Use of GENOPT and BIGBOSOR4 to obtain optimum designs of a double-walled inflatable cylindrical vacuum chamber

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GENOPT/BIGBOSOR4 is applied to the problem of a perfect elastic cylindrical "shell" the complex inflatable wall of which is a webbed sandwich. The cylindrical "shell" is stabilized by uniform pressure applied between its inner and outer walls. General buckling of the entire cross section of the cylindrical shell and local buckling of some of the individual segments of its complex wall cross section can occur because an external pressure applied outside the outer wall is greater than an internal pressure applied inside the inner wall. The complex wall cross section is prismatic (uniform in the axial direction). It consists of the following segments: outer curved (small cylindrical) segments that bulge outward locally from the average outer radius of the double-walled cylindrical "shell", inner curved (small cylindrical) segments that bulge inward locally from the average inner radius of the cylindrical "shell", straight segments oriented in the circumferential direction that connect the ends of the outer curved segments, straight segments oriented in the circumferential direction that connect the ends of the inner curved segments, and webs that connect the inner and outer walls. There are two geometries: one in which the webs are oriented radially and the other in which the webs are slanted, forming a kind of truss. The entire structure consists of segments that act like membranes. The structure is a kind of balloon. The radial distance from the inner wall to the outer wall is the same order as the average radius of the entire cylindrical vacuum chamber. The decision variables of the optimization problem are the radial distance from the inner wall to the outer wall, the radius of curvature of the outer curved membrane segments, the radius of curvature of the inner curved membrane segments, and the thicknesses of the various segments that comprise each repeating module of the complex wall. Design constraints are the lowest buckling load factor (whether general buckling or local buckling) and various membrane stress components in each of the segments of the complex wall cross section. The objective is the weight per axial length of the cylindrical vacuum chamber. Optimum designs of the balloons with the truss-like (slanted) webs weigh significantly less than optimum designs of the balloons with the radial webs. The balloon with the overall minimum weight has 17 modules over 90 degrees of circumference of the cylindrical vacuum chamber.

SECTION 1 INTRODUCTION AND PURPOSE

The effort that resulted in this paper was motivated by Michael Mayo, Lockheed Martin Advanced Technology

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Center, Palo Alto, California, who found Ref.[1]. In [1] the stability of a double-walled cylindrical vacuum chamber of one of the types treated in this paper is investigated: the balloon with radial webs (Fig. 1). The aim of the analysis in [1] is to determine the pressure between the inner and outer walls such that the vacuum chamber subjected to a given external pressure will remain stable.

The purpose of the work reported in the present paper is to create a capability to optimize (find the minimum weight of) cylindrical vacuum chambers of the types depicted in Fig. 1 and in Fig. 7(a) of [1]. Figures 1 and 2 of the present paper show the geometries of the cross sections of 90 degrees of circumference of the cylindrical vacuum chamber that are analogous to those shown in Figs. 1 of [1] and in Fig. 7(a) of [1], respectively.

To solve the problem for a given balloon wall cross section and to provide a means to optimize that wall cross section, the GENOPT system [2,3] is combined with the most recent version of the BOSOR4 code [4,5], a shell-of-revolution analyzer called "BIGBOSOR4" [6]. (BIGBOSOR4 handles many more shell segments than does the much older BOSOR4). In this paper the system created to optimize shells of revolution is called "**GENOPT/BIGBOSOR4**" [6 and 9-11].

The double-walled cylindrical vacuum chamber is not, in the ordinary sense, a shell of revolution. However, it can be modeled as such by treating it as part of a huge torus, as shown in Fig. 3. The purpose of the "huge torus" model is to extend the capability of a shell-of-revolution code to handle not only shells of revolution but also to handle prismatic shells and panels [7,8]. This "trick" has been used recently in the work reported in [9] and [10]. The present paper is analogous to [9] and [10].

A brief overview of the GENOPT program [2] is given here. Extensive details about how to use GENOPT in connection with BIGBOSOR4 are presented in [9] and [11] and will not be repeated here. Brief summaries of the "huge torus" model and the "true prismatic shell model" are provided here.

The GENOPT computer program [2] performs optimization with the use of a gradient-based optimizer called "ADS", created many years ago by Vanderplaats and his colleagues [12,13]. In [2, 3, 6 and 9-11] ADS is "hard wired" in a "modified-method-of-steepest-descent" (1-5-7) mode. In GENOPT a matrix of behavior constraint gradients is computed from finite differences of the behavior constraints for the perturbed design minus the behavior constraints for the current design in which the decision variables are perturbed one at a time by a certain percentage, usually five per cent. By "behavior" is meant buckling, stress, displacement, vibration frequency, clearance, and any other phenomena that may affect the evolution of a design during optimization cycles. In this paper the behavior constraints are buckling and stress.

The objective of the optimization is to minimize the weight per axial length of the cylindrical vacuum chamber subjected to a set of specified requirements (behavior constraints such as buckling and stress). A brief description of GENOPT [2] appears in SECTION 2. SECTIONS 3 and 4 explain the "huge torus" and "true prismatic shell" models. SECTION 5 describes how GENOPT is used by what is called here "**the GENOPT user**" to produce a user-friendly capability to optimize any cylindrical shell in the generic class called "**balloon**" with a sandwich wall configured as shown in Fig. 1 or in Fig. 2. SECTION 6 describes material properties, geometry, and decision variables. SECTIONS 7 – 11 pertain to the geometry displayed in Fig. 2: the balloon with the truss-like (slanted) webs. SECTION 7 discusses the run stream executed by what is called here the "**end user**", who chooses to analyze and optimize a specific member, "**try4**", of the generic class, "**balloon**". SECTION 8 describes selected items that appear in the GENOPT/BIGBOSOR4 output file, "try4.OPM", for an optimized design. SECTION 9 gives numerical results for the configuration with truss-like (slanted) webs such as that displayed in Fig. 2. SECTION 10 demonstrates the capability of GENOPT to perform design sensitivity analyses of an optimized design. SECTION 11 presents minimum weights as a function of the number of modules over 90 degrees of the circumference of the cylindrical vacuum chamber. SECTION 12 gives results for optimized balloons with radial webs (Fig. 1). Conclusions are enumerated in SECTION 13.

The models used here for the optimization of the sandwich wall are BIGBOSOR4 models [4,5,6]. Therefore, the discretization is one-dimensional (strip method), which causes solution times on the computer to be much less than for the usual two-dimensionally discretized finite-element models generated in general-purpose computer programs for the analysis of structures. This property of one-dimensional discretization leads to efficient optimization. Even so, computer run times for “global” optimization via the GENOPT processor called “SUPEROPT” [14] are long.

SECTION 2 ABOUT GENOPT

GENOPT [2] is a system by means of which one can convert any analysis into a user-friendly analysis and into an optimization capability. GENOPT is not limited to the field of structural mechanics [3]. In the GENOPT "universe" there are considered to be two types of user: 1. **the "GENOPT user"**, and 2. **the "end user"**. The GENOPT user creates the user-friendly analysis and optimization capability for a class of problems with a generic name (“**balloon**” in this paper), and the end user uses that capability to find optimum designs for a member of that class with a specific name (“**try4**” in this paper). For the work reported here **the GENOPT user** and **the end user** are the same person: the author.

References [6, 9-11] and this paper provide examples in which GENOPT is used with BIGBOSOR4 to find optimum designs of complex cylindrical and ellipsoidal shells. Many details are given in Ref. [11] about what GENOPT does and how to use it. These details will not be repeated here.

It is the duty of **the GENOPT user** to create user-friendly names, one-line definitions, and "help" paragraphs for the variables to be used in the analysis or analyses. These are listed in Tables 1 and 2 for the generic class, “**balloon**”. **The GENOPT user** must also supply software (subroutines and/or FORTRAN statements) that perform the analysis or analyses that generate the behavior constraints, such as buckling, stress, vibration, etc. The tasks performed by **the GENOPT user** are listed in Part 1 of Table 4. Tables 5 – 7 list FORTRAN coding created partly by GENOPT and partly by **the GENOPT user** for the generic class, “**balloon**”. **The GENOPT user** must decide what behaviors will constrain the design during optimization cycles, behaviors such as general buckling, local buckling, stress, vibration, etc. [5]. While identifying each variable to be used in the generic case, **the GENOPT user** must decide which of seven roles each of these variables plays. The seven possible roles are:

1. decision variable candidate (such as a structural dimension)
2. parameter that is not a decision variable candidate (such as a material property)
3. environmental variable (such as a load)
4. behavioral variable (such as a stress)
5. allowable variable (such as a maximum allowable effective stress)
6. factor of safety (such as a factor of safety for stress)
7. objective (such as weight)

Table 1 lists the variable names, one-line definitions, and roles established by **the GENOPT user** for the generic case, "**balloon**". This table is created automatically by the GENOPT processor called "GENTEXT". Table 2 lists the file called "balloon.PRO" that is created automatically by GENTEXT from the text provided by **the GENOPT user** during his long interactive GENTEXT execution. **The GENOPT user's** input provided during this long interactive GENTEXT session are preserved in the file called "balloon.INP", which is listed in Table 3. The FORTRAN statements provided by **the GENOPT user** for the generic case called "**balloon**" are listed in bold face in Tables 5 – 7. The FORTRAN coding in regular face is generated automatically by the GENTEXT processor of GENOPT.

It is the duty of **the end user** to provide, for a specific case, a starting design, loads, material properties allowables, and factors of safety (Table 8), to choose decision variables, lower and upper bounds, equality constraints, and inequality constraints (Table 9), and to choose strategy indices and whether to optimize or simply to analyze an existing design or both (Table 10). Examples of these data provided by **the end user** are listed in many tables, among them Tables 8 – 10 and Tables 12 – 16.

Please read [2] first, followed by the first part of [11], which contains many details about how to use GENOPT. Also read [9, 10], which are analogous to this paper. If you have access to the GENOPT system, please read the file, /home/progs/genopt/doc/getting.started [15].

SECTION 3 MODELING A CYLINDRICAL SHELL AS A HUGE TORUS

In this paper the double-walled cylindrical vacuum chamber is modeled as part of a huge torus [7]. This concept is displayed in Fig. 3. In general part of any cylindrical shell (180 degrees or less of its circumference) can be modeled as part of a huge torus.

Modeling a cylindrical shell or a sector of a cylindrical shell as a part of a huge toroidal shell is a "trick" that permits, for example, the detailed modeling of a cylindrical shell with an axially oriented truss-core sandwich wall [10] while working within the restriction that BIGBOSOR4 [6] only handles shells of revolution. In this "**balloon**" application 90 degrees of the circumference of the cylindrical vacuum chamber are included in the model, as displayed in Figs. 1 and 2.

With the "huge torus" model [7, 9, 10] the shell coordinates are exchanged: what was the axial coordinate of the cylindrical shell modeled in the usual way becomes the circumferential coordinate of the "huge torus" model, and what was the circumferential coordinate of the cylindrical shell modeled in the usual way becomes the meridional coordinate of the "huge torus" model. In the "huge torus" model of a stiffened cylindrical shell [9] what were stringers become rings, and what were rings become stringers. What were axial half waves in the buckling mode of the cylindrical shell become circumferential half waves in the "huge torus" model of that cylindrical shell, and what were circumferential waves in the buckling mode of the cylindrical shell become meridional waves in the "huge torus" model of that cylindrical shell.

If the radius (RBIG in Fig. 3) from the axis of revolution to the center of meridional curvature of the huge torus is large enough, then under certain circumstances the behavior of the huge torus or a segment of the huge torus should be close to that of the cylindrical shell that the "huge torus" model simulates. The "certain circumstances" are:

Item 1: The cylindrical shell must be simply supported at its ends. In the "huge torus" model of a cylindrical shell meridionally oriented buckling nodal lines, two of which are shown in Fig. 3, occur at buckling modal half-wavelength intervals around the circumference of the huge torus. The conditions at a buckling nodal line are identical to simple support (antisymmetry of the buckling modal displacement components, u, v, w, and shell wall rotation). In the "huge torus" model of a cylindrical shell the BIGBOSOR4 user must choose the circumferential wave numbers, N0B, NMINB, NMAXB, INCRB, such that buckling nodal lines will always fall at intervals around the circumference of the huge torus that correspond to an integral number of half waves over the circumferential length of the huge torus that is equal to the axial length of the cylindrical shell that is being simulated by the "huge torus" model.

Item 2: The radius, RBIG in Fig. 3 must be much larger than the meridional radius of curvature, RADIUS.

In **Item 1** reference is made to "the BIGBOSOR4 user". In the GENOPT/BIGBOSOR4 optimization context there is no "BIGBOSOR4 user". The requirements just stipulated are satisfied automatically.

In the "huge torus" model, as in any shell-of-revolution model, the meridional coordinate is discretized and variation of the buckling mode in the circumferential coordinate direction (normal to the plane of the paper in Figs. 1 and 2) is trigonometric with n (or N) circumferential waves around the circumference of the huge torus. There is a relationship between the axial length of the cylindrical shell and the smallest “huge torus” circumferential wave number that corresponds to one half wavelength spanning that axial length. As mentioned above, simple support end conditions on the cylindrical shell are implied because simple support (antisymmetry) is what naturally occurs along the meridionally oriented buckling nodal lines (Fig. 3) in the "huge torus" model of the cylindrical shell.

In the discussion above it is implied that the "huge torus" model only applies to circular cylindrical shells. However, this is not so. The "huge torus" model can be used for shells or panels of any cross section, such as the "oval-shaped" cylindrical shells and corrugated panels discussed in [7]. The same holds for true prismatic shells and for true prismatic assemblages of shells to be described in the next section.

For more details of the “huge torus” model, see [7 – 9].

SECTION 4 TRUE PRISMATIC SHELLS

In principle, predictions for a "huge torus" model of a cylindrical shell should approach those for a true prismatic shell as the radius from the axis of revolution of the center of meridional curvature (RBIG in Fig. 3) approaches infinity. Reference [8] describes modifications that were made to BIGBOSOR4 [6] in order to generate a valid model of buckling of a truly prismatic, segmented shell, not just a “huge torus” approximation of a prismatic structure. All the GENOPT/BIGBOSOR4 results in this paper were generated with use of the new "true prismatic shell" formulation in BIGBOSOR4 [8-10]. In the “true prismatic shell” formulation n or N is the number of axial half-waves over a reduced length, FACLEN x LENGTH for local buckling [10] and over the entire length, LENGTH, for general buckling, not the number of full waves around the circumference of a huge torus.

In this paper the “length reduction factor”, FACLEN [10], is not used. Instead of the two buckling contraints, local buckling and general buckling, that exist in [9] and [10] there is only one buckling constraint called "GENBUCK" in the "**balloon**" formulation (Table 1). Although the name, "GENBUCK", and its one-line definition, "`general buckling load factor`" (Table 1), imply only general buckling, the buckling mode shape corresponding to the lowest buckling eigenvalue in the "**balloon**" formulation can be either general or local buckling of the wall of the cylindrical vacuum chamber. Examples of both types of buckling are displayed in Figs. 11 – 14.

SECTION 5 PRODUCTION OF THE PROGRAM SYSTEM TO OPTIMIZE THE COMPLEX WALL OF THE CYLINDRICAL BALLOON

The generic case is called "**balloon**".

Table 4 lists a typical run stream for obtaining and analyzing optimum designs for configurations in which GENOPT [2] is used in connection with BIGBOSOR4 [6]. Table 4 is divided into two parts. Part 1 pertains to tasks performed by **the GENOPT user**, and Part 2 pertains to tasks performed by **the end user**.

The GENOPT user first provides input during the long GENTEXT interactive session as listed in Table 3. During the interactive GENTEXT session, GENOPT automatically produces the files balloon.DEF and balloon.PRO. Table 1 of this paper contains a small part of the balloon.DEF file: a glossary of the GENOPT user's variable names, one-line definitions, the role of each variable, and other properties of each variable. Table 2 of this paper lists the entire balloon.PRO file for the generic case called "**balloon**". The balloon.PRO file contains the prompts and "help" paragraphs, created by the GENOPT user. These prompts and "help" paragraphs will be seen by the end user. The program system created by GENOPT for the "**balloon**" generic case will be end-user friendly if the one-line prompting phrases and "help" paragraphs created by the GENOPT user, such as the following (taken from Table 2) are written clearly by the GENOPT user and are reasonably free of jargon with which the end user might not be familiar:

```
20.1 number of modules over 90 degrees: NMODUL                      |one-line
                                                                    |prompt
20.2
     This is the number of triangular "trusses" with two points     | "help"
     on the inner membrane and one point on the outer membrane      |paragraph
     over a 90-degree sector of the circumference of the            |
     cylindrical balloon. See Figs. 2 and 5 of [1]. For the         |
     configuration in which the webs are radial rather than         |
     slanted, the number of modules is equal to the number          |
     of radial webs over 90 degrees of the circumference            |
     of the cylindrical balloon. See Figs. 1 and 4 of [1].          |
```

The string, "[1]" in the text just listed refers to this paper, not to Reference [1] cited in Section 14 of this paper. Although it is long, Table 2 is included here because the "help" paragraphs will help the reader better to understand the generic "**balloon**" model.

GENTEXT also produces FORTRAN fragments, balloon.*, analogous to the weldland.* files listed on page 1 of Table 4 of [9**b**] and described on pages 2 and 3 of Table 6 of [9**b**]. GENOPT automatically assembles these FORTRAN fragments into various programs (BEGIN.NEW, STOGET.NEW, STRUCT.NEW, BEHAVIOR.NEW, CHANGE.NEW) described on page 2 of Table 6 of [9**b**]. BEGIN.NEW, STOGET.NEW, and CHANGE.NEW are complete programs and subroutines, created automatically entirely by GENOPT. The

GENOPT user does not have to be concerned about the creation of them at all.

It is a different matter in the case of STRUCT.NEW and BEHAVIOR.NEW. As created automatically by GENOPT during the GENTEXT interactive session conducted by the GENOPT user, these are "**skeletal**" subroutine libraries either or both of which must be "fleshed out" by the GENOPT user. In this particular application (as in [9] and [10]) the GENOPT user adds merely three statements to the skeletal version of STRUCT.NEW automatically created by GENOPT. (See the lines in bold face in Table 6). In this particular application the GENOPT user does more to "flesh out" the BEHAVIOR.NEW library. (See the lines in bold face in Table 5).

In preparing for the GENTEXT interactive session the GENOPT user here decided to introduce four behaviors:

1. buckling (GENBUK in Table 1; could be either general or local buckling of the wall of the vacuum shell)
2. a maximum of five stress components in material number 1 (STRM1 in Table 1)
3. a maximum of five stress components in material number 2 (STRM2 in Table 1)
4. a maximum of five stress components in material number 3 (STRM3 in Table 1)

The locations of materials 1 – 3 are indicated in Fig. 5.

Corresponding to these four behaviors, GENTEXT automatically creates four **skeletal** "behavioral" subroutines, SUBROUTINE BEHX1 (local or general buckling), SUBROUTINE BEHX2 (five components of stress in material no. 1), SUBROUTINE BEHX3 (five components of stress in material no. 2), and SUBROUTINE BEHX4 (five components of stress in material no. 3). The GENOPT user has to "flesh out" each of these four "behavioral" subroutines, as listed in Table 5. The GENOPT user also has to "flesh out" the subroutine that computes the objective, SUBROUTINE OBJECT in Table 5. The FORTRAN coding created by the GENOPT user is listed in bold face in Tables 5 and 6. The FORTRAN coding listed in regular face is created automatically by the GENTEXT processor of GENOPT.

In the "fleshed out" version of SUBROUTINE BEHX1 there are calls to SUBROUTINE BOSDEC. SUBROUTINE BOSDEC must entirely be written by the GENOPT user. SUBROUTINE BOSDEC ("BOSorDECk") creates a valid input file for BIGBOSOR4. A general guideline on how to go about creating SUBROUTINE BOSDEC is provided in the file associated with the GENOPT sample case called "cylinder":
/home/progs/genopt/case/cylinder/howto.bosdec.

For the present application SUBROUTINE BOSDEC is listed in Table 7 for the balloon with the truss-like (slanted) webs shown in Fig. 2.

Updated versions of the FORTRAN coding created by GENOPT and "fleshed out" by the GENOPT user are maintained in the following files:

/home/progs/genopt/case/balloon/behavior.balloon (SUBROUTINEs BEHX1, BEHX2, etc. and OBJECT)
/home/progs/genopt/case/balloon/struct.balloon (SUBROUTINE STRUCT)
/home/progs/genopt/case/balloon/bosdec.balloon (SUBROUTINE BOSDEC for the Fig. 2 configuration)
/home/progs/genopt/case/balloon/bosdec.balloon2 (SUBROUTINE BOSDEC for the Fig. 1 configuration)

SECTION 6 MATERIAL, GEOMETRY, DECISION VARIABLES

The balloon material is assumed to be polyethylene terephthalate. The material is assumed to be isotropic with a Young's modulus of 435100 psi, Poisson ratio of 0.3, and density of 0.1 lb/inch^3. (The density, 0.1 lb/inch^3, is probably too high. However a change of the density would have no effect on the optimum design. Only the objective, weight/axial length, would be affected, and that is proportional to the density.) Note that the material, although in this particular case isotropic, is handled in the same manner as if it were a ply made of elastic composite material with five different maximum allowable stress components [10]. Here the maximum allowable stress is assumed to be 10000 psi for all 5 stress components: Component 1 = tension along the fibers, Component 2 = compression along the fibers, Component 3 = tension normal to the fibers, Component 4 = compression normal to the fibers, and Component 5 = in-plane shear stress.

In [10] models for local buckling and for general buckling consist of a number of repeating modules chained together. Each module consists of several shell segments. Each shell segment is discretized in the plane of the cross section of the truss-core sandwich wall of the cylindrical shell. The "**balloon**" models explored in this paper are analogous to the truss-core sandwich models in [10]. A number of modules (called NMODUL in Table 1) is chained together. Here always 90 degrees of the circumference of the cylindrical "shell" are included in the model, as displayed in Figs. 1 and 2 and in Figs. 4 and 5.

Figure 4 shows a 90-degree sector of a vacuum chamber with radial webs and with only three modules. The conventions for shell segment numbering and direction of "travel" along each shell segment are identified. Figure 5 is analogous to Fig. 4 and shows the 90-degree sector of a vacuum chamber with truss-like (slanted) webs. In addition to the segment-numbering and direction-of-"travel" conventions, Fig. 5 also shows which parts of the structure are fabricated from which material. It is assumed in the generic "**balloon**" case that the vacuum chamber is fabricated from three different materials. The outer and inner membrane segments with curvature are made of Material No. 1; the outer and inner straight membrane segments are made of Material No. 2; the webs are made of Material No. 3. **In the studies reported in this paper all three materials have the same properties. The entire balloon is essentially made of one material.**

As is indicated in Figs. 1 and 2, symmetry boundary conditions are applied at circumferential coordinates 0 and 90 degrees.

Figure 1 identifies the eight decision variable candidates listed in Table 1:

1. HEIGHT is the radial distance from the inner to outer walls of the vacuum chamber.
2. RINNER is the radius of curvature of each segment of the curved innermost wall.
3. ROUTER is the radius of curvature of each segment of the curved outermost wall.
4. TINNER is the thickness of each segment of the curved innermost wall.
5. TOUTER is the thickness of each segment of the curved outermost wall.
6. TFINNR is the thickness of each segment of the inner wall that consists entirely of straight segments.
7. TFOUTR is the thickness of each segment of the outer wall that consists entirely of straight segments.
8. TWEBS (should be spelled "TFWEBS") is the thickness of each web.

The end user selects which of the eight decision variable candidates are to be actual decision variables for optimization.

SECTION 7 THE END USER PERFORMS HIS INITIAL TASKS

Part 2 of Table 4 lists the many activities of the **end user**. In this section we concentrate on what the end user does as listed in the first few pages of Part 2 of Table 4, when the end user first establishes a name for the specific case (**try4** in the example described here) and then runs the GENOPT processors, BEGIN, DECIDE, MAINSETUP, and OPTIMIZE, followed by one or more executions of BIGBOSOR4 outside the GENOPT context, followed by execution of the "global" optimization script, SUPEROPT [14].

The end user first executes the GENOPT processor, BEGIN. Here the end user provides a starting design, material properties, loading, allowables, and factors of safety (Table 8). Unless there is an existing file, try4.BEG, the end user conducts an interactive "BEGIN" session, providing input data one item at a time when prompted. The one-line prompts and "help" paragraphs are among those listed in Table 2. When the interactive "BEGIN" session is over, there exists a file called "try4.BEG" (Table 8) that can be used for documentation or for future executions of BEGIN for the same specific case. Also, if the end user wishes, he or she can edit the try4.BEG file and then use that edited version as input to BEGIN rather than provide each datum interactively. Table 8 lists a typical try4.BEG file generated by BEGIN.

Notice that in the file, try4.BEG (Table 8) the pressure between the inner and outer walls, PMIDDL = 60 psi. This value of PMIDDL is used in keeping with Eq. (13) of [1]: P = (4/3)R/(R-1), in which P is the ratio, PMIDDL/(POUTER-PINNER) and R is the ratio, (RADIUS + HEIGHT)/RADIUS. With the use of RADIUS = 120 inches and HEIGHT = 60 inches, we have R/(R-1) = 3.0 and therefore, PMIDDL/(POUTER-PINNER) = 4.0. In this study we intend that PINNER = 0 psi and POUTER = 15 psi. However, note that in Table 8 we have the following data entries:

```
5.00000      $ pressure outside the outer membrane: POUTER(  1)
3.000000     $ general buckling factor of safety: GENBUKF(  1)
```

From the point of view of buckling the combination, POUTER = 5.0 psi and GENBUCF(1) = 3.0, is the same as the combination, POUTER = 15 psi and GENBUCF(1) = 1.0. We use POUTER = 5.0 psi and GENBUKF(1) = 3.0 during optimization cycles in order to avoid failure of convergence of the pre-buckling nonlinear equilibrium analysis in BIGBOSOR4.

Next, the end user executes the GENOPT processor, DECIDE, which operates in a manner analogous to BEGIN. Table 9 lists a typical try4.DEC file generated by DECIDE in this particular case.

The end user then executes the GENOPT processor, MAINSETUP, which (although a much shorter interactive session) operates in a manner analogous to BEGIN and DECIDE. Table 10 lists a typical try4.OPT file generated by MAINSETUP.

The end user then executes the GENOPT processor, OPTIMIZE, with the analysis type index, ITYPE = 2 (analysis of a fixed design, not optimization). OPTIMIZE is not interactive (except OPTIMIZE, like BEGIN, DECIDE, and MAINSETUP, asks the end user for the specific name of the case: "**try4**"). OPTIMIZE produces an output file, try4.OPM, which, if ITYPE = 2 (analysis of a fixed design), the end user should inspect. This try4.OPM file (not listed in this paper for the starting design but completely analogous to the try4.OPM file listed for an optimized design in Table 11) corresponds to the "starting" design plotted in Fig. 6a.

The end user then executes BIGBOSOR4 in a context independent of the GENOPT system. The main purpose of doing this is to obtain plots such as those in Figs. 1, 2, 4, 5, 6a. and 11 – 14. During the execution of OPTIMIZE for the fixed design (ITYPE = 2 in the try4.OPT file, as listed in Table 10), GENOPT/BIGBOSOR4 produces a file called try4.BEHX1. This file is valid input for BIGBOSOR4. In this particular execution of BIGBOSOR4 and BOSORPLOT Fig. 6a is produced, as indicated in Table 4.

The end user then executes the GENOPT processor, SUPEROPT [14], with the analysis type index, ITYPE = 1 (optimization) in the try4.OPT file. SUPEROPT requires about 20 hours of computer time on the writer's desktop computer for the configuration shown in Fig. 6a, which has 15 modules over 90 degrees of circumference. Figure 6b shows the objective versus design iterations obtained during this first execution of SUPEROPT.

After the end of the long SUPEROPT run the end user then performs the analysis of the fixed, now optimized, design by changing the analysis type index from ITYPE = 1 to ITYPE = 2 in the try4.OPT file and then executing OPTIMIZE.

NOTE: Although the command is "OPTIMIZE", the GENOPT processor called "OPTIMIZE" does not only optimize a configuration (ITYPE=1), but, depending on the value of the analysis type index, ITYPE, also performs the analysis of a fixed design (ITYPE=2) or performs design sensitivity calculations (ITYPE=3).

Table 11 lists a slightly edited version of the try4.OPM file corresponding to ITYPE = 2. Details are discussed in the next section.

Listed in Part 2 of Table 4 are the commands and their meanings that correspond to the above paragraphs.

SECTION 8 DISCUSSION OF THE RESULTS LISTED IN THE try4.OPM FILE (Table 11)

This section contains a brief description of some of the calculations that lead to the some of the results listed in the try4.OPM file (Table 11) for the optimized design obtained after the first execution of the GENOPT processor, SUPEROPT [14]. This output file pertains to an optimized design of the vacuum chamber with truss-like (slanted) webs (Fig. 2). The configuration of the starting design is shown in Fig. 6a. The items discussed here are identified with the strings, "**Item 1**", "**Item 2"**, "**Item 3"**, …, "**Item14"**. These items are listed in bold face in Table 11.

Item 1: The try4.OPM file listed in Table 11 is the result of the analysis of a fixed design (ITYPE = 2). The fixed design in this example is the optimum design that emerges from the first execution of SUPEROPT [14]. See Part 2 of Table 4 for where that first execution of SUPEROPT occurs in the overall run stream.

Item 2: The optimized values of the decision variable candidates, HEIGHT, RINNER, ROUTER, TINNER, TOUTER, TFINNR, TFOUTR, and TFWEBS are listed. Note that for this initial optimization RINNER and ROUTER are not decision variables. This strategy is used to prevent early termination of SUPEROPT caused by possible failure of convergence of the nonlinear pre-buckling equilibrium analysis. Later, RINNER and ROUTER are introduced as additional decision variables (Table 13) before SUPEROPT is executed a second time, as listed in Part 2 of Table 4.

Item 3: The file, try4.BEHX1, is a valid input file for BIGBOSOR4. It is used for the execution of BIGBOSOR4 in a context independent of GENOPT. (See Part 2 of Table 4). The main purpose of execution of BIGBOSOR4 independently of GENOPT is to obtain plots of the type shown in Fig. 6a and Figs. 11 – 14, for examples.

Item 4: With the "huge torus" model (Fig. 3) or the "true prismatic shell" model [8] there is no way in BIGBOSOR4 directly to apply a mechanical load in the circumferential direction of the huge torus, that is, in the direction of the axis of the true prismatic shell. That is because there is no shell boundary in that coordinate direction. In the "**balloon**" case such a load is needed because the internal pressure, PMIDDL, between the inner and outer walls of the cylindrical vacuum chamber must be reacted where that cavity is sealed at the two ends of the vacuum chamber. Fortunately, the total end load over the 90-degree sector shown in Figs. 1 and 2, called "ENDFCE" below, can in this particular case be produced thermally. The temperature difference from ambient, DELTAT, required to produce the correct reaction, ENDFCE, is computed in SUBROUTINE BOSDEC (Table 7) as follows:

```
      DELTAT = -ENDFCE/ARCTOT                                              (1)
```

in which the total reaction force at the ends of the vacuum chamber is given by:

```
      ENDFCE = PMIDDL(ILOADX)*PI*((RADIUS+HEIGHT)**2 -RADIUS**2)/4.        (2)
```

and ARCTOT is given by:

```
      ARCTOT = ARCOUT*C22OUT*ALPHA2(1)                                     (3)
     1        +ARCINR*C22INR*ALPHA2(1)
     1        +ARCFOT*C22FOT*ALPHA2(2)
     1        +ARCFIN*C22FIN*ALPHA2(2)
     1        +ARCWEB*C22WEB*ALPHA2(3)
```

where `ARCOUT, ARCINR, ARCFOT, ARCFIN, ARCWEB, C22OUT, C22INR, C22FOT, C22FIN,` and `C22WEB` are given by:

```
      FMODS = NMODUL
      ARCOUT = 2.*PHIOUT*ROUTER*FMODS                                      (4)
      ARCINR = 2.*PHIINR*RINNER*FMODS
      SLANT = SQRT((R2J(5)-R1J(5))**2 +(Z2J(5)-Z1J(5))**2)
      ARCFOT = FLOUTR*FMODS
      ARCFIN = FLINNR*FMODS
      ARCWEB = 2.*SLANT*FMODS
      ENDFCE = PMIDDL(ILOADX)*PI*((RADIUS+HEIGHT)**2 -RADIUS**2)/4.
      FNU21O = EMOD1(1)*NU(1)/EMOD2(1)
      FNU21I = EMOD1(2)*NU(2)/EMOD2(2)
      FNU21M = EMOD1(3)*NU(3)/EMOD2(3)
      C22OUT = EMOD2(1)*TOUTER/(1.-NU(1)*FNU21O)
      C22INR = EMOD2(1)*TINNER/(1.-NU(1)*FNU21O)
      C22FOT = EMOD2(2)*TFOUTR/(1.-NU(2)*FNU21I)
      C22FIN = EMOD2(2)*TFINNR/(1.-NU(2)*FNU21I)
      C22WEB = EMOD2(3)*TFWEBS/(1.-NU(3)*FNU21M)
      C44FIN = (EMOD1(2)*TFINNR**3)/(12.*(1.-NU(2)*FNU21I))
```

with

```
      DANGLE = 0.5*PI/FLOAT(2*MODULS)                                  (5)
      FLOUTR = 2.*(RADIUS + HEIGHT)*SIN(DANGLE)
      FLINNR = 2.*RADIUS*SIN(DANGLE)
      PHIOUT = ASIN(0.5*FLOUTR/ROUTER)
      PHIINR = ASIN(0.5*FLINNR/RINNER)
```

and `ALPHA2(1), ALPHA2(2), ALPHA2(3)` are the coefficients of thermal expansion in the circumferential direction of the huge torus (axial direction of the true prismatic shell) for material types 1, 2, 3, respectively. These coefficients of thermal expansion are input data, as are the material properties, `EMOD1(i), EMOD2(i), NU(i), i = 1,2,3,` the number of modules, `NMODUL,` the radius of the inner wall, `RADIUS,` and the decision variable candidates, `HEIGHT, RINNER, ROUTER, TINNER, TOUTER, TFINNR, TFOUTR, and TFWEBS.` (See Table 8).

Note that the objective, `WEIGHT,` is also computed in SUBROUTINE BOSDEC (Table 7) as follows:

```
      WEIGHT = 4.*(ARCOUT*TOUTER*DENSTY(1) +ARCINR*TINNER*DENSTY(1)    (6)
     1             +ARCFOT*TFOUTR*DENSTY(2) +ARCFIN*TFINNR*DENSTY(2)
     1             +ARCWEB*TFWEBS*DENSTY(3))
```

in which `DENSTY(i), i = 1,2,3,` are the weight densities of materials 1, 2, 3 (Table 8). The factor, 4.0, is used on the right-hand side of Eq. (6) because `ARCOUT, ARCINR, ARCFOT, ARCFIN, ARCWEB` pertain only to 90 degrees of the circumference of the cylindrical vacuum chamber.

The quantity, DELT, is analogous to DELTAT. DELT is the temperature rise that would be required to eliminate the axial reaction caused by application of the external pressure, POUTER, in Load Step 2 of the pre-buckling analysis. It was decided not to eliminate this axial reaction, which is compressive and therefore gives rise to buckling load factors that are probably slightly conservative. Hence, DELT = 0.0. (See **Item 6**.)

Item 5a: For the optimized vacuum chamber the buckling load factor, computed in SUBROUTINE BEHX1, is very close to 3.0 (2.9921). Since the factor of safety for buckling is set equal to 3.0 (Table 8 and **Item 14**), the buckling margin is very close to 0.0 [margin = (buckling load factor)/(buckling factor of safety) – 1.0]. Why are the buckling load factor and buckling factor of safety so high? This strategy is followed in order to avoid possible failure of convergence of the nonlinear pre-buckling equilibrium analysis. The actual applied external pressure, POUTER, that we are interested in is POUTER = 15 psi, not the POUTER = 5.0 psi that is listed in Table 8. If we had applied POUTER = 15 psi we would have used a factor of safety of 1.0. However, if we had applied POUTER = 15 psi instead of POUTER = 5.0 psi, we might well have caused failure of convergence of the nonlinear pre-buckling equilibrium solution at Load Step No. 2 during one of the optimization cycles. The pre-buckling equilibrium solution is obtained from nonlinear theory (Newton iteration) as described in **Item 9**. If nonlinear pre-buckling convergence failure had occurred during any of the optimization cycles, the SUPEROPT process would have terminated early. It would therefore have been much more difficult to find a "global" optimum design because not enough design cycles would have been previously successfully processed when the failure of convergence occurred.

Item 5b: The "critical number of axial half-waves, NWVCRT = 1". Ordinarily in the analysis of buckling of shells of revolution there is a search over the number of circumferential waves for the minimum buckling load factor as a function of the circumferential wave number [6]. In this case we perform no search because we are interested only in buckling with a very long wave in the direction normal to the plane of the paper in Fig. 6a and in Figs. 10 – 14, for examples. Hence, we compute buckling with only one half-wave over the very long axial length, LENGTH = 6000 inches (**Item 13**). The buckling load factor corresponding to one axial half-wave over 6000 inches is almost the same as the buckling load factor corresponding to a buckling mode that is prismatic (zero axial half-waves over 6000 inches). This prismatic mode is analogous to the general buckling mode shown in Fig. 5 of [1] and in Fig. 11 of this paper.

In the generic "**balloon**" case, wall configurations that have been optimized usually have many local buckling eigenvalues clustered near the fundamental (lowest) buckling eigenvalue, especially if the lowest buckling eigenvalue (fundamental buckling load factor) happens to correspond to local buckling as it does in the optimized configuration obtained after the first execution of SUPEROPT in this particular case. For the specific case, **try4**, that produces Table 11, the first 50 eigenvalues, all of which correspond to local buckling of the inner segmented straight membrane with thickness, TFINNR, are computed by BIGBOSOR4 if the input datum, NVEC (Part 2 of Table 4), is set equal to 50 in the try4.ALL file, the input file for BIGBOSOR4. The 50 eigenvalues, obtained from the BIGBOSOR4 execution and listed in the try4.OUT file generated by BIGBOSOR4 executed independently of the GENOPT context, are as follows:

```
BUCKLING LOADS FOLLOW
 AXIAL HALF WAVE NUMBER, N = 1

 EIGENVALUES =
    2.99211E+00    2.99210E+00    2.99209E+00    2.99208E+00    2.99210E+00
    2.99207E+00    2.99211E+02    2.99210E+00    2.99212E+00    2.99211E+00
    2.99211E+00    2.99212E+00    2.99209E+00    3.00226E+00    3.00226E+00
    3.02104E+00    3.02104E+00    3.02105E+00    3.02107E+00    3.02107E+00
    3.02106E+00    3.02109E+00    3.02108E+00    3.02108E+00    3.02115E+00
    3.02113E+00    3.02112E+00    3.02115E+00    3.04111E+00    3.04110E+00
    3.07006E+00    3.07006E+00    3.07008E+00    3.07006E+00    3.07008E+00
    3.07007E+00    3.07009E+00    3.07011E+00    3.07012E+00    3.07015E+00
    3.07007E+00    3.07013E+00    3.07014E+00    3.10037E+00    3.10039E+00
    3.14019E+00    3.14019E+00    3.14020E+00    3.14016E+00    3.14017E+00
```

NOTE: Multiple eigenvalues are computed in the generic "**balloon**" case only when BIGBOSOR4 is executed outside the GENOPT context in the manner specified in Part 2 of Table 4. When BIGBOSOR4 is executed within the GENOPT context in the generic "**balloon**" case, only the lowest eigenvalue is computed. To obtain multiple eigenvalues for a given wave number (`AXIAL HALF WAVE NUMBER, N = 1` in this example) the end user has to edit the try4.ALL file by changing NVEC from 1 to some number higher than 1 and less than or equal to 50. (See Part 2 of Table 4.) Fifty eigenvalues per wave number N is the maximum number of eigenvalues that are permitted by BIGBOSOR4.

Item 6: In the pre-buckling analysis there are only two load steps of interest:

Load Step No. 1: The applied loads are PINNER=0 psi, PMIDDL = 60 psi, DELTAT = -112.1 degrees
Load Step No. 2: The applied loads are those in Load Step No. 1 plus POUTER = 5.0 psi.

The stress resultants along the axis of the prismatic balloon, N2DIFF(J), J = 1,6, in the six segments of Module No. 1 (Fig. 5) are negative because of the Poisson effect. The applied external pressure, POUTER = 5.0 psi, would ordinarily cause Poisson expansion in the axial direction of the cylindrical vacuum chamber. However, the symmetry boundary condition at 0 degrees (Figs. 1 and 2) prevents that Poisson axial expansion, thereby inducing negative internal stress resultants, N2DIFF(J). (J is the shell segment number in the first module, as displayed in Fig. 5.) The role of the temperature difference, DELT (**Item 4**, Load Step No. 2), would have been to remove the total "Poisson" negative axial force induced by POUTER in a manner analogous to the creation of axial tension by application of DELTAT as described in **Item 4**. However, we have set DELT equal to zero and so have not applied an axial thermal tension that would cancel the total Poisson-ratio-induced axial compression generated by the application of POUTER in Load Step No. 2. This is a conservative strategy with regard to buckling of the wall of the cylindrical vacuum chamber.

Item 7: The pre-buckling stress resultants from Load Step No. 1 (N1FIX in the meridional direction and N2FIX in the circumferential direction in the huge torus shown in Fig. 3) and from Load Step No. 2 (N1VAR, N2VAR) are listed for the six segments in Module No. 1 (Figs. 2 and 5). These "fixed" and "variable" stress resultants are the same at every nodal point in a segment and in every repeating module of the 90-degree model. The quantities, N1FIX and N2FIX, affect the stiffness matrix but are not "eigenvalue quantities", that is, they are not to be multiplied by the eigenvalue (buckling load factor) in the computation of the buckling load. The differences, N1VAR – N1FIX and N2VAR – N2FIX, are "eigenvalue quantities", that is, they are to be multiplied by the eigenvalue in the computation of the buckling load. In this particular case the most critical module segment from the point of view of local buckling is Segment No. 3, the inner straight segment of thickness, TFINNR (Fig. 5):

```
       PREBUCKLING STRESS RESULTANTS IN THE FIRST MODULE
           "fixed" from Load Step No. 1 total from Load Step No. 2
 Seg.J  Node I  N1FIX(I,J)   N2FIX(I,J)   N1VAR(I,J)   N2VAR(I,J)
    3      1   2.0636E+03   1.2948E+03   1.3718E+03   1.0872E+03
```

The stress resultant decrement that causes buckling is the difference, N1VAR(I,3) – N1FIX(I,3) = -691.8 lb/in, in which N1VAR and N1FIX are the stress resultants in the **meridional coordinate direction**, that is, the stress resultants in the plane of the cross section of the wall of the cylindrical vacuum chamber. The buckling load factor (Item 5a) is 2.9921. Therefore, the total meridional stress resultant at the buckling of Segment No. 3 is given by N1FIX(I,3) + 2.9921x[N1VAR(I,3) – N1FIX(I,3)] = 2063.6 + 2.9921 x (-691.8) = -6.3347 lb/in. This is a very small value compared to the meridional tension, 2063.6 lb/in, caused by the application of the fixed loads, PINNER = 0 psi, PMIDDL = 60 psi, DELTAT = -112.1 degrees. If segment 3 were a membrane and not a shell segment with finite bending stiffness the quantity, N1FIX(I,3) + 2.9921x[N1VAR(I,3) – N1FIX(I,3)] would be zero. Because the quantity, N1FIX(I,3) + 2.9921x[N1VAR(I,3) – N1FIX(I,3)], is very small compared to N1FIX(I,3) and compared to N1VAR(I,3) – N1FIX(I,3), Segment 3 acts essentially like a membrane, not like a shell segment with finite bending stiffness: buckling (or rather wrinkling) occurs as soon as there is no remaining pre-buckling meridional tension in the segment. The other shell segments in each module also act like membranes. However, these other shell segments are not as critical as is Segment No. 3 with regard to buckling (loss of tension due to the application of the external pressure, POUTER). For example, the remaining meridional tension at the buckling load factor, 2.9921, in Segment No. 1, the outer straight segment of thickness, TFOUTR (Fig. 5), is given by: N1FIX(I,1) + 2.9921x[N1VAR(I,1) – N1FIX(I,1)] = 409.41 + 2.9921 x (-109.15) = +82.8223 lb/in. For Segment No. 1 to buckle, the buckling load factor would have to be at least 409.41/109.15 = 3.7509 rather than 2.9921. In all the cases explored in this paper local buckling of the inner segmented straight membrane of the wall of the cylindrical vacuum chamber occurs before local buckling of any of the other segments of the balloon wall.

Item 8: The pre-buckling stresses in Segments 1 – 5 of the first module (and of all the other repeating modules in the 90-degree model) are listed for Load Step No. 1 and for Load Step No. 2. Notice the following:

1. The meridional "fixed" and "variable" stress components, STRS1F and STRS1V, are higher than the axial stress components, STRS2F and STRS2V. (The axial stress components in the true prismatic shell are the circumferential stress components in the "huge torus" model of the cylindrical vacuum chamber shown in Fig. 3.)

2. The stress components at Load Step No. 1 are higher than those at Load Step No. 2. That is because the external pressure, POUTER, applied only in Load Step No. 2, acts to counteract the internal pressure, PMIDDL.

Even though the stress components at Load Step No. 1 are higher than those at Load Step No. 2, we use the stress components at Load Step No. 2 in computing the stress margins listed in **Item 11**. Notice that the meridional stress components, STRS1V(I,J), are all "critical", that is, they are very close to the maximum allowable value, 10000 psi, listed in Table 8 for all three materials.

On October 7, 2010 an important change was made to the "**balloon**" software, in particular to SUBROUTINE BEHX2. The most recent version of SUBROUTINE BEHX2 is listed in Table 5. Previous to October 7 the stresses in the balloon wall were computed by BIGBOSOR4. These stresses included the bending stresses in the walls of the shell segments. The following comments in SUBROUTINE BEHX2 explain the change and the reason for it:

```
C  NOTE IMPORTANT CHANGE:
C
C  October 7, 2010: Use the MEMBRANE stresses computed in
C  SUBROUTINE BEHX1 because the
C  meridional curvature change, KAPPA1, from BIGBOSOR4 is
C  sometimes much too large in the immediate neighborhoods
C  of the ends of the shell segments, generating maximum
C  stress components that are much too high in this particular
C  case that involves a balloon-like (membrane) structure.
C  This is especially true for the outer and inner curved
C  membranes, that is, segments 2 (outer) and 4 (inner)
C  of each module of the multi-module model. For example,
C  here is some BIGBOSOR4 output for Segment 4 (inner
C  curved membrane) for Load Step No. 1 (applied loads
C  are PINNER, PMIDDL, and DELTAT):
C
C      AXISYMMETRIC PRESTRESS DISTRIBUTION FOR SEGMENT 4
C POINT   EPSILON 1  EPSILON 2   KAPPA 1      KAPPA 2
C           MERID.     CIRCUMF.    MERID.      CIRCUMF.
C           STRAIN     STRAIN     CHANGES IN CURVATURE
C   1    1.070E-01   7.019E-15   5.784E-01   1.795E-08
C   2    1.062E-01  -1.695E-09  -1.015E+01  -4.307E-10
```

```
C   3    1.059E-01   1.154E-08   2.981E+00  -8.949E-09
C   4    1.074E-01   1.021E-08  -1.251E+00  -2.625E-09
C   5    1.077E-01   1.483E-08   6.715E-01  -5.319E-09
C   6    1.085E-01   1.647E-08  -2.629E-01  -3.690E-09
C   7    1.090E-01   1.910E-08   1.910E-01  -4.184E-09
C   8    1.096E-01   2.095E-08  -3.076E-02  -3.533E-09
C   9    1.100E-01   2.283E-08   7.706E-02  -3.398E-09
C  10    1.104E-01   2.441E-08   2.394E-02  -2.950E-09
C
C  Note that the meridional change in curvature, KAPPA 1,
C  is very large at nodal points 2, 3, 4, especially at
C  nodal point 2. This very local edge effect gives rise to
C  artificially high local bending meridional strain, which
C  probably does not exist in a balloon (membrane
C  pressure-stabilized "shell" structure). The extreme
C  fiber meridional strain from BIGBOSOR4 is given by:
C  EPS1 = EPSILON1 + THICK*KAPPA1/2.
C  in which EPSILON1 is the reference (middle) surface
C  meridional strain and THICK is the thickness of the
C  shell segment.
C
C  Because of this spurious and extremely high meridional
C  bending strain predicted by BIGBOSOR4 (which has difficulty
C  predicting accurate bending stresses in membrane-like structures
C  but which works well for shell structures with "finite"
C  bending stiffness), the previous FORTRAN statement:
C     IF (JCOL.GT.1) GO TO 500
C  has been commented out and replaced by the following
C  statement, "GO TO 500". Because of this important change
C  the file, *.BEHX2, is no longer created and you can
C  therefore no longer obtain plots of the pre-buckled states
C  at Load Steps 1 and 2 unless you remove the "C" in column
C  1 of the statement, "IF (JCOL.GT.1) GO TO 500", and insert
C  a "C" in column 1 of the following statement, "GO TO 500".
C  and then re-compile via the GENOPT command, "genprograms".
```

Stress components in the various segments of a module are therefore computed from membrane theory, that is, the stress component is equal to the appropriate stress resultant divided by the thickness of the segment. The stress components are computed in SUBROUTINE BEHX1 (Table 5) and are available to SUBROUTINES BEHX2, BEHX3, and BEHX4 by means of a labeled common block.

Note that in computing the stresses from membrane theory we are being unconsersvative, especially with regard to the neglect of bending stress concentrations that doubtless occur at and in the immediate neighborhoods of the junctions between shell segments. If a balloon were to be fabricated in a configuration equal to an optimized design generated as described in this paper, the seams at the junctions between segments would have to be reinforced. Inclusion of the effect of local reinforcement at seams is beyond the scope of this paper.

Item 9: In BIGBOSOR4 the axisymmetric pre-buckling equilibrium state is computed from nonlinear theory when the BIGBOSOR4 analysis type index, INDIC, equals zero or unity [4,5]. Convergence is obtained with the use of Newton's method. Table 18 lists some output from BIGBOSOR4 from the nonlinear pre-buckling analysis of the optimized balloon depicted in Figs. 11 and 12. Shell structures that behave in a manner similar to membranes usually require many small load steps in order to obtain converged results at operating load levels. **In the "balloon" formulation the pre-buckling nonlinearity is considerably reduced by the fact that the various shell segments, such as those displayed in Figs. 4 and 5, are hinged at their junctions.** This hinging considerably reduces the very local bending that would otherwise occur in the neighborhoods of these junctions if continuity of the rotation of the shell wall across junctions were enforced there. Still, several Newton iterations are required, especially when Load Step No. 1 is applied.

The output listed in Table 11 corresponds to an earlier version of SUBROUTINE BEHX1 than that listed in Table 5. In this earlier version of SUBROUTINE BEHX1 the entire "fixed" (non-eigenvalue) loads, PINNER, PMIDDL, and DELTAT, were applied in a single load step. This earlier strategy often led to failure of convergence of the nonlinear pre-buckling equilibrium state during optimization cycles, a characteristic that causes early termination of SUPEROPT executions.

In the present improved version of SUBROUTINE BEHX1, which is that listed in Table 5, a rather elaborate strategy has been introduced. There are now three attempts to obtain the nonlinear pre-buckled equilibrium state that exists after the "fixed" (non-eigenvalue) loads, PINNER, PMIDDL, and DELTAT, have been applied:

Try no. 1: the "fixed" (non-eigenvalue) loads, PINNER, PMIDDL, and DELTAT, are applied in 10 sub-steps, with 0.1 x PINNER, 0.1 x PMIDDL, and 0.1 x DELTAT added to the total load in the previous sub-step in each successive sub-step. This new strategy requires about twice the total computer time of the "one sub-step" earlier strategy. However, the increase in reliability of the optimization runs is worth the sacrifice in the speed of execution. From the latest version of SUBROUTINE BEHX1 (Table 5) the following lines now appear in the try4.OPM file for the same case as that from which Table 11 was generated:

```
Newton iterations required to solve the nonlinear
 axisymmetric pre-buckling equilibrium state for the
 "fixed" loads,
PINNER=  0.0000E+00, PMIDDL=  6.0000E+01, DELTAT= -1.1210E+02

 LOAD STEP   Newton iterations  Maximum displacement
     1             5               4.405314E-01
     2             2               8.816130E-01
     3             2               1.323535E+00
     4             2               1.766536E+00
     5             1               2.210816E+00
     6             1               2.656576E+00
     7             1               3.103976E+00
     8             1               3.553154E+00
     9             1               4.004255E+00
    10             1               4.457406E+00
```

These data are now obtained from the section in SUBROUTINE BEHX1 (Table 5) in which SUBROUTINE BOSDEC (Table 7) is called with INDIC = 0 and with INDX = 0 (the first argument in CALL BOSDEC(0, etc), as follows:

```
C
C   Obtain nonlinear equilibrium for Load Set B by itself.
C   Use 10 load steps to assure convergence.
C
       INDIC = 0
       WRDCOL = '                                                    '
       IFTOTS = 0
       CALL MOVER(0.,0,FTOTX,1,40000)
       CALL BOSDEC(0,24,ILOADX,INDIC)
C
```

(lines skipped to save space and enhance readability)

```
C
       CALL B4READ
       CALL B4MAIN
C
```

Try no. 2: If Try no. 1 fails, the "fixed" (non-eigenvalue) loads, PINNER, PMIDDL, DELTAT, are applied in a single load "sub-step". This is done as a second try (rather than increasing the number of load "sub-steps" which seems at first to be more logical) because BIGBOSOR4 often finds that with the generic "**balloon**" configurations it is easier to obtain a converged nonlinear pre-buckling solution with a one-sub-step formulation than with a multi-sub-step formulation.

Try no. 3: If Try no. 2 fails, the "fixed" (non-eigenvalue loads, PINNER, PMIDDL, DELTAT, are applied in 50 sub-steps. If Try no. 3 fails, the run aborts with the message printed at the end of this section. "Try no. 3" was introduced after several cases involving the balloon with the radial webs (Figs. 1 and 26) bombed. The results plotted in Figs. 26 and 27, for example, were generated before "Try no. 3" was introduced: the SUPEROPT run aborted early due to the failure of both Try no. 1 and Try no. 2. In several optimizations since "Try no. 3" was introduced there has not been a single failure of convergence of the nonlinear pre-buckling equilibrium analysis. The computer times for optimization does not seem to have increased very much since "Try no. 3" was introduced because "Try no. 3" may not be required very often during optimization cycles in an execution of SUPEROPT.

Item 10: At the optimum design the load factor for general buckling is very close to 3.0. The strategy of applying a low value of the external pressure, POUTER = 5.0 psi (one third the desired value, POUTER = 15 psi), coupled with the compensating use of a factor of safety for buckling of 3.0 rather than 1.0 is described in **Item 5a**. The purpose of this strategy is to avoid failure of convergence of the nonlinear pre-buckling equilibrium solution at Load Step No. 2. As has been mentioned previously, failure of nonlinear pre-buckling convergence causes an execution of SUPEROPT to abort. Unlike the fixed loads, which are now applied in 10 steps as described in **Item 9**, the additional load in Load Step No. 2 (the external pressure, POUTER) is still always applied in a single load step.

Item 11: At the optimum design several of the margins are critical, that is, they are close to zero and even slightly negative. NOTE: Designs with negative margins are accepted as "FEASIBLE" provided that no margin is less than -0.01 and as "ALMOST FEASIBLE" provided that no margin is less than -0.05.

Item 12: The objective to be minimized is the weight/axial length of the cylindrical vacuum chamber. The formula for this objective is given as Eq. (6) in **Item 4**. The objective is computed in SUBROUTINE BOSDEC (Table 7) and available in SUBROUTINE OBJECT (Table 5) by means of a labeled common block.

Item 13: The length, LENGTH, of the cylindrical vacuum chamber is taken to be 6000 inches. The purpose of this extremely long LENGTH is to permit the computation of buckling load factors that are close to those that would exist if the cylindrical vacuum chamber were infinitely long [1].

Item 14: The factor of safety for buckling is taken in this example to be 3.0 rather than 1.0. As explained in Item 5a, this high factor of safety is used in connection with a correspondingly low value for the external pressure, POUTER = 5.0 psi. Again, it is emphasized that this strategy leads to the avoidance of failure of convergence of the nonlinear pre-buckling equilibrium state at Load Step No. 2, which is always applied in a single load sub-step.

Another point to emphasize relative to this Item: The GENOPT-user-established name for the buckling behavior is GENBUK (Table 1), and the definition of the buckling behavior contains the string, "general buckling". It is emphasized that the critical (lowest) buckling eigenvalue (load factor) may correspond either to general buckling or to local buckling. For the optimized configuration shown in Fig. 11 the lowest buckling eigenvalue happens to correspond to general buckling (ovalization of the cylindrical vacuum chamber in a mode similar to that displayed in Fig. 5 of [1]). However, for the starting design shown in Fig. 6a the lowest buckling eigenvalue happens to correspond to local buckling (buckling of the segments of the inner straight membrane of thickness TFINNR). During optimization cycles the optimizer, ADS, seeks to force the margin for buckling into "FEASIBLE" or "ALMOST FEASIBLE" territory. As long as that margin is positive (or only slightly negative: greater than -0.01 for a "FEASIBLE" design and greater than -0.05 for an "ALMOST FEASIBLE" design) it does not matter what type of buckling, general or local, the buckling margin corresponds to. Both types of buckling, local and general, will correspond to a positive or only a slightly negative margin. At optimum designs the buckling load factors for local and general buckling are usually fairly close to each other. In fact, as listed at the end of **Item 5b**, there are almost always many, many eigenvalues corresponding to various modes of local buckling clustered together near the fundamental (lowest) buckling eigenvalue.

What happens when a SUPEROPT run aborts? The end of Table 4 lists what appears in the *.OPM file just before and at the moment of an error termination of an execution of SUPEROPT. The unrecoverable error is caused by the failure of the nonlinear pre-buckling iterations to converge after three attempts: **Try no. 1**, **Try no. 2**, and **Try no. 3** as listed in **Item 9**. If that happens the following message appears at the end of the *.OPM file:

```
************* ABORT **********
 INITIAL LOADS TOO HIGH FOR THIS STRUCT
 This is an unrecoverable error because we have already
 tried and failed to obtain nonlinear pre-buckling convergence
 by changing from a nonlinear solution with 10 load steps to
 a nonlinear solution with 1 load step and then changing from
 1 load step to 50 load steps:three tries. That strategy just
```

```
failed. You may well have performed enough design iterations
to have a good optimum design now. Look near the end of the
*.OPP file at the "FEASIBLE" and "ALMOST FEASIBLE" designs.
If you are not satisfied that you have performed enough
design iterations, then look at the thicknesses of the
various segments. If any thicknesses seem too small, then
increase them and also increase the corresponding lower
bounds of them. Another thing you can try that has worked
for Bushnell is to look near the end of the *.OPM file for
the last successfully obtained design. Use the GENOPT
processor, CHANGE, to reset the values of the decision
variables to those of the last successfully obtained design
and then launch a new execution of SUPEROPT, probably
leaving the lower bounds unchanged, or perhaps also changing
them if you wish (before launching SUPEROPT, of course).
The run is now aborting: IMODX= 1
********************************
```

SECTION 9 NUMERICAL RESULTS FOR THE BALLOON WITH TRUSS-LIKE (SLANTED) WEBS

Figures 7 – 20 and Tables 11 – 18 pertain to this section. Although the name of the specific case used in this paper is "**try4**" for all of the cases reported in this paper, both for the balloon with the truss-like webs (Fig. 2) and for the balloon with the radial webs (Fig. 1), most of the figures and Tables associated with the truss-like configuration are associated with the specific name, "**try41**", and most of the figures associated with the radial configuration are associated with the specific name, "**try42**".

Note that the input data are the same for either the truss-like configuration or for the radial configuration. The different configurations are established, not by the input data, but by the choice of SUBROUTINE BOSDEC: **bosdec.balloon** for the truss-like configuration and **bosdec.balloon2** for the radial configuration. The files, **bosdec.balloon** and **bosdec.balloon2** are stored as follows:

/home/progs/genopt/case/balloon/bosdec.balloon   (SUBROUTINE BOSDEC for the Fig. 2 configuration)
/home/progs/genopt/case/balloon/bosdec.balloon2 (SUBROUTINE BOSDEC for the Fig. 1 configuration)

The optimum design obtained after the first execution of SUPEROPT (Fig. 6b and Tables 11 and 12) is not the final optimum design because the decision variable candidates, RINNER and ROUTER, were not initially selected as decision variables (Table 9). Part 2 of Table 4 lists additional elements of the run stream that lead to generation of most of the remaining figures pertaining to the balloon with the truss-like (slanted) webs (Fig.2). Table 13 lists new input for the GENOPT processor, DECIDE, in which RINNER and ROUTER are now selected as decision variables. Figures 7 and 8 are plots of the objective versus design iterations generated from a second (Fig. 7) and from a third (Fig. 8, Table 16) execution of SUPEROPT. The third execution of SUPEROPT does not produce in this particular case an optimum design that is better than that produced by the second (partial) execution of SUPEROPT.

Tables 14 and 17 list the final optimum design for this particular balloon configuration, in which there are 15 modules. The results listed in Table 14 correspond to the external pressure, POUTER = 5.0 psi and factor of safety for buckling of 3.0. The results listed in Table 17 correspond to the external pressure, POUTER = 15.0 psi and factor of safety for buckling of 1.0. Figures 9 – 14 pertain to this final optimum design. Figures 9 and 10 show the pre-buckled equilibrium states from application of only the "fixed" loads, PINNER = 0 psi, PMIDDL = 60 psi, DELTAT = -99.623 degrees (Fig. 9) and from application of the total loads, PINNER = 0 psi, PMIDDLE = 60 psi, DELTAT = -99.623 degrees, and POUTER = 15 psi (Fig. 10). As written in the run stream (Part 2 of Table 4) and in **Item 8** in the previous section, the pre-buckled states such as those displayed in Figs. 9 and 10 can no longer be generated without the two simple changes in SUBROUTINE BEHX2 identified in **Item 8** of the previous section.

Figures 11 – 14 show general and local buckling modes corresponding to the final optimum design listed in Table 14. In this particular case it turns out that the fundamental buckling mode (the buckling mode that corresponds to the lowest eigenvalue) is general buckling, that is, ovalization of the entire cross section of the balloon. This general buckling mode is analogous to the general buckling mode presented as Fig. 5 in [1]. The "higher" buckling modes (Figs. 12 – 14) correspond to various combinations of general and local buckling. Note that the first 10 eigenvalues are very, very close to one another. This clustering of eigenvalues near the fundamental eigenvalue is typical for optimized balloons of the type analyzed in this paper. **Item 5b** in the previous section lists the first 50 eigenvalues, the maximum number of eigenvalues permitted by BIGBOSOR4, of the preliminary optimum design obtained after the first execution of SUPEROPT. All of these local buckling eigenvalues are very close to each other. The clustering of eigenvalues occurs because there are many, many eigenvalues that correspond to local buckling (wrinkling) of the inner segmented straight membrane with thickness, TFINNR.

Table 18 lists a small part of the try4.OUT file generated by an execution of BIGBOSOR4 outside the GENOPT context. The optimized design and the loading are those listed in Table 17.

SECTION 10 DESIGN SENSITIVITY OF THE FINAL OPTIMUM DESIGN OF THE BALLOON WITH THE TRUSS-LIKE (SLANTED) WEBS

Figures 15 – 17 pertain to this section. These results are obtained with the use of ITYPE = 3 in the try4.OPT file, as described near the end of Part 2 of Table 4. As is typical of optimized designs, several design margins tend to cluster near zero at the optimum designs, that is, at the optimum value of whichever decision variable, HEIGHT (Fig.15) or RINNER (Fig. 16) or ROUTER (Fig. 17), is varied during the design sensitivity (ITYPE = 3) analysis.

SECTION 11 OPTIMIZED WEIGHT AS A FUNCTION OF THE NUMBER OF MODULES IN THE BALLOON WITH THE TRUSS-LIKE (SLANTED) WEBS

Figures 18 – 20 pertain to this section. The minimum weight/axial length corresponds to about 17 modules over 90 degrees of circumference of the vacuum chamber. The procedure used to obtain the final optimum designs for each number of modules is the same as that listed in Part 2 of Table 4. In the first execution of SUPEROPT the decision variable candidates, RINNER and ROUTER, are usually not selected as decision variables. In successive executions of SUPEROPT the radii, RINNER and ROUTER, are decision variables the lower and upper bounds of which depend on results from previous executions of SUPEROPT for that particular number, NMODUL, of modules. The curve displayed in Fig. 18 required many days to generate because each execution of SUPEROPT requires many hours of computer time, especially the executions of SUPEROPT corresponding to the balloons with high numbers of modules. For example, the datum in Fig. 18 corresponding to 30 modulus required about 50 hours for each execution of SUPEROPT.

It is possible, although unlikely in the generic "**balloon**" class, that somewhat lower optimum weights would have been found if SUPEROPT had been executed perhaps several additional times for each number, NMODUL, of modules in each 90-degree model. This was not done because of the long computer times required for each execution of SUPEROPT in this case. For other optimization problems explored in the past that require less computer time the writer has strongly recommended that SUPEROPT be executed several times to obtain a "global" optimum design. If the engineer or researcher has enough calendar time this should be done in the "**balloon**" case also.

In the paragraph above the word, "global", is in quotes because GENOPT does not seek the actual global optimum design. It is expected that many attempts at optimization starting from many different points in design space will produce a locally optimum design that is almost a good as the true global optimum design. Often several successive executions of SUPEROPT are needed to accomplish this.

Figures 19 and 20 show the local and general buckling modes for the optimized balloon with 12 modules over 90 degrees of circumference. In this particular case the fundamental buckling mode corresponds to local buckling. The 49th buckling mode corresponds to general buckling.

SECTION 12 BALLOON WITH RADIAL WEBS (FIG. 1)

This section pertains to the balloon with the radial webs (Fig. 1). For this configuration with radial webs the fundamental buckling mode (the buckling mode corresponding to the lowest eigenvalue) of the optimized balloons always corresponds to general buckling in the particular cases explored in this paper.

Figures 21 and 22 show the objective versus design iterations during the first and second executions of SUPEROPT. Figures 23 and 24 show the general and local buckling modes of the optimized balloon with 15 modules over 90 degrees of circumference (15 radial webs).

Figure 25 shows the optimized weight/axial length as a function of the number of modules over 90 degrees of circumference. Two curves pertain to the balloon with the radial webs: the second and the fourth traces. The second trace is for optimized designs obtained with the index, IDESGN, in the try4.OPT file (Table 10) set equal to 2. The fourth trace is for optimized designs obtained with the index, IDESGN, set equal to 1. IDESIGN = 2 means that either "ALMOST FEASIBLE" or "FEASIBLE" designs are accepted by GENOPT as feasible designs. IDESIGN = 1 means that only "FEASIBLE" designs are accepted by GENOPT as feasible designs. As mentioned above, a design is "ALMOST FEASIBLE" if the most critical margin is greater than -0.05, and a design is "FEASIBLE" if the most critical margin is greater than -0.01. The curve from Fig. 18, which pertains to the balloon with the truss-like (slanted) webs, is also included as the first trace in Fig. 25.

The best optimum design of the balloon with truss-like (slanted) webs weighs considerably less than the best optimum design of the balloon with the radial webs.

Figures 26 and 27 contain plots of the objective and the design margins, respectively, from a partial SUPEROPT run of the balloon with 30 modules over 90 degrees of circumference. This SUPEROPT run was made **before** the strategy for avoiding failure of convergence of the pre-buckling equilibrium solution received its final improvement: the introduction of the third try (**Try no. 3** in **Item 9** in **Section 8**) for a solution in which Load Step No. 1 is divided into 50 sub-increments. Figures 27b and 27c contain plots analogous to those in Figs. 26 and 27, respectively. These two plots (Figs. 27b,c) were generated following an execution of SUPEROPT made **after** the final refinement of the strategy used to obtain converged solutions of the nonlinear pre-buckling equilibrium analysis. The introduction of **Try no. 3** to the strategy seems to have completely eliminated unintended early terminations of SUPEROPT caused by failure of convergence of the nonlinear pre-buckling equilibrium equations during Newton iterations. In every case run since the introduction of **Try no. 3** SUPEROPT has either been terminated early on purpose or has run to a successful completion when the total number of design iterations is about 470.

Figures 28 and 29 show the general and local buckling modes of the optimized design with 8 modules. Figures 30 and 31 show the general and local buckling modes of the optimized design with 35 modules. The general and local buckling modes of the optimized design with 15 modules are displayed in Figs. 23 and 24.

Most of the optimization runs, results for which are displayed in Figs. 27b and 27c, for example, were conducted with the strategy index, IMOVE = 1 in the try4.OPT file (Table 10). IMOVE = 1 means that each decision variable has a "move limit" that is 10 per cent of its current value. It is often difficult to close in precisely on a minimum weight design with IMOVE = 1 because the values of the design margins "oscillate" from design iteration to iteration as shown, for example, in Fig. 27. In order to close in more precisely on a local minimum weight it is sometimes a good idea to set IMOVE = 4 in the try4.OPT file. IMOVE = 4 means that each decision variables has a "move limit" that is 2 per cent of its current value. The more restrictive move limit that exists when IMOVE = 4 rather than IMOVE = 1 leads to smoother aspects of the objective versus design iteration and design margins versus design iteration, as displayed in Figs. 32 and 33, respectively.

Ordinarily the more restrictive move limit, IMOVE = 4, should not be used in connection with SUPEROPT. Instead, it should be used only in connection with OPTIMIZE (working in the optimization mode: analysis type, ITYPE = 1 in the try4.OPT file). The processor, OPTIMIZE, can be executed by the end user many times in succession if that is required in order to close in on an acceptable optimum design.

An example of the use of IMOVE = 4 combined with an execution of SUPEROPT is shown in Figs. 34 and 35. In this particular case the end user has specified 12 executions of OPTIMIZE for every execution of AUTOCHANGE, which is the GENOPT processor that generates a new "starting" design by a random process [14]. With the restrictive IMOVE = 4 many successive executions of OPTIMIZE are required for a design to "settle down" to a local optimum after each execution of AUTOCHANGE. Therefore, a successfully completed SUPEROPT execution with the use of IMOVE = 4 would correspond to fewer "starting" designs than is exhibited in Figs 6b and. 27b, for examples, for which IMOVE = 1 and the end user has specified that there be 5 executions of OPTIMIZE for each execution of AUTOCHANGE. With more "starting" designs it is more likely that a "global" optimum design will be found during an execution of SUPEROPT.

An example of the effect on minimum weight of IMOVE appears in Fig. 25 for the balloon with radial webs and 25 modules. The little triangle is the minimum weight determined with IMOVE = 1. The point at 25 modules on the second trace in Fig. 25 is the minimum weight determined after two successive executions of

OPTIMIZE with IMOVE = 4 and with the use of a "starting" design equal to the optimized design of the balloon with 30 modules.

In optimized balloons with truss-like (slanted) webs the fundamental buckling mode is sometimes local buckling (Fig. 19) with the general buckling mode corresponding to a higher eigenvalue (Fig. 20), and the fundamental buckling mode is sometimes general buckling (Fig. 11) with the local buckling mode corresponding to a higher eigenvalue (Fig. 12). In contrast, in optimized balloons with radial webs the fundamental buckling mode is general buckling in every case explored during this project.

SECTION 13 CONCLUSIONS

The following conclusions may be drawn:

1. The best optimum design of the balloon with truss-like (slanted) webs weights considerably less than the best optimum design of the balloon with the radial webs.

2. For the balloon with truss-like (slanted) webs the lowest buckling eigenvalue sometimes corresponds to local buckling and sometimes corresponds to general buckling (ovalization). In the cases explored here the lowest local buckling mode always involves one or more of the straight segments that form the inner membrane of thickness, TFINNR. No other parts of the wall of the balloon buckle locally at a lower local buckling load factor.

3. For the optimized balloons with radial webs the lowest buckling eigenvalue corresponds to general buckling (ovalization) in every case explored in this project.

4. In spite of the fact that the segments of the balloon wall behave like membranes that have no bending stiffness rather than like shells that have a finite bending stiffness, BIGBOSOR4, which is designed to handle segmented shell structures with finite bending stiffness, seems to be capable of solving both the nonlinear pre-buckling phase and the linear bifurcation buckling phase of the analysis.

5. A strategy is established by means of which failure of convergence of the nonlinear pre-buckling analysis is minimized.

6. Stress components in the various segments of a module are computed from membrane theory, that is, the stress component is equal to the appropriate stress resultant divided by the thickness of the segment. This is an unconservative strategy. An actual balloon fabricated in a configuration that corresponds to an optimized design developed here by GENOPT/BIGBOSOR4 should have reinforced seams at the junctions between segments.

7. The capability to analyze and design double-walled cylindrical vacuum chambers (balloons) is established within the GENOPT/BIGBOSOR4 framework. Enough information is provided in this paper and in [2] and in [6 – 11] so that researchers can use GENOPT/BIGBOSOR4 to analyze and design other shell structures of a similar nature.

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