Cheat Sheet PROE (parte 1)

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\{\underline{\mathbf{E}}(\mathbf{r})e^{j\omega t}\}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}, \quad \nabla\times\mathbf{H} = \mathbf{J}_{\text{ext}} + \sigma\mathbf{E} + \frac{\partial\mathbf{D}}{\partial t}$$

$$\nabla\times\underline{\mathbf{E}} = -j\omega\underline{\mathbf{B}}, \quad \nabla\times\underline{\mathbf{H}} = \underline{\mathbf{J}}_{\text{ext}} + j\omega\varepsilon_{\text{ef}}(\omega)\underline{\mathbf{E}}$$

$$\varepsilon_{\text{ef}}(\omega) = \varepsilon_{\text{d}}(\omega) + \frac{\sigma_c(\omega)}{j\omega} = \varepsilon'_{\text{ef}}(\omega) - j\varepsilon''_{\text{ef}}(\omega)$$

$$\tan\theta_{\text{loss}} = \frac{\varepsilon''_{\text{ef}}(\omega)}{\varepsilon'_{\text{ef}}(\omega)} = \frac{\omega\varepsilon''_{\text{d}}(\omega) + \sigma_c(\omega)}{\omega\varepsilon'_{\text{d}}(\omega)} \cong \frac{\sigma}{\omega\varepsilon_r\varepsilon_0}$$

$$\lambda = \frac{2\pi}{k} = \frac{c}{f}, \quad c = \frac{1}{\sqrt{\mu\varepsilon}},$$

$$c = \frac{c_0}{n}, \quad \eta = \frac{\eta_0}{n}, \quad \lambda = \frac{\lambda_0}{n}, \quad k = nk_0$$

$$\eta = \frac{\|\mathbf{E}\|}{\|\mathbf{H}\|} = \sqrt{\frac{\mu}{\varepsilon}}, \quad k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \omega\sqrt{\mu\varepsilon}$$

$$\mathbf{J}_{\text{ext}} = 0 \implies \mathbf{H} = \frac{1}{\eta}\hat{\mathbf{d}}\times\mathbf{E}, \quad \mathbf{E} = \eta\mathbf{H}\times\hat{\mathbf{d}}$$

$$\gamma = jk = \alpha + j\beta, \quad \delta_{\text{skin}} = \frac{1}{\alpha}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_{ph}}{f}$$

$$\eta = \sqrt{\frac{\mu(\omega)}{\varepsilon_{\text{ef}}(\omega)}}, \quad k = \omega\sqrt{\mu(\omega)\varepsilon_{\text{ef}}(\omega)}, \quad v_g = \frac{d\omega}{d\beta}$$

$$\underline{\mathbf{J}}_{\text{ext}} = 0 \implies \underline{\mathbf{H}} = \frac{1}{\eta}\hat{\mathbf{d}}\times\underline{\mathbf{E}}, \quad \underline{\mathbf{E}} = \eta\underline{\mathbf{H}}\times\hat{\mathbf{d}},$$

$$\varepsilon_0 \approx 8.854\cdot 10^{-12}\ [\text{F/m}], \quad \mu_0 \approx 4\pi\cdot 10^{-7}\ [\text{H/m}]$$

$$c_0 = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = 3\cdot 10^8\ [\text{m/s}], \quad \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 120\pi\ [\Omega]$$

$$\varepsilon_{\text{ef}}(\omega) \cong \varepsilon + \frac{\sigma}{j\omega}, \quad \varepsilon = \varepsilon_r\varepsilon_0; \quad \mu(\omega) \cong \mu$$

$$\gamma = j\omega\sqrt{\mu\left(\varepsilon + \frac{\sigma}{j\omega}\right)} = j\omega\sqrt{\mu\varepsilon}\sqrt{1 + \frac{\sigma}{j\omega\varepsilon}}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon + \frac{\sigma}{j\omega}}} = \sqrt{\frac{\mu}{\varepsilon}}\frac{1}{\sqrt{1 + \frac{\sigma}{j\omega\varepsilon}}}$$

- $\sigma = 0$:

$$\gamma = j\omega\sqrt{\mu\varepsilon}, \quad \eta = \sqrt{\frac{\mu}{\varepsilon}} = \eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

- $\sigma \neq 0$, $\sigma/\omega\varepsilon \ll 1$:

$$\gamma_{\text{g.d.}} \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} + j\frac{\omega}{c_0}\sqrt{\mu_r\varepsilon_r}, \quad \eta_{\text{g.d.}} \approx \sqrt{\frac{\mu}{\varepsilon}}\left(1 + j\frac{\sigma}{2\omega\varepsilon}\right)$$

- $\sigma \neq 0$, $\sigma/\omega\varepsilon \gg 1$:

$$\gamma_{\text{g.c.}} \approx \sqrt{\frac{\sigma\omega\mu}{2}}(1+j), \quad \eta_{\text{g.c.}} \approx \sqrt{\frac{\omega\mu}{2\sigma}}(1+j)$$

$$x \ll 1: \quad 1/\sqrt{1+x} \approx 1 - x, \quad \sqrt{1+x} \approx 1 + x/2$$

$$\hat{\mathbf{d}} = \hat{\mathbf{u}}_{\mathbf{1}}\times\hat{\mathbf{u}}_{\mathbf{2}} \rightarrow \frac{\underline{E}_2}{\underline{E}_1} = |p|e^{j\phi}$$

$\bigcirc$ $\phi = \pm\pi/2$ e $|p| = 1$

$\rightarrow$ $\phi = 0$, $\phi = \pi$, $|p| = 0$, $|p| = \infty$

$\circlearrowleft$ $0 < \phi < \pi$

$\circlearrowright$ $-\pi < \phi < 0$

$$\mathbf{S} = \mathbf{E}\times\mathbf{H} \quad [\text{W/m}^2]$$

$$\mathcal{P}_\Sigma = \int_\Sigma \mathbf{S}\cdot\hat{\mathbf{n}}\,dA \overset{\hat{\mathbf{d}}=\hat{\mathbf{n}}}{=} \|\mathbf{S}\|\cdot A$$

$$\mathbf{S}_{\text{av}} = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{\eta}\right\}\|\underline{\mathbf{E}}\|^2\hat{\mathbf{d}} = \frac{1}{2}\mathrm{Re}\{\eta\}\|\underline{\mathbf{H}}\|^2\hat{\mathbf{d}}$$

$$\underline{\mathbf{E}}_0^{\text{ref}} = \rho\underline{\mathbf{E}}_0^{\text{inc}} \qquad \underline{\mathbf{E}}_0^{\text{tx}} = \tau\underline{\mathbf{E}}_0^{\text{inc}}$$

$$\rho = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} \qquad \tau = 1 + \rho = \frac{2\eta_2}{\eta_1 + \eta_2}$$

$$\hat{\mathbf{u}}_{\|}^{l}\times\hat{\mathbf{u}}_{\perp} = \hat{\mathbf{d}}^{l}, \quad l = i, r, t$$

$$\boxed{\begin{aligned}\underline{\mathbf{E}}^{\text{inc}} &= \left(\underline{E}_{0\|}^{i}\hat{\mathbf{u}}_{\|} + \underline{E}_{0\perp}^{i}\hat{\mathbf{u}}_{\perp}\right)e^{-\gamma_1\hat{\mathbf{d}}^i\cdot\mathbf{r}},\\ \underline{\mathbf{E}}^{\text{ref}} &= \left(\rho_{\|}\,\underline{E}_{0\|}^{i}\hat{\mathbf{u}}_{\|}^{r} + \rho_{\perp}\,\underline{E}_{0\perp}^{i}\hat{\mathbf{u}}_{\perp}\right)e^{-\gamma_1\hat{\mathbf{d}}^r\cdot\mathbf{r}},\\ \underline{\mathbf{E}}^{\text{tx}} &= \left(\tau_{\|}\,\underline{E}_{0\|}^{i}\hat{\mathbf{u}}_{\|}^{t} + \tau_{\perp}\,\underline{E}_{0\perp}^{i}\hat{\mathbf{u}}_{\perp}\right)e^{-\gamma_2\hat{\mathbf{d}}^t\cdot\mathbf{r}}.\end{aligned}}$$

$$\rho_{\|} = \frac{\eta_1\cos\theta_i - \eta_2\cos\theta_t}{\eta_1\cos\theta_i + \eta_2\cos\theta_t}, \quad \tau_{\|} = \frac{\eta_2}{\eta_1}(1 + \rho_{\|})$$

$$\rho_{\perp} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}, \quad \tau_{\perp} = 1 + \rho_{\perp}$$

$$\theta_i = \theta_r \qquad n_1\sin\theta_i = n_2\sin\theta_t$$

$$\theta_B = \arctan\left(\frac{n_2}{n_1}\right) \rightarrow |\rho_{\|}| = 0$$

$$n_1\sin\theta_B = n_2\cos\theta_B$$

$$n_1 > n_2, \quad \theta_t = 90^\circ \rightarrow \theta_c = \arcsin\left(\frac{n_2}{n_1}\right)$$

$$\theta_i \geqslant \theta_c \implies |\rho_{\|}| = |\rho_{\perp}| = 1$$

$$\|\mathbf{S}_{\text{av}}^{\text{ref}}\| \overset{\bigcirc}{=} \|\mathbf{S}_{\text{av}}^{\text{inc}}\|\left(\frac{|\rho_{\perp}|^2 + |\rho_{\|}|^2}{2}\right)$$

$$\|\mathbf{S}_{\text{av}}^{\text{tx}}\| \overset{\bigcirc}{=} \|\mathbf{S}_{\text{av}}^{\text{inc}}\|\frac{\eta_1}{\eta_2}\left(\frac{|\tau_{\perp}|^2 + |\tau_{\|}|^2}{2}\right)$$

$$\%\mathcal{P}_{\text{ref}} = \frac{\|\mathbf{S}_{\text{av}}^{\text{ref}}\|}{\|\mathbf{S}_{\text{av}}^{\text{inc}}\|}\times 100\%$$

$$\%\mathcal{P}_{\text{tx}} = (100\% - \%\mathcal{P}_{\text{ref}})$$

† João Gonçalves 99995, Teresa Nogueira 100029, nov. 23
https://github.com/Kons-5/IST-PROE-Notes

Cheat Sheet PROE (parte 2)

$$\frac{\partial v(x,t)}{\partial x} = -Ri(x,t) - L\frac{\partial i(x,t)}{\partial t}$$
$$\frac{\partial i(x,t)}{\partial x} = -Gv(x,t) - C\frac{\partial v(x,t)}{\partial t}$$

$$\gamma = \alpha + j\beta = \sqrt{Z_l Y_l} = \sqrt{(R+j\omega L)(G+j\omega C)}$$
$$Z_0 = \frac{Z_l}{\gamma} = \sqrt{\frac{Z_l}{Y_l}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\omega L \gg R, \quad \omega C \gg G:$$
$$\alpha \approx \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}, \qquad \beta \approx \omega\sqrt{LC}.$$
$$x \ll 1: \quad 1/\sqrt{1+x} \approx 1-x, \quad \sqrt{1+x} \approx 1 + x/2$$

$$\frac{R}{L} = \frac{G}{C}$$
$$Z_0 = \sqrt{\frac{L}{C}}, \qquad \alpha = R\sqrt{\frac{C}{L}}, \qquad \beta = \omega\sqrt{LC}.$$

$$\underline{V}(x) = \underline{V}_0^+ e^{-\gamma x} + \underline{V}_0^- e^{\gamma x} \qquad = \underline{V}^+(x) + \underline{V}^-(x)$$
$$\underline{I}(x) = \frac{\underline{V}_0^+}{Z_0} e^{-\gamma x} - \frac{\underline{V}_0^-}{Z_0} e^{\gamma x} \qquad = \underline{I}^+(x) + \underline{I}^-(x)$$

$$Z(x) = \frac{\underline{V}(x)}{\underline{I}(x)} \qquad \rho_V(x) = \frac{\underline{V}^-(x)}{\underline{V}^+(x)} = \rho_{V0} e^{2\gamma x}$$
$$Z(x) = Z_0\frac{1+\rho_V(x)}{1-\rho_V(x)} \qquad \rho_V(x) = \frac{Z(x)-Z_0}{Z(x)+Z_0}$$
$$0 \leqslant |\rho_V| \leqslant 1$$

$$d = x_2 - x_1:$$
$$\rho_{V1} = \rho_{V2} e^{-2\gamma d} \iff \rho_{V2} = \rho_{V1} e^{2\gamma d}$$
$$Z_1 = Z_0\frac{Z_2 + Z_0\tanh(\gamma d)}{Z_0 + Z_2\tanh(\gamma d)}, \qquad Y_1 = Y_0\frac{Y_2 + Y_0\tanh(\gamma d)}{Y_0 + Y_2\tanh(\gamma d)}$$

$$Z_2 = Z_0 \implies Z_1 = Z_0, \quad \rho_V(x)\big|_{\text{matched}} = 0.$$
$$Z_2 = 0 \implies \rho_{V\,c.c.} = -1$$
$$Z_2 \to +\infty \implies \rho_{V\,o.c.} = +1$$

$$\gamma = j\beta, \quad Z_0 \in \mathbb{R}_{>0}$$
$$|\underline{V}^+(x)| = \text{const.}, \qquad |\rho_V(x)| = \text{const.}$$
$$Z_1 = Z_0\frac{Z_2 + jZ_0\tan(\beta d)}{Z_0 + jZ_2\tan(\beta d)}, \quad \rho_{V1} = \rho_{V2}e^{-j2\beta d}.$$

$$\begin{bmatrix}\underline{V}_1\\ \underline{I}_1\end{bmatrix} = \begin{bmatrix}\cos(\beta d) & jZ_0\sin(\beta d)\\ j\sin(\beta d)/Z_0 & \cos(\beta d)\end{bmatrix}\begin{bmatrix}\underline{V}_2\\ \underline{I}_2\end{bmatrix}$$

$$d = \lambda/2: \quad Z_1 = Z_2, \quad \rho_{V1} = \rho_{V2}$$
$$d = \lambda/4: \quad Z_1 = Z_0^2/Z_2, \quad \rho_{V1} = -\rho_{V2}$$
$$\boxed{Z_1\big|_{d=l} = Z_1\big|_{d=l+n\frac{\lambda}{2}}, \quad \forall n \in \mathbb{Z}}$$

$$\underline{V}_1 = \underline{V}_g - Z_g\underline{I}_1,$$
$$\underline{V}_1^+ = \frac{\underline{V}_g}{1 + Z_g/Z_0 + (1 - Z_g/Z_0)\rho_{V1}}.$$

If $Z_g = Z_0$, then $\underline{V}_1^+ = \underline{V}_g/2$.

$$V_{\max} = \max|\underline{V}(x)|, \quad V_{\min} = \min|\underline{V}(x)|.$$
$$\text{SWR} \equiv \frac{V_{\max}}{V_{\min}} = \frac{1+|\rho_V|}{1-|\rho_V|} \implies |\rho_V| = \frac{\text{SWR}-1}{\text{SWR}+1}$$
$$Z_{\max} = \text{SWR}\cdot Z_0, \qquad Z_{\min} = Z_0/\text{SWR}$$

$$d_{V_{\max},I_{\min},Z_{\max}} = \frac{\arg(\rho_{V\,\text{load}})\cdot\lambda}{4\pi} + n\frac{\pi}{2}$$
$$d_{V_{\min},I_{\max},Z_{\min}} = \frac{\arg(\rho_{V\,\text{load}})\cdot\lambda}{4\pi} + \frac{\lambda}{4} + n\frac{\pi}{2}$$

$$\mathcal{P}(x) = \frac{1}{2}\operatorname{Re}\{\underline{V}(x)\underline{I}^*(x)\}$$
$$= \frac{1}{2}\operatorname{Re}\left\{\frac{|\underline{V}^+(x)|^2}{Z_0^*} - \frac{|\underline{V}^-(x)|^2}{Z_0^*} + \frac{1}{Z_0^*}\left[\underline{V}^{+*}(x)\underline{V}^-(x) - \underline{V}^+(x)\underline{V}^{-*}(x)\right]\right\}.$$

$$\gamma = j\beta, \quad Z_0 \in \mathbb{R}_{>0}$$
$$\mathcal{P} = \mathcal{P}^+ - \mathcal{P}^- = \mathcal{P}^+\left(1 - |\rho_{V2}|^2\right),$$
$$\mathcal{P}^+ = \frac{1}{2}\frac{|\underline{V}^+|^2}{Z_0}, \quad \mathcal{P}^- = \frac{1}{2}\frac{|\underline{V}^-|^2}{Z_0}.$$

$$\gamma = \alpha + j\beta$$
$$\mathcal{P}_1 \approx \frac{1}{2}\frac{|\underline{V}_1^+|^2}{Z_0}(1 - |\rho_{V1}|^2), \quad \mathcal{P}_2 \approx \frac{1}{2}\frac{|\underline{V}_2^+|^2}{Z_0}(1 - |\rho_{V2}|^2).$$
$$\frac{\mathcal{P}_1}{\mathcal{P}_2} = \frac{(1 - e^{-4\alpha d}|\rho_{V2}|^2)}{e^{-2\alpha d}(1 - |\rho_{V2}|^2)} = \frac{e^{2\alpha d} + e^{-2\alpha d}|\rho_{V2}|^2}{1 - |\rho_{V2}|^2}$$
$$\approx \boxed{1 + 2\alpha d\frac{1 + |\rho_{V2}|^2}{1 - |\rho_{V2}|^2}}.$$
$$\mathcal{P}_{\text{dis}} = \mathcal{P}_1 - \mathcal{P}_2 \simeq \mathcal{P}_2\cdot\alpha d\left(\text{SWR} + \frac{1}{\text{SWR}}\right)$$

$$P_L = \frac{|\underline{V}_g|^2}{2}\frac{R_L}{(R_L + R_g)^2 + (X_L + X_g)^2}$$
$$P_{L,\max} = P_a = \frac{|\underline{V}_g|^2}{8R_g}, \text{ for } Z_L = Z_g^* = R_g - jX_g.$$

$$\lambda_0 = \frac{c}{f_0} \to l_T = \frac{\lambda_0}{4} \to Z_{0T} = \sqrt{Z_0 Z_L}, \quad Z_L \in \mathbb{R}_{>0}$$

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† João Gonçalves 99995, Teresa Nogueira 100029, dec. 23
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Cheat Sheet PROE (parte 3)

Near-field region (Fresnel)

$$\nabla^2\underline{\mathbf{A}} + k_0^2\underline{\mathbf{A}} = -\mu_0\underline{\mathbf{J}}_{\text{Hertz}}, \quad \boxed{\underline{\mathbf{A}} = \mu_0\underline{I}_0\, dl\, \frac{e^{-jkr}}{4\pi r}\hat{\mathbf{z}}.}$$

$$\boxed{\begin{aligned}\underline{\mathbf{E}} &= \frac{1}{j\omega\varepsilon_0}(\nabla\times\underline{\mathbf{H}} - \underline{\mathbf{J}}_{\text{Hertz}})\\ \underline{\mathbf{H}} &= \frac{1}{\mu_0}\nabla\times\underline{\mathbf{A}}.\end{aligned}} \quad \text{for } \begin{cases} i = \text{Re}\{\underline{I}_0 e^{j\omega t}\} \text{ and} \\ \underline{\mathbf{J}}_{\text{Hertz}} = \underline{I}_0\, dl\, \hat{\mathbf{z}}\,\delta(\mathbf{r}).\end{cases}$$

$$\underline{\mathbf{H}} = \underline{H}_\phi\hat{\boldsymbol{\phi}}, \quad \boxed{\underline{H}_\phi = jk_0\underline{I}_0\, dl\, \sin\theta\frac{e^{-jk_0r}}{4\pi r}\left[1 + \frac{1}{jk_0r}\right].}$$

$$\underline{\mathbf{E}} = \underline{E}_r\hat{\mathbf{r}} + \underline{E}_\theta\hat{\boldsymbol{\theta}}, \quad \boxed{\underline{E}_r = \eta_0\underline{I}_0\, dl\, \frac{\cos\theta}{r}\frac{e^{-jk_0r}}{4\pi r}\left[1 + \frac{1}{jk_0r}\right],}$$

$$\boxed{\underline{E}_\theta = j\eta_0k_0\underline{I}_0\, dl\, \sin\theta\frac{e^{-jk_0r}}{4\pi r}\left[1 + \frac{1}{jk_0r} + \frac{1}{(jk_0r)^2}\right].}$$

Far-field region (Fraunhofer)

$$\boxed{r \gg L, \quad r \gg \lambda_0, \quad r > 2L^2/\lambda_0.}$$

$$\underline{\mathbf{E}}\Big|_{\text{far-field}}\cdot\hat{\mathbf{r}} = 0, \quad \underline{\mathbf{H}}\Big|_{\text{far-field}}\cdot\hat{\mathbf{r}} = 0.$$

$$\underline{\mathbf{E}}\Big|_{\text{far-field}} = \underline{E}_\theta\hat{\boldsymbol{\theta}} + \underline{E}_\phi\hat{\boldsymbol{\phi}}, \quad \underline{\mathbf{H}}\Big|_{\text{far-field}} = \underline{H}_\theta\hat{\boldsymbol{\theta}} + \underline{H}_\phi\hat{\boldsymbol{\phi}}.$$

$$\boxed{\underline{\mathbf{E}}\Big|_{\text{far-f.}} = \eta_0\underline{\mathbf{H}}(\mathbf{r})\Big|_{\text{far-f.}}\times\hat{\mathbf{r}}, \quad \underline{\mathbf{H}}\Big|_{\text{far-f.}} = \frac{1}{\eta_0}\hat{\mathbf{r}}\times\underline{\mathbf{E}}\Big|_{\text{far-f.}}.}$$

$$\left[\frac{\|\underline{\mathbf{E}}\|}{\|\underline{\mathbf{H}}\|}\right]_{\text{far}} = \eta_0, \quad \mathbf{S}_{\text{av}}\Big|_{\text{far}} = S_r\hat{\mathbf{r}} = \frac{\|\underline{\mathbf{E}}\|^2}{2\eta_0}\hat{\mathbf{r}} = \frac{\|\underline{\mathbf{H}}\|^2\eta_0}{2}\hat{\mathbf{r}}.$$

$$\underline{\mathbf{E}}\Big|_{\text{far-field}} = j\eta_0k_0\underline{I}_0\mathbf{h_e}(\theta,\phi)\frac{e^{-jk_0r}}{4\pi r},$$

$$\boxed{\mathbf{h_e}(\theta,\phi) = h_{e,\theta}\hat{\boldsymbol{\theta}} + h_{e,\phi}\hat{\boldsymbol{\phi}}}$$

$$\mathcal{P}_{rad} \equiv \iint \mathbf{S}_{av}\cdot\hat{\mathbf{n}}\, dS \overset{\hat{\mathbf{r}}=\hat{\mathbf{n}}}{=} \iint S_r r^2\sin\theta\, d\theta\, d\phi$$

$$U(\theta,\phi)\Big|_{\text{far-field}} \equiv S_r r^2 = \frac{\eta_0|\underline{I}_0|^2}{2}\left(\frac{k_0}{4\pi}\right)^2\|\mathbf{h}_e(\theta,\phi)\|^2$$

$$\therefore \mathcal{P}_{rad} = \iint U(\theta,\phi)\, d\Omega, \quad d\Omega = \sin\theta\, d\theta\, d\phi.$$

$$U(\theta,\phi) = \frac{d\mathcal{P}_{rad}}{d\Omega}\ [\text{W/sr}] \rightarrow \text{HPBW: } U(\theta,\phi) = U_{\max}/2.$$

$$\text{FNBW} = 2\cdot\text{HPBW}, \quad \text{SLL} = U_{s,\max}/U_{\max}.$$

$$\underline{V}_0 = Z_a\underline{I}_0, \quad Z_a = R_a + jX_a, \quad X_a = -Z_0\cot(k_0L/2).$$

$$\boxed{\mathcal{P}_{in} = \mathcal{P}_{rad} + \mathcal{P}_{loss}}$$

$$\mathcal{P}_{in} = \frac{1}{2}\text{Re}\{\underline{V}_0\underline{I}_0^*\} \rightarrow \mathcal{P}_{in} = \frac{1}{2}R_a|\underline{I}_0|^2.$$

$$\mathcal{P}_{rad} = \frac{\eta_0}{4\pi}|I_m|^2\int_0^\pi |P(\theta)|^2\sin\theta\, d\theta$$

$$\mathcal{P}_{\text{loss}} = \int_{-L/2}^{L/2}\frac{1}{2}R|\underline{I}(z')|^2dz', \quad R\ [\Omega/\text{m}].$$

$$\boxed{R_a = R_{rad} + R_{loss}}$$

$$R_{rad} = 2\mathcal{P}_{rad}/|\underline{I}_0|^2, \quad R_{\text{loss}} = 2\mathcal{P}_{\text{loss}}/|\underline{I}_0|^2.$$

$$R_{rad} = \frac{\eta_0}{2\pi}\sin^{-2}\left(\frac{k_0L}{2}\right)\int_0^\pi|P(\theta)|^2\sin\theta\, d\theta.$$

$$R_{\text{loss}} = \int_{-L/2}^{L/2}R\frac{|\underline{I}(z')|^2}{|\underline{I}_0|^2}\, dz', \quad R\ [\Omega/\text{m}].$$

$$e = \frac{\mathcal{P}_{rad}}{\mathcal{P}_{in}} = \frac{\mathcal{P}_{rad}}{\mathcal{P}_{rad}+\mathcal{P}_{loss}}, \quad e = \frac{R_{rad}}{R_{in}} = \frac{R_{rad}}{R_{rad}+R_{loss}}.$$

$$g(\theta,\phi) = \frac{U_{\theta,\phi}}{U_{iso}} = \frac{4\pi U_{\theta,\phi}}{\mathcal{P}_{rad}}, \quad D = \max g(\theta,\phi) = \frac{4\pi U_{\max}}{\mathcal{P}_{rad}}.$$

$$S_{\text{av}} = \frac{\mathcal{P}_{in}}{4\pi r^2}G(\theta,\phi), \quad G(\theta,\phi) = eg(\theta,\phi), \quad G_m = eD.$$

Linear Dipoles

$$\boxed{\underline{I}(z') = \underline{I}_m\sin\left[k_0(L/2 - |z'|)\right], \ \underline{I}_m = \underline{I}_0/\sin(k_0L/2)}$$

$$h_e(\theta) = \sin\theta\int_{-L/2}^{L/2}\frac{\underline{I}(z')}{\underline{I}_0}e^{jkz'\cos\theta}\, dz'.$$

$$\text{Hertz dip. } (L = dl) \implies \underline{I}(z') = \underline{I}_0 = \text{const.}$$

$$h_e(\theta) \approx \sin\theta\int_{-L/2}^{L/2}\frac{\underline{I}(z')}{\underline{I}_0}\, dz' = dl\,\sin\theta.$$

$$L \ll \lambda_0 \implies \underline{I}(z') \approx \underline{I}_0(1 - |2z'|), \quad \underline{I}_0 = \underline{I}_mk_0L/2.$$

$$h_e(\theta) \approx \sin\theta\int_{-L/2}^{L/2}\frac{\underline{I}(z')}{\underline{I}_0}\, dz' = \frac{L}{2}\sin\theta.$$

$$P(\theta) = \frac{\cos([k_0L/2]\cos\theta) - \cos(k_0L/2)}{\sin\theta}.$$

$$h_e(\theta) = \frac{2\underline{I}_m}{\underline{I}_0k_0}P(\theta), \quad U(\theta) = \frac{\eta_0}{8\pi^2}|\underline{I}_m|^2|P(\theta)|^2.$$

$$R_{rad}\Big|_{\text{Hertz dip.}} = \frac{2\pi}{3}\eta_0\left(\frac{L}{\lambda_0}\right)^2 = 80\pi^2\left(\frac{L}{\lambda_0}\right)^2$$

$$R_{rad}\Big|_{\text{short dip.}} = R_{rad}\Big|_{\text{Hertz dip.}}/4 = 20\pi^2\left(\frac{L}{\lambda_0}\right)^2$$

$$R_{rad}\Big|_{\lambda_0/2} \approx 60\int_0^\pi|P(\theta)|^2\sin\theta\, d\theta$$

Length (L)	Radiation Resistance (R_{rad})
λ_0	$\rightarrow\infty$
$\lambda_0/2$	73.13 [Ω]
$3\lambda_0/2$	105.3 [Ω]

$$g\Big|_{\text{short/H dip.}} = \frac{3}{2}\sin^2\theta, \quad D\Big|_{\text{short/H dip.}} = \frac{3}{2}.$$

$$g(\theta)\Big|_{\lambda/2} = 1.64\left|\cos\left([\pi/2]\cos\theta\right)/\sin\theta\right|^2, \quad D\Big|_{\lambda/2} = 1.64.$$

† João Gonçalves 99995, Teresa Nogueira 100029, jan. 24
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