Cheat Sheet PROE (parte 2)

$$\frac{\partial v(x,t)}{\partial x} = -Ri(x,t) - L\frac{\partial i(x,t)}{\partial t}$$
$$\frac{\partial i(x,t)}{\partial x} = -Gv(x,t) - C\frac{\partial v(x,t)}{\partial t}$$

$$\gamma = \alpha + j\beta = \sqrt{Z_l Y_l} = \sqrt{(R+j\omega L)(G+j\omega C)}$$
$$Z_0 = \frac{Z_l}{\gamma} = \sqrt{\frac{Z_l}{Y_l}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\omega L \gg R, \quad \omega C \gg G:$$
$$\alpha \approx \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}, \qquad \beta \approx \omega\sqrt{LC}.$$
$$x \ll 1: \quad 1/\sqrt{1+x} \approx 1-x, \quad \sqrt{1+x} \approx 1 + x/2$$

$$\frac{R}{L} = \frac{G}{C}$$
$$Z_0 = \sqrt{\frac{L}{C}}, \qquad \alpha = R\sqrt{\frac{C}{L}}, \qquad \beta = \omega\sqrt{LC}.$$

$$\underline{V}(x) = \underline{V}_0^+ e^{-\gamma x} + \underline{V}_0^- e^{\gamma x} \quad = \underline{V}^+(x) + \underline{V}^-(x)$$
$$\underline{I}(x) = \frac{\underline{V}_0^+}{Z_0} e^{-\gamma x} - \frac{\underline{V}_0^-}{Z_0} e^{\gamma x} \quad = \underline{I}^+(x) + \underline{I}^-(x)$$

$$Z(x) = \frac{\underline{V}(x)}{\underline{I}(x)} \qquad \rho_V(x) = \frac{\underline{V}^-(x)}{\underline{V}^+(x)} = \rho_{V\,0} e^{2\gamma x}$$
$$Z(x) = Z_0 \frac{1+\rho_V(x)}{1-\rho_V(x)} \qquad \rho_V(x) = \frac{Z(x) - Z_0}{Z(x) + Z_0}$$
$$0 \leqslant |\rho_V| \leqslant 1$$

$$d = x_2 - x_1:$$
$$\rho_{V\,1} = \rho_{V\,2} e^{-2\gamma d} \iff \rho_{V\,2} = \rho_{V\,1} e^{2\gamma d}$$
$$Z_1 = Z_0 \frac{Z_2 + Z_0 \tanh(\gamma d)}{Z_0 + Z_2 \tanh(\gamma d)}, \quad Y_1 = \frac{Y_2 + Y_0 \tanh(\gamma d)}{Y_0 + Y_2 \tanh(\gamma d)}$$

$$Z_2 = Z_0 \implies Z_1 = Z_0, \quad \rho_V(x)\big|_{\text{matched}} = 0.$$
$$Z_2 = 0 \implies \rho_{V\,\text{c.c.}} = -1$$
$$Z_2 \to +\infty \implies \rho_{V\,\text{o.c.}} = +1$$

$$\gamma = j\beta, \quad Z_0 \in \mathbb{R}_{>0}$$
$$|\underline{V}^+(x)| = \text{const.}, \qquad |\rho_V(x)| = \text{const.}$$
$$Z_1 = Z_0 \frac{Z_2 + jZ_0\tan(\beta d)}{Z_0 + jZ_2\tan(\beta d)}, \quad \rho_{V\,1} = \rho_{V\,2} e^{-j2\beta d}.$$

$$\begin{bmatrix} \underline{V}_1 \\ \underline{I}_1 \end{bmatrix} = \begin{bmatrix} \cos(\beta d) & jZ_0\sin(\beta d) \\ j\sin(\beta d)/Z_0 & \cos(\beta d) \end{bmatrix} \begin{bmatrix} \underline{V}_2 \\ \underline{I}_2 \end{bmatrix}$$

$$d = \lambda/2: \quad Z_1 = Z_2, \quad \rho_{V\,1} = \rho_{V\,2}$$
$$d = \lambda/4: \quad Z_1 = Z_0^2/Z_2, \quad \rho_{V\,1} = -\rho_{V\,2}$$
$$\boxed{Z_1\big|_{d=l} = Z_1\big|_{d=l+n\frac{\lambda}{2}}, \quad \forall n \in \mathbb{Z}}$$

$$\underline{V}_1 = \underline{V}_g - Z_g \underline{I}_1,$$
$$\underline{V}_1^+ = \frac{\underline{V}_g}{1 + Z_g/Z_0 + (1 - Z_g/Z_0)\rho_{V\,1}}.$$

If $Z_g = Z_0$, then $\underline{V}_1^+ = \underline{V}_g/2$.

$$V_{\max} = \max|\underline{V}(x)|, \quad V_{\min} = \min|\underline{V}(x)|.$$
$$\text{SWR} \equiv \frac{V_{\max}}{V_{\min}} = \frac{1+|\rho_V|}{1-|\rho_V|} \implies |\rho_V| = \frac{\text{SWR}-1}{\text{SWR}+1}$$
$$Z_{\max} = \text{SWR}\cdot Z_0, \qquad Z_{\min} = Z_0/\text{SWR}$$

$$d_{V_{\max}, I_{\min}, Z_{\max}} = \frac{\arg(\rho_{V\,\text{load}})\cdot\lambda}{4\pi} + n\frac{\pi}{2}$$
$$d_{V_{\min}, I_{\max}, Z_{\min}} = \frac{\arg(\rho_{V\,\text{load}})\cdot\lambda}{4\pi} + \frac{\lambda}{4} + n\frac{\pi}{2}$$

$$\mathcal{P}(x) = \frac{1}{2}\operatorname{Re}\{\underline{V}(x)\underline{I}^*(x)\}$$
$$= \frac{1}{2}\operatorname{Re}\left\{\frac{|\underline{V}^+(x)|^2}{Z_0^*} - \frac{|\underline{V}^-(x)|^2}{Z_0^*} + \frac{1}{Z_0^*}\left[\underline{V}^{+*}(x)\underline{V}^-(x) - \underline{V}^+(x)\underline{V}^{-*}(x)\right]\right\}.$$

$$\gamma = j\beta, \quad Z_0 \in \mathbb{R}_{>0}$$
$$\mathcal{P} = \mathcal{P}^+ - \mathcal{P}^- = \mathcal{P}^+\left(1 - |\rho_{V\,2}|^2\right),$$
$$\mathcal{P}^+ = \frac{1}{2}\frac{|\underline{V}^+|^2}{Z_0}, \quad \mathcal{P}^- = \frac{1}{2}\frac{|\underline{V}^-|^2}{Z_0}.$$

$$\gamma = \alpha + j\beta$$
$$\mathcal{P}_1 \approx \frac{1}{2}\frac{|\underline{V}_1^+|^2}{Z_0}(1 - |\rho_{V\,1}|^2), \quad \mathcal{P}_2 \approx \frac{1}{2}\frac{|\underline{V}_2^+|^2}{Z_0}(1 - |\rho_{V\,2}|^2).$$
$$\frac{\mathcal{P}_1}{\mathcal{P}_2} = \frac{(1 - e^{-4\alpha d}|\rho_{V\,2}|^2)}{e^{-2\alpha d}(1 - |\rho_{V\,2}|^2)} = \frac{e^{2\alpha d} + e^{-2\alpha d}|\rho_{V\,2}|^2}{1 - |\rho_{V\,2}|^2}$$
$$\approx \boxed{1 + 2\alpha d\frac{1 + |\rho_{V\,2}|^2}{1 - |\rho_{V\,2}|^2}}.$$
$$\mathcal{P}_{\text{dis}} = \mathcal{P}_1 - \mathcal{P}_2 \simeq \mathcal{P}_2 \cdot \alpha d\left(\text{SWR} + \frac{1}{\text{SWR}}\right)$$

$$P_L = \frac{|\underline{V}_g|^2}{2}\frac{R_L}{(R_L + R_g)^2 + (X_L + X_g)^2}$$
$$P_{L,\max} = P_a = \frac{|\underline{V}_g|^2}{8R_g}, \text{ for } Z_L = Z_g^* = R_g - jX_g.$$

$$\lambda_0 = \frac{c}{f_0} \to l_T = \frac{\lambda_0}{4} \to Z_{0T} = \sqrt{Z_0 Z_L}, \quad Z_L \in \mathbb{R}_{>0}$$

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https://github.com/Kons-5/IST-PROE-Notes