Cheat Sheet PROE (parte 3)

Near-field region (Fresnel)

$$\nabla^2 \underline{\mathbf{A}} + k_0^2 \underline{\mathbf{A}} = -\mu_0 \underline{\mathbf{J}}_{\text{Hertz}}, \qquad \boxed{\underline{\mathbf{A}} = \mu_0 \underline{I}_0 \, dl \, \frac{e^{-jkr}}{4\pi r} \hat{\mathbf{z}}.}$$

$$\boxed{\begin{aligned} \underline{\mathbf{E}} &= \frac{1}{j\omega\varepsilon_0}(\nabla \times \underline{\mathbf{H}} - \underline{\mathbf{J}}_{\text{Hertz}}) \\ \underline{\mathbf{H}} &= \frac{1}{\mu_0} \nabla \times \underline{\mathbf{A}}. \end{aligned}} \quad \text{for} \begin{cases} i = \text{Re}\{\underline{I}_0 e^{j\omega t}\} \text{ and} \\ \underline{\mathbf{J}}_{\text{Hertz}} = \underline{I}_0 \, dl \, \hat{\mathbf{z}} \, \delta(\mathbf{r}). \end{cases}$$

$$\underline{\mathbf{H}} = \underline{H}_\phi \hat{\boldsymbol{\phi}}, \quad \boxed{\underline{H}_\phi = jk_0 \underline{I}_0 \, dl \, \sin\theta \frac{e^{-jk_0 r}}{4\pi r}\left[1 + \frac{1}{jk_0 r}\right].}$$

$$\underline{\mathbf{E}} = \underline{E}_r \hat{\mathbf{r}} + \underline{E}_\theta \hat{\boldsymbol{\theta}}, \quad \boxed{\underline{E}_r = \eta_0 \underline{I}_0 \, dl \, \frac{\cos\theta}{r} \frac{e^{-jk_0 r}}{4\pi r}\left[1 + \frac{1}{jk_0 r}\right],}$$

$$\boxed{\underline{E}_\theta = j\eta_0 k_0 \underline{I}_0 \, dl \, \sin\theta \frac{e^{-jk_0 r}}{4\pi r}\left[1 + \frac{1}{jk_0 r} + \frac{1}{(jk_0 r)^2}\right].}$$

Far-field region (Fraunhofer)

$$\boxed{r \gg L, \quad r \gg \lambda_0, \quad r > 2L^2/\lambda_0.}$$

$$\underline{\mathbf{E}}\Big|_{\text{far-field}} \cdot \hat{\mathbf{r}} = 0, \quad \underline{\mathbf{H}}\Big|_{\text{far-field}} \cdot \hat{\mathbf{r}} = 0.$$

$$\underline{\mathbf{E}}\Big|_{\text{far-field}} = \underline{E}_\theta \hat{\boldsymbol{\theta}} + \underline{E}_\phi \hat{\boldsymbol{\phi}}, \quad \underline{\mathbf{H}}\Big|_{\text{far-field}} = \underline{H}_\theta \hat{\boldsymbol{\theta}} + \underline{H}_\phi \hat{\boldsymbol{\phi}}.$$

$$\boxed{\underline{\mathbf{E}}\Big|_{\text{far-f.}} = \eta_0 \underline{\mathbf{H}}(\mathbf{r})\Big|_{\text{far-f.}} \times \hat{\mathbf{r}}, \quad \underline{\mathbf{H}}\Big|_{\text{far-f.}} = \frac{1}{\eta_0} \hat{\mathbf{r}} \times \underline{\mathbf{E}}\Big|_{\text{far-f.}}.}$$

$$\left[\frac{\|\underline{\mathbf{E}}\|}{\|\underline{\mathbf{H}}\|}\right]_{\text{far}} = \eta_0, \quad \mathbf{S}_{\text{av}}\Big|_{\text{far}} = S_r \hat{\mathbf{r}} = \frac{\|\underline{\mathbf{E}}\|^2}{2\eta_0} \hat{\mathbf{r}} = \frac{\|\underline{\mathbf{H}}\|^2 \eta_0}{2} \hat{\mathbf{r}}.$$

$$\underline{\mathbf{E}}\Big|_{\text{far-field}} = j\eta_0 k_0 \underline{I}_0 \mathbf{h_e}(\theta,\phi) \frac{e^{-jk_0 r}}{4\pi r},$$

$$\boxed{\mathbf{h_e}(\theta,\phi) = h_{e,\theta} \hat{\boldsymbol{\theta}} + h_{e,\phi} \hat{\boldsymbol{\phi}}}$$

$$\mathcal{P}_{rad} \equiv \iint \mathbf{S}_{av} \cdot \hat{\mathbf{n}} \, dS \overset{\hat{\mathbf{r}} = \hat{\mathbf{n}}}{=} \iint S_r r^2 \sin\theta \, d\theta \, d\phi$$

$$U(\theta,\phi)\Big|_{\text{far-field}} \equiv S_r r^2 = \frac{\eta_0 |\underline{I}_0|^2}{2} \left(\frac{k_0}{4\pi}\right)^2 \|\mathbf{h}_e(\theta,\phi)\|^2$$

$$\therefore \mathcal{P}_{rad} = \iint U(\theta,\phi) \, d\Omega, \quad d\Omega = \sin\theta \, d\theta \, d\phi.$$

$$U(\theta,\phi) = \frac{d\mathcal{P}_{rad}}{d\Omega} \text{ [W/sr]} \rightarrow \text{HPBW: } U(\theta,\phi) = U_{\max}/2.$$

$$\text{FNBW} = 2 \cdot \text{HPBW}, \quad \text{SLL} = U_{s,\max}/U_{\max}.$$

$$\underline{V}_0 = Z_a \underline{I}_0, \quad Z_a = R_a + jX_a, \quad X_a = -Z_0 \cot(k_0 L/2).$$

$$\boxed{\mathcal{P}_{in} = \mathcal{P}_{rad} + \mathcal{P}_{loss}}$$

$$\mathcal{P}_{in} = \frac{1}{2}\text{Re}\{\underline{V}_0 \underline{I}_0^*\} \rightarrow \mathcal{P}_{in} = \frac{1}{2} R_a |\underline{I}_0|^2.$$

$$\mathcal{P}_{rad} = \frac{\eta_0}{4\pi} |I_m|^2 \int_0^\pi |P(\theta)|^2 \sin\theta \, d\theta$$

$$\mathcal{P}_{\text{loss}} = \int_{-L/2}^{L/2} \frac{1}{2} R |\underline{I}(z')|^2 dz', \quad R \text{ [}\Omega\text{/m]}.$$

$$\boxed{R_a = R_{rad} + R_{loss}}$$

$$R_{rad} = 2\mathcal{P}_{rad}/|\underline{I}_0|^2, \quad R_{\text{loss}} = 2\mathcal{P}_{\text{loss}}/|\underline{I}_0|^2.$$

$$R_{rad} = \frac{\eta_0}{2\pi} \sin^{-2}\left(\frac{k_0 L}{2}\right) \int_0^\pi |P(\theta)|^2 \sin\theta \, d\theta.$$

$$R_{\text{loss}} = \int_{-L/2}^{L/2} R \frac{|\underline{I}(z')|^2}{|\underline{I}_0|^2} dz', \quad R \text{ [}\Omega\text{/m]}.$$

$$e = \frac{\mathcal{P}_{rad}}{\mathcal{P}_{in}} = \frac{\mathcal{P}_{rad}}{\mathcal{P}_{rad} + \mathcal{P}_{loss}}, \quad e = \frac{R_{rad}}{R_{in}} = \frac{R_{rad}}{R_{rad} + R_{loss}}.$$

$$g(\theta,\phi) = \frac{U_{\theta,\phi}}{U_{iso}} = \frac{4\pi U_{\theta,\phi}}{\mathcal{P}_{rad}}, \quad D = \max g(\theta,\phi) = \frac{4\pi U_{\max}}{\mathcal{P}_{rad}}.$$

$$S_{\text{av}} = \frac{\mathcal{P}_{in}}{4\pi r^2} G(\theta,\phi), \quad G(\theta,\phi) = e g(\theta,\phi), \quad G_m = eD.$$

Linear Dipoles

$$\boxed{\underline{I}(z') = \underline{I}_m \sin\Big[k_0(L/2 - |z'|)\Big], \; \underline{I}_m = \underline{I}_0 / \sin(k_0 L/2)}$$

$$h_e(\theta) = \sin\theta \int_{-L/2}^{L/2} \frac{\underline{I}(z')}{\underline{I}_0} e^{jkz'\cos\theta} dz'.$$

$$\text{Hertz dip. } (L = dl) \implies \underline{I}(z') = \underline{I}_0 = \text{const.}$$

$$h_e(\theta) \approx \sin\theta \int_{-L/2}^{L/2} \frac{\underline{I}(z')}{\underline{I}_0} dz' = dl \sin\theta.$$

$$L \ll \lambda_0 \implies \underline{I}(z') \approx \underline{I}_0(1 - |2z'|), \quad \underline{I}_0 = \underline{I}_m k_0 L/2.$$

$$h_e(\theta) \approx \sin\theta \int_{-L/2}^{L/2} \frac{\underline{I}(z')}{\underline{I}_0} dz' = \frac{L}{2} \sin\theta.$$

$$P(\theta) = \frac{\cos([k_0 L/2]\cos\theta) - \cos(k_0 L/2)}{\sin\theta}.$$

$$h_e(\theta) = \frac{2\underline{I}_m}{\underline{I}_0 k_0} P(\theta), \quad U(\theta) = \frac{\eta_0}{8\pi^2} |\underline{I}_m|^2 |P(\theta)|^2.$$

$$R_{rad}\Big|_{\text{Hertz dip.}} = \frac{2\pi}{3} \eta_0 \left(\frac{L}{\lambda_0}\right)^2 = 80\pi^2 \left(\frac{L}{\lambda_0}\right)^2$$

$$R_{rad}\Big|_{\text{short dip.}} = R_{rad}\Big|_{\text{Hertz dip.}}/4 = 20\pi^2 \left(\frac{L}{\lambda_0}\right)^2$$

$$R_{rad}\Big|_{\lambda_0/2} \approx 60 \int_0^\pi |P(\theta)|^2 \sin\theta \, d\theta$$

Length (L)	Radiation Resistance (R_{rad})
λ_0	$\rightarrow \infty$
$\lambda_0/2$	73.13 [Ω]
$3\lambda_0/2$	105.3 [Ω]

$$g\Big|_{\text{short/H dip.}} = \frac{3}{2} \sin^2\theta, \quad D\Big|_{\text{short/H dip.}} = \frac{3}{2}.$$

$$g(\theta)\Big|_{\lambda/2} = 1.64 \Big|\cos\left([\pi/2]\cos\theta\right)/\sin\theta\Big|^2, \quad D\Big|_{\lambda/2} = 1.64.$$

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https://github.com/Kons-5/IST-PROE-Notes