HW: Extension of revised simplex algorithm

Use either the Big-M method or two-phase method to solve the following LP problem.

$$\begin{aligned} \min \quad & z = 2x_1 + 3x_2 \\ \text{s.t.} \quad & \tfrac{1}{2}x_1 + \tfrac{1}{4}x_2 \le 4 \\ & x_1 + 3x_2 \ge 36 \\ & x_1 + x_2 = 10 \\ & x_1, x_2 \ge 0 \end{aligned}$$

$z = C_{BV} X_{BV} + C_{NBV} X_{NBV}$

$B X_{BV} + N X_{NBV} = b$

$X_{BV} = B^{-1}b - B^{-1}N X_{NBV}$

$C_{BV} B^{-1} b + (C_{NBV} - C_{BV} B^{-1} N) X_{NBV}$

$\min \quad z = 2x_1 + 3x_2 - Ma_2 - Ma_3$

$\text{s.t.} \quad \frac{1}{2}x_1 + \frac{1}{4}x_2 + s_1 = 4$

$x_1 + 3x_2 - s_2 + a_2 = 36$

$x_1 + x_2 + a_3 = 10$

$x_1, x_2, s_1, s_2, a_1, a_2 \ge 0$

① $BV = (s_1, a_2, a_3)$ $\quad NBV = (x_1, x_2, s_2)$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad N = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & 0 \\ 1 & 3 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$C_{NBV} - C_{BV} B^{-1} N = [0, -M, -M] - [2, 3, 0] \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & 0 \\ 1 & 3 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= [0, -M, -M] - [4, \tfrac{19}{2}, -3]$$

$$= [-4, -M - \tfrac{19}{2}, -M + 3]$$

x_2 入

$$B^{-1}a_2 = \begin{bmatrix} \frac{1}{4} \\ 3 \\ 1 \end{bmatrix} \qquad B^{-1}b = \begin{bmatrix} 4 \\ 36 \\ 10 \end{bmatrix} \qquad \begin{bmatrix} 16 \\ 12 \\ 10 \end{bmatrix}$$

a_3 出