Project in Operations Research

Team: one or two students in a group
Each group submits a report online. Please clearly write out the names of team members.

The **objective** of the project is to solve the simulated real-world problems by modeling, solving, and analyzing problems using OR, especially linear programming. You are trained to develop linear programming models that consider the key elements of the real problem; solve the models for their optimal solutions; interpret the models' solutions and infer solutions to the real-world problems.

Requirements:

a. Formulate the problems.
b. Use LINGO/LINDO or others to solve your problems and perform sensitivity analysis.
c. Interpret the solutions and infer the solutions and sensitivity analysis to answer the questions at the end of the problem description. In addition to the optimal solutions for the decisions and the optimal objective values, you also need to make the conclusions and suggestions to the decision makers based on the optimal solutions and sensitivity analysis.

Project report: The final report should be a well-typed printed document with a cover page. Handwriting report is not acceptable. Please use Times New Roman font, 12 pt in size, the single line spacing, and 1 in (2.54cm) for margins. Each paragraph should be correctly indented. The final report should at least include:

a. LP model and its illustration: decision variables, objective function, and constraints. Please provide the detailed (verbal) illustrations of the models, e.g., the objective function and constraints.
b. Computational results: the outputs from the Solver including clear solution report and sensitivity report. You should have them as appendix and answer the questions based on them. Do not include the entire solution report and sensitivity report in the main context in your report. Please try to present and organize the optimal solutions in the tables if appropriate.
c. Detailed interpretations, if applicable: interpretation of results, reduced cost for basic variables and non-basic variables, shadow prices for binding constraints and nonbinding constraints, the sensitivity analysis on the objective coefficients and the right-hand-side of constraints.
d. DO NOT submit the solution report, sensitivity report, etc. that are generated by the solver. Summarize and analyze the optimal solutions based on these reports.

Problem 1. Golf-Sport: Managing Operations

Golf-Sport is a small-sized company that produces high-quality components for people who build their own golf clubs and prebuilt sets of clubs. There are five components—steel shafts, graphite shafts, forged iron heads, metal wood heads, and metal wood heads with titanium inserts—made in three plants—Chandler, Glendale, and Tucson—in the Golf-Sport system. Each plant can produce any of the components, although each plant has a different set of individual constraints and unit costs. These constraints cover labor and packaging machine time (the machine is used by all components); the specific values for each component-plant combination are given in Tables 1–3. Note that even though the components are identical in the three plants, different production processes are used, and therefore the products use different amounts of resources in different plants.

TABLE 1
Product-Resource Constraints: Chandler

	Resources		
Products	Labor (Minutes/Unit)	Packing (Minutes/Unit)	Advertising ($/Unit)
Steel shafts	1	4	1.0
Graphite shafts	1.5	4	1.5
Forged iron heads	1.5	5	1.1
Metal wood heads	3	6	1.5
Titanium insert heads	4	6	1.9
Monthly availability (minutes)	12,000	20,000	—

TABLE 2
Product-Resource Constraints: Glendale

	Resources		
Products	Labor (Minutes/Unit)	Packing (Minutes/Unit)	Advertising ($/Unit)
Steel shafts	3.5	7	1.1
Graphite shafts	3.5	7	1.1
Forged iron heads	4.5	8	1.1
Metal wood heads	4.5	9	1.2
Titanium insert heads	5.0	7	1.9
Monthly availability (minutes)	15,000	40,000	—

TABLE 3
Product-Resource Constraints: Tucson

	Resources		
Products	Labor (Minutes/Unit)	Packing (Minutes/Unit)	Advertising ($/Unit)
Steel shafts	3	7.5	1.3
Graphite shafts	3.5	7.5	1.3
Forged iron heads	4	8.5	1.3
Metal wood heads	4.5	9.5	1.3
Titanium insert heads	5.5	8.0	1.9
Monthly availability (minutes)	22,000	35,000	—

Besides component sales, the company takes the components and manufactures sets of golf clubs. Each set requires 13 shafts, 10 iron heads, and 3 wood heads. All of the shafts in a set must be the same type (steel or graphite), and all of the wood heads must be the same type (metal or metal with inserts). Assembly times per month for the sets at each plant are shown in Table 4.

TABLE 4

Plant	Time (Minutes per set)	Total Time Available (Minutes)
Chandler	65	5,500
Glendale	60	5,000
Tucson	65	6,000

Each plant of Golf-Sport has a retail outlet to sell components and sets, and the specific plant is the only supplier for its retail outlet. The minimum and maximum amounts of demand for each plant–product pair are given in Table 5. Note that, although the minimums must be satisfied, you do not need to satisfy demand up to the maximum amount.

TABLE 5
Minimum and Maximum Product Demand per Month

	Store (or Plant)		
Products	Chandler	Glendale	Tucson
Steel shafts	[0, 2,000]	[0, 2,000]	[0, 2,000]
Graphite shafts	[100, 2,000]	[100, 2,000]	[50, 2,000]
Forged iron heads	[200, 2,000]	[200, 2,000]	[100, 2,000]
Metal wood heads	[30, 2,000]	[30, 2,000]	[15, 2,000]
Titanium insert heads	[100, 2,000]	[100, 2,000]	[100, 2,000]
Set: Steel, metal	[0, 200]	[0, 200]	[0, 200]
Set: Steel, insert	[0, 100]	[0, 100]	[0, 100]
Set: Graphite, metal	[0, 300]	[0, 300]	[0, 300]
Set: Graphite, insert	[0, 400]	[0, 400]	[0, 400]

This planning problem is for two months. The costs in Table 6 increase by 12% for the second month, and production times are stationary. Inventory costs are based on end-of-period inventory for each product set and cost out at 8% of the cost values in Table 6. Table 7 lists the revenue generated by each product. Initially, there is no inventory.

TABLE 6
Material, Production, and Assembly Costs ($) per Part or Set

	Plants		
Products	Chandler	Glendale	Tucson
Steel shafts	6	5	7
Graphite shafts	19	18	20
Forged iron heads	4	5	5
Metal wood heads	10	11	12
Titanium insert heads	26	24	27
Set: Steel, metal	178	175	180
Set: Steel, insert	228	220	240
Set: Graphite, metal	350	360	370
Set: Graphite, insert	420	435	450

TABLE 7
Revenue per Part or Set ($)

	Plants		
Products	Chandler	Glendale	Tucson
Steel shafts	10	10	12
Graphite shafts	25	25	30
Forged iron heads	8	8	10
Metal wood heads	18	18	22
Titanium insert heads	40	40	45
Set: Steel, metal	290	290	310
Set: Steel, insert	380	380	420
Set: Graphite, metal	560	560	640
Set: Graphite, insert	650	650	720

The corporation controls the capital available for expenses; the cash requirements for each product are given in the last column of Tables 1–3. There is a total of $20,000 available for advertising the production for the entire system during each month, and any money not spent in a month is not available the next month. The corporation also controls graphite. Each graphite shaft requires 4 ounces of graphite; a total of 1,000 pounds is available for each of the two months.

Your job is to determine a recommendation for the company. A recommendation must include a plan for production and sales. In addition, you should also address the following sensitivity-analysis issues in your recommendation:

- If you could get more graphite or advertising cash, how much would you like, how would you use it, and what would you be willing to pay?

- At what site(s) would you like to add extra packing machine hours, assembly hours, and/or extra labor hours? How much would you be willing to pay per hour and how many extra hours would you like?
- Marketing is trying to get Golf-Sport to consider an advertising program that promises a 50% increase in their maximum demand. Can we handle this with the current system or do we need more resources? How much more is the production going to cost if we take on the additional demand?

Note that (1) The production costs for the sets (products 6, 7, 8, 9) in the objective account for the assembly cost (the costs in Table 6 – the costs for the components) (2) The unit inventory costs for the sets are based on the total cost in Table 6 (i.e., the assembly cost and the costs of components). (3) 1 pound = 16 ounce (4) Decision variables are treated as continuous.

Problem 2. Production planning and inventory control

Consider a production line with four workstations, labelled j = 1, 2, 3, and 4, in tandem (all products flow through all four machines in order). Three different products, labelled i = A, B, and C, are produced on the line. The hours required on each workstation for each product and the net profits per unit sold (r_i) are given as follows:

i\j	1	2	3	4	r_i
A	2.4	1.1	0.8	3.0	$50
B	2.0	2.2	1.2	2.1	$65
C	0.9	0.9	1.0	2.5	$70

The number of hours available (c_{jt}) and the upper and lower limits on demand ($\bar{d}_{jt}$ and $\underline{d}_{jt}$) for each product over the next four quarters are:

t	1	2	3	4
c_{1t}	640	640	1280	1280
c_{2t}	640	640	640	640
c_{3t}	1920	1920	1920	1920
c_{4t}	1280	1280	1280	2560
$\bar{d}_{At}$	100	50	50	75
$\underline{d}_{At}$	0	0	0	0
$\bar{d}_{Bt}$	100	100	100	100
$\underline{d}_{Bt}$	20	20	20	25
$\bar{d}_{Ct}$	300	250	250	400
$\underline{d}_{Ct}$	0	0	0	50

The initial inventory is zero. All the decision variables are continuous.

a. Suppose we use a quarterly holding cost of $5 and a quarterly backorder cost of $10 per item on all products and allow backordering. Formulate an LP to maximize profit minus holding and backorder costs subject to the constraints on workstation capacity and min/max sales.
b. Solve your formulation in part (a). Which constraints are binding in your solution? Give a brief illustration of your solution.

c. Suppose that all the stockouts are lost sales. Use a quarterly holding cost of $5 and a one-time lost-sale cost of $5 per item (for counting the loss of reputation) on all the products. Formulate an LP to maximize profit minus holding and lost-sale costs subject to the constraints on workstation capacity and min/max sales.
d. Solve your formulation in part (c). Which constraints are binding in your solution? Give a brief illustration of your solution.
e. Compare and discuss the results from the models from part (a)-(b) and part (c)-(d).

Problem 3 (Worker scheduling) The Gotham City Police Department employs 30 police officers. Each officer works 5 days per week. The crime rate fluctuates with the day of the week, so the number of police officers required each day is different: Monday (day 1), 17; Tuesday (day 2), 23; Wednesday, 25; Thursday, 16; Friday, 21; Saturday, 28; Sunday (day 7), 18. The police department wants to schedule police officers to minimize the number whose days off are not consecutive. Formulate an LP that will accomplish this goal.