2022-2023 Autumn Semester

Operation Research

Assignment 1

-LP modeling-

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目录

1 Introductions

Please present the linear programming models for each of the following problems, including the definition of decision variables, objective function, and constraints and sign restrictions. You do not need to solve the models.

2 Problem-1 A Diet Problem.

2.1 Question

My diet requires that all the food I eat come from one of the four “basic food groups”(chocolate cake, ice cream, soda, and cheesecake). At present, the following four foods are available for consumption: brownies, chocolate ice cream, cola, pineapple cheesecake. Each brownie costs 50¢, each scoop of chocolate ice cream cost 20¢, each bottle of cola costs 30¢, and each piece of pineapple cheesecake costs 80¢. Each day, I must ingest at least 500 calories, 6 oz of chocolate, 10 oz of sugar, and 8 oz of fat. The nutritional content per unit of each food in shown in the table 1. Formulate a LP model that can be used to satisfy my daily nutritional requirements at minimum cost.

表 1: Nutritional Content

	CALORIES	CHOCOLATE	SUGAR	FAT
Brownie	400	3	2	2
Chocolate ice cream	200	2	2	4
Cola	150	0	4	1
Pineapple cheese-cake	500	0	4	5

2.2 Solution

Decision variables:

X_1: the number of browine consumed;

X_2: the number of scoops of chocolate ice cream consumed;

X_3: the number of bottles of cola consumed;

X_4: the number of pieces of pineapple cheesecake consumed.

Objective function:

$$\min z = 50X_1 + 20X_2 + 30X_3 + 80X_4.$$

Constraints:

- $400X_1 + 200X_2 + 150X_3 + 500X_4 \geq 500$ (Calories) ;
- $3X_1 + 2X_2 \geq 6$ (Chocolate) ;
- $2X_1 + 2X_2 + 4X_3 + 4X_4 \geq 10$ (Sugar) ;
- $2X_1 + 4X_2 + X_3 + 5X_4 \geq 8$ (Fat) .

Sign restrictions:

$X_i \geq 0,\ integer\ ,\ i = 1, 2, 3, 4.$

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3 Problrm-2 Short-Term Financial Planning.

3.1 Question

Semicond is a small electronics company that manufactures tape recorders and radios. The per-unit labor costs, raw material costs, and selling price of each product are given in table 2. On January 1, Semicond has available raw material that is sufficient to manufacture 100 tape recorders and 100 radios. On the same date, the company's balance sheet is as shown in the table 3, and Semicond's asset-liability ratio (called the current ratio) is 20,000/10,000 = 2.

Semicond must determine how many tape recorders and radios should be produced during January. Demand is large enough to ensure that all goods produced will be sold. All sales are on credit, however, and payment for goods produced in January will not be received until March 1. During January, Semicond will collect \$2,000 in accounts receivable, and Semicond must pay off \$1,000 of the outstanding loan and a monthly rent of \$1,000. On February 1, Semicond will receive a shipment of raw material worth \$2,000, which will be paid for on March 1. Semicond's management has decided that the cash balance on February 1, must be at least \$4,000. Semicond's bank requires that the current ratio at the beginning of February be at least 2. To maximize the contribution to profit from January production, (revenues to be received) (variable production costs), what should Semicond produce during January?

表 2: Product Information

	Tape Recorder	Radio
Selling price	\$100	\$90
Labor cost	\$50	\$35
Raw material cost	\$30	\$40

表 3: balance sheet(January)

	Assets	Liabilities
Cash	\$10000	
Accounts receivable	\$3000	
Inventory outstanding	\$7000	
Bank loan		\$10000

3.2 Solution

Decision variables:

X_1: the number of Tape recorders produced during January;

X_2: the number of Radios produced during January.

Objectiive function:

$$\max z = 20X_1 + 15X_2.$$

Constraints:

In February, cash will be

$$10000 + 2000 - 1000 - 1000 - 50X_1 - 35X_2;$$

since the receivable accounts, monthly rent, outstanding loan and labor cost in January production.

Accounts receivable will change to

$$3000 - 2000 + 100X_1 + 90X_2;$$

because of the product produced and accounts received in January.

In cosequence of material used in production and new stock obtained in January, raw material cost will turn into

$$7000 + 2000 - 30X_1 - 40X_2.$$

In light of some loan paid in January and the spendence of new stock, the bank loan will be updated to

$$10000 + 2000 - 1000.$$

In summary, the balance sheet in February should be what is shown in table 4.

表 4: balance sheet(February)

	Assets	Liabilities
Cash	$10000 - 50X_1 - 35X_2$	
Accounts receivable	$1000 + 100X_1 + 90X_2$	
Inventory outstanding	$9000 - 30X_1 - 40X_2$	
Bank loan		11000

The constraints of this LP problem are list as follow:

- $X_1 \leq 100$ (Resource limits of tape recorders) ;
- $X_2 \leq 100$ (Resource limits of radios);

- $10000 - 50X_1 - 35X_2 \geq 4000$ (cash flow requirement) ;
- $20000 + 20X_1 + 15X_2 \geq 22000$ (current ratio requirement) .

Sign restrictions:

$X_i \geq 0,\ integer\ ,\ i = 1, 2.$

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4 Problem-3 Multiperiod Financial Models.

4.1 Question

Finco Investment Corporation must determine investment strategy for the firm during the next three years. Currently (time 0), $100,000 is available for investment. Investments A, B, C, D, and E are available, whose details is in table 5. The cash flow associated with investing $1 in each investment is given. For example, $1 invested in investment B requires a $1 cash outflow at time 1 and returns 50¢ at time 2 and $1 at time 3. To ensure that the company's portfolio is diversified, Finco requires that at most $75,000 be placed in any single investment. In addition to investments A−E, Finco can earn interest at 8% per year by keeping uninvested cash in money market funds. Returns from investments may be immediately reinvested. For example, the positive cash flow received from investment C at time 1 may immediately be reinvested in investment B. Finco cannot borrow funds, so the cash available for investment at any time is limited to cash on hand. Formulate an LP that will maximize cash on hand at time 3.

表 5: Cash Flow of Five Investments

	0	1	2	3
A	-1	+0.50	+1	0
B	0	-1	+0.50	+1
C	-1	+1.2	0	0
D	-1	0	0	+1.9
E	0	0	-1	+1.5

4.2 Solution

Decision Variables:

X_i : the money invested into i, $i = A,\ B,\ C,\ D,\ E$;

S_t : the money uninvested at time t, $t = 0,\ 1,\ 2$.

Objective Function:

maximize the money in total at time 3:

$$\max z = X_B + 1.9X_D + 1.5X_E + 1.08S_2.$$

Constraints:

In order to maximize the objective function, we could suppose that the money at any time is invented to either the investment product(A, B, C, D, E) or the the money market funds.

At time 0, we could invest A, C, D and the money market funds, thus

$$100000 = X_A + X_C + X_D + S_0.$$

At time 1, we could receive the profit from A, C and money market funds invested at time 0. Also, the product B and the money market funds could be chosen to invested. Hence,

$$0.5X_A + 1.2X_C + 1.08S_0 = S_1 + X_B.$$

At time 2, the income from the investment at time 0 and time 1 will received, and E along with money market funds are good choice.

$$X_A + 0.5X_B + 1.08S_1 = X_E + S_2.$$

Considering the single investment restrictions, we could list the constraints in summary as below.

- $100000 = X_A + X_C + X_D + S_0$ (time 0) ;

- $0.5X_A + 1.2X_C + 1.08S_0 = S_1 + X_B$ (time 1) ;
- $X_A + 0.5X_B + 1.08S_1 = X_E + S_2$ (time 2) ;
- $X_A \leq 75000$ (single investment restriction) ;
- $X_B \leq 75000$ (single investment restriction) ;
- $X_C \leq 75000$ (single investment restriction) ;
- $X_D \leq 75000$ (single investment restriction) ;
- $X_E \leq 75000$ (single investment restriction) ;

Sign restrictions:

$X_i,\ S_t \geq 0,\ i = A,\ B,\ C,\ D,\ E \ and \ t = 0,\ 1,\ 2.$

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5 Problem-4 Aggregate Planning.

5.1 Question

Consider an aggregate planning problem involving a single workstation but m products for $m > 1$ over $\bar{t}$ periods (planning horizon). The maximum demand of product i in period $t(\overline{d}_{it})$ and the minimum sales of product i that must be met in period $t(\overline{d}_{it})$ are specified. The problem is to determine the amount of product i produced, sold, and stocked for each period t over $\bar{t}$ periods. The objective is to maximize the total net profits. Introduce the following notation:

- m = total number of products;
- $\bar{t}$ = number of periods in a planning horizon;
- $\overline{d}_{it}$ = maximum demand of product i in period t;
- $\underline{d}_{it}$ = minimum sales of product i allowed in period t;

- a_i = time required to produce one unit of product i;
- c_t = capacity of workstation in period t;
- r_i = net profit per unit of product i sold;
- h_i = cost to hold one unit of product i for one period.

Decision variables are

- X_{it} = amount of product i produced in period t;
- S_{it} = amount of product i sold in period t;
- I_{it} = inventory of product i at the end of period t.

a. Formulate a product mix planning as an LP model. Define objective function and constraints clearly.

b. Suppose that the plant can supplement its capacity by subcontracting part of or all the production of certain parts. Show how to modify the LP formulation in part (a) to include this option, where we define for $t = 1, 2, \cdots, \bar{t}$,

   - V_{it} = amount of product i received from a subcontractor in period t;
   - k_{it} = premium paid for subcontracting product i in period t (i.e., cost above variable cost of making it in-house) ;
   - $\overline{v}_{it}$ = maximum amount of product i that can be purchased in period t (e.g., due to capacity constraints on supplier, as specified in long-term contract) ;
   - $\underline{v}_{it}$ = minimum amount of product i that must be purchased in period t (e.g., specified as part of long-term contract with supplier).

c. How would you modify the formulation in part (b) if the contract with a supplier stipulated only that total purchases of product i over the time horizon must be at least 300?

d. How would you modify the formulation in part (b) if the supplier contract, instead of specifying $\overline{v}_{it}$ and $\underline{v}_{it}$, stipulated that the firm specifying a base amount of product i (needs to be determined by the firm), to be purchased every month, and that the maximum purchase in a given month can exceed the base amount by no more than 20 percent?

5.2 Solution

a.

According to the decision variables denoted above, we could define objective function and constraints as below.

Objective function:

$$\max z = \sum_i \sum_t r_i S_{it} - \sum_i \sum_t h_i I_{it}.$$

Constraints:

- $\underline{d}_{it} \leq S_{it} \leq \overline{d}_{it}$ (sales demand) ;
- $\sum_i a_i X_i t \leq c_t$ (capacity of workstation restriction) ;
- $I_{i,t} = I_{i,t-1} + X_{it} - S_{it}$ (Inventory of product i in period t).

Sign restrictions:

$X_{it}, S_{it}, I_{it} \geq 0,\ i = 1, 2, \cdots m,\ t = 1, 2, \cdots, \bar{t}.$

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b.

We could modify the formulation in part (a) as below.

Objective function:

$$\max z = \sum_i \sum_t r_i S_{it} - \sum_i \sum_t h_i I_{it} - \sum_i \sum_t k_{it} V_{it}.$$

Constraints:

- $\underline{d}_{it} \leq S_{it} \leq \overline{d}_{it}$ (sales demand) ;

- $\sum_i a_i X_i t \leq c_t$ (capacity of workstation restriction) ;
- $I_{i,t} = I_{i,t-1} + X_{it} + V_{it} - S_{it}$ (Inventory of product i in period t) ;
- $\underline{v}_{it} \leq V_{it} \leq \overline{v}_{it}$ (purchase demand).

Sign restrictions:

$X_{it}, S_{it}, I_{it}, V_{it} \geq 0, \ i = 1, 2, \cdots m, \ t = 1, 2, \cdots, \bar{t}.$

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c.

We could modify the formulation in part (b) as below.

Objective function:

$$\max z = \sum_i \sum_t r_i S_{it} - \sum_i \sum_t h_i I_{it} - \sum_i \sum_t k_{it} V_{it}.$$

Constraints:

- $\underline{d}_{it} \leq S_{it} \leq \overline{d}_{it}$ (sales demand) ;
- $\sum_i a_i X_i t \leq c_t$ (capacity of workstation restriction) ;
- $I_{i,t} = I_{i,t-1} + X_{it} + V_{it} - S_{it}$ (Inventory of product i in period t) ;
- $\sum_t V_{it} \geq 300$ (product i purchase demand).

Sign restrictions:

$X_{it}, S_{it}, I_{it}, V_{it} \geq 0, \ i = 1, 2, \cdots m, \ t = 1, 2, \cdots, \bar{t}.$

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d.

We denote the base amount of product i as B_i, then we could modify the formulation in part (b) as below.

Objective function:

$$\max z = \sum_i \sum_t r_i S_{it} - \sum_i \sum_t h_i I_{it} - \sum_i \sum_t k_{it} V_{it}.$$

Constraints:

- $\underline{d}_{it} \leq S_{it} \leq \overline{d}_{it}$ (sales demand) ;
- $\sum_i a_i X_i t \leq c_t$ (capacity of workstation restriction) ;
- $I_{i,t} = I_{i,t-1} + X_{it} + V_{it} - S_{it}$ (Inventory of product i in period t) ;
- $B_i \leq V_{it} \leq 1.2 B_i$ (purchase demand according to the base amount).

Sign restrictions:

$X_{it}, S_{it}, I_{it}, V_{it}, B_i \geq 0,\ i = 1, 2, \cdots m,\ t = 1, 2, \cdots, \bar{t}.$

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