Introduction to Operations Research (OR) / Management Science (MS)

1. History of MS/OR

As a formal discipline, MS/OR originated in the efforts of military planners during World War II. In the decades after the war, the techniques were more widely applied to problems in business, industry and society. Since that time, operational research has expanded into a field widely used in industries ranging from petrochemicals to airlines, finance, logistics, and government, moving to a focus on the development of mathematical models that can be used to analyze and optimize complex systems, and has become an area of active academic and industrial research.

The term OR was coined during WWII when British military leaders asked scientists and engineers to analyze several military operations such as the deployment of radar and management of convoy, bombing, antisubmarine operations, etc.

During WWII, merchant ships were used to transport soldiers, weapons, medical supplies, etc. Merchant ships are not armed and could be easily attacked by German warships, especially submarines. Britain introduced the convoy system to reduce shipping losses. The principal is to use warships to accompany merchant ships. However, it is unclear whether it was better for convoys to be small or large. Convoys travel at the speed of the slowest member, so small convoys can travel faster. In addition, small convoys might be harder to be detected by German submarines. On the other hand, large convoys could deploy more warships against the attacker. A few large convoys are more defensible than many small ones.

Booming: Britain Royal Air Force conducted bombing raids over Germany during WWII. Over a particular period, the RAF inspected all bombers returning from the bombing raids. All damages caused by German Anti defenses were recorded. The survey report recommended to add armor to the most heavily and frequently damaged areas. Does it make sense? However, OR scientists provided a surprising and counter-intuitive recommendation that the armor be placed in the areas which were completely untouched by damage in the bombers which returned. They reasoned that the survey was biased, since it only included aircraft that returned. The untouched areas of returning aircraft were probably vital areas, which, if hit, would result in the loss of the aircraft.

2. Definition of OR/MS

MS can be described by the alternative name of operational research (OR). Some of you may have met the terms MS, operational/operations research (OR), and OR/MS before. Often some, or all, of these terms are used interchangeably. In this course we use the term MS throughout.

MS can be defined as the application of scientific and systematic procedures, techniques and tools to operational, strategic and policy problems in order to help develop and evaluate solutions to problems encountered within management. MS includes all rational approaches to management decision-making that are based on an application of scientific and systematic methodologies.

It is hard to give a noncontroversial definition of MS. There are many versions of definition of MS. This is a rather new field that is renewing itself and changing constantly. It has benefited from contributions originating in the social and natural sciences, econometrics, and mathematics, much of which escape the rigidity of a definition. Nonetheless it is possible to provide a general statement about the basic elements of the management-science approach.

> MS is characterized by the use of <u>mathematical models (scientific methods)</u> in providing guidelines to managers for <u>making effective decisions</u> within the state of the current information, or in seeking further information if current knowledge is insufficient to reach a proper decision in <u>management</u>.

First, the essence of MS is the <u>model-building approach</u>—that is, an attempt to capture the most significant features of the decision under consideration by means of a mathematical abstraction. Models are simplified representations of the real world. Models have to provide a complete and realistic representation of the decision environment by incorporating all the elements required to characterize the essence of the problem under study. This is not an easy task since we need the information and data, which is hard to obtain and might be even not available. But, if done properly, it will supply managers with a formidable tool to be used in complex decision situations. It is the complexity of the decision under study, and not the tool being used to investigate the decision-making process, that should determine the amount of information needed to handle that decision effectively.

Second, through this model-design effort, MS tries to provide guidelines to managers or, in other words, to increase managers' understanding of the consequences of their actions. There is never an attempt to replace or substitute for managers, but rather the aim is to <u>support management decisions</u>. It is critical, then, to recognize the strong interaction required between managers and models. Models can expediently and effectively account for the many interrelationships that might be present among the alternatives being considered, and can explicitly evaluate the economic consequences of the actions available to managers within the constraints imposed by the existing resources and the demands placed upon the use of those resources. Managers, on the other hand, should formulate the basic questions to be addressed by the model, and then interpret the model's results in light of their own experience and intuition, recognizing the model's limitations. The complementarity between the superior computational capabilities provided by the model and the higher judgmental capabilities of the human decision-maker is the key to a successful MS approach.

3. Scope and tools of MS/OR

The management scientist's mandate is to use rational, systematic, science-based techniques to inform and improve decisions of all kinds, with strong links with economics, business, engineering, and other sciences. Of course, the techniques of MS are not restricted to business applications but may be applied to military, medical, public administration, charitable groups, political groups or community groups.MS is concerned with developing and applying scientific modeling and concepts that may prove useful in helping to illuminate management issues and solve managerial problems, as well as designing and developing new and better models of organizational excellence.

Applications of MS are abundant in industry as airlines, manufacturing companies, service organizations, military branches, and in government. The range of problems and issues to which MS has contributed insights and solutions is vast. It includes:

- managing inventory to minimize the inventory levels,
- determining the routes of school buses so that as few buses are needed as possible,
- scheduling airlines, including both planes and crew,
- scheduling manufacturing steps,
- deciding the place to site new facilities such as a warehouse, factory or fire station,
- managing the flow of water from reservoirs,
- designing the layout of a factory for efficient flow of materials,
- identifying possible future development paths for parts of the telecommunications industry,
- establishing the information needs and appropriate systems to supply them within the health service,
- identifying and understanding the strategies adopted by companies for their information systems, and so on.

MS uses various scientific research-based principles, strategies, and analytical methods including mathematical model, statistics and numerical algorithms to improve an organization's ability to enact rational and meaningful management decisions by arriving at optimal or near optimal solutions to complex decision problems.

Table 1 Related fields in MS

Mathematical programming	Forecasting	Inventory theory
Meta-heuristics	Scheduling	Revenue management
Probability and statistics	Queuing theory	Supply chain management
Game theory	Business Analytics	Reliability engineering
Graph theory	Business process management	Maintenance engineering
Network	Logistics	Quality Management
Data mining	Project management	System engineering
Stochastic models	Policy analysis	Management engineering
Stochastic processes	Social network	Managerial economics
Decision analysis	Behavioral operations research	Financial engineering
Simulation	Econometrics	Marketing Engineering
	Transportation	…

Introduction to Optimization or Mathematical Programming (MP)

Optimization or mathematical programming is only a tool of the broad discipline known as MS. Mathematical programming, and especially linear programming, is one of the best developed and most used branches of MS. It concerns the optimum allocation of limited resources among competing activities, under a set of constraints imposed by the nature of the problem being studied. These constraints could reflect financial, technological, marketing, organizational, or many other considerations. In broad terms, mathematical programming can be defined as a mathematical representation aimed at programming or planning the best possible allocation of scarce resources.

The critical emphasis in the part is on developing those principles and techniques that lead to good formulations of actual decision problems and solution procedures that are efficient for solving these formulations.

Definition of mathematical programming

Optimization, or **mathematical programming**, refers to the study of problems in which one seeks to minimize or maximize a real function by systematically choosing the values of real or integer variables from an allowed set. An **optimization problem** or a **mathematical programming problem** (a term not directly related to computer programming, but still in use for example in linear programming) can be formulated as

Given: a function $f : X \to \mathbf{R}$ from some set X to the real numbers
Sought: an element $\mathbf{x}_0$ in X such that $f(\mathbf{x}_0) \leq f(\mathbf{x})$ for all $\mathbf{x}$ in X ("minimization") or such that $f(\mathbf{x}_0) \geq f(\mathbf{x})$ for all $\mathbf{x}$ in X ("maximization").

- **x**: **decision variables**
- The function f is called an **objective function**, or cost function. A feasible solution that minimizes (or maximizes, if that is the goal) the objective function is called an **optimal solution**.
- X: The **domain** X of f is called the **search space** or **feasible region (set)** or **solution space**. Typically, X is some subset of the Euclidean space $\boldsymbol{R}^n$, often specified by a set of **constraints**, equalities or inequalities that the members of X have to satisfy.
- The elements of X are called **feasible solutions**.

Many real-world and theoretical problems may be modeled in this general framework.

A simple example of MP

In management decision making it is helpful to consider whether we are deciding under conditions of certainty or uncertainty. The Two Toys problem considered below was one where we had certainty, we knew all data values precisely. Uncertainty can arise for two basic reasons:

- we are not sure of the exact numeric value for a data item
- probability (chance) plays a natural role in the decision problem

As to the first of these, this might occur if we were unsure as to the precise costs per train and solider in the Two Toys problem and only had imprecise information as to these costs. As to the second of these, this might occur if we had a decision problem relating to the price to bid on a contract and we are unsure, for a given price, of the probability that our bid would be accepted.

It is sometimes believed in management that decision problems only arise due to uncertainty. This is not true. As the Two Toys problem indicates it may be difficult to make a decision even when we are absolutely certain about everything. For example, do you know what the maximum profit solution to that problem is?

The inclusion of uncertainty (or not) generally makes a substantial difference in the type and complexity of the techniques that are employed. Problems involving uncertainty are inherently more difficult to formulate well and to solve efficiently. But you will see many examples throughout this subject guide where we are certain about the problem being considered and all data items, but still have a difficult decision problem to solve.

Two Toys Problem. Giapetto's Woodcarving, Inc., manufactures two types of wooden toys: soldiers and trains. A soldier sells for $27 and uses $10 worth of raw materials. Each soldier that is manufactured increases Giapetto's variable labor and overhead costs by $14. A train sells for $21 and uses $9 worth of raw materials. Each train built increases Giapetto's variable labor and overhead costs by $10. The manufacture of wooden soldiers and trains requires two types of skilled labor: carpentry and finishing. A soldier requires 2 hours of finishing labor and 1 hour of carpentry labor. A train requires 1 hour of finishing labor and 1 hour of carpentry labor. Each week, Giapetto can obtain all the needed raw material but only 100 finishing hours and 80 carpentry hours. Demand for the trains is unlimited. But at most 40 soldiers are bought each week. Giapetto wishes to maximize weekly profit (revenues-costs). Formulate a mathematical model of Giapetto's situation that can be used to maximize Giapetto's weekly profit.

Major subfields

Optimization models can be subject to various classifications from different points of view (i.e., objective function, constraints, variables, and parameters).

- **Linear programming** studies the case in which the objective function *f* is linear and the set *X* is specified using only linear equalities and inequalities. Such a set is called a polyhedron or a polytope if it is bounded.
- **Nonlinear programming** studies the general case in which the objective function or the constraints or both contain nonlinear parts.
- **Convex programming** studies the case when the objective function is convex and the constraints, if any, form a convex set. This can be viewed as a particular case of nonlinear programming or as generalization of linear or convex quadratic programming.
- **Quadratic programming** allows the objective function to have quadratic terms, while the set *X* must be specified with linear equalities and inequalities.
- **Integer programming and mixed integer programming** studies programs in which some or all variables are constrained to take on integer values. When there are some integer variables and some continuous variables in the problem, the optimization model is said to be mixed. In general, problems with integer variables are significantly more difficult to solve than those with continuous variables.
- **Combinatorial optimization** is concerned with problems where the set of feasible solutions is discrete or can be reduced to a discrete one. Refer to the typical problems in combinatorial optimization, such as traveling salesman problem (TSP) and the minimum spanning tree problem (MST).
- **Network models** are of special structures and can be solved by adequate computational techniques.
- **Stochastic programming** studies the case in which some of the constraints or parameters depend on random variables. If the parameters are specified as uncertain quantities, whose values are characterized by probability distributions, the optimization model is said to be stochastic. Although important theoretical and practical contributions have been made in the areas of stochastic programming, there are still no effective general procedures that cope with these problems.
- **Robust programming** is, as stochastic programming, an attempt to capture uncertainty in the data underlying the optimization problem. This is not done through the use of random variables, but instead, the problem is solved taking into account inaccuracies in the input data.

- **Dynamic programming** is a method for solving a complex problem by breaking it down into a collection of simpler subproblems, solving each of those subproblems just once, and storing their solutions – ideally, using a memory-based data structure. The next time the same subproblem occurs, instead of recomputing its solution, one simply looks up the previously computed solution, thereby saving computation time at the expense of a (hopefully) modest expenditure in storage space. (Each of the subproblem solutions is indexed in some way, typically based on the values of its input parameters, so as to facilitate its lookup.) The technique of storing solutions to subproblems instead of recomputing them is called "memorization". Dynamic programming algorithms are often used for optimization.

- etc.

4. Methodological issues in MS/OR (Anderson, Chapter 1, start– Section 1.4)

There are a number of methodological issues that arise in MS work that we need to consider. These relate to:

- consultancy
- cost versus decision quality
- optimization
- implementation

Consultancy

It often happens that there is a client who has some problem on which they need help and they decide to call in an 'expert' to provide that help. The 'expert' is called a consultant and the process in which they engage (tackling the client's problem) is called consultancy. Clearly a client might well engage an MS worker as a consultant (for example, because the MS worker has skills that the client lacks). Clients can be drawn from a wide range of organizations (for example, private companies, public companies, and governmental departments).

There is no consultant without a client, and the consultant needs to be clear who the client is, the nature of the problem, and what kind of help is needed. Often the answer to these questions will be covered in some form of contract between the client and consultant. However, this may change over the course of the project and therefore should be kept under review. Close contact between the client and the consultant is a key determinant of the success of the project and will be discussed further below.

Problems have a number of characteristics:

- things are not as they should be, or understanding is incomplete
- the problem owner wants to do something about it
- the problem owner either: does not know what to do or knows what needs to be done but lacks the time/skills to do it themselves.

The client should be the 'problem owner' (that is, the person or group who is responsible and who can make changes). However, it may be that a consultant is called in by another person, and in these circumstances the consultant will need to address the 'problem owner' through the client. In addition, the consultant should be aware what changes the client can feasibly make. Some problems cross departmental boundaries, and the client may only be able to make changes in their department.

In situations where the client is not the person commissioning the work or where the problem crosses departmental boundaries, the consultant must ensure that they can give advice on the options available so that the client can make changes. Otherwise, the consultant's work is likely to be in vain.

The consultant therefore helps the client decide what to do. Defining the problem should involve finding and agreeing some activity that will be useful in helping the client decide what to do. Note that problems are subjective, and therefore so are solutions. It is the client's problem and the desired solution should be the client's, not the consultant's. The consultant must respond to the client's concerns and value systems, or risk the whole enterprise – these can be debated/negotiated. However, ultimately the client decides.

Clearly the consultant needs to acquire an understanding of the context in which the problem is set. There is usually some obvious technical context that needs to be understood (for example, the client's organization is providing services of a particular type, using these particular resources, to particular customers, and it is, for example, a non-profit making organization). There is also the social context (who is involved/affected and how these things are articulated, how the actors interact with one another, etc.); and a cultural one (what rules and beliefs are core to the client/ organization, what is the power pattern, how do things get done, etc.).

An organization or individual may employ consultants because:

- they lack the skills within the organization to find a resolution to the problem: the consultant is the expert
- they lack the time/resources to find a resolution to the problem: the consultant as a hired body or temporary employee
- they need an 'independent' person to help resolve the problem: either to act as an arbitrator between two or more groups, or to provide external justification for a decision, or to audit recommendations of an internal project
- they need to be seen to be doing something and employing a consultant/firm of consultants will provide a positive image
- they need a 'scapegoat', someone else to blame if things go wrong.

Obviously in any particular case there may be a mix of reasons for employing a consultant rather than any single reason.

Organizations may have their own internal group of consultants or they may employ an outside consultancy firm. The internal consultant has many advantages (e.g. shared organizational objectives, familiarity with context, probable ease of working relationships) but may not be seen as independent and may not have special skills. External consultants may be employed as perceptibly independent and for their special technical skills or relevant previous experience (e.g. with similar organizations in other countries), but their value system may differ from the client's (e.g. their goal may be to maximize their own profit).

Cost versus decision quality

Consider a simple decision problem, should you ask someone to marry you or not? You could reach a solution to this decision problem very easily (very quickly and cheaply) by tossing a coin – heads I ask, tails I do not. Alternatively, you might consider it worthwhile spending time and money finding out whether you are compatible before deciding whether or not to ask that person to marry you. The point is perhaps clear – if we wish to reach good quality decisions then we have to take our time and incur costs.

MS projects use resources – the consultant's, the client's, other staff's – and there is always a tension between minimizing the cost and time taken to reach a decision and making the 'perfect' decision. This tension comes in the development of the model, in the collection of the data used in the model, and in the accuracy of the results.

Models are representations of the system under investigation, but are simplified representations. The more time, and cost, put into building a model, the more accurately it should represent the system. The model is also likely to be more complex, including more variables or more relationships between variables, which is likely to make it less accessible to the client and likely to take longer to solve on the computer. Therefore there is a trade-off between an accurate model and the cost and time involved in building the model. The same is true for the collection of data used in the model. It is always possible to collect more data, and spend more time validating the data, and the more time and effort spent in this area the more accurate the data are likely to be.

Note here that although the issue of the time taken to solve a model on a computer is less important today than it was in the past (with modern PCs), there are still many applications where solutions to decision problems have to be produced very quickly. An example of this is the airline industry where decisions as to whether or not to sell a potential passenger a ticket, or how to reconfigure the schedule to cope with disruptions to the pre-planned schedule, must be made very quickly.

Optimization

The purpose of carrying out a project is usually to provide advice to the client on what to do, based on the construction and experimentation with a model. Traditional MS models have used optimization to determine what is the best action the client should take (i.e. mathematical models where the optimum value of the controllable variables can be determined). Such a model was seen above for the Two Toys problem.

In general, optimization assumes that:

- The model accurately represents the system, and therefore the optimal solution for the model is also the optimal solution for the system. This may not be the case since models rarely represent accurately the system under investigation.
- There is one objective or, where there is more than one objective, they can be translated into a common unit, usually monetary values: for instance, giving time a monetary value and so being able to optimize over cost and time.
- There is consensus over what the objective of the system is.
- The problem will not change over time (at least in the short-term) and therefore one optimal solution can be found (i.e. the solution given is the best advice available now for the near future).
- All data can be quantified (i.e. assigned numerical values). In some cases it is not possible to quantify some factors, and therefore only qualitative information can be provided.

As an alternative to giving the client the optimal solution the consultant could attempt to scan available options (i.e. attempt to look at all the feasible options in the solution space and evaluate their performance based on the various different objectives of the client). Some of the options could be removed from consideration because other options are better overall objectives (i.e. some options are dominated by other options), but those remaining could be presented to the client for them to determine the trade-off between the objectives, and non-quantifiable factors can then be taken into account.

The effects of inaccurate data and minor changes in the systems' environment can be overcome by carrying out <u>sensitivity analysis</u> using the model. This involves changing assumptions and input data in the model to determine how sensitive the solution is to minor, and major, changes in data.

The accuracy and usefulness of a solution not only depend on the model's ability to represent the system and the accuracy of the data, it also depends on the <u>robustness</u> of the solution obtained. A solution is said to be robust if the option identified as being 'best' remains the best option (or if not best at least an extremely good option) even when the situation changes (e.g., environmental changes, such as cost and demand data). More informative and robust results are likely to be obtained by putting extra time into experimentation with the model (e.g. changing input data, changing assumptions about the future environment). It is always important to carry out some sensitivity analysis on the results from a model to check the robustness of the solution, and also to identify the cost of choosing an option which, while it may not be the 'best', is more robust.

Many of the models presented in this subject guide use optimization. You should be aware of the limitations of optimization and also conscious of the need to carry out sensitivity analysis. We will look at robustness and sensitivity analysis with regard to one of the optimization techniques, linear programming.

Implementation

To a large extent the success of an MS project is not determined by whether the project produces an elegant model, or by the size of the benefits of the recommended course of action, but by whether the project affects the decisions made by the client, including whether any action recommended is undertaken by the client.

In order for an organization to implement a proposed course of action, it must be possible to implement the solution (technologically and culturally), and the person(s) with the power to implement the recommended course of action must be committed to it.

In order for an MS project to produce a solution which it is possible to implement (a feasible solution), it is necessary to ensure, when formulating the problem, that all the relevant technological and cultural constraints are known and, where appropriate, included in the model. Continuous contact with the organization, and specifically the client in the organization (the person/people who have the problem), including regular discussions on the progress of the project and the model being produced, should ensure that a feasible solution is produced.

Gaining the commitment of the person(s) with the power to implement the solution requires the consultant to persuade them that the changes recommended are worth making. Approaches which are likely to gain such commitment include:

- Regular and positive communication with the client, to get information as well as to ensure the problem has not changed, and to make sure that the client knows how the problem is being approached. Problems can change over time; they may go away of their own accord; the objectives of the organization may change; the constraints may change. A technically brilliant solution to a problem the client no longer faces, or to the wrong problem, will have no value to the client and will not be implemented, or may be impossible to implement.
- Continually involving the client during the project. A good practical rule is that the consultant should never surprise the client by what they do – the result may surprise the client (i.e. it may not fit with their previously conceived ideas of what the solution would be), but the work that is being done should not.
- Giving the solution, and justification for the solution, in plain language, using terms the client will understand. This may require the use, at least in persuading the client of the validity of the solution, of simplified, transparent models and possibly examples showing the benefits of the solution. A common criticism of MS work over the years has been that some of the models used are ‘opaque’ (they cannot be understood by the client) which can lead to non-implementation because the client does not believe in the model and therefore cannot confidently implement results, especially where the results are counter-intuitive or very radical.
- Ensuring that the action recommended in the solution is within the power of the client (not recommending action which the client is unable to implement).