Homework: **Sensitivity Analysis and Duality Theory**

Problem 1: The following questions refer to the Giapetto problem. Giapetto's LP was

$$\begin{aligned} \max\ z &= 3x_1 + 2x_2 \\ \text{s.t.}\quad 2x_1 + x_2 &\le 100 \quad \text{(Finishing constraint)} \\ x_1 + x_2 &\le 80 \quad \text{(Carpentry constraint)} \\ x_1 &\le 40 \quad \text{(Limited demand for soldiers)} \\ x_1, x_2 &\ge 0 \end{aligned}$$

(x_1=number of soldiers and x_2=number of trains). After adding slack variables s_1, s_2, and s_3, the solution report from LINDO is given below. Based on those and using the sensitivity analysis, answer the following questions:

(a) Show the range of profit that soldiers (x_1) contribute (i.e., the range of c_1) such that the current basis remains optimal. If soldiers contribute \$3.50 to profit, find the new optimal solution to the Giapetto problem.
(b) Show the range of profit that trains (x_2) contribute (i.e., the range of c_2) such that the current basis remains optimal.
(c) Show that the range of the available finishing hours such that the current basis remains optimal. Find the new optimal solution to the Giapetto problem if 90 finishing hours are available.
(d) Show the range of the demand for soldiers such that the current basis remains optimal.

```
---------------------------------------------------------------------------------
LP OPTIMUM FOUND AT STEP      2

     OBJECTIVE FUNCTION VALUE
     1)      180.0000

 VARIABLE        VALUE          REDUCED COST
      X1        20.000000        0.000000
      X2        60.000000        0.000000

    ROW   SLACK OR SURPLUS    DUAL PRICES
     2)        0.000000         1.000000
     3)        0.000000         1.000000
     4)       20.000000         0.000000

 NO. ITERATIONS=       2

 RANGES IN WHICH THE BASIS IS UNCHANGED:

                     OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT       ALLOWABLE       ALLOWABLE
                       COEF         INCREASE        DECREASE
     X1                3.000000       1.000000        1.000000
     X2                2.000000       1.000000        0.500000

                     RIGHTHAND SIDE RANGES
    ROW        CURRENT      ALLOWABLE       ALLOWABLE
                  RHS               INCREASE        DECREASE
     2           100.000000      20.000000       20.000000
     3           80.000000       20.000000       20.000000
     4           40.000000       INFINITY        20.000000
---------------------------------------------------------------------------------
```

Problem 2: Use the rules given in class to find the dual of the following LP **directly**.

(a) Max $z = 4x_1 - x_2 + 2x_3$

St
$$
\begin{aligned}
x_1 + x_2 \quad\quad &\leq 5 \\
2x_1 + x_2 \quad\quad &\leq 7 \\
2x_2 + x_3 &\geq 6 \\
x_1 \quad\quad + x_3 &= 4 \\
x_1 \geq 0,\ x_2,\ x_3 &\text{ urs}
\end{aligned}
$$

(b) Min $w = 4y_1 + 2y_2 - y_3$

St
$$
\begin{aligned}
y_1 + 2y_2 \quad\quad &\leq 6 \\
y_1 - y_2 + 2y_3 &= 8 \\
y_1, y_2 \geq 0,\ y_3 &\text{ urs}
\end{aligned}
$$

Problem 3. For the following LP

$$
\begin{aligned}
\max\ z &= -x_1 + 5x_2 \\
\text{s.t.}\quad & x_1 + 2x_2 \leq 0.5 \\
& -x_1 + 3x_2 \leq 0.5 \\
& x_1,\ x_2 \geq 0
\end{aligned}
$$

the optimal rof is $z = ? -0.4s_1 - 1.4s_2$ Determine the optimal z-value for the given LP **based on the current information (i.e., WITHOUT solving the model)**.