2022-2023 Autumn Semester

Operation Research

Assignment 2

-Revised simplex algorithm-

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1 Problem 1

1.1 Question:

Use the revised simplex algorithm DIRECTLY to solve the following LP problem.Please use the discussed method in class for updating the inverse of B.

$$
\begin{array}{llll}
\max & z = & 3x_1 + 5x_2 & \\
s.t. & & x_1 & \leq & 4 \\
& & x_2 & \leq & 6 \\
& & 3x_1 + 2x_2 & \leq & 18 \\
& & x_1, x_2 & \geq & 0
\end{array} \tag{1}
$$

1.2 Solution 1 (Revised Simplex Method):

Step 1 Convert LP to Canonical Form

We need to add slack variables s_1, s_2, s_3 to obtain the Canonical Form as below.

$$
\begin{array}{llllll}
\max & z = & 3x_1 + 5x_2 & & & \\
s.t. & & x_1 & +s_1 & = & 4 \\
& & x_2 & +s_2 & = & 6 \\
& & 3x_1 + 2x_2 & +s_3 & = & 18 \\
& & x_1, x_2, s_1, s_2, s_3 & & \geq & 0
\end{array} \tag{2}
$$

Step 2 Obtain a bfs

We choose basic variables $BV = \{s_1, s_2, s_3\}$ and nonbasic variables $NBV = \{x_1, x_2\}$, then we can obtain a basic fessible solution $x_1 = 0, x_2 = 0, s_1 = 4, s_2 = 6, s_3 = 18$.

Step 3 Determine if the current bfs is optimal and choose the entering variable

Since we have decided the basic variables and nonbasic variables, we can know

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C_{BV} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix};$$

$$N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}, \quad C_{NBV} = \begin{bmatrix} 3 & 5 \end{bmatrix}.$$

According to these matrices and vectors, we could calculate $C_{NBV} - C_{BV}B^{-1}N$, and the result is

$$C_{NBV} - C_{BV}B^{-1}N = \begin{bmatrix} 3 & 5 \end{bmatrix}. \tag{3}$$

Hence, the bfs obtained initially is not the optimal solution and we choose x_2 as the entering variable.

Step 4 The Ratio Test for determining the leaving variable

Because we choose x_2 as entering variable, we need to calculate $B^{-1}a_2$ and $B^{-1}b$ respectively.

$$B^{-1}a_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \quad B^{-1}b = \begin{bmatrix} 4 \\ 6 \\ 18 \end{bmatrix}. \tag{4}$$

So the ratio test result is shown in table 2.

表 1: Result of ratio test

variables	result
s_1	-
s_2	6
s_3	9

In light of the result, we choose s_2 as the leaving variable. And then we repeat the Step 3 - Step 4.

Step 5 Determine if the current bfs is optimal and choose the entering variable

Now we need to update the basic variables and nonbasic variables as $BV = \{s_1, x_2, s_3\}, NBV = x_1, s_2$.By the new variables, we can update correspond matrices and vectors as below.

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \quad , \quad C_{BV} = \begin{bmatrix} 0 & 5 & 0 \end{bmatrix};$$
$$N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 3 & 0 \end{bmatrix} \quad , \quad C_{NBV} = \begin{bmatrix} 3 & 0 \end{bmatrix}.$$

Using what we learned in the course, we can get the inverse matrix of B quickly.

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}, \ B_{new}^{-1} = EB_{old}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

According to the updated matrices and vectors, we could calculate $C_{NBV} - C_{BV}B^{-1}N$, and the result is

$$C_{NBV} - C_{BV}B^{-1}N = \begin{bmatrix} 3 & -5 \end{bmatrix}. \tag{5}$$

Hence, the bfs obtained is not the optimal solution and we choose x_1 as the entering variable.

Step 6 The Ratio Test for determining the leaving variable

Because we choose x_1 as entering variable, we need to calculate $B^{-1}a_1$ and $B^{-1}b$ respectively.

$$B^{-1}a_2 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \quad B^{-1}b = \begin{bmatrix} 4 \\ 6 \\ 6 \end{bmatrix}. \tag{6}$$

So the ratio test result is shown in table 2.

表 2: Result of ratio test

variables	result
s_1	4
x_2	-
s_3	2

In light of the result, we choose s_3 as the leaving variable. And then we repeat the Step 3 - Step 4 again.

Step 7 Determine if the current bfs is optimal and choose the entering variable

Now we need to update the basic variables and nonbasic variables as $BV = \{s_1, x_2, x_1\}, NBV = s_3, s_2$.By the new variables, we can update correspond matrices and vectors as below.

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 2 & 3 \end{bmatrix} \quad , \quad C_{BV} = \begin{bmatrix} 0 & 5 & 3 \end{bmatrix};$$
$$N = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad , \quad C_{NBV} = \begin{bmatrix} 0 & 0 \end{bmatrix}.$$

Using what we learned in the course, we can get the inverse matrix of B quickly.

$$E = \begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}, \; B_{new}^{-1} = EB_{old}^{-1} = \begin{bmatrix} 1 & \frac{2}{3} & -\frac{1}{3} \\ 0 & 1 & 0 \\ 0 & -\frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

According to the updated matrices and vectors, we could calculate $C_{NBV} - C_{BV}B^{-1}N$, and the result is

$$C_{NBV} - C_{BV}B^{-1}N = \begin{bmatrix} -1 & -3 \end{bmatrix}. \tag{7}$$

Hence, the bfs obtained now is the optimal solution and the final result is

$$X_{BV} = \begin{bmatrix} s_1 \\ x_2 \\ x_1 \end{bmatrix} = B^{-1}b = \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix} \tag{8}$$

and $\max z = 36(x_1 = 2, x_2 = 6)$ at this time.

1.3 Solution 2 (Graphic Method):

We can also solve this LP problem by graphic method. The result is shown in picture 1

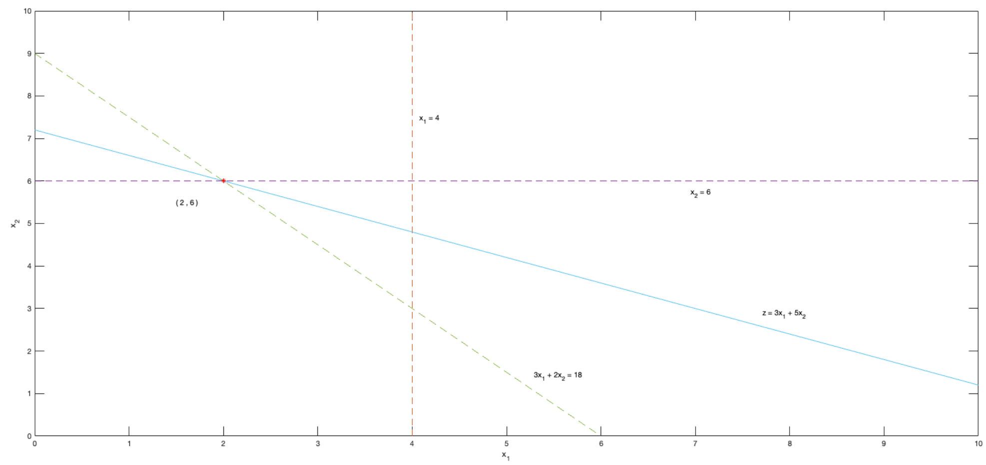

图 1: Results according to graphic method