2022-2023 Autumn Semester

Operation Research

Assignment 4

- Sensitivity Analysis and Duality Theory-

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Contents

1 Problem 1

1.1 Question

The following questions refer to the Giapetto problem. Giapetto's LP was

$$
\begin{aligned}
\max \mathrm{z} = \quad & 3x_1 + 2x_2 & & \\
\text{s.t.} \quad & 2x_1 + x_2 & \leq 100 \quad & \text{(Finishing constraint)} \\
& x_1 + x_2 & \leq 80 \quad & \text{(Carpentry constraint)} \\
& x_1 & \leq 40 \quad & \text{(Limited demand for soldiers)} \\
& x_1, x_2 \geq 0 & &
\end{aligned}
$$

(x_1 = number of soldiers and x_2 = number of trains). After adding slack variables s_1, s_2, and s_3, the solution report from LINDO is given below. Based on those and using the sensitivity analysis, answer the following questions:

(a) Show the range of profit that soldiers (x_1) contribute (i.e., the range of c_1) such that the current basis remains optimal. If soldiers contribute \$3.50 to profit, find the new optimal solution to the Giapetto problem.

(b) Show the range of profit that trains (x_2) contribute (i.e., the range of c_2) such that the current basis remains optimal.

(c) Show that the range of the available finishing hours such that the current basis remains optimal. Find the new optimal solution to the Giapetto problem if 90 finishing hours are available.

(d) Show the range of the demand for soldiers such that the current basis remains optimal.

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LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 180.0000

VARIABLE VALUE	REDUCED COST	
X1	20.000000	0.000000
X2	60.000000	0.000000

ROW	SLACK OR SURPLUS	
2)	0.000000	1.000000
3)	0.000000	1.000000
4)	20.000000	0.000000

NO. ITERATIONS = 2

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT	ALLOWABLE	ALLOWABLE
	COEF	INCREASE	DECREASE
X1	3.000000	1.000000	1.000000
X2	2.000000	1.000000	0.500000

RIGHTHAND SIDE RANGES

ROW	CURRENT	ALLOWABLE	ALLOWABLE
	RHS	INCREASE	DECREASE
2	100.000000	20.000000	20.000000
3	80.000000	20.000000	20.000000
4	40.000000	INFINITY	20.000000

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1.2 Solution

(a)

According to the solution report from LINDO, the range of c_1 is $[2, 4]$ such that the current basis remains optimal. If soldiers contribute \$3.50 to profit, the value of c_1 is still in $[2, 4]$, so the optimal solution is still $X_1 = 20, X_2 = 60$ and the value of Z is \$190.

(b)

According to the solution report from LINDO, the range of c_2 is $[1.5, 3]$ such that the current basis remains optimal.

(c)

According to the solution report from LINDO, the range of the available finishing hours is $[80, 120]$ such that the current basis remains optimal. When 90 finishing hours are available now, the optimal solution is still $X_1 = 20, X_2 = 60$ and the value of Z is \$180.

(d)

According to the solution report from LINDO, the range of the demand for soldiers is $[20, +\infty)$ such that the current basis remains optimal.

2 Problem 2

2.1 Question

Use the rules given in class to find the dual of the following LP **directly**.

(a)

$$\begin{aligned}
\text{Max}\, z = \quad & 4x_1 - x_2 + 2x_3 \\
\text{St} \quad & x_1 + x_2 \quad \leq 5 \\
& 2x_1 + x_2 \quad \leq 7 \\
& 2x_2 + x_3 \quad \geq 6 \\
& x_1 + x_3 \quad = 4 \\
& x_1 \geq 0, x_2, x_3 \text{ urs}
\end{aligned}$$

(b)

$$\begin{aligned}
\text{Min}\, w = \quad & 4y_1 + 2y_2 - y_3 \\
\text{St} \quad & y_1 + 2y_2 \quad \leq 6 \\
& y_1 - y_2 + 2y_3 \quad = 8 \\
& y_1, y_2 \geq 0, y_3 \text{ urs}
\end{aligned}$$

2.2 Solution

(a)

The dual of the LP problem is

$$\begin{aligned}
\text{Min}\, w = \quad & 5y_1 + 7y_2 + 6y_3 + 4y_4 \\
\text{St} \quad & y_1 + 2y_2 + y_4 \quad \geq 4 \\
& y_1 + y_2 + 2y_3 \quad = -1 \\
& y_3 + y_4 \quad = 2 \\
& y_1, y_2 \geq 0, y_3 \leq 0, y_4 \text{ urs}
\end{aligned}$$

(b)

The dual of the LP problem is

$$\begin{aligned}
\text{Max}\, z = \quad & 6x_1 + 8x_2 \\
\text{St} \quad & x_1 + x_2 \quad \leq 4 \\
& 2x_1 - x_2 \quad \leq 2 \\
& 2x_2 \quad = -1 \\
& x_1 \leq 0, x_2 \text{ urs}
\end{aligned}$$

3 Problem 3

3.1 Question

For the following LP

$$\begin{aligned} \text{Max}\, z = \quad & -x_1 + 5x_2 \\ \text{St} \quad & \begin{aligned} x_1 + 2x_2 &\quad \leq 0.5 \\ -x_1 + 3x_2 &\quad \leq 0.5 \end{aligned} \\ & x_1, x_2 \geq 0 \end{aligned}$$

the optimal rof is $z = ? - 0.4s_1 - 1.4s_2$ Determine the optimal z-value for the given LP **based on the current information (i.e., WITHOUT solving the model)**.

3.2 Solution

The value of 0.4 and 1.4 in the optimal rof is the reduced cost of the two constraints, and they are also the optimal solution of the dual LP problem. So the value of ? is $0.4 * 0.5 + 0.5 * 1.4 = 0.9$, which is also the maximum value of the primary LP problem.