2022-2023 Autumn Semester

Operation Research

Assignment 6

-Integer Programming-

By 承子杰

Student ID：202228000243001

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目录

1 Problem 1

1.1 Question

The Cubs are trying to determine which of the following free agent pitchers should be signed: Rick Sutcliffe (RS), Bruce Sutter (BS), Dennis Eckersley (DE), Steve Trout (ST), Tim Stoddard (TS). The cost of signing each pitcher and the number of victories each pitcher will add to the Cubs are shown below. Subject to the following restrictions, the Cubs want to sign the pitchers who will add the most victories to the team.

Pitcher	Cost of Signing Pitcher (Millions $)	Victories Added to Cubs
RS	6	6 (righty)
BS	4	5 (righty)
DE	3	3 (righty)
ST	2	3 (lefty)
TS	2	2 (righty)

(a) At most, $12 million can be spent.

(b) If both DE and ST are signed, then BS cannot be signed.

(c) At most two right-handed pitchers can be signed.

(d) The Cubs cannot sign both BS and RS.

Formulate an IP to help the Cubs determine who they should sign.

1.2 Solution

Objective Function

We denote $x_i = 1, 0$ to demonstrate whther we choose the ith [1] Picher. Our aim is to maximize the Victories added to Cubs, so the Objective Function is as eq 1.

$$\max z = 6x_1 + 5x_2 + 3x_3 + 3x_4 + 2x_5. \tag{1}$$

[1] 1-RS,2-BS,3-DE,4-ST,5-TS

Constraints

For (a) restriction,

$$6x_1 + 4x_2 + 3x_3 + 2x_4 + 2x_5 \leq 12;$$

For (b) restriction,

$$x_2 \leq 2 - x_3 - x_4;$$

For (c) restriction,

$$x_1 + x_2 + x_3 + x_5 \leq 2;$$

For (d) restriction,

$$x_1 + x_2 \leq 1.$$

In summary, the constraints of the IP as list below.

- $6x_1 + 4x_2 + 3x_3 + 2x_4 + 2x_5 \leq 12;$
- $x_2 \leq 2 - x_3 - x_4;$
- $x_1 + x_2 + x_3 + x_5 \leq 2;$
- $x_1 + x_2 \leq 1.$

Sign Restrictions

Obviously, we have the following sign restrictions:

$$x_1, \cdots, x_5 \text{ binary variables of value 0,1}$$

1.3 Result

By the help of LINGO software, we obtain the result $x_1 = x_3 = x_4 = 1, x_2 = x_5 = 0, z = 12$. In other words, we sign the Pitcher RS、DE and ST, and the values of the victories added to Cubs are 12.

2 Problem 2

2.1 Question

State University must purchase 1,100 computers from three vendors. Vendor 1 charges $500 per computer plus a delivery charge of $5,000. Vendor 2 charges $350 per computer plus a delivery charge of $4,000. Vendor 3 charges $250 per computer plus a delivery charge of $6,000. Vendor 1 will sell the university at most 500 computers; vendor 2, at most 900; and vendor 3, at most 400. Formulate an IP to minimize the cost of purchasing the needed computers.

2.2 Solution

Objective Function

We denote $x_j, j = 1, 2, 3$ as the number of computers purchased from Vendor j. Our aim is to minimize the cost of purchasing the needed computers, so we could obtain the Objective Function as equation 2.

$$\min z = 500x_1 + 350x_2 + 250x_3 + 15000 \tag{2}$$

Constraints

There is major one constraint: the amount of computers each Vendor could provide. Hence, we have the equation 3

$$\begin{aligned} &x_1 + x_2 + x_3 = 1100 \\ &x_1 \leq 500 \\ &x_2 \leq 900 \\ &x_3 \leq 400 \end{aligned} \tag{3}$$

Sign Restrictions

Obviously, the value of the decision variables should be integer. Hence,

$$x_1, x_2, x_3 \geq 0, integer \tag{4}$$

2.3 Result

By the help of LINGO software, we obtain the result $x_1 = 0, x_2 = 700, x_3 = 400, z = 360000$. In other words, we should purchase 700 computers from Vendor 2 and 400 computers from Vendor 3. The total cost is 360000.

3 Problem 3

3.1 Question

Use the branch-and-bound method to solve the following IP:

$$\begin{aligned} \max z = \quad & 3x_1 + x_2 \\ \text{s.t.} \quad & 5x_1 + x_2 \leq 12 \\ & 2x_1 + x_2 \leq 8 \\ & x_1, x_2 \geq 0; x_1, x_2 \text{ integer} \end{aligned}$$

3.2 Soluion

We could use Branch & Bound Method to solve the IP as follow.

Subproblem 1

$$\begin{aligned} \max z = \quad & 3x_1 + x_2 \\ \text{s.t.} \quad & 5x_1 + x_2 \leq 12 \\ & 2x_1 + x_2 \leq 8 \\ & x_1, x_2 \geq 0 \end{aligned}$$

We could solve the LP relaxation of the IP at first. By Revised Simplex Method, we could easily obtain the optimal solution: $z = 9.3, x_1 = 1.3, x_2 = 5.3$. Since x_1 and x_2 are not integer, we could add constraints $x_1 \leq 1$ and $x_1 \geq 2$ respectively to turn subproblem 1 to subproblem 2 and subproblem 3.

Subproblem 2($x_1 \le 1$)

$$\begin{aligned} \max z = \quad & 3x_1 + x_2 \\ \text{s.t.} \quad & 5x_1 + x_2 \le 12 \\ & 2x_1 + x_2 \le 8 \\ & x_1 \le 1 \\ & x_1, x_2 \ge 0 \end{aligned}$$

By Revised Simplex Method, we could easily obtain the optimal solution: $z = 9, x_1 = 1, x_2 = 6$. Hence, we get a candidate solution and the LB=9. Now we solve the subproblem 3.

Subproblem 3($x_1 \ge 2$)

$$\begin{aligned} \max z = \quad & 3x_1 + x_2 \\ \text{s.t.} \quad & 5x_1 + x_2 \le 12 \\ & 2x_1 + x_2 \le 8 \\ & x_1 \ge 2 \\ & x_1, x_2 \ge 0 \end{aligned}$$

By Revised Simplex Method, we could easily obtain the optimal solution: $z = 8, x_1 = 2, x_2 = 2$. Since the value of z is less than LB, so the result is not the optimal solution.

In summary, the optimal solution of this IP is $x_1 = 1, x_2 = 6, z = 9$. The process above is illustrate as the figure 1.

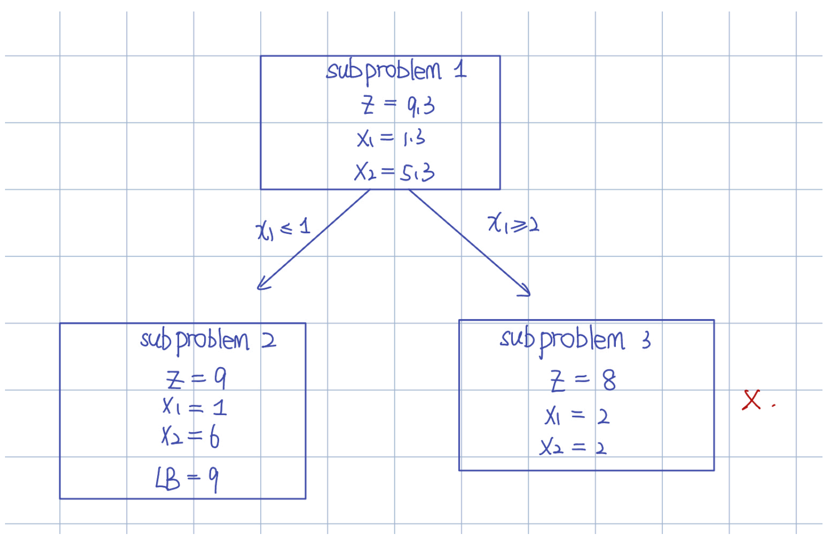

图 1: The Procee of B&B Solution

4 Problem 4

4.1 Question

Use the branch-and-bound method to solve the following IP:

$$\begin{aligned} \max z = \quad & 3x_1 + x_2 \\ \text{s.t.} \quad & 2x_1 + x_2 \leq 6 \\ & x_1 + x_2 \leq 4 \\ & x_1, x_2 \geq 0; x1 \text{ integer} \end{aligned}$$

4.2 Solution

We could use Branch & Bound Method to solve the IP as follow.

Subproblem 1

$$\begin{aligned} \max z = \quad & 3x_1 + x_2 \\ \text{s.t.} \quad & 2x_1 + x_2 \leq 6 \\ & x_1 + x_2 \leq 4 \\ & x_1, x_2 \geq 0; x1 \text{ integer} \end{aligned}$$

We could solve the LP relaxation of the IP at first. By Revised Simplex Method, we could easily obtain the optimal solution: $z = 9, x_1 = 3, x_2 = 0$. Since x_1 is integer, then the result is the optimal solution.