N drones 1 package

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July 1, 2019

Abstract

We consider the problem of package delivery using multiple drones in which each of their speeds can be different. There is one package and one destination, we need to find a good path and combination of drones so that the delivery time is as small as possible.

1 Introduction

Let d_0 to d_{n-1} depict n drones. Their starting positions are p_0 to p_{n-1} and their speeds are v_0 to v_{n-1}. Without the loss of generality, assume that $v_0 < v_1 < ... < v_{n-1}$. The package is located at point q_0 and the location for delivery is q_n. Additionally, assume that v_0 can reach q_0 before any other drone (1). Two drones can exchange the package at some locations q_i. The problem here is to find the ordered set of optimal drones, their paths and their exchange locations so that the delivery time is the smallest. A note here is that in the optimal ordered set of drones will be monotonically increasing in speed as there is no point in handing the package to a slower drone (unless we are taking other factors such as battery or maximum traveling distance into account).

2 Analysis for two drones

Let:

- t_0 be the time for d_0 to reach q_0 to get the package from the warehouse.
- q_1 be the location where d_0 and d_1 meet.
- t_1 be the time for d_0 to fly from q_0 to q_1. It's also the time for d_1 to fly from p_1 to q_1.
- t_2 be the time for d_1 to fly from q_1 to q_n to deliver the package.

The details above are shown in Fig. 1.

The goal is to minimize $t_0 + t_1 + t_2$ or $t_1 + t_2$ because t_0 is a constant under assumption (1). The objective, written in term of speed and locations of the drones, is as follow:

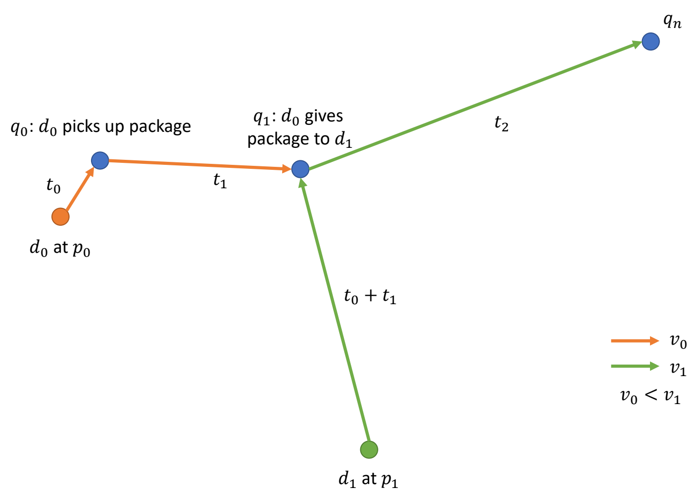

Figure 1: Illustration of the case where there are only two drones.

$$\begin{aligned} \min_{q_1} \ & \frac{||q_1 - p_1||}{v_1} + \frac{||q_1 - q_n||}{v_1} \\ \text{s.t.} \ & \frac{||q_1 - q_0||}{v_0} + t_0 = \frac{||q_1 - p_1||}{v_1} \end{aligned}$$

The constraint is there to ensure that the time it takes for d_0 and d_1 to go from their starting points to q_1 is the same. Let r_{01} be the set of all possible q_1. The set r_{01} is shown as a ring in Fig. 2. Another assumption here is that q_n lies outside of r_{01}, otherwise there is no point in using d_1.

Geometrically, the objective is to find a point q_1 on the ring r_{01} so that the distance $||q_1 - p_1|| + ||q_n - q_1||$ is the smallest.

3 Analysis for three drones

In case there are three drones d_0, d_1 and d_2, one thing we can do at the start is checking if r_{02} is completely contained by r_{01}. If it is, then we don't have to use d_1 since d_2 can meet d_0 faster. Otherwise, we can always use d_1 before d_2 (even if the ordered set (d_0, d_2) gives the best result, inserting d_1 in-between can also give us the same best result). The reasoning is as follow:

4 Analysis for the general case

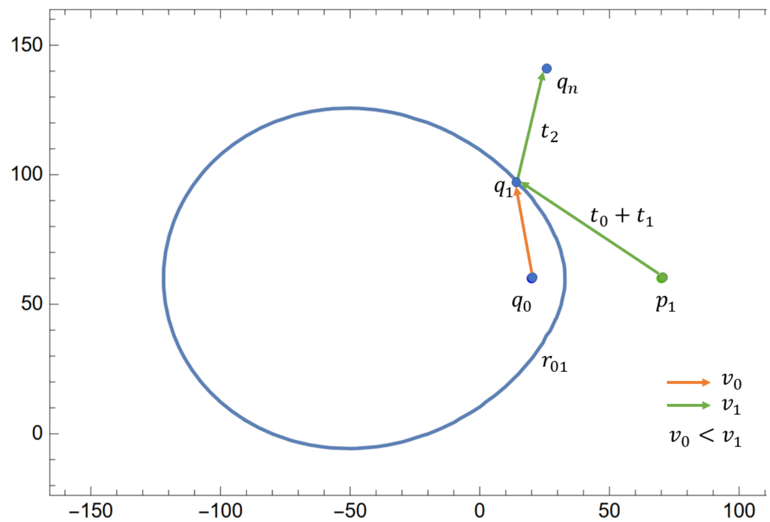

Figure 2: An example constraint: $\frac{1}{10}\sqrt{(x-20)^2+(y-60)^2} + 1.8 = \frac{1}{12}\sqrt{(x-70)^2+(y-60)^2}$, $q_0 = (20,60)$, $p1 = (70,60)$. r_{01} is the set of all possible points where d_0 and d_1 could meet based on their speed.