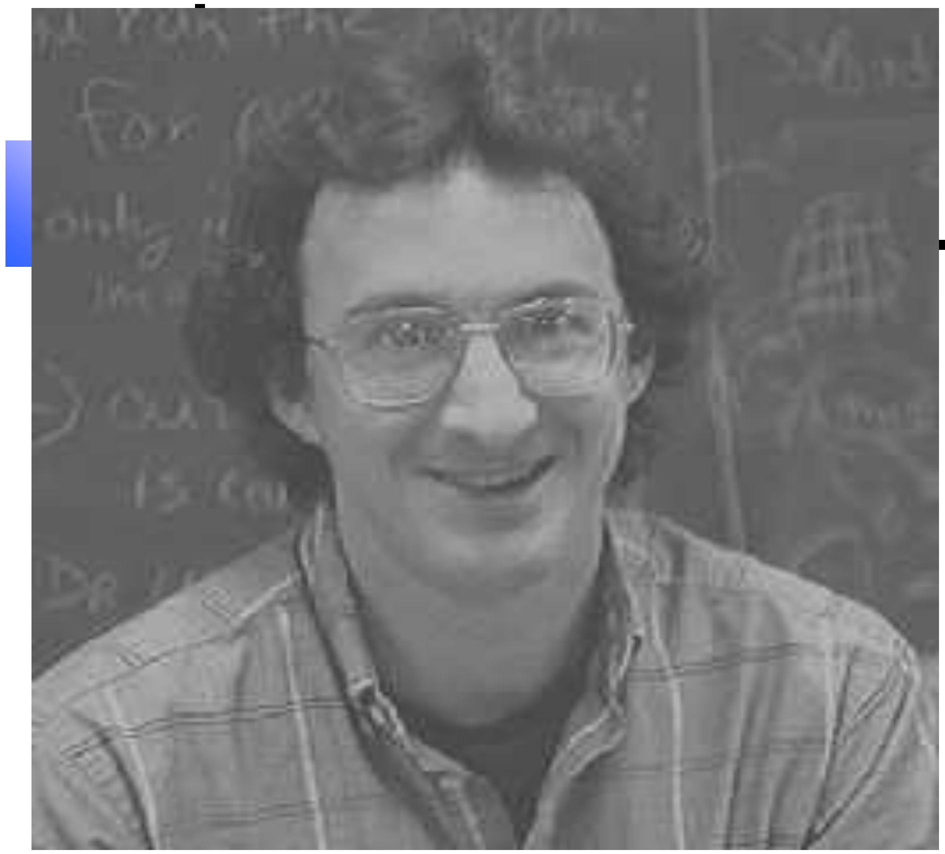
The mapping function for histogram stretch can be found by the
following equation:

$$\text{Stretch}(I(r,c)) = \left[\frac{I(r,c) - I(r,c)_{\min}}{I(r,c)_{\max} - I(r,c)_{\min}}\right] [\text{MAX} - \text{MIN}] + \text{MIN}$$

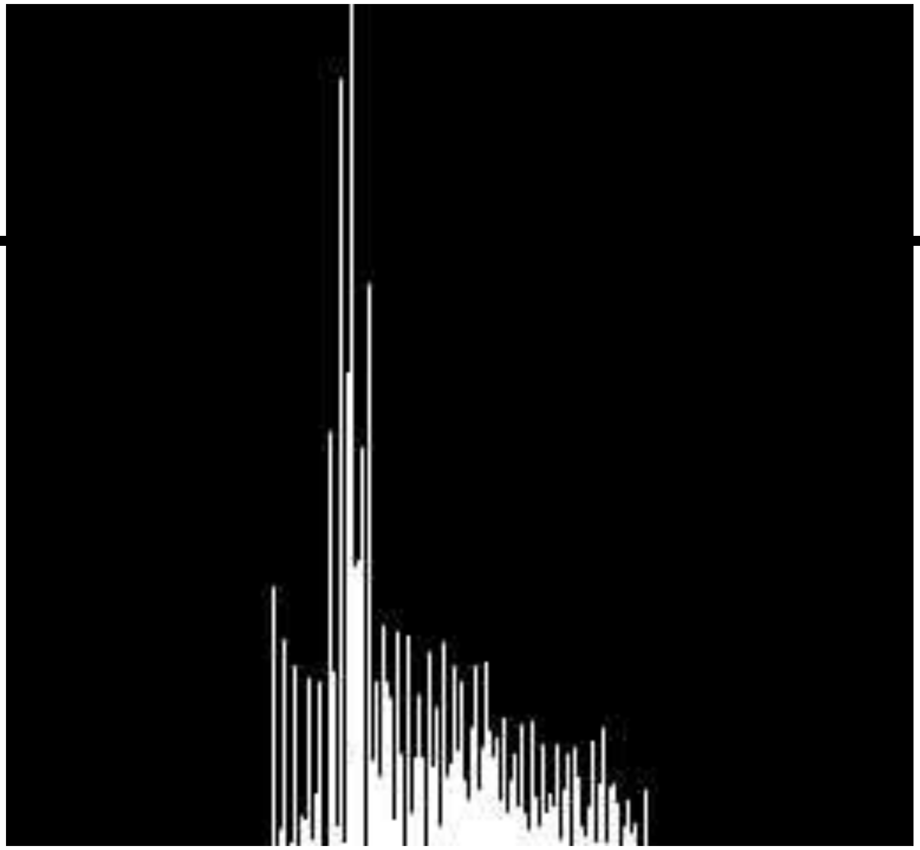
Where,

- $I(r,c)_{\max}$ is the largest gray- level in the image $I(r,c)$.
- $I(r,c)_{\min}$ is the smallest gray- level in the image $I(r,c)$.
- MAX and MIN correspond to the maximum and minimum gray – level values possible (for an 8-bit image these are 255 and 0).

This equation will take an image and stretch the histogram a cross the entire gray-level range which has the effect of increasing the contrast of a low contrast image of histogram stretching).



Low-contrast image



Histogram of low-contrast image

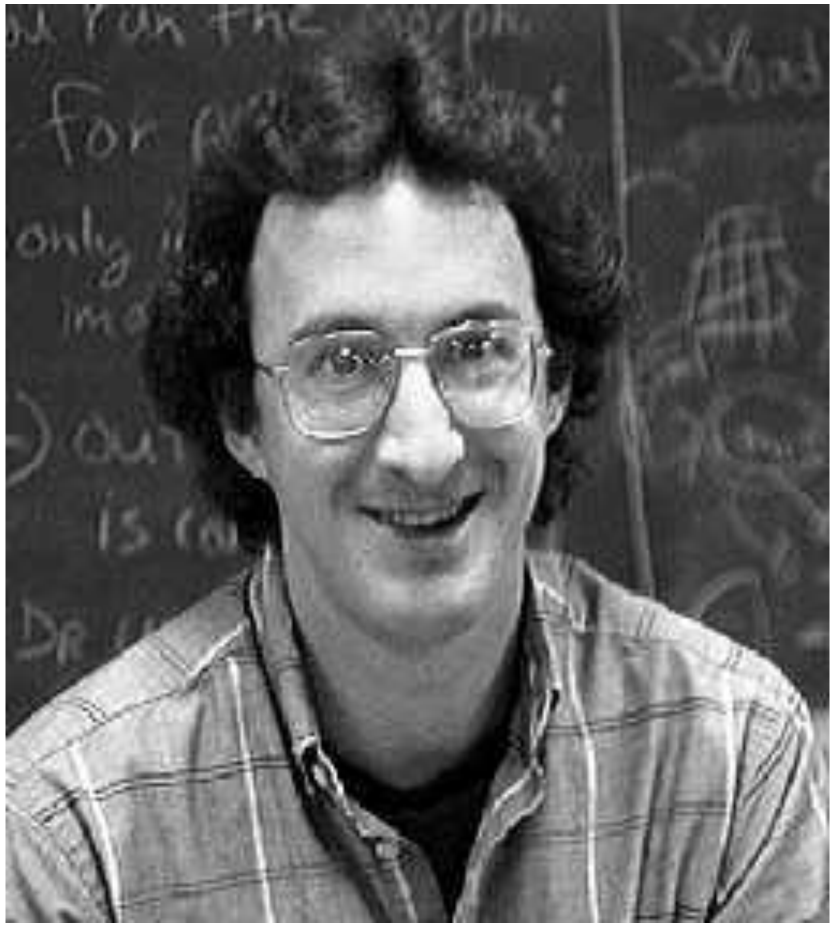
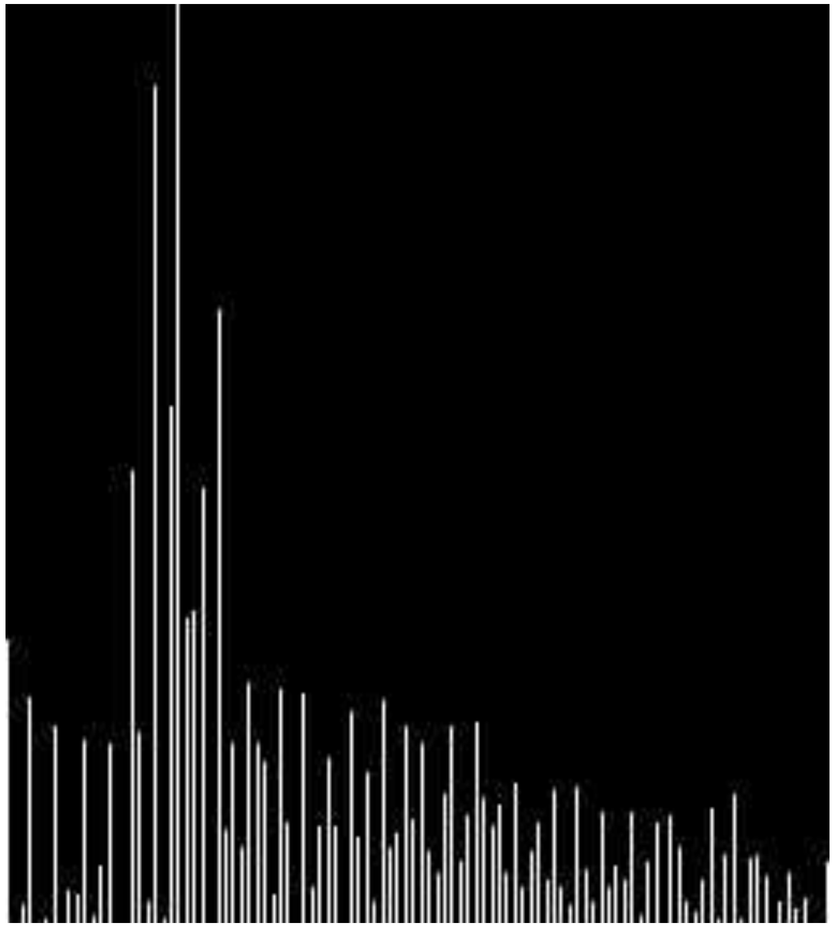


Image after histogram stretching

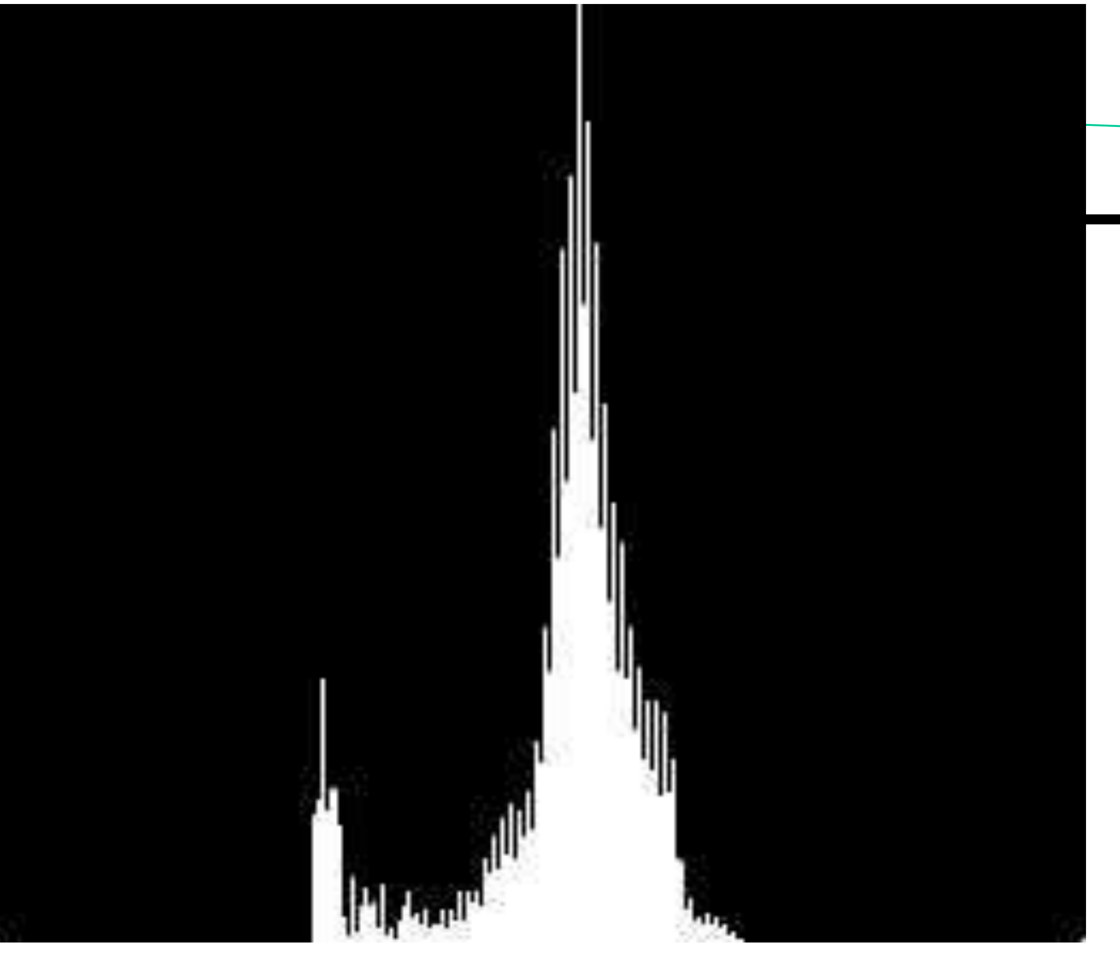


Histogram of image after stretching

Mapping function cont

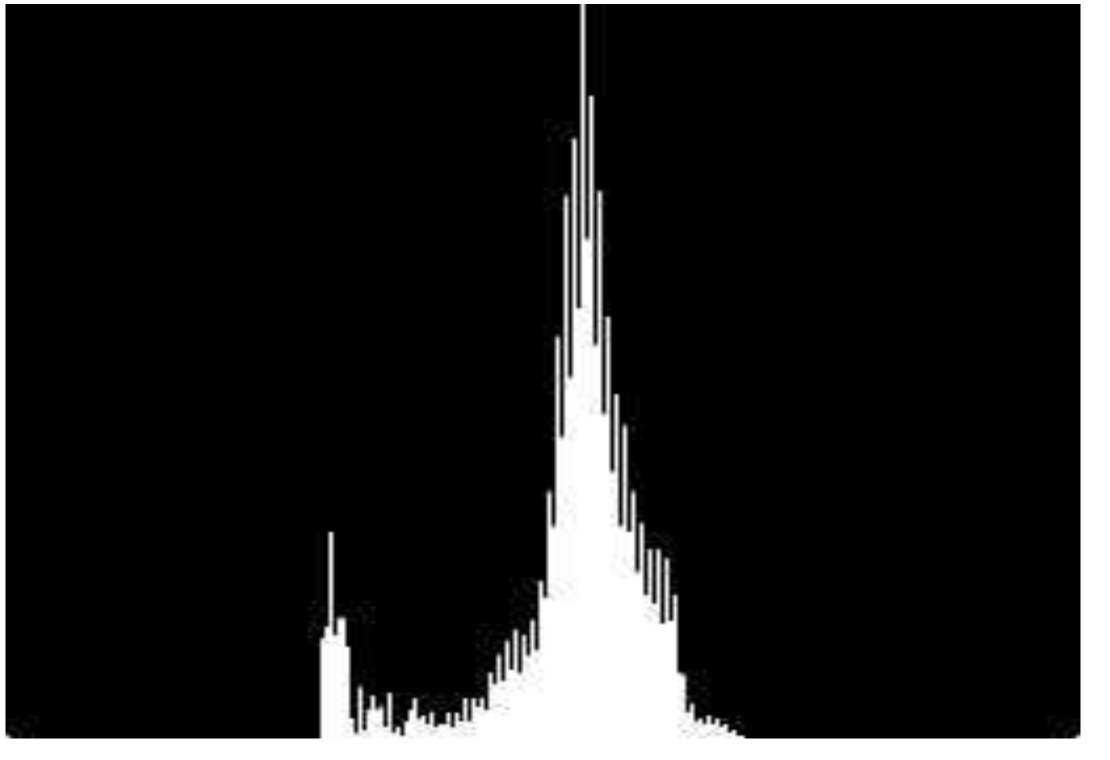
In most of the pixel values in an image fall within small range, but a few outlines force the histogram to span the entire range, a pure histogram stretch will not improve the image.
In this case it is useful to allow a small proceeding of the pixel values to be aliped at the low and high end of the range (for an 8-bit image this means truncating at 0 and 255).

Original Image



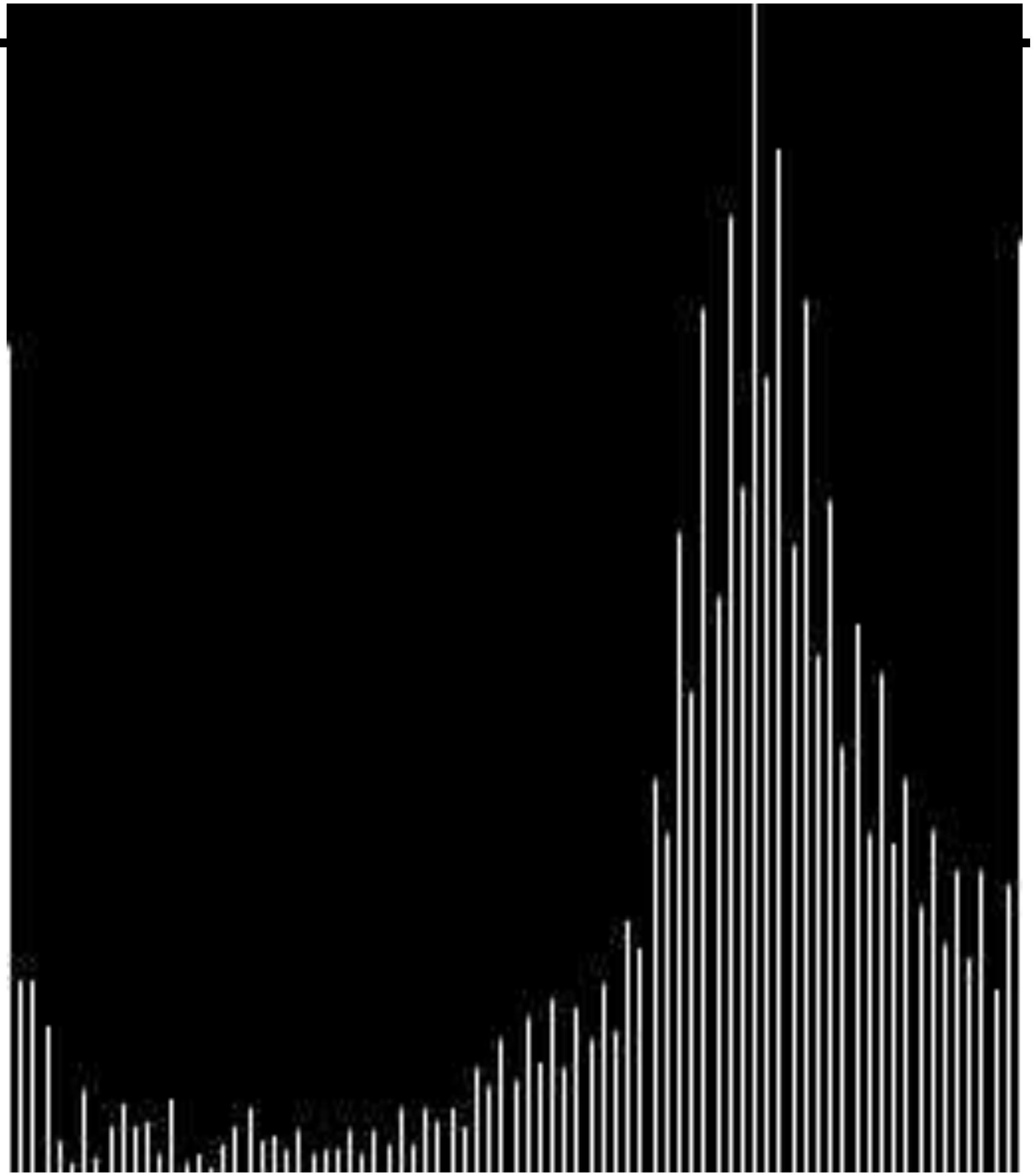
Histogram of the original image

Image after histogram stretching without clipping



Histogram of the image

Image after histogram stretching with clipping 3% low and high value



Histogram of the image

Histogram Shrink

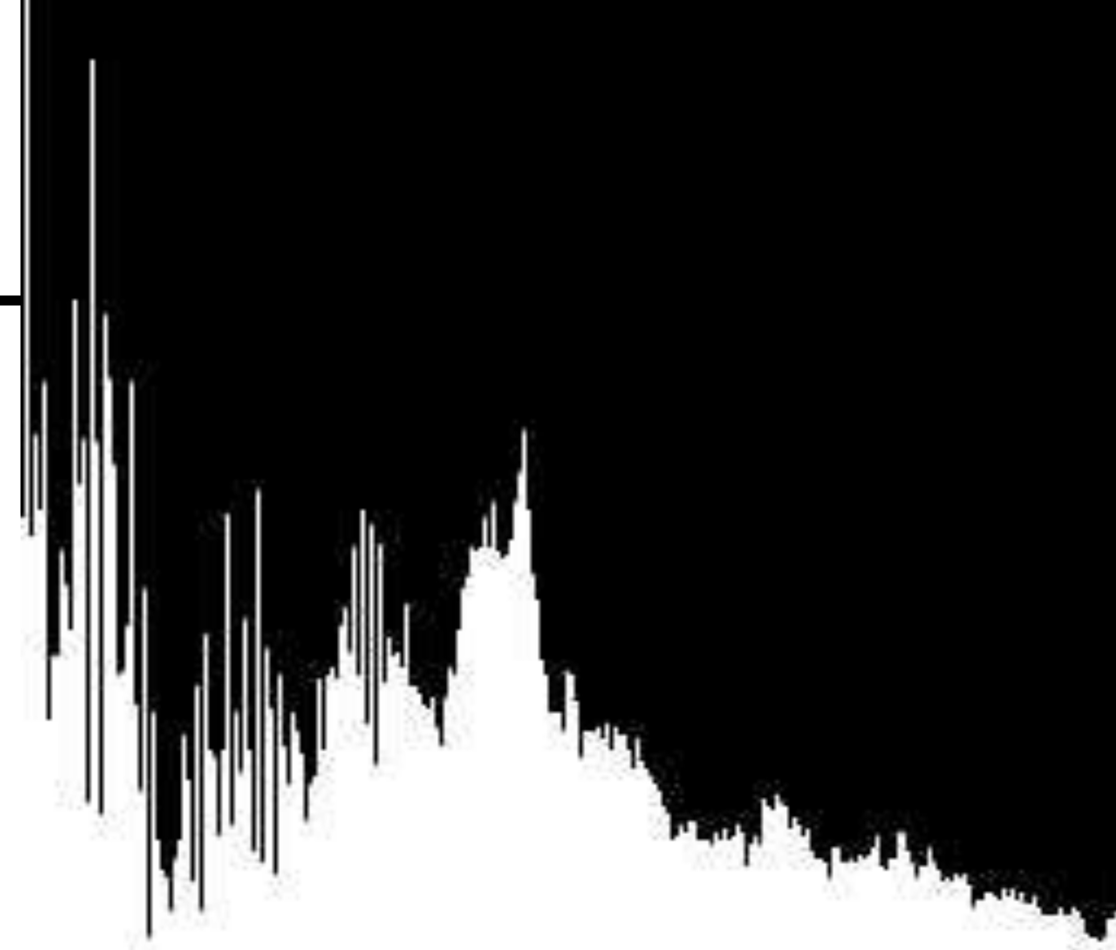
The opposite of a histogram stretch is a histogram shrink, which will decrease image contrast by compressing the gray levels. The mapping function for a histogram shrinking can be found by the following equation:

$$\text{Shrink}((r,c)) = \left[\frac{Shrink_{\max} - Shrink_{\min}}{I(r,c)_{\max} - I(r,c)_{\min}}\right]\left[I(r,c) - I(r,c)_{min}\right] + Shrink_{min}$$

Shrink $_{max}$ and shrink $_{min}$ correspond to the maximum and minimum desired in the compressed histogram.

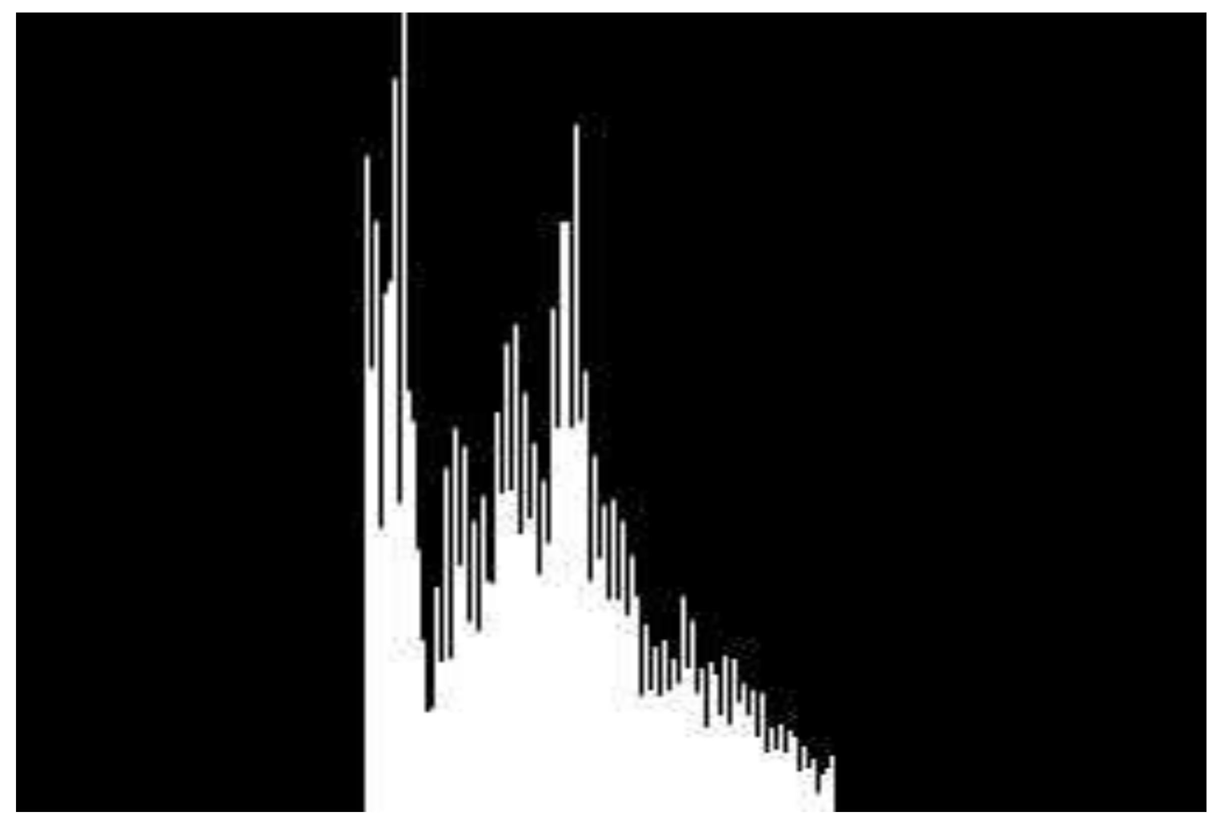
In general, this process produces an image of reduced contrast and may not seem to be useful an image enhancement

Original image



Histogram of original image

Image after histogram shrink to the range [75, 175]



Histogram of the image

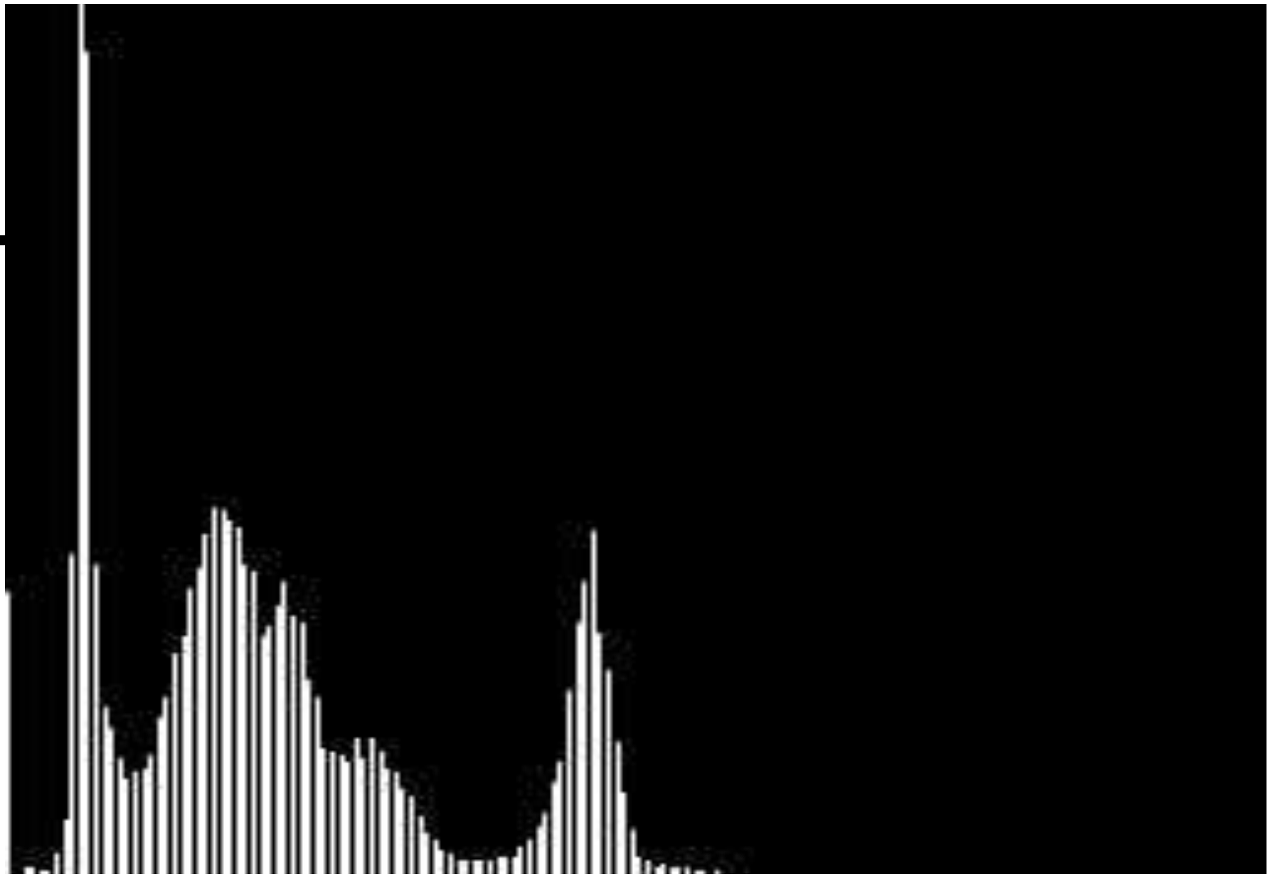
Histogram Slide Techniques

The histogram slide techniques can be used to make an image either darker or lighter but retain the relationship between gray-level values. This can be a accomplished by simply adding or subtracting a fixed number for all the gray-level values, as follows:

Slide (I(r,c)) = I (r,c)+ OFFSET.

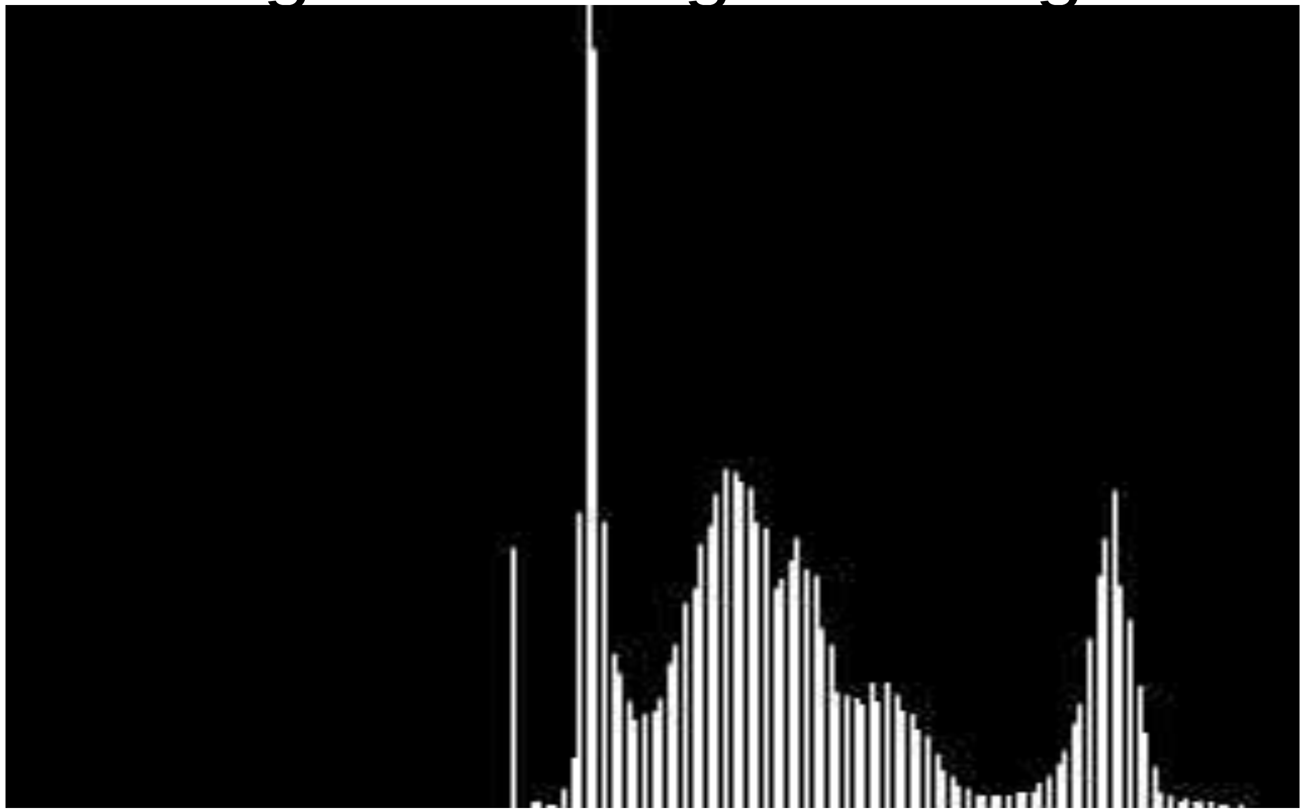
Where OFFSET values is the amount to slide the In this equation, a positive OFFSET value will increase the overall brightness; where as a negative OFFSET will create a darker image

Original image



Histogram of original image

Image after positive-value histogram sliding



Histogram of image after sliding

Problem

•Gray level histogram of an image represented in 3 bit system is given below

Gray level	1	2	3	4	5	6	7
Frequency	0	0	50	200	250	40	0

Stretch the contrast of histogram over the entire range.

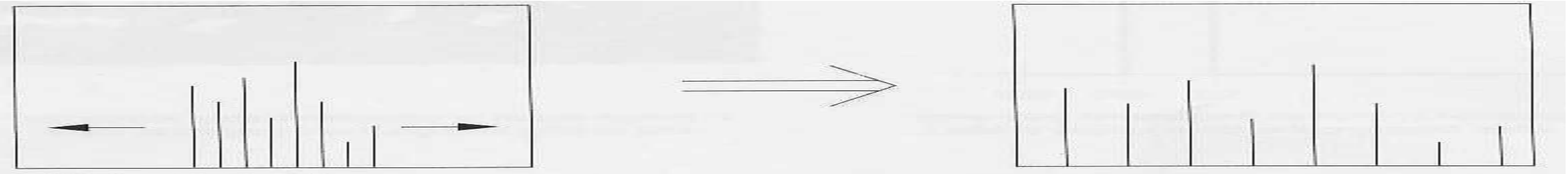
$$\text{Stretch}\,(I\,(r,c)) = \left[\frac{I(r,c) - I(r,c)_{\text{min}}}{I(r,c)_{\text{max}} - I(r,c)_{\text{min}}}\right][\text{MAX-MIN}] + \text{MIN}$$

$I\,(r, c)_{max}$

$I\,(r, c)_{min}$ =

MAX=

MIN=



a. Histogram stretch.

Problem

- Gray level histogram of an image represented in 3 bit system is given below

Gray level	1	2	3	4	5	6	7
Frequency	0	0	50	200	250	40	0

Stretch the contrast of histogram over the entire range.

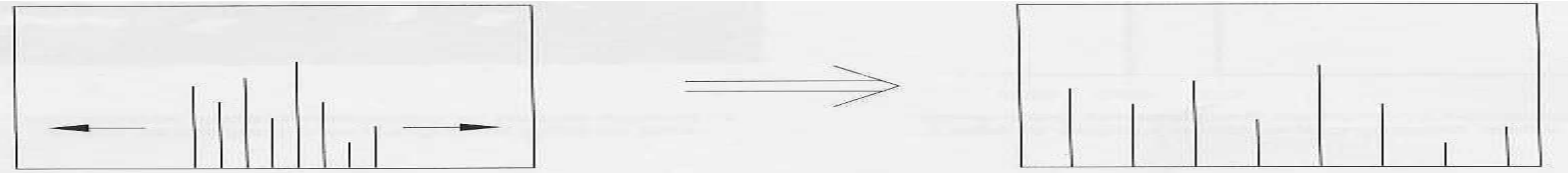
$$\text{Stretch}\,(I\,(r,c)) = \left[\frac{I(r,c) - I(r,c)_{\min}}{I(r,c)_{\max} - I(r,c)_{\min}}\right][MAX-MIN] + MIN$$

$I\,(r,c)_{max} = 6$

$I\,(r,c)_{min} = 3$

MAX=0

MIN=7



a. Histogram stretch.

Solution

$I\,(r, c)_{max}$ = 6 $I\,(r, c)_{min}$ = 3 MAX=7 MIN=0

I (r, c)	Stretch (I (r, c)) = $\left[\frac{I(r,c)-I(r,c)_{min}}{I(r,c)_{max}-I(r,c)_{min}}\right]$ [MAX-MIN] + MIN	Modified Gray Level
3	$\frac{3-3}{6-3} * (7-0)+0$	**0**
4	$\frac{4-3}{6-3} * (7-0)+0$	**2.33~2**
5	$\frac{5-3}{6-3} * (7-0)+0$	**4.67~5**
6	$\frac{6-3}{6-3} * (7-0)+0$	**7**

Modified Histogram

Gray level	0	1	2	3	4	5	6	7
Frequency	50	0	200	0	0	250	0	40

Histogram Equalisation

- Histogram equalization is a process for increasing the contrast in an image by spreading the histogram out to be approximately uniformly distributed
- The gray levels of an image that has been subjected to histogram equalization are spread out and always reach white

– The increase of dynamic range produces an increase in contrast

- For images with low contrast, histogram equalization has the adverse effect of increasing visual graininess

Histogram Equalisation

- The intensity transformation function we are constructing is of the form

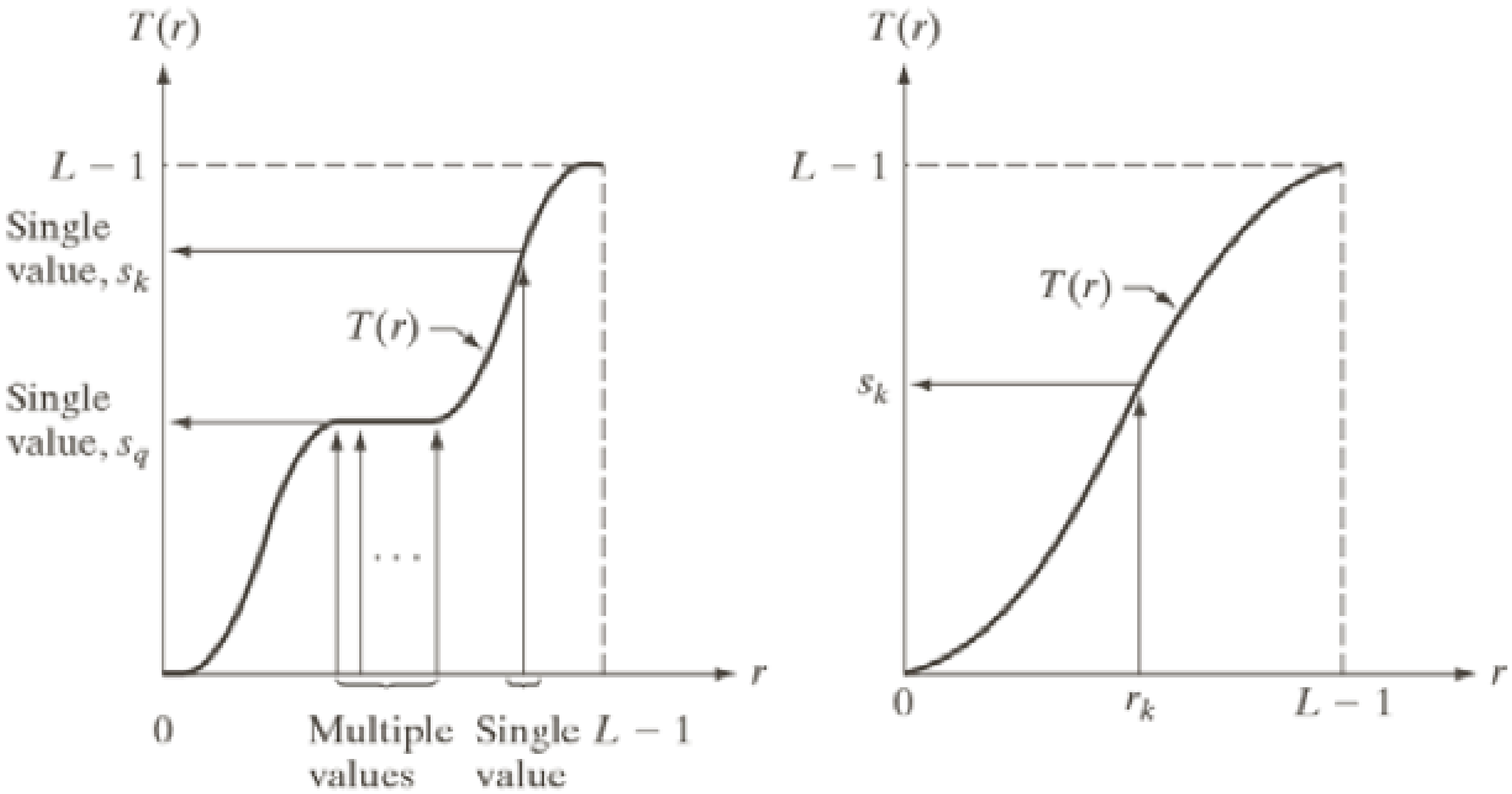
$$s = T(r) \qquad 0 \le r \le L-1$$

- An output intensity level *s* is produced for every pixel in the input image having intensity *r*
- We assume
  - $T(r)$ is monotonically increasing in the interval $0 \le r \le L-1$
    - $0 \le T(r) \le L-1$ for $0 \le r \le L-1$
- If we define the inverse

$$r = T^{-1}(s) \qquad 0 \le s \le L-1$$

- Then $T(r)$ should be strictly monotonically increasing

Histogram Equalisation



a b

FIGURE 3.17 (a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

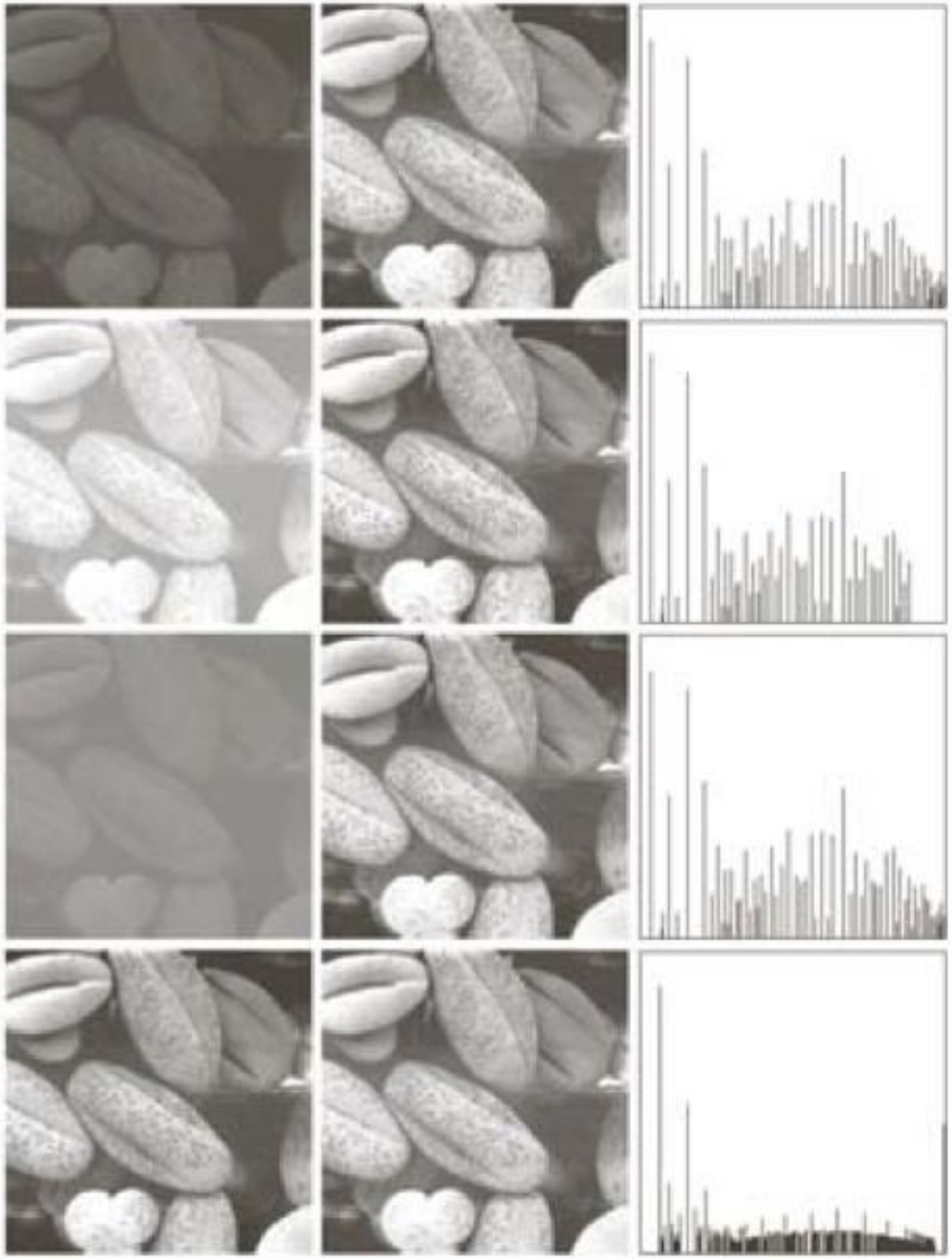
Histogram Equalisation

- Histogram equalization requires construction of a transformation function s_k

$$s_k = T(r_k) = \sum_{j=0}^{k} \frac{n_j}{M \times N} \qquad s_k = T(r_k) = \frac{(L-1)}{M \times N} \sum_{j=0}^{k} n_j$$

- where r_k is the kth gray level, n_k is the number of pixels with that gray level, MxN is the number of pixels in the image, and k=0,1,...,L-1
- This yields an s with as many elements as the original image's histogram (normally 256 for our test images)
- The values of s will be in the range [0,1]. For constructing a new image, s would be scaled to the range [1,256]

Histogram Equalisation



Histogram Equalisation

Spreading out the frequencies in an image (or equalising the image) is a simple way to improve dark or washed out images

The formula for histogram equalisation is given where

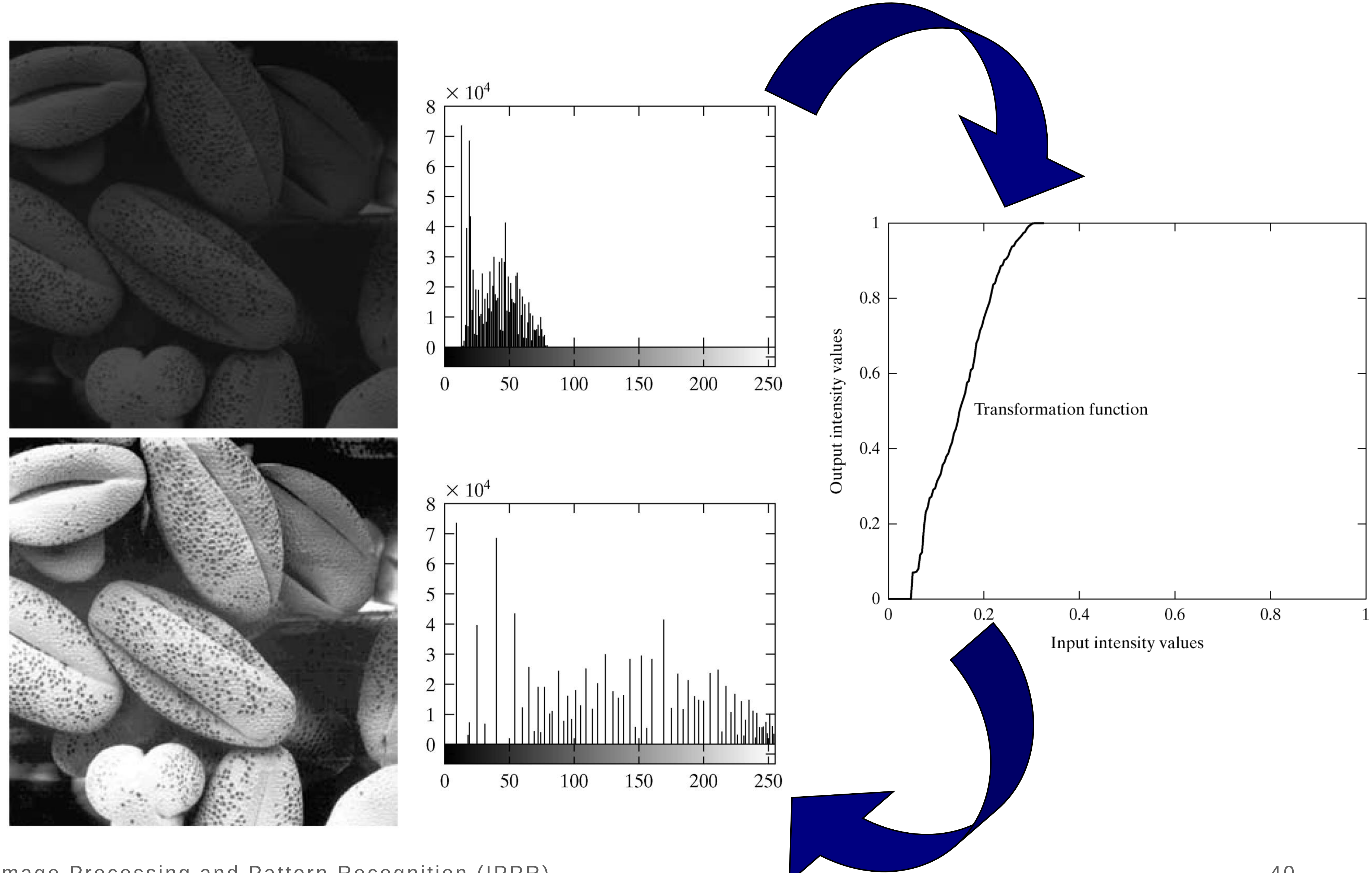
- r_k: input intensity
- s_k: processed intensity
- k: the intensity range (e.g 0.0 – 1.0)
- n_j: the frequency of intensity j
- n: the sum of all frequencies

$$s_k = T(r_k) = \sum_{j=1}^{k} p_r(r_j) = \sum_{j=1}^{k} \frac{n_j}{n}$$

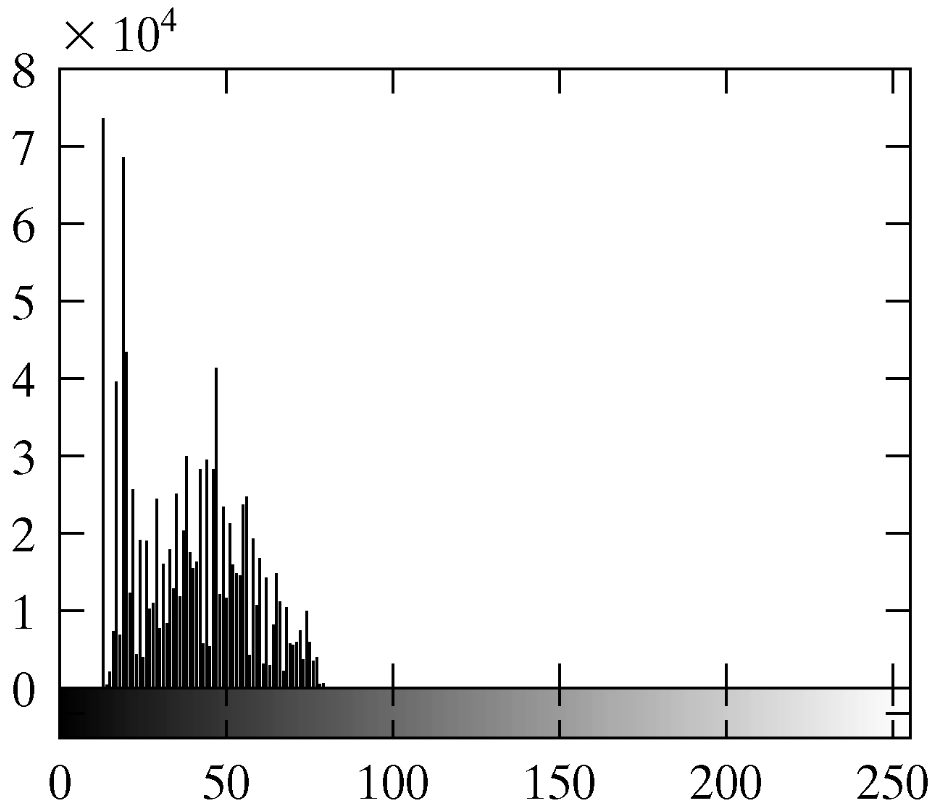
The transformation function in the next slide is Obtained by applying the RHS formula to the Original image in the next side.

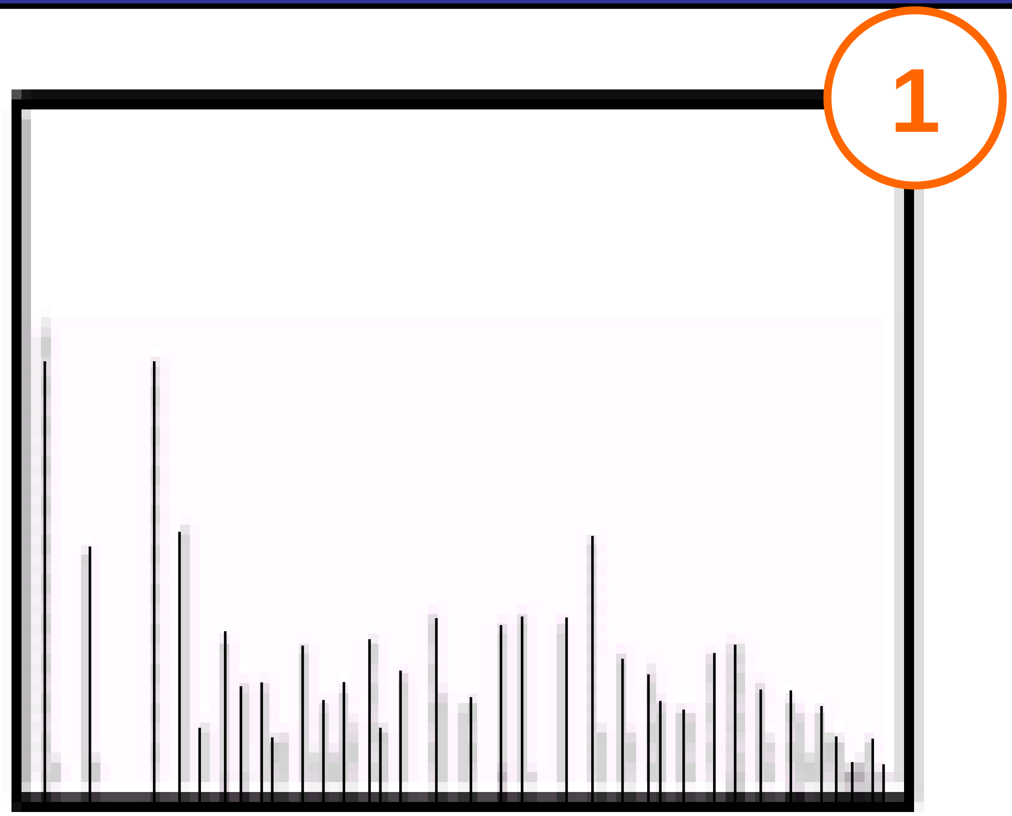
Equalisation Transformation Function

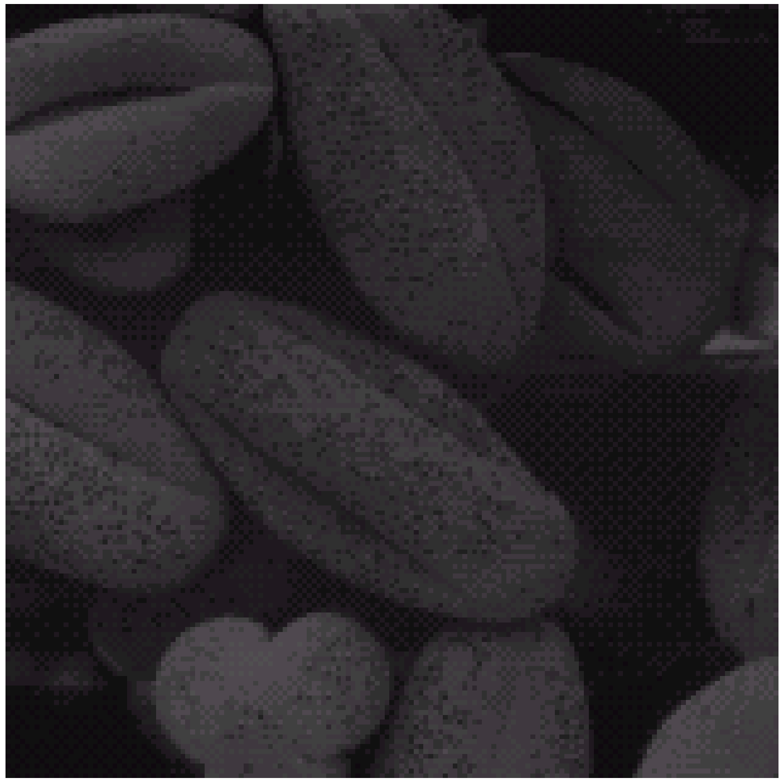
Images taken from Gonzalez & Woods, Digital Image Processing (2002)



Equalisation Examples



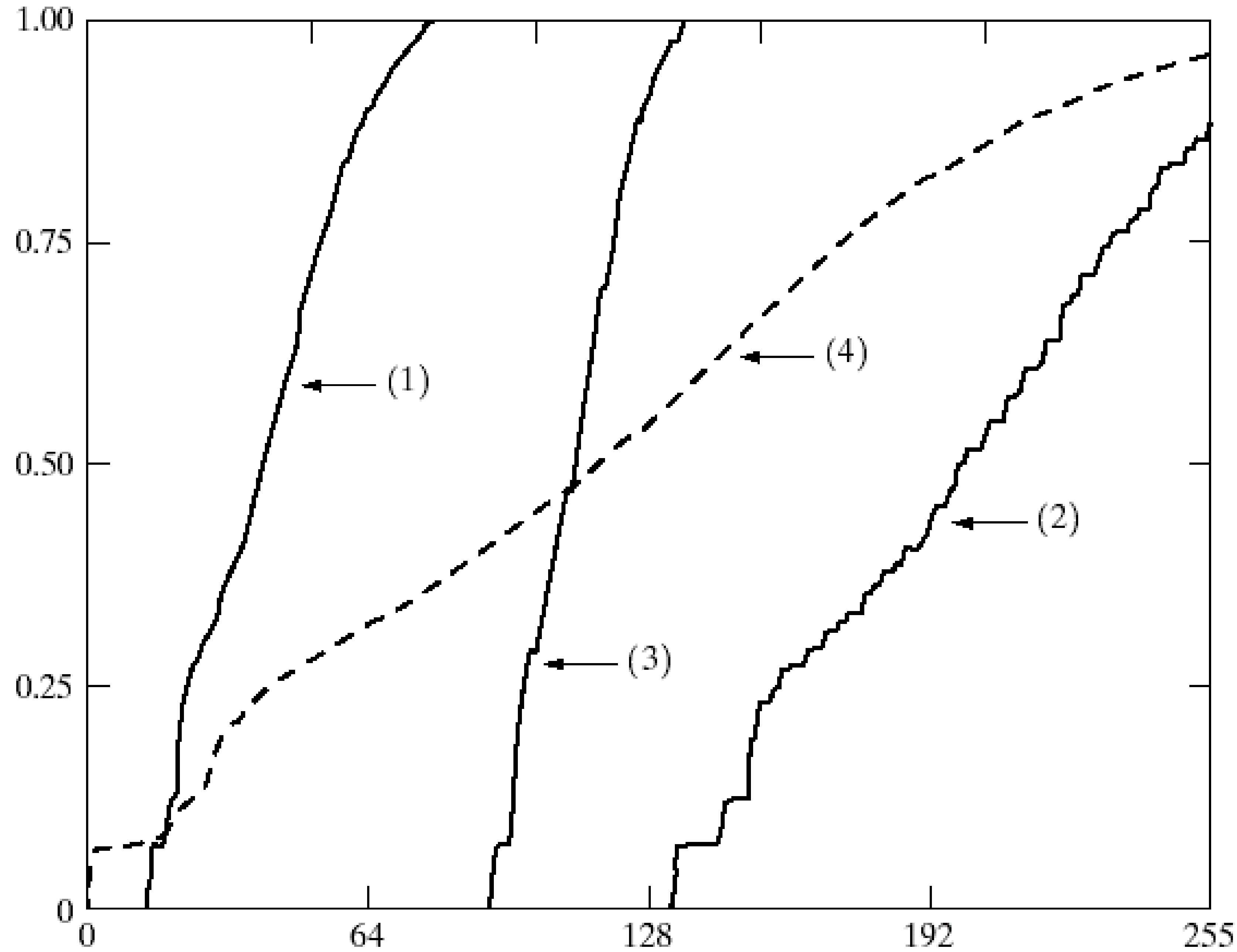




Images taken from Gonzalez & Woods, Digital Image Processing (2002)

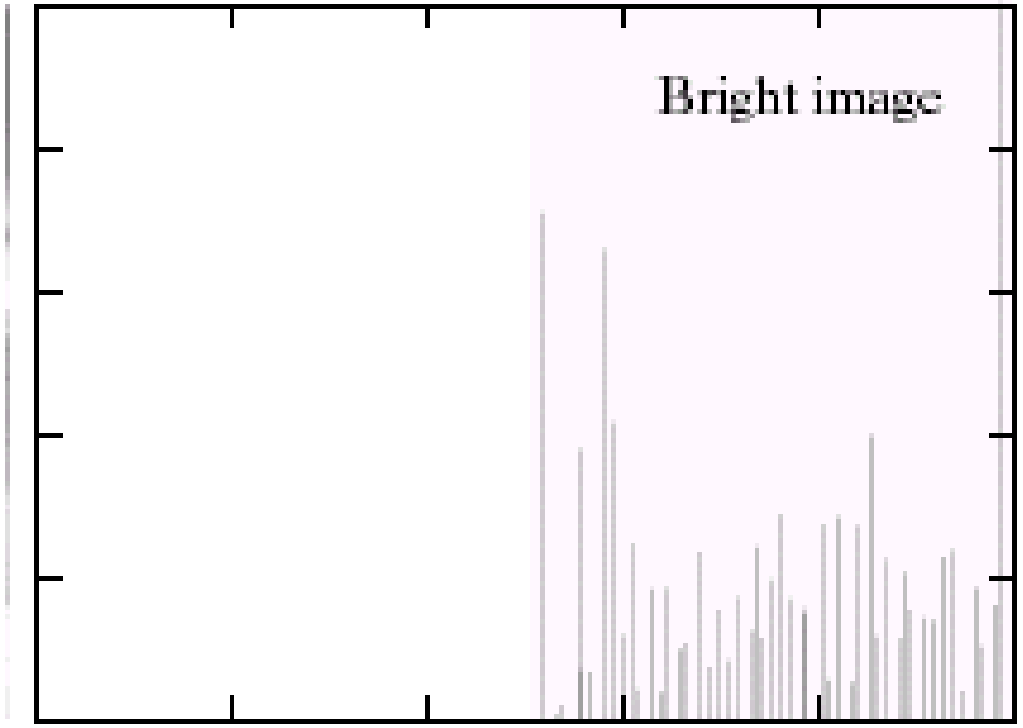
Equalisation Transformation Functions

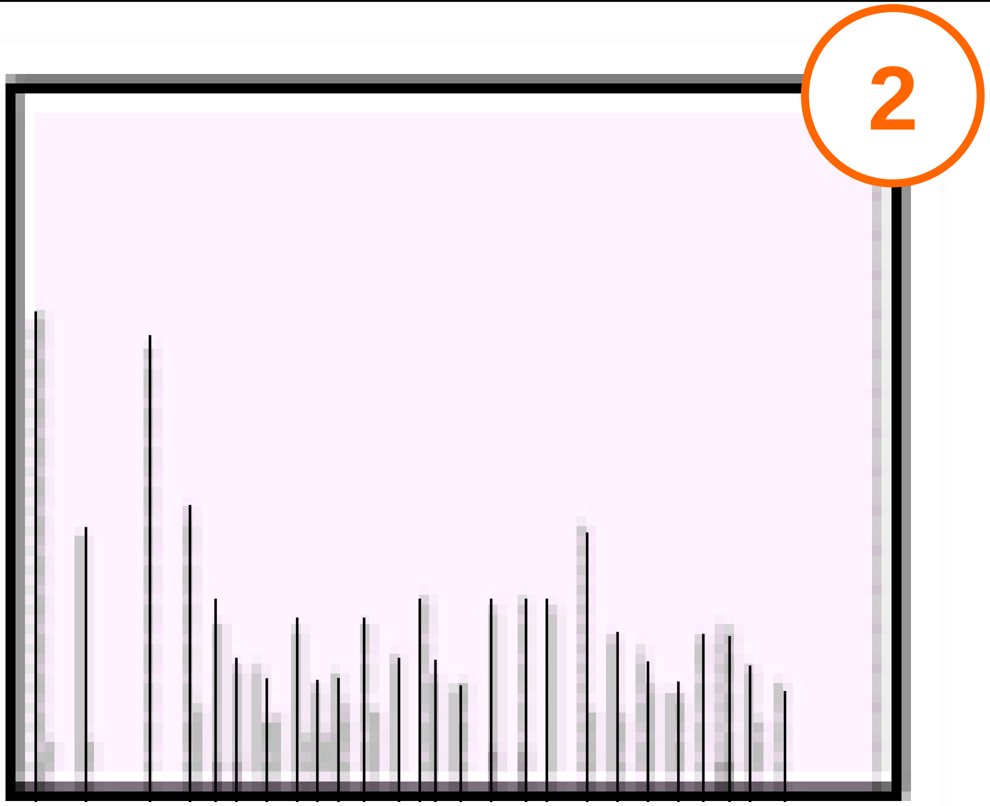
The functions used to equalise the images in the previous example



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Equalisation Examples





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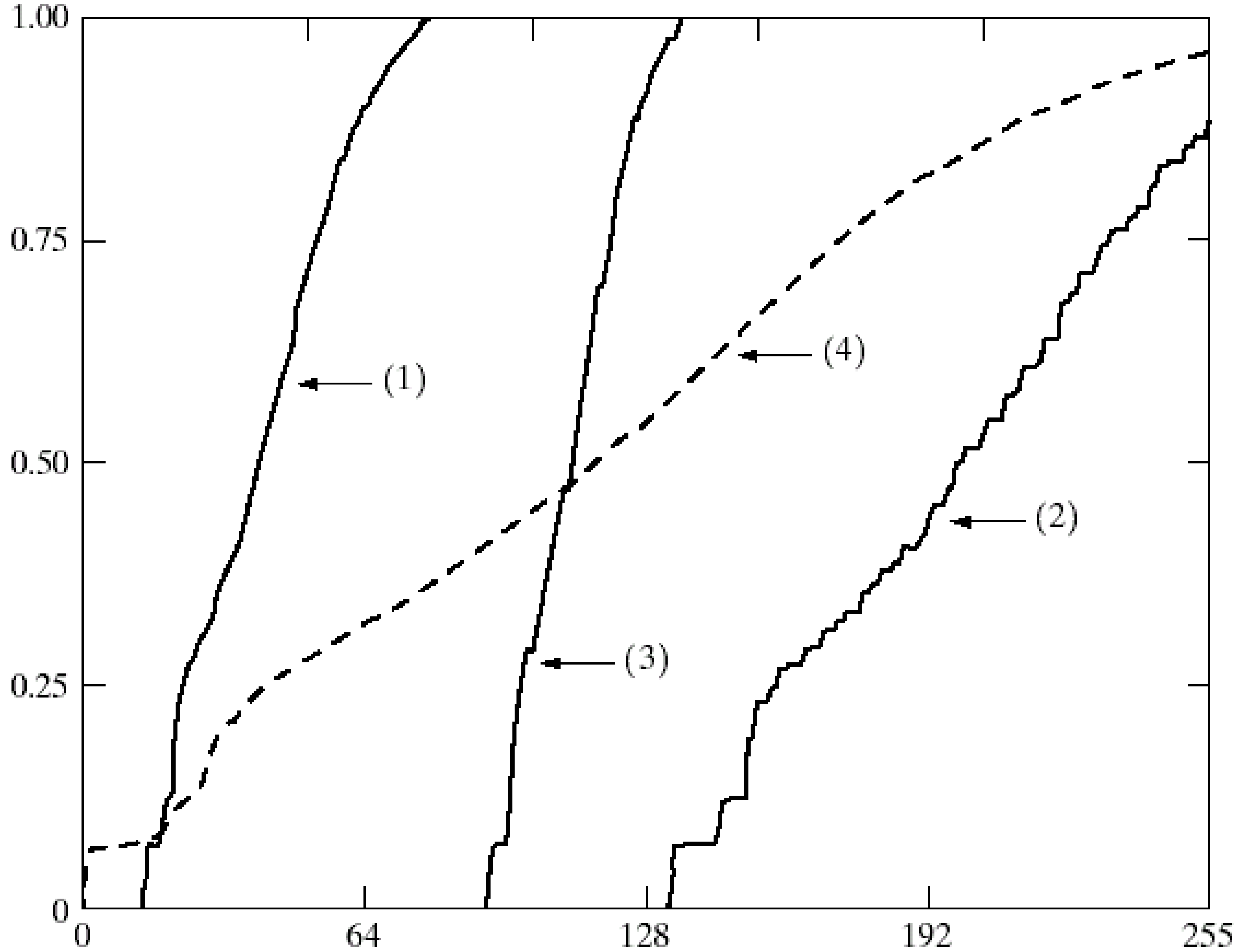
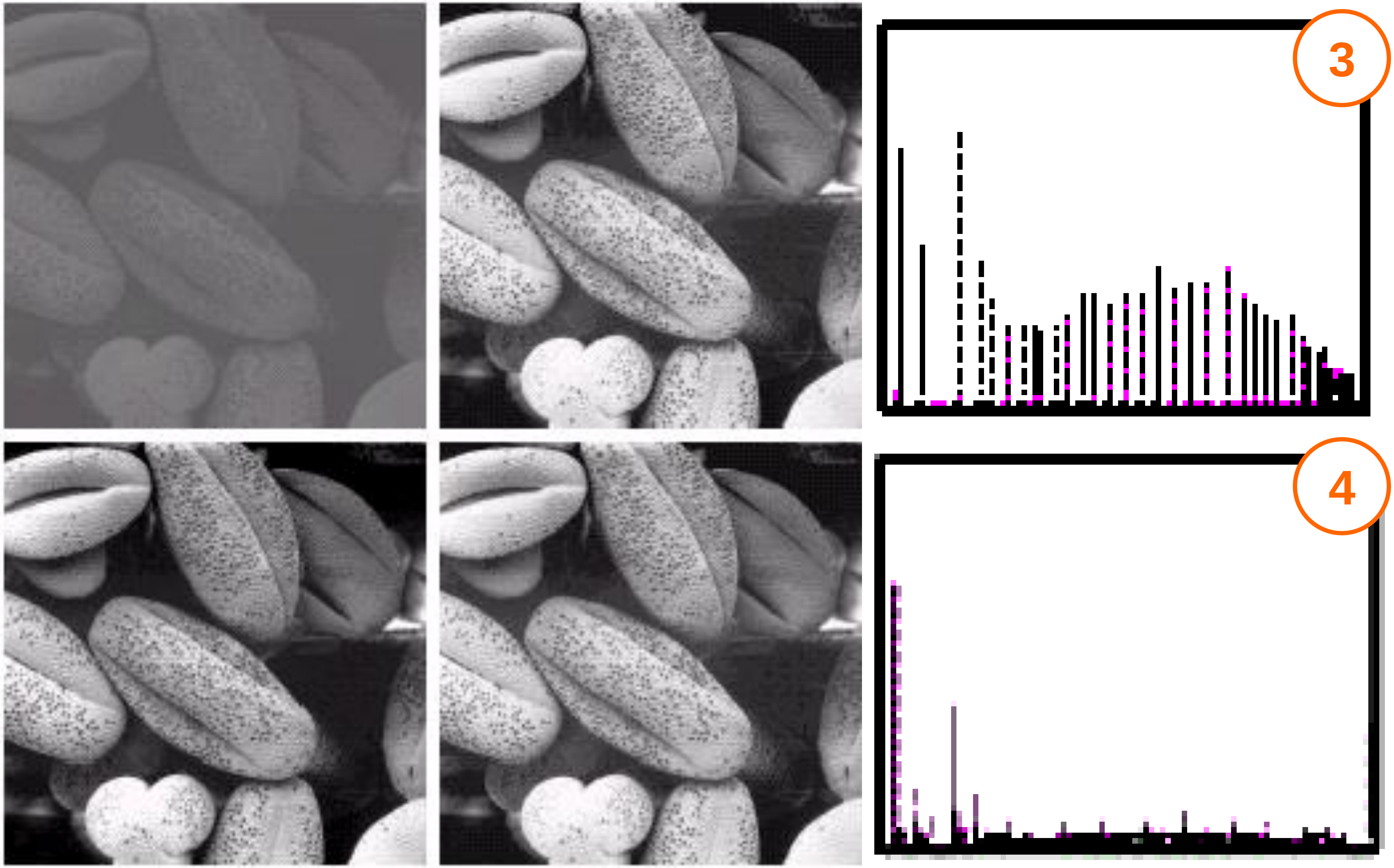


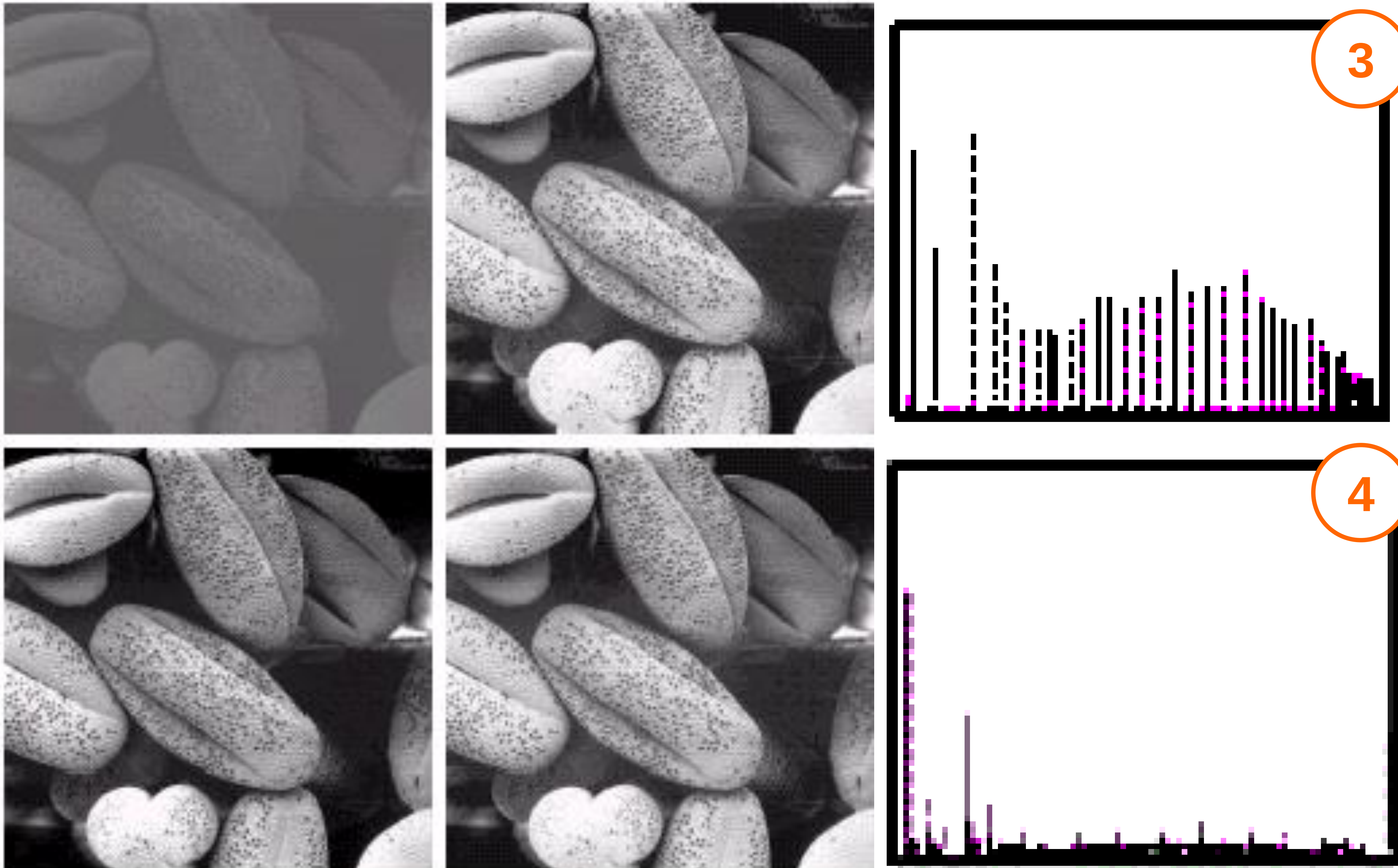
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Equalisation Examples (cont…)



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

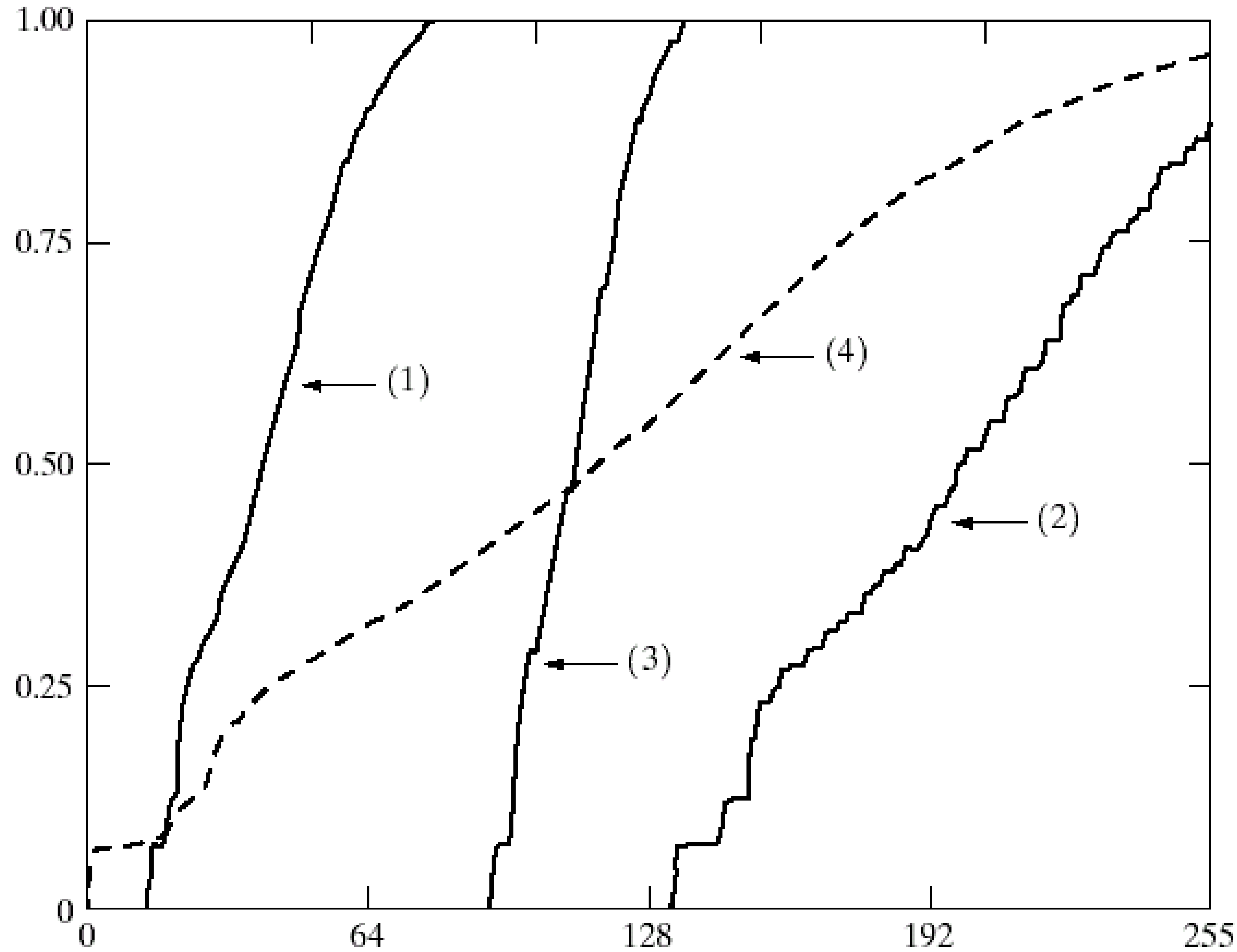
Equalisation Examples (cont…)



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Equalisation Transformation Functions

The functions used to equalise the images in the previous examples



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Problem

•Gray level histogram of an 3-bit image (L=8) of size 64 × 64 pixels (MN = 4096) system is given below

Gray level:	0	1	2	3	4	5	6	7
Frequency:	790	1023	850	656	329	245	122	81

Compute the Histogram equalization.

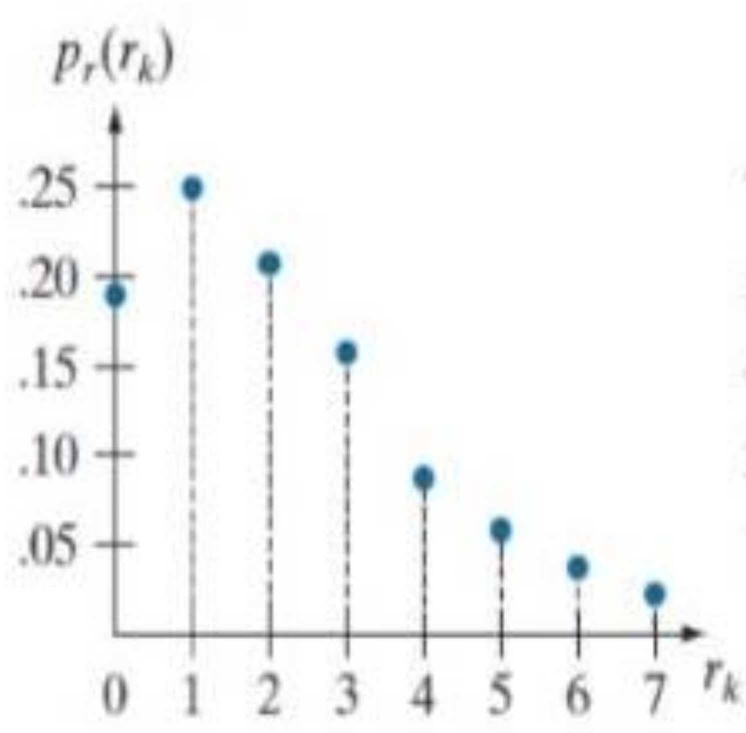
$$s_k = T(r_k) = \frac{(L-1)}{M \times N}\sum_{j=0}^{k} n_j = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

Example: Histogram Equalization

Gray level:	0	1	2	3	4	5	6	7
Frequency:	790	1023	850	656	329	245	122	81

Get the histogram equalization transformation function and give the $p_s(s_k)$ for each s_k.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02



Example: Histogram Equalization

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7 \times 0.19 = 1.33 \quad \rightarrow 1$$

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7 \times (0.19 + 0.25) = 3.08 \quad \rightarrow 3$$

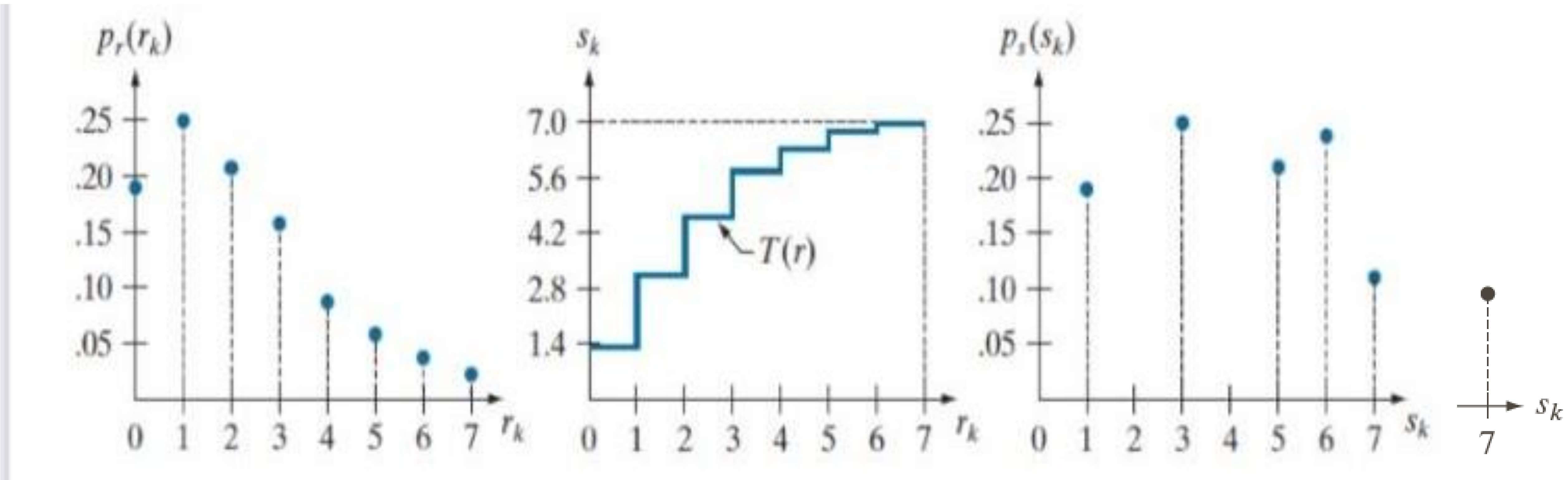
$$s_2 = 4.55 \quad \rightarrow 5 \qquad s_3 = 5.67 \quad \rightarrow 6$$

$$s_4 = 6.23 \quad \rightarrow 6 \qquad s_5 = 6.65 \quad \rightarrow 7$$

$$s_6 = 6.86 \quad \rightarrow 7 \qquad s_7 = 7.00 \quad \rightarrow 7$$

Example: Histogram Equalization

Gray level:	0	1	2	3	4	5	6	7
Frequency:	0	790	0	1023	0	850	985	448+

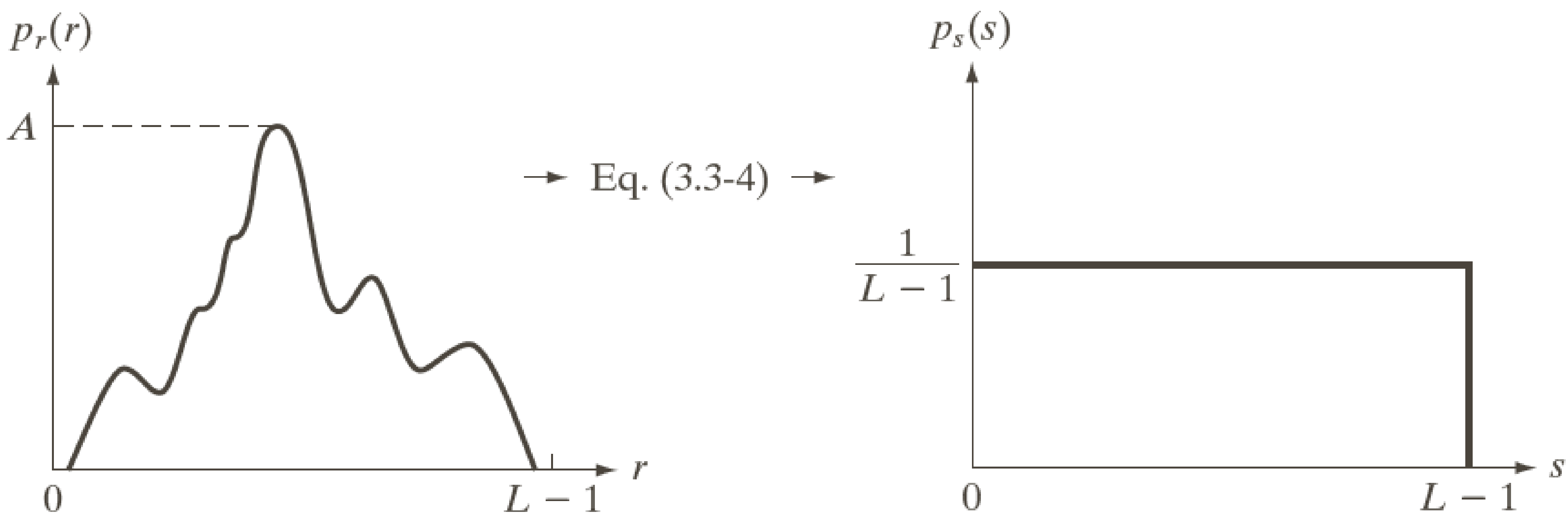


a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Histogram Equalization

- Histogram Equalization or Linearization is used to obtain a uniform histogram.
- Image is not only spread over the dynamic range but also have equal number of pixel in all gray labels.



a b

FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Histogram Specification/ Matching

- Histogram equalization produces (**in theory**) image with uniform distribution of pixel intensities
- **It is sometimes desirable to have some interactive methods in which certain gray labels are highlighted.**
- To enhance image based on a specified histogram: **Histogram Specification**
- **Histogram matching:** **transform** a given **image into a similar image** that has a pre-defined histogram
- A desired histogram can be specified according to various needs
- Allows **interactive image enhancement**

Histogram Matching: Discrete Cases

- Obtain $p_r(r_j)$ from the input image and then obtain the values of s_k, round the value to the integer range [0, L-1].

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j$$

- Use the specified PDF and obtain the transformation function $G(z_q)$, round the value to the integer range [0, L-1].

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i) = s_k$$

- Mapping from s_k to z_q

$$z_q = G^{-1}(s_k)$$

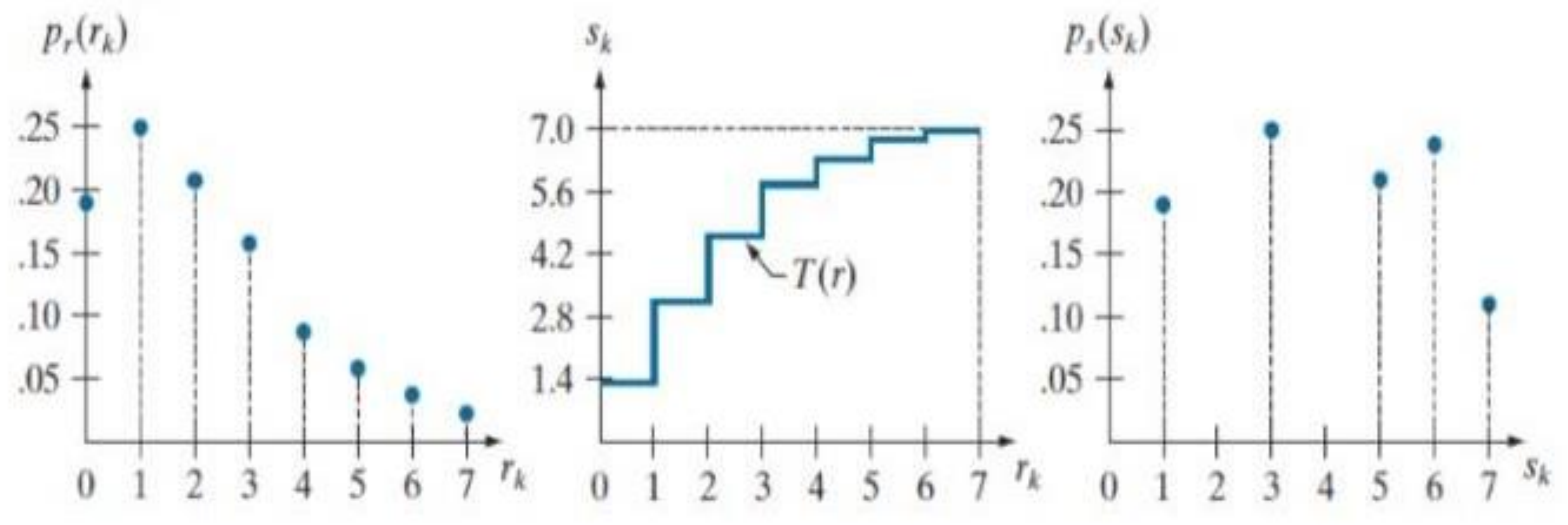
Example: Histogram Matching

Suppose that a 3-bit image (L=8) of size 64 × 64 pixels (MN = 4096) has the intensity distribution shown in the following table (on the left). Get the histogram transformation function and make the output image with the specified histogram, listed in the table on the right.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

Result: Histogram Equalization



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Example: Histogram Matching

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

Obtain the scaled histogram-equalized values,

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$

$$s_5 = 7, s_6 = 7, s_7 = 7.$$

Compute all the values of the transformation function G,

$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j) = 0.00 \quad \rightarrow 0$$

$$G(z_1) = 0.00 \quad \rightarrow 0 \qquad G(z_2) = 0.00 \rightarrow 0$$

$$G(z_3) = 1.05 \quad \rightarrow 1 \qquad G(z_4) = 2.45 \rightarrow 2$$

$$G(z_5) = 4.55 \quad \rightarrow 5 \qquad G(z_6) = 5.95 \rightarrow 6$$

$$G(z_7) = 7.00 \quad \rightarrow 7$$

z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7

TABLE 3.3 All possible values of the transformation function G scaled, rounded, and ordered with respect to z.

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

TABLE 3.4 Mappings of all the values of s_k into corresponding values of z_q.

Example: Histogram Matching

Obtain the scaled histogram-equalized values,

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$
$$s_5 = 7, s_6 = 7, s_7 = 7.$$

Compute all the values of the transformation function G,

$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j) = 0.00 \rightarrow 0$$

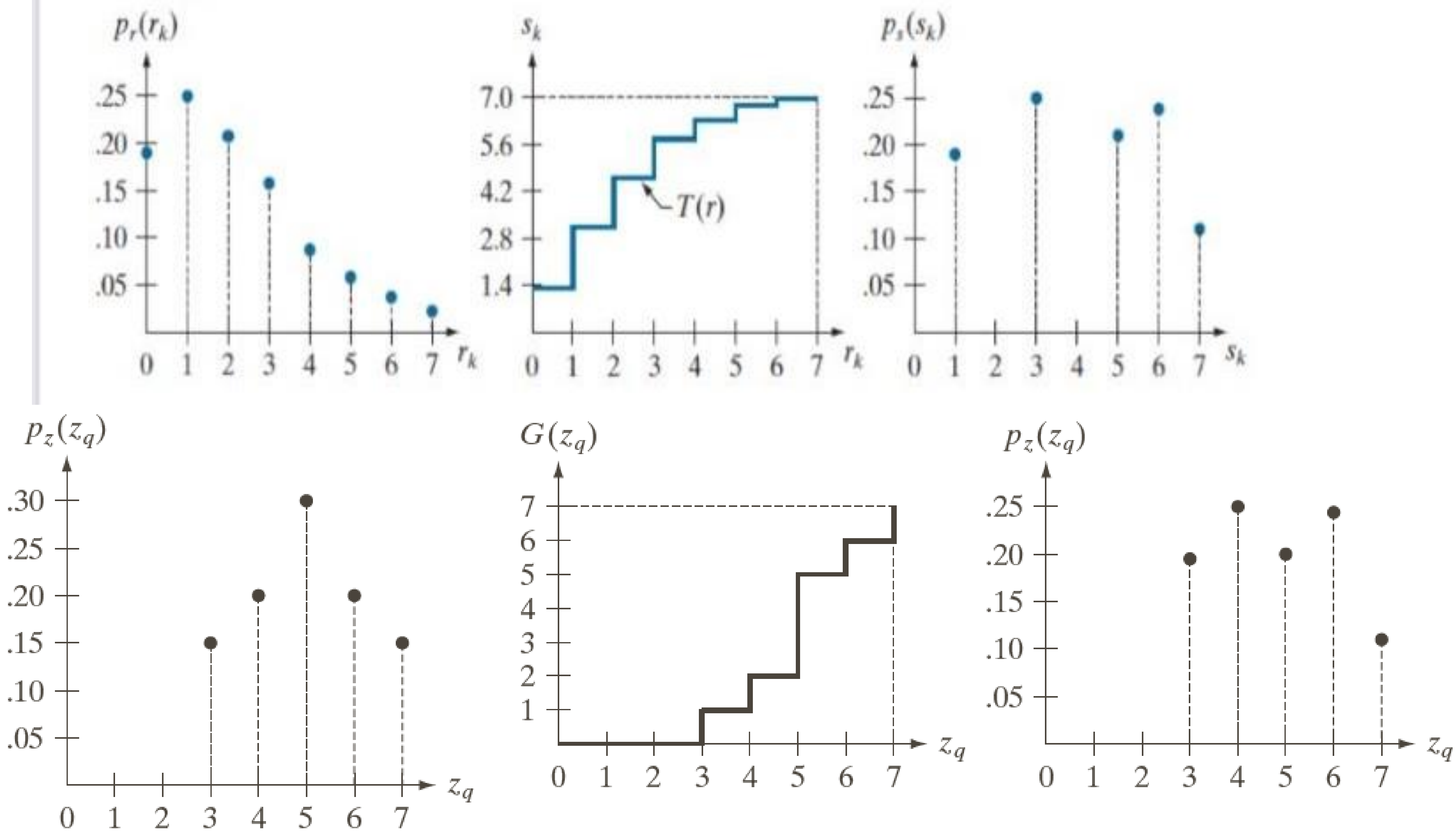
$G(z_1) = 0.00 \rightarrow 0$ $\quad$ $G(z_2) = 0.00 \rightarrow 0$

$G(z_3) = 1.05 \rightarrow 1$ **s_0** $\quad$ $G(z_4) = 2.45 \rightarrow 2$ **s_1**

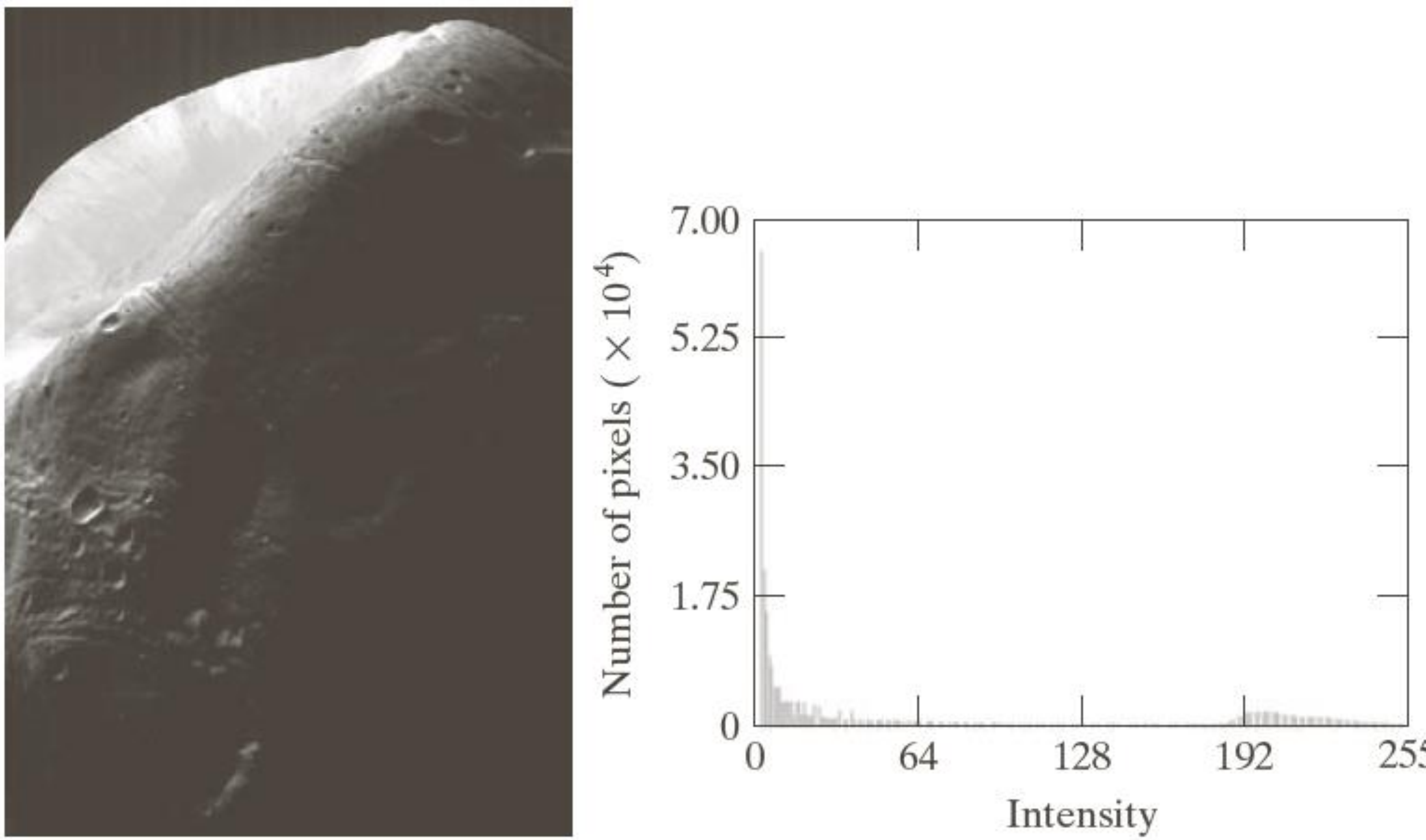
$G(z_5) = 4.55 \rightarrow 5$ **s_2** $\quad$ $G(z_6) = 5.95 \rightarrow 6$ **s_3**

$G(z_7) = 7.00 \rightarrow 7$ **s_4 s_5 s_6 s_7**

Result: Histogram Equalization



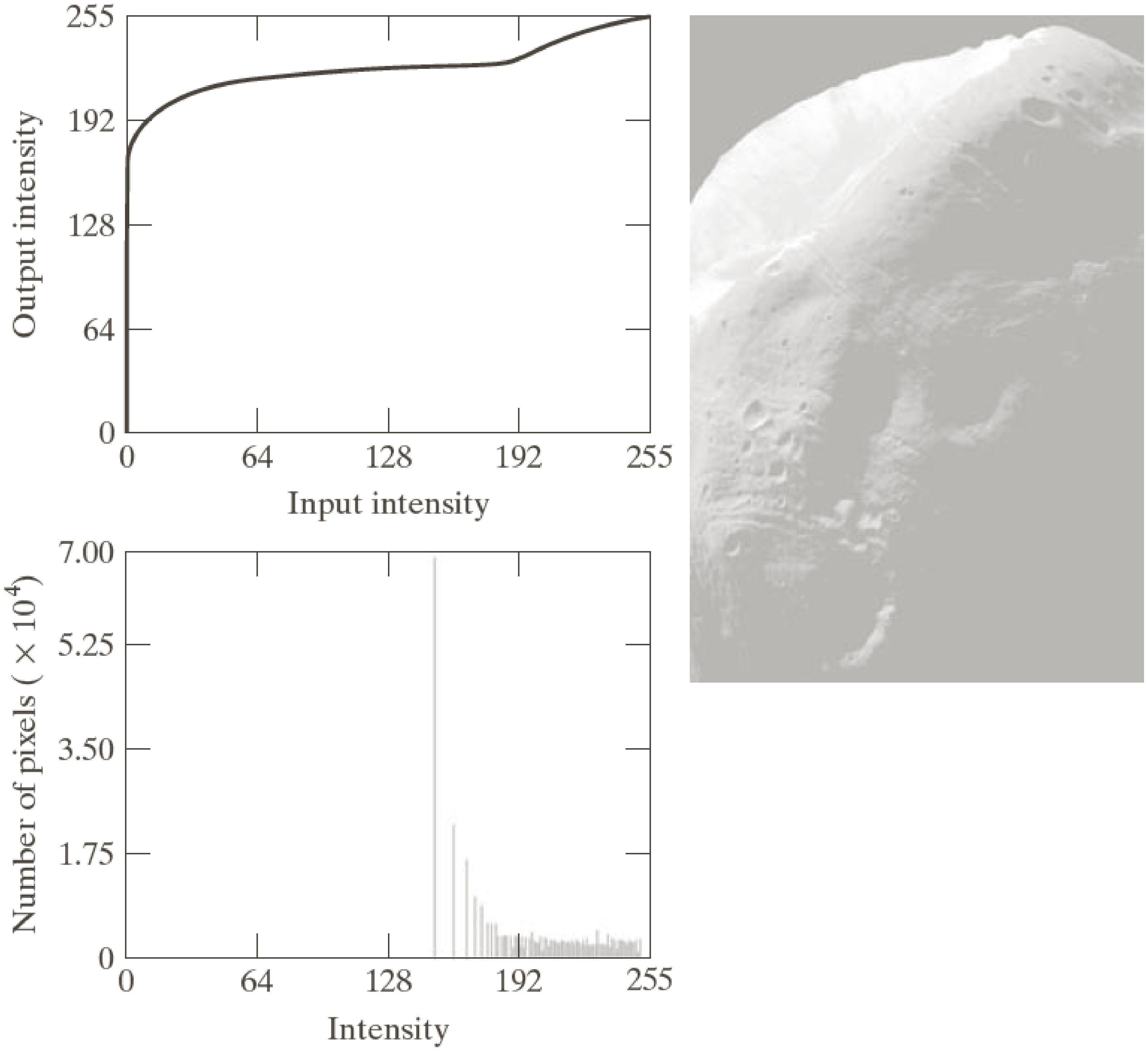
Example: Histogram Matching



a b

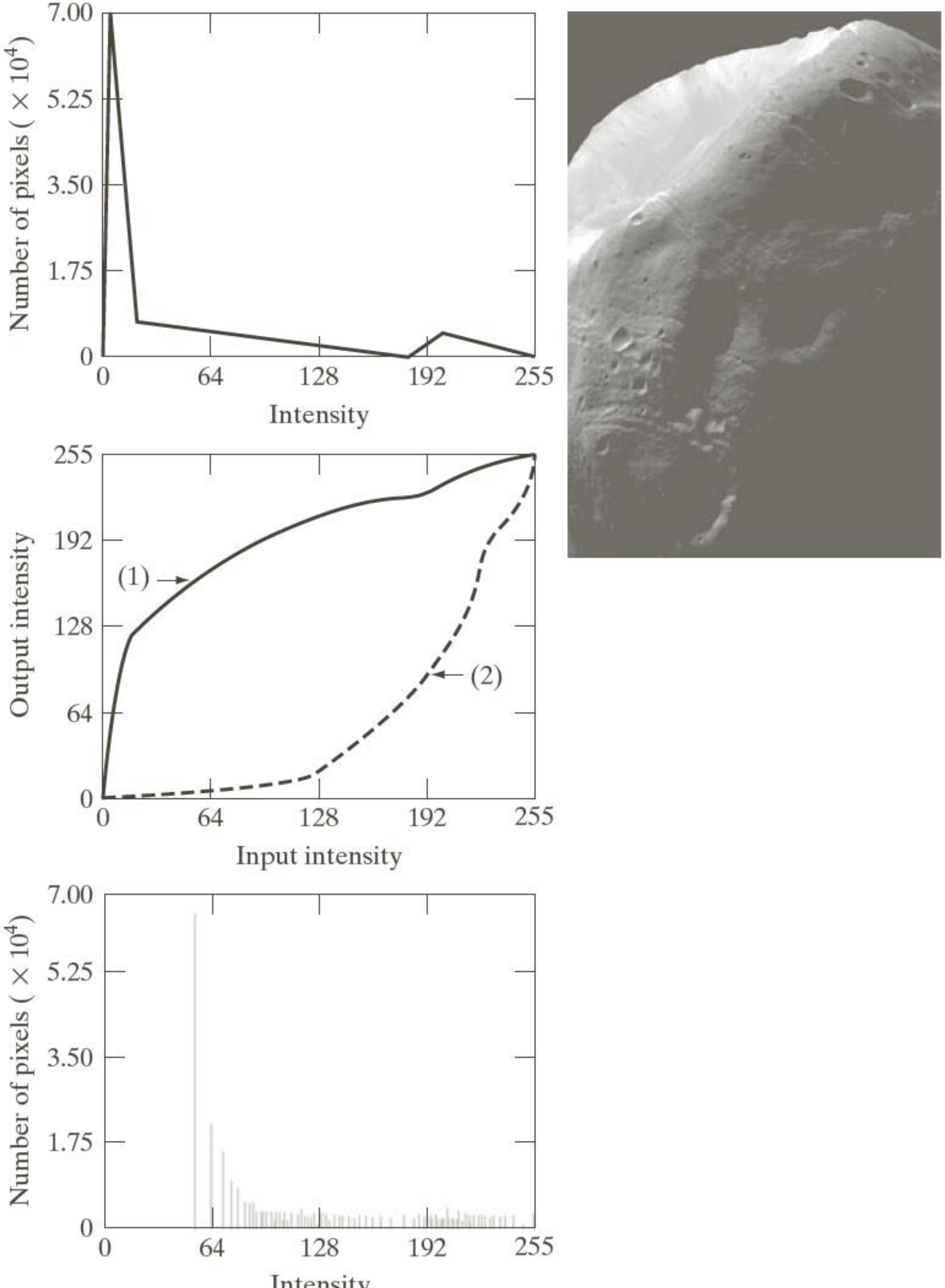
FIGURE 3.23 (a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)

Example: Histogram Matching



a b
c

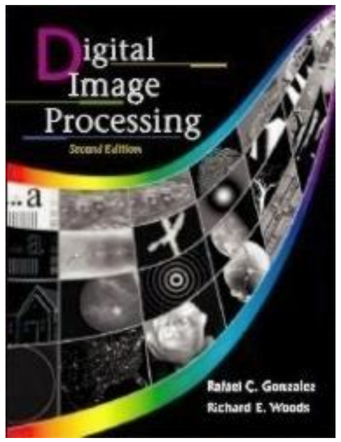
FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).



a c
b
d

FIGURE 3.25
(a) Specified histogram.
(b) Transformations.
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).

References



"Digital Image Processing", Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002

- –Much of the material that follows is taken from this book

–Image Processing and Pattern Recognition Slides of Dr. Sanjeeb Prasad Panday

Thank you !!!