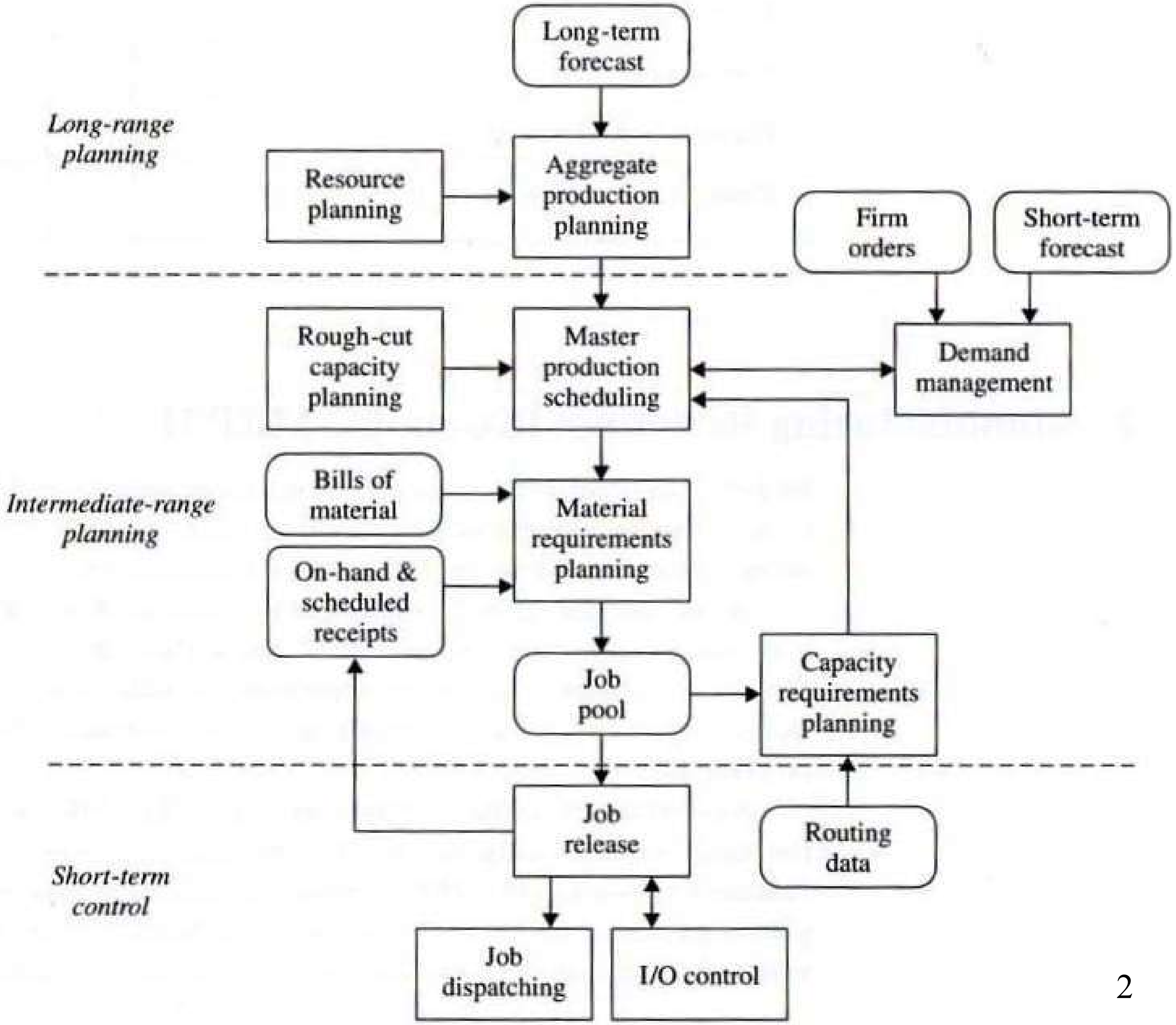
Aggregate and Workforce Planning

Material covered: Chapter 16, *Factory Physics* by W. J. Hopp and M. I. Spearman, 3rd Edition, McGraw Hill.

Production planning and control hierarchy for MRP II



1. Introduction to AP

- Aggregate planning (AP) generates an *aggregate plan* that specifies what and how much of each product to produce for each time period *over planning horizon*.

  Planning horizon: 1-3 years typically
- AP begins with the forecast of demand.

  Assume that the demand is deterministic.
- AP methodology is designed to translate demand forecasts into planning in production level over planning horizon.
- The tool used is normally linear programming.

Aggregate Plan

- *Aggregate plan*: determining quantity and timing of production of "product families (many types, styles)" that will meet estimated demand.

  Not single item but group of products

  Production plan: specification of the quantity and timing of each individual final product.
- Deal with product mix issue.
- AP occupies a central position in the production planning control hierarchy.

2. A simple AP model

- Assumptions:
  - A single product
  - A single workstation with limited capacity
  - Demands = customer orders that are due at the end of the period (given)
  - No randomness
  - No yield loss
  - No setup cost
  - No backorder

Notation

$\bar{t} =$ planning horizon
t = an index of time periods, $t = 1, \ldots, \bar{t}$
d_t = demand in period t
c_t = capacity in period t
r = profit per unit of product sold
h = cost to hold one unit of inventory for one period
X_t = quantity produced during period t
S_t = quantity sold during period t
I_t = inventory at end of period t; assume I_0 is given

LP Model

$$\max \quad \sum_{t=1}^{\bar{t}} rS_t - hI_t \qquad \text{max net profit: revenue minus inventory carrying cost}$$

$$
\begin{aligned}
\text{s.t.} \quad & S_t \le d_t, && t = 1, \dots, \bar{t} && \text{demand constraints} \\
& X_t \le c_t, && t = 1, \dots, \bar{t} && \text{capacity constraints} \\
& I_t = I_{t-1} + X_t - S_t, && t = 1, \dots, \bar{t} && \text{inventory balance constraints} \\
& X_t, S_t, I_t \ge 0, && t = 1, \dots, \bar{t} && \text{nonnegative constraints}
\end{aligned}
$$

Analysis

Constraints:

binding (tight): LHS = RHS, otherwise nonbinding (slack)

- If $S_t = d_t$ (binding) $\Rightarrow$ demand is satisfied;
  If $S_t < d_t$ (nonbinding) $\Rightarrow$ demand can not be met under current capacity levels, i.e., capacity−infeasible.
- If $X_t = c_t$ (binding) $\Rightarrow$ full capacity;
  If $X_t < c_t$ (nonbinding) $\Rightarrow$ excess capacity;

- Case 1
  - If $d_t \leq c_t \; \forall t = 1,2 \ldots \bar{t}$, i.e., the demand is less than the capacity in every period, then the optimal solution is to produce amount equal to the demand in every period.

$$\left.\begin{aligned} X_t &= d_t \\ S_t &= d_t \\ I_t &= 0 \end{aligned}\right\} \Rightarrow \text{ maximal profit} = \sum_{t=1}^{\bar{t}} rS_t - hI_t = \sum_{t=1}^{\bar{t}} rd_t$$

    The optimal solution meets all the demands just-in-time and therefore no inventory build up.
- Case 2
  - If $\exists t_0$ s.t., $d_{t_0} > c_{t_0}$, then we must work ahead (i.e., produce more than we need in some previous periods). Some inventory will build up.
  - If $I_0 + \sum_{t=1}^{t_0} c_t < \sum_{t=1}^{t_0} d_t$, it implies that demand in some period cannot be met even by working ahead. $\Rightarrow S_{t_0} < d_{t_0}$, profit lost!
- Other general cases: solvers

Example

- To make the above formulation concrete and to illustrate the mechanics of solving it via LP, consider a simple example.
- Excel file: The unit profit *r* of \$10, the one-period unit holding cost h of \$1, the initial inventory I_0 of 0, and capacity and demand data *ct* and *dt* for the next six months.
- Excel solver can solve the small-size LP problems.

$$\text{Maximize} \quad 10(S_1 + S_2 + S_3 + S_4 + S_5 + S_6) - 1(I_1 + I_2 + I_3 + I_4 + I_5 + I_6)$$

Subject to:

Demand constraints	Capacity constraints	Inventory balance constraints
$S_1 \le 80$	$X_1 \le 100$	$I_1 - X_1 + S_1 = 0$
$S_2 \le 100$	$X_2 \le 100$	$I_2 - I_1 - X_2 + S_2 = 0$
$S_3 \le 120$	$X_3 \le 100$	$I_3 - I_2 - X_3 + S_3 = 0$
$S_4 \le 140$	$X_4 \le 120$	$I_4 - I_3 - X_4 + S_4 = 0$
$S_5 \le 90$	$X_5 \le 120$	$I_5 - I_4 - X_5 + S_5 = 0$
$S_6 \le 140$	$X_6 \le 120$	$I_6 - I_5 - X_6 + S_6 = 0$

nonnegative constraints： $X_t, S_t, I_t \ge 0$， $t = 1, \dots, 6$

3. Product Mix Planning

- Multiple products
- Multiple workstations

Notation

$m =$ number of products
$n =$ number of workstations
$i =$ index of product, $i = 1, \ldots, m$
$j =$ index of workstations, $j = 1, \ldots, n$
$\bar{d}_{it} =$ maximum demand for product i in period t
$\underline{d}_{it} =$ minimum demand that must be satisfied in period t
$a_{ij} =$ time required on workstation j to produce one unit of product i
$c_{jt} =$ capacity of workstation j in period t in units consistent with a_{ij}
$r_i =$ net profit from one unit of product i
$h_i =$ cost to hold one unit of product i for one period
$h_i = \alpha p_i$($\alpha =$ one−period interest rate; p_i=production cost of product i)

LP Model

X_{it} = quantity of product i produced during period t
S_{it} = quantity of product i sold during period t
I_{it} = inventory of product i at end of period t; assume I_{i0} is given

max $\sum_{t=1}^{\bar{t}} \sum_{i=1}^{m} r_i S_{it} - h_i I_{it}$ max net profit:

revenue minus inventory carrying cost

s.t. $\underline{d}_{it} \leq S_{it} \leq \bar{d}_{it}, \quad \forall i, t$ demand constraints

$\sum_{\text{i}=1}^{m} a_{ij} X_{it} \leq c_{jt}, \quad \forall j, t$ capacity constraints for each workstation

$I_{it} = I_{i,t-1} + X_{it} - S_{it}, \quad \forall i, t$ inventory balance constraints

$X_{it}, S_{it}, I_{it} \geq 0, \quad \forall i, t$ nonnegative constraints

Sensitivity Analysis

1. demand feasibility

   $S_{it} = \bar{d}_{it} \quad \Rightarrow$ demand satisfied

   $\underline{d}_{it} < S_{it} < \bar{d}_{it} \quad \Rightarrow$ insufficient capacity

   $S_{it} < \underline{d}_{it} \quad \Rightarrow$ capacity$-$infeasible

2. bottleneck locations:

   > workstations at which capacity constraint is binding

3. product mix issue

   > due to capacity limitation

4. Extensions to the basic model

4.1. Backorders

I_{it} : inventory position for product i at end of period t

where $I_{it} = I_{it}^{+} - I_{it}^{-}$.

$$I_{it}^{+} = \begin{cases} I_{it} & \text{if } I_{it} \geq 0 \\ 0 & \text{o.w.} \end{cases}$$

represents the *inventory* of product i carried from period t to t+1

$$I_{it}^{-} = \begin{cases} -I_{it} & \text{if } I_{it} < 0 \\ 0 & \text{o.w.} \end{cases}$$

represents the number of *backorders* carried from period t to t+1

$I_{it}^{+}, I_{it}^{-} \geq 0, I_{it}$ nonrestrictive.

I_{it}^{+}, I_{it}^{-} cannot be both positive,

LP can automatically guarantee that.

Coefficient π_{i} is analog to h_i, representing the penalty of carrying one unit of product i on backorder for one period.

$$
\begin{array}{ll}
\max & \sum_{t=1}^{\bar{t}} \sum_{i=1}^{m} r_i S_{it} - h_i I_{it}^{+} - \pi_i I_{it}^{-} \\
\text{s.t.} & \underline{d}_{it} \le S_{it} \le \bar{d}_{it}, \quad \forall i, t \\
& \sum_{i=1}^{m} a_{ij} X_{it} \le c_{jt}\,, \quad \forall j, t \\
& I_{it} = I_{i,t-1} + X_{it} - S_{it}, \quad \forall i, t \\
& I_{it} = I_{it}^{+} - I_{it}^{-}, \qquad \forall i, t \\
& X_{it}, S_{it}, I_{it}^{+}, I_{it}^{-} \ge 0, \quad \forall i, t
\end{array}
$$

- Example in the handout: page 601
  - Objective function

    $$\max\ 50 \sum_{t=1}^{4} S_{A,t} + 65 \sum_{t=1}^{4} S_{B,t} + 70 \sum_{t=1}^{4} S_{C,t} - 5 \sum_{i=A,B,C} \sum_{t=1}^{4} I_{it}^{+} - 10 \sum_{i=A,B,C} \sum_{t=1}^{4} I_{it}^{-}$$

Partial backorder policy

- The demands of product i during stockout in period t are backorder with probability ρ_{it}^{B} and are lost forever with probability $1-\rho_{it}^{B}$.

$$I_{i,t-1}^{+}-\rho_{i,t-1}^{B}I_{i,t-1}^{-}+X_{it}-S_{it}=I_{it}^{+}-I_{it}^{-}$$

$\rho_{i,t-1}^{B}I_{i,t-1}^{-}$: backorder quantity

4.2. Overtime

cost parameter: ℓ'_j = cost of one hour of overtime at workstation j

decision variable: O_{jt} = overtime at workstation j in period t

$$\max \quad \sum_{t=1}^{\bar{t}} \sum_{i=1}^{m} r_i S_{it} - h_i I_{it}^{+} - \pi_i I_{it}^{-} - \sum_{t=1}^{\bar{t}} \sum_{j=1}^{n} \ell'_j O_{jt}$$

$$\text{s.t.} \quad \underline{d}_{it} \le S_{it} \le \bar{d}_{it}, \quad \forall i,t$$

$$\sum_{i=1}^{m} a_{ij} X_{it} \le c_{jt} + O_{jt}, \quad \forall j,t$$

$$I_{it} = I_{i,t-1} + X_{it} - S_{it}, \quad \forall i,t$$

$$I_{it} = I_{it}^{+} - I_{it}^{-}, \quad \forall i,t$$

$$X_{it}, S_{it}, I_{it}^{+}, I_{it}^{-}, O_{jt} \ge 0, \quad \forall i,t$$

4.3. Utilization matching

- Randomness (e.g., machines failures, setups, errors in the scheduling process): diminish utilization.
- Production quota below full average capacity: to avoid excessive overtime utilization
- Let q = fraction of full capacity, then

$$\sum_{i=1}^{m} a_{ij} X_{it} \leq q c_{jt} \quad \text{for all } j, t$$

4.4. Other resource constraints

- The format is similar to capacity constraints.

let b_{ik} =units of resource k required per unit of product i

u_{kt} =number of units of resource k available in period t

X_{it} = amount of product i produced in period t

then, the resource constraint on resource type k in period t:

$$\sum_{i=1}^{m} b_{ik} X_{it} \le u_{kt} \quad \text{for each } k, t.$$

- Additionally, $b_{ijk}, u_{jkt}, X_{ijt}$ with j representing jth workstation.

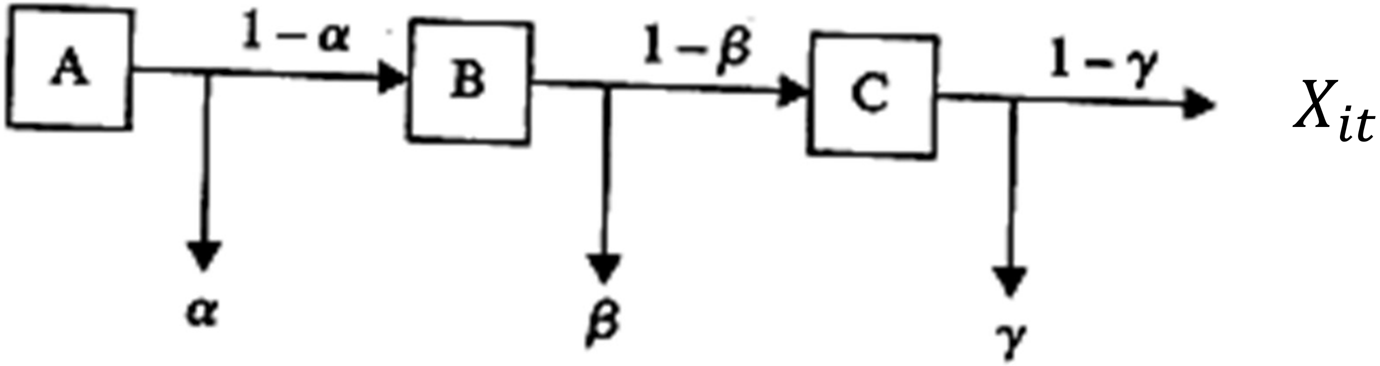
$$\sum_{i=1}^{m} b_{ijk} X_{ijt} \le u_{jkt} \quad \text{for each } j,\ k,\ t.$$

- Resources: capacity (time), people, raw materials, money, and transport devices etc.
- **Example**: suppose an inspector must check products 1,2, and 3, which require 1, 2, and 1.5 hours, respectively, per unit to inspect. If the inspector is available a total of 160 hours per month, then the constraint on this people's time in month *t*:

$$X_{1t} + 2X_{2t} + 1.5X_{3t} \le 160.$$

4.5. Yield loss

- For example, the yield of wafer is about 10% due to quality problems.
- Let constant y_{ij}= cumulative yield from workstation j onward (including workstation j) for product i.
- $\sum_{i=1}^{m} \frac{a_{ij} X_{it}}{y_{ij}} \leq c_{jt}$ for all j, t.



Fraction of product that is lost.

$$y_{iC} = (1-\gamma)$$
$$y_{iB} = (1-\beta)(1-\gamma)$$
$$y_{iA} = (1-\alpha)(1-\beta)(1-\gamma)$$

5. Workforce Planning

- Workforce Planning (WP) deals with the staffing problem, e.g.,
  - how and when to resize the labor pool
  - how many and what types of workers to hire or fire in order to meet production needs;
  - whether to use overtime instead of workforce additions.
- Alternatives

  - Change workforce size by hiring or firing

  - Overtime load
- LP can be used to support the decisions.

5.1. A simple WP model

- Assumptions:
  - A single product
  - Multiple workstations
  - No backorder cost
- Note that the labor resource could be evaluated by *worker-hours*.

Notation

$\bar{t}$ = planning horizon

t = an index of period, $t = 1, \ldots, \bar{t}$

n = total number of workstations

j = an index of workstation, $j = 1, \ldots, n$

$\bar{d}_t$ = maximum demand in period t

$\underline{d}_t$ = minimum sales allowed in period t

a_j = time required on workstation j to produce one unit of product

c_{jt} = capacity of workstation j in period t

b=number of worker-hours required to produce one unit product

r= net profit per unit of product sold

h= cost to hold one unit of product for one period

ℓ= cost of regular time in dollars per worker-hour

ℓ'= cost of overtime in dollars per worker-hour

e= cost to increase workforce by one worker-hour per period

e'= cost to decrease workforce by one worker-hour per period

Decision variables

For $t = 1, 2, \cdots, \bar{t}$,

X_t = amount produced in period t

S_t = amount sold in period t

I_t = inventory at end of t (I_0 is given as data)

W_t = workforce in period t in worker-hours of regular time

(W_0 is given as data)

H_t = increase (hires) in workforce from period $t-1$ to t in worker-hours

F_t = decrease (fires) in workforce from period $t-1$ to t in worker-hours

O_t =overtime in period t in hours

LP Model

$$
\begin{aligned}
\max \quad & \sum_{t=1}^{\bar{t}} \{ rS_t - hI_t - \ell W_t - \ell' O_t - eH_t - e' F_t \} \\
\text{s.t.} \quad & \underline{d}_t \le S_t \le \bar{d}_t, \quad \forall t \\
& a_j X_t \le c_{jt}, \quad \forall j,t \\
& I_t = I_{t-1} + X_t - S_t, \quad \forall t \\
& W_t = W_{t-1} + H_t - F_t, \ \forall t \ \text{workforce balance constraints} \\
& bX_t \le W_t + O_t, \quad \forall t \\
& X_t, S_t, I_t, O_t, W_t, H_t, F_t \ge 0, \quad \forall t
\end{aligned}
$$

Alternative LP Model

ℓ''= cost of idle time in dollars per worker-hour

decision variable U_t= idle time in period t in hours

$$\max \quad \sum_{t=1}^{\bar{t}} \{ rS_t - hI_t - \ell W_t - \ell' O_t - \ell'' U_t - eH_t - e' F_t \}$$

$$\text{s.t.} \quad \underline{d}_t \le S_t \le \bar{d}_t, \quad \forall t$$

$$a_j X_t \le c_{jt}, \quad \forall j, t$$

$$I_t = I_{t-1} + X_t - S_t, \quad \forall t$$

$$W_t = W_{t-1} + H_t - F_t, \quad \forall t$$

$$W_t = bX_t - O_t + U_t, \quad \forall t$$

$$X_t, S_t, I_t, O_t, W_t, H_t, F_t, U_t \ge 0, \quad \forall t$$

- Parameter b relates workforce requirements to production needs.
- Two options
  - It can schedule overtime, using variable O_t and incurring cost at rate l', or
  - It can resize the workforce, using variables H_t and F_t and incurring a cost of e (e') for every worker added (laid off).
- $H_t \cdot F_t = 0$. That is, we cannot hire and fire workers at the same time. LP can guarantee it since the objective function is to maximize the net profit and the coefficients of H_t and F_t are negative.
- Similarly, $U_t \cdot O_t = 0$.

5.2 Other constraints

$O_t \le \alpha W_t$ for some constant α (e.g., $= 0.2$) :

to control the load of overtime by setting up an overtime limitation.

$F_t = 0$:

to avoid firing workers since firing worker not only is costly but also damage the reputation of company.