Introduction to Computer Graphics

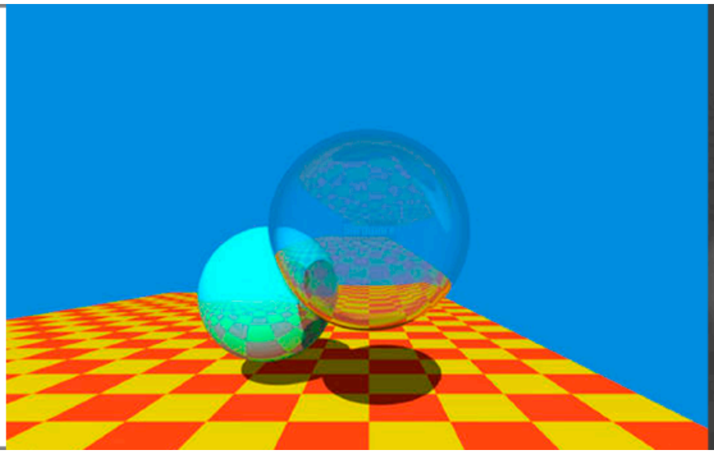
GAMES101, Lingqi Yan, UC Santa Barbara

反走样与深度缓冲

Lecture 6:

Rasterization 2
(Antialiasing and Z-Buffering)






Announcements

- Homework 1
  - Already 49 submissions so far!
  - In general, start early
- Today's topics are not easy
  - Having knowledge on Signal Processing is appreciated
  - But no worries if you don't

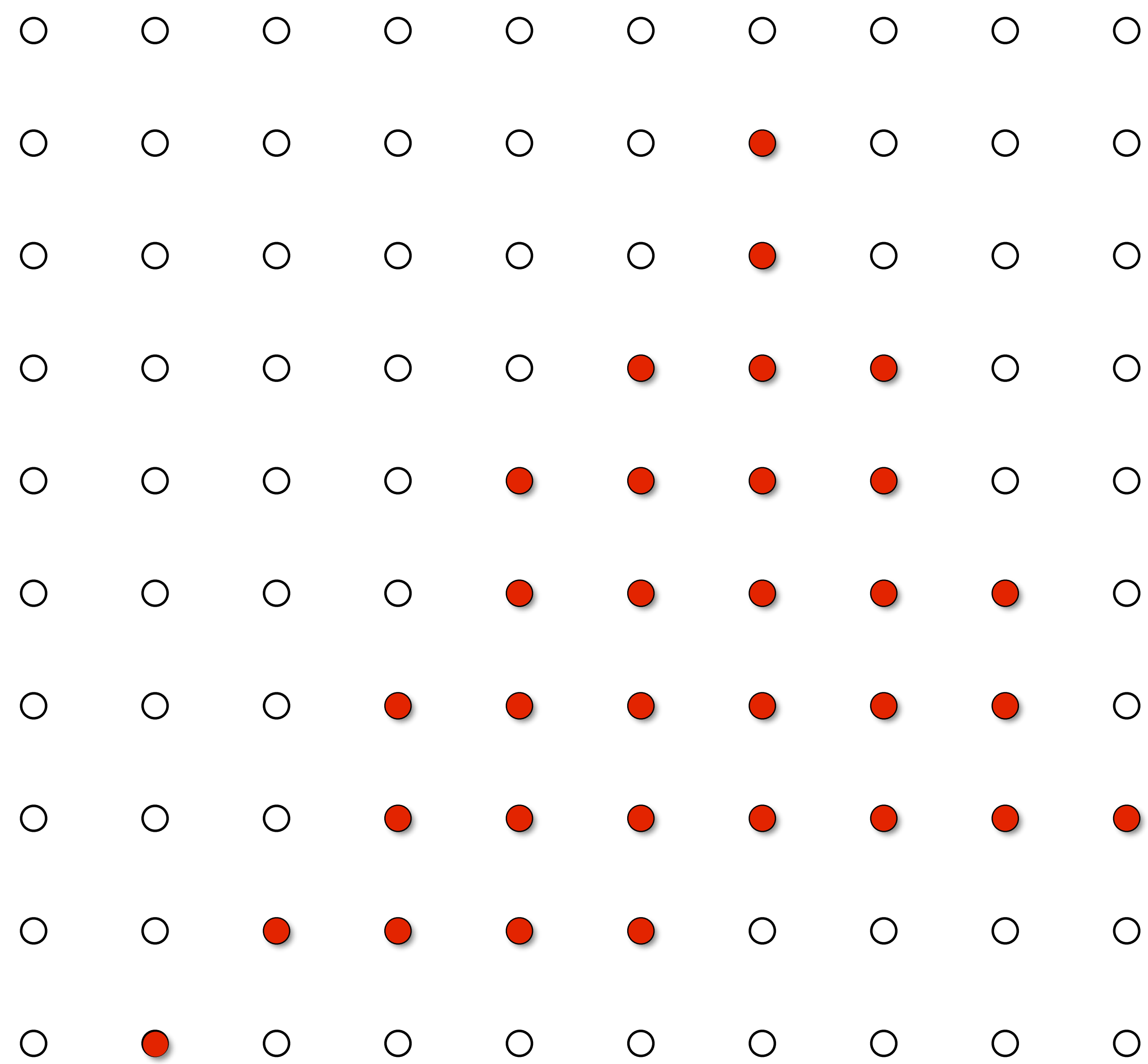
Last Lectures

- Viewing
  - View + Projection + Viewport 视口变换
- Rasterizing triangles 三角光栅化
  - Point-in-triangle test
  - Aliasing

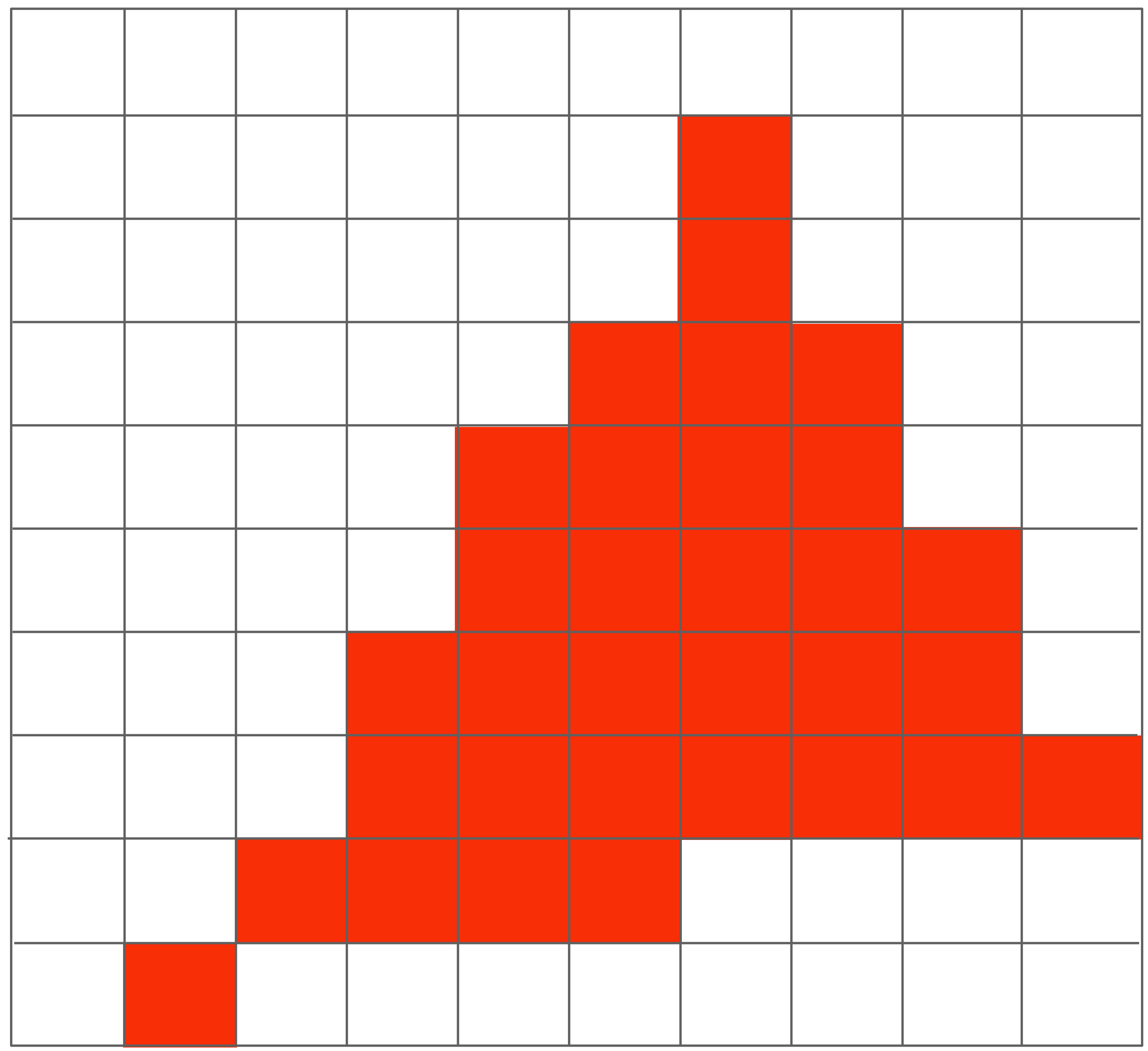
Today

- Antialiasing 反走样
  - Sampling theory
  - Antialiasing in practice
- Visibility / occlusion
  - Z-buffering

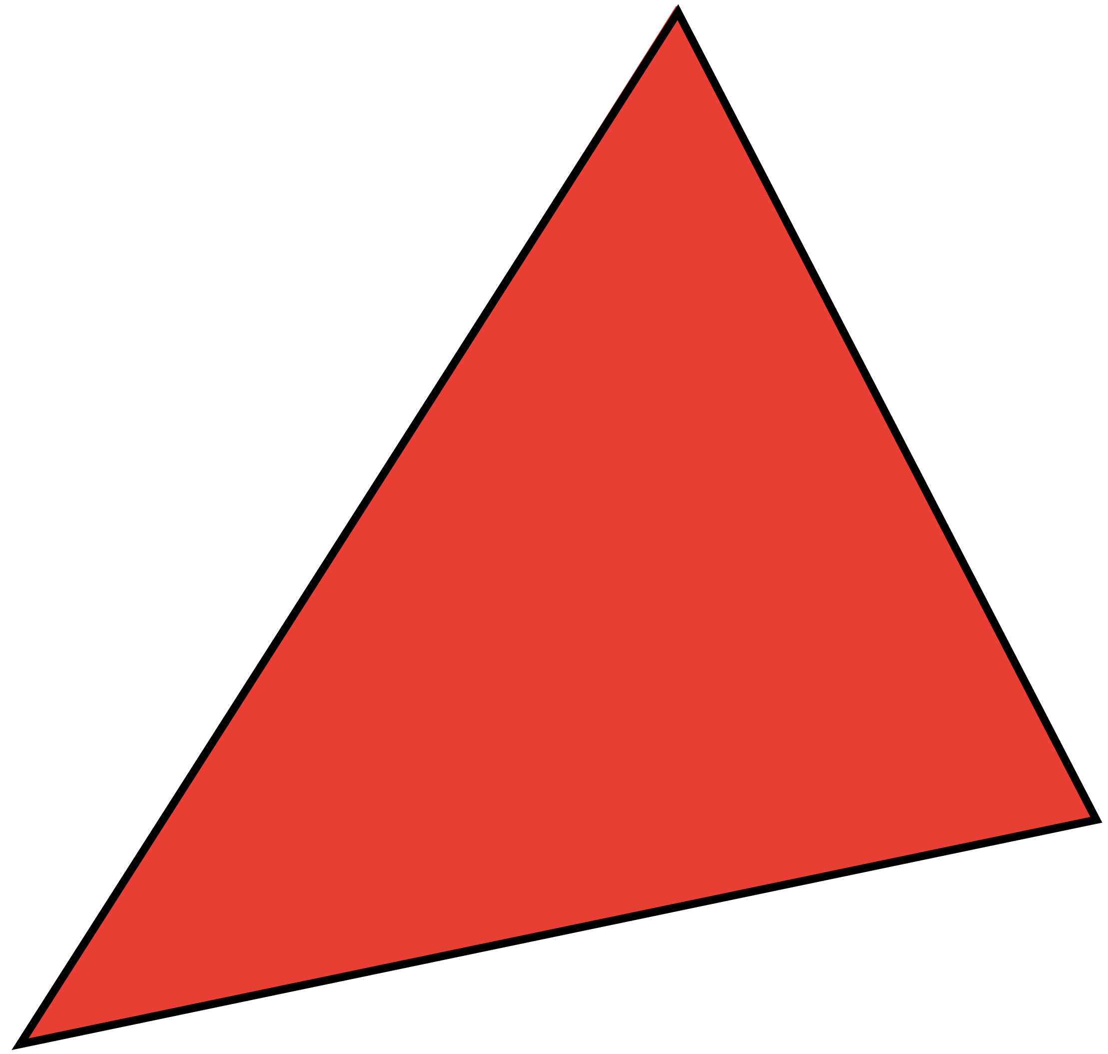
Recap: Testing in/out △ at pixels' centers



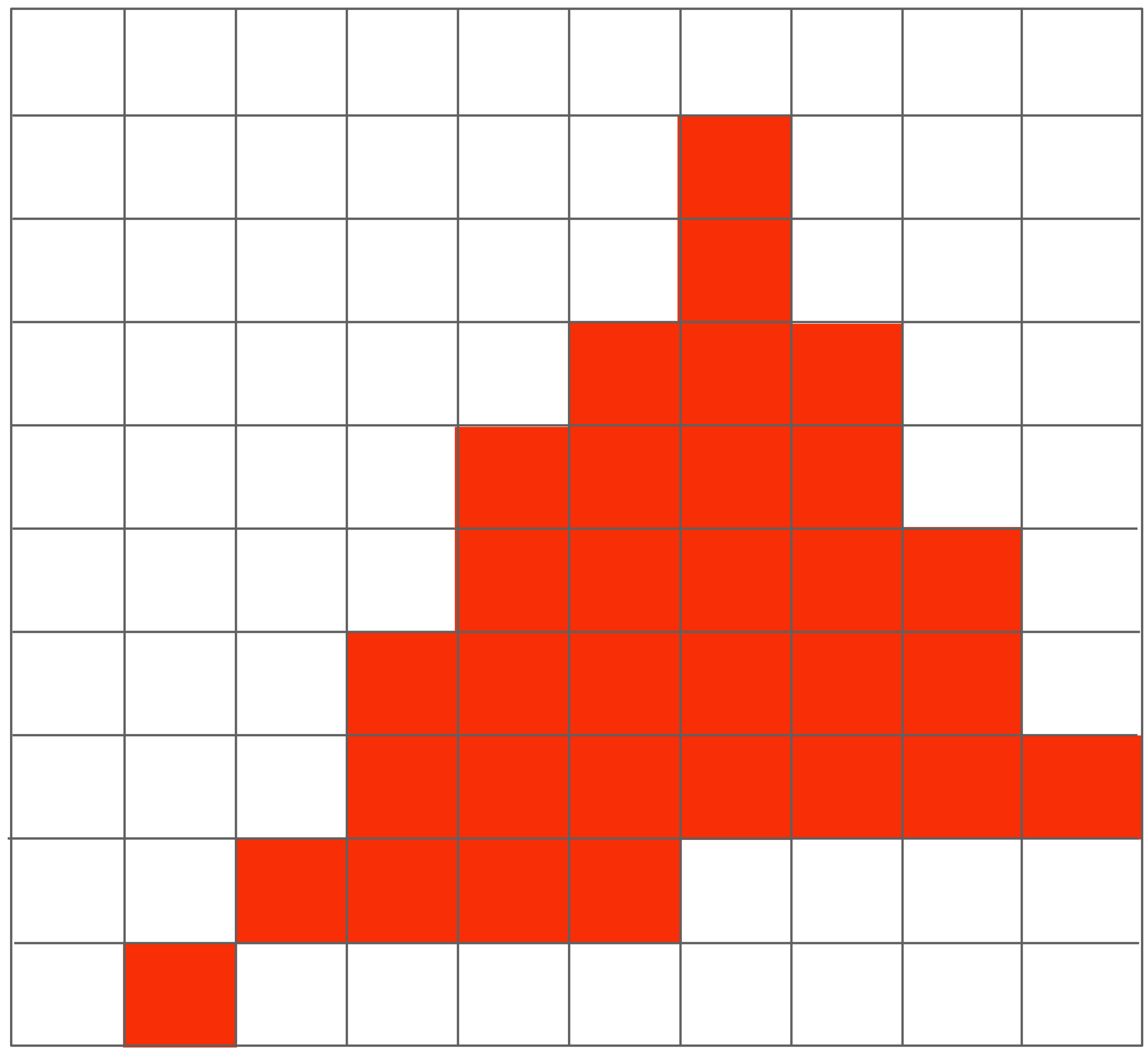
Pixels are uniformly-colored squares



Compare: The Continuous Triangle Function

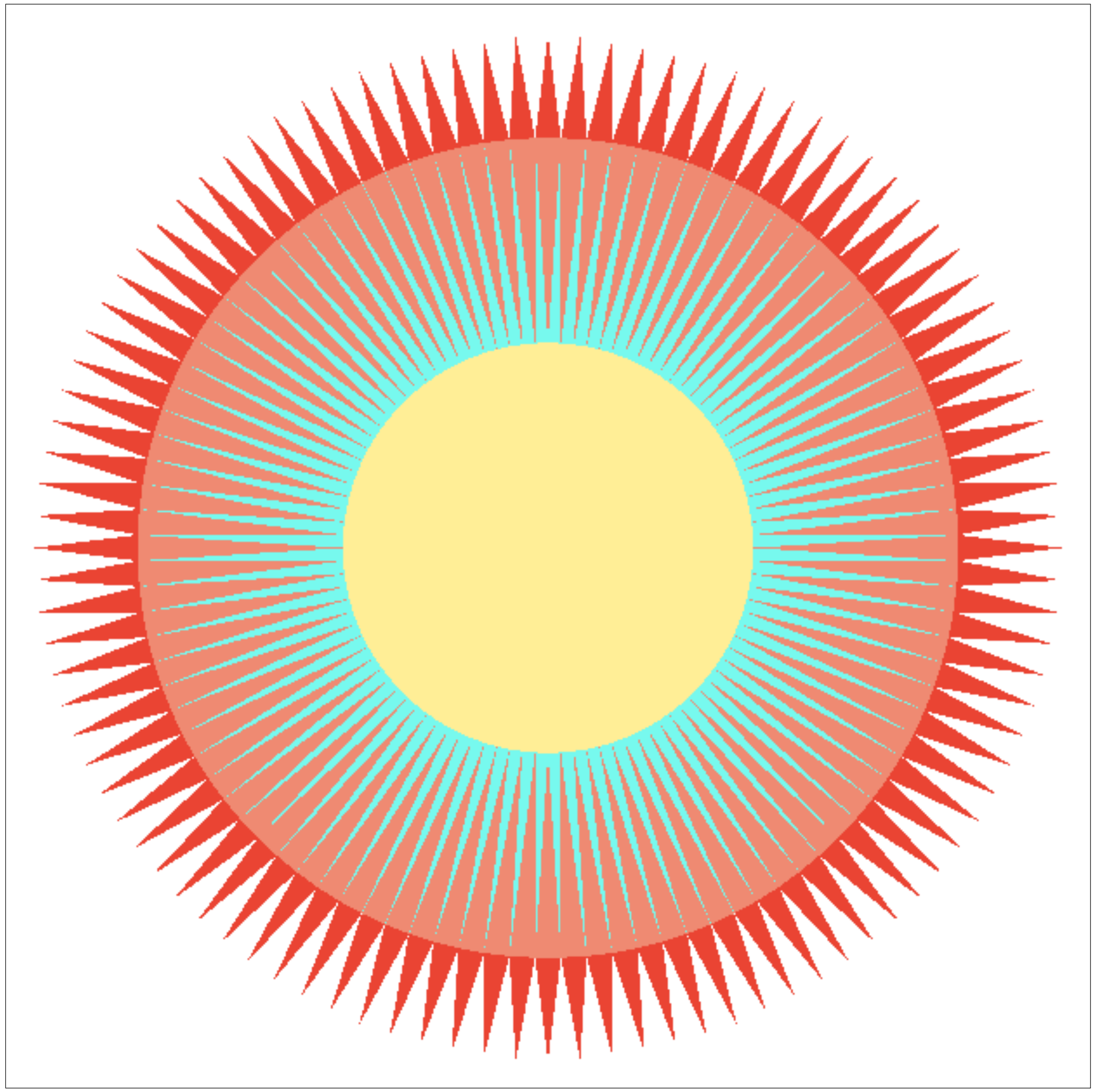


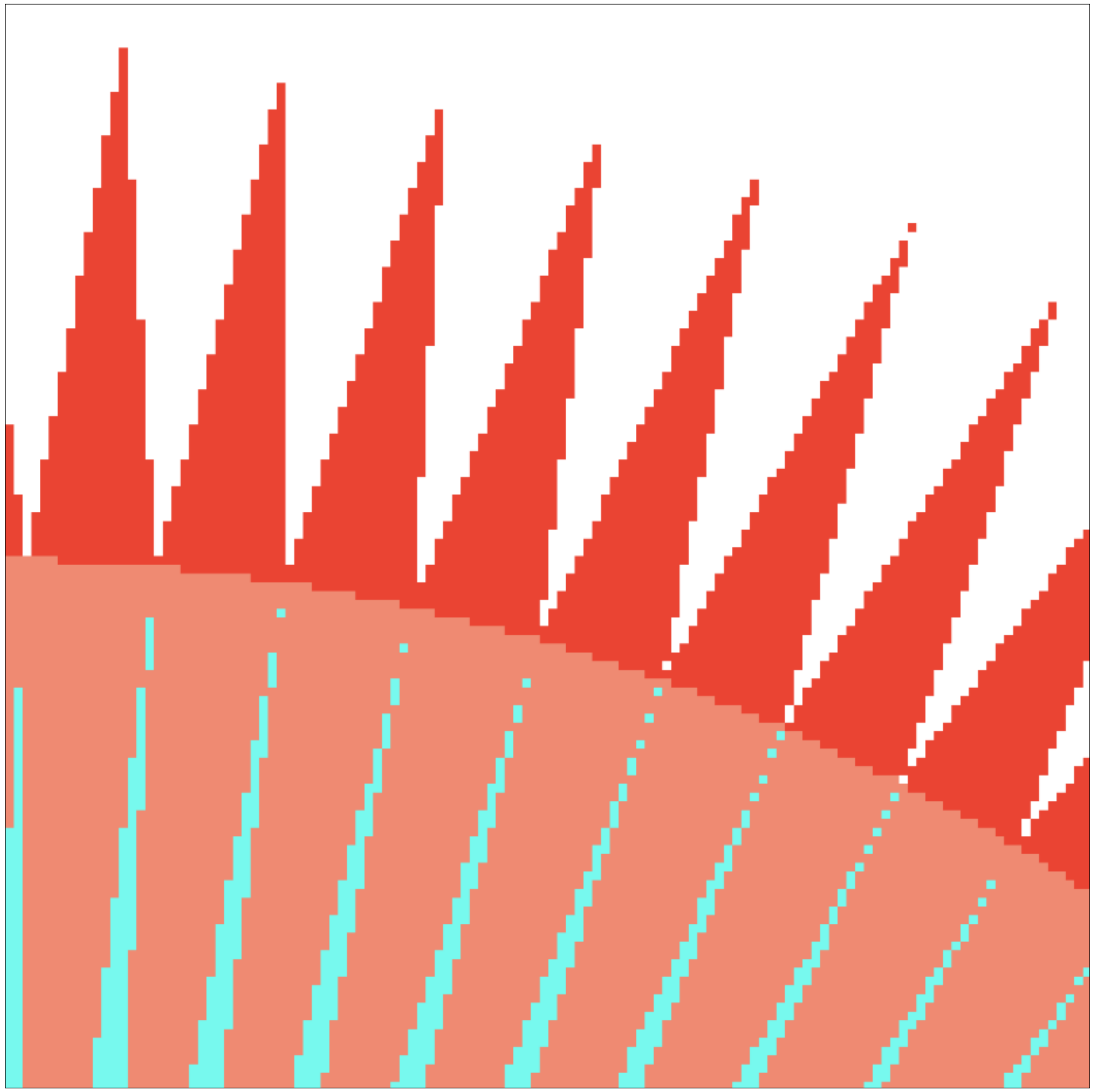
What's Wrong With This Picture?



Jaggies!

Aliasing





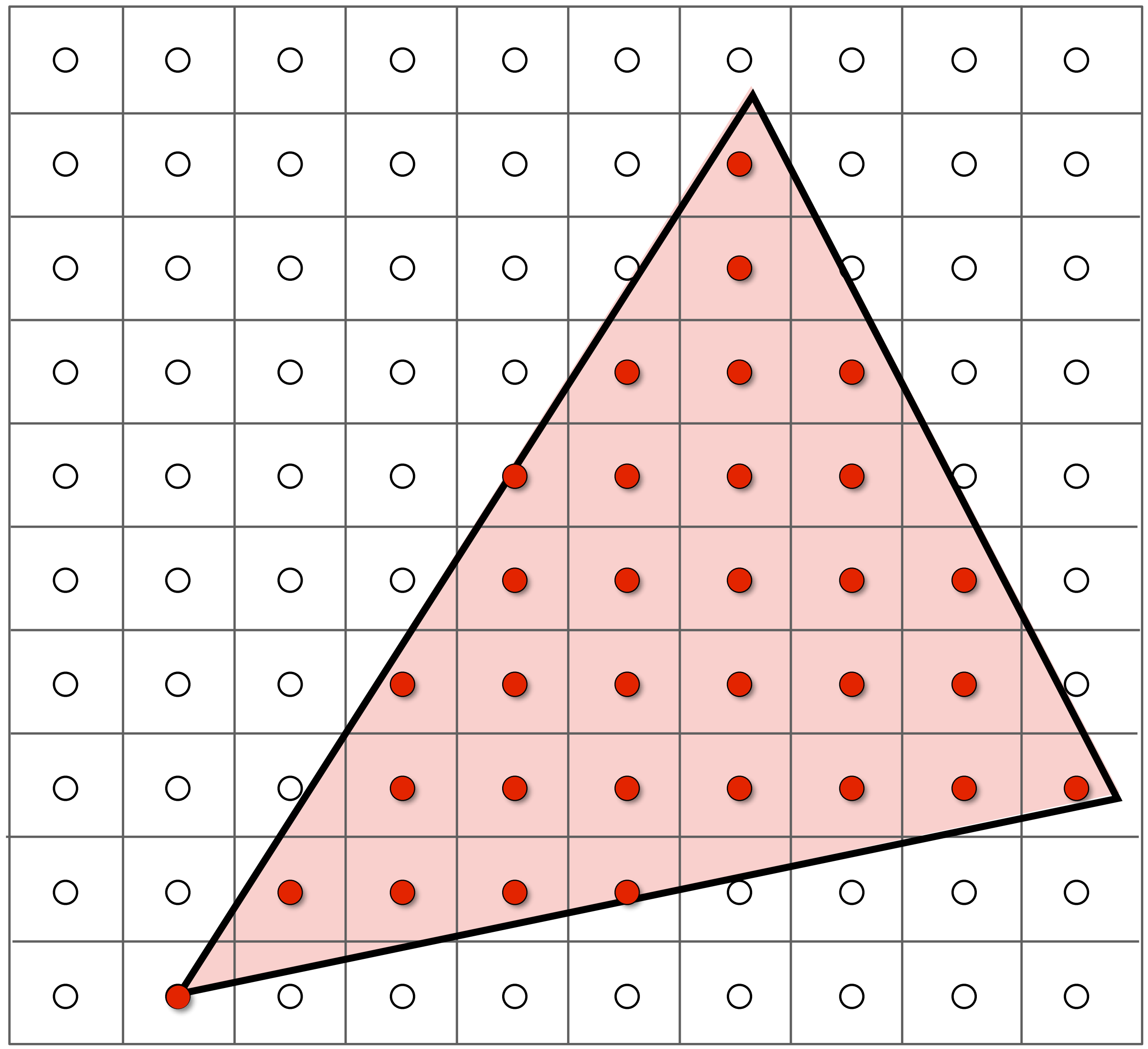
Is this the best we can do?

Slide courtesy of Prof. Ren Ng, UC Berkeley

普遍存在

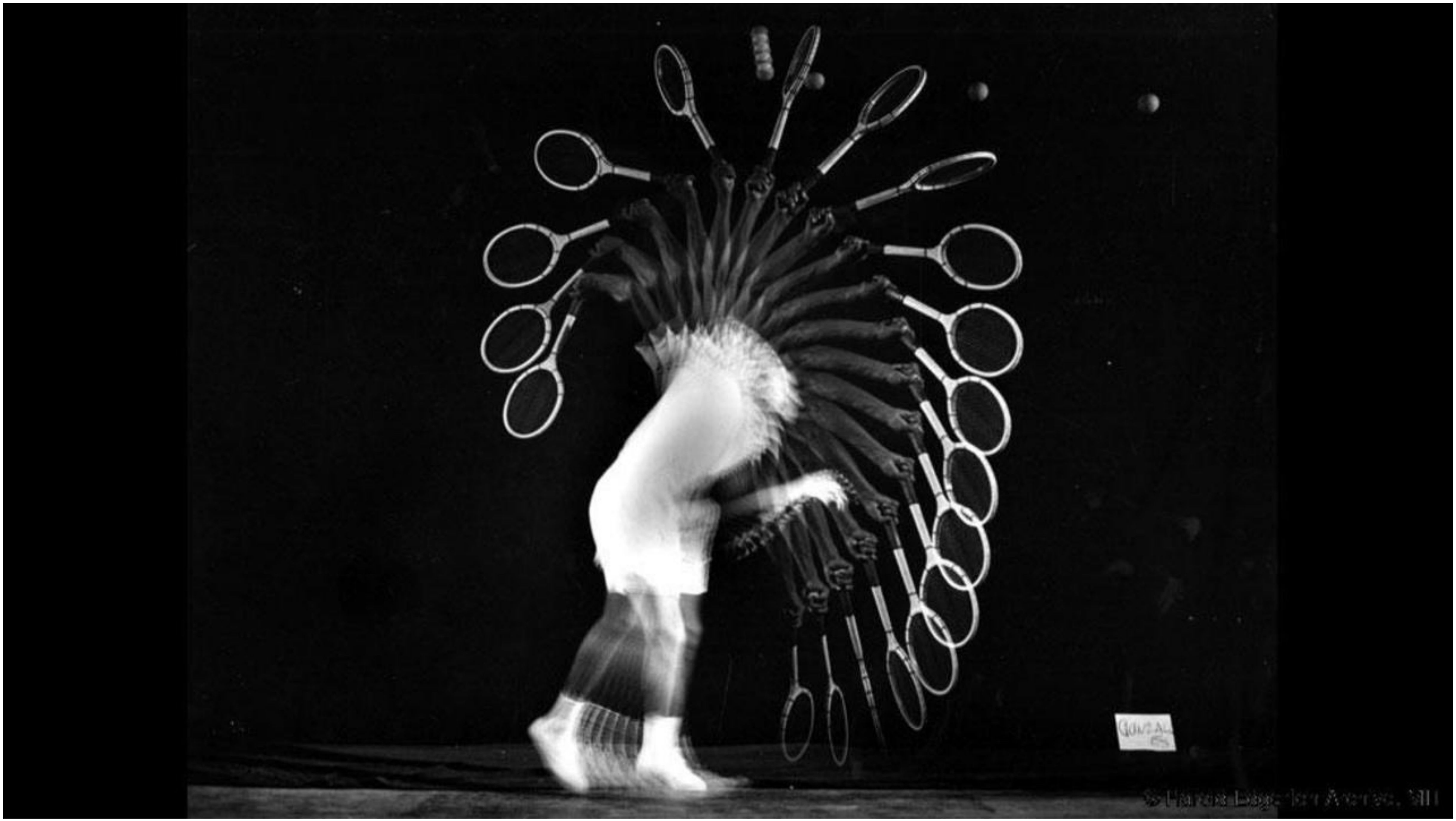
Sampling is Ubiquitous in Computer Graphics

Rasterization = Sample 2D Positions



Photograph = Sample Image Sensor Plane

Video = Sample Time



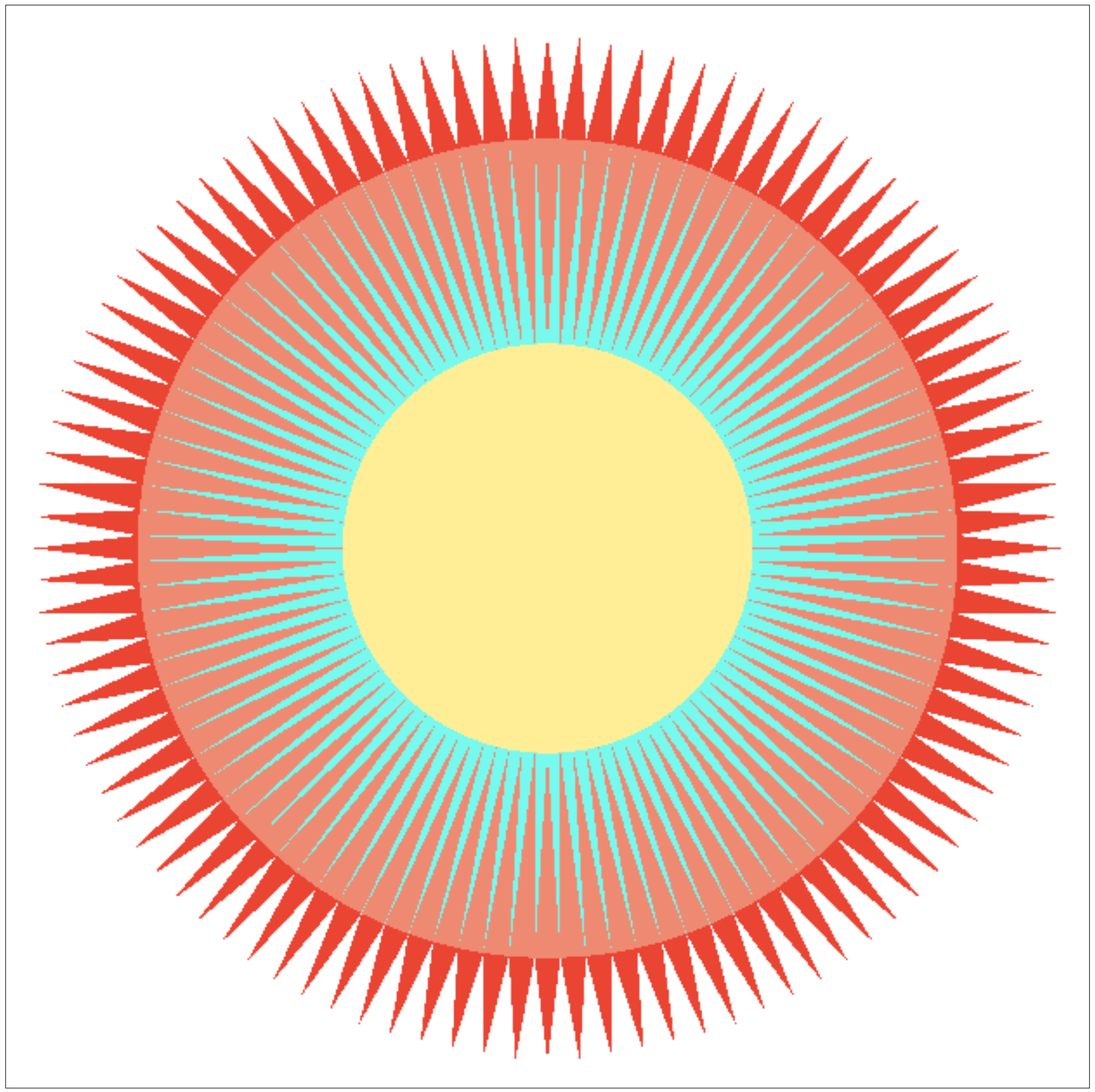
Harold Edgerton Archive, MIT

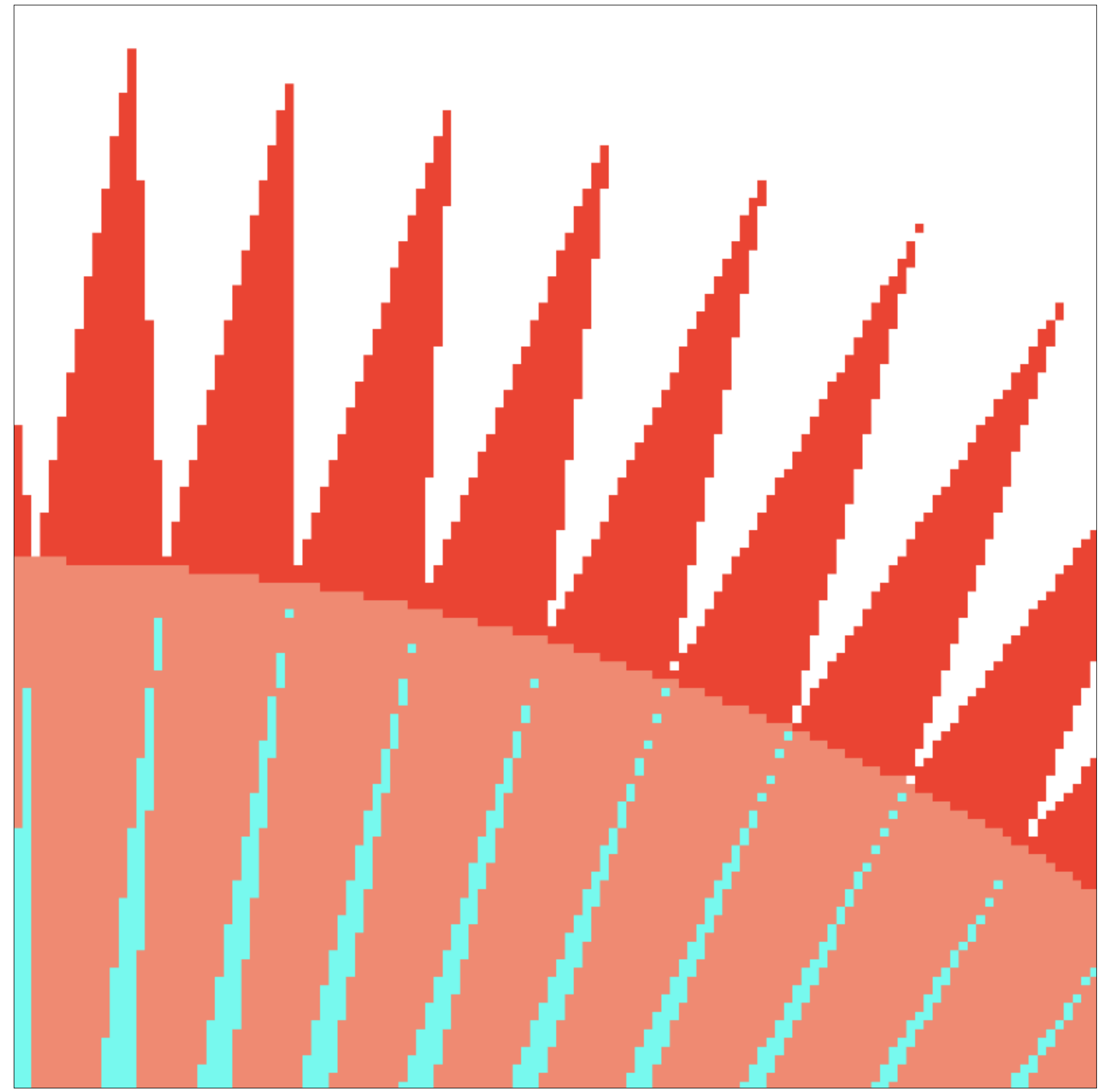
Sampling Artifacts

(Errors / Mistakes / Inaccuracies) in

Computer Graphics

Jaggies (Staircase Pattern)



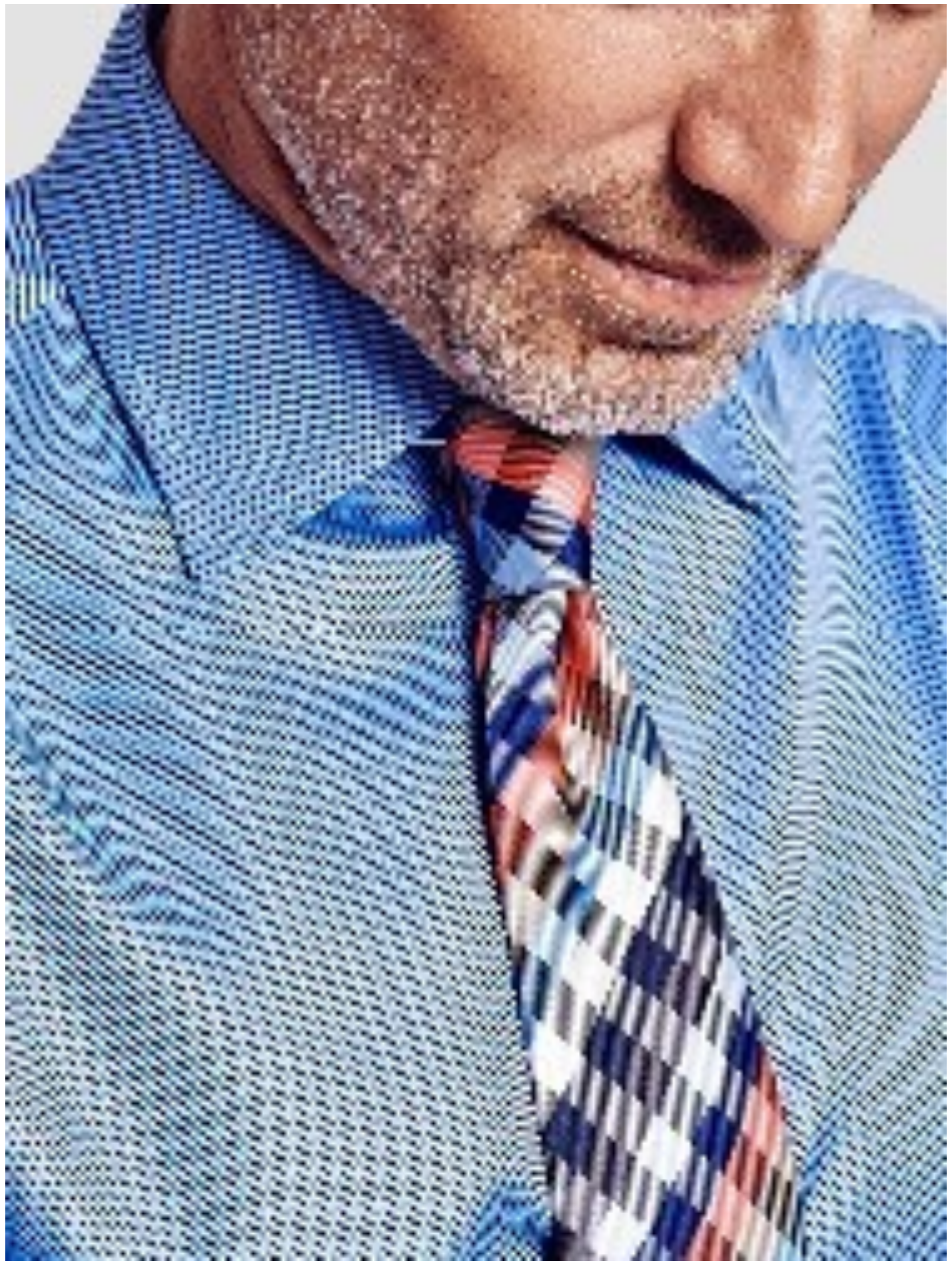


This is also an example of “aliasing” – a sampling error

Moiré Patterns in Imaging

[mwɑ:]

摩尔纹



lystit.com

Skip odd rows and columns

跳过奇数行与奇数列

人眼时间采样跟不上运动速度

Wagon Wheel Illusion (False Motion)

Sampling Artifacts in Computer Graphics

Artifacts due to sampling - "Aliasing"

- Jaggies – sampling in space
- Moire – undersampling images

空间

- Wagon wheel effect – sampling in time 时间
- [Many more] …

Behind the Aliasing Artifacts

- Signals are changing too fast (high frequency), but sampled too slowly

Antialiasing Idea:
Blurring (Pre-Filtering) Before Sampling

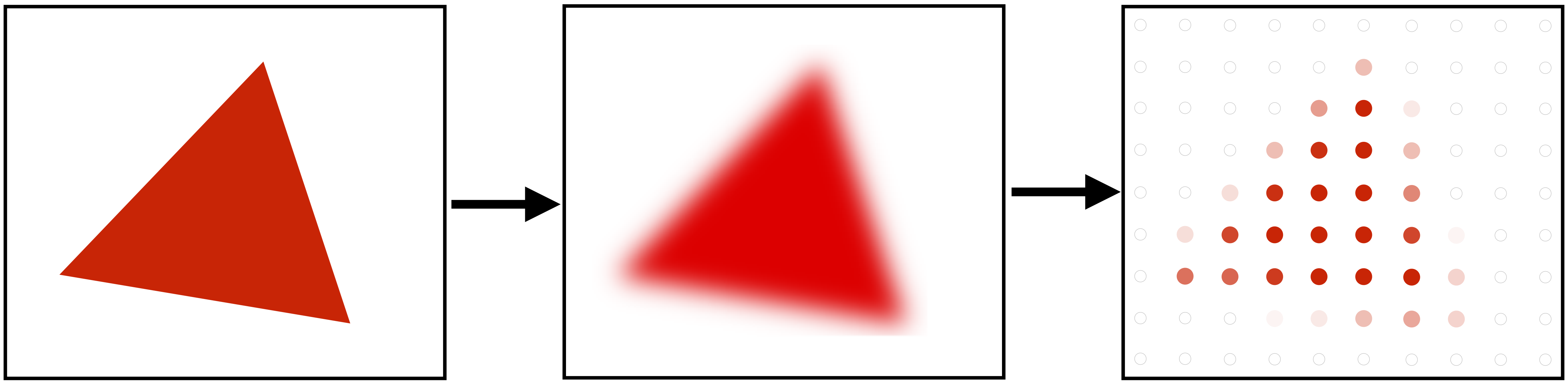
先模糊再采样

Rasterization: Point Sampling in Space

Note jaggies in rasterized triangle
where pixel values are **pure red or white**

值变化太快

Rasterization: Antialiased Sampling



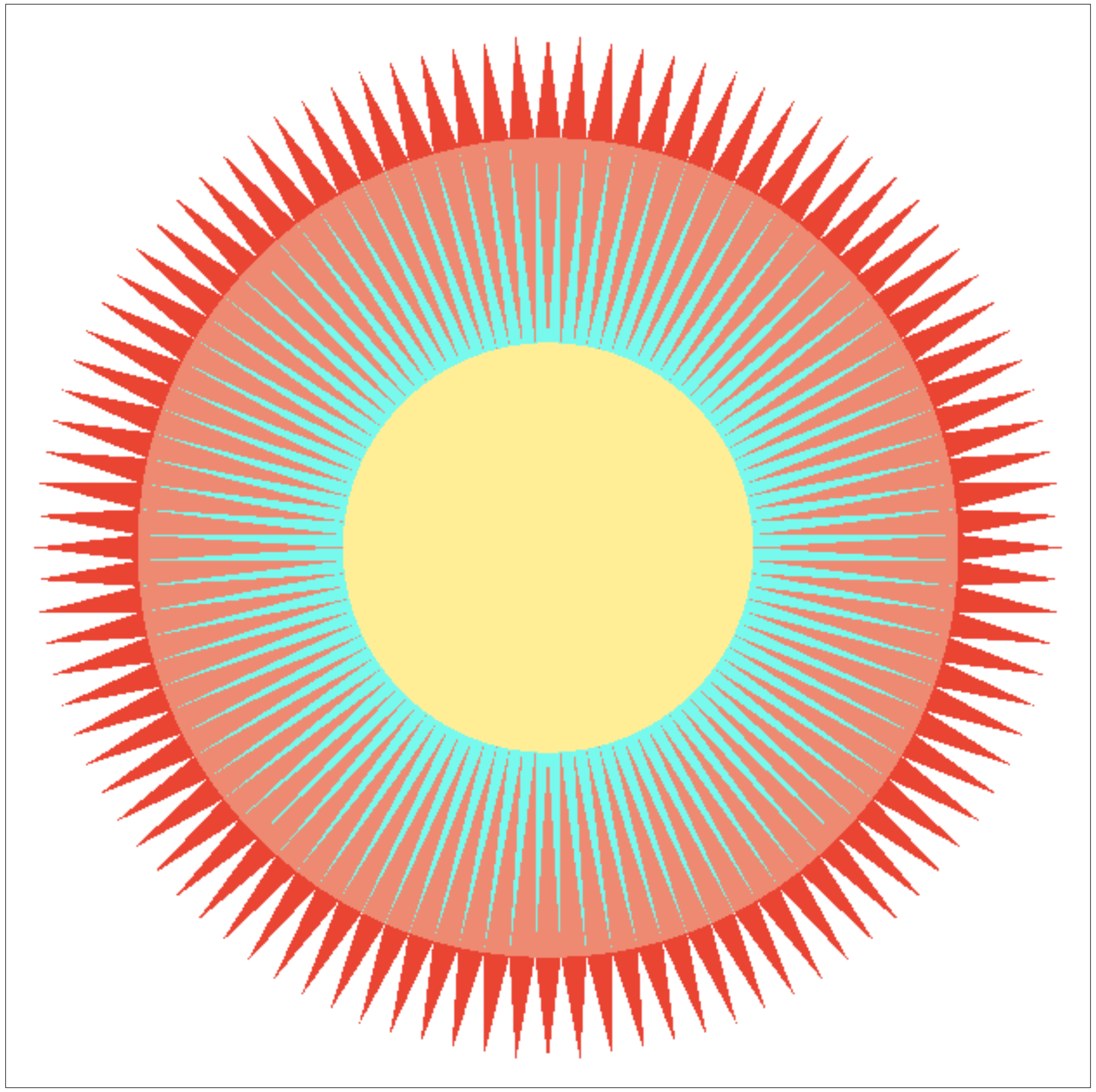
Pre-Filter

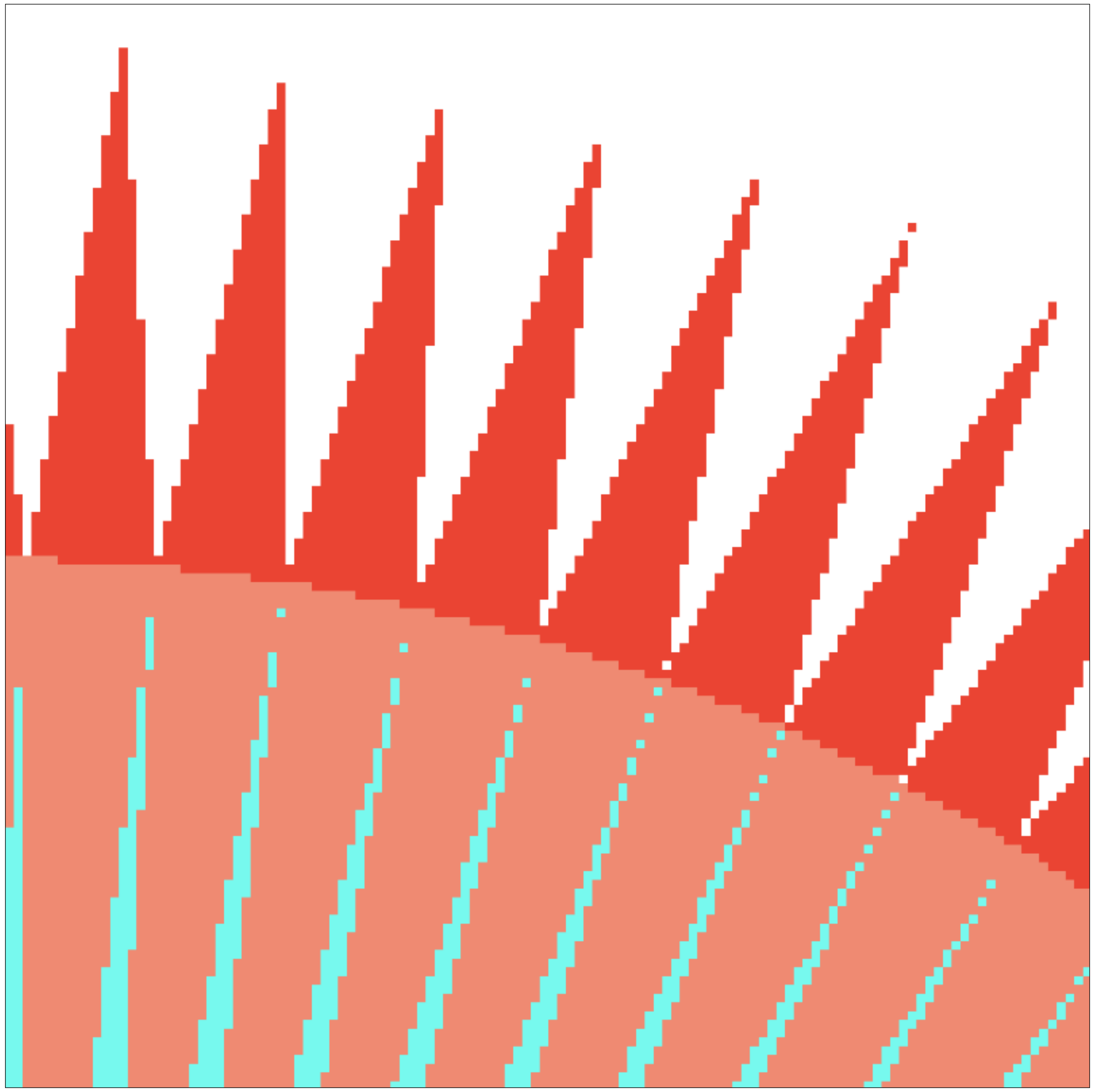
(remove frequencies above Nyquist) (?)

Sample

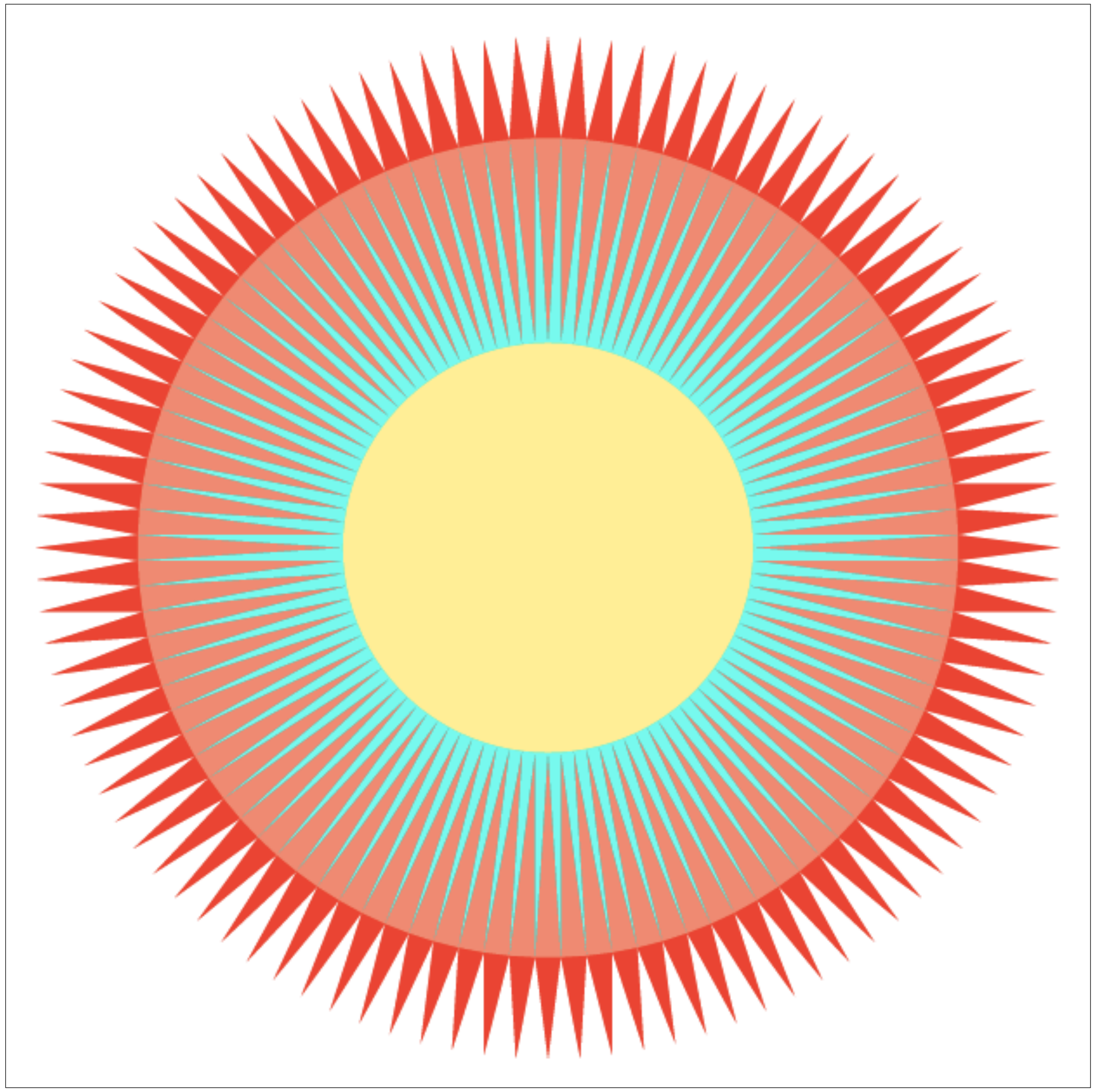
Note antialiased edges in rasterized triangle where pixel values take intermediate values

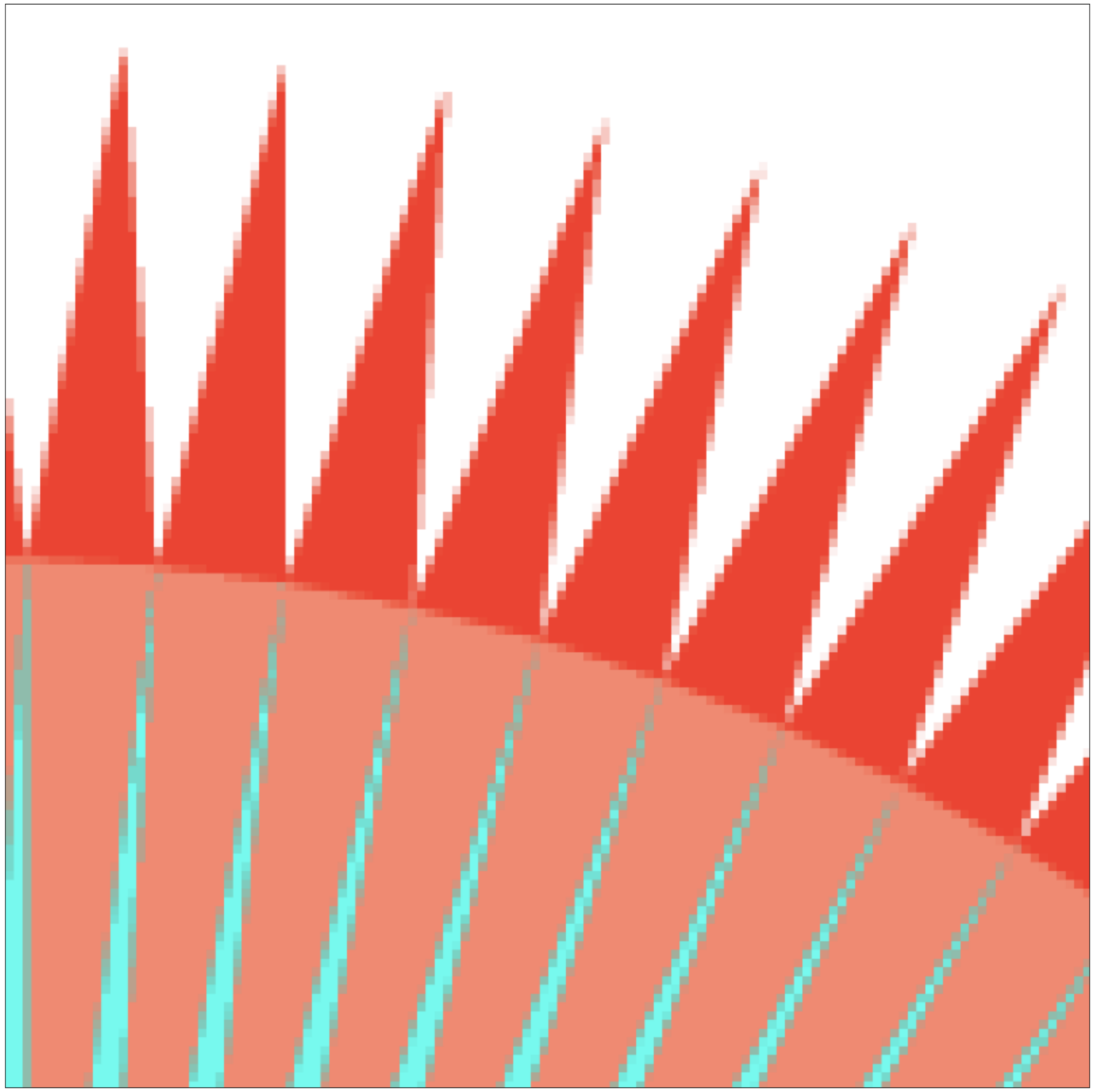
Point Sampling



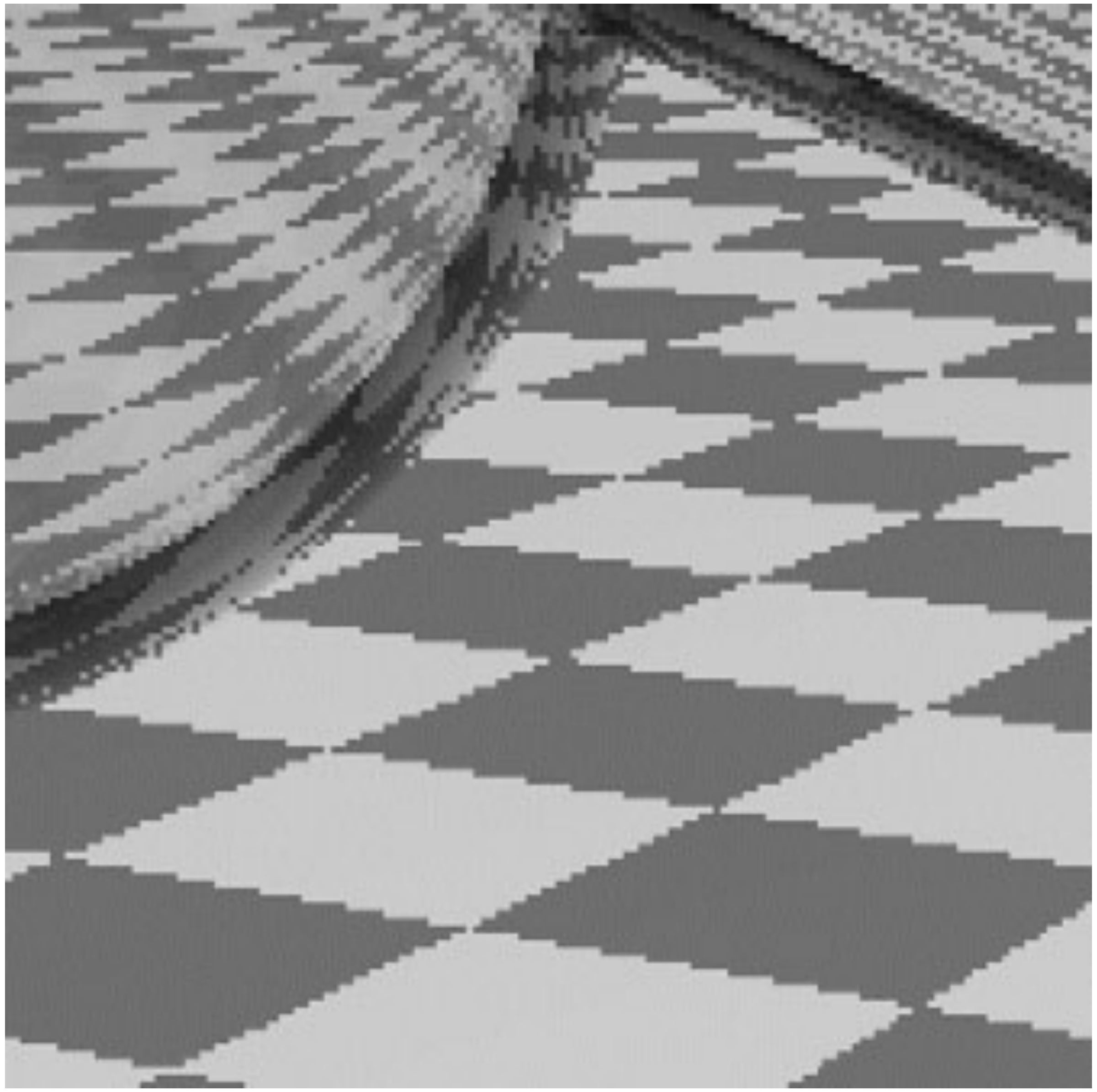


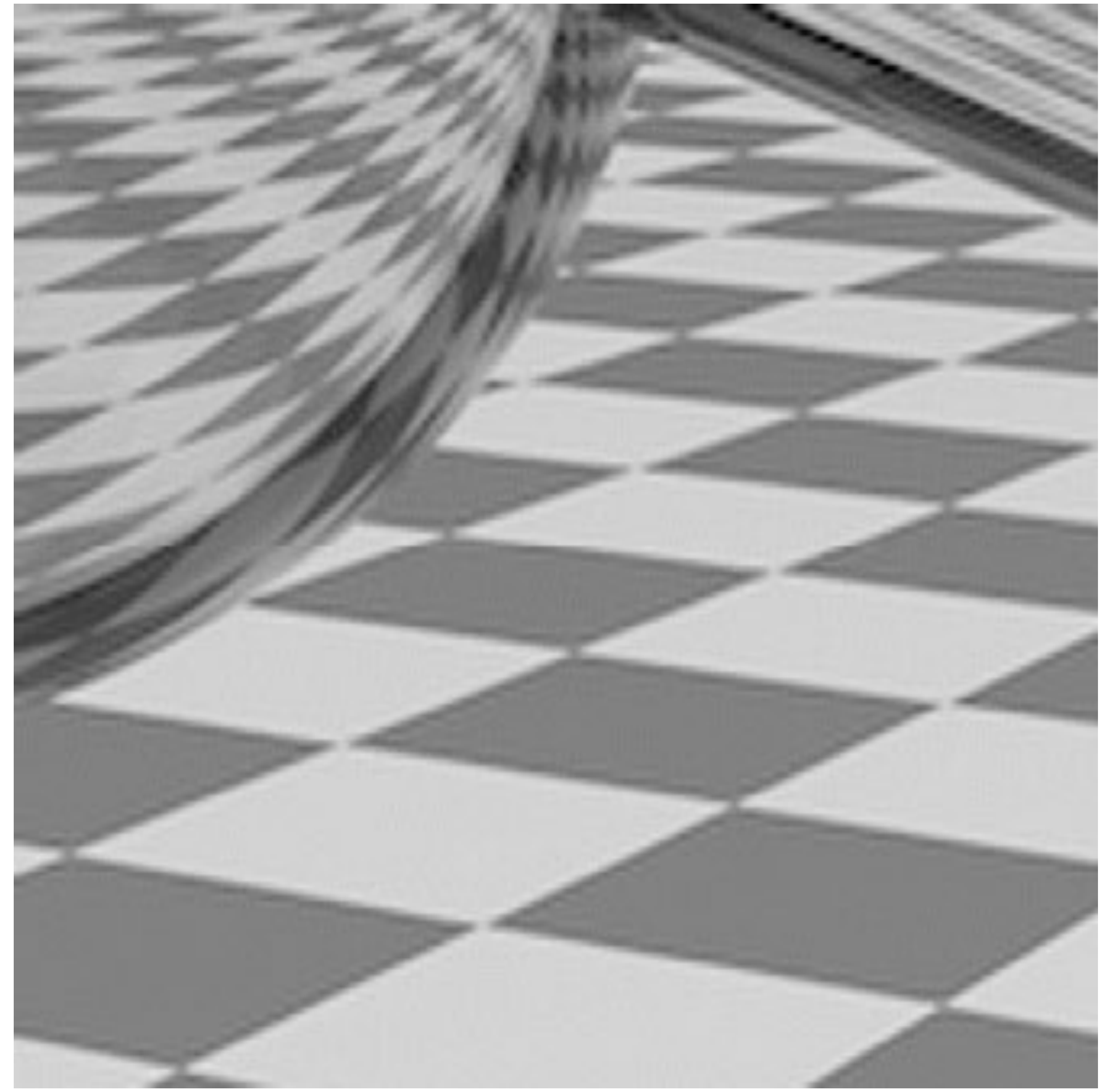
Antialiasing





Point Sampling vs Antialiasing

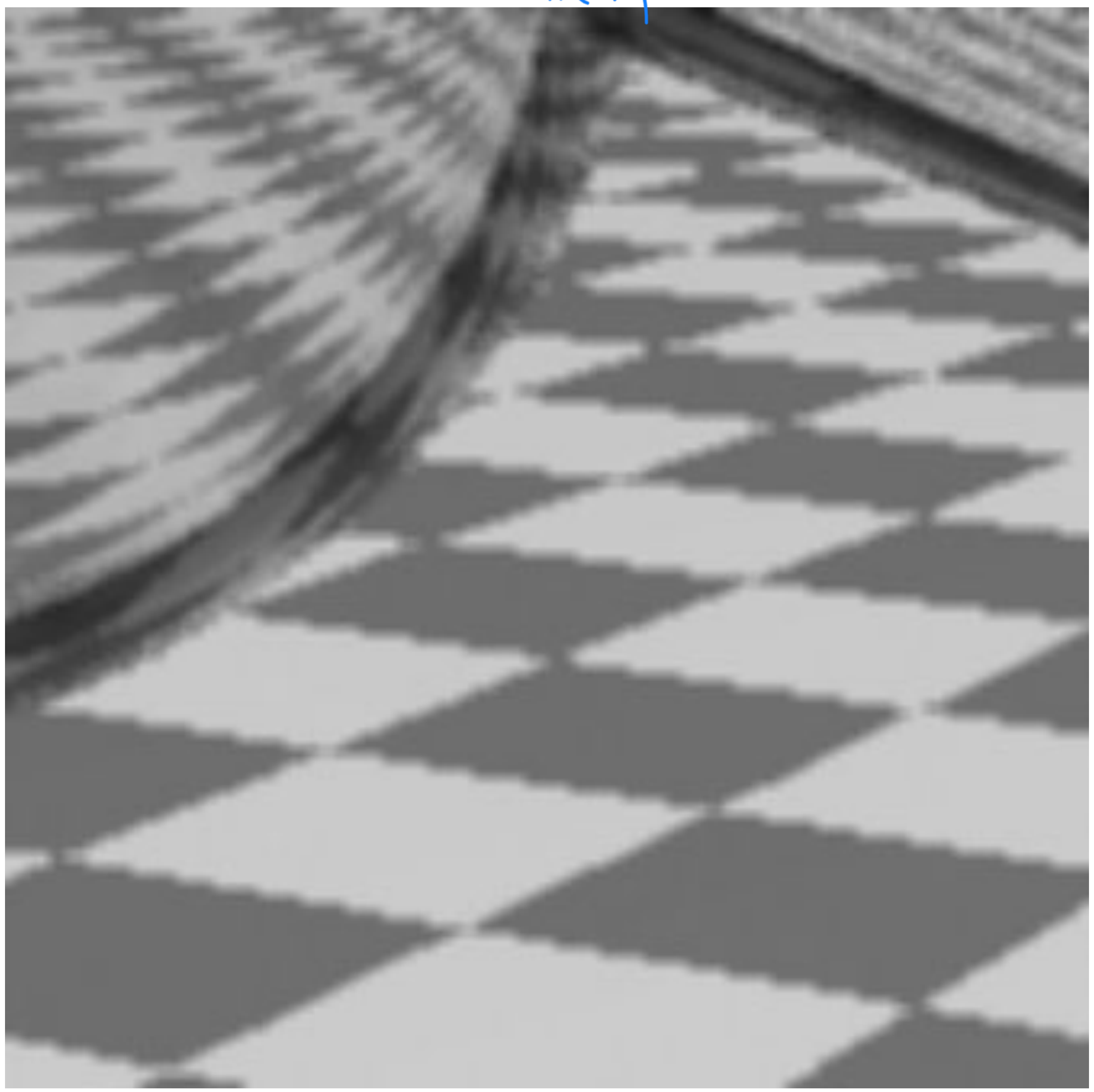




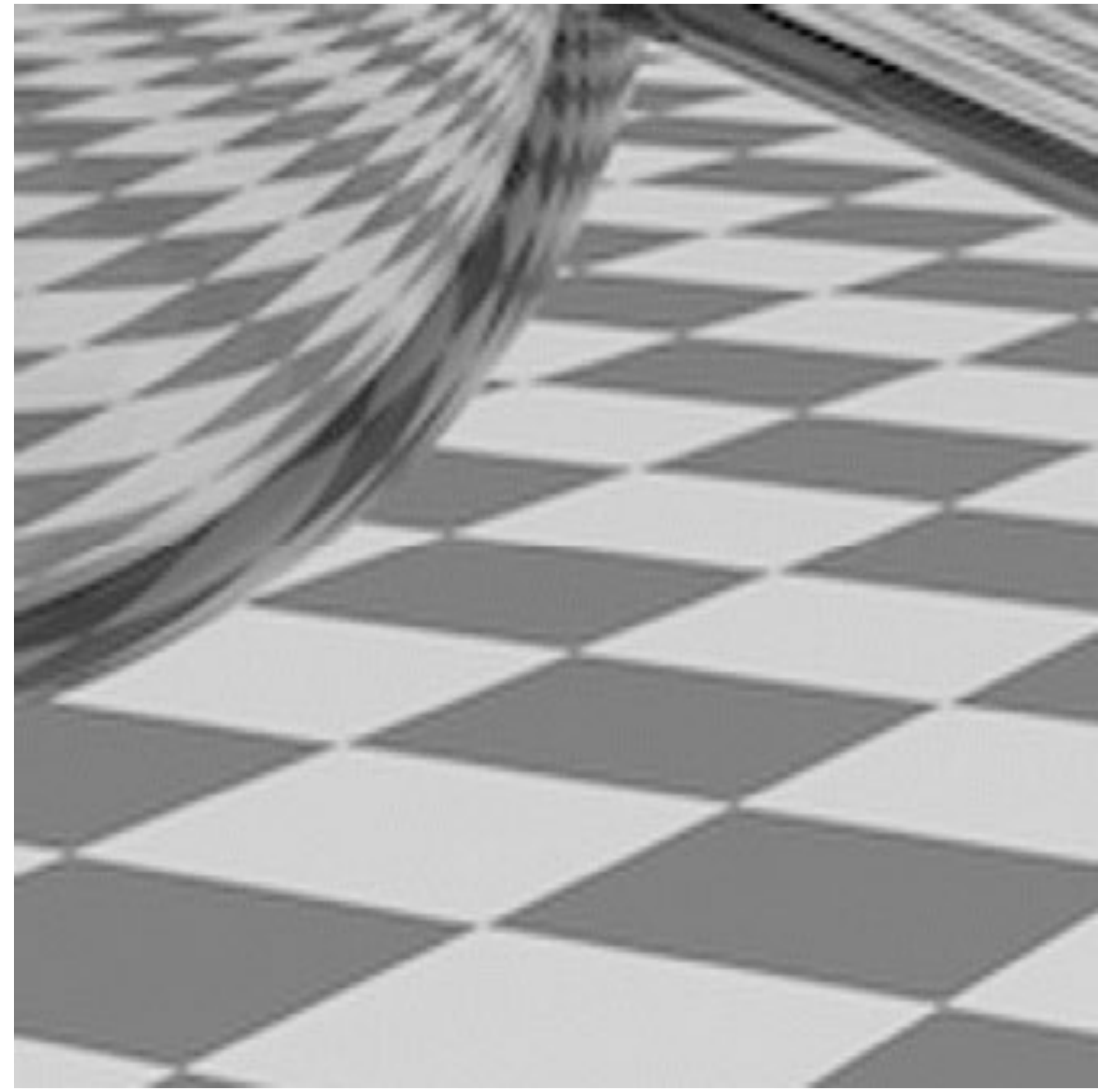
Antialiasing vs Blurred Aliasing

先采样再Blur

先blur再采样



(Sample then filter, WRONG!)



(Filter then sample)

But why?

1. Why undersampling introduces aliasing?
2. Why pre-filtering then sampling can do antialiasing?

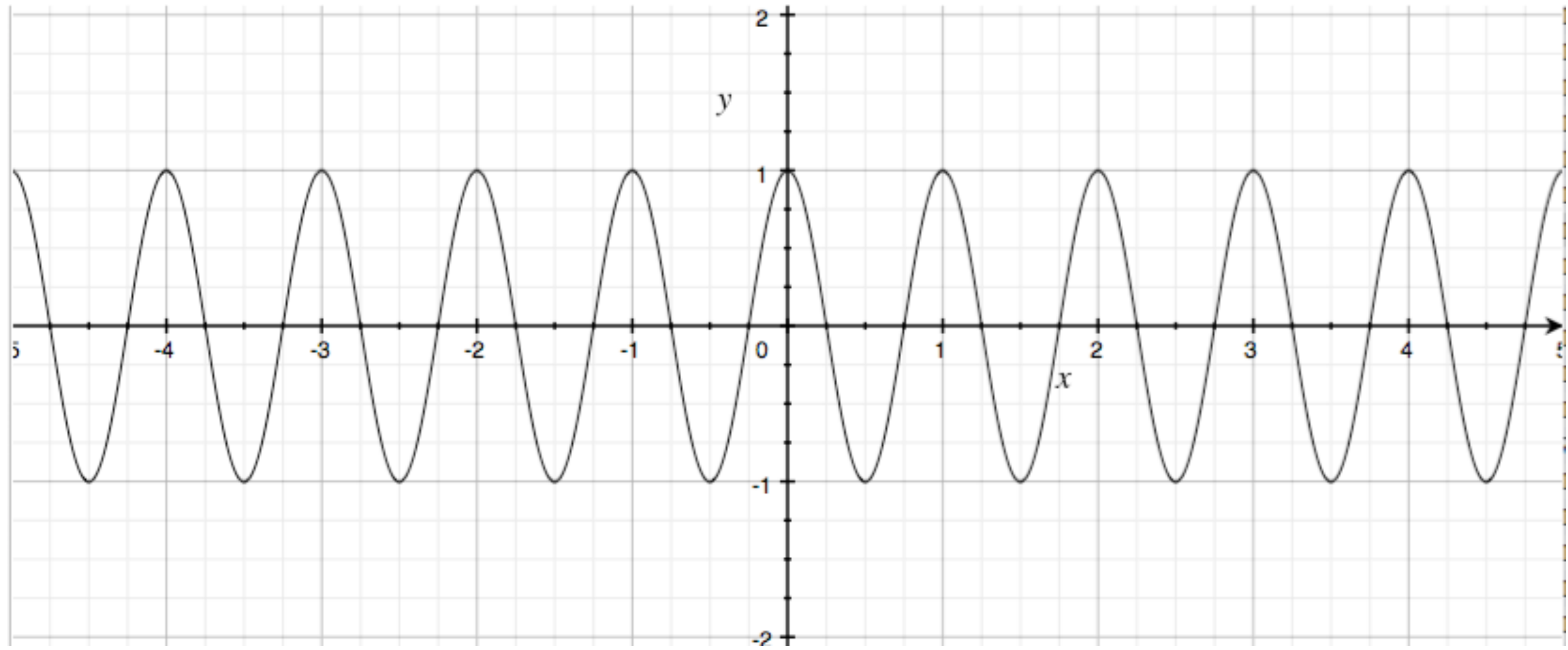
Let's dig into fundamental reasons

And look at how to implement antialiased rasterization

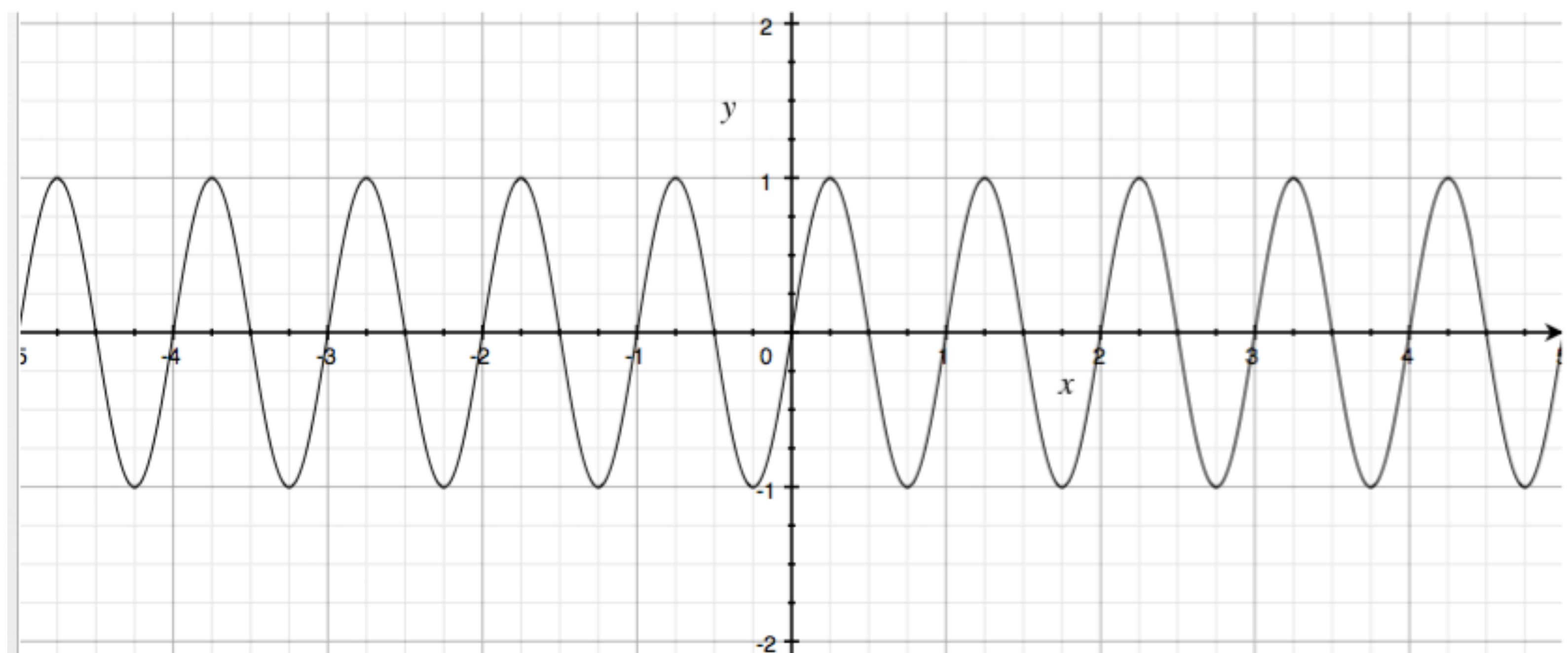
频域

Frequency Domain

Sines and Cosines



$$\cos 2\pi x$$

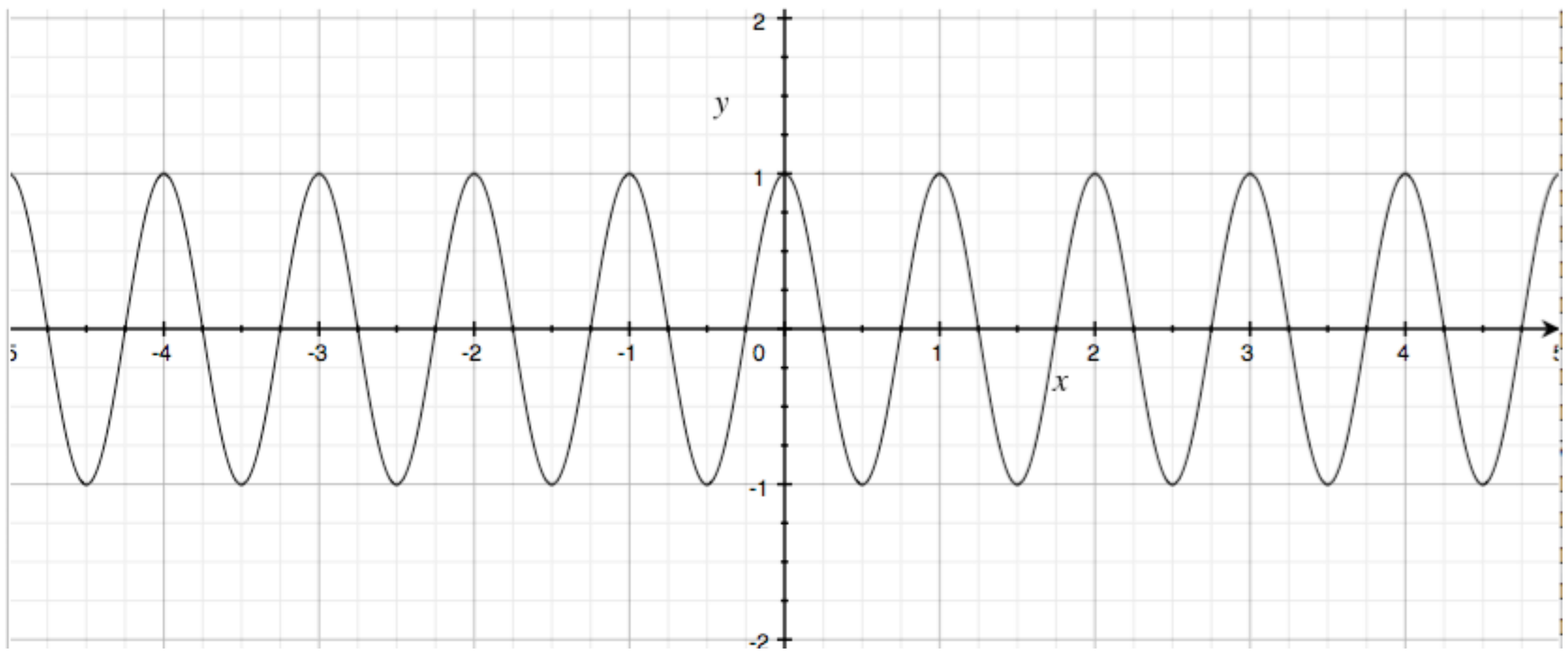


$$\sin 2\pi x$$

Frequencies $\cos 2\pi f x$

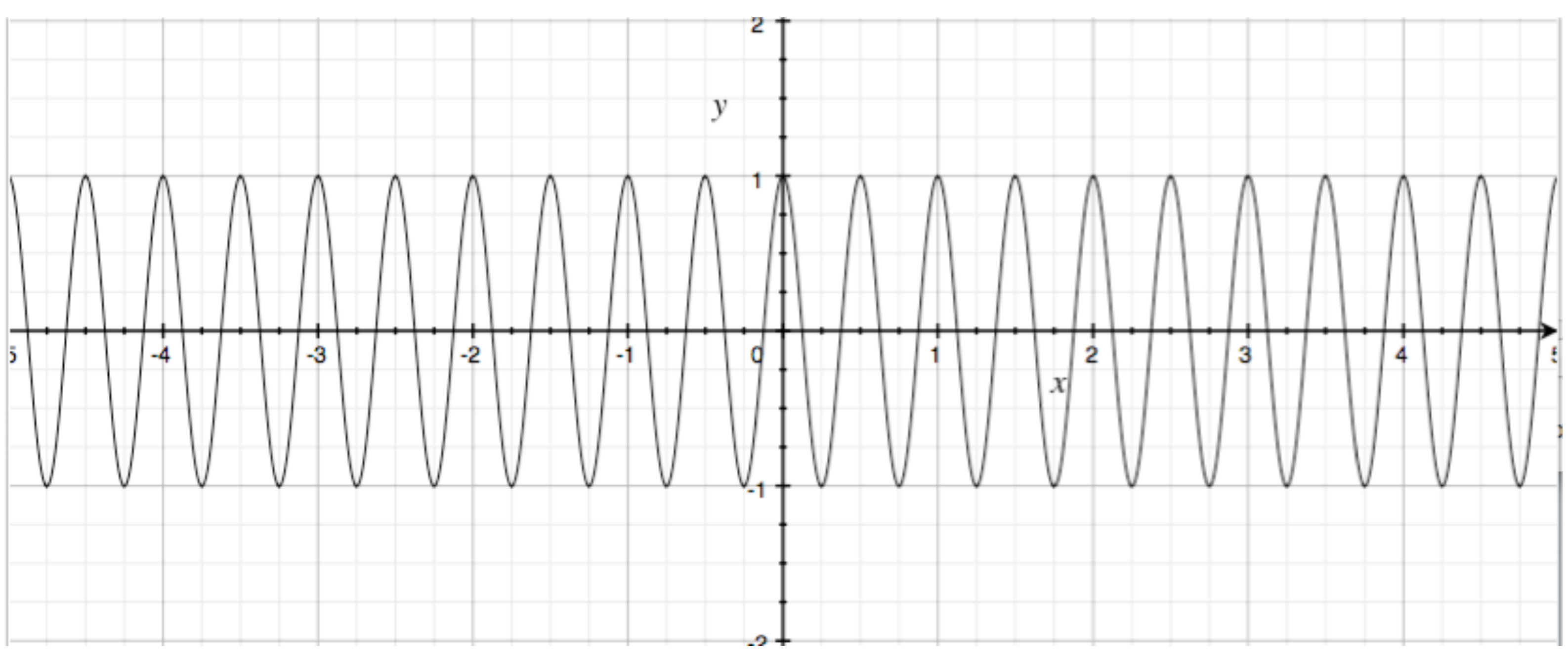
$$f = \frac{1}{T}$$

频率　周期



$\cos 2\pi x$

$f = 1$



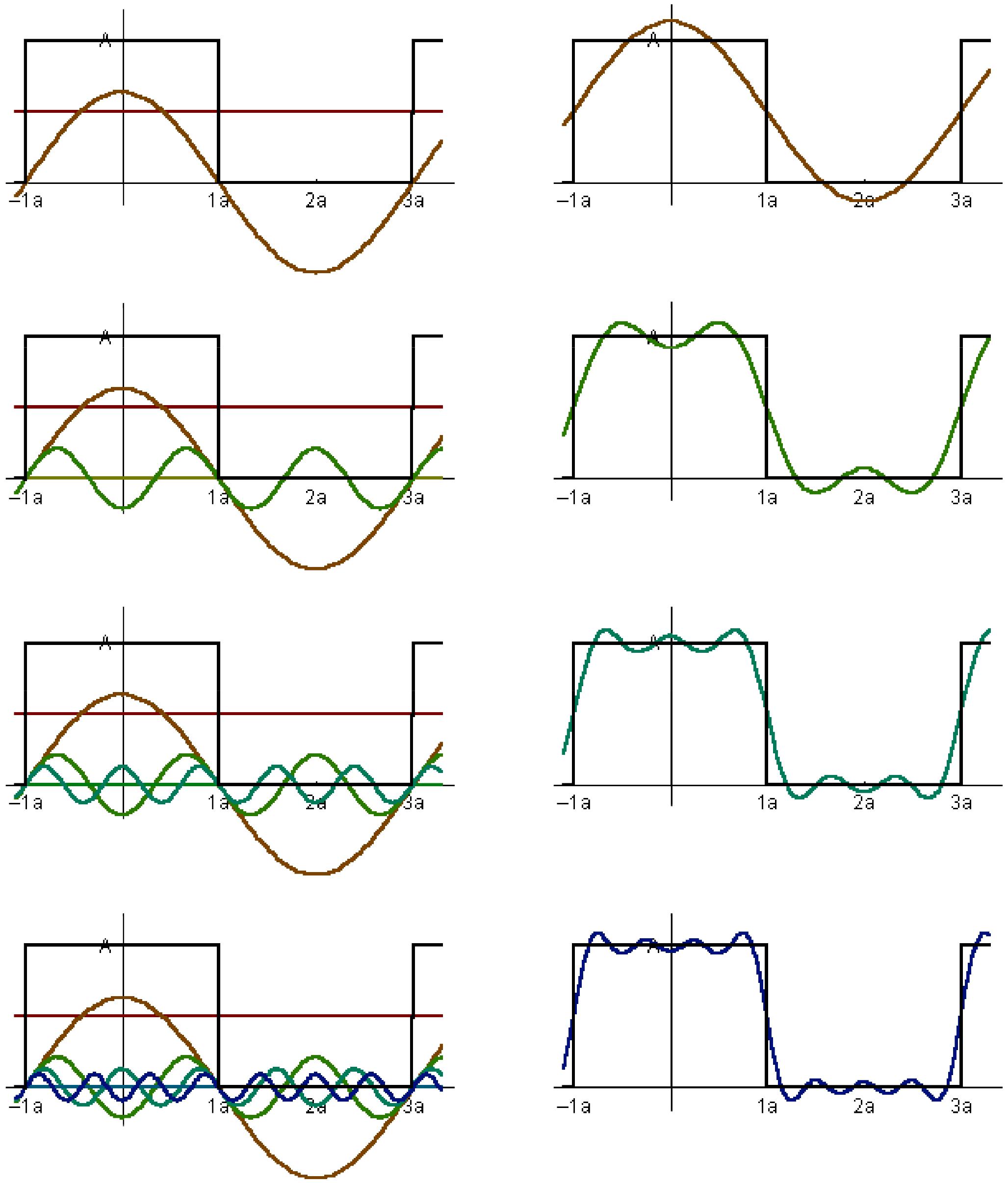
$\cos 4\pi x$

$f = 2$

Fourier Transform

Represent a function as a weighted sum of sines and cosines

Joseph Fourier 1768 - 1830



$$f(x) = \frac{A}{2} + \frac{2A\cos(t\omega)}{\pi} - \frac{2A\cos(3t\omega)}{3\pi} + \frac{2A\cos(5t\omega)}{5\pi} - \frac{2A\cos(7t\omega)}{7\pi} + \cdots$$

Fourier Transform Decomposes A Signal Into Frequencies

$$f(x) \qquad F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i \omega x} dx \qquad F(\omega)$$

spatial domain → **Fourier transform** → **frequency domain**

spatial domain ← **Inverse transform** ← **frequency domain**

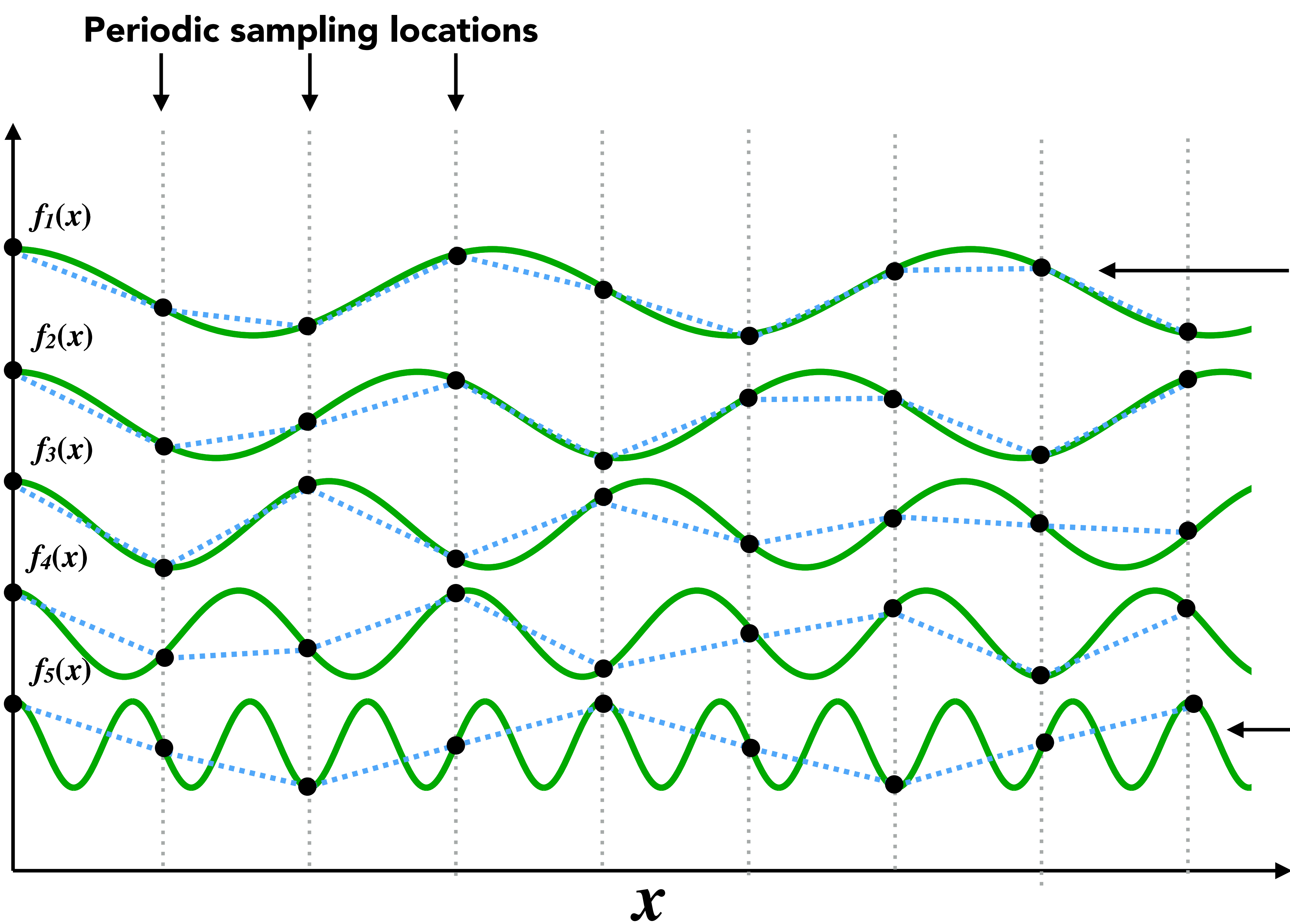
时域

频域

$$f(x) = \int_{-\infty}^{\infty} F(\omega) e^{2\pi i \omega x} d\omega$$

Recall $e^{ix} = \cos x + i \sin x$

Higher Frequencies Need Faster Sampling



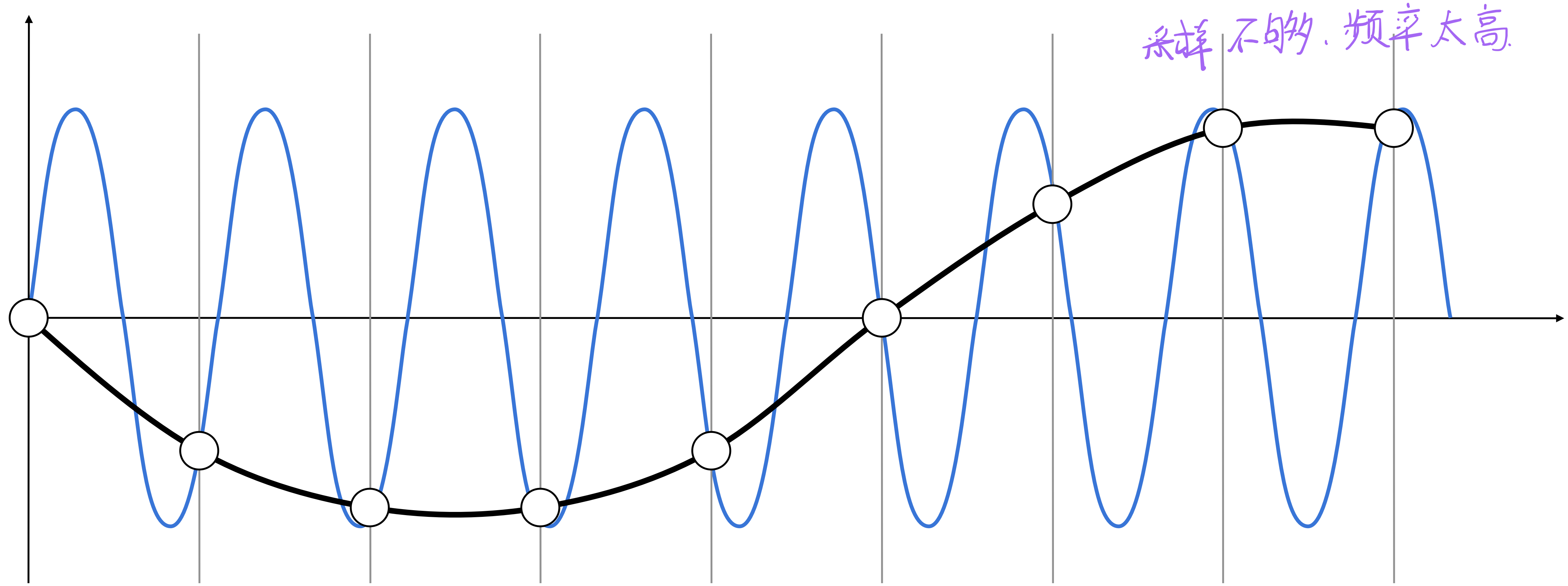
Low-frequency signal: sampled adequately for reasonable reconstruction

High-frequency signal is insufficiently sampled: reconstruction incorrectly appears to be from a low frequency signal

通过分析频率理解走样

Undersampling Creates Frequency Aliases

采样不够，频率太高



High-frequency signal is insufficiently sampled: samples erroneously appear to be from a low-frequency signal

Two frequencies that are indistinguishable at a given sampling rate are called "aliases"

同样采样方法采样两种不同函数，我们无法区分（走样）

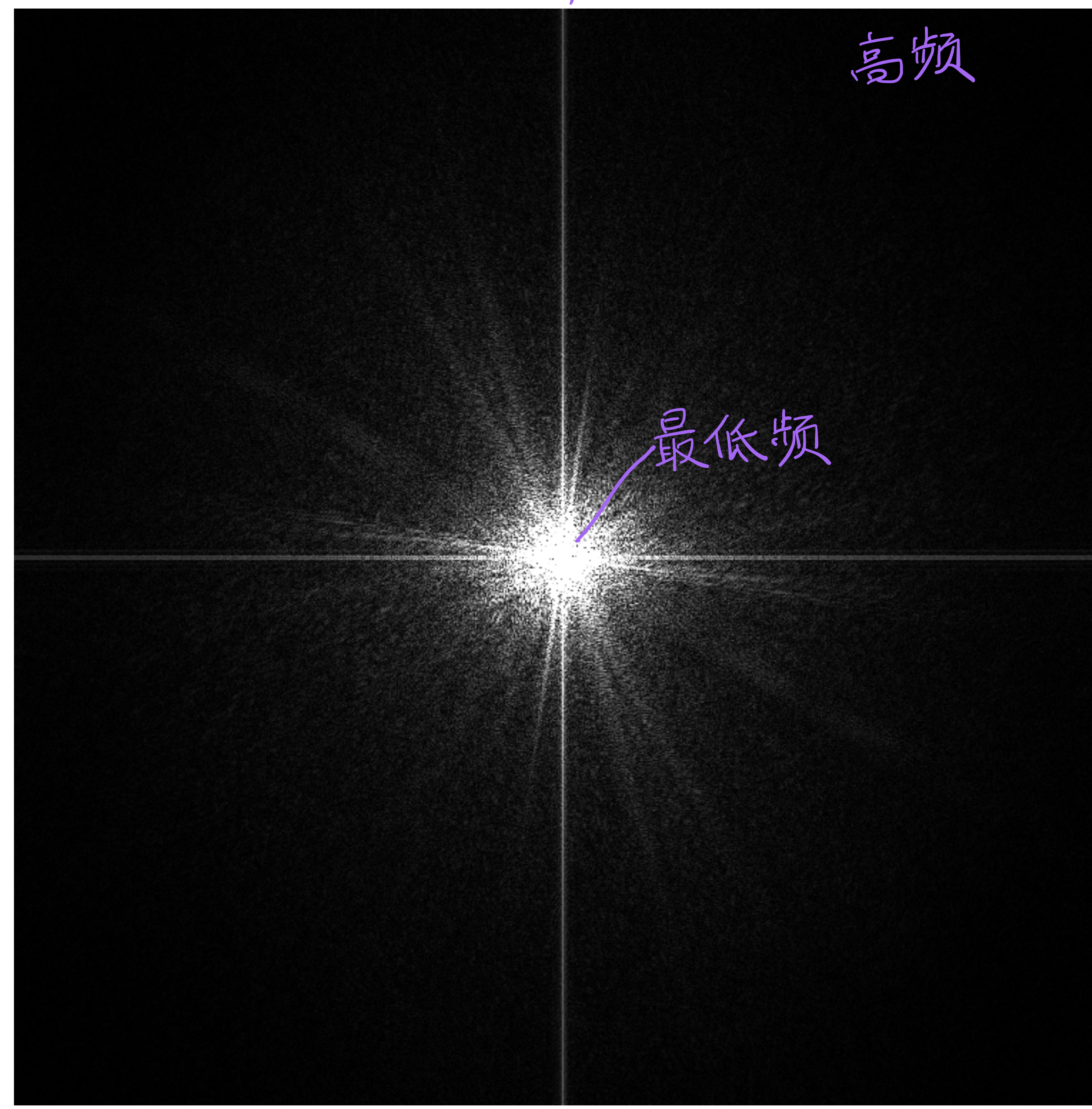
滤波 删掉特定频率的信号

Filtering = Getting rid of certain frequency contents

Visualizing Image Frequency Content

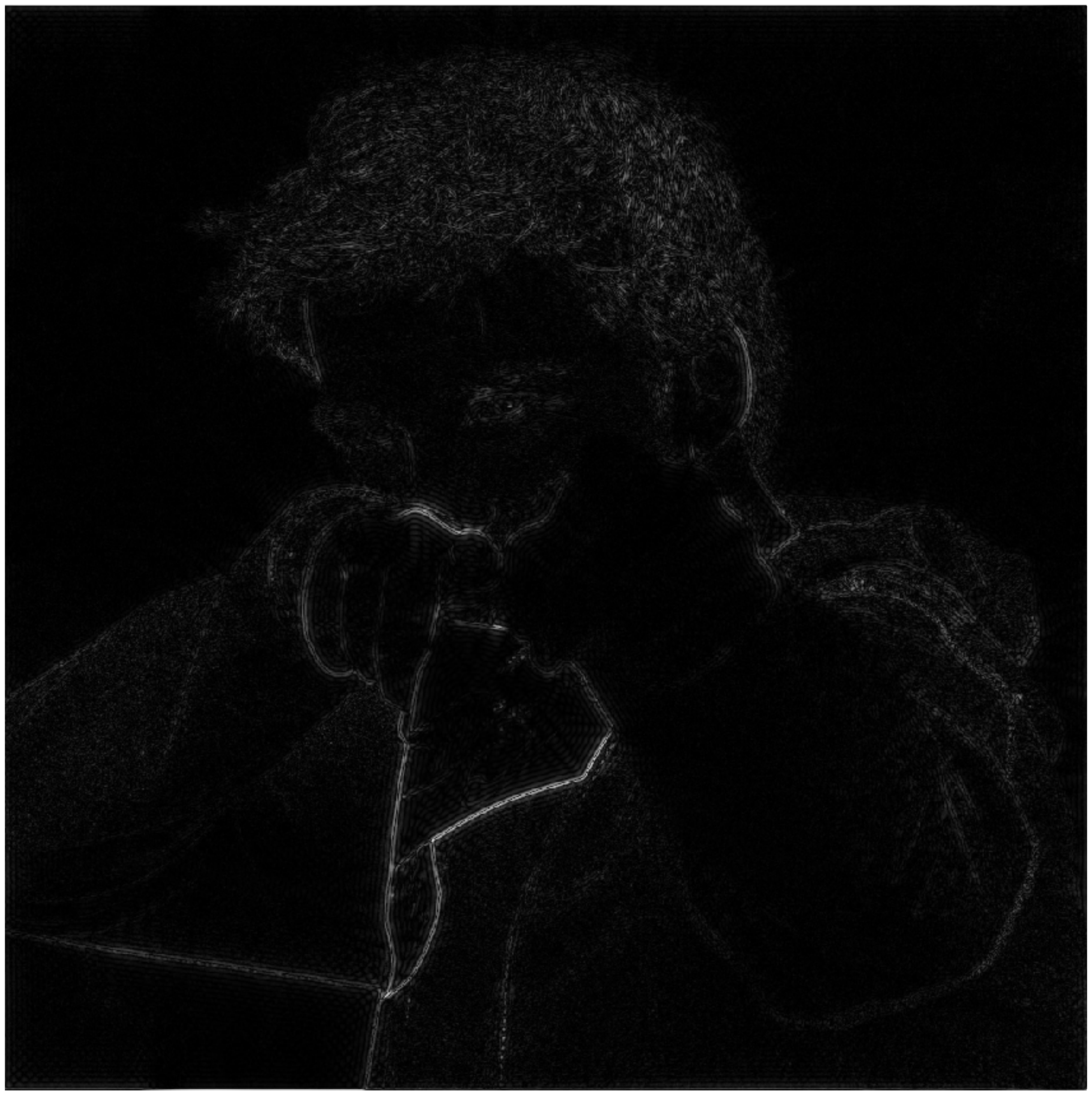
时域 → 频域

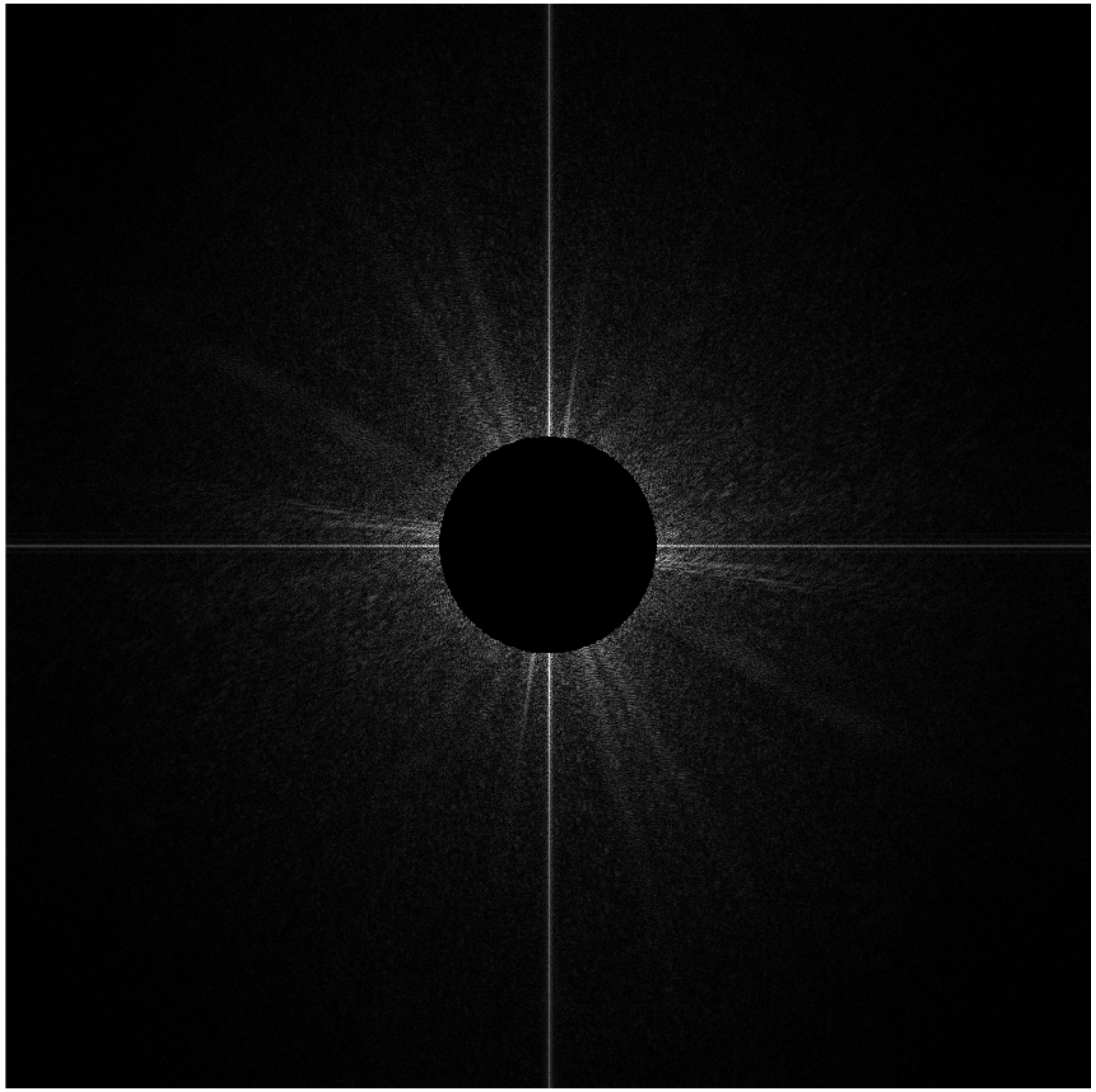
边界剧烈变化



亮度高 信息多
一般图片低频信息多.
高频信息少

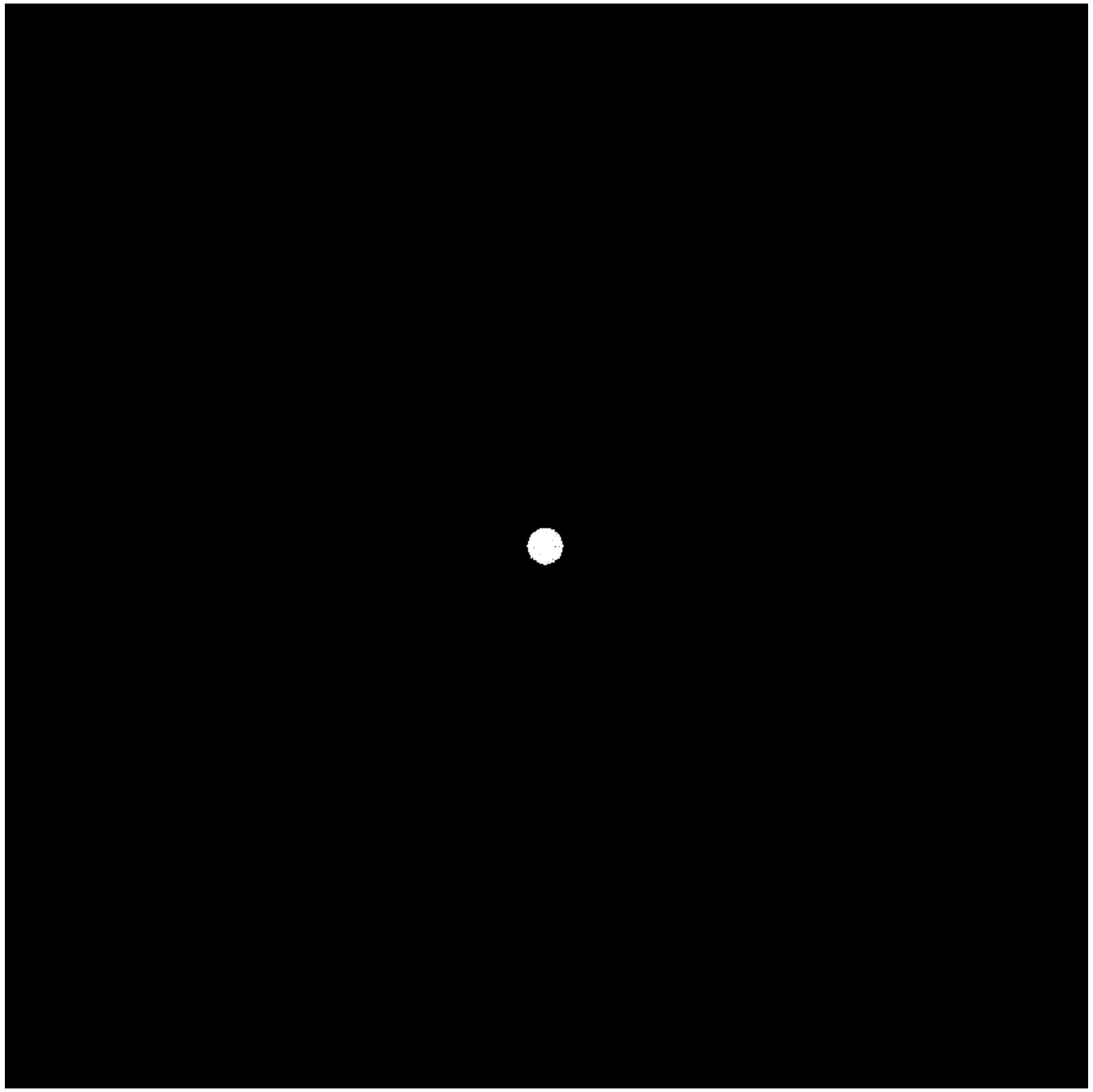
Filter Out Low Frequencies Only (Edges)





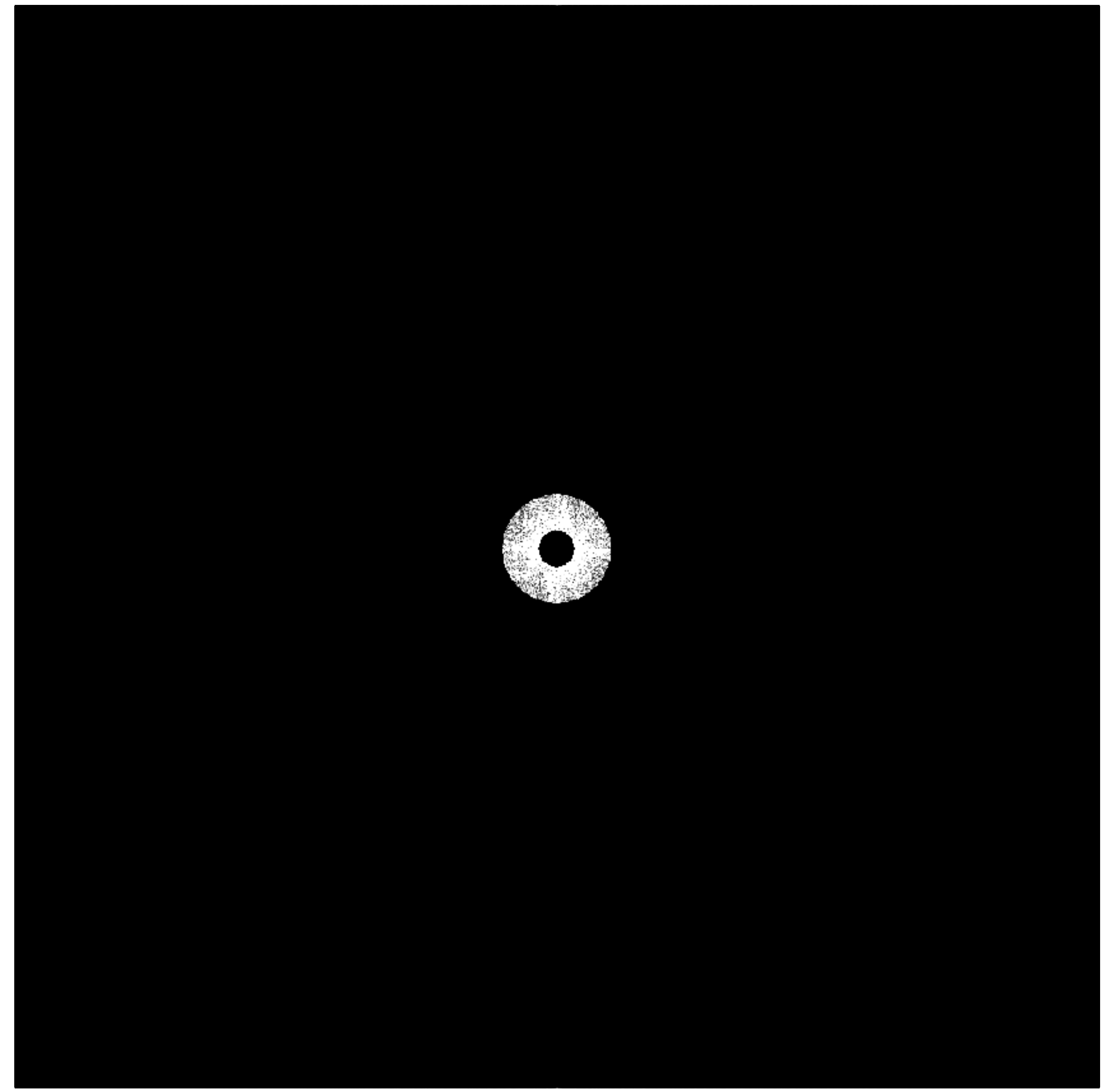
High-pass filter (图像边界) 高通滤波

Filter Out High Frequencies (Blur)

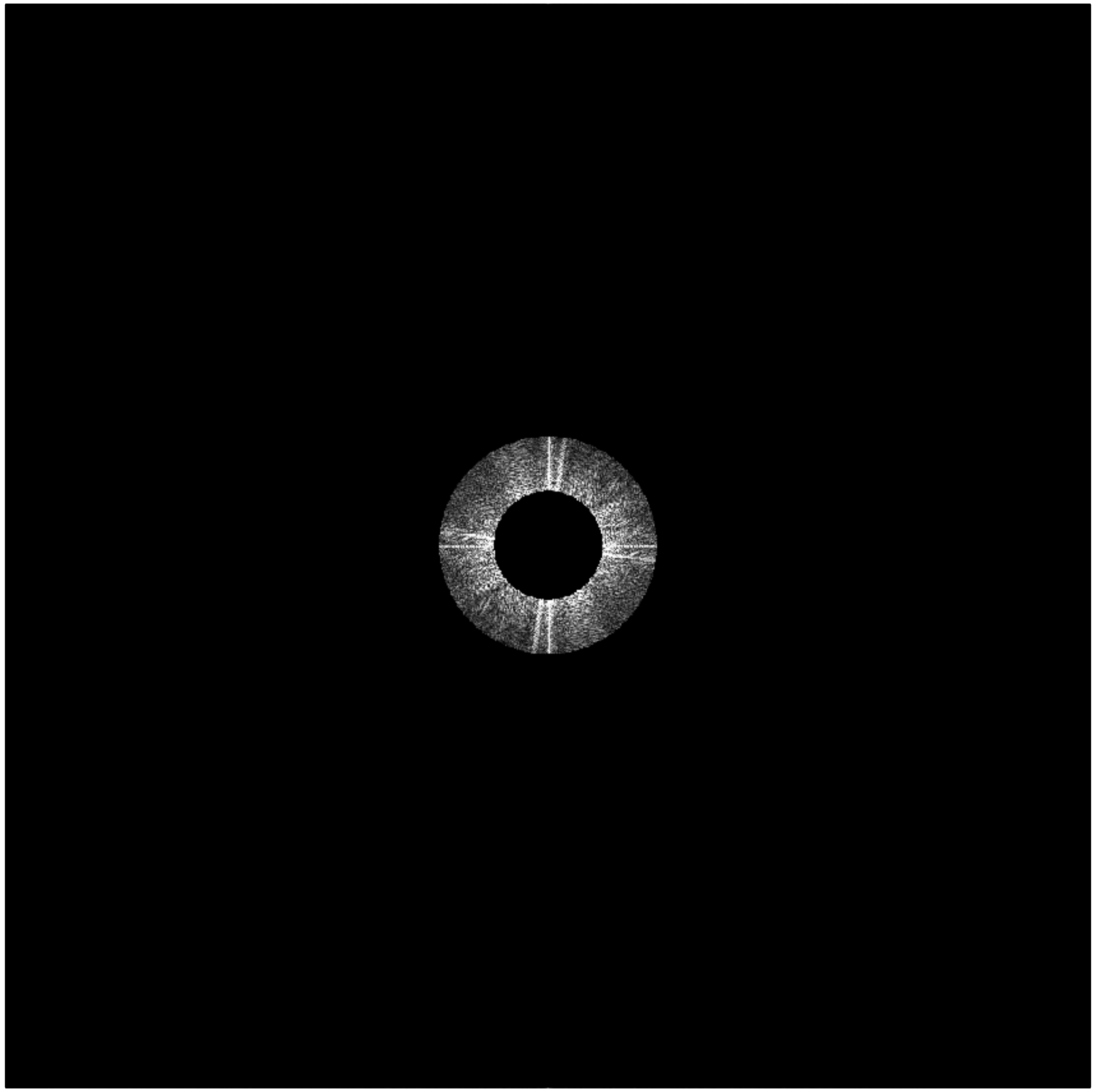


Low-pass filter 低通滤波 (只保留低频信号)

Filter Out Low and High Frequencies



Filter Out Low and High Frequencies



卷积

Filtering = Convolution

(= Averaging)

平均

Convolution

Signal

1	3	5	3	7	1	3	8	6	4

Filter

1/4	1/2	1/4

Point-wise local averaging in a “sliding window”

Convolution

Signal

1	3	5	3	7	1	3	8	6	4

Filter

1/4	1/2	1/4

1 x (1/4) + 3 x (1/2) + 5 x (1/4) = 3

Result

	3								

Convolution

Signal

1	3	5	3	7	1	3	8	6	4

Filter

1/4	1/2	1/4

$$3 \times (1/4) + 5 \times (1/2) + 3 \times (1/4) = 4$$

Result

	3	4							

Convolution Theorem

时域上卷积 ⟺ 频域上乘积

Convolution in the spatial domain is **equal to multiplication in the frequency domain**, and vice versa

Option 1:

- Filter by convolution in the spatial domain

Option 2:

- Transform to frequency domain (Fourier transform)
- Multiply by Fourier transform of convolution kernel
- Transform back to spatial domain (inverse Fourier)

Convolution Theorem

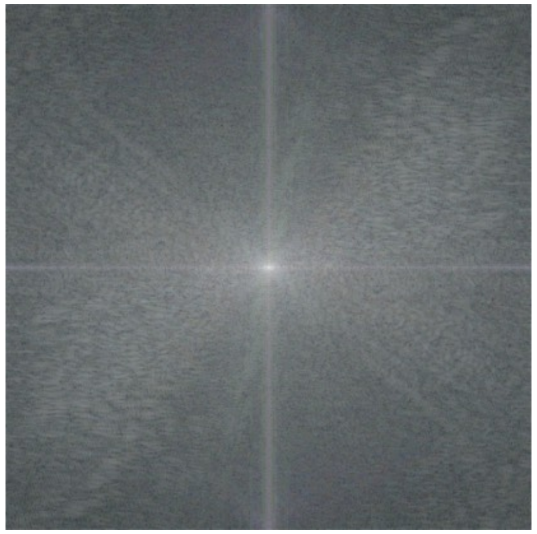
Spatial Domain

$$* \ \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$

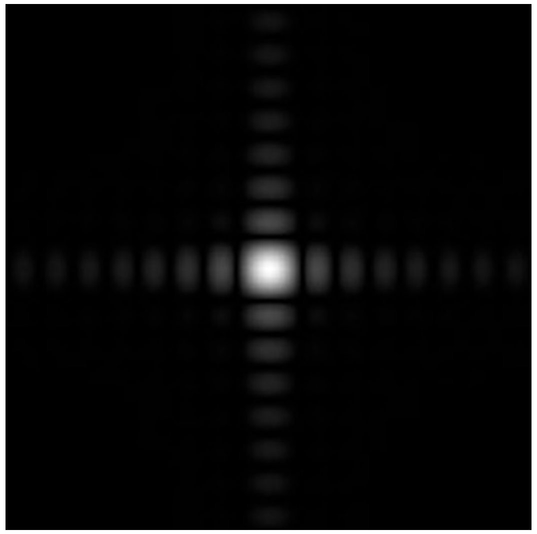
Fourier Transform $\downarrow$ $\downarrow$

Inv. Fourier Transform $\uparrow$

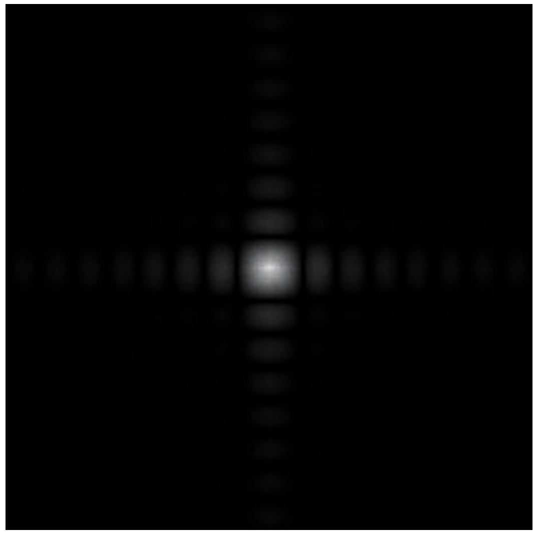
Frequency Domain



$\times$



$=$



Box Filter

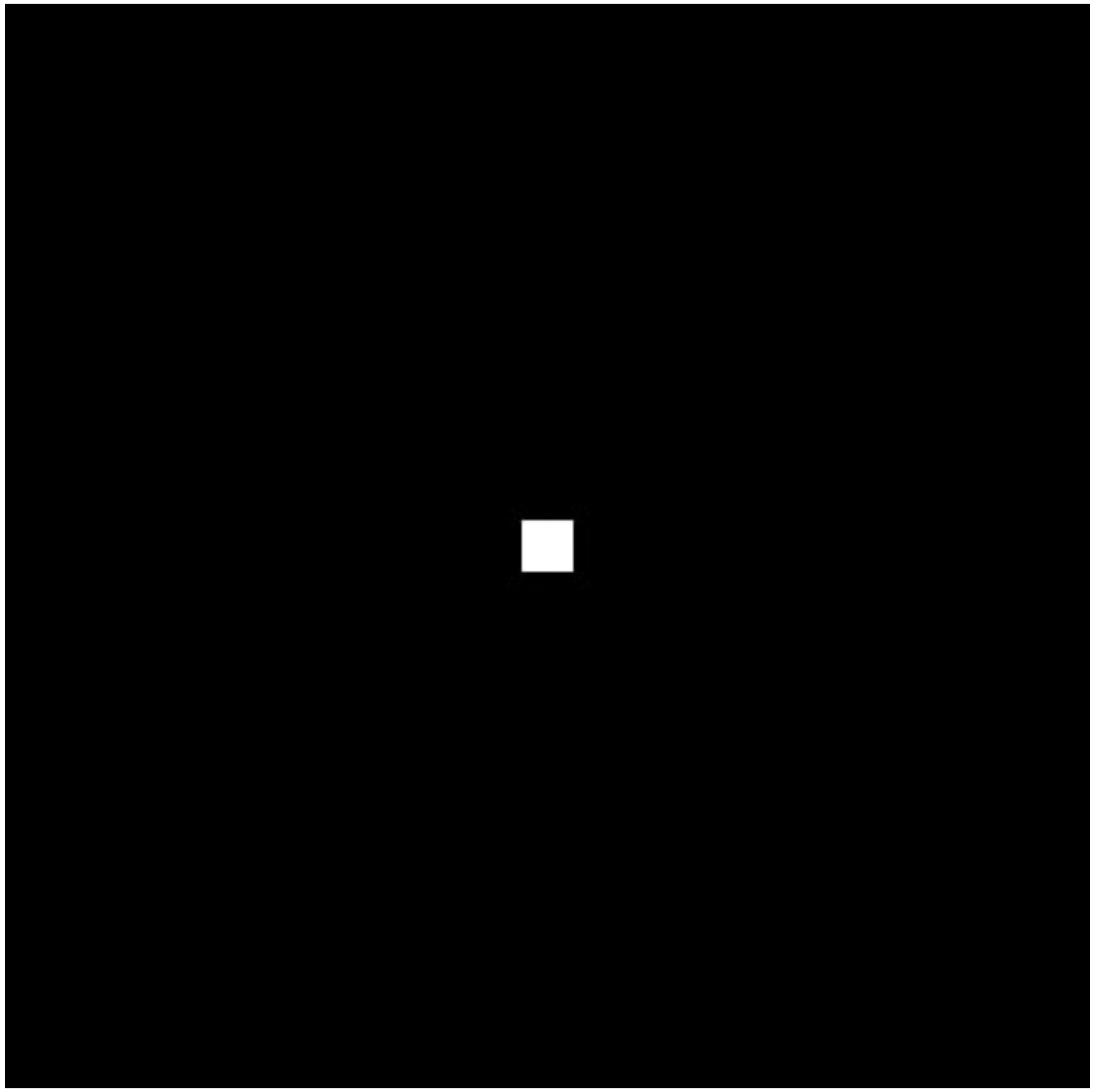
$$\frac{1}{9}\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline\end{array}$$

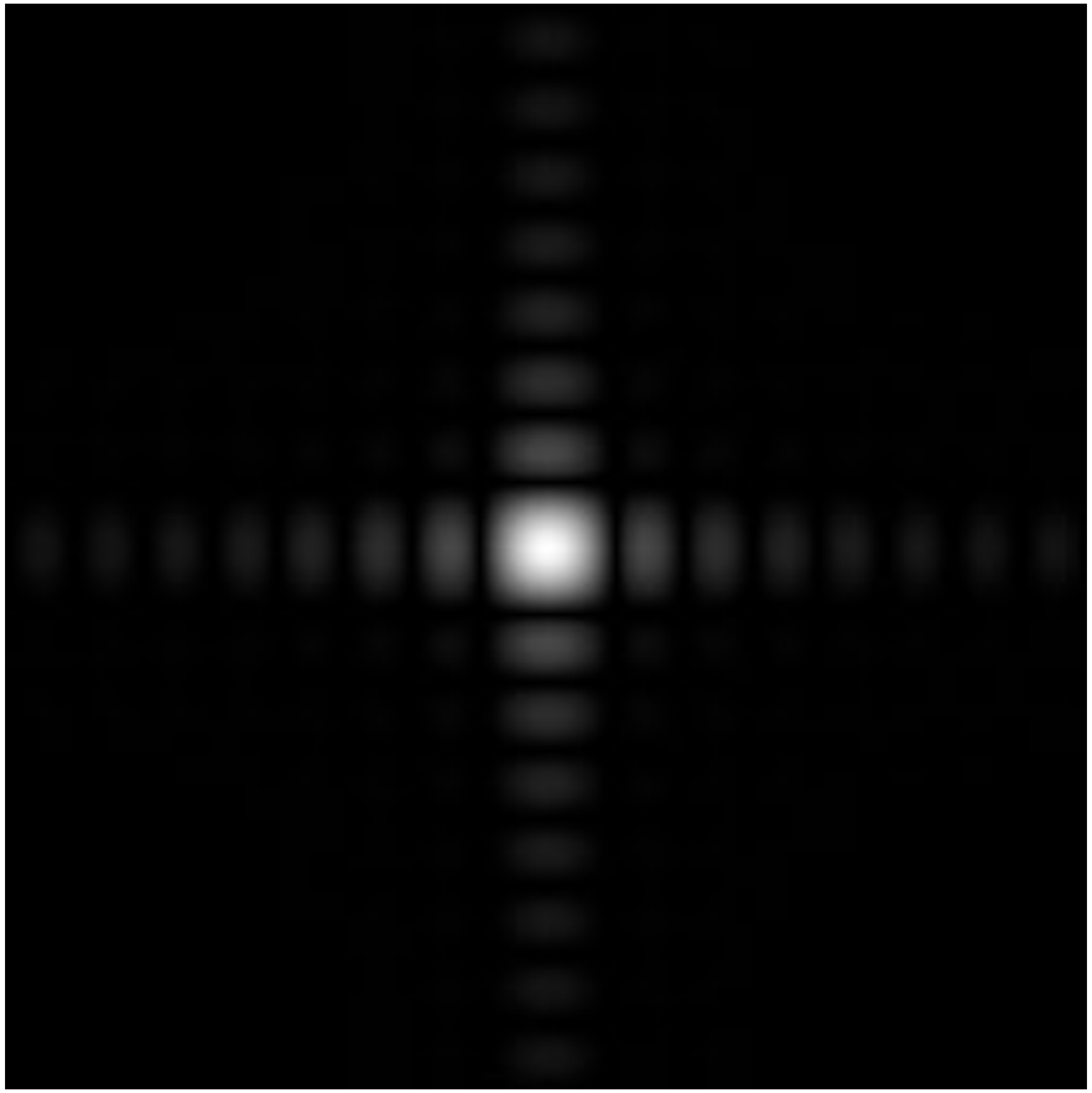
低通滤器

越大越模糊

Example: 3x3 box filter

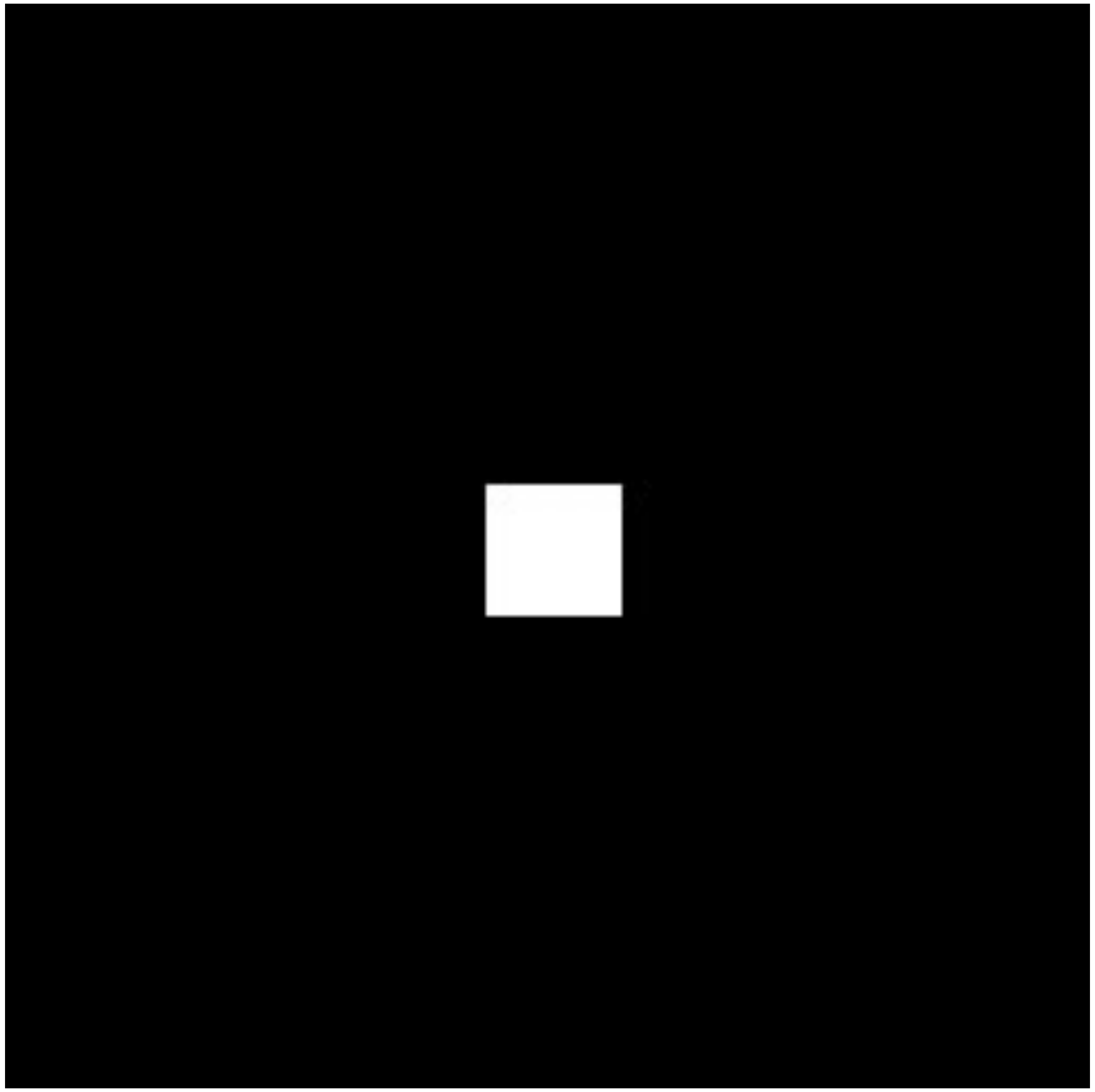
Box Function = “Low Pass” Filter



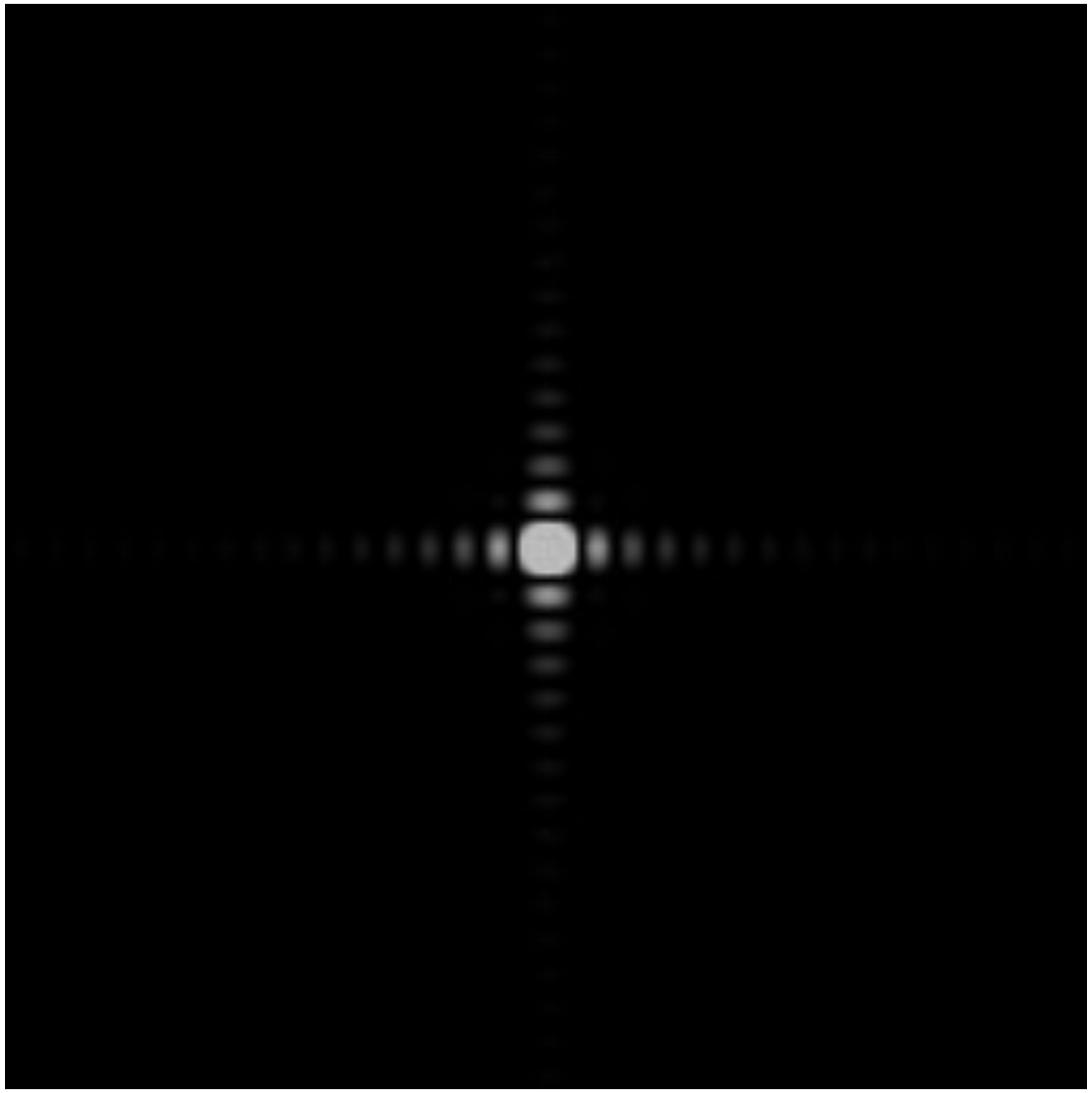


Wider Filter Kernel = Lower Frequencies

变大



变小



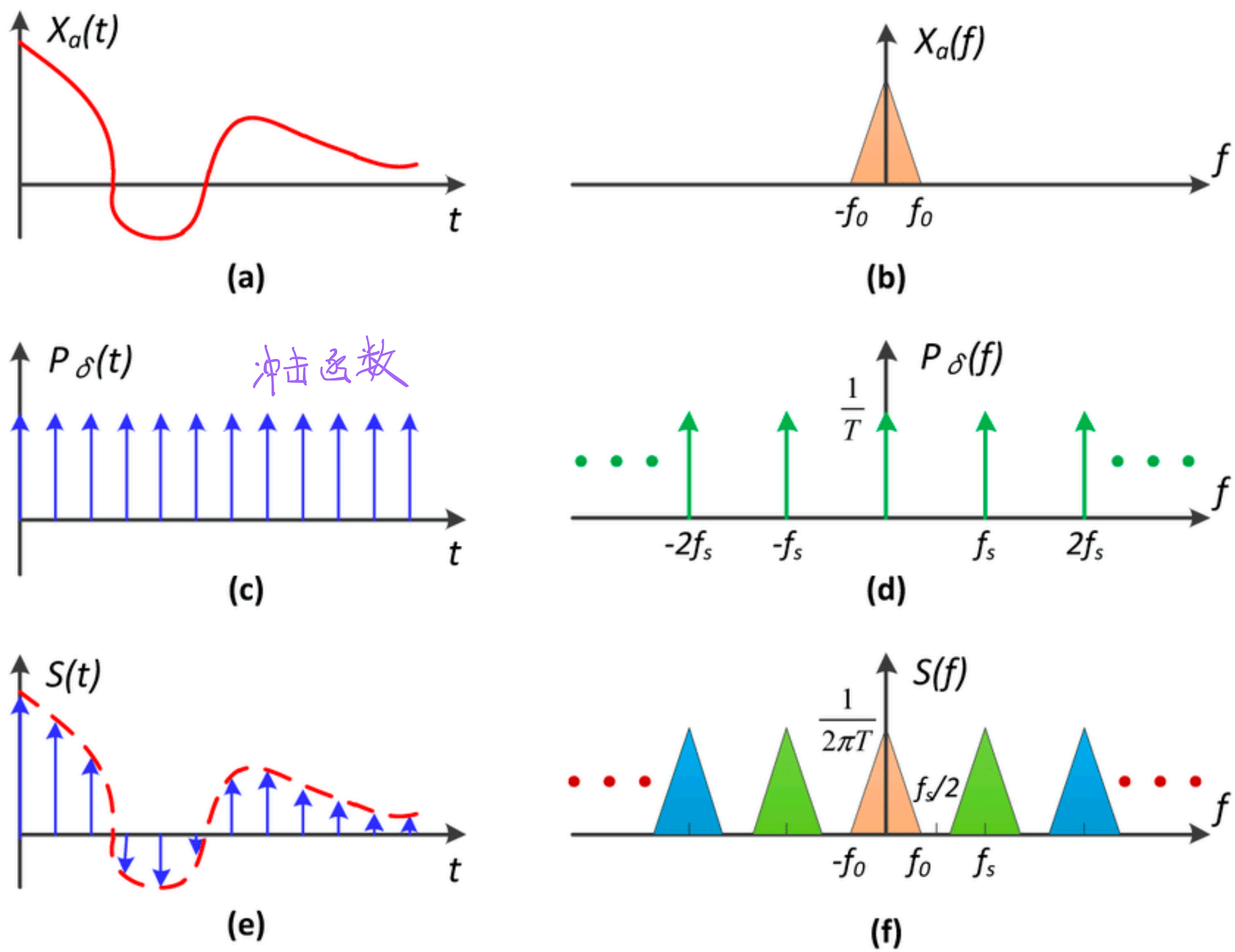
采样 重复频域上的内容

Sampling = Repeating Frequency Contents

Sampling = Repeating Frequency Contents

乘积

卷积



冲击函数

复制.粘贴

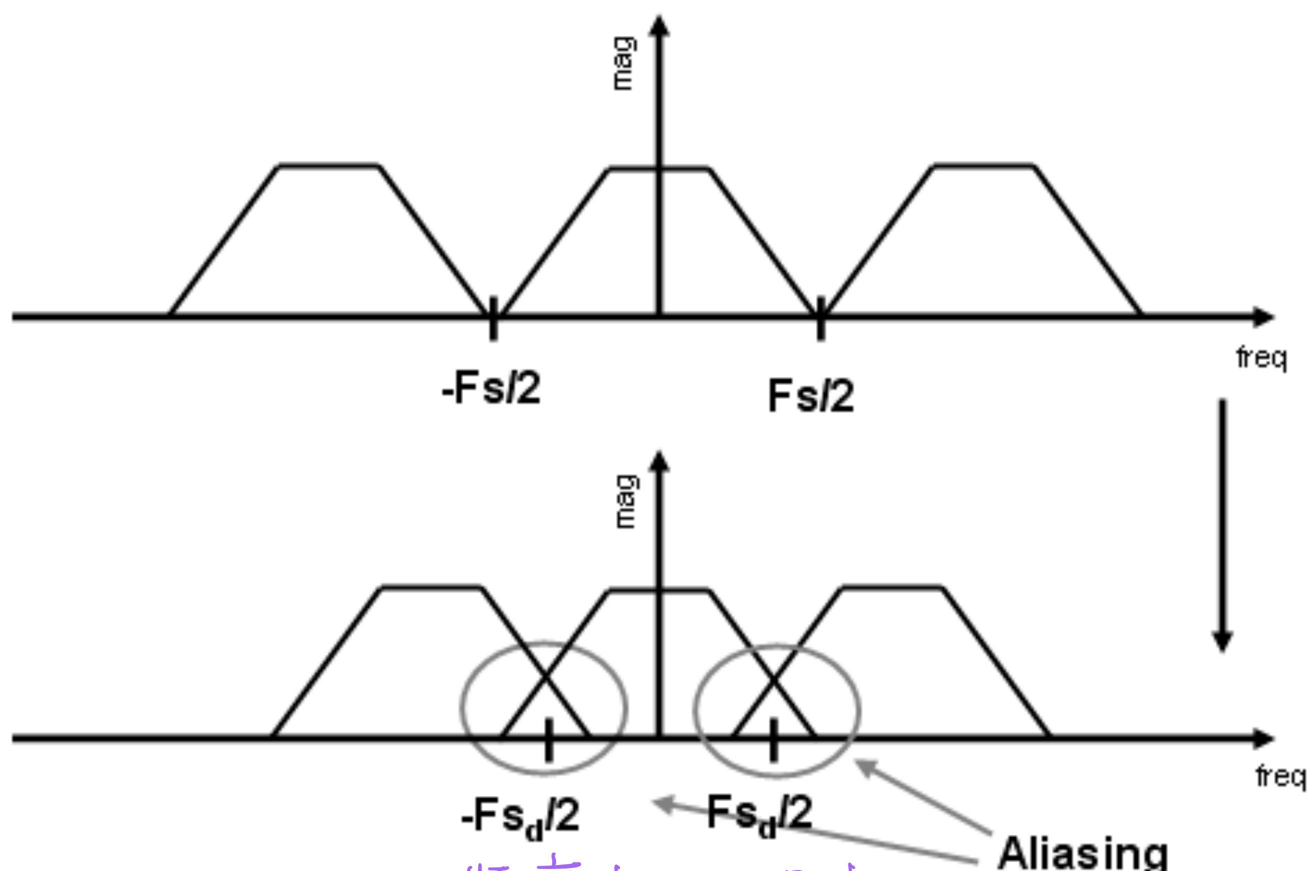
https://www.researchgate.net/figure/The-evolution-of-sampling-theorem-a-The-time-domain-of-the-band-limited-signal-and-b_fig5_301556095

Aliasing = Mixed Frequency Contents

密集采样

Dense sampling:

Sparse sampling:



频率太大，混在一起

Antialiasing

How Can We Reduce Aliasing Error?

Option 1: Increase sampling rate 提高采样率（增加分辨率）

- Essentially increasing the distance between replicas in the Fourier domain
- Higher resolution displays, sensors, framebuffers…
- But: costly & may need very high resolution

Option 2: Antialiasing 反走样

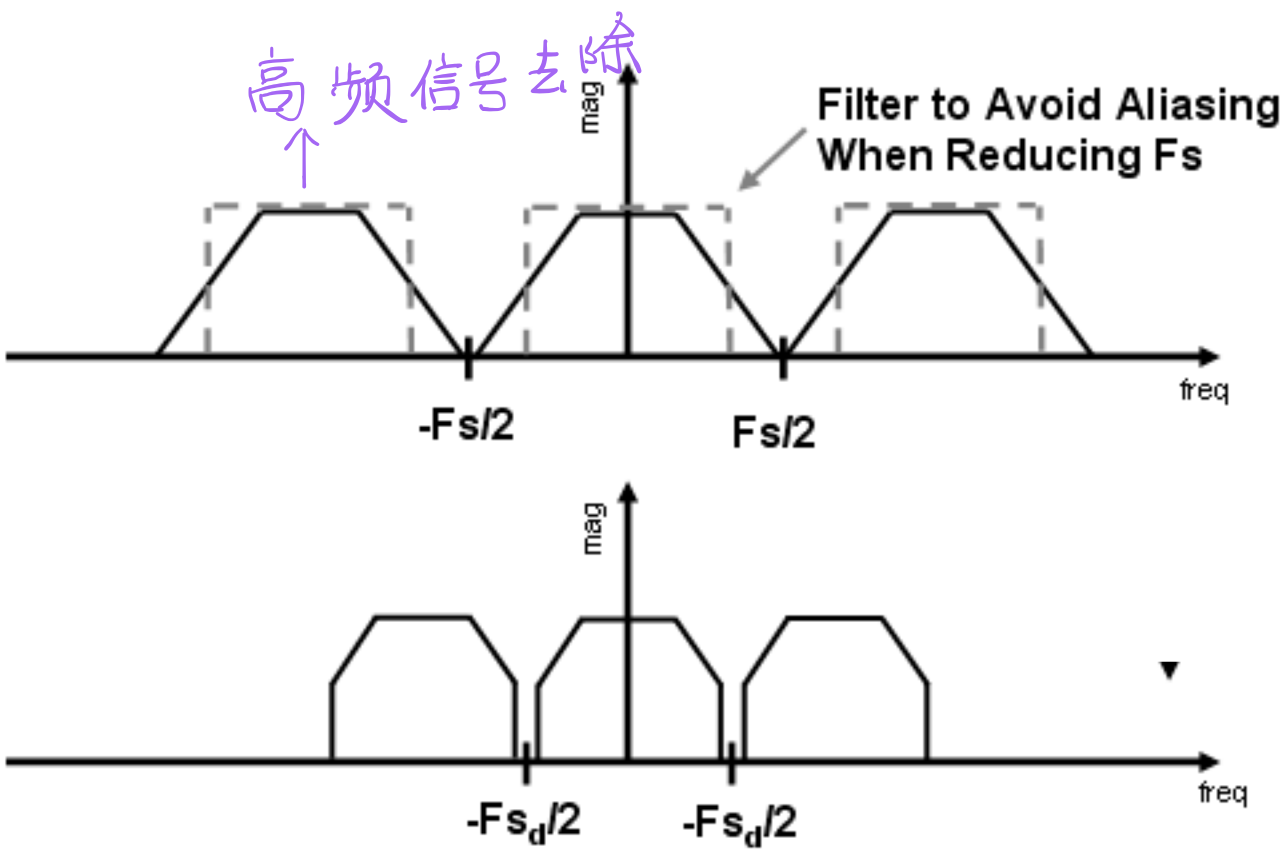
- Making Fourier contents "narrower" before repeating
- i.e. Filtering out high frequencies before sampling

模糊 ⇔ 低通滤波，消除高频信号，再采样

Antialiasing = Limiting, then repeating

Filtering

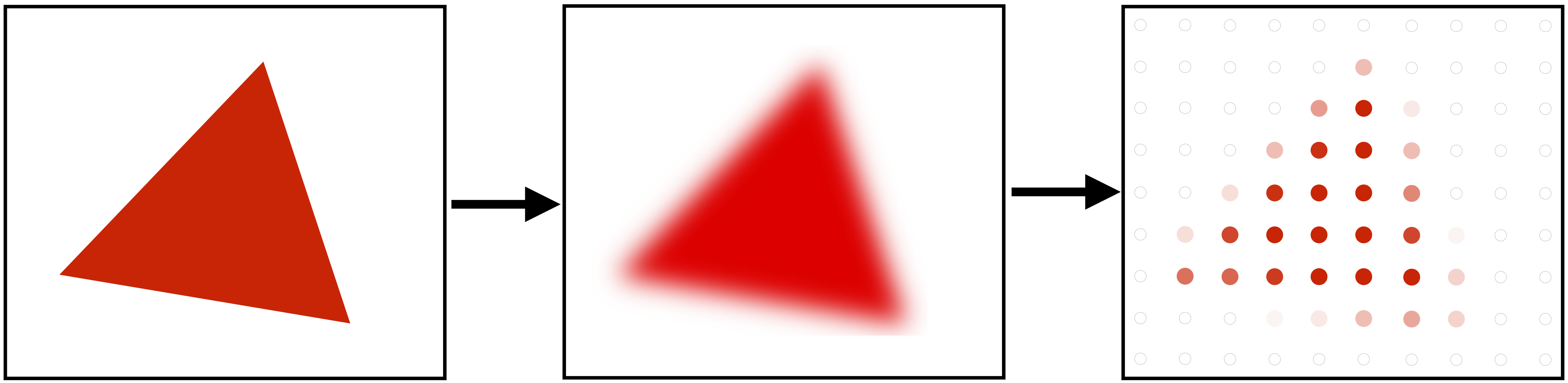
Then sparse sampling



Regular Sampling

Note jaggies in rasterized triangle
where pixel values are pure red or white

Antialiased Sampling



Pre-Filter

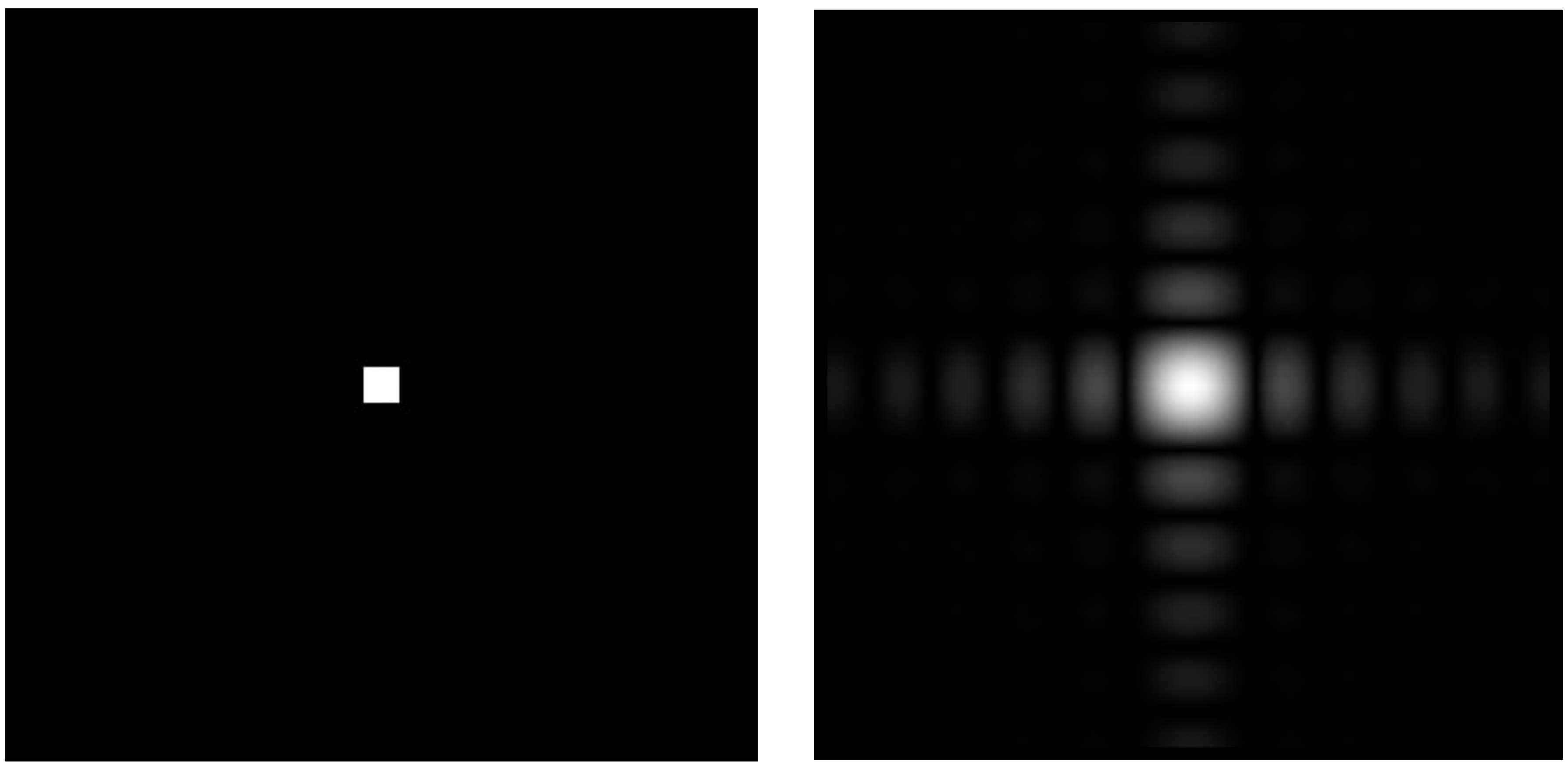
(remove frequencies above Nyquist)

Sample

Note antialiased edges in rasterized triangle where pixel values take intermediate values

A Practical Pre-Filter

A 1 pixel-width box filter (low pass, blurring)



Spatial Domain Frequency Domain

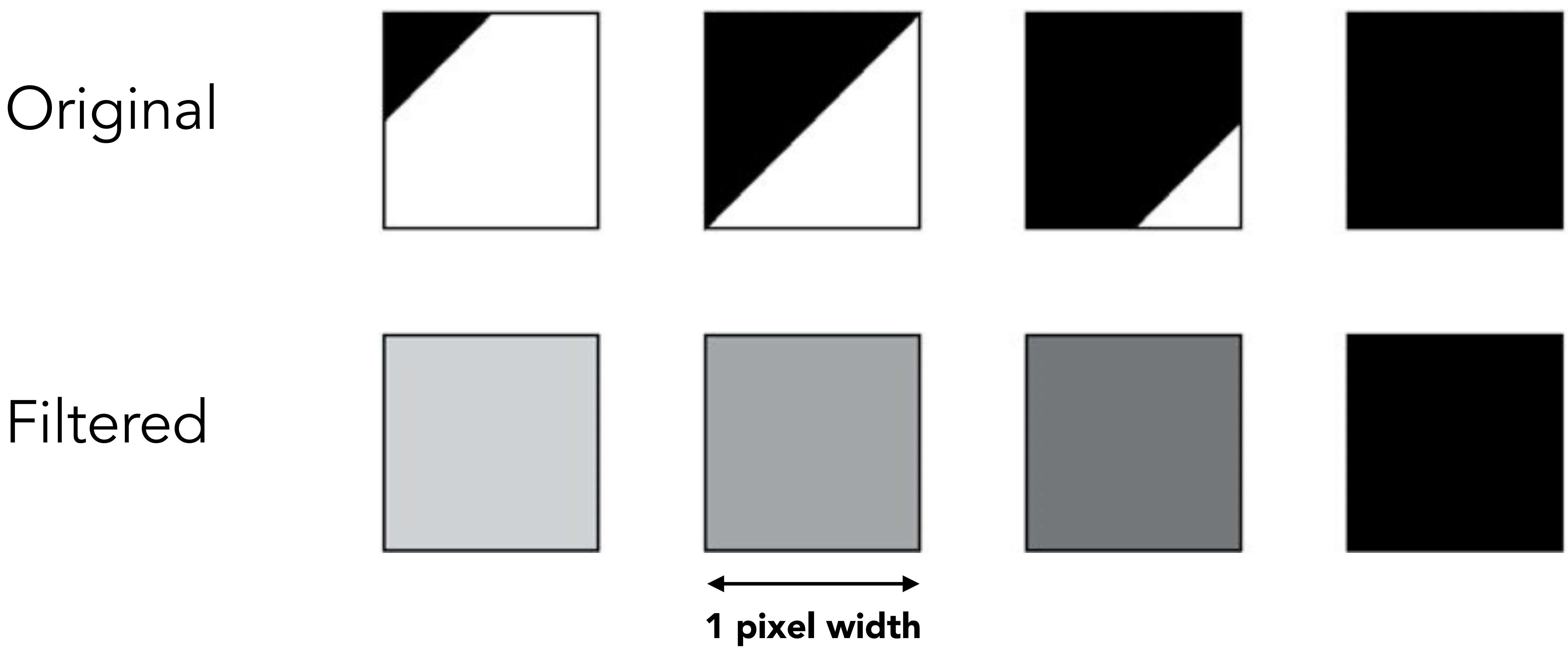
Antialiasing By Averaging Values in Pixel Area

Solution:

- **Convolve** f(x,y) by a 1-pixel box-blur
  - Recall: convolving = filtering = averaging
- **Then sample** at every pixel's center

Antialiasing by Computing Average Pixel Value

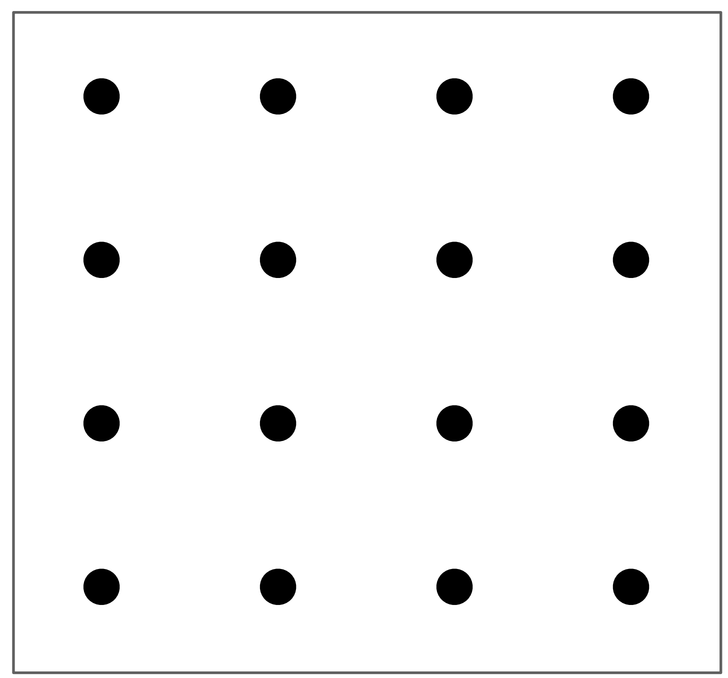
In rasterizing one triangle, the average value inside a pixel area of f(x,y) = inside(triangle,x,y) is equal to the area of the pixel covered by the triangle.



Antialiasing By Supersampling (MSAA)

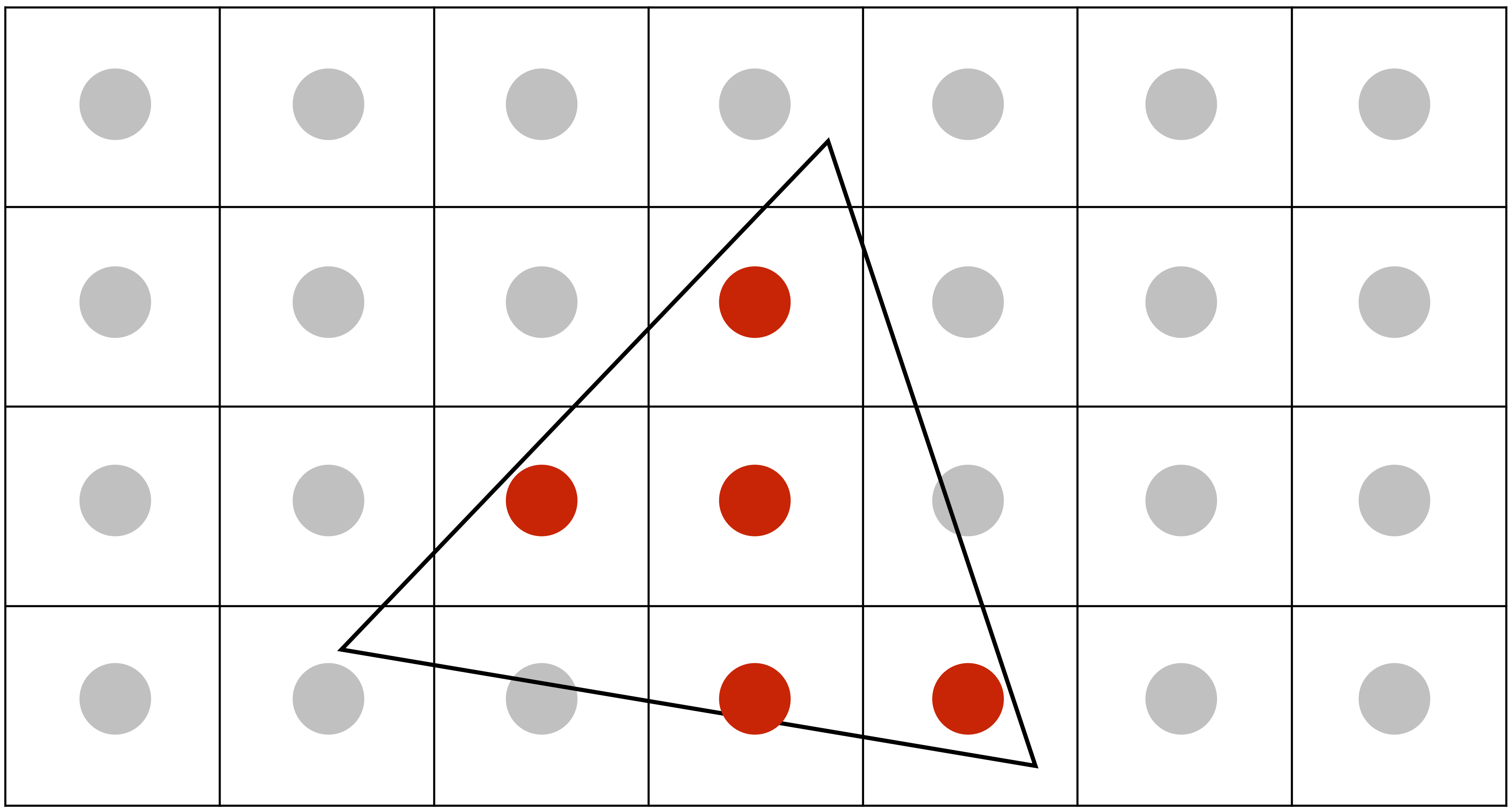
Supersampling

Approximate the effect of the 1-pixel box filter by sampling multiple locations within a pixel and averaging their values:



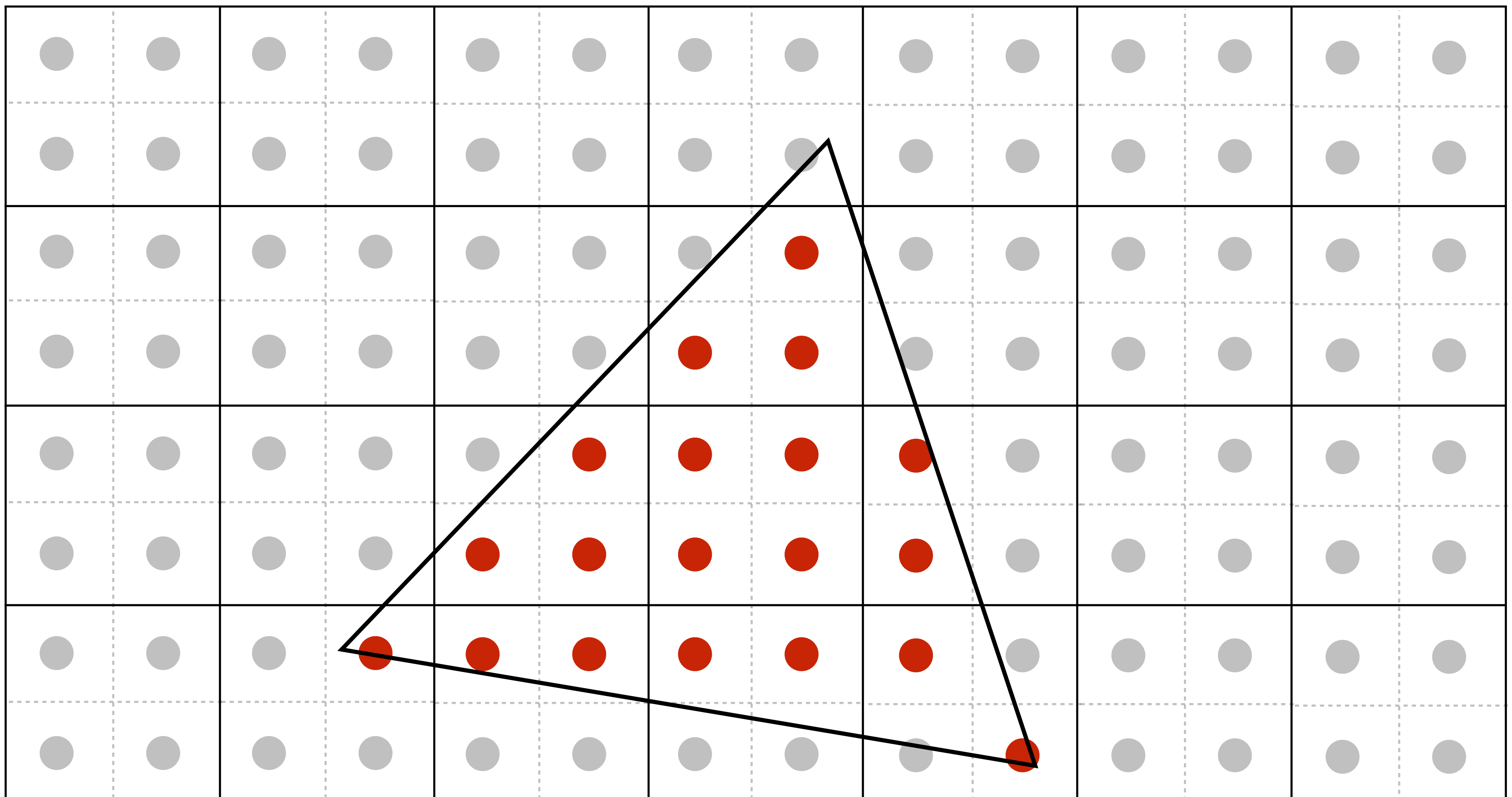
4x4 supersampling

Point Sampling: One Sample Per Pixel



Supersampling: Step 1

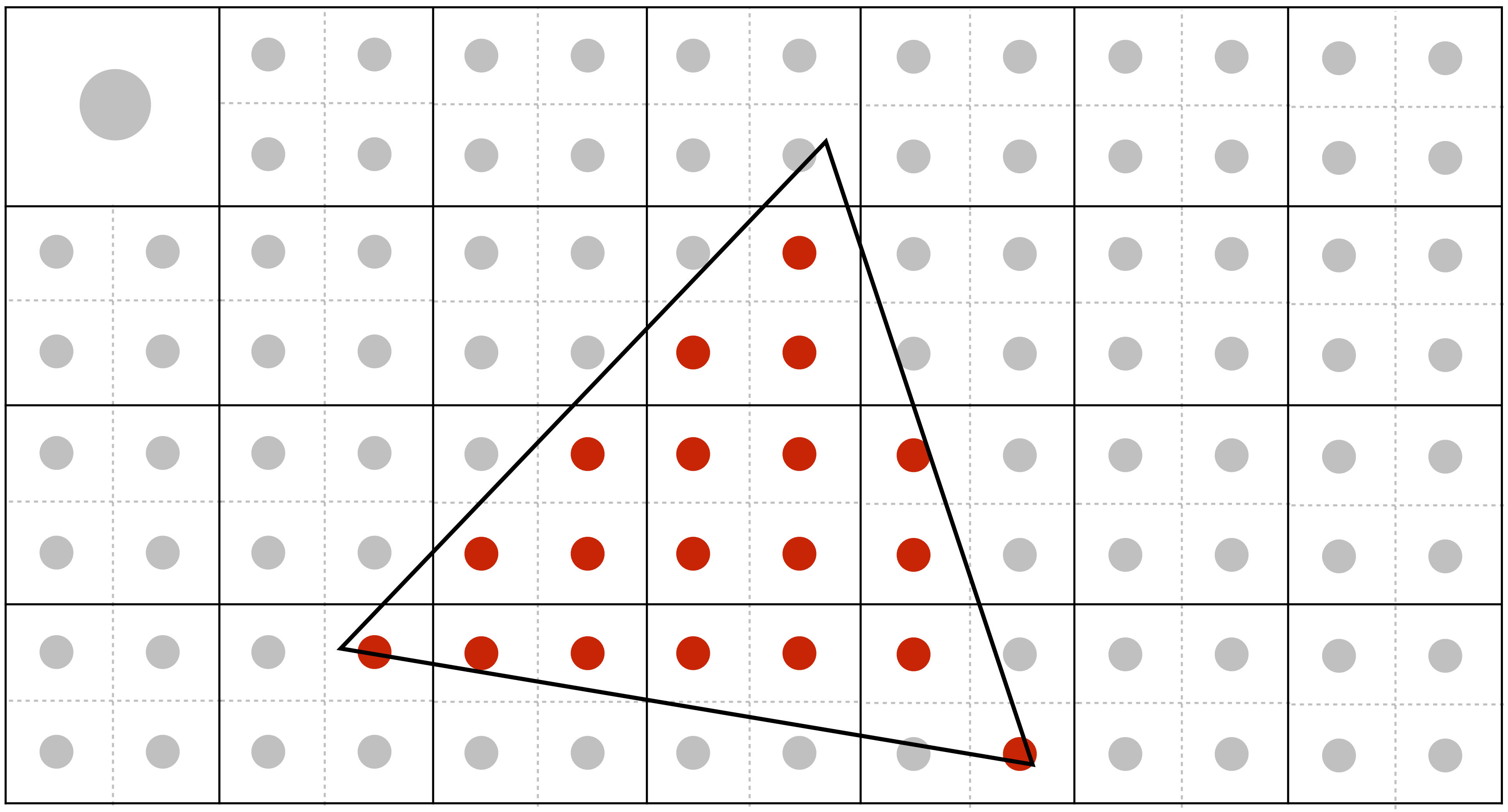
Take NxN samples in each pixel.



2x2 supersampling

Supersampling: Step 2

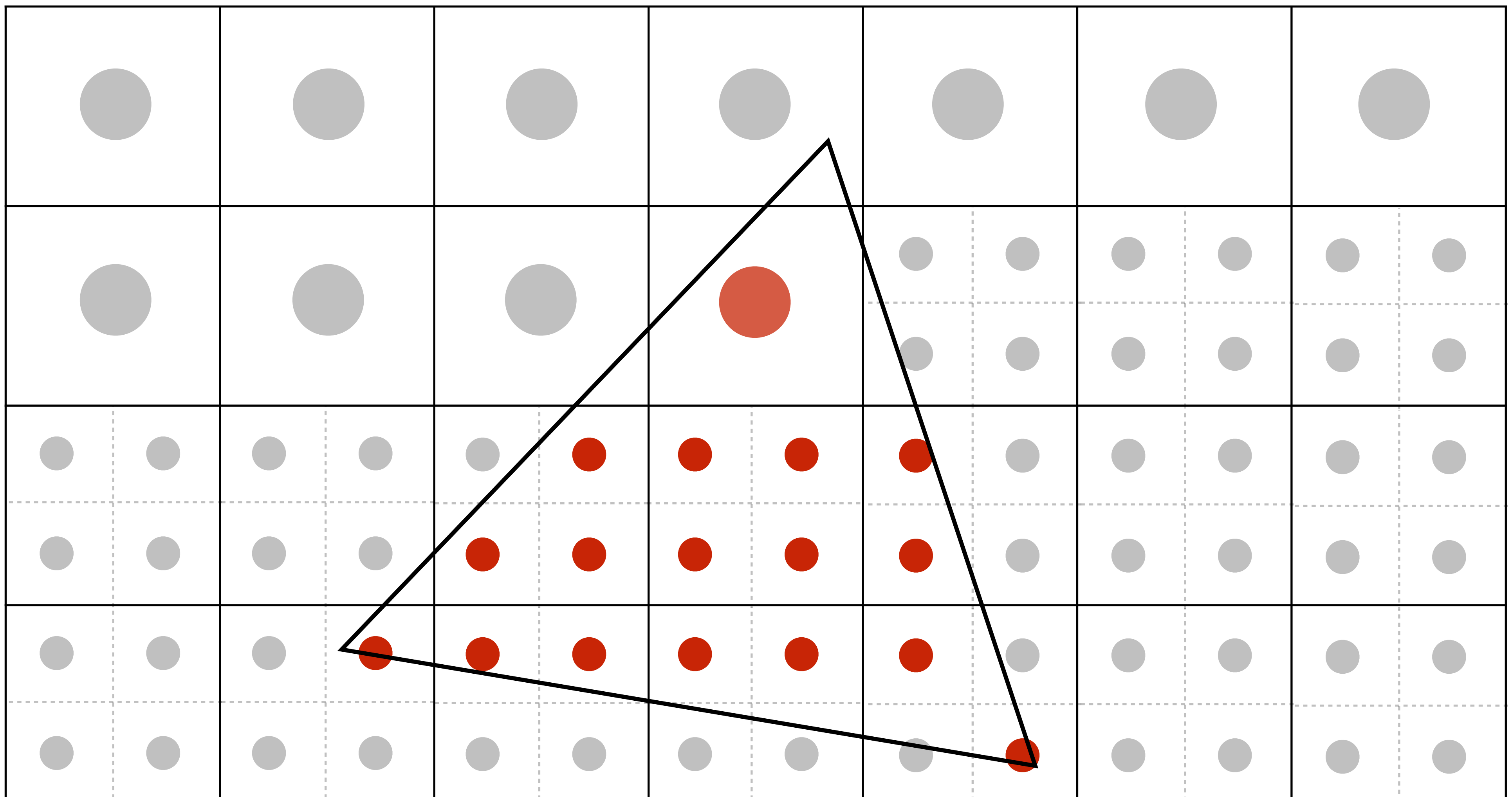
Average the NxN samples “inside” each pixel.



Averaging down

Supersampling: Step 2

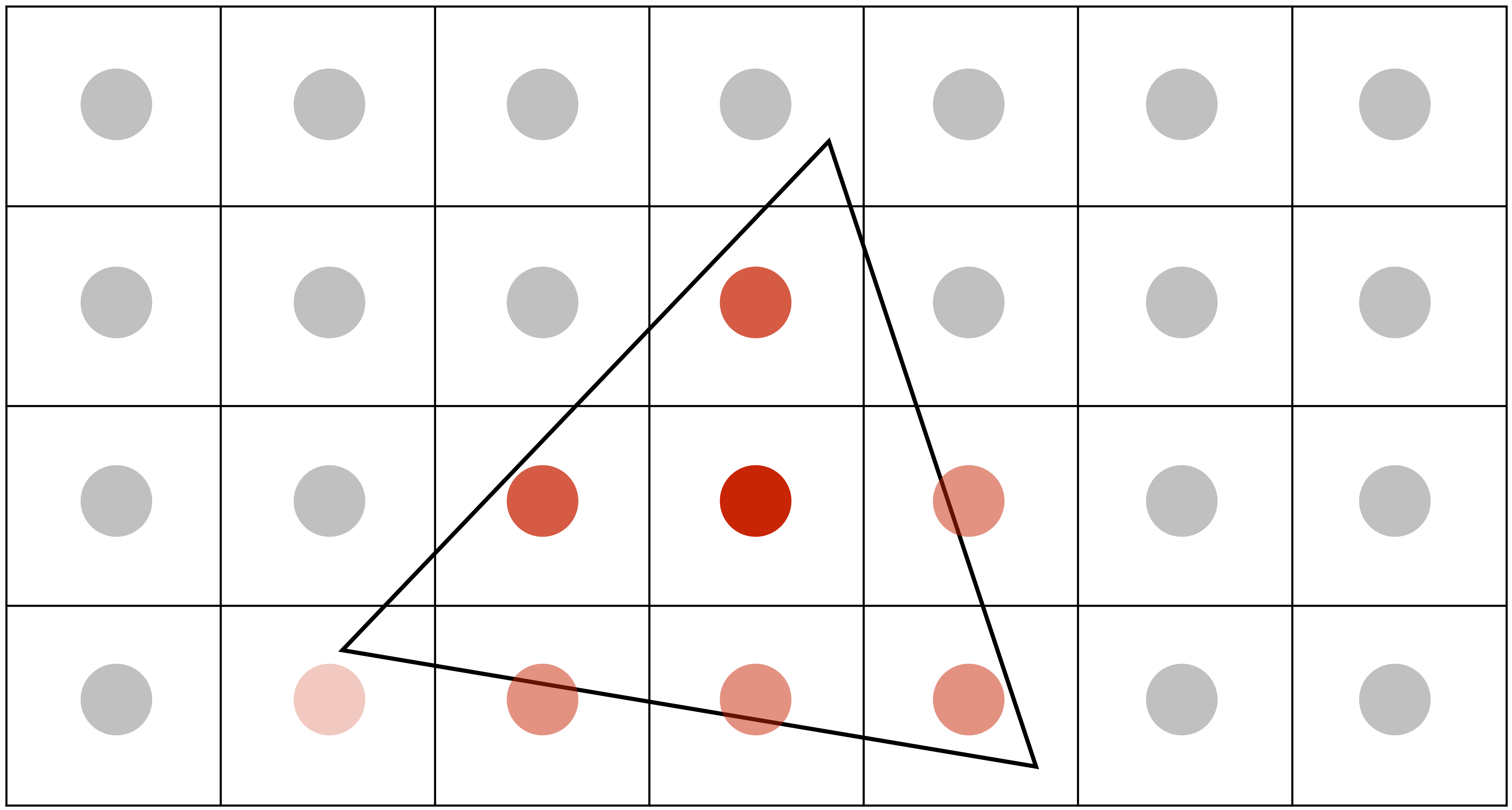
Average the NxN samples "inside" each pixel.



Averaging down

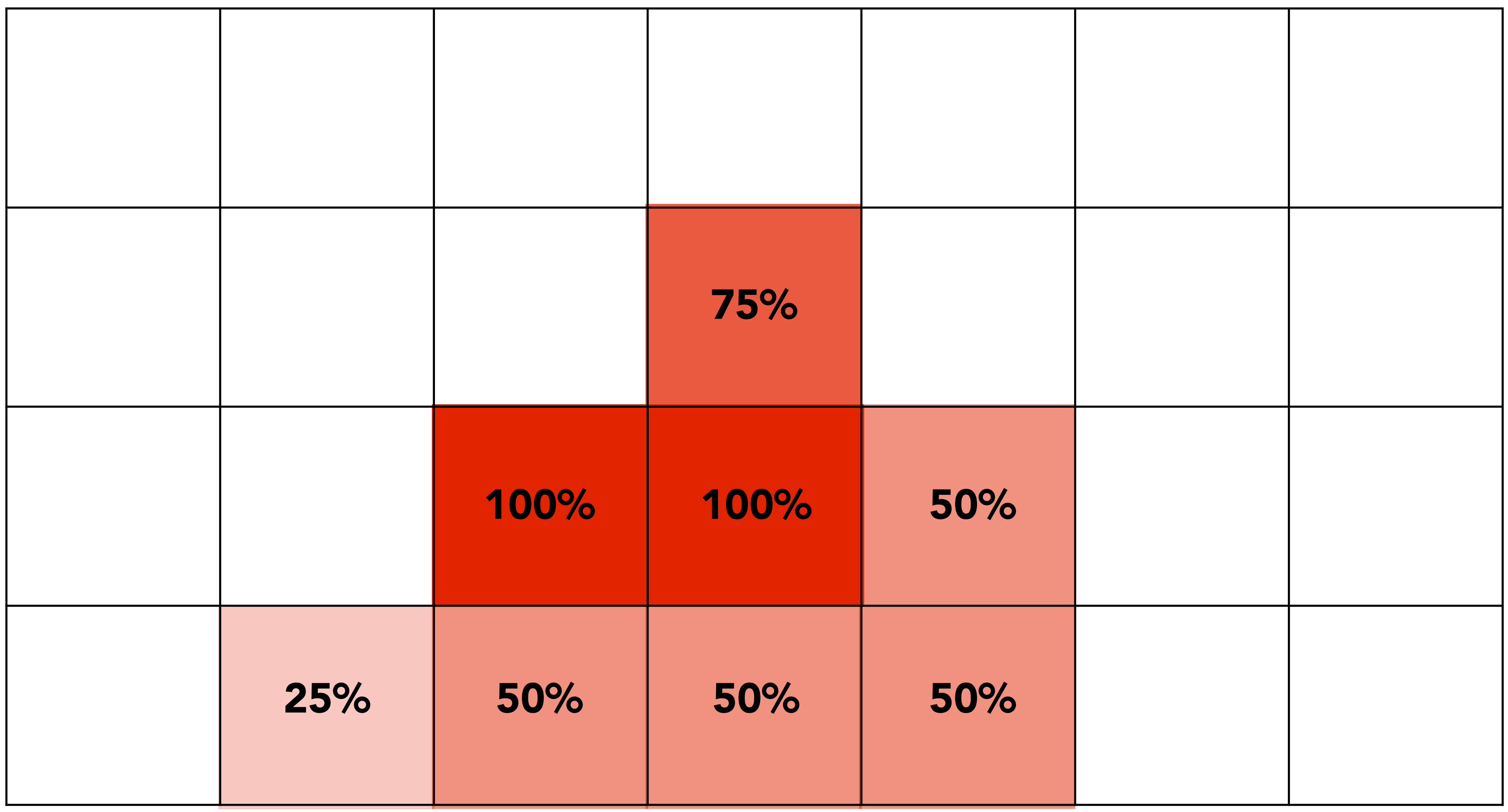
Supersampling: Step 2

Average the NxN samples "inside" each pixel.

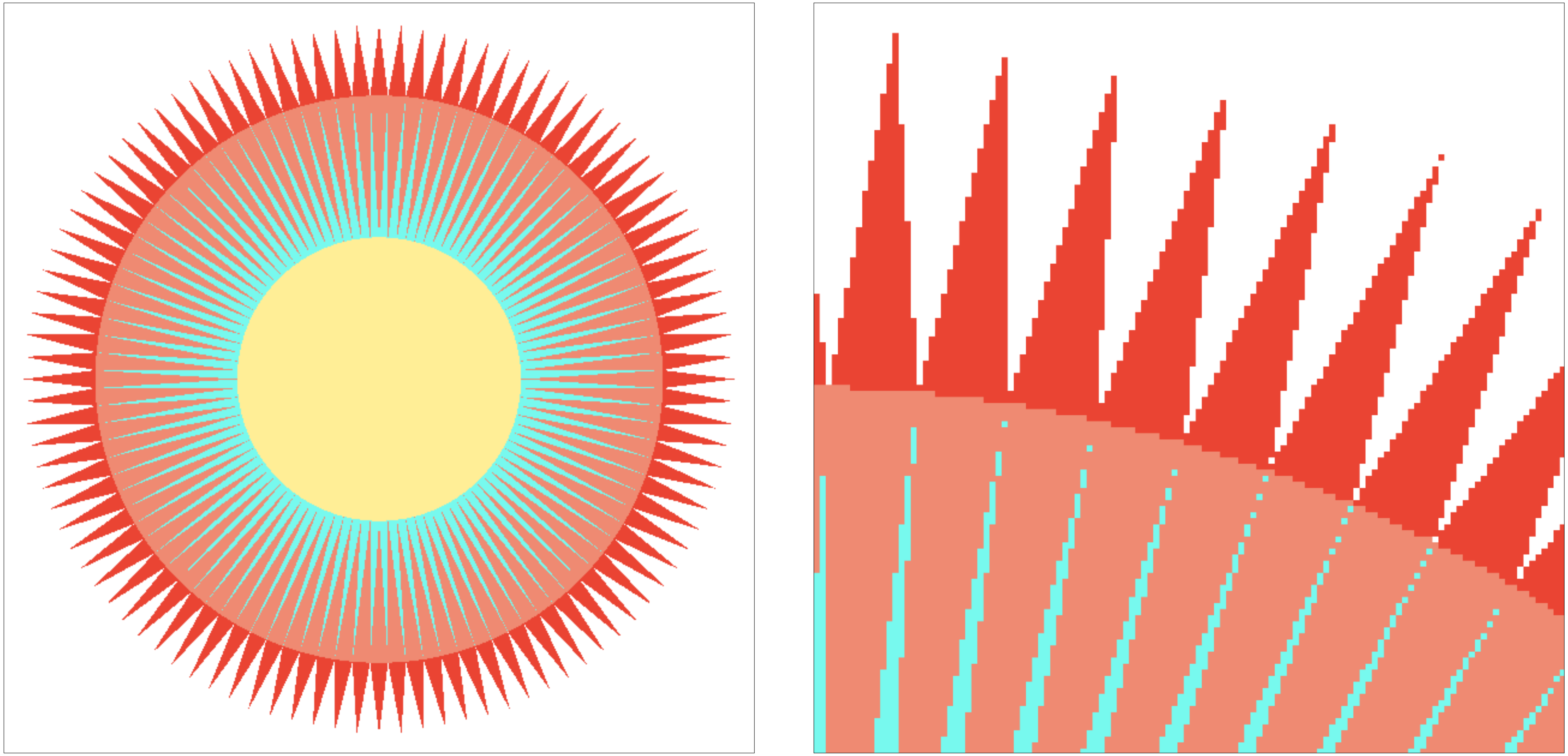


Supersampling: Result

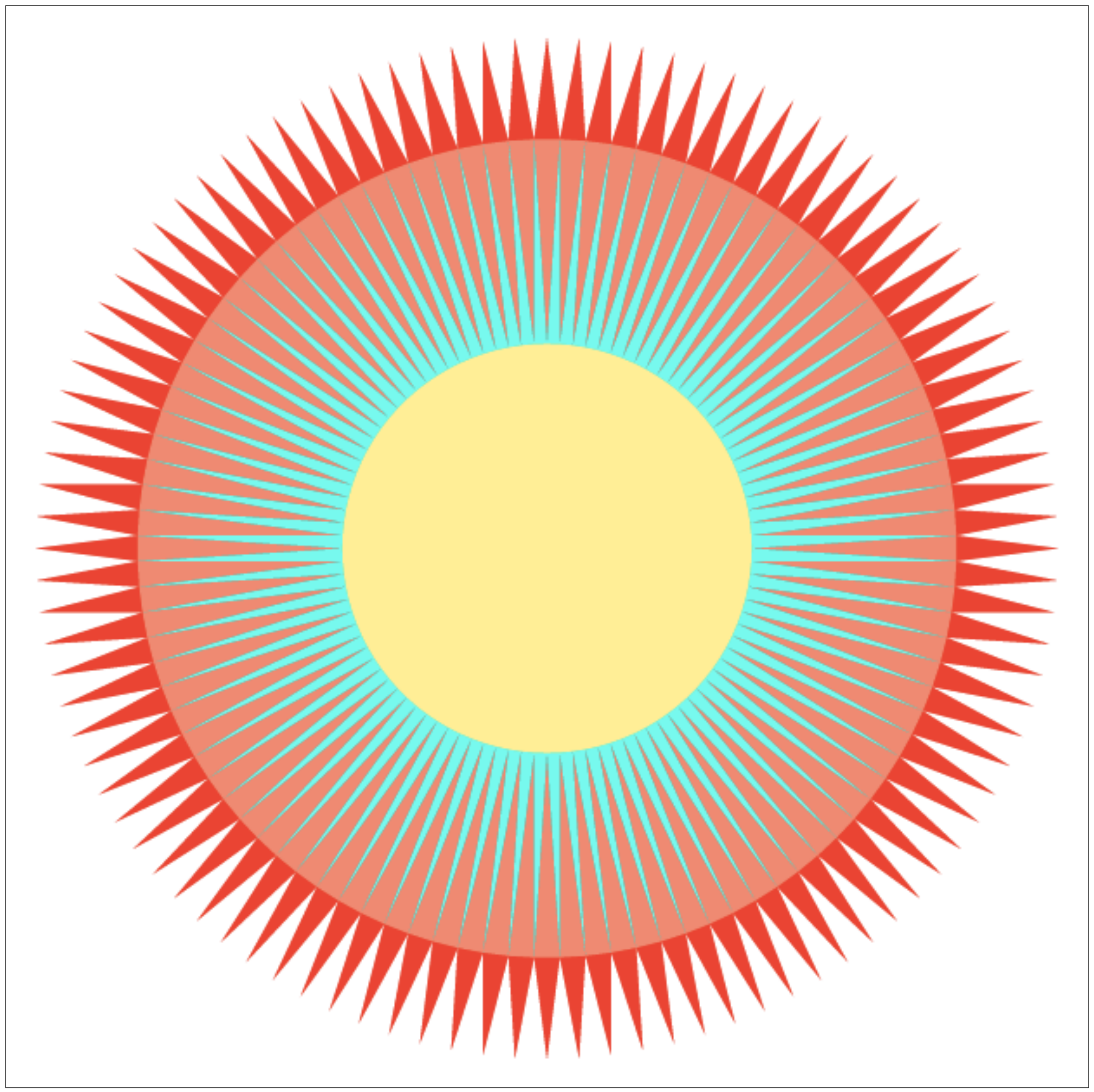
This is the corresponding signal emitted by the display

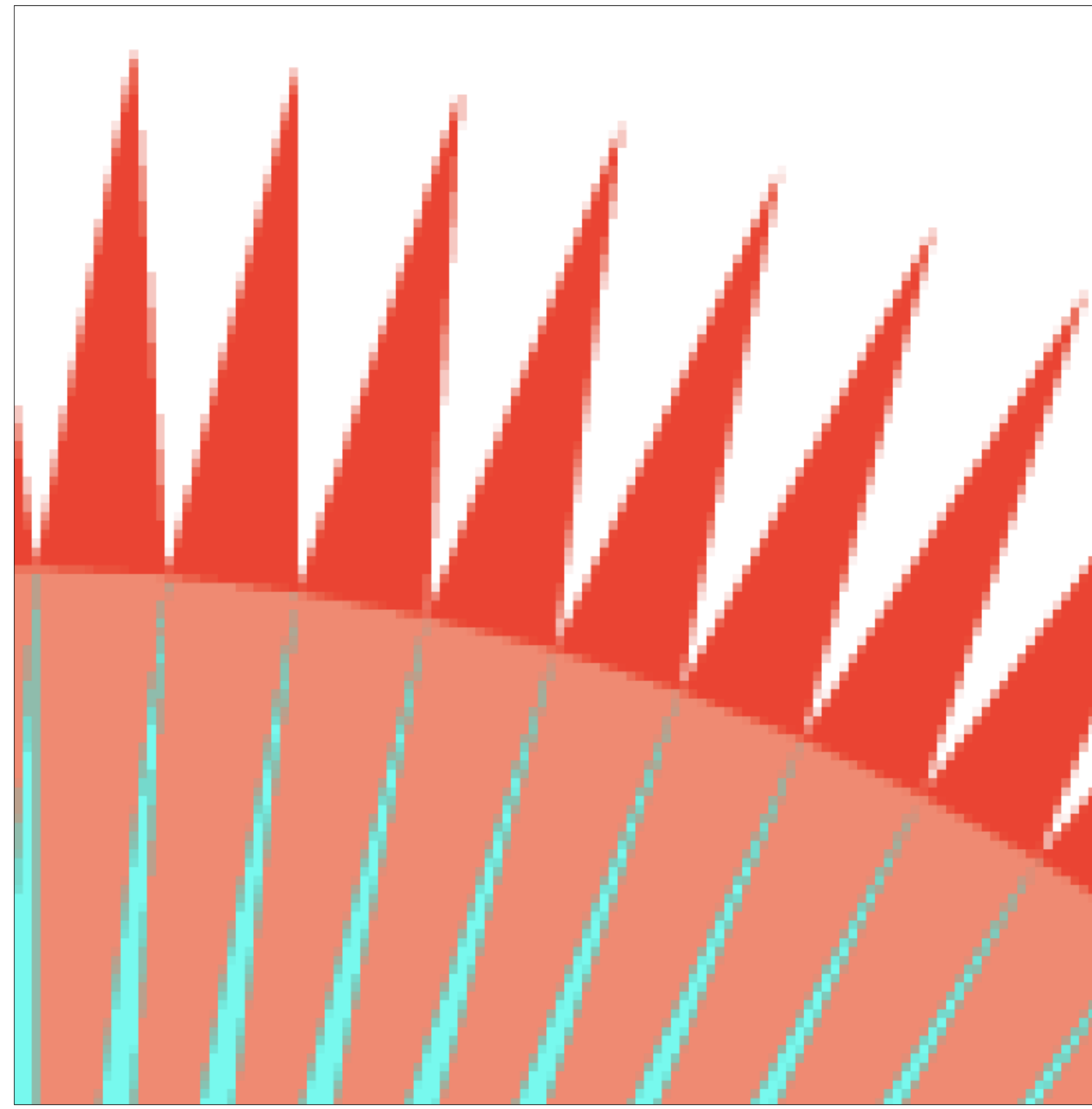


Point Sampling



4x4 Supersampling





Antialiasing Today

No free lunch!

- What's the cost of MSAA? 增大计算量

Milestones (personal idea)

- FXAA (Fast Approximate AA) 找到锯齿，更换掉
- TAA (Temporal AA) 复用上一帧

超分辨率

Super resolution / super sampling 超采样 (本质相同)

- From low resolution to high resolution
- Essentially still "not enough samples" problem
- DLSS (Deep Learning Super Sampling)

Thank you!