Operations Research II

Lecture 1: Introduction of OR with Uncertainty

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Lecture 1

Outline

1. What is Operations Research (OR)? Old and New Views
2. Operations Research (OR) and Uncertainty
3. Growing Interests in Uncertainty
4. Brief History of Linear Optimization
5. Contents of OR II

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What is OR?

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- **OR** is about Planning: plan your actions.
- **OR** is about Smartness: How to do things smartly by planning.

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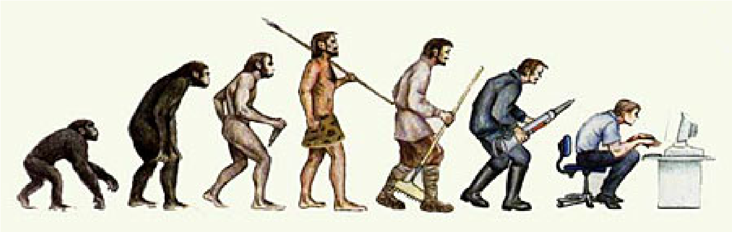
$$\textbf{OR} = \textbf{Modelling} + \textbf{Optimization}$$



Classic OR problems?

- **Manufacturing**: Production Planning, Capacity Planning, Scheduling, Lot sizing, Inventory, etc.
- **Transportation and Logistics**: Location, Vehicle Routing, TSP, etc.
- **Engineering Design**: System Reliability, Structure Reliability, Network design, etc.
- **Business Planning:** Assortment, Ordering (inventory), Project Management, etc.
- **Finance:** Portfolio Management, Risk Management, etc.
- **Others:**

What is OR?

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- **OR** is about Planning: plan your actions.
- **OR** is about Smartness: How to do things smartly by planning.

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Today, OR is closely related with Data Science, i.e.,

$$\textbf{OR} = \underbrace{\textbf{Statistics/Machine Learning} + \textbf{Modelling}}_{\text{Dealing with ``Uncertainty''}} + \textbf{Optimization}$$

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♡Life is full of uncertainty.

♠ How do you explain it, let alone plan for it?

Real world data are almost always uncertain:

- **Estimation errors**: part of the data is measured/estimated.
- **Prediction errors**: part of the data (e.g., future demands/prices) does not exist when problem is solved.
- **Implementation errors**: some components of a solution cannot be implemented exactly as computed, which in many models can be mimicked by appropriate data uncertainty

OR under uncertainty I: Do we understand the market uncertainty?!

Example 1: Portfolio Investment

We look at a portfolio investment using the following real data:

- 24 small cap stocks from different industry categories of the S&P 600 index.
- Historical returns of 10 years.
- Return and Covariance estimated from initial 80% of the data. Evaluate performance on last 20%.

Example 1: Portfolio Investment

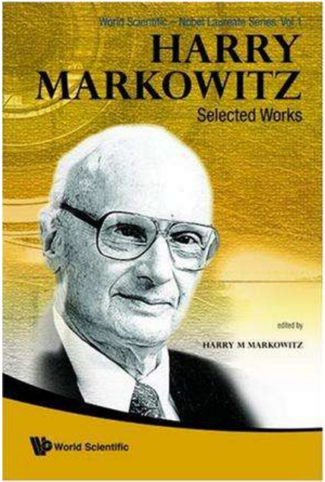


Figure 1: Harry Markowitz (1927-), Nobel laureates (1990) for Modern Portfolio Theory

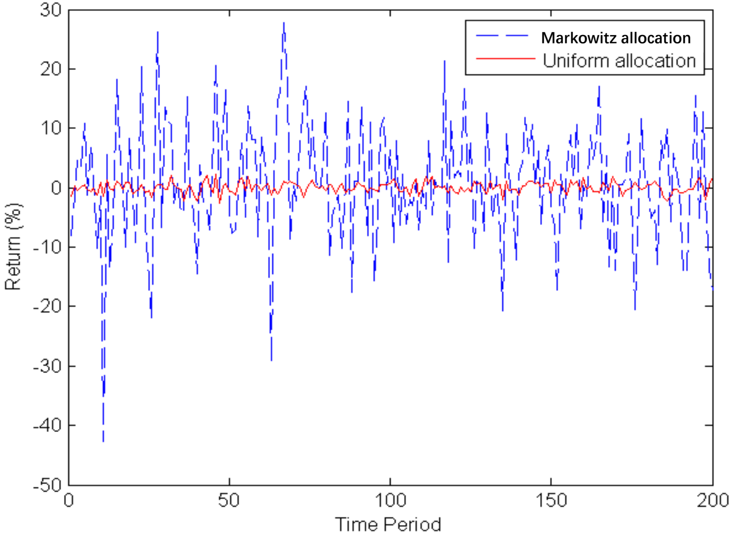
Example 1: Portfolio Investment

We use the famous Markowitz (mean-variance) model:

$$
\begin{array}{ll}
\underset{\boldsymbol{x}}{\text{Min}} & \boldsymbol{x}'\boldsymbol{\Sigma}\boldsymbol{x} \\
\text{s.t.} & \boldsymbol{x}'\boldsymbol{\mu} = \dfrac{1}{24}\displaystyle\sum_{i=1}^{24}\mu_i, \ \sum_{i=1}^{24} x_i = 1.
\end{array}
$$

- $\tilde{\boldsymbol{R}} = (\tilde{R}_1, \tilde{R}_2, \cdots, \tilde{R}_{24})$: vector of stock returns.
- The estimated mean: $\boldsymbol{\mu} = (\mu_1, \mu_2, \cdots, \mu_{24})$; and covariance matrix of $\tilde{\boldsymbol{R}}$: $\boldsymbol{\Sigma}$.
- $\boldsymbol{x} = (x_1, x_2, \cdots, x_{24})$: decision variables, the (regularized) investment budget allocation.
- Portfolio return: $\boldsymbol{x}'\tilde{\boldsymbol{R}}$; portfolio variance: $\boldsymbol{x}'\boldsymbol{\Sigma}\boldsymbol{x}$.

Example 1: Portfolio Investment



Example 1: Portfolio Investment

- Q: How can we model the portfolio optimization problem so as to reduce the fluctuations?

OR under uncertainty II: Engineering design

Example 2: Engineering design under uncertainty

- NETLIB includes about 100 not very large LOs, mostly of real-world origin, used as the standard benchmark for LO solvers.

Example 2: NETLIB example

Constraint # 372 of the problem PILOT4 from NETLIB:

$$
\begin{aligned}
\boldsymbol{a}'\boldsymbol{x} \quad := \quad & -15.79081x_{826} - 8.598819x_{827} - 1.88789x_{828} - 1.362417x_{829} \\
& -1.526049x_{830} - 0.031883x_{849} - 28.725555x_{850} - 10.792065x_{851} \\
& -0.19004x_{852} - 2.757176x_{853} - 12.290832x_{854} + 717.562256x_{855} \\
& -0.057865x_{856} - 3.785417x_{857} - 78.30661x_{858} - 122.163055x_{859} \\
& -6.46609x_{860} - 0.48371x_{861} - 0.615264x_{862} - 1.353783x_{863} \\
& -84.644257x_{864} - 122.459045x_{865} - 43.15593x_{866} - 1.712592x_{870} \\
& -0.401597x_{871} - x_{880} - 0.946049x_{898} - 0.946049x_{916} \\
& \geq b =: 23.387405
\end{aligned}
$$

- NETLIB includes about 100 not very large LOs, mostly of real-world origin, used as the standard benchmark for LO solvers.
- Most coefficients are ugly real numbers and highly unlikely that real-life parameters are known to high accuracy.

Let us look at a typical linear optimization problem (LOP):

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \boldsymbol{c}'\boldsymbol{x} \\ \text{s.t.} \quad & \boldsymbol{A}\boldsymbol{x} \geq \boldsymbol{b} \end{aligned}$$

- $\boldsymbol{x}$: decision variable
- $(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c})$: data (potentially uncertainty).

Estimation/Prediction:

$$\longmapsto (\hat{\boldsymbol{A}}, \hat{\boldsymbol{b}}, \hat{\boldsymbol{c}}).$$

Decision making (to obtain the "nominal" solution):

$$(\hat{\boldsymbol{A}}, \hat{\boldsymbol{b}}, \hat{\boldsymbol{c}}) \longmapsto \quad \hat{\boldsymbol{x}}^*.$$

Implementation (of the solution):

$$(\tilde{\boldsymbol{A}}, \tilde{\boldsymbol{b}}, \tilde{\boldsymbol{c}}) \quad \overrightarrow{\text{apply}} \quad \hat{\boldsymbol{x}}^* \longmapsto \quad \begin{cases} \tilde{\boldsymbol{c}}'\hat{\boldsymbol{x}}^*, & \text{if } \boldsymbol{A}\hat{\boldsymbol{x}}^* \geq \boldsymbol{b} \\ \infty, & \text{otherwise} \end{cases}$$

Gap between the model and the reality:

$$\boldsymbol{c}'\boldsymbol{x} \text{ vs. } \tilde{\boldsymbol{c}}'\hat{\boldsymbol{x}}^*,$$

and

$$\boldsymbol{A}\boldsymbol{x} \geq \boldsymbol{b} \text{ vs. } \boldsymbol{A}\hat{\boldsymbol{x}}^* \geq \boldsymbol{b}.$$

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- NETLIB includes about 100 not very large LOs, mostly of real-world origin, used as the standard benchmark for LO solvers.
- Most coefficients are ugly real numbers and highly unlikely that real-life parameters are known to high accuracy.

- What happened if data accuracy is only 0.1%?

$$\frac{\left|a_i^{\text{true}} - a_i\right|}{|a_i|} \leq 0.1\%$$

- In the worst case, the constraints can be violated by 450%!! (relative to RHS term b)

Danger!!

- Small perturbation in coefficients can make the constraint severely infeasible with respect to the RHS.
- Perturbation in objective can lead to large deviation.

Bad Designs under Uncertainty

- Bridges collapse due to the "**poor**" engineering design and development...

Figure 2: Jiu-Jiang Bridge (China) crashed by a ship, June 15, 2007, **8 died**

Bad Designs under Uncertainty

Figure 3: Morandi Bridge (Italy) collapsed, Aug 14, 2018, **43 died**

Bad Designs under Uncertainty

Figure 4: A new pedestrian bridge collapsed, March 15, 2018, **10 died**

Yet another example: System Reliability Design

Figure 5: Wellhead control system, Hitec Products Singapore

Yet another example: System Reliability Design

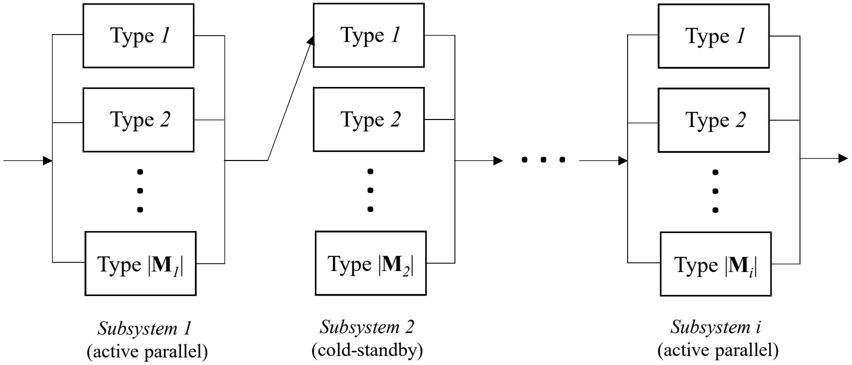


Figure 6: Typology of a series-parallel system with mixed active parallel and cold-standby redundancy strategies (Wang, et al. 2018)

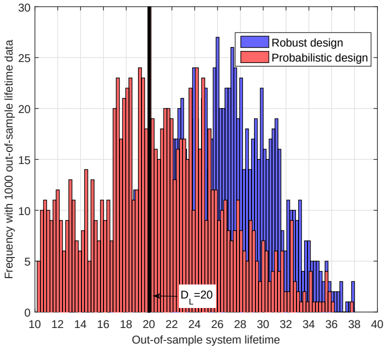
Yet another example: System Reliability Design

The OR model:

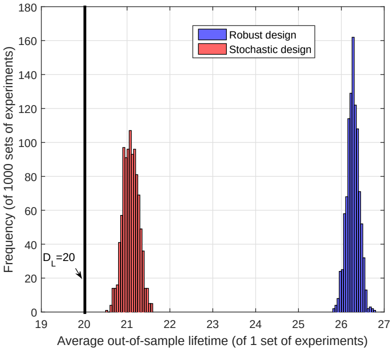
$$
\begin{aligned}
\operatorname*{Min}_{\mathbf{y}} \quad & \sum_{i\in\mathbf{N}}\sum_{j\in\mathbf{M}_i} y_{ij}c_{ij} \\
\text{s.t.} \quad & \mathbb{P}\left[\operatorname*{Min}_{i\in\mathbf{N}}\left(\sum_{j\in\mathbf{M}_i^{\mathrm{c}}}\sum_{k=1}^{y_{ij}}\tilde{z}_{ijk}\bigvee\operatorname*{Max}_{j\in\mathbf{M}_i^{\mathrm{a}}}\operatorname*{Max}_{k=1}^{y_{ij}}\tilde{z}_{ijk}\right) > D_L\right] \geq R_0 \\
& L_i \leq \sum_{j\in\mathbf{M}_i} y_{ij} \leq U_i, \forall i\in\mathbf{N} \\
& y_{ij}\in\mathbb{Z}_+, \forall i\in\mathbf{N}, j\in\mathbf{M}_i
\end{aligned}
\tag{1}
$$

where the c_{ij} is the unit cost of type-j components in subsystem i, L_i and U_i are the minimum and maximum numbers of components specified for subsystem i, respectively.

Yet another example: System Reliability Design



(a) System lifetimes with P-design and R-design (Wang, et al. 2020)



(b) Average System lifetimes with P-design and R-design (Wang, et al. 2020)

Yet another example: System Reliability Design

Table 1: Out-of-sample reliability levels comparison ($R_0 = 0.9$), where the 'Design' specifies the number of redundant components allocated in each of 5 subsystems, 'P-Model' and 'R-Model' refer to the probabilistic model and robust model, respectively. (Wang, et al. 2018)

D_L	Model	Design	Designed reliability level	Average $\mathbb{P}\left[\tilde{\mathcal{L}}_{\text{syst.}} \geq D_L\right]$	StD
18	P-Model	(1,1,1,1,1)	$R_0 = 0.9$	**0.617**	0.015
	R-Model	(2,1,2,2,2)		0.873	0.010
19	P-Model	(2,1,1,1,1)	$R_0 = 0.9$	**0.564**	0.015
	R-Model	(2,1,2,2,2)		0.884	0.009
20	P-Model	(2,1,1,3,2)	$R_0 = 0.9$	**0.554**	0.016
	R-Model	(2,2,3,3,2)		0.909	0.009

- S. Wang, Y.-F. Li, T. Jia, Distributionally robust design for redundancy allocation, *INFORMS Journal on Computing*, vol. 32, no. 3, 620–640, 2020.

"In real-world applications of Linear Programming, one cannot ignore the possibility that a small uncertainty in the data can make the usual optimal solution completely meaningless from practical viewpoint."

— Ben-Tal & Nemirovski (2000)

- Q: Is it possible to immunize the design to the uncertainty?
- Q: How? Modelling and computing

OR under uncertainty III: Business Analytics

Example 4: Flight booking management

Figure 7: Flight booking management

Example 4: Flight booking management

Figure 8: UA passenger dragged off the overbooked flight, 2017/4/9

Example 4: Flight booking management

- 2017/4/11: shares of United Continental Holdings (ual, +0.46%) fell as much as 4%, knocking off more than 1 billion in market value.
- The sell off was also costly for Uniteds biggest shareholder, Warren Buffett, who owns more than 9% of United. By market close, United stock was down slightly more than 1%. Buffetts total hit from the controversy: 24 million.

Figure 9: Uniteds biggest shareholder: Warren Buffett

Ordering Management

Example (Newsvendor Problem)

1. A newsboy needs to decide how many papers to order in weekend for the sales in Monday.
2. The newsboy pays \$ 0.20 for each paper, and sells each for \$ 0.50.
3. He has collected sales data over a few months and had found that on average each Monday 90 papers were sold with a standard deviation (StD) of 10 papers. (Here we assume during this time the papers are overstock.)

Figure 10: Newsvendor problem

Ordering Management

Example (Newsvendor Problem)

1. In other words, we have

$$\mu_{\text{demand}} = 90, \quad \sigma_{\text{demand}} = 10.$$

2. For an ease of exposition, we make the normality assumption, i.e.,

$$\widetilde{\text{demand}} \sim \mathcal{N}(90, 10^2)$$

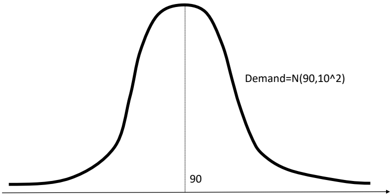


Figure 11: The demand distribution

Newsvendor Problem: A Statistical Picture

1. Q: How much to order?
2. A1: If you want to control the stocking out risk $\leq 50\%$, then you should order more than 90.
3. A2: If you want to control the stocking out risk $\leq 20\%$, then you should order more than $\mu + \Phi^{-1}(0.8)\sigma = 99$.

$$\mathbb{P}\left\{\tilde{d} \leq \text{Order}\right\} = \mathbb{P}\left\{\frac{\tilde{d}-\mu}{\sigma} \leq \frac{\text{Order}-\mu}{\sigma}\right\} \geq 0.8,$$

that is

$$\text{Order} \geq \mu + \Phi^{-1}(0.8)\sigma = 90 + 8.416 \approx 99.$$

Ordering Management

Newsvendor Problem: A Statistical Picture

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3. A2: If you want to control the stocking out risk $\leq 20\%$, then you should order more than 99.
4. A3: If you want to control the stocking out risk $\leq (1-p)$, then you should order more than $\mu + \Phi^{-1}(p)\sigma$.

Newsvendor Problem: A Statistical Picture

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- It does not tell much about profit or loss on the ordering, which however is what the ordering decision is about!

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- The statistical pic deals with the risk of stocking out (only the statistical information is used)!
- It does not tell much about profit or loss on the ordering, which however is what the ordering decision is about!
- A DM Q: Which stocking-out probability $(1-p)$ (or which p) is the best?

Ordering Management

Newsvendor Problem: A Decision-Making Picture

- Q: How much to order?
- A: The best ordering level:

$$\text{Order}^* = \mu + \Phi^{-1}\left(\frac{C_u}{C_o + C_u}\right)\sigma = 93,$$

where

$$p^* = \frac{C_u}{C_o + C_u} = 0.6$$

is the 'best' stocking safe probability level (DM point of view).

- The DM pic provides the 'best' ordering decision. It also determines the 'best' stocking probability level, via a non statistical approach (independent of the normality)!

Ordering Management

Newsvendor Problem: The OR Model

- Q: How much y to order?
- A: Solve the following optimization problem:

$$\operatorname*{Max}_{y\geq 0} \mathbb{E}\left[(C_o + C_u)\,\mathrm{Min}\{\tilde{d}, y\}\right] - C_o y,$$

by first-order condition, we have the best ordering decision y^*:

$$\underbrace{\left(1 - \mathbb{P}\{\tilde{d} \leq y^*\}\right) C_u - \mathbb{P}\{\tilde{d} \leq y^*\} C_o = 0.}_{\text{zero expected marginal profit condition}}$$

That is

$$\underbrace{\mathbb{P}\{\tilde{d} \leq y^*\} = F(y^*) = \frac{C_u}{C_o + C_u}}_{\text{Best stocking probability}}, \text{ or } \underbrace{y^* = F^{-1}\left(\frac{C_u}{C_o + C_u}\right)}_{\text{Best order}}$$

Applications of Newsvendor Model:

- Ordering of fashion items
- Hotel booking management
- Overbooking of airline flights
- Financial liquidity planning

OR under Uncertainty IV: Decision with dynamics

Example 5: On-line order fulfillment

A myopic order fulfillment policy

- The Etailer Fulfills each order the cheapest way possible based on its current inventory position

Example 5: On-line order fulfillment

A myopic order fulfillment policy

Example

- Two fulfillment centers (FCs): one in LA, one in Nashville.
- LA FC: 3 textbooks in stock; Nashville FC: 1 textbook and 9 CDs.
- An order comes from Dallas: 1 textbook

Example 5: On-line order fulfillment

A myopic order fulfillment policy

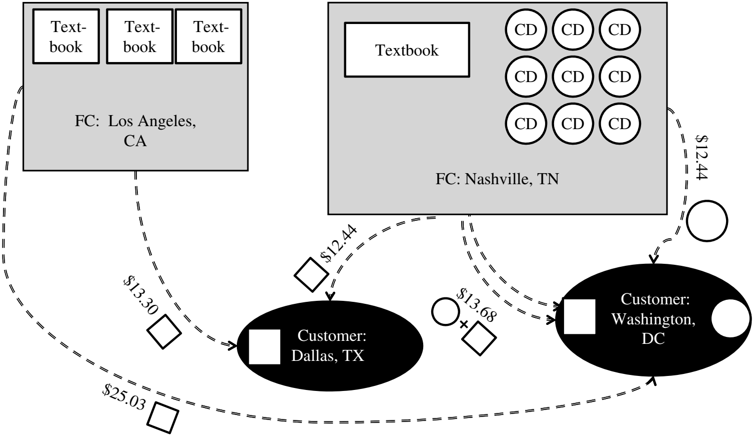

Figure 12: Myopic fulfillment with shipping cost

Example

- Two fulfillment centers (FCs): one in LA, one in Nashville.
- LA FC: 3 textbooks in stock; Nashville FC: 1 textbook and 9 CDs.
- An order from Dallas: 1 textbook
- Myopic policy: ship 1 textbook from Nashville FC to Dallas customer.

Example 5: On-line order fulfillment

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Example

- Two fulfillment centers (FCs): one in LA, one in Nashville.
- LA FC: 3 textbooks in stock; Nashville FC: 1 textbook and 9 CDs.
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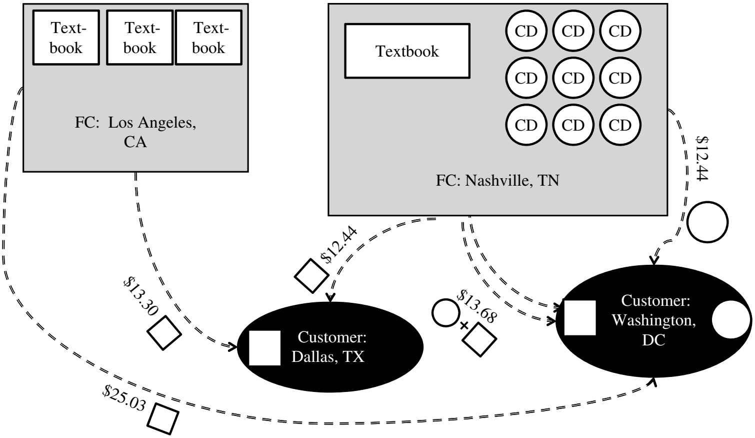
- Myopic policy: ship 1 textbook from Nashville FC to Dallas customer.
- An order from Washington DC: 1 textbook and 1 CD.

Example 5: On-line order fulfillment

A myopic order fulfillment policy



Example 5: On-line order fulfillment

A myopic order fulfillment policy

Example

- Two fulfillment centers (FCs): one in LA, one in Nashville.
- LA FC: 3 textbooks in stock; Nashville FC: 1 textbook and 9 CDs.
- An order from Dallas: 1 textbook
- Myopic policy: ship 1 textbook from Nashville FC to Dallas customer.
- An order from Washington DC: 1 textbook and 1 CD.
- Myopic policy: ship 1 textbook from LA FC and 1 CD from Nashville FC to the W. DC customer.

Example 5: On-line order fulfillment

A myopic order fulfillment policy

The total cost of myopic policy

- Total cost of myopic policy: \$12.44 + \$25.03 +\$12.44 = \$ 49.91.

- What if we know the next order information in advance?

The perfect hindsight policy

- Hindsight policy: ship 1 textbook from LA to Dallas customer, and ship 1 textbook + 1 CD together from Nashville to W. DC customer
- Total cost of myopic policy: \$13.30 + \$13.68 = \$ 26.98.

Modeling Order Fulfillment as Dynamic Optimization

An Nested Optimization Model

The minimum cost for the order fulfillment with current inventory state S_t given the order $\o_t$ at t:

$$J_t(\boldsymbol{S}_t|\o_t) = \underset{\boldsymbol{x}\in\boldsymbol{X}(\boldsymbol{S}_t)}{\text{Min}} \left[\underbrace{C_t(\boldsymbol{x}, \o_t)}_{\text{Current cost}} + \underbrace{\mathbb{E}_{\tilde{\o}_{t+1}}\left[J_{t+1}\Big(\boldsymbol{S}_{t+1}(\boldsymbol{S}_t, \boldsymbol{x})|\tilde{\o}_{t+1}\Big)\right]}_{\text{Future expected cost}} \right]$$

- $\boldsymbol{x}$: the order fulfillment decision
- $\boldsymbol{X}(S_t)$: order fulfillment constraints affected by S_t
- $S_{t+1}(S_t, \boldsymbol{x})$: next stage inventory level
- $\tilde{\o}_{t+1}$: future order uncertainty

Modeling Order Fulfillment as Dynamic Optimization

An Nested Optimization Model

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- Solving this Stochastic Dynamic Programming is a Mission Impossible !
- Curse of Dimensionality!

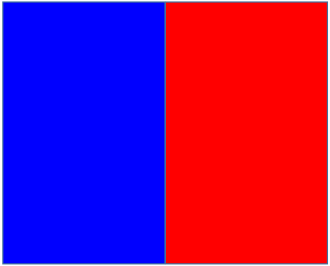
OR under Uncertainty V: Ambiguity and Risk in Decision

Ambiguity in Decision Making

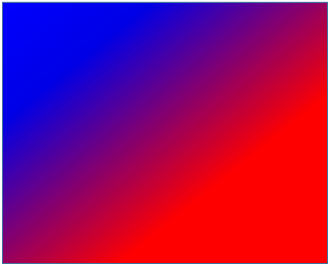
- **Elephant in the room**: The exact distributions (of uncertainty) in many practical situations are not available.
- **Q**: Do people make decisions under ambiguity using subject probability?

Ellsberg's (1961) Two-Color Experiment

Box 1 contains 50 red balls and 50 blue balls.
Box 2 contains 100 red and blue balls in unknown proportions.



Box 1: 50 blue balls, 50 red balls



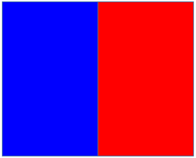
Box 2: 100 blue and red balls

Ellsberg's (1961) Two-Color Experiment

TEST-I. Subjects are given two choices as below:

- Gamble A: Win \$1000 if ball drawn from **box 1** is red.
- Gamble B: Win \$1000 if ball drawn from **box 2** is red.

Which box will you choose?



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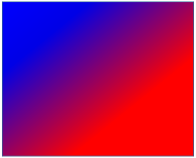

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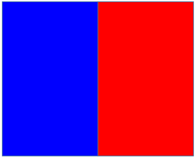
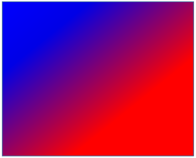
Which box will you choose?

Result: More subjects choose box 1.

$$\mathbb{P}_S\{\text{ball drawn from box-2 is red}\} < 0.5,$$



Box 1: 50 blue balls, 50 red balls



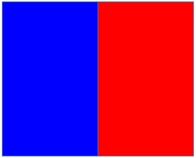
Box 2: 100 blue and red balls

Ellsberg's (1961) Two-Color Experiment

TEST-II. Subjects are given two choices as below:

- Gamble C: Win \$1000 if ball drawn from **box 1** is blue.
- Gamble D: Win \$1000 if ball drawn from **box 2** is blue.

Which box will you choose?



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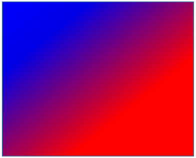

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Ellsberg's (1961) Two-Color Experiment

TEST-II. Subjects are given two choices as below:

- Gamble C: Win \$1000 if ball drawn from **box 1** is blue.
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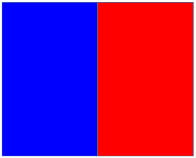
Which box will you choose?

Result: More subjects choose box 1.

$$\mathbb{P}_S\{\text{ball drawn from box-2 is blue}\} < 0.5, \text{ or}$$
$$\mathbb{P}_S\{\text{ball drawn from box-2 is red}\} \geq 0.5$$



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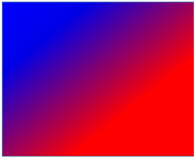


Box 2: 100 blue and red balls

Ellsberg's (1961) Two-Color Experiment

Contradiction!!!

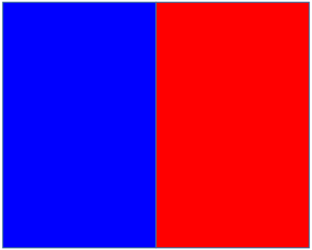
- TEST-I: $\mathbb{P}_S\{\text{ball drawn from box-2 is red}\} < 0.5$.
- TEST-II: $\mathbb{P}_S\{\text{ball drawn from box-2 is red}\} \geq 0.5$.

The subjective probability does not work!!

Conclusion

The drive behind the decision-making under ambiguity is not **subjective probability**, but **ambiguity aversion**

Ambiguity $\neq$ Risk



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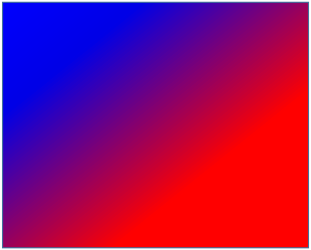
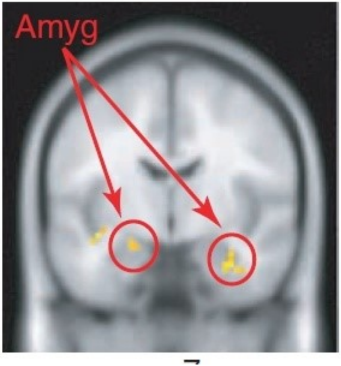


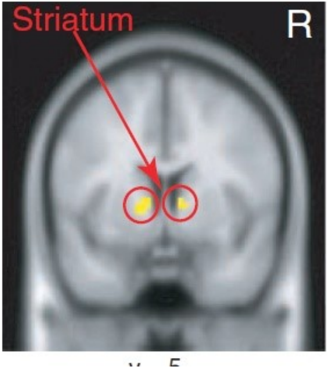
Box 2: 100 blue and red balls

Physiological Evidence

Ambiguity $\neq$ Risk



(a) Response to Ambiguity



(b) Response to Risk

- Hsu, Bhatt, Adolphs, Tranel and Camerer, *Science*, 310, pp. 1680-1683, 2005.

Ambiguity in Decision

- Q: How to model ambiguity aversion or ambiguity hedging?
- Q: What information do we need?
- Q: How do we compute the ambiguity models?

Outline

1 What is Operations Research (OR)? Old and New Views

2 Operations Research (OR) and Uncertainty

3 Growing Interests in Uncertainty

4 Brief History of Linear Optimization

5 Contents of OR II

Growing Interests in Uncertainty

Economic community: Maximin Expected Utility (Gilboa and Schmeidler); Variational Preferences (Maccheroni, Marinacci, and Rustichini).

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Operation Research community: Robust and Stochastic Optimization (Nemirovski, Ben-Tal, Bertsimas, El-Ghoui, Kuhn, Sim, $\cdots$).

At the center of DM under uncertainty is playing with the **decision criterion**, *i.e.*, a **payoff function** $f(\boldsymbol{x}, \tilde{\boldsymbol{z}})$ housed in a **utility function or preference** $\mathbb{U}$:

$$\operatorname*{Min}_{\boldsymbol{x} \in \mathcal{X}} \mathbb{U}\Big[f(\boldsymbol{x}, \tilde{\boldsymbol{z}})\Big]$$

- $\boldsymbol{x} \in \mathcal{X}$: decision (region, feasible set).
- $\tilde{\boldsymbol{z}}$: parameters, uncertainty. A random variable with known or unknown distribution.
- $\mathbb{U}$: decision preference. For instance, $\mathbb{E}_{\mathbb{P}}[\cdot], \mathrm{VaR}_{\mathbb{P}}, \mathrm{CVaR}_{\mathbb{P}}, \sup_{\mathbb{P} \in \mathcal{F}} \mathbb{E}_{\mathbb{P}}[\cdot]$, etc.
- **A Key Q: How to compute it?**

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Brief History of Linear Optimization

Linear Programming & Simplex Algorithm

"Father of the Linear Programming"

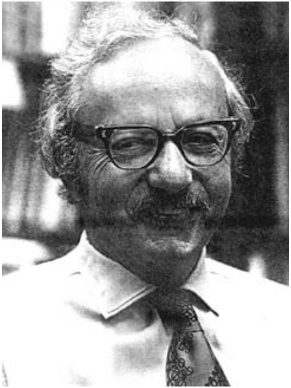


Figure 13: George Bernard Dantzig (1914 – 2005)

Ellipsoid Algorithm

Geniuses from Russia...

(a) Arkadi Nemirovski (1947–)

(b) Leonid Khachiyan (1952–2005)

Interior Point Algorithm

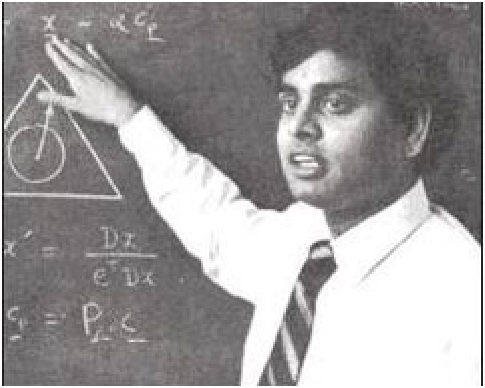


Figure 14: Narendra Krishna Karmarkar (1957–)

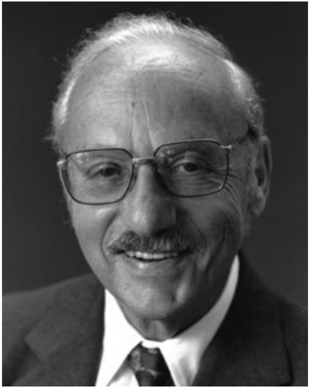
Conic Linear Programming

$$\text{Interior Point Algorithm} \Longrightarrow \left.\begin{array}{l}\text{Linear Programming}\\ \text{Second Order Conic Programming}\\ \text{Semi-Definite Programming}\end{array}\right\}$$

- CVX
- IBM ILOG CPLEX
- MOSEK

Polynomial Time Solvable !!

Optimization under Uncertainty



"...I work on planning under uncertainty . . . that's the future."

— George B. Dantzig 1999

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- Lecture 2: Basic Convex Analysis
- Lecture 3: Duality Theory
- Lecture 4: Chance-Constrained Programming
- Lecture 5: Robust Optimization-I
- Lecture 6: Robust Optimization-II
- Lecture 7: Large Scale Optimization
- Lecture 8: Two-Stage Stochastic Programming

Evaluation

- Course attendance (25%)
- Homework (25%)
- Final Exam (40%-50%)
- ♣ (0%-10%)

Reference and Further Reading

1. **D. Bertsimas, J.N. Tsitsiklis, Introduction to Linear Optimization, Athena Scientific, Nashua, 1997.**
2. S. Boyd, L. Vandenberghe, Convex Optimization, Cambridge University Press, 2009.
3. J.R. Birge, F. Louveaux. Introduction to Stochastic Programming. 2nd Edition, Springer-Verlag, NY, 2011.
4. L.F. Ackert, R. Deaves, Behavioral Finance: Psychology, Decision-Making, and Markets. Cengage Learning, Singapore, 2010.
5. Many papers