2022-2023 Autumn Semester

Operation Research

Assignment 5

-MCNFP-

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1 Problem

1.1 Question

State University has three professors who each teach four courses per year. Each year, four sections of marketing, finance, and production must be offered. At least one section of each class must be offered during each semester (fall and spring). Each professor's time preference and preference for teaching various courses are given in table 1. The total satisfaction a professor earns teaching a class is the sum of the semester satisfaction and the course satisfaction. For example, professor 1 derives a satisfaction of $3 + 6 = 9$ from teaching marketing during the fall semester.

Table 1: Preference

	Professor 1	Professor 2	Professor 3
Fall Preference	3	5	4
Spring Preference	4	3	4
Marketing	6	4	5
Finance	5	6	4
Production	4	5	6

Formulate an MCNFP that can be used to assign professors to courses so as to maximize the total satisfaction of the three professors and solve the model.

1.2 Solution

1.2.1 Analysis

We could construct a graph as figure 1 according to the problem.

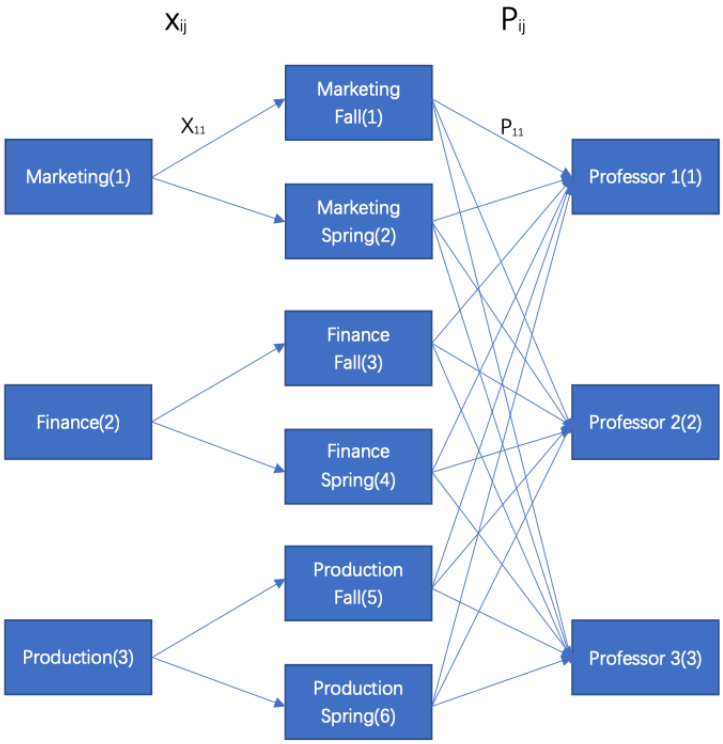

Figure 1: The graph of the LP Problem

This is a two-layer figure. In the left part, the nodes are three course: Marketing, Finance and Production, which is represented by number 1, 2, 3 respectively. In the center part, the nodes stand for the courses belong to different semesters, and we also use integer $1, \cdots, 6$ to represent from up to down. In the right part, the nodes are three professors, ordered by number 1, 2, 3. The arches between the first(left) layer are noted by $x_{i,j}$, which means that we arrange three courses to different semesters. The arches between the second(right) layer are noted by $p_{i,j}$, which means that professors choose the courses in the detailed semester to teach.

1.2.2 Model

Since we have to maximize the total satisfaction of the three professors. So we could obtain the objective function (1-5).

$$\max z = \sum_{i=1}^{6} \sum_{j=1}^{3} s_{i.j} p_{i,j}; \tag{1}$$

where $s_{i,j}$ is the preference of professor i to teach the corresponding course j in the graph 1.

By the general model of an MCNFP, we could have the constraints (2).

$$\sum_j x_{i,j} = 4, \ \forall \ i. \tag{2}$$

The constraints (2) means that each course should be offered four sections a year in different semesters.

$$x_{i,j} = \sum_k p_{j,k}, \ \forall \ i, j. \tag{3}$$

The constraints (3) means that the amount flow in should equal to the amount flow out in the general MCNFP. In this question, it means that the number of courses allocated to different semesters should be equal to the course arrangement of professors.

$$\sum_i p_{i,j} = 4, \ \forall \ j. \tag{4}$$

The constraints (4) means that each professor has to teach four courses per year.

$$x_{i,j} \geq 1, \ \forall \ i, j. \tag{5}$$

The constraints (5) means that At least one section of each class must be offered during each semester.

Apparently, all of the decision variables $x_{i,j}, \ p_{i,j}$ should be nonnegative.

In summary, we have the following MCNFP model.

$$\max z = \sum_{i=1}^{6} \sum_{j=1}^{3} s_{i.j} p_{i,j}$$

$$\begin{aligned} s.t. \quad & \sum_j x_{i,j} = 4, \ \forall \ i; \\ & x_{i,j} = \textstyle\sum_k p_{j,k}, \ \forall \ i, j; \\ & \sum_i p_{i,j} = 4, \ \forall \ j; \\ & x_{i,j} \geq 1, \ \forall \ i, j \\ & x_{i,j}, \ p_{i,j} \geq 0. \end{aligned}$$

1.3 Result

We could solve the detailed problem easily by the use of LINGO, and the concrete arrangement is in table 2.

Table 2: The course arrangement

Professors	Courses	Number of Section
1	Marketing(Spring)	3
	Finance(Spring)	1
2	Marketing(Fall)	1
	Finance(Fall)	3
3	Production(Fall)	3
	Production(Spring)	1

The preference of this arrangement is 121 in total.