Cheat Sheet PROE (parte 1)

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\{\underline{\mathbf{E}}(\mathbf{r})e^{j\omega t}\}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t},\quad \nabla\times\mathbf{H} = \mathbf{J}_{\text{ext}} + \sigma\mathbf{E} + \frac{\partial\mathbf{D}}{\partial t}$$

$$\nabla\times\underline{\mathbf{E}} = -j\omega\underline{\mathbf{B}},\quad \nabla\times\underline{\mathbf{H}} = \underline{\mathbf{J}}_{\text{ext}} + j\omega\varepsilon_{\text{ef}}(\omega)\underline{\mathbf{E}}$$

$$\varepsilon_{\text{ef}}(\omega) = \varepsilon_{\text{d}}(\omega) + \frac{\sigma_c(\omega)}{j\omega} = \varepsilon'_{\text{ef}}(\omega) - j\varepsilon''_{\text{ef}}(\omega)$$

$$\tan\theta_{\text{loss}} = \frac{\varepsilon''_{\text{ef}}(\omega)}{\varepsilon'_{\text{ef}}(\omega)} = \frac{\omega\varepsilon''_{\text{d}}(\omega) + \sigma_c(\omega)}{\omega\varepsilon'_{\text{d}}(\omega)} \cong \frac{\sigma}{\omega\varepsilon_r\varepsilon_0}$$

$$\lambda = \frac{2\pi}{k} = \frac{c}{f},\quad c = \frac{1}{\sqrt{\mu\varepsilon}},\quad \frac{T}{\lambda} = \frac{2\pi}{\omega} = \frac{2\pi}{k}\frac{n}{c} = \lambda\frac{n}{c}$$

$$\mathbf{J}_{\text{ext}} = 0 \implies \mathbf{H} = \frac{1}{\eta}\hat{\mathbf{d}}\times\mathbf{E},\quad \mathbf{E} = \eta\mathbf{H}\times\hat{\mathbf{d}}$$

$$\eta = \frac{\|\mathbf{E}\|}{\|\mathbf{H}\|} = \sqrt{\frac{\mu}{\varepsilon}},\quad k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \omega\sqrt{\mu\varepsilon}$$

$$\gamma = jk = \alpha + j\beta,\quad \delta_{\text{skin}} = \frac{1}{\alpha}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_{ph}}{f}$$

$$\underline{\mathbf{J}}_{\text{ext}} = 0 \implies \underline{\mathbf{H}} = \frac{1}{\eta}\hat{\mathbf{d}}\times\underline{\mathbf{E}},\quad \underline{\mathbf{E}} = \eta\underline{\mathbf{H}}\times\hat{\mathbf{d}},$$

$$\eta = \sqrt{\frac{\mu(\omega)}{\varepsilon_{\text{ef}}(\omega)}},\quad k = \omega\sqrt{\mu(\omega)\varepsilon_{\text{ef}}(\omega)},\quad v_g = \frac{d\omega}{d\beta}$$

$$\varepsilon_0 \approx 8.854\cdot 10^{-12}\ [\text{F/m}],\qquad \mu_0 \approx 4\pi\cdot 10^{-7}\ [\text{H/m}]$$

$$c_0 = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = 3\cdot 10^8\ [\text{m/s}],\quad \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 120\pi\ [\Omega]$$

$$c = \frac{c_0}{n},\quad \eta = \frac{\eta_0}{n},\quad \lambda = \frac{\lambda_0}{n},\quad k = nk_0$$

$$\varepsilon_{\text{ef}}(\omega) \cong \varepsilon + \frac{\sigma}{j\omega},\quad \varepsilon = \varepsilon_r\varepsilon_0;\quad \mu(\omega)\cong\mu$$

$$\gamma = j\omega\sqrt{\mu\left(\varepsilon + \frac{\sigma}{j\omega}\right)} = j\omega\sqrt{\mu\varepsilon}\sqrt{1 + \frac{\sigma}{j\omega\varepsilon}}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon + \frac{\sigma}{j\omega}}} = \sqrt{\frac{\mu}{\varepsilon}}\frac{1}{\sqrt{1 + \frac{\sigma}{j\omega\varepsilon}}}$$

- $\sigma = 0$:

$$\gamma = j\omega\sqrt{\mu\varepsilon},\quad \eta = \sqrt{\frac{\mu}{\varepsilon}} = \eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

- $\sigma \neq 0$, $\sigma/\omega\varepsilon \ll 1$:

$$\gamma_{\text{g.d.}} \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} + j\frac{\omega}{c_0}\sqrt{\mu_r\varepsilon_r},\quad \eta_{\text{g.d.}} \approx \sqrt{\frac{\mu}{\varepsilon}}\left(1 + j\frac{\sigma}{2\omega\varepsilon}\right)$$

- $\sigma \neq 0$, $\sigma/\omega\varepsilon \gg 1$:

$$\gamma_{\text{g.c.}} \approx \sqrt{\frac{\sigma\omega\mu}{2}}(1+j),\quad \eta_{\text{g.c.}} \approx \sqrt{\frac{\omega\mu}{2\sigma}}(1+j)$$

$$x \ll 1:\quad 1/\sqrt{1+x} \approx 1 - x,\quad \sqrt{1+x} \approx 1 + x/2$$

$$\hat{\mathbf{d}} = \hat{\mathbf{u}}_{\mathbf{1}}\times\hat{\mathbf{u}}_{\mathbf{2}} \to \frac{\underline{\mathrm{E}}_2}{\underline{\mathrm{E}}_1} = |p|e^{j\phi}$$

○ $\phi = \pm\pi/2$ e $|p| = 1$

→ $\phi = 0$, $\phi = \pi$, $|p| = 0$, $|p| = \infty$

↺ $0 < \phi < \pi$

↻ $-\pi < \phi < 0$

$$\mathbf{S} = \mathbf{E}\times\mathbf{H}\quad [\text{W/m}^2]$$

$$\mathcal{P}_\Sigma = \int_\Sigma \mathbf{S}\cdot\hat{\mathbf{n}}\,dA \overset{\hat{\mathbf{d}}=\hat{\mathbf{n}}}{=} \|\mathbf{S}\|\cdot A$$

$$\mathbf{S}_{\text{av}} = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{\eta}\right\}\|\underline{\mathbf{E}}\|^2\hat{\mathbf{d}} = \frac{1}{2}\mathrm{Re}\{\eta\}\|\underline{\mathbf{H}}\|^2\hat{\mathbf{d}}$$

$$\underline{\mathbf{E}}^{\text{ref}} = \rho\underline{\mathbf{E}}^{\text{inc}}\qquad \underline{\mathbf{E}}^{\text{tx}} = \tau\underline{\mathbf{E}}^{\text{inc}}$$

$$\rho = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}\qquad \tau = 1 + \rho = \frac{2\eta_2}{\eta_1 + \eta_2}$$

$$\hat{\mathbf{u}}^l_\parallel \times \hat{\mathbf{u}}_\perp = \hat{\mathbf{d}}^l,\quad l = i, r, t$$

$$\boxed{\begin{aligned}
\underline{\mathbf{E}}^{\text{inc}} &= \left(\underline{E}^i_{0\parallel}\hat{\mathbf{u}}_\parallel + \underline{E}^i_{0\perp}\hat{\mathbf{u}}_\perp\right)e^{-\gamma_1\hat{\mathbf{d}}^i\cdot\mathbf{r}},\\
\underline{\mathbf{E}}^{\text{ref}} &= \left(\rho_\parallel\,\underline{E}^i_{0\parallel}\hat{\mathbf{u}}^r_\parallel + \rho_\perp\,\underline{E}^i_{0\perp}\hat{\mathbf{u}}_\perp\right)e^{-\gamma_1\hat{\mathbf{d}}^r\cdot\mathbf{r}},\\
\underline{\mathbf{E}}^{\text{tx}} &= \left(\tau_\parallel\,\underline{E}^i_{0\parallel}\hat{\mathbf{u}}^t_\parallel + \tau_\perp\,\underline{E}^i_{0\perp}\hat{\mathbf{u}}_\perp\right)e^{-\gamma_2\hat{\mathbf{d}}^t\cdot\mathbf{r}}.
\end{aligned}}$$

$$\rho_\parallel = \frac{\eta_1\cos\theta_i - \eta_2\cos\theta_t}{\eta_1\cos\theta_i + \eta_2\cos\theta_t},\qquad \tau_\parallel = \frac{\eta_2}{\eta_1}(1 + \rho_\parallel)$$

$$\rho_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t},\qquad \tau_\perp = 1 + \rho_\perp$$

$$\theta_i = \theta_r\qquad n_1\sin\theta_i = n_2\sin\theta_t$$

$$\theta_B = \arctan\left(\frac{n_2}{n_1}\right) \to |\rho_\parallel| = 0$$

$$n_1\sin\theta_B = n_2\cos\theta_B$$

$$n_1 > n_2,\quad \theta_t = 90^\circ \to \theta_c = \arcsin\left(\frac{n_2}{n_1}\right)$$

$$\theta_i \geqslant \theta_c \implies |\rho_\parallel| = |\rho_\perp| = 1$$

$$\|\mathbf{S}^{\text{ref}}_{\text{av}}\| \overset{\bigcirc}{=} \|\mathbf{S}^{\text{inc}}_{\text{av}}\|\left(\frac{|\rho_\perp|^2 + |\rho_\parallel|^2}{2}\right)$$

$$\|\mathbf{S}^{\text{tx}}_{\text{av}}\| \overset{\bigcirc}{=} \|\mathbf{S}^{\text{inc}}_{\text{av}}\|\frac{\eta_1}{\eta_2}\left(\frac{|\tau_\perp|^2 + |\tau_\parallel|^2}{2}\right)$$

$$\%\mathcal{P}_{\text{ref}} = \frac{\|\mathbf{S}^{\text{ref}}_{\text{av}}\|}{\|\mathbf{S}^{\text{inc}}_{\text{av}}\|}\times 100\%$$

$$\%\mathcal{P}_{\text{tx}} = (100\% - \%\mathcal{P}_{\text{ref}})$$

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https://github.com/Kons-5/IST-PROE-Notes