2022-2023 Autumn Semester

Operation Research

Group Project

-LP Problems and Sensitive Analysis-

By *Cheng Zijie: 202228000243001*

& Zheng Wenzhen: 202228000243004

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1 Problem 1. Golf-Sport: Managing Operations

1.1 Question:

Golf-Sport is a small-sized company that produces high-quality components for people who build their own golf clubs and prebuilt sets of clubs. There are five components—steel shafts, graphite shafts, forged iron heads, metal wood heads, and metal wood heads with titanium inserts—made in three plants—Chandler, Glendale, and Tucson—in the Golf-Sport system. Each plant can produce any of the components, although each plant has a different set of individual constraints and unit costs. These constraints cover labor and packaging machine time (the machine is used by all components); the specific values for each component-plant combination are given in Tables 1,2,3. Note that even though the components are identical in the three plants, different production processes are used, and therefore the products use different amounts of resources in different plants.

Besides component sales, the company takes the components and manufactures sets of golf clubs. Each set requires 13 shafts, 10 iron heads, and 3 wood heads. All of the shafts in a set must be the same type (steel or graphite), and all of the wood heads must be the same type (metal or metal with inserts). Assembly times per month for the sets at each plant are shown in Table 4.

Table 1: Product-Resource Constraints:Chandler

	Resources		
Products	Labor (Minutes Unit)	Packing (Minutes Unit)	Advertising ($ Unit)
Steel shafts	1	4	1.0
Graphite shafts	1.5	4	1.5
Forged iron heads	1.5	5	1.1
Metal wood heads	3	6	1.5
Titanium insert heads	4	6	1.9
Monthly availability (minutes)	12,000	20,000	—

Table 2: Product-Resource Constraints:Glendale

	Resources		
Products	Labor (Minutes Unit)	Packing (Minutes Unit)	Advertising ($ Unit)
Steel shafts	3.5	7	1.1
Graphite shafts	3.5	7	1.1
Forged iron heads	4.5	8	1.1
Metal wood heads	4.5	9	1.2
Titanium insert heads	5.0	7	1.9
Monthly availability (minutes)	15,000	40,000	—

Table 3: Product-Resource constraints:Tucson

	Resources		
Products	Labor (Minutes Unit)	Packing (Minutes Unit)	Advertising ($ Unit)
Steel shafts	3	7.5	1.3
Graphite shafts	3.5	7.5	1.3
Forged iron heads	4	8.5	1.3
Metal wood heads	4.5	9.5	1.3
Titanium insert heads	5.5	8.0	1.9
Monthly availability (minutes)	22,000	35,000	—

Each plant of Golf-Sport has a retail outlet to sell components and sets, and the specific plant is the only supplier for its retail outlet. The minimum and maximum amounts of demand for each plant–product pair are given in Table 5. Note that, although the minimums must be satisfied, you do not need to satisfy demand up to the maximum amount.

Table 4: Assembly Times per Month for Four Sets

Plant	Time (Minutes per set)	Total Time Avaiable (Minutes)
Chandler	65	5,500
Glendale	60	5,000
Tucson	65	6,000

Table 5: Minimum and Maximum Product Demand per Month

	Store(for Plant)		
Products	Chandler	Glendale	Tucson
Steel shafts	[0,2,000]	[0,2,000]	[0,2,000]
Graphite shafts	[100,2,000]	[100,2,000]	[50,2,000]
Forged iron heads	[200,2,000]	[200,2,000]	[100,2,000]
Metal wood heads	[30,2,000]	[30,2,000]	[15,2,000]
Titanium insert heads	[100,2,000]	[100,2,000]	[100,2,000]
Set:Steel, metal	[0,200]	[0,200]	[0,200]
Set:Steel, insert	[0,100]	[0,100]	[0,100]
Set:Graphite, metal	[0,300]	[0,300]	[0,300]
Set:Graphite, insert	[0,400]	[0,400]	[0,400]

This planning problem is for two months. The costs in Table 6 increase by 12% for the second month, and production times are stationary. Inventory costs are based on end-of-period inventory for each product set and cost out at 8% of the cost values in Table 6. Table 7 lists the revenue generated by each product. Initially, there is no inventory.

Table 6: Material, Production, and Assembly Costs ($) per Part or Set

	Store(for Plant)		
Products	Chandler	Glendale	Tucson
Steel shafts	6	5	7
Graphite shafts	19	18	20
Forged iron heads	4	5	5
Metal wood heads	10	11	12
Titanium insert heads	26	24	27
Set:Steel, metal	178	175	180
Set:Steel, insert	228	220	240
Set:Graphite, metal	350	360	370
Set:Graphite, insert	420	435	450

Table 7: Revenue per Part or Set ($)

	Store(for Plant)		
Products	Chandler	Glendale	Tucson
Steel shafts	10	10	12
Graphite shafts	25	25	30
Forged iron heads	8	8	10
Metal wood heads	18	18	22
Titanium insert heads	40	40	45
Set:Steel, metal	290	290	310
Set:Steel, insert	380	380	420
Set:Graphite, metal	560	560	640
Set:Graphite, insert	650	650	720

The corporation controls the capital available for expenses; the cash requirements for each product are given in the last column of Tables 1, 2, 3. There is a total of $20,000 available for advertising the production for the entire system during each month, and any money not spent in a month is not available the next month. The corporation also controls graphite. Each graphite shaft requires 4 ounces of graphite; a total of 1,000 pounds is available for each of the two months.

Your job is to determine a recommendation for the company. A recommendation must include a plan for production and sales. In addition, you should also address the following sensitivity-analysis issues in your recommendation:

- If you could get more graphite or advertising cash, how much would you like, how would you use it, and what would you be willing to pay?
- At what site(s) would you like to add extra packing machine hours, assembly hours, and/or extra labor hours? How much would you be willing to pay per hour and how many extra hours would you like?
- Marketing is trying to get Golf-Sport to consider an advertising program that promises a 50% increase in their maximum demand. Can we handle this with the current system or do we need more resources? How much more is the production going to cost if we take on the additional demand?

Note that:

1. The production costs for the sets (products 6, 7, 8, 9) in the objective account for the assembly cost (the costs in Table 6 – the costs for the components)
2. The unit inventory costs for the sets are based on the total cost in Table 6 (i.e., the assembly cost and the costs of components).
3. 1 pound = 16 ounce
4. Decision variables are treated as continuous.

1.2 Solution:

1.2.1 Notioin Explanation

The notions used in this problem are list in the table 8, and its detailed meanings are shown together as well.

Table 8: Notion Explanation in Problem 3

Notion	Explanation
i	period i between $(i-1)$th month and ith month, i=1,2;
j	company j, $j = 1, 2$[1];
k	product or set k, $k = 1, \cdots, 9$[2];
$lbt_{i,j,k}$	labor time required to produce a product k from company j of period i; $i = 1, 2,\ j = 1, 2, 3,\ k = 1, \cdots, 5$;
$c_{i,j,k}$	cost from producing a product or a set k from company j of period i; $i = 1, 2,\ j = 1, 2, 3,\ k = 1, \cdots, 5$;
$r_{i,j,k}$	revenue from selling a product or a set k from company j of period i; $i = 1, 2,\ j = 1, 2, 3,\ k = 1, \cdots, 9$;
$ad_{i,j,k}$	advertisement cost from producing a product k from company j of period i; $i = 1, 2,\ j = 1, 2, 3,\ k = 1, \cdots, 5$;

[1] 1–Chandler, 2–Glendale, 3–Tucson;

[2] 1–Steel shafts, 2–Graphite shafts, 3–Forged iron heads, 4–Metal wood heads, 5– Titanium insert heads, 6–Set: Steel, metal, 7–Set: Steel, insert, 8–Set: Graphite, metal, 9–Set: Graphite, insert.

$h_{i,j,k}$	inventory cost of holding a product or a set k from company j of period i; $i = 1, 2,\ j = 1, 2, 3,\ k = 1, \cdots, 9$;
$pt_{i,j,k}$	packing time required to produce a product k from company j of period i; $i = 1, 2,\ j = 1, 2, 3,\ k = 1, \cdots, 5$;
$minD_{i,j,k}$	the minimum demand of product or set k from company j of period i; $i = 1, 2,\ j = 1, 2, 3,\ k = 1, \cdots, 9$;
$maxD_{i,j,k}$	the maximum demand of product or set k from company j of period i; $i = 1, 2,\ j = 1, 2, 3,\ k = 1, \cdots, 9$;
$g_{i,j,k}$	graphite required to produce a product k from company j of period i; $i = 1, 2,\ j = 1, 2, 3,\ k = 1, \cdots, 5$;
$ast_{i,j,k}$	assembly time equired to produce a set k from company j of period i; $i = 1, 2,\ j = 1, 2, 3,\ k = 6, \cdots, 9$;
$A_{i,j}$	Total assembly time offered for company j of period i; $i = 1, 2,\ j = 1, 2, 3$;
$L_{i,j}$	Total labour time offered for company j of period i; $i = 1, 2,\ j = 1, 2, 3$;
$P_{i,j}$	Total packing time offered for company j of period i; $i = 1, 2,\ j = 1, 2, 3$;
AD_i	Total budget for advertising of period i; $i = 1, 2$;
G_i	Total graphite could be obtained in period i; $i = 1, 2$.

1.2.2 Decision Variables

The decision variables for the LP problem primaryly consist of three kinds. They are

- $x_{i,j,k}$: the number of the product k **produced** from company j of period i; $i = 1, 2;\ j = 1, 2, 3;\ k = 1, \cdots, 9$;
- $y_{i,j,k}$: the number of the product k **sold** from company j of period i; $i = 1, 2;\ j = 1, 2, 3;\ k = 1, \cdots, 9$;
- $s_{i,j,k}$: the number of the product k **hold** from company j of period i; $i = 1, 2;\ j = 1, 2, 3;\ k = 1, \cdots, 9$.

1.2.3 Objective Function

We are expected to make a plan to obtain maximum revenues. Obviously, selling products could make profits, and producing and holding products need respective cost as well. Hence, our objective

function is equation (1).

$$\max z = \sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{9}[r_{i,j,k}y_{i,j,k} - c_{i,j,k}x_{i,j,k} - h_{i,j,k}s_{i,j,k}]. \tag{1}$$

1.2.4 Constraints

The first restriction is the production capacity of the basic products, which is mainly made up of available labour time and offered packing time. Thus, we have constraints (2) and (3).

$$\sum_{k=1}^{5} lbt_{i,j,k}x_{i,j,k} \leq L_{i,j} \quad \forall\, i = 1,2; j = 1,2,3; \tag{2}$$

$$\sum_{k=1}^{5} pt_{i,j,k}x_{i,j,k} \leq P_{i,j} \quad \forall i = 1,2; j = 1,2,3. \tag{3}$$

In the Meanwhile, the production of the sets is also restricted with assembly time, whose form of constraint is equation (4).

$$\sum_{k=6}^{9} ast_{i,j,k}x_{i,j,k} \leq A_{i,j} \quad \forall i = 1,2; j = 1,2,3. \tag{4}$$

For sales, we contemporaryly have corresponding restrictions. We are required to meet the minimum demand and are now allowed to excess the maximum demand as well. In other words, we have constraints (5), (6).

$$y_{i,j,k} \geq minD_{i,j,k} \quad \forall i = 1,2; j = 1,2,3, k = 1,\cdots,9; \tag{5}$$

$$y_{i,j,k} \leq maxD_{i,j,k} \quad \forall i = 1,2; j = 1,2,3, k = 1,\cdots,9. \tag{6}$$

What we are supposed to take in consider is inventory, which is equal to the value that current production minus current sales plus previous inventory. Since the initial inventory is empty, then it is reasonable to denote $s_{0,j,k} = 0$ in order to simplify the constraints formulation. Attention should be pay to the inventory of basic products ($k = 1,\cdots,5$), in light of that the set products ($k = 6,\cdots,9$) are composed by corresponding basic products. The detailed representations have been demonstrated in equation(7-12).

$$x_{i,j,1} - y_{i,j,1} + s_{i-1,j,1} - \sum_{k=6,7} 13x_{i,j,k} = s_{i,j,1}, \ \ \forall\, i = 1,2,\ j = 1,2,3; \tag{7}$$

$$x_{i,j,2} - y_{i,j,2} + s_{i-1,j,2} - \sum_{k=8,9} 13x_{i,j,k} = s_{i,j,2}, \ \ \forall\, i = 1,2,\ j = 1,2,3; \tag{8}$$

$$x_{i,j,3} - y_{i,j,3} + s_{i-1,j,3} - \sum_{k=6}^{9} 10x_{i,j,k} = s_{i,j,3}, \ \ \forall\, i = 1,2,\ j = 1,2,3; \tag{9}$$

$$x_{i,j,4} - y_{i,j,4} + s_{i-1,j,4} - \sum_{k=6,8} 3x_{i,j,k} = s_{i,j,4}, \ \ \forall\, i = 1,2,\ j = 1,2,3; \tag{10}$$

$$x_{i,j,5} - y_{i,j,5} + s_{i-1,j,5} - \sum_{k=7,9} 3x_{i,j,k} = s_{i,j,5}, \ \ \forall\, i = 1,2,\ j = 1,2,3; \tag{11}$$

$$x_{i,j,k} - y_{i,j,k} + s_{i-1,j,k} = s_{i,j,k}, \ \ \forall\, i = 1,2,\ j = 1,2,3,\ k = 6,7,8,9. \tag{12}$$

The corporation controls the capital available for advertising expenses every month, and any money not spent in a month cannot be cumulated to next month. So we can get the constraints (13).

$$\sum_{j=1}^{3}\sum_{k=1}^{5} ad_{i,j,k}x_{i,j,k} \le AD_i, \ \ \forall\, i = 1,2. \tag{13}$$

Finally, the supply for the graphite is also limited. Since only the graphite shaft requires graphite, we denote other basic products cost $g_{i,j,k} = 0,\ k = 1,3,4,5$ in order to represent conveniently. What needs to be focused on is that the surplus amount could be used in the following production. The formal expression is equation (14), (15) .

$$\sum_{j=1}^{3}\sum_{k=1}^{5} g_{1,j,k}x_{1,j,k} \le G_1; \tag{14}$$

$$\sum_{j=1}^{3}\sum_{k=1}^{5} g_{2,j,k}x_{2,j,k} \le G_2 + G_1 - \sum_{j=1}^{3}\sum_{k=1}^{5} g_{1,j,k}x_{1,j,k}. \tag{15}$$

In summary, the total constraints for the LP problem are list below.

- $\sum_{k=1}^{5} lbt_{i,j,k}x_{i,j,k} \le L_{i,j} \quad \forall\, i = 1,2; j = 1,2,3;$
- $\sum_{k=1}^{5} pt_{i,j,k}x_{i,j,k} \le P_{i,j} \quad \forall i = 1,2; j = 1,2,3;$
- $\sum_{k=6}^{9} ast_{i,j,k}x_{i,j,k} \le A_{i,j} \quad \forall i = 1,2; j = 1,2,3;$
- $y_{i,j,k} \ge minD_{i,j,k} \quad \forall i = 1,2; j = 1,2,3, k = 1,\cdots,9;$

- $y_{i,j,k} \leq maxD_{i,j,k} \quad \forall i = 1,2; j = 1,2,3, k = 1, \cdots, 9;$
- $x_{i,j,1} - y_{i,j,1} + s_{i-1,j,1} - \sum_{k=6,7} 13x_{i,j,k} = s_{i,j,1}, \ \forall\, i = 1,2, \ j = 1,2,3;$
- $x_{i,j,2} - y_{i,j,2} + s_{i-1,j,2} - \sum_{k=8,9} 13x_{i,j,k} = s_{i,j,2}, \ \forall\, i = 1,2, \ j = 1,2,3;$
- $x_{i,j,3} - y_{i,j,3} + s_{i-1,j,3} - \sum_{k=6}^{9} 10x_{i,j,k} = s_{i,j,3}, \ \forall\, i = 1,2, \ j = 1,2,3;$
- $x_{i,j,4} - y_{i,j,4} + s_{i-1,j,4} - \sum_{k=6,8} 3x_{i,j,k} = s_{i,j,4}, \ \forall\, i = 1,2, \ j = 1,2,3;$
- $x_{i,j,5} - y_{i,j,5} + s_{i-1,j,5} - \sum_{k=7,9} 3x_{i,j,k} = s_{i,j,5}, \ \forall\, i = 1,2, \ j = 1,2,3;$
- $x_{i,j,k} - y_{i,j,k} + s_{i-1,j,k} = s_{i,j,k}, \ \forall\, i = 1,2, \ j = 1,2,3, \ k = 6,7,8,9;$
- $\sum_{j=1}^{3} \sum_{k=1}^{5} ad_{i,j,k} x_{i,j,k} \leq AD_i, \ \forall\, i = 1,2;$
- $\sum_{j=1}^{3} \sum_{k=1}^{5} g_{1,j,k} x_{1,j,k} \leq G_1;$
- $\sum_{j=1}^{3} \sum_{k=1}^{5} g_{2,j,k} x_{2,j,k} \leq G_2 + G_1 - \sum_{j=1}^{3} \sum_{k=1}^{5} g_{1,j,k} x_{1,j,k}.$

1.2.5 Sign Restriction

Obviously, all the decision variables should be nonnegative, so

$$x_{i,j,k}, \ y_{i,j,k}, \ s_{i,j,k} \geq 0, \ i = 1,2, \ j = 1,2,3, \ k = 1, \cdots, 9. \tag{16}$$

1.3 Result

By the use of Lingo, we could easily obtain the result of this LP problem, which is shown in table 9 - 12.

Table 9: The Production Arrangement of period 1

	Chandler	Glendale	Tucson
Steel shafts	0	0	0
Graphite shafts	795.5	545.4502	2189.334
Forged iron heads	735	542.654	245.6416
Metal wood heads	30	30	15
Titanium insert heads	2160.5	2102.796	2043.692
Set:Steel, metal	0	0	0
Set:Steel, insert	0	0	0
Set:Graphite, metal	0	0	0
Set:Graphite, insert	53.5	34.2654	14.56416

Table 10: The Production Arrangement of period 2

	Chandler	Glendale	Tucson
Steel shafts	0	0	0
Graphite shafts	795.5	545.4502	1136.363
Forged iron heads	735	542.654	935.6635
Metal wood heads	30	30	265.6991
Titanium insert heads	2160.5	2102.796	2000
Set:Steel, metal	0	0	0
Set:Steel, insert	0	0	0
Set:Graphite, metal	0	0	83.56635
Set:Graphite, insert	53.5	34.2654	0

Table 11: The Sale Arrangement of period 1

	Chandler	Glendale	Tucson
Steel shafts	0	0	0
Graphite shafts	100	100	2000
Forged iron heads	200	200	100
Metal wood heads	30	30	15
Titanium insert heads	2000	2000	2000
Set:Steel, metal	0	0	0
Set:Steel, insert	0	0	0
Set:Graphite, metal	0	0	0
Set:Graphite, insert	53.5	34.2654	14.56416

Table 12: The Sale Arrangement of period 2

	Chandler	Glendale	Tucson
Steel shafts	0	0	0
Graphite shafts	100	100	50
Forged iron heads	200	200	100
Metal wood heads	30	30	15
Titanium insert heads	2000	2000	2000
Set:Steel, metal	0	0	0
Set:Steel, insert	0	0	0
Set:Graphite, metal	0	0	83.56635
Set:Graphite, insert	53.5	34.2654	0

The inventory of all products of three companys in two periods is both empty. In summary, the profit under this arrangement is $ 258326.8.

1.4 Sensitivity Analysis

1.4.1 Graphite and Advertising Cash

According to the solution report of LINGO software, we could find that the constraints of Graphite and Advertising Cash in two periods are both slack. Hence, any increasement of the two resources could not increase the last profit, and there is no need to get more graphite or advertising cash.

1.4.2 Packing Machine Hours, Assembly Hours and Extra Labor Hours

Packing Machine Hours

According to the solution report of LINGO software, we could obtain that the constraints of $P_{1,1}$, $P_{1,3}$, $P_{2,1}$, $P_{2,3}$ is tight. For the packing time of company Chandler in period 1, we would like to increase 2509.032 hours, and we would like to pay for $ 1.916667 per hour. For the packing time of company Tucson in period 1, we would like to increase 4144.081 hours, and we would like to pay for $ 1.307506 per hour. For the packing time of company Chandler in period 2, we would like to increase 2509.032 hours, and we would like to pay for $ 1.496667 per hour as the cost increased. For the packing time of company Tucson in period 2, we would like to increase 1844.423 hours, and we would like to pay for $ 1.069194 per hour as the cost increased. In conclusion, the sensitivity analysis of packinng machine hours are shown in table 13.

Table 13: Sensitivity Analysis of Packing Machine Hours

	Increasement	Payment(per hour)
$P_{1,1}$	2509.03	$1.92
$P_{1,3}$	4144.08	$1.31
$P_{2,1}$	2509.03	$1.50
$P_{2,3}$	1844.42	$1.07

Assembly Hours

According to the solution report of LINGO software, we could obtain that the constraints of Assembly Hours are all slack. Hence, we have no needs to increase the assembly hours.

Labor Hours

According to the solution report of LINGO software, we could obtain that the constraints of $L_{1,2}$, $L_{2,2}$ is tight. For the labor time of company Glendale in period 1, we would like to increase 3811.923 hours, and we are willing to pay for $ 2.037915 per hour. For the labor time of company Glendale in period 2, we would like to increase 5176.667 hours, and we are willing to pay for $ 1.543128 per hour. In conclusion, the sensitivity analysis of labor hours are shown in table 14.

Table 14: Sensitivity Analysis of Labor Hours

	Increasement	Payment(per hour)
$L_{1,2}$	3811.92	\$2.04
$P_{2,2}$	5176.67	\$1.54

1.4.3 Advertising Program

If a 50% increase happpens in maximum demand, our current system can handle this by just adjusting some amount of production. The arrangement under the new situation is in table.

Table 15: The Production Arrangement of period 1

	Chandler	Glendale	Tucson
Steel shafts	0	0	0
Graphite shafts	418.2	100	1334.333
Forged iron heads	444.7761	200	100
Metal wood heads	30	30	15
Titanium insert heads	2653.881	2723	3000
Set:Steel, metal	0	0	0
Set:Steel, insert	0	0	0
Set:Graphite, metal	0	0	0
Set:Graphite, insert	24.47761	0	0

Table 16: The Production Arrangement of period 2

	Chandler	Glendale	Tucson
Steel shafts	0	0	0
Graphite shafts	418.209	100	643.4716
Forged iron heads	444.7761	200	556.5166
Metal wood heads	30	30	151.955
Titanium insert heads	2653.881	2723	3000
Set:Steel, metal	0	0	0
Set:Steel, insert	0	0	0
Set:Graphite, metal	0	0	45.65166
Set:Graphite, insert	24.47761	0	0

Table 17: The Sale Arrangement of period 1

	Chandler	Glendale	Tucson
Steel shafts	0	0	0
Graphite shafts	100	100	1334.333
Forged iron heads	200	200	100
Metal wood heads	30	30	15
Titanium insert heads	2580.448	2723	3000
Set:Steel, metal	0	0	0
Set:Steel, insert	0	0	0
Set:Graphite, metal	0	0	0
Set:Graphite, insert	24.47761	0	0

Table 18: The Sale Arrangement of period 2

	Chandler	Glendale	Tucson
Steel shafts	0	0	0
Graphite shafts	100	100	50
Forged iron heads	200	200	100
Metal wood heads	30	30	15
Titanium insert heads	2580.448	2723	3000
Set:Steel, metal	0	0	0
Set:Steel, insert	0	0	0
Set:Graphite, metal	0	0	45.65166
Set:Graphite, insert	24.47761	0	0

If we take on the additional demand, the production cost now is $ 535457.3. At the meanwhile, if we not take on the additional demand, the production will cost $ 502456.9, so the overpayment is $ 33000.4 .

2 Problem 2. Production planning and inventory control

2.1 Question:

Consider a production line with four workstations, labelled $j = 1, 2, 3,$ and $4,$ in tandem (all products flow through all four machines in order). Three different products, labelled $i = A, B,$ and $C,$ are produced on the line. The hours required on each workstation for each product and the net profits per unit sold (r_i) are given in table 19.

Table 19: The Hours Required on Each Workstation

	1	2	3	4	r_i
A	2.4	1.1	0.8	3.0	\$50
B	2.0	2.2	1.2	2.1	\$65
C	0.9	0.9	1.0	2.5	\$70

The number of hours available (c_{jt}) and the upper and lower limits on demand ($\overline{d}_{jt}$ and $\underline{d}_{jt}$) for each product over the next four quarters are in table 20.

Table 20: The Number of Hours Available and Limits on Demand

	1	2	3	4
c_{1t}	640	640	1280	1280
c_{2t}	640	640	640	640
c_{3t}	1920	1920	1920	1920
c_{4t}	1280	1280	1280	2560
$\overline{d}_{At}$	100	50	50	75
$\underline{d}_{At}$	0	0	0	0
$\overline{d}_{Bt}$	100	100	100	100
$\underline{d}_{Bt}$	20	20	20	25
$\overline{d}_{Ct}$	300	250	250	400
$\underline{d}_{Ct}$	0	0	0	50

The initial inventory is zero. All the decision variables are continuous.

a. Suppose we use a quarterly holding cost of \$5 and a quarterly backorder cost of \$10 per item

on all products and allow backordering. Formulate an LP to maximize profit minus holding and backorder costs subject to the constraints on workstation capacity and min/max sales.

b. Solve your formulation in part (a). Which constraints are binding in your solution? Give a brief illustration of your solution.

c. Suppose that all the stockouts are lost sales. Use a quarterly holding cost of $5 and a one-time lost-sale cost of $5 per item (for counting the loss of reputation) on all the products. Formulate an LP to maximize profit minus holding and lost-sale costs subject to the constraints on workstation capacity and min/max sales.

d. Solve your formulation in part (c). Which constraints are binding in your solution? Give a brief illustration of your solution.

e. Compare and discuss the results from the models from part (a)-(b) and part (c)-(d).

2.2 Solution:

2.2.1 Notioin Explanation

The notions used in this problem are consistent with those in the question.

2.2.2 Decision variables

The decision variables for the LP problem primaryly consist of three kinds. They are

- $x_{i,j}$ The number of product i **produced** in period j, $i = A, B, C,\ j = 1, 2, 3, 4$;
- $y_{i,j}$ The number of product i **sold** in period j, $i = A, B, C,\ j = 1, 2, 3, 4$;
- $i^{+}_{i,j}$ The number of product i **hold** in period j, $i = A, B, C,\ j = 1, 2, 3, 4$;
- $i^{-}_{i,j}$ The number of product i **backordered** in period j, $i = A, B, C,\ j = 1, 2, 3, 4$;

2.2.3 Objective Function of Question (a)

Our aim is to maximize the profit by offering a production and sale plan. It is apparent that selling products is only way to obtain revenue. Holding products needs some cost, and backorder will make

some profit loss although it could expand current sales. Thus, our objective function is equation (17).

$$\max z = \sum_{i=A,B,C} \sum_{j=1}^{4} r_i y_{i,j} - 5 \sum_{i=A,B,C} \sum_{j=1}^{4} i_{i,j}^{+} - 10 \sum_{i=A,B,C} \sum_{j=1}^{3} i_{i,j}^{-} \tag{17}$$

2.2.4 Constraints of Question (a)

The frist restriction is the production capacity. If we denote $h_{i,t}$ as the hours which product i requires on workstation t , we could get the constraint (18).

$$\sum_{i=A,B,C} h_{i,t} x_{i,j} \leq c_{jt}, \ \forall\, t = 1,2,3,4, \ j = 1,2,3,4. \tag{18}$$

The Limits on demand is what we need to take in consider. So we have the constraint (19).

$$\underline{d}_{i,j} \leq y_{i,j} \leq \overline{d}_{i,j}, \ \forall\, i = A,B,C, \ j = 1,2,3,4. \tag{19}$$

Since the inital inventory is empty and the backorder in the last month should be zero, we could denote $i_{i,0}^{+}, i_{i,0}^{-}, i^{i,4}$. Hence, the constraint of inventory is equation(20)

$$i_{i,j}^{+} - i_{i,j}^{-} = i_{i,j-1}^{+} + x_{i,j} - y_{i,j}, \ \forall\, i = A,B,C, \ j = 1,2,3,4. \tag{20}$$

In summary, the all constraints of the Question (a) are list below.

- $\sum_{i=A,B,C} h_{i,t} x_{i,j} \leq c_{jt}, \ \forall\, t = 1,2,3,4, \ j = 1,2,3,4.$
- $\underline{d}_{i,j} \leq y_{i,j} \leq \overline{d}_{i,j}, \ \forall\, i = A,B,C, \ j = 1,2,3,4.$
- $i_{i,j}^{+} - i_{i,j}^{-} = i_{i,j-1}^{+} + x_{i,j} - y_{i,j}, \ \forall\, i = A,B,C, \ j = 1,2,3,4.$

2.2.5 Sign Restrictions of Question (a)

Obviously, all the decision variables should be nonnegative, so

$$x_{i,j}, y_{i,j}, i_{i,j}^{+}, i_{i,j}^{-} \geq 0, \ i = A,B,C, \ j = 1,2,3,4. \tag{21}$$

2.2.6 Results of Question (b)

By the use of the solver LINGO, we could easily obtain the results. The arrangement of the company is in the table 23, 24.

Table 21: The Production Arrangment in each period

	1	2	3	4
A	70.8333	50	50	75
B	100	100	110.2273	89.77273
C	300	250	250	400

Table 22: The Sale Arrangment in each period

	1	2	3	4
A	100	50	50	75
B	100	100	100	100
C	300	250	250	400

The prodct A in period 1 has 29.16667 ammount backorder, and the product B in period 3 has 10.22727 amount inventory. The total profit under this arrangement is $ 123407.2.

By the analysis of the sensitivity report from LINGO, we could know that the constraint of capacity of workstaion 1 in period 1, the minimum demand of product B in period 1, the maximum demand of product B and C in period 1, the minimum demand of product B in period 2, the maximum demand of product A, B and C in period 2, the minimum demand of product B in period 3, the maximum demand of product A, B and C in period 3, the capacity of workstation 2 in period 4, the minimum demand of product B, C in period 4, the maximum demand of product A, B and C in period 4 are tight. Hence the profit of the company is primaryly restricted by the maximum demand.

2.2.7 Objective Function of Question (c)

What is different with problem (a) is the lost sale in problem (c). In this situation, we could not expand our sales by backorder. In contrast, if we cannot meet the maximum demand in current period, we have to face the loss from lost sale. Hence, the Objective function in problem (c) is

equation (22)

$$\max z = \sum_{i=A,B,C} \sum_{j=1}^{4} r_i y_{i,j} - 5 \sum_{i=A,B,C} \sum_{j=1}^{4} i_{i,j} - 5 \sum_{i=A,B,C} \sum_{j=1}^{3} (\bar{d}_{i,j} - y_{i,j}) \tag{22}$$

2.2.8 Constraints of Question (c)

The capacity constraints and the limits on demand are the same as the question (a). However, there is a slight difference in inventory and lose sale constraints. In this situation, the formulations should be 23.

$$i_{i,j} = i_{i,j-1} + x_{i,j} - y_{i,j}, \ \forall\, i = A, B, C, \ j = 1, 2, 3, 4. \tag{23}$$

and $i_{i,0}$ is agreed as 0 here.

2.2.9 Sign Restrictions of Question (c)

Obviously, all the decision variables should be nonnegative, so

$$x_{i,j}, y_{i,j}, i_{i,j} \geq 0, \ i = A, B, C, \ j = 1, 2, 3, 4. \tag{24}$$

2.2.10 The Resuts of Question (d)

By the use of the solver LINGO, we could easily obtain the results. The arrangement of the company is in the table 23, 24.

Table 23: The Production Arrangment in each period

	1	2	3	4
A	70.8333	50	50	75
B	100	100	110.2273	89.77273
C	300	250	250	400

Table 24: The Sale Arrangment in each period

	1	2	3	4
A	70.8333	50	50	75
B	100	100	100	100
C	300	250	250	400

The product B in period 3 has 10.22727 amount inventory. The total profit under this arrangement is $ 122094.7.

By the analysis of the sensitivity report from LINGO, we could know that the constraint of capacity of workstaion 1 in period 1, the minimum demand of product B in period 1, the maximum demand of product B and C in period 1, the minimum demand of product B in period 2, the maximum demand of product A, B and C in period 2, the minimum demand of product B in period 3, the maximum demand of product A, B and C in period 3, the capacity of workstation 2 in period 4, the minimum demand of product B, C in period 4, the maximum demand of product A, B and C in period 4 are tight. Hence the profit of the company is primaryly restricted by the maximum demand.

2.2.11 Some analysis of problem (e)

The result of (a)-(b) is similar with that of (c)-(d). Since the backorder of products, the first question could meet the maximum demand of product A. In consequence, the first question meets all maximum demands and large amounts of production capacity are waste. The company should broaden the market and make more advertisement. For the second question, since we cannot sell the production by backorder, we lose sale in product A of period 1. So the comapny could slightly add some labor time(70 hours by sensitivity analysis) on workstaion 1 to solve the lost sale. At other periods, the sales are saturated.

3 Problem 3. Worker scheduling

3.1 Question

The Gotham City Police Department employs 30 police officers. Each officer works 5 days per week. The crime rate fluctuates with the day of the week, so the number of police officers required each day is different: Monday (day 1), 17; Tuesday (day 2), 23; Wednesday, 25; Thursday, 16; Friday, 21; Saturday, 28; Sunday (day 7), 18. The police department wants to schedule police officers to minimize the number whose days off are not consecutive. Formulate an LP that will accomplish this goal.

3.2 Solution:

3.2.1 Notion Explanation

The notions used in this problem are list in the table 25, and its detailed meanings are shown together as well.

Table 25: Notion Explanation in Problem 3

Notion	Explanation
p_{ij}	the number of police officers whose days off are day i and day j

3.2.2 Objective Function

According to the question describition, we are expected to minimize the number whose days off are not consecutive. Hence, the objective function has the formulation below.

$$\min z = p_{13} + p_{14} + p_{15} + p_{16} + p_{24} + p_{25} + p_{26} + p_{27} + p_{35} + p_{36} + p_{37} + p_{46} + p_{47} + p_{57}. \quad (25)$$

3.2.3 Constraints

Obviously, we needs to meet the demand for police officers each days. In details, we are supposed to schedule at most 13 officers to rest on Monday:

$$p_{12} + p_{13} + p_{14} + p_{15} + p_{16} + p_{17} \leq 13; \quad (26)$$

on Tuesday, we are supposed to schedule at most 7 officers to rest:

$$p_{12} + p_{23} + p_{24} + p_{25} + p_{26} + p_{27} \leq 7; \quad (27)$$

on Wednesday, we are supposed to schedule at most 5 officers to rest:

$$p_{13} + p_{23} + p_{34} + p_{35} + p_{36} + p_{37} \leq 5; \quad (28)$$

on Thursday, we are supposed to schedule at most 14 officers to rest:

$$p_{14} + p_{24} + p_{34} + p_{45} + p_{46} + p_{47} \leq 14; \quad (29)$$

on Friday, we are supposed to schedule at most 9 officers to rest:

$$p_{15} + p_{25} + p_{35} + p_{45} + p_{56} + p_{57} \leq 9; \quad (30)$$

on Saturday, we are supposed to schedule at most 2 officers to rest:

$$p_{16} + p_{26} + p_{36} + p_{46} + p_{56} + p_{67} \leq 2; \tag{31}$$

on Sunday, we are supposed to schedule at most 12 officers to rest:

$$p_{17} + p_{27} + p_{37} + p_{47} + p_{57} + p_{67} \leq 12; \tag{32}$$

We should let all 30 police officers to rest two days a week:

$$\sum_{i=1\cdots7, j=1\cdots7, i<j} p_{ij} = 30. \tag{33}$$

In summary, the total constraints for the problem are list below:

- $p_{12} + p_{13} + p_{14} + p_{15} + p_{16} + p_{17} \leq 13$; (day 1)
- $p_{12} + p_{23} + p_{24} + p_{25} + p_{26} + p_{27} \leq 7$; (day 2)
- $p_{13} + p_{23} + p_{34} + p_{35} + p_{36} + p_{37} \leq 5$; (day 3)
- $p_{14} + p_{24} + p_{34} + p_{45} + p_{46} + p_{47} \leq 14$; (day 4)
- $p_{15} + p_{25} + p_{35} + p_{45} + p_{56} + p_{57} \leq 9$; (day 5)
- $p_{16} + p_{26} + p_{36} + p_{46} + p_{56} + p_{67} \leq 2$; (day 6)
- $p_{17} + p_{27} + p_{37} + p_{47} + p_{57} + p_{67} \leq 12$; (day 7)
- $\sum_{i=1\cdots7, j=1\cdots7, i<j} p_{ij} = 30$. (every police officer rest 2 day a week)

3.2.4 Sign Restrictions

Since the decision variables —number of the police officers— p_{ij} should be nonnegative, so we could obtain sign restrictions as below: $p_{ij} \geq 0, i, j = 1...7, i < j$.

3.3 Result

By the use of Lingo, we could easily obtain the result of the LP problem, which is shown in table 26.

Table 26: The result of Problem 3

Days arranged to rest	Number
p_{47}	1
p_{12}	6
p_{17}	7
p_{23}	1
p_{34}	4
p_{45}	9
p_{67}	2

and at this situation, $z = 1$.

In another words, the Gotham City Police Department should arrange 6 police officers to rest in day 1 and day 2, 1 police officers to rest in day 2 and day 3, 4 police officers to rest in day 3 and day 4, 9 police officers to rest in day 4 and day 5, 2 police officers to rest in day 6 and day 7, and 1 police officers to rest in day 4 and day 7. Hence, just **1** police officer have no consecutive rest in total.

References

[1] Winston W L, Goldberg J B. Operations research: applications and algorithms[M]. Belmont: Thomson Brooks/Cole, 2004.

[2] Hopp W J, Spearman M L. Factory physics[M]. Waveland Press, 2011.

[3] Taha H A. Operations research: an introduction[M]. Pearson Educación, 2003.

Appendix

A Problem 1

A. I LINGO Code

```
sets:
period;
company;
product;
info(period,product,company):labortime, cost,revenue,ad, inventory,packtime,
mindemand,maxdemand,graphite,x,y,i;
assemble(period,company):num1;
totalassemble(period,company):num2;
totallabor(period,company):num3;
totalpack(period,company):num4;
endsets

data:
period = 1 2;
company = 1 2 3;
product = 1 2 3 4 5 6 7 8 9;

cost = 6 5 7 19 18 20 4 5 5
10.00 11.00 12.00 26.00 24.00 27.00
30.00 27.00 3.00 32.00 33.00 18.00
33.00 43.00 24.00 55.00 79.00 59.00
6.72 5.60 7.84 21.28 20.16 22.40
4.48 5.60 5.60 11.20 12.32 13.44
29.12 26.88 30.24 33.60 30.24 3.36
35.84 36.96 20.16 36.96 48.16 26.88
61.60 88.48 66.08;

labortime =
1.00 3.50 3.00 1.50 3.50 3.50
1.50 4.50 4.00 3.00 4.50 4.50
4.00 5.00 5.50 0 0 0
0 0 0 0 0 0
0 0 0 1.00 3.50 3.00
1.50 3.50 3.50 1.50 4.50 4.00
3.00 4.50 4.50 4.00 5.00 5.50
0 0 0 0 0 0
```

```
0       0       0       0       0       0;

revenue=
10.00   10.00   12.00   25.00   25.00   30.00
8.00    8.00    10.00   18.00   18.00   22.00
40.00   40.00   45.00   290.00  290.00  310.00
380.00  380.00  420.00  560.00  560.00  640.00
650.00  650.00  720.00  10.00   10.00   12.00
25.00   25.00   30.00   8.00    8.00    10.00
18.00   18.00   22.00   40.00   40.00   45.00
290.00  290.00  310.00  380.00  380.00  420.00
560.00  560.00  640.00  650.00  650.00  720.00;

ad=
1.00    1.10    1.30    1.50    1.10    1.30
1.10    1.10    1.30    1.50    1.20    1.30
1.90    1.90    1.90    0       0       0
0       0       0       0       0       0
0       0       0       1.00    1.10    1.30
1.50    1.10    1.30    1.10    1.10    1.30
1.50    1.20    1.30    1.90    1.90    1.90
0       0       0       0       0       0
0       0       0       0       0       0;

inventory=
 0.48    0.40    0.56    1.52    1.44    1.60
0.32    0.40    0.40    0.80    0.88    0.96
2.08    1.92    2.16    14.24   14.00   14.40
18.24   17.60   19.20   28.00   28.80   29.60
33.60   34.80   36.00   0.54    0.45    0.63
1.70    1.61    1.79    0.36    0.45    0.45
0.90    0.99    1.08    2.33    2.15    2.42
15.95   15.68   16.13   20.43   19.71   21.50
31.36   32.26   33.15   37.63   38.98   40.32;

packtime=
4.00    7.00    7.50    4.00    7.00    7.50
5.00    8.00    8.50    6.00    9.00    9.50
6.00    7.00    8.00    0       0       0
0       0       0       0       0       0
0       0       0       4.00    7.00    7.50
4.00    7.00    7.50    5.00    8.00    8.50
6.00    9.00    9.50    6.00    7.00    8.00
0       0       0       0       0       0
0       0       0       0       0       0;
```

```
82
83  mindemand=
84  0              0              0          100.00         100.00         50.00
85  200.00         200.00         100.00      30.00          30.00         15.00
86  100.00         100.00         100.00          0              0             0
87  0              0              0           0              0             0
88  0              0              0           0              0             0
89  100.00         100.00          50.00      200.00         200.00        100.00
90  30.00          30.00          15.00       100.00         100.00        100.00
91  0              0              0           0              0             0
92  0              0              0           0              0             0;
93
94  maxdemand=
95   2000.00        2000.00        2000.00     2000.00        2000.00       2000.00       2000.00
96  2000.00        2000.00        2000.00     2000.00        2000.00       2000.00       2000.00
97  2000.00         200.00         200.00      200.00         100.00        100.00        100.00
98  300.00         300.00         300.00      400.00         400.00        400.00        2000.00
99  2000.00        2000.00        2000.00     2000.00        2000.00       2000.00       2000.00
100 2000.00        2000.00        2000.00     2000.00        2000.00       2000.00       2000.00
101 200.00         200.00         200.00      100.00         100.00        100.00        300.00
102 300.00         300.00         400.00      400.00         400.00;
103
104  graphite =
105 0              0              0          0.25           0.25          0.25          0
106 0              0              0           0              0             0             0
107 0              0              0           0              0             0             0
108 0              0              0           0              0             0             0
109 0              0             0.25        0.25           0.25          0             0
110 0              0              0           0              0             0             0
111 0              0              0           0              0             0             0
112 0              0              0           0              0;
113
114 num1= 65 60 65 65 60  65;
115 num2= 5500  5000 6000  5500 5000 6000;
116 num3= 12000 15000 22000 12000 15000 22000;
117 num4= 20000  40000 35000 20000 40000 35000;
118 enddata;
119
120 max= @sum(info(a,b,c):revenue(a,b,c)*y(a,b,c)-cost(a,b,c)*x(a,b,c)-inventory(a,b,c)*i(a,b,c));
121 @for(period(a):@for(company(b):@sum(product(c):labortime(a,c,b)*x(a,c,b))<num3(a,b)));
122 @for(period(a):@for(company(b):@sum(product(c):packtime(a,c,b)*x(a,c,b))<num4(a,b)));
123 @for(period(a):@for(company(b):num1(a,b)*(x(a,6,b)+x(a,7,b)+x(a,8,b)+x(a,9,b))< num2(a,b)));
124 @for(period(a):@for(company(b):@for(product(c):mindemand(a,c,b)<y(a,c,b))));
125 @for(period(a):@for(company(b):@for(product(c):maxdemand(a,c,b)*1.5>y(a,c,b))));
126 @for(company(b):x(1,1,b)-y(1,1,b)-13*x(1,6,b)-13*x(1,7,b)=i(1,1,b));
```

```
@for(company(b):x(1,2,b)-y(1,2,b)-13*x(1,8,b)-13*x(1,9,b)=i(1,2,b));
@for(company(b):x(1,3,b)-y(1,3,b)-10*x(1,6,b)-10*x(1,7,b)-10*x(1,8,b)-10*x(1,9,b)=i(1,3,b));
@for(company(b):x(1,4,b)-y(1,4,b)-3*x(1,6,b)-3*x(1,8,b)=i(1,4,b));
@for(company(b):x(1,5,b)-y(1,5,b)-3*x(1,7,b)-3*x(1,9,b)=i(1,5,b));
@for(company(b):x(1,6,b)-y(1,6,b)=i(1,6,b));
@for(company(b):x(1,7,b)-y(1,7,b)=i(1,7,b));
@for(company(b):x(1,8,b)-y(1,8,b)=i(1,8,b));
@for(company(b):x(1,9,b)-y(1,9,b)=i(1,9,b));

@for(company(b):x(2,1,b)-y(2,1,b)-13*x(2,6,b)-13*x(2,7,b)+i(1,1,b)=i(2,1,b));
@for(company(b):x(2,2,b)-y(2,2,b)-13*x(2,8,b)-13*x(2,9,b)+i(1,2,b)=i(2,2,b));
@for(company(b):x(2,3,b)-y(2,3,b)-10*x(2,6,b)-10*x(2,7,b)-10*x(2,8,b)-
10*x(2,9,b)+i(1,3,b)=i(2,3,b));
@for(company(b):x(2,4,b)-y(2,4,b)-3*x(2,6,b)-3*x(2,8,b)+i(1,4,b)=i(2,4,b));
@for(company(b):x(2,5,b)-y(2,5,b)-3*x(2,7,b)-3*x(2,9,b)+i(1,5,b)=i(2,5,b));
@for(company(b):x(2,6,b)-y(2,6,b)+i(1,6,b)=i(2,6,b));
@for(company(b):x(2,7,b)-y(2,7,b)+i(1,7,b)=i(2,7,b));
@for(company(b):x(2,8,b)-y(2,8,b)+i(1,8,b)=i(2,8,b));
@for(company(b):x(2,9,b)-y(2,9,b)+i(1,9,b)=i(2,9,b));

@for(period(a):@sum(company(b):@sum(product(c):ad(a,c,b)*x(a,c,b)))<20000);
@sum(company(b):@sum(product(c):graphite(1,c,b)*x(1,c,b)))<1000;
@sum(company(b):@sum(product(c):graphite(2,c,b)*x(2,c,b)))
<1000+1000-@sum(company(b):@sum(product(c):graphite(1,c,b)*x(1,c,b)));
end model;
```

A. II Solution Report

```
LINGO/OSX64 19.0.46 (1 Sep 2021 ), LINDO API 13.0.4099.299

Licensee info: chengzijie22@mails.ucas.ac.cn
License expires: 23 JAN 2023

Global optimal solution found.
Objective value:                              258326.8
  Infeasibilities:                            0.000000
Total solver iterations:                            66
Elapsed runtime seconds:                          0.17

Model Class:                                        LP

Total variables:                  162
```

Nonlinear variables:	0
Integer variables:	0
Total constraints:	185
Nonlinear constraints:	0
Total nonzeros:	654
Nonlinear nonzeros:	0

Variable	Value	Reduced Cost
LABORTIME(1, 1, 1)	1.000000	0.000000
LABORTIME(1, 1, 2)	3.500000	0.000000
LABORTIME(1, 1, 3)	3.000000	0.000000
LABORTIME(1, 2, 1)	1.500000	0.000000
LABORTIME(1, 2, 2)	3.500000	0.000000
LABORTIME(1, 2, 3)	3.500000	0.000000
LABORTIME(1, 3, 1)	1.500000	0.000000
LABORTIME(1, 3, 2)	4.500000	0.000000
LABORTIME(1, 3, 3)	4.000000	0.000000
LABORTIME(1, 4, 1)	3.000000	0.000000
LABORTIME(1, 4, 2)	4.500000	0.000000
LABORTIME(1, 4, 3)	4.500000	0.000000
LABORTIME(1, 5, 1)	4.000000	0.000000
LABORTIME(1, 5, 2)	5.000000	0.000000
LABORTIME(1, 5, 3)	5.500000	0.000000
LABORTIME(1, 6, 1)	0.000000	0.000000
LABORTIME(1, 6, 2)	0.000000	0.000000
LABORTIME(1, 6, 3)	0.000000	0.000000
LABORTIME(1, 7, 1)	0.000000	0.000000
LABORTIME(1, 7, 2)	0.000000	0.000000
LABORTIME(1, 7, 3)	0.000000	0.000000
LABORTIME(1, 8, 1)	0.000000	0.000000
LABORTIME(1, 8, 2)	0.000000	0.000000
LABORTIME(1, 8, 3)	0.000000	0.000000
LABORTIME(1, 9, 1)	0.000000	0.000000
LABORTIME(1, 9, 2)	0.000000	0.000000
LABORTIME(1, 9, 3)	0.000000	0.000000
LABORTIME(2, 1, 1)	1.000000	0.000000
LABORTIME(2, 1, 2)	3.500000	0.000000
LABORTIME(2, 1, 3)	3.000000	0.000000
LABORTIME(2, 2, 1)	1.500000	0.000000
LABORTIME(2, 2, 2)	3.500000	0.000000
LABORTIME(2, 2, 3)	3.500000	0.000000

60	LABORTIME(2, 3, 1)	1.500000	0.000000
61	LABORTIME(2, 3, 2)	4.500000	0.000000
62	LABORTIME(2, 3, 3)	4.000000	0.000000
63	LABORTIME(2, 4, 1)	3.000000	0.000000
64	LABORTIME(2, 4, 2)	4.500000	0.000000
65	LABORTIME(2, 4, 3)	4.500000	0.000000
66	LABORTIME(2, 5, 1)	4.000000	0.000000
67	LABORTIME(2, 5, 2)	5.000000	0.000000
68	LABORTIME(2, 5, 3)	5.500000	0.000000
69	LABORTIME(2, 6, 1)	0.000000	0.000000
70	LABORTIME(2, 6, 2)	0.000000	0.000000
71	LABORTIME(2, 6, 3)	0.000000	0.000000
72	LABORTIME(2, 7, 1)	0.000000	0.000000
73	LABORTIME(2, 7, 2)	0.000000	0.000000
74	LABORTIME(2, 7, 3)	0.000000	0.000000
75	LABORTIME(2, 8, 1)	0.000000	0.000000
76	LABORTIME(2, 8, 2)	0.000000	0.000000
77	LABORTIME(2, 8, 3)	0.000000	0.000000
78	LABORTIME(2, 9, 1)	0.000000	0.000000
79	LABORTIME(2, 9, 2)	0.000000	0.000000
80	LABORTIME(2, 9, 3)	0.000000	0.000000
81	COST(1, 1, 1)	6.000000	0.000000
82	COST(1, 1, 2)	5.000000	0.000000
83	COST(1, 1, 3)	7.000000	0.000000
84	COST(1, 2, 1)	19.00000	0.000000
85	COST(1, 2, 2)	18.00000	0.000000
86	COST(1, 2, 3)	20.00000	0.000000
87	COST(1, 3, 1)	4.000000	0.000000
88	COST(1, 3, 2)	5.000000	0.000000
89	COST(1, 3, 3)	5.000000	0.000000
90	COST(1, 4, 1)	10.00000	0.000000
91	COST(1, 4, 2)	11.00000	0.000000
92	COST(1, 4, 3)	12.00000	0.000000
93	COST(1, 5, 1)	26.00000	0.000000
94	COST(1, 5, 2)	24.00000	0.000000
95	COST(1, 5, 3)	27.00000	0.000000
96	COST(1, 6, 1)	30.00000	0.000000
97	COST(1, 6, 2)	27.00000	0.000000
98	COST(1, 6, 3)	3.000000	0.000000
99	COST(1, 7, 1)	32.00000	0.000000
100	COST(1, 7, 2)	33.00000	0.000000
101	COST(1, 7, 3)	18.00000	0.000000
102	COST(1, 8, 1)	33.00000	0.000000
103	COST(1, 8, 2)	43.00000	0.000000
104	COST(1, 8, 3)	24.00000	0.000000

105	COST(1, 9, 1)	55.00000	0.000000
106	COST(1, 9, 2)	79.00000	0.000000
107	COST(1, 9, 3)	59.00000	0.000000
108	COST(2, 1, 1)	6.720000	0.000000
109	COST(2, 1, 2)	5.600000	0.000000
110	COST(2, 1, 3)	7.840000	0.000000
111	COST(2, 2, 1)	21.28000	0.000000
112	COST(2, 2, 2)	20.16000	0.000000
113	COST(2, 2, 3)	22.40000	0.000000
114	COST(2, 3, 1)	4.480000	0.000000
115	COST(2, 3, 2)	5.600000	0.000000
116	COST(2, 3, 3)	5.600000	0.000000
117	COST(2, 4, 1)	11.20000	0.000000
118	COST(2, 4, 2)	12.32000	0.000000
119	COST(2, 4, 3)	13.44000	0.000000
120	COST(2, 5, 1)	29.12000	0.000000
121	COST(2, 5, 2)	26.88000	0.000000
122	COST(2, 5, 3)	30.24000	0.000000
123	COST(2, 6, 1)	33.60000	0.000000
124	COST(2, 6, 2)	30.24000	0.000000
125	COST(2, 6, 3)	3.360000	0.000000
126	COST(2, 7, 1)	35.84000	0.000000
127	COST(2, 7, 2)	36.96000	0.000000
128	COST(2, 7, 3)	20.16000	0.000000
129	COST(2, 8, 1)	36.96000	0.000000
130	COST(2, 8, 2)	48.16000	0.000000
131	COST(2, 8, 3)	26.88000	0.000000
132	COST(2, 9, 1)	61.60000	0.000000
133	COST(2, 9, 2)	88.48000	0.000000
134	COST(2, 9, 3)	66.08000	0.000000
135	REVENUE(1, 1, 1)	10.00000	0.000000
136	REVENUE(1, 1, 2)	10.00000	0.000000
137	REVENUE(1, 1, 3)	12.00000	0.000000
138	REVENUE(1, 2, 1)	25.00000	0.000000
139	REVENUE(1, 2, 2)	25.00000	0.000000
140	REVENUE(1, 2, 3)	30.00000	0.000000
141	REVENUE(1, 3, 1)	8.000000	0.000000
142	REVENUE(1, 3, 2)	8.000000	0.000000
143	REVENUE(1, 3, 3)	10.00000	0.000000
144	REVENUE(1, 4, 1)	18.00000	0.000000
145	REVENUE(1, 4, 2)	18.00000	0.000000
146	REVENUE(1, 4, 3)	22.00000	0.000000
147	REVENUE(1, 5, 1)	40.00000	0.000000
148	REVENUE(1, 5, 2)	40.00000	0.000000
149	REVENUE(1, 5, 3)	45.00000	0.000000

150	REVENUE(1, 6, 1)	290.0000	0.000000
151	REVENUE(1, 6, 2)	290.0000	0.000000
152	REVENUE(1, 6, 3)	310.0000	0.000000
153	REVENUE(1, 7, 1)	380.0000	0.000000
154	REVENUE(1, 7, 2)	380.0000	0.000000
155	REVENUE(1, 7, 3)	420.0000	0.000000
156	REVENUE(1, 8, 1)	560.0000	0.000000
157	REVENUE(1, 8, 2)	560.0000	0.000000
158	REVENUE(1, 8, 3)	640.0000	0.000000
159	REVENUE(1, 9, 1)	650.0000	0.000000
160	REVENUE(1, 9, 2)	650.0000	0.000000
161	REVENUE(1, 9, 3)	720.0000	0.000000
162	REVENUE(2, 1, 1)	10.00000	0.000000
163	REVENUE(2, 1, 2)	10.00000	0.000000
164	REVENUE(2, 1, 3)	12.00000	0.000000
165	REVENUE(2, 2, 1)	25.00000	0.000000
166	REVENUE(2, 2, 2)	25.00000	0.000000
167	REVENUE(2, 2, 3)	30.00000	0.000000
168	REVENUE(2, 3, 1)	8.000000	0.000000
169	REVENUE(2, 3, 2)	8.000000	0.000000
170	REVENUE(2, 3, 3)	10.00000	0.000000
171	REVENUE(2, 4, 1)	18.00000	0.000000
172	REVENUE(2, 4, 2)	18.00000	0.000000
173	REVENUE(2, 4, 3)	22.00000	0.000000
174	REVENUE(2, 5, 1)	40.00000	0.000000
175	REVENUE(2, 5, 2)	40.00000	0.000000
176	REVENUE(2, 5, 3)	45.00000	0.000000
177	REVENUE(2, 6, 1)	290.0000	0.000000
178	REVENUE(2, 6, 2)	290.0000	0.000000
179	REVENUE(2, 6, 3)	310.0000	0.000000
180	REVENUE(2, 7, 1)	380.0000	0.000000
181	REVENUE(2, 7, 2)	380.0000	0.000000
182	REVENUE(2, 7, 3)	420.0000	0.000000
183	REVENUE(2, 8, 1)	560.0000	0.000000
184	REVENUE(2, 8, 2)	560.0000	0.000000
185	REVENUE(2, 8, 3)	640.0000	0.000000
186	REVENUE(2, 9, 1)	650.0000	0.000000
187	REVENUE(2, 9, 2)	650.0000	0.000000
188	REVENUE(2, 9, 3)	720.0000	0.000000
189	AD(1, 1, 1)	1.000000	0.000000
190	AD(1, 1, 2)	1.100000	0.000000
191	AD(1, 1, 3)	1.300000	0.000000
192	AD(1, 2, 1)	1.500000	0.000000
193	AD(1, 2, 2)	1.100000	0.000000
194	AD(1, 2, 3)	1.300000	0.000000

195	AD(1, 3, 1)	1.100000	0.000000
196	AD(1, 3, 2)	1.100000	0.000000
197	AD(1, 3, 3)	1.300000	0.000000
198	AD(1, 4, 1)	1.500000	0.000000
199	AD(1, 4, 2)	1.200000	0.000000
200	AD(1, 4, 3)	1.300000	0.000000
201	AD(1, 5, 1)	1.900000	0.000000
202	AD(1, 5, 2)	1.900000	0.000000
203	AD(1, 5, 3)	1.900000	0.000000
204	AD(1, 6, 1)	0.000000	0.000000
205	AD(1, 6, 2)	0.000000	0.000000
206	AD(1, 6, 3)	0.000000	0.000000
207	AD(1, 7, 1)	0.000000	0.000000
208	AD(1, 7, 2)	0.000000	0.000000
209	AD(1, 7, 3)	0.000000	0.000000
210	AD(1, 8, 1)	0.000000	0.000000
211	AD(1, 8, 2)	0.000000	0.000000
212	AD(1, 8, 3)	0.000000	0.000000
213	AD(1, 9, 1)	0.000000	0.000000
214	AD(1, 9, 2)	0.000000	0.000000
215	AD(1, 9, 3)	0.000000	0.000000
216	AD(2, 1, 1)	1.000000	0.000000
217	AD(2, 1, 2)	1.100000	0.000000
218	AD(2, 1, 3)	1.300000	0.000000
219	AD(2, 2, 1)	1.500000	0.000000
220	AD(2, 2, 2)	1.100000	0.000000
221	AD(2, 2, 3)	1.300000	0.000000
222	AD(2, 3, 1)	1.100000	0.000000
223	AD(2, 3, 2)	1.100000	0.000000
224	AD(2, 3, 3)	1.300000	0.000000
225	AD(2, 4, 1)	1.500000	0.000000
226	AD(2, 4, 2)	1.200000	0.000000
227	AD(2, 4, 3)	1.300000	0.000000
228	AD(2, 5, 1)	1.900000	0.000000
229	AD(2, 5, 2)	1.900000	0.000000
230	AD(2, 5, 3)	1.900000	0.000000
231	AD(2, 6, 1)	0.000000	0.000000
232	AD(2, 6, 2)	0.000000	0.000000
233	AD(2, 6, 3)	0.000000	0.000000
234	AD(2, 7, 1)	0.000000	0.000000
235	AD(2, 7, 2)	0.000000	0.000000
236	AD(2, 7, 3)	0.000000	0.000000
237	AD(2, 8, 1)	0.000000	0.000000
238	AD(2, 8, 2)	0.000000	0.000000
239	AD(2, 8, 3)	0.000000	0.000000

AD(2, 9, 1)	0.000000	0.000000	
AD(2, 9, 2)	0.000000	0.000000	
AD(2, 9, 3)	0.000000	0.000000	
INVENTORY(1, 1, 1)	0.4800000	0.000000	
INVENTORY(1, 1, 2)	0.4000000	0.000000	
INVENTORY(1, 1, 3)	0.5600000	0.000000	
INVENTORY(1, 2, 1)	1.520000	0.000000	
INVENTORY(1, 2, 2)	1.440000	0.000000	
INVENTORY(1, 2, 3)	1.600000	0.000000	
INVENTORY(1, 3, 1)	0.3200000	0.000000	
INVENTORY(1, 3, 2)	0.4000000	0.000000	
INVENTORY(1, 3, 3)	0.4000000	0.000000	
INVENTORY(1, 4, 1)	0.8000000	0.000000	
INVENTORY(1, 4, 2)	0.8800000	0.000000	
INVENTORY(1, 4, 3)	0.9600000	0.000000	
INVENTORY(1, 5, 1)	2.080000	0.000000	
INVENTORY(1, 5, 2)	1.920000	0.000000	
INVENTORY(1, 5, 3)	2.160000	0.000000	
INVENTORY(1, 6, 1)	14.24000	0.000000	
INVENTORY(1, 6, 2)	14.00000	0.000000	
INVENTORY(1, 6, 3)	14.40000	0.000000	
INVENTORY(1, 7, 1)	18.24000	0.000000	
INVENTORY(1, 7, 2)	17.60000	0.000000	
INVENTORY(1, 7, 3)	19.20000	0.000000	
INVENTORY(1, 8, 1)	28.00000	0.000000	
INVENTORY(1, 8, 2)	28.80000	0.000000	
INVENTORY(1, 8, 3)	29.60000	0.000000	
INVENTORY(1, 9, 1)	33.60000	0.000000	
INVENTORY(1, 9, 2)	34.80000	0.000000	
INVENTORY(1, 9, 3)	36.00000	0.000000	
INVENTORY(2, 1, 1)	0.5400000	0.000000	
INVENTORY(2, 1, 2)	0.4500000	0.000000	
INVENTORY(2, 1, 3)	0.6300000	0.000000	
INVENTORY(2, 2, 1)	1.700000	0.000000	
INVENTORY(2, 2, 2)	1.610000	0.000000	
INVENTORY(2, 2, 3)	1.790000	0.000000	
INVENTORY(2, 3, 1)	0.3600000	0.000000	
INVENTORY(2, 3, 2)	0.4500000	0.000000	
INVENTORY(2, 3, 3)	0.4500000	0.000000	
INVENTORY(2, 4, 1)	0.9000000	0.000000	
INVENTORY(2, 4, 2)	0.9900000	0.000000	
INVENTORY(2, 4, 3)	1.080000	0.000000	
INVENTORY(2, 5, 1)	2.330000	0.000000	
INVENTORY(2, 5, 2)	2.150000	0.000000	
INVENTORY(2, 5, 3)	2.420000	0.000000	

285	INVENTORY(2, 6, 1)	15.95000	0.000000
286	INVENTORY(2, 6, 2)	15.68000	0.000000
287	INVENTORY(2, 6, 3)	16.13000	0.000000
288	INVENTORY(2, 7, 1)	20.43000	0.000000
289	INVENTORY(2, 7, 2)	19.71000	0.000000
290	INVENTORY(2, 7, 3)	21.50000	0.000000
291	INVENTORY(2, 8, 1)	31.36000	0.000000
292	INVENTORY(2, 8, 2)	32.26000	0.000000
293	INVENTORY(2, 8, 3)	33.15000	0.000000
294	INVENTORY(2, 9, 1)	37.63000	0.000000
295	INVENTORY(2, 9, 2)	38.98000	0.000000
296	INVENTORY(2, 9, 3)	40.32000	0.000000
297	PACKTIME(1, 1, 1)	4.000000	0.000000
298	PACKTIME(1, 1, 2)	7.000000	0.000000
299	PACKTIME(1, 1, 3)	7.500000	0.000000
300	PACKTIME(1, 2, 1)	4.000000	0.000000
301	PACKTIME(1, 2, 2)	7.000000	0.000000
302	PACKTIME(1, 2, 3)	7.500000	0.000000
303	PACKTIME(1, 3, 1)	5.000000	0.000000
304	PACKTIME(1, 3, 2)	8.000000	0.000000
305	PACKTIME(1, 3, 3)	8.500000	0.000000
306	PACKTIME(1, 4, 1)	6.000000	0.000000
307	PACKTIME(1, 4, 2)	9.000000	0.000000
308	PACKTIME(1, 4, 3)	9.500000	0.000000
309	PACKTIME(1, 5, 1)	6.000000	0.000000
310	PACKTIME(1, 5, 2)	7.000000	0.000000
311	PACKTIME(1, 5, 3)	8.000000	0.000000
312	PACKTIME(1, 6, 1)	0.000000	0.000000
313	PACKTIME(1, 6, 2)	0.000000	0.000000
314	PACKTIME(1, 6, 3)	0.000000	0.000000
315	PACKTIME(1, 7, 1)	0.000000	0.000000
316	PACKTIME(1, 7, 2)	0.000000	0.000000
317	PACKTIME(1, 7, 3)	0.000000	0.000000
318	PACKTIME(1, 8, 1)	0.000000	0.000000
319	PACKTIME(1, 8, 2)	0.000000	0.000000
320	PACKTIME(1, 8, 3)	0.000000	0.000000
321	PACKTIME(1, 9, 1)	0.000000	0.000000
322	PACKTIME(1, 9, 2)	0.000000	0.000000
323	PACKTIME(1, 9, 3)	0.000000	0.000000
324	PACKTIME(2, 1, 1)	4.000000	0.000000
325	PACKTIME(2, 1, 2)	7.000000	0.000000
326	PACKTIME(2, 1, 3)	7.500000	0.000000
327	PACKTIME(2, 2, 1)	4.000000	0.000000
328	PACKTIME(2, 2, 2)	7.000000	0.000000
329	PACKTIME(2, 2, 3)	7.500000	0.000000

330	PACKTIME(2, 3, 1)	5.000000	0.000000
331	PACKTIME(2, 3, 2)	8.000000	0.000000
332	PACKTIME(2, 3, 3)	8.500000	0.000000
333	PACKTIME(2, 4, 1)	6.000000	0.000000
334	PACKTIME(2, 4, 2)	9.000000	0.000000
335	PACKTIME(2, 4, 3)	9.500000	0.000000
336	PACKTIME(2, 5, 1)	6.000000	0.000000
337	PACKTIME(2, 5, 2)	7.000000	0.000000
338	PACKTIME(2, 5, 3)	8.000000	0.000000
339	PACKTIME(2, 6, 1)	0.000000	0.000000
340	PACKTIME(2, 6, 2)	0.000000	0.000000
341	PACKTIME(2, 6, 3)	0.000000	0.000000
342	PACKTIME(2, 7, 1)	0.000000	0.000000
343	PACKTIME(2, 7, 2)	0.000000	0.000000
344	PACKTIME(2, 7, 3)	0.000000	0.000000
345	PACKTIME(2, 8, 1)	0.000000	0.000000
346	PACKTIME(2, 8, 2)	0.000000	0.000000
347	PACKTIME(2, 8, 3)	0.000000	0.000000
348	PACKTIME(2, 9, 1)	0.000000	0.000000
349	PACKTIME(2, 9, 2)	0.000000	0.000000
350	PACKTIME(2, 9, 3)	0.000000	0.000000
351	MINDEMAND(1, 1, 1)	0.000000	0.000000
352	MINDEMAND(1, 1, 2)	0.000000	0.000000
353	MINDEMAND(1, 1, 3)	0.000000	0.000000
354	MINDEMAND(1, 2, 1)	100.0000	0.000000
355	MINDEMAND(1, 2, 2)	100.0000	0.000000
356	MINDEMAND(1, 2, 3)	50.00000	0.000000
357	MINDEMAND(1, 3, 1)	200.0000	0.000000
358	MINDEMAND(1, 3, 2)	200.0000	0.000000
359	MINDEMAND(1, 3, 3)	100.0000	0.000000
360	MINDEMAND(1, 4, 1)	30.00000	0.000000
361	MINDEMAND(1, 4, 2)	30.00000	0.000000
362	MINDEMAND(1, 4, 3)	15.00000	0.000000
363	MINDEMAND(1, 5, 1)	100.0000	0.000000
364	MINDEMAND(1, 5, 2)	100.0000	0.000000
365	MINDEMAND(1, 5, 3)	100.0000	0.000000
366	MINDEMAND(1, 6, 1)	0.000000	0.000000
367	MINDEMAND(1, 6, 2)	0.000000	0.000000
368	MINDEMAND(1, 6, 3)	0.000000	0.000000
369	MINDEMAND(1, 7, 1)	0.000000	0.000000
370	MINDEMAND(1, 7, 2)	0.000000	0.000000
371	MINDEMAND(1, 7, 3)	0.000000	0.000000
372	MINDEMAND(1, 8, 1)	0.000000	0.000000
373	MINDEMAND(1, 8, 2)	0.000000	0.000000
374	MINDEMAND(1, 8, 3)	0.000000	0.000000

375	MINDEMAND(1, 9, 1)	0.000000	0.000000
376	MINDEMAND(1, 9, 2)	0.000000	0.000000
377	MINDEMAND(1, 9, 3)	0.000000	0.000000
378	MINDEMAND(2, 1, 1)	0.000000	0.000000
379	MINDEMAND(2, 1, 2)	0.000000	0.000000
380	MINDEMAND(2, 1, 3)	0.000000	0.000000
381	MINDEMAND(2, 2, 1)	100.0000	0.000000
382	MINDEMAND(2, 2, 2)	100.0000	0.000000
383	MINDEMAND(2, 2, 3)	50.00000	0.000000
384	MINDEMAND(2, 3, 1)	200.0000	0.000000
385	MINDEMAND(2, 3, 2)	200.0000	0.000000
386	MINDEMAND(2, 3, 3)	100.0000	0.000000
387	MINDEMAND(2, 4, 1)	30.00000	0.000000
388	MINDEMAND(2, 4, 2)	30.00000	0.000000
389	MINDEMAND(2, 4, 3)	15.00000	0.000000
390	MINDEMAND(2, 5, 1)	100.0000	0.000000
391	MINDEMAND(2, 5, 2)	100.0000	0.000000
392	MINDEMAND(2, 5, 3)	100.0000	0.000000
393	MINDEMAND(2, 6, 1)	0.000000	0.000000
394	MINDEMAND(2, 6, 2)	0.000000	0.000000
395	MINDEMAND(2, 6, 3)	0.000000	0.000000
396	MINDEMAND(2, 7, 1)	0.000000	0.000000
397	MINDEMAND(2, 7, 2)	0.000000	0.000000
398	MINDEMAND(2, 7, 3)	0.000000	0.000000
399	MINDEMAND(2, 8, 1)	0.000000	0.000000
400	MINDEMAND(2, 8, 2)	0.000000	0.000000
401	MINDEMAND(2, 8, 3)	0.000000	0.000000
402	MINDEMAND(2, 9, 1)	0.000000	0.000000
403	MINDEMAND(2, 9, 2)	0.000000	0.000000
404	MINDEMAND(2, 9, 3)	0.000000	0.000000
405	MAXDEMAND(1, 1, 1)	2000.000	0.000000
406	MAXDEMAND(1, 1, 2)	2000.000	0.000000
407	MAXDEMAND(1, 1, 3)	2000.000	0.000000
408	MAXDEMAND(1, 2, 1)	2000.000	0.000000
409	MAXDEMAND(1, 2, 2)	2000.000	0.000000
410	MAXDEMAND(1, 2, 3)	2000.000	0.000000
411	MAXDEMAND(1, 3, 1)	2000.000	0.000000
412	MAXDEMAND(1, 3, 2)	2000.000	0.000000
413	MAXDEMAND(1, 3, 3)	2000.000	0.000000
414	MAXDEMAND(1, 4, 1)	2000.000	0.000000
415	MAXDEMAND(1, 4, 2)	2000.000	0.000000
416	MAXDEMAND(1, 4, 3)	2000.000	0.000000
417	MAXDEMAND(1, 5, 1)	2000.000	0.000000
418	MAXDEMAND(1, 5, 2)	2000.000	0.000000
419	MAXDEMAND(1, 5, 3)	2000.000	0.000000

420	MAXDEMAND(1, 6, 1)	200.0000	0.000000
421	MAXDEMAND(1, 6, 2)	200.0000	0.000000
422	MAXDEMAND(1, 6, 3)	200.0000	0.000000
423	MAXDEMAND(1, 7, 1)	100.0000	0.000000
424	MAXDEMAND(1, 7, 2)	100.0000	0.000000
425	MAXDEMAND(1, 7, 3)	100.0000	0.000000
426	MAXDEMAND(1, 8, 1)	300.0000	0.000000
427	MAXDEMAND(1, 8, 2)	300.0000	0.000000
428	MAXDEMAND(1, 8, 3)	300.0000	0.000000
429	MAXDEMAND(1, 9, 1)	400.0000	0.000000
430	MAXDEMAND(1, 9, 2)	400.0000	0.000000
431	MAXDEMAND(1, 9, 3)	400.0000	0.000000
432	MAXDEMAND(2, 1, 1)	2000.000	0.000000
433	MAXDEMAND(2, 1, 2)	2000.000	0.000000
434	MAXDEMAND(2, 1, 3)	2000.000	0.000000
435	MAXDEMAND(2, 2, 1)	2000.000	0.000000
436	MAXDEMAND(2, 2, 2)	2000.000	0.000000
437	MAXDEMAND(2, 2, 3)	2000.000	0.000000
438	MAXDEMAND(2, 3, 1)	2000.000	0.000000
439	MAXDEMAND(2, 3, 2)	2000.000	0.000000
440	MAXDEMAND(2, 3, 3)	2000.000	0.000000
441	MAXDEMAND(2, 4, 1)	2000.000	0.000000
442	MAXDEMAND(2, 4, 2)	2000.000	0.000000
443	MAXDEMAND(2, 4, 3)	2000.000	0.000000
444	MAXDEMAND(2, 5, 1)	2000.000	0.000000
445	MAXDEMAND(2, 5, 2)	2000.000	0.000000
446	MAXDEMAND(2, 5, 3)	2000.000	0.000000
447	MAXDEMAND(2, 6, 1)	200.0000	0.000000
448	MAXDEMAND(2, 6, 2)	200.0000	0.000000
449	MAXDEMAND(2, 6, 3)	200.0000	0.000000
450	MAXDEMAND(2, 7, 1)	100.0000	0.000000
451	MAXDEMAND(2, 7, 2)	100.0000	0.000000
452	MAXDEMAND(2, 7, 3)	100.0000	0.000000
453	MAXDEMAND(2, 8, 1)	300.0000	0.000000
454	MAXDEMAND(2, 8, 2)	300.0000	0.000000
455	MAXDEMAND(2, 8, 3)	300.0000	0.000000
456	MAXDEMAND(2, 9, 1)	400.0000	0.000000
457	MAXDEMAND(2, 9, 2)	400.0000	0.000000
458	MAXDEMAND(2, 9, 3)	400.0000	0.000000
459	GRAPHITE(1, 1, 1)	0.000000	0.000000
460	GRAPHITE(1, 1, 2)	0.000000	0.000000
461	GRAPHITE(1, 1, 3)	0.000000	0.000000
462	GRAPHITE(1, 2, 1)	0.2500000	0.000000
463	GRAPHITE(1, 2, 2)	0.2500000	0.000000
464	GRAPHITE(1, 2, 3)	0.2500000	0.000000

465	GRAPHITE(1, 3, 1)	0.000000	0.000000
466	GRAPHITE(1, 3, 2)	0.000000	0.000000
467	GRAPHITE(1, 3, 3)	0.000000	0.000000
468	GRAPHITE(1, 4, 1)	0.000000	0.000000
469	GRAPHITE(1, 4, 2)	0.000000	0.000000
470	GRAPHITE(1, 4, 3)	0.000000	0.000000
471	GRAPHITE(1, 5, 1)	0.000000	0.000000
472	GRAPHITE(1, 5, 2)	0.000000	0.000000
473	GRAPHITE(1, 5, 3)	0.000000	0.000000
474	GRAPHITE(1, 6, 1)	0.000000	0.000000
475	GRAPHITE(1, 6, 2)	0.000000	0.000000
476	GRAPHITE(1, 6, 3)	0.000000	0.000000
477	GRAPHITE(1, 7, 1)	0.000000	0.000000
478	GRAPHITE(1, 7, 2)	0.000000	0.000000
479	GRAPHITE(1, 7, 3)	0.000000	0.000000
480	GRAPHITE(1, 8, 1)	0.000000	0.000000
481	GRAPHITE(1, 8, 2)	0.000000	0.000000
482	GRAPHITE(1, 8, 3)	0.000000	0.000000
483	GRAPHITE(1, 9, 1)	0.000000	0.000000
484	GRAPHITE(1, 9, 2)	0.000000	0.000000
485	GRAPHITE(1, 9, 3)	0.000000	0.000000
486	GRAPHITE(2, 1, 1)	0.000000	0.000000
487	GRAPHITE(2, 1, 2)	0.000000	0.000000
488	GRAPHITE(2, 1, 3)	0.000000	0.000000
489	GRAPHITE(2, 2, 1)	0.2500000	0.000000
490	GRAPHITE(2, 2, 2)	0.2500000	0.000000
491	GRAPHITE(2, 2, 3)	0.2500000	0.000000
492	GRAPHITE(2, 3, 1)	0.000000	0.000000
493	GRAPHITE(2, 3, 2)	0.000000	0.000000
494	GRAPHITE(2, 3, 3)	0.000000	0.000000
495	GRAPHITE(2, 4, 1)	0.000000	0.000000
496	GRAPHITE(2, 4, 2)	0.000000	0.000000
497	GRAPHITE(2, 4, 3)	0.000000	0.000000
498	GRAPHITE(2, 5, 1)	0.000000	0.000000
499	GRAPHITE(2, 5, 2)	0.000000	0.000000
500	GRAPHITE(2, 5, 3)	0.000000	0.000000
501	GRAPHITE(2, 6, 1)	0.000000	0.000000
502	GRAPHITE(2, 6, 2)	0.000000	0.000000
503	GRAPHITE(2, 6, 3)	0.000000	0.000000
504	GRAPHITE(2, 7, 1)	0.000000	0.000000
505	GRAPHITE(2, 7, 2)	0.000000	0.000000
506	GRAPHITE(2, 7, 3)	0.000000	0.000000
507	GRAPHITE(2, 8, 1)	0.000000	0.000000
508	GRAPHITE(2, 8, 2)	0.000000	0.000000
509	GRAPHITE(2, 8, 3)	0.000000	0.000000

510	GRAPHITE(2, 9, 1)	0.000000	0.000000
511	GRAPHITE(2, 9, 2)	0.000000	0.000000
512	GRAPHITE(2, 9, 3)	0.000000	0.000000
513	X(1, 1, 1)	0.000000	0.000000
514	X(1, 1, 2)	0.000000	0.000000
515	X(1, 1, 3)	0.000000	0.000000
516	X(1, 2, 1)	795.5000	0.000000
517	X(1, 2, 2)	545.4502	0.000000
518	X(1, 2, 3)	2189.334	0.000000
519	X(1, 3, 1)	735.0000	0.000000
520	X(1, 3, 2)	542.6540	0.000000
521	X(1, 3, 3)	245.6416	0.000000
522	X(1, 4, 1)	30.00000	0.000000
523	X(1, 4, 2)	30.00000	0.000000
524	X(1, 4, 3)	15.00000	0.000000
525	X(1, 5, 1)	2160.500	0.000000
526	X(1, 5, 2)	2102.796	0.000000
527	X(1, 5, 3)	2043.692	0.000000
528	X(1, 6, 1)	0.000000	0.000000
529	X(1, 6, 2)	0.000000	0.000000
530	X(1, 6, 3)	0.000000	0.000000
531	X(1, 7, 1)	0.000000	0.000000
532	X(1, 7, 2)	0.000000	0.000000
533	X(1, 7, 3)	0.000000	0.000000
534	X(1, 8, 1)	0.000000	0.000000
535	X(1, 8, 2)	0.000000	0.000000
536	X(1, 8, 3)	0.000000	0.000000
537	X(1, 9, 1)	53.50000	0.000000
538	X(1, 9, 2)	34.26540	0.000000
539	X(1, 9, 3)	14.56416	0.000000
540	X(2, 1, 1)	0.000000	2.706667
541	X(2, 1, 2)	0.000000	1.000948
542	X(2, 1, 3)	0.000000	3.858957
543	X(2, 2, 1)	795.5000	0.000000
544	X(2, 2, 2)	545.4502	0.000000
545	X(2, 2, 3)	1136.363	0.000000
546	X(2, 3, 1)	735.0000	0.000000
547	X(2, 3, 2)	542.6540	0.000000
548	X(2, 3, 3)	935.6635	0.000000
549	X(2, 4, 1)	30.00000	0.000000
550	X(2, 4, 2)	30.00000	0.000000
551	X(2, 4, 3)	265.6991	0.000000
552	X(2, 5, 1)	2160.500	0.000000
553	X(2, 5, 2)	2102.796	0.000000
554	X(2, 5, 3)	2000.000	0.000000

X(2, 6, 1)	0.000000	0.000000
X(2, 6, 2)	0.000000	53.47299
X(2, 6, 3)	0.000000	67.03355
X(2, 7, 1)	0.000000	19.77333
X(2, 7, 2)	0.000000	16.18768
X(2, 7, 3)	0.000000	19.42218
X(2, 8, 1)	0.000000	11.60000
X(2, 8, 2)	0.000000	0.000000
X(2, 8, 3)	83.56635	0.000000
X(2, 9, 1)	53.50000	0.000000
X(2, 9, 2)	34.26540	0.000000
X(2, 9, 3)	0.000000	0.000000
Y(1, 1, 1)	0.000000	3.666667
Y(1, 1, 2)	0.000000	2.132701
Y(1, 1, 3)	0.000000	4.806295
Y(1, 2, 1)	100.0000	0.000000
Y(1, 2, 2)	100.0000	0.000000
Y(1, 2, 3)	2000.000	0.000000
Y(1, 3, 1)	200.0000	0.000000
Y(1, 3, 2)	200.0000	0.000000
Y(1, 3, 3)	100.0000	0.000000
Y(1, 4, 1)	30.00000	0.000000
Y(1, 4, 2)	30.00000	0.000000
Y(1, 4, 3)	15.00000	0.000000
Y(1, 5, 1)	2000.000	0.000000
Y(1, 5, 2)	2000.000	0.000000
Y(1, 5, 3)	2000.000	0.000000
Y(1, 6, 1)	0.000000	118.0000
Y(1, 6, 2)	0.000000	96.94313
Y(1, 6, 3)	0.000000	145.8838
Y(1, 7, 1)	0.000000	78.00000
Y(1, 7, 2)	0.000000	55.00000
Y(1, 7, 3)	0.000000	90.00000
Y(1, 8, 1)	0.000000	20.00000
Y(1, 8, 2)	0.000000	11.94313
Y(1, 8, 3)	0.000000	5.883777
Y(1, 9, 1)	53.50000	0.000000
Y(1, 9, 2)	34.26540	0.000000
Y(1, 9, 3)	14.56416	0.000000
Y(2, 1, 1)	0.000000	0.000000
Y(2, 1, 2)	0.000000	0.000000
Y(2, 1, 3)	0.000000	0.000000
Y(2, 2, 1)	100.0000	0.000000
Y(2, 2, 2)	100.0000	0.000000
Y(2, 2, 3)	50.00000	0.000000

600	Y(2, 3, 1)	200.0000	0.000000
601	Y(2, 3, 2)	200.0000	0.000000
602	Y(2, 3, 3)	100.0000	0.000000
603	Y(2, 4, 1)	30.00000	0.000000
604	Y(2, 4, 2)	30.00000	0.000000
605	Y(2, 4, 3)	15.00000	0.000000
606	Y(2, 5, 1)	2000.000	0.000000
607	Y(2, 5, 2)	2000.000	0.000000
608	Y(2, 5, 3)	2000.000	0.000000
609	Y(2, 6, 1)	0.000000	53.77333
610	Y(2, 6, 2)	0.000000	0.000000
611	Y(2, 6, 3)	0.000000	0.000000
612	Y(2, 7, 1)	0.000000	0.000000
613	Y(2, 7, 2)	0.000000	0.000000
614	Y(2, 7, 3)	0.000000	0.000000
615	Y(2, 8, 1)	0.000000	0.000000
616	Y(2, 8, 2)	0.000000	3.685308
617	Y(2, 8, 3)	83.56635	0.000000
618	Y(2, 9, 1)	53.50000	0.000000
619	Y(2, 9, 2)	34.26540	0.000000
620	Y(2, 9, 3)	0.000000	4.788626
621	I(1, 1, 1)	0.000000	4.146667
622	I(1, 1, 2)	0.000000	2.532701
623	I(1, 1, 3)	0.000000	5.366295
624	I(1, 2, 1)	0.000000	0.9200000
625	I(1, 2, 2)	0.000000	1.011754
626	I(1, 2, 3)	0.000000	0.9873381
627	I(1, 3, 1)	0.000000	1.940000
628	I(1, 3, 2)	0.000000	2.026540
629	I(1, 3, 3)	0.000000	1.825650
630	I(1, 4, 1)	0.000000	2.120000
631	I(1, 4, 2)	0.000000	1.786540
632	I(1, 4, 3)	0.000000	1.783962
633	I(1, 5, 1)	0.000000	1.480000
634	I(1, 5, 2)	0.000000	1.513934
635	I(1, 5, 3)	0.000000	0.8264939
636	I(1, 6, 1)	0.000000	78.46667
637	I(1, 6, 2)	0.000000	110.9431
638	I(1, 6, 3)	0.000000	160.2838
639	I(1, 7, 1)	0.000000	96.24000
640	I(1, 7, 2)	0.000000	72.60000
641	I(1, 7, 3)	0.000000	109.2000
642	I(1, 8, 1)	0.000000	48.00000
643	I(1, 8, 2)	0.000000	37.05782
644	I(1, 8, 3)	0.000000	35.48378

645	I(1, 9, 1)	0.000000	33.60000
646	I(1, 9, 2)	0.000000	34.80000
647	I(1, 9, 3)	0.000000	31.21137
648	I(2, 1, 1)	0.000000	10.54000
649	I(2, 1, 2)	0.000000	10.45000
650	I(2, 1, 3)	0.000000	12.63000
651	I(2, 2, 1)	0.000000	28.96667
652	I(2, 2, 2)	0.000000	27.17095
653	I(2, 2, 3)	0.000000	32.20896
654	I(2, 3, 1)	0.000000	12.32333
655	I(2, 3, 2)	0.000000	12.99408
656	I(2, 3, 3)	0.000000	15.13815
657	I(2, 4, 1)	0.000000	21.08000
658	I(2, 4, 2)	0.000000	20.25408
659	I(2, 4, 3)	0.000000	24.67735
660	I(2, 5, 1)	0.000000	40.43000
661	I(2, 5, 2)	0.000000	36.74564
662	I(2, 5, 3)	0.000000	41.21355
663	I(2, 6, 1)	0.000000	359.7233
664	I(2, 6, 2)	0.000000	305.6800
665	I(2, 6, 3)	0.000000	326.1300
666	I(2, 7, 1)	0.000000	400.4300
667	I(2, 7, 2)	0.000000	399.7100
668	I(2, 7, 3)	0.000000	441.5000
669	I(2, 8, 1)	0.000000	591.3600
670	I(2, 8, 2)	0.000000	595.9453
671	I(2, 8, 3)	0.000000	673.1500
672	I(2, 9, 1)	0.000000	687.6300
673	I(2, 9, 2)	0.000000	688.9800
674	I(2, 9, 3)	0.000000	765.1086
675	NUM1(1, 1)	65.00000	0.000000
676	NUM1(1, 2)	60.00000	0.000000
677	NUM1(1, 3)	65.00000	0.000000
678	NUM1(2, 1)	65.00000	0.000000
679	NUM1(2, 2)	60.00000	0.000000
680	NUM1(2, 3)	65.00000	0.000000
681	NUM2(1, 1)	5500.000	0.000000
682	NUM2(1, 2)	5000.000	0.000000
683	NUM2(1, 3)	6000.000	0.000000
684	NUM2(2, 1)	5500.000	0.000000
685	NUM2(2, 2)	5000.000	0.000000
686	NUM2(2, 3)	6000.000	0.000000
687	NUM3(1, 1)	12000.00	0.000000
688	NUM3(1, 2)	15000.00	0.000000
689	NUM3(1, 3)	22000.00	0.000000

NUM3(2, 1)	12000.00	0.000000
NUM3(2, 2)	15000.00	0.000000
NUM3(2, 3)	22000.00	0.000000
NUM4(1, 1)	20000.00	0.000000
NUM4(1, 2)	40000.00	0.000000
NUM4(1, 3)	35000.00	0.000000
NUM4(2, 1)	20000.00	0.000000
NUM4(2, 2)	40000.00	0.000000
NUM4(2, 3)	35000.00	0.000000

A. III Sensitivity Report

Row	Slack or Surplus	Dual Price
1	258326.8	1.000000
2	972.2500	0.000000
3	0.000000	2.037915
4	2046.955	0.000000
5	972.2500	0.000000
6	0.000000	1.543128
7	2084.431	0.000000
8	0.000000	1.916667
9	16851.04	0.000000
10	0.000000	1.307506
11	0.000000	1.496667
12	16851.04	0.000000
13	0.000000	1.069194
14	2022.500	0.000000
15	2944.076	0.000000
16	5053.329	0.000000
17	2022.500	0.000000
18	2944.076	0.000000
19	568.1872	0.000000
20	0.000000	0.000000
21	0.000000	1.666667
22	0.000000	5.583333
23	0.000000	3.500000
24	1900.000	0.000000
25	0.000000	0.000000
26	0.000000	0.000000
27	0.000000	0.000000
28	53.50000	0.000000
29	0.000000	0.000000

30	0.000000	0.1327014
31	0.000000	6.170616
32	0.000000	2.170616
33	1900.000	0.000000
34	0.000000	0.000000
35	0.000000	0.000000
36	0.000000	0.000000
37	34.26540	0.000000
38	0.000000	0.000000
39	1950.000	0.000000
40	0.000000	6.113801
41	0.000000	2.421308
42	1900.000	0.000000
43	0.000000	0.000000
44	0.000000	0.000000
45	0.000000	0.000000
46	14.56416	0.000000
47	0.000000	0.000000
48	0.000000	2.266667
49	0.000000	3.963333
50	0.000000	2.180000
51	1900.000	0.000000
52	0.000000	0.000000
53	0.000000	0.000000
54	0.000000	0.000000
55	53.50000	0.000000
56	0.000000	0.000000
57	0.000000	0.5609479
58	0.000000	4.544076
59	0.000000	1.264076
60	1900.000	0.000000
61	0.000000	0.000000
62	0.000000	0.000000
63	0.000000	0.000000
64	34.26540	0.000000
65	0.000000	0.000000
66	0.000000	0.4189573
67	0.000000	4.688152
68	0.000000	1.597346
69	1900.000	0.000000
70	0.000000	0.000000
71	0.000000	0.000000
72	83.56635	0.000000
73	0.000000	0.000000
74	2000.000	0.000000

76	75	1900.000	0.000000
77	76	1800.000	0.000000
78	77	1970.000	0.000000
79	78	0.000000	−2.500000
80	79	200.0000	0.000000
81	80	100.0000	0.000000
82	81	300.0000	0.000000
83	82	346.5000	0.000000
84	83	2000.000	0.000000
85	84	1900.000	0.000000
86	85	1800.000	0.000000
87	86	1970.000	0.000000
88	87	0.000000	−5.810427
89	88	200.0000	0.000000
90	89	100.0000	0.000000
91	90	300.0000	0.000000
92	91	365.7346	0.000000
93	92	2000.000	0.000000
94	93	0.000000	−0.1937046
95	94	1900.000	0.000000
96	95	1985.000	0.000000
97	96	0.000000	−7.539952
98	97	200.0000	0.000000
99	98	100.0000	0.000000
100	99	300.0000	0.000000
101	100	385.4358	0.000000
102	101	2000.000	0.000000
103	102	1900.000	0.000000
104	103	1800.000	0.000000
105	104	1970.000	0.000000
106	105	0.000000	−1.900000
107	106	200.0000	0.000000
108	107	100.0000	0.000000
109	108	300.0000	0.000000
110	109	346.5000	0.000000
111	110	2000.000	0.000000
112	111	1900.000	0.000000
113	112	1800.000	0.000000
114	113	1970.000	0.000000
115	114	0.000000	−5.404360
116	115	200.0000	0.000000
117	116	100.0000	0.000000
118	117	300.0000	0.000000
119	118	365.7346	0.000000
120	119	2000.000	0.000000

121	120	1950.000	0.000000
122	121	1900.000	0.000000
123	122	1985.000	0.000000
124	123	0.000000	−6.206445
125	124	200.0000	0.000000
126	125	100.0000	0.000000
127	126	216.4336	0.000000
128	127	400.0000	0.000000
129	128	0.000000	−13.66667
130	129	0.000000	−12.13270
131	130	0.000000	−16.80630
132	131	0.000000	−26.66667
133	132	0.000000	−25.13270
134	133	0.000000	−29.80630
135	134	0.000000	−13.58333
136	135	0.000000	−14.17062
137	136	0.000000	−16.11380
138	137	0.000000	−21.50000
139	138	0.000000	−20.17062
140	139	0.000000	−24.42131
141	140	0.000000	−37.50000
142	141	0.000000	−34.18957
143	142	0.000000	−37.46005
144	143	0.000000	−408.0000
145	144	0.000000	−386.9431
146	145	0.000000	−455.8838
147	146	0.000000	−458.0000
148	147	0.000000	−435.0000
149	148	0.000000	−510.0000
150	149	0.000000	−580.0000
151	150	0.000000	−571.9431
152	151	0.000000	−645.8838
153	152	0.000000	−650.0000
154	153	0.000000	−650.0000
155	154	0.000000	−720.0000
156	155	0.000000	−10.00000
157	156	0.000000	−10.00000
158	157	0.000000	−12.00000
159	158	0.000000	−27.26667
160	159	0.000000	−25.56095
161	160	0.000000	−30.41896
162	161	0.000000	−11.96333
163	162	0.000000	−12.54408
164	163	0.000000	−14.68815
165	164	0.000000	−20.18000

165	0.000000	−19.26408
166	0.000000	−23.59735
167	0.000000	−38.10000
168	0.000000	−34.59564
169	0.000000	−38.79355
170	0.000000	−343.7733
171	0.000000	−290.0000
172	0.000000	−310.0000
173	0.000000	−380.0000
174	0.000000	−380.0000
175	0.000000	−420.0000
176	0.000000	−560.0000
177	0.000000	−563.6853
178	0.000000	−640.0000
179	0.000000	−650.0000
180	0.000000	−650.0000
181	0.000000	−724.7886
182	1552.088	0.000000
183	1781.030	0.000000
184	117.4289	0.000000
185	498.1007	0.000000

Ranges in which the basis is unchanged:

Objective Coefficient Ranges:

Current Variable	Allowable Coefficient	Allowable Increase	Decrease
X(1, 1, 1)	−6.000000	3.666667	INFINITY
X(1, 1, 2)	−5.000000	2.132701	INFINITY
X(1, 1, 3)	−7.000000	4.806295	INFINITY
X(1, 2, 1)	−19.00000	1.623529	2.276923
X(1, 2, 2)	−18.00000	0.2333333	2.457231
X(1, 2, 3)	−20.00000	1.870507	0.3669725
X(1, 3, 1)	−4.000000	3.325714	2.760000
X(1, 3, 2)	−5.000000	3.533884	0.4000000
X(1, 3, 3)	−5.000000	0.5333333	2.133387
X(1, 4, 1)	−10.00000	2.120000	INFINITY
X(1, 4, 2)	−11.00000	1.786540	INFINITY
X(1, 4, 3)	−12.00000	1.783962	INFINITY
X(1, 5, 1)	−26.00000	1.741176	2.941176
X(1, 5, 2)	−24.00000	1.764862	1.333333
X(1, 5, 3)	−27.00000	0.9351835	1.919431
X(1, 6, 1)	−30.00000	78.46667	INFINITY
X(1, 6, 2)	−27.00000	96.94313	INFINITY

211	X(1, 6, 3)	−3.000000	145.8838	INFINITY
212	X(1, 7, 1)	−32.00000	78.00000	INFINITY
213	X(1, 7, 2)	−33.00000	55.00000	INFINITY
214	X(1, 7, 3)	−18.00000	90.00000	INFINITY
215	X(1, 8, 1)	−33.00000	20.00000	INFINITY
216	X(1, 8, 2)	−43.00000	11.94313	INFINITY
217	X(1, 8, 3)	−24.00000	5.883777	INFINITY
218	X(1, 9, 1)	−55.00000	50.00000	20.00000
219	X(1, 9, 2)	−79.00000	122.6000	4.000000
220	X(1, 9, 3)	−59.00000	5.333333	5.758294
221	X(2, 1, 1)	−6.720000	2.706667	INFINITY
222	X(2, 1, 2)	−5.600000	1.000948	INFINITY
223	X(2, 1, 3)	−7.840000	3.858957	INFINITY
224	X(2, 2, 1)	−21.28000	2.276923	1.623529
225	X(2, 2, 2)	−20.16000	0.9863333	1.779000
226	X(2, 2, 3)	−22.40000	0.7788546	1.835492
227	X(2, 3, 1)	−4.480000	2.760000	3.325714
228	X(2, 3, 2)	−5.600000	3.049714	1.690857
229	X(2, 3, 3)	−5.600000	2.179878	1.178667
230	X(2, 4, 1)	−11.20000	2.180000	2.120000
231	X(2, 4, 2)	−12.32000	1.228436	1.786540
232	X(2, 4, 3)	−13.44000	1.846795	1.630993
233	X(2, 5, 1)	−29.12000	9.200000	1.741176
234	X(2, 5, 2)	−26.88000	10.16571	1.246154
235	X(2, 5, 3)	−30.24000	1.596209	0.8264939
236	X(2, 6, 1)	−33.60000	53.77333	78.46667
237	X(2, 6, 2)	−30.24000	53.47299	INFINITY
238	X(2, 6, 3)	−3.360000	67.03355	INFINITY
239	X(2, 7, 1)	−35.84000	19.77333	INFINITY
240	X(2, 7, 2)	−36.96000	16.18768	INFINITY
241	X(2, 7, 3)	−20.16000	19.42218	INFINITY
242	X(2, 8, 1)	−36.96000	11.60000	INFINITY
243	X(2, 8, 2)	−48.16000	3.685308	37.05782
244	X(2, 8, 3)	−26.88000	21.79878	4.892978
245	X(2, 9, 1)	−61.60000	27.60000	11.60000
246	X(2, 9, 2)	−88.48000	30.49714	3.738462
247	X(2, 9, 3)	−66.08000	4.788626	31.21137
248	Y(1, 1, 1)	10.00000	3.666667	INFINITY
249	Y(1, 1, 2)	10.00000	2.132701	INFINITY
250	Y(1, 1, 3)	12.00000	4.806295	INFINITY
251	Y(1, 2, 1)	25.00000	1.666667	INFINITY
252	Y(1, 2, 2)	25.00000	0.1327014	INFINITY
253	Y(1, 2, 3)	30.00000	INFINITY	0.1937046
254	Y(1, 3, 1)	8.000000	5.583333	INFINITY
255	Y(1, 3, 2)	8.000000	6.170616	INFINITY

256	Y(1, 3, 3)	10.00000	6.113801	INFINITY
257	Y(1, 4, 1)	18.00000	3.500000	INFINITY
258	Y(1, 4, 2)	18.00000	2.170616	INFINITY
259	Y(1, 4, 3)	22.00000	2.421308	INFINITY
260	Y(1, 5, 1)	40.00000	INFINITY	2.500000
261	Y(1, 5, 2)	40.00000	INFINITY	5.810427
262	Y(1, 5, 3)	45.00000	INFINITY	7.539952
263	Y(1, 6, 1)	290.0000	118.0000	INFINITY
264	Y(1, 6, 2)	290.0000	96.94313	INFINITY
265	Y(1, 6, 3)	310.0000	145.8838	INFINITY
266	Y(1, 7, 1)	380.0000	78.00000	INFINITY
267	Y(1, 7, 2)	380.0000	55.00000	INFINITY
268	Y(1, 7, 3)	420.0000	90.00000	INFINITY
269	Y(1, 8, 1)	560.0000	20.00000	INFINITY
270	Y(1, 8, 2)	560.0000	11.94313	INFINITY
271	Y(1, 8, 3)	640.0000	5.883777	INFINITY
272	Y(1, 9, 1)	650.0000	50.00000	20.00000
273	Y(1, 9, 2)	650.0000	122.6000	4.000000
274	Y(1, 9, 3)	720.0000	5.333333	5.758294
275	Y(2, 1, 1)	10.00000	2.706667	1.521026
276	Y(2, 1, 2)	10.00000	1.000948	1.245206
277	Y(2, 1, 3)	12.00000	3.858957	1.494014
278	Y(2, 2, 1)	25.00000	2.266667	INFINITY
279	Y(2, 2, 2)	25.00000	0.5609479	INFINITY
280	Y(2, 2, 3)	30.00000	0.4189573	INFINITY
281	Y(2, 3, 1)	8.000000	3.963333	INFINITY
282	Y(2, 3, 2)	8.000000	4.544076	INFINITY
283	Y(2, 3, 3)	10.00000	4.688152	INFINITY
284	Y(2, 4, 1)	18.00000	2.180000	INFINITY
285	Y(2, 4, 2)	18.00000	1.264076	INFINITY
286	Y(2, 4, 3)	22.00000	1.597346	INFINITY
287	Y(2, 5, 1)	40.00000	INFINITY	1.900000
288	Y(2, 5, 2)	40.00000	INFINITY	5.404360
289	Y(2, 5, 3)	45.00000	INFINITY	6.206445
290	Y(2, 6, 1)	290.0000	53.77333	INFINITY
291	Y(2, 6, 2)	290.0000	53.47299	305.6800
292	Y(2, 6, 3)	310.0000	67.03355	326.1300
293	Y(2, 7, 1)	380.0000	19.77333	400.4300
294	Y(2, 7, 2)	380.0000	16.18768	399.7100
295	Y(2, 7, 3)	420.0000	19.42218	441.5000
296	Y(2, 8, 1)	560.0000	11.60000	591.3600
297	Y(2, 8, 2)	560.0000	3.685308	INFINITY
298	Y(2, 8, 3)	640.0000	21.79878	4.892978
299	Y(2, 9, 1)	650.0000	27.60000	11.60000
300	Y(2, 9, 2)	650.0000	30.49714	3.738462

301	Y(2, 9, 3)	720.0000	4.788626	INFINITY
302	I(1, 1, 1)	−0.4800000	4.146667	INFINITY
303	I(1, 1, 2)	−0.4000000	2.532701	INFINITY
304	I(1, 1, 3)	−0.5600000	5.366295	INFINITY
305	I(1, 2, 1)	−1.520000	0.9200000	INFINITY
306	I(1, 2, 2)	−1.440000	1.011754	INFINITY
307	I(1, 2, 3)	−1.600000	0.9873381	INFINITY
308	I(1, 3, 1)	−0.3200000	1.940000	INFINITY
309	I(1, 3, 2)	−0.4000000	2.026540	INFINITY
310	I(1, 3, 3)	−0.4000000	1.825650	INFINITY
311	I(1, 4, 1)	−0.8000000	2.120000	INFINITY
312	I(1, 4, 2)	−0.8800000	1.786540	INFINITY
313	I(1, 4, 3)	−0.9600000	1.783962	INFINITY
314	I(1, 5, 1)	−2.080000	1.480000	INFINITY
315	I(1, 5, 2)	−1.920000	1.513934	INFINITY
316	I(1, 5, 3)	−2.160000	0.8264939	INFINITY
317	I(1, 6, 1)	−14.24000	78.46667	INFINITY
318	I(1, 6, 2)	−14.00000	110.9431	INFINITY
319	I(1, 6, 3)	−14.40000	160.2838	INFINITY
320	I(1, 7, 1)	−18.24000	96.24000	INFINITY
321	I(1, 7, 2)	−17.60000	72.60000	INFINITY
322	I(1, 7, 3)	−19.20000	109.2000	INFINITY
323	I(1, 8, 1)	−28.00000	48.00000	INFINITY
324	I(1, 8, 2)	−28.80000	37.05782	INFINITY
325	I(1, 8, 3)	−29.60000	35.48378	INFINITY
326	I(1, 9, 1)	−33.60000	33.60000	INFINITY
327	I(1, 9, 2)	−34.80000	34.80000	INFINITY
328	I(1, 9, 3)	−36.00000	31.21137	INFINITY
329	I(2, 1, 1)	−0.5400000	10.54000	INFINITY
330	I(2, 1, 2)	−0.4500000	10.45000	INFINITY
331	I(2, 1, 3)	−0.6300000	12.63000	INFINITY
332	I(2, 2, 1)	−1.700000	28.96667	INFINITY
333	I(2, 2, 2)	−1.610000	27.17095	INFINITY
334	I(2, 2, 3)	−1.790000	32.20896	INFINITY
335	I(2, 3, 1)	−0.3600000	12.32333	INFINITY
336	I(2, 3, 2)	−0.4500000	12.99408	INFINITY
337	I(2, 3, 3)	−0.4500000	15.13815	INFINITY
338	I(2, 4, 1)	−0.9000000	21.08000	INFINITY
339	I(2, 4, 2)	−0.9900000	20.25408	INFINITY
340	I(2, 4, 3)	−1.080000	24.67735	INFINITY
341	I(2, 5, 1)	−2.330000	40.43000	INFINITY
342	I(2, 5, 2)	−2.150000	36.74564	INFINITY
343	I(2, 5, 3)	−2.420000	41.21355	INFINITY
344	I(2, 6, 1)	−15.95000	359.7233	INFINITY
345	I(2, 6, 2)	−15.68000	305.6800	INFINITY

I(2, 6, 3)	−16.13000	326.1300	INFINITY
I(2, 7, 1)	−20.43000	400.4300	INFINITY
I(2, 7, 2)	−19.71000	399.7100	INFINITY
I(2, 7, 3)	−21.50000	441.5000	INFINITY
I(2, 8, 1)	−31.36000	591.3600	INFINITY
I(2, 8, 2)	−32.26000	595.9453	INFINITY
I(2, 8, 3)	−33.15000	673.1500	INFINITY
I(2, 9, 1)	−37.63000	687.6300	INFINITY
I(2, 9, 2)	−38.98000	688.9800	INFINITY
I(2, 9, 3)	−40.32000	765.1086	INFINITY

Righthand Side Ranges:

Current Row	Allowable RHS	Allowable Increase	Allowable Decrease
2	12000.00	INFINITY	972.2500
3	15000.00	3811.923	3615.000
4	22000.00	INFINITY	2046.955
5	12000.00	INFINITY	972.2500
6	15000.00	5176.667	3615.000
7	22000.00	INFINITY	2084.431
8	20000.00	2509.032	6420.000
9	40000.00	INFINITY	16851.04
10	35000.00	4144.081	3007.500
11	20000.00	2509.032	6420.000
12	40000.00	INFINITY	16851.04
13	35000.00	1844.423	17632.50
14	5500.000	INFINITY	2022.500
15	5000.000	INFINITY	2944.076
16	6000.000	INFINITY	5053.329
17	5500.000	INFINITY	2022.500
18	5000.000	INFINITY	2944.076
19	6000.000	INFINITY	568.1872
20	0.000000	INFINITY	0.000000
21	−100.0000	100.0000	828.9099
22	−200.0000	200.0000	1284.000
23	−30.00000	30.00000	1070.000
24	−100.0000	INFINITY	1900.000
25	0.000000	INFINITY	0.000000
26	0.000000	INFINITY	0.000000
27	0.000000	INFINITY	0.000000
28	0.000000	INFINITY	53.50000
29	0.000000	INFINITY	0.000000
30	−100.0000	100.0000	825.9166
31	−200.0000	200.0000	803.3333

391	32	−30.00000	30.00000	803.3333
392	33	−100.0000	INFINITY	1900.000
393	34	0.000000	INFINITY	0.000000
394	35	0.000000	INFINITY	0.000000
395	36	0.000000	INFINITY	0.000000
396	37	0.000000	INFINITY	34.26540
397	38	0.000000	INFINITY	0.000000
398	39	−50.00000	INFINITY	1950.000
399	40	−100.0000	100.0000	353.8235
400	41	−15.00000	15.00000	316.5789
401	42	−100.0000	INFINITY	1900.000
402	43	0.000000	INFINITY	0.000000
403	44	0.000000	INFINITY	0.000000
404	45	0.000000	INFINITY	0.000000
405	46	0.000000	INFINITY	14.56416
406	47	0.000000	INFINITY	0.000000
407	48	−100.0000	100.0000	1605.000
408	49	−200.0000	200.0000	1284.000
409	50	−30.00000	30.00000	1070.000
410	51	−100.0000	INFINITY	1900.000
411	52	0.000000	INFINITY	0.000000
412	53	0.000000	INFINITY	0.000000
413	54	0.000000	INFINITY	0.000000
414	55	0.000000	INFINITY	53.50000
415	56	0.000000	INFINITY	0.000000
416	57	−100.0000	100.0000	1032.857
417	58	−200.0000	200.0000	803.3333
418	59	−30.00000	30.00000	803.3333
419	60	−100.0000	INFINITY	1900.000
420	61	0.000000	INFINITY	0.000000
421	62	0.000000	INFINITY	0.000000
422	63	0.000000	INFINITY	0.000000
423	64	0.000000	INFINITY	34.26540
424	65	0.000000	INFINITY	0.000000
425	66	−50.00000	50.00000	1950.000
426	67	−100.0000	100.0000	1900.000
427	68	−15.00000	15.00000	1856.053
428	69	−100.0000	INFINITY	1900.000
429	70	0.000000	INFINITY	0.000000
430	71	0.000000	INFINITY	0.000000
431	72	0.000000	INFINITY	83.56635
432	73	0.000000	INFINITY	0.000000
433	74	−2000.000	2000.000	INFINITY
434	75	−2000.000	1900.000	INFINITY
435	76	−2000.000	1800.000	INFINITY

77	−2000.000	1970.000	INFINITY
78	−2000.000	622.3077	580.4478
79	−200.0000	200.0000	INFINITY
80	−100.0000	100.0000	INFINITY
81	−300.0000	300.0000	INFINITY
82	−400.0000	346.5000	INFINITY
83	−2000.000	2000.000	INFINITY
84	−2000.000	1900.000	INFINITY
85	−2000.000	1800.000	INFINITY
86	−2000.000	1970.000	INFINITY
87	−2000.000	762.3846	723.0000
88	−200.0000	200.0000	INFINITY
89	−100.0000	100.0000	INFINITY
90	−300.0000	300.0000	INFINITY
91	−400.0000	365.7346	INFINITY
92	−2000.000	2000.000	INFINITY
93	−2000.000	1950.000	401.0000
94	−2000.000	1900.000	INFINITY
95	−2000.000	1985.000	INFINITY
96	−2000.000	932.6565	375.9375
97	−200.0000	200.0000	INFINITY
98	−100.0000	100.0000	INFINITY
99	−300.0000	300.0000	INFINITY
100	−400.0000	385.4358	INFINITY
101	−2000.000	2000.000	INFINITY
102	−2000.000	1900.000	INFINITY
103	−2000.000	1800.000	INFINITY
104	−2000.000	1970.000	INFINITY
105	−2000.000	622.3077	580.4478
106	−200.0000	200.0000	INFINITY
107	−100.0000	100.0000	INFINITY
108	−300.0000	300.0000	INFINITY
109	−400.0000	346.5000	INFINITY
110	−2000.000	2000.000	INFINITY
111	−2000.000	1900.000	INFINITY
112	−2000.000	1800.000	INFINITY
113	−2000.000	1970.000	INFINITY
114	−2000.000	1035.333	723.0000
115	−200.0000	200.0000	INFINITY
116	−100.0000	100.0000	INFINITY
117	−300.0000	300.0000	INFINITY
118	−400.0000	365.7346	INFINITY
119	−2000.000	2000.000	INFINITY
120	−2000.000	1950.000	INFINITY
121	−2000.000	1900.000	INFINITY

481	122	−2000.000	1985.000	INFINITY
482	123	−2000.000	230.5529	1193.528
483	124	−200.0000	200.0000	INFINITY
484	125	−100.0000	100.0000	INFINITY
485	126	−300.0000	216.4336	INFINITY
486	127	−400.0000	400.0000	INFINITY
487	128	0.000000	1605.000	0.000000
488	129	0.000000	1032.857	0.000000
489	130	0.000000	401.0000	0.000000
490	131	0.000000	828.9099	933.4615
491	132	0.000000	825.9166	959.0833
492	133	0.000000	401.0000	2140.538
493	134	0.000000	1284.000	746.7692
494	135	0.000000	803.3333	847.0940
495	136	0.000000	353.8235	417.4897
496	137	0.000000	1070.000	30.00000
497	138	0.000000	803.3333	30.00000
498	139	0.000000	316.5789	15.00000
499	140	0.000000	580.4478	622.3077
500	141	0.000000	723.0000	762.3846
501	142	0.000000	375.9375	932.6565
502	143	0.000000	53.50000	0.000000
503	144	0.000000	34.75962	0.000000
504	145	0.000000	14.25355	0.000000
505	146	0.000000	53.50000	0.000000
506	147	0.000000	34.26540	0.000000
507	148	0.000000	14.56416	0.000000
508	149	0.000000	53.50000	0.000000
509	150	0.000000	34.75962	0.000000
510	151	0.000000	14.25355	0.000000
511	152	0.000000	53.50000	346.5000
512	153	0.000000	34.26540	365.7346
513	154	0.000000	14.56416	385.4358
514	155	0.000000	0.000000	2000.000
515	156	0.000000	0.000000	2000.000
516	157	0.000000	0.000000	2000.000
517	158	0.000000	1605.000	933.4615
518	159	0.000000	1032.857	959.0833
519	160	0.000000	2351.000	245.9231
520	161	0.000000	1284.000	746.7692
521	162	0.000000	803.3333	946.2810
522	163	0.000000	2074.412	216.9910
523	164	0.000000	1070.000	30.00000
524	165	0.000000	803.3333	30.00000
525	166	0.000000	1856.053	194.1498

167	0.000000	580.4478	622.3077
168	0.000000	723.0000	1035.333
169	0.000000	1193.528	230.5529
170	0.000000	0.000000	0.000000
171	0.000000	0.000000	200.0000
172	0.000000	0.000000	200.0000
173	0.000000	0.000000	100.0000
174	0.000000	0.000000	100.0000
175	0.000000	0.000000	100.0000
176	0.000000	0.000000	300.0000
177	0.000000	34.75962	0.000000
178	0.000000	83.56635	216.4336
179	0.000000	53.50000	346.5000
180	0.000000	34.26540	365.7346
181	0.000000	85.38741	0.000000
182	20000.00	INFINITY	1552.088
183	20000.00	INFINITY	1781.030
184	1000.000	INFINITY	117.4289
185	2000.000	INFINITY	498.1007

B Problem 2

B. I LINGO Code of Question (a)-(b)

```
model:
max = 50*(ya1+ya2+ya3+ya4)+65*(yb1+yb2+yb3+yb4)+70*(yc1+yc2+yc3+yc4)-
5*(ia11+ia21+ia31+ia41+ib11+ib21+ib31+ib41+ic11+ic21+ic31+ic41)-
10*(ia12+ia22+ia32+ia42+ib12+ib22+ib32++ib42+ic12+ic22+ic32+ic42);
2.4*xa1+2*xb1+0.9*xc1<640;
1.1*xa1+2.2*xb1+0.9*xc1<640;
0.8*xa1+1.2*xb1+xc1<1920;
3*xa1+2.1*xb1+2.5*xc1<1280;
ya1<100;
yb1>20;
yb1<100;
yc1<300;
ia11-ia12 = xa1-ya1;
ib11-ib12 = xb1-yb1;
ic11-ic12 = xc1-yc1;
2.4*xa2+2*xb2+0.9*xc2<640;
```

```
1.1*xa2+2.2*xb2+0.9*xc2<640;
0.8*xa2+1.2*xb2+xc2<1920;
3*xa2+2.1*xb2+2.5*xc2<1280;
ya2<50;
yb2>20;
yb2<100;
yc2<250;
ia21-ia22 = ia11+xa2-ya2;
ib21-ib22 = ib11+xb2-yb2;
ic21-ic22 = ic11+xc2-yc2;
2.4*xa3+2*xb3+0.9*xc3<1280;
1.1*xa3+2.2*xb3+0.9*xc3<640;
0.8*xa3+1.2*xb3+xc3<1920;
3*xa3+2.1*xb3+2.5*xc3<1280;
ya3<50;
yb3>20;
yb3<100;
yc3<250;
ia31-ia32 = ia21+xa3-ya3;
ib31-ib32 = ib21+xb3-yb3;
ic31-ic32 = ic21+xc3-yc3;

2.4*xa4+2*xb4+0.9*xc4<1280;
1.1*xa4+2.2*xb4+0.9*xc4<640;
0.8*xa4+1.2*xb4+xc4<1920;
3*xa4+2.1*xb4+2.5*xc4<2560;
ya4<75;
yb4>25;
yb4<100;
yc4<400;
yc4>50;
ia41-ia42 = ia31+xa4-ya4;
ib41-ib42 = ib31+xb4-yb4;
ic41-ic42 = ic31+xc4-yc4;
```

B. II Result Report of Question (a)-(b)

```
LINGO/OSX64 19.0.46 (1 Sep 2021 ), LINDO API 13.0.4099.299

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```

```
Global optimal solution found.
Objective value:                              123407.2
  Infeasibilities:                            0.000000
Total solver iterations:                           14
Elapsed runtime seconds:                         0.04

Model Class:                                       LP

Total variables:                   48
Nonlinear variables:                0
Integer variables:                  0

Total constraints:                 46
Nonlinear constraints:              0

Total nonzeros:                   158
Nonlinear nonzeros:                 0


  Variable           Value        Reduced Cost
  YA1            100.0000            0.000000
  YA2            50.00000            0.000000
  YA3            50.00000            0.000000
  YA4            75.00000            0.000000
  YB1            100.0000            0.000000
  YB2            100.0000            0.000000
  YB3            100.0000            0.000000
  YB4            100.0000            0.000000
  YC1            300.0000            0.000000
  YC2            250.0000            0.000000
  YC3            250.0000            0.000000
  YC4            400.0000            0.000000
  IA11           0.000000            15.00000
  IA21           0.000000            5.000000
  IA31           0.000000            2.500000
  IA41           0.000000            7.500000
  IB11           0.000000            13.33333
  IB21           0.000000            5.000000
  IB31           10.22727            0.000000
  IB41           0.000000            10.00000
  IC11           0.000000            8.750000
  IC21           0.000000            5.000000
  IC31           0.000000            2.954545
  IC41           0.000000            7.045455
```

IA12	29.16667	0.000000
IA22	0.000000	10.00000
IA32	0.000000	10.00000
IA42	0.000000	7.500000
IB12	0.000000	1.666667
IB22	0.000000	10.00000
IB32	0.000000	10.00000
IB42	0.000000	5.000000
IC12	0.000000	6.250000
IC22	0.000000	10.00000
IC32	0.000000	10.00000
IC42	0.000000	7.954545
XA1	70.83333	0.000000
XB1	100.0000	0.000000
XC1	300.0000	0.000000
XA2	50.00000	0.000000
XB2	100.0000	0.000000
XC2	250.0000	0.000000
XA3	50.00000	0.000000
XB3	110.2273	0.000000
XC3	250.0000	0.000000
XA4	75.00000	0.000000
XB4	89.77273	0.000000
XC4	400.0000	0.000000

Row	Slack or Surplus	Dual Price
1	123407.2	1.000000
2	0.000000	4.166667
3	72.08333	0.000000
4	1443.333	0.000000
5	107.5000	0.000000
6	0.000000	40.00000
7	80.00000	0.000000
8	0.000000	56.66667
9	0.000000	66.25000
10	0.000000	10.00000
11	0.000000	8.333333
12	0.000000	3.750000
13	95.00000	0.000000
14	140.0000	0.000000
15	1510.000	0.000000
16	295.0000	0.000000
17	0.000000	50.00000
18	80.00000	0.000000
19	0.000000	65.00000

```
20      0.000000        70.00000
21      0.000000        0.000000
22      0.000000        0.000000
23      0.000000        0.000000
24      714.5455        0.000000
25      117.5000        0.000000
26      1497.727        0.000000
27      273.5227        0.000000
28      0.000000        50.00000
29      80.00000        0.000000
30      0.000000        65.00000
31      0.000000        70.00000
32      0.000000        0.000000
33      0.000000        0.000000
34      0.000000        0.000000
35      560.4545        0.000000
36      0.000000        2.272727
37      1352.273        0.000000
38      1146.477        0.000000
39      0.000000        47.50000
40      75.00000        0.000000
41      0.000000        60.00000
42      0.000000        67.95455
43      350.0000        0.000000
44      0.000000        2.500000
45      0.000000        5.000000
46      0.000000        2.045455
```

B. III LINGO Code of Question (c)-(d)

```
model:
max = 50*(ya1+ya2+ya3+ya4)+65*(yb1+yb2+yb3+yb4)+70*(yc1+yc2+yc3+yc4)-
5*(ia11+ia21+ia31+ia41+ib11+ib21+ib31+ib41+ic11+ic21+ic31+ic41)-
5*(1875-ya1-yb1-yc1-ya2-yb2-yc2-ya3-yb3-yc3-ya4-yb4-yc4);
2.4*xa1+2*xb1+0.9*xc1<640;
1.1*xa1+2.2*xb1+0.9*xc1<640;
0.8*xa1+1.2*xb1+xc1<1920;
3*xa1+2.1*xb1+2.5*xc1<1280;
ya1<100;
yb1>20;
yb1<100;
yc1<300;
```

```
ia11 = xa1-ya1;
ib11 = xb1-yb1;
ic11 = xc1-yc1;
2.4*xa2+2*xb2+0.9*xc2<640;
1.1*xa2+2.2*xb2+0.9*xc2<640;
0.8*xa2+1.2*xb2+xc2<1920;
3*xa2+2.1*xb2+2.5*xc2<1280;
ya2<50;
yb2>20;
yb2<100;
yc2<250;
ia21 = ia11+xa2-ya2;
ib21 = ib11+xb2-yb2;
ic21 = ic11+xc2-yc2;
2.4*xa3+2*xb3+0.9*xc3<1280;
1.1*xa3+2.2*xb3+0.9*xc3<640;
0.8*xa3+1.2*xb3+xc3<1920;
3*xa3+2.1*xb3+2.5*xc3<1280;
ya3<50;
yb3>20;
yb3<100;
yc3<250;
ia31 = ia21+xa3-ya3;
ib31 = ib21+xb3-yb3;
ic31 = ic21+xc3-yc3;
2.4*xa4+2*xb4+0.9*xc4<1280;
1.1*xa4+2.2*xb4+0.9*xc4<640;
0.8*xa4+1.2*xb4+xc4<1920;
3*xa4+2.1*xb4+2.5*xc4<2560;
ya4<75;
yb4>25;
yb4<100;
yc4<400;
yc4>50;
ia41 = ia31+xa4-ya4;
ib41 = ib31+xb4-yb4;
ic41 = ic31+xc4-yc4;
```

B. IV Result Report of Question (c)-(d)

```
LINGO/OSX64 19.0.46 (1 Sep 2021 ), LINDO API 13.0.4099.299

```

```
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Global optimal solution found.
Objective value:                              122094.7
  Infeasibilities:                            0.000000
Total solver iterations:                            11
Elapsed runtime seconds:                          0.05

Model Class:                                        LP

Total variables:                   36
Nonlinear variables:                0
Integer variables:                  0

Total constraints:                 46
Nonlinear constraints:              0

Total nonzeros:                   134
Nonlinear nonzeros:                 0


  Variable           Value        Reduced Cost
  YA1            70.83333            0.000000
  YA2            50.00000            0.000000
  YA3            50.00000            0.000000
  YA4            75.00000            0.000000
  YB1            100.0000            0.000000
  YB2            100.0000            0.000000
  YB3            100.0000            0.000000
  YB4            100.0000            0.000000
  YC1            300.0000            0.000000
  YC2            250.0000            0.000000
  YC3            250.0000            0.000000
  YC4            400.0000            0.000000
  IA11           0.000000            60.00000
  IA21           0.000000            5.000000
  IA31           0.000000            2.500000
  IA41           0.000000            7.500000
  IB11           0.000000            50.83333
  IB21           0.000000            5.000000
  IB31           10.22727            0.000000
  IB41           0.000000            10.00000
  IC11           0.000000            25.62500
```

IC21	0.000000	5.000000
IC31	0.000000	2.954545
IC41	0.000000	7.045455
XA1	70.83333	0.000000
XB1	100.0000	0.000000
XC1	300.0000	0.000000
XA2	50.00000	0.000000
XB2	100.0000	0.000000
XC2	250.0000	0.000000
XA3	50.00000	0.000000
XB3	110.2273	0.000000
XC3	250.0000	0.000000
XA4	75.00000	0.000000
XB4	89.77273	0.000000
XC4	400.0000	0.000000

Row	Slack or Surplus	Dual Price
1	122094.7	1.000000
2	0.000000	22.91667
3	72.08333	0.000000
4	1443.333	0.000000
5	107.5000	0.000000
6	29.16667	0.000000
7	80.00000	0.000000
8	0.000000	24.16667
9	0.000000	54.37500
10	0.000000	55.00000
11	0.000000	45.83333
12	0.000000	20.62500
13	95.00000	0.000000
14	140.0000	0.000000
15	1510.000	0.000000
16	295.0000	0.000000
17	0.000000	55.00000
18	80.00000	0.000000
19	0.000000	70.00000
20	0.000000	75.00000
21	0.000000	0.000000
22	0.000000	0.000000
23	0.000000	0.000000
24	714.5455	0.000000
25	117.5000	0.000000
26	1497.727	0.000000
27	273.5227	0.000000
28	0.000000	55.00000

```
29          80.00000            0.000000
30          0.000000            70.00000
31          0.000000            75.00000
32          0.000000            0.000000
33          0.000000            0.000000
34          0.000000            0.000000
35          560.4545            0.000000
36          0.000000            2.272727
37          1352.273            0.000000
38          1146.477            0.000000
39          0.000000            52.50000
40          75.00000            0.000000
41          0.000000            65.00000
42          0.000000            72.95455
43          350.0000            0.000000
44          0.000000            2.500000
45          0.000000            5.000000
46          0.000000            2.045455
```

C Problem 3

C. I LINGO Code

```
min=p13+p14+p15+p16+p24+p25+p26+p27+p35+p36+p37+p46+p47+p57;
p12+p13+p14+p15+p16+p17<13;
p12+p23+p24+p25+p26+p27<7;
p23+p34+p35+p36+p13+p37<5;
p14+p24+p34+p45+p46+p47<14;
p15+p25+p35+p45+p56+p57<9;
p16+p26+p36+p46+p56+p67<2;
p17+p27+p37+p47+p57+p67<12;
p12+p13+p14+p15+p16+p17+p23+p24+p25+p26+p27+
p34+p35+p36+p13+p37+p45+p46+p47+p56+p57+p67=30;
```

C. II Solution Report

```
LINGO/OSX64 19.0.46 (1 Sep 2021 ), LINDO API 13.0.4099.299

```

```
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Global optimal solution found.
Objective value:                              1.000000
  Infeasibilities:                            0.000000
Total solver iterations:                             8
Elapsed runtime seconds:                          0.08

Model Class:                                        LP

Total variables:                    21
Nonlinear variables:                 0
Integer variables:                   0

Total constraints:                   9
Nonlinear constraints:               0

Total nonzeros:                     77
Nonlinear nonzeros:                  0


  Variable           Value        Reduced Cost
  P13            0.000000            1.000000
  P14            0.000000            1.000000
  P15            0.000000            2.000000
  P16            0.000000            2.000000
  P24            0.000000            0.000000
  P25            0.000000            1.000000
  P26            0.000000            1.000000
  P27            0.000000            0.000000
  P35            0.000000            2.000000
  P36            0.000000            2.000000
  P37            0.000000            1.000000
  P46            0.000000            1.000000
  P47            1.000000            0.000000
  P57            0.000000            1.000000
  P12            6.000000            0.000000
  P17            7.000000            0.000000
  P23            1.000000            0.000000
  P34            4.000000            0.000000
  P45            9.000000            0.000000
  P56            0.000000            1.000000
  P67            2.000000            0.000000
```

Row	Slack or Surplus	Dual Price
1	1.000000	−1.000000
2	0.000000	1.000000
3	0.000000	0.000000
4	0.000000	1.000000
5	0.000000	0.000000
6	0.000000	1.000000
7	0.000000	1.000000
8	2.000000	0.000000
9	0.000000	−1.000000