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A REPORT ON MINIMUM WEIGHT DESIGN OF IMPERFECT ISOGRID-STIFFENED ELLIPSOIDAL SHELLS UNDER UNIFORM EXTERNAL PRESSURE

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ABSTRACT

GENOPT, a program that can be used to optimize anything, and BIGBOSOR4, a program for stress, buckling, and vibration analysis of segmented, branched, stiffened, elastic shells of revolution, are combined to create a capability to optimize a specific kind of shell of revolution: an internally isogrid-stiffened elastic ellipsoidal shell subjected to uniform external pressure. Optimum designs are obtained for isogrid-stiffened and unstiffened axisymmetrically imperfect and perfect titanium 2:1 ellipsoidal shells. The decision variables are the shell skin thickness at several user-selected meridional stations, the height of the isogrid stiffeners at the same meridional stations, the spacing of the isogrid stiffeners (constant over the entire shell), and the thickness of the isogrid stiffeners (also constant over the entire shell). The design constraints involve maximum stress in the isogrid stiffeners, maximum stress in the shell skin, local buckling of an isogrid stiffener, local buckling of the shell skin between isogrid stiffeners, general nonlinear bifurcation buckling, nonlinear axisymmetric collapse, and maximum normal displacement at the apex of the dome. Optimum designs first obtained by GENOPT are subsequently evaluated by the use of STAGS, a general-purpose finite element computer program. It is found that in order to obtain reasonably good agreement between predictions from BIGBOSOR4 and STAGS it is necessary to model the ellipsoidal shell as an "equivalent" ellipsoidal shell consisting of a spherical cap and a series of toroidal shell segments that closely approximates the true ellipsoidal meridional shape. The equivalent ellipsoidal shell is optimized with up to four axisymmetric buckling modal imperfections, each imperfection shape assumed to be present by itself. Computations include both plus and minus axisymmetric buckling modal imperfection shapes. At each design cycle and for the plus and minus version of each axisymmetric imperfection shape the following analyses are conducted: 1. linear general axisymmetric bifurcation buckling analysis (in order to obtain the axisymmetric linear buckling modal imperfection shapes), 2. nonlinear axisymmetric stress analysis at the design pressure, 3. nonlinear axisymmetric collapse analysis, and 4. nonlinear non-axisymmetric bifurcation buckling analysis. For each axisymmetric imperfection shape the design margins include an axisymmetric collapse margin, a general buckling margin, a margin involving the normal displacement of the apex of the shell, and local skin and stiffener stress margins and local skin and stiffener buckling margins within two approximately equal meridional regions of the equivalent ellipsoidal shell. There is

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generally good agreement of the predictions from STAGS and from BIGBOSOR4 for the elastic behavior of the perfect stiffened and unstiffened optimized shells and for the behavior of the imperfect stiffened optimized shells with axisymmetric buckling modal imperfections. Optimization with the use of only axisymmetric buckling modal imperfections has a disadvantage in the case of the unstiffened imperfect shell under certain conditions: the optimum design of the axisymmetrically imperfect unstiffened shell evolves in such a way that, according to predictions from STAGS, a non-axisymmetric buckling modal imperfection with the same amplitude as an axisymmetric buckling modal imperfection causes collapse of the shell at an external pressure far below the design pressure. This disadvantage is easily overcome if, during optimization cycles, the unstiffened shell wall in the neighborhood of the apex is forced to remain thick enough so that local axisymmetric buckling does not occur primarily at and near the apex but instead occurs primarily in the remainder of the shell. An extensive study of some of the previously optimized elastic shells is conducted with STAGS including elastic-plastic material properties. The effect on collapse pressure of initial imperfections in the form of off-center residual dents produced by load cycles applied before application of the uniform external pressure is determined and compared with the effect on collapse pressure of imperfections in the form of non-axisymmetric and axisymmetric linear buckling modes, especially the non-axisymmetric linear buckling modal imperfection with n=1 circumferential wave, which seems to be the most harmful imperfection shape for optimized externally pressurized ellipsoidal shells. For the optimized unstiffened shell it is found that a residual dent that locally resembles the n=1 linear buckling modal imperfection shape is just as harmful as the entire n=1 linear buckling modal imperfection shape.

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1.0 INTRODUCTION

Cohen and Haftka [1] were the first to create a capability that could be used for the automated design of imperfect shells of revolution. In this report a further step is taken by combination of GENOPT [2-7], which incorporates the optimizer ADS written by Vanderplaats and his colleagues [8,9], with a version of BOSOR4 [10-12] called "BIGBOSOR4" [7] to permit optimization of a certain class of shells of revolution: an elastic isogrid-stiffened ellipsoidal shell of revolution subjected to uniform external pressure. BIGBOSOR4 requires the same input data and performs the same analyses as BOSOR4, but BIGBOSOR4 will handle shells of revolution with more segments and more degrees of freedom than can be handled by BOSOR4.

GENOPT is described in detail in [2]. It has previously been used to obtain optimum designs of various systems [3-7]. Optimum designs are obtained via the optimizer, ADS, created many years ago by Vanderplaats and his colleagues [8,9]. In GENOPT the optimizer, ADS, is "hardwired" in the "0-5-7" mode, which is a modified method of steepest descent. This optimization method requires as input the gradients of the design constraints (stress, buckling, collapse, displacement) with respect to each of the decision variables.

As described in [2] and [7], GENOPT creates a system of processors called "BEGIN", "DECIDE", "MAINSETUP", "OPTIMIZE", "SUPEROPT", "CHANGE", "CHOOSEPLOT", "DIPLOT", by means of which optimum designs can be obtained. The architecture of this system of processors is analogous to that previously generated for specific applications [13,14]. "SUPEROPT" is a script by means of which "global" optimum designs can be obtained, as described in [7] and [15] and briefly on the second page of Table 34. “Global” is enclosed in quotation marks because SUPEROPT does not actually find the global optimum design but, with repeated executions, will find a design that is very likely to have an objective that is very close to the global objective.

2.0 PURPOSE AND SUMMARY OF THIS REPORT

2.1 Purpose of the report

The author was motivated to create a program to optimize an elastic shell of revolution **the behavior of which is significantly nonlinear**. In this application the nonlinearity is entirely caused by moderately large axisymmetric prebuckling meridional rotations of the elastic externally pressurized 2:1 ellipsoidal (or "equivalent" ellipsoidal) shell. (NOTE: some of the STAGS models used to evaluate the optimum designs produced by GENOPT include plastic flow.)

The author wanted to generate another application of GENOPT. **He hopes that in the future GENOPT will be used by others to obtain optimum designs of entirely different systems**. In this long version of this document (called “report” in the four file folders and names for text, figures, tables, and appendix), the text, the figures, the tables, and the appendix are stored electronically in four separate folders:

text = sdm50.report.pdf;
figures = sdm50pdf.report.figures;
tables = sdm50pdf.report.tables;
appendix = sdm50pdf.report.appendix.

This electronic report is long. In particular there are many, many figures and tables. Some of the tables are very long. **They are all included here in order that a researcher or designer will be able to use this information to obtain, by analogy, the optimum design of any different system.**

The captions of the figures and tables are unusually long. The intention is to minimize the need for the reader to flip back and forth from text to figure or from text to table to learn the meaning of the data presented there. A significant amount of the textual material in this report is contained in those captions. A nomenclature section is not included in this report because there are few equations, the equations are simple, and the meanings of the symbols in them are explained with them.

2.2 Summary of this report

GENOPT [2, 7] is used to obtain optimum designs of externally pressurized, perfect or imperfect, isogrid-stiffened or unstiffened, elastic titanium 2:1 ellipsoidal shells subjected to uniform external pressure, p = 460 psi (called the "design pressure").

GENOPT [2, 7] is described in Section 3. **BIGBOSOR4** [7] and **STAGS** [20 – 24] are described in Section 4.

The necessity to use an "**equivalent**" ellipsoidal shell profile rather than a **true** ellipsoidal shell profile is explained in Section 5.

Minimum-weight optimum designs are obtained in the presence of **stress, collapse, bifurcation buckling and displacement design constraints** derived from the various analyses described in Section 6.

Numerical results for **optimized isogrid-stiffened and unstiffened perfect and imperfect elastic titanium 2:1 equivalent ellipsoidal shells** are presented in Sections 7 and 8. The shells are first optimized by GENOPT in the presence of **axisymmetric buckling modal imperfections**. Then the optimum designs are evaluated by means of STAGS models that include both axisymmetric and non-axisymmetric buckling modal imperfection shapes.

It turns out that the pressure-carrying capacity of the optimum design of the **unstiffened, imperfect** shell is extremely sensitive to non-axisymmetric buckling modal imperfections, a type of imperfection that cannot be accounted for in the GENOPT model, which can only handle axisymmetric buckling modal imperfections because the BIGBOSOR4 computer program can only handle axisymmetric imperfections. Therefore, the optimized unstiffened shell is severely under-designed. **This deficiency is avoided by a simple reformulation of the optimization problem in which a higher lower bound is specified for the shell wall thickness in the neighborhood of the apex of the shell.**

Improved optimum designs of **unstiffened** axisymmetrically imperfect equivalent ellipsoidal shells are derived in Section 9. These improved optimum designs, the so-called "**thick-apex**" optimum designs, are evaluated by means of STAGS [20 – 24]. The STAGS models account for elastic-plastic material, axisymmetric and non-axisymmetric buckling modal imperfections, and imperfections in the form of **off-center residual dents** produced via a "Load Set B" load cycle involving concentrated normal inward-directed loads or imposed displacements. By "**off-center**" is meant dents located at some radius from the axis of revolution of the shell. Collapse of the dented shells is determined after completion of the Load Set B load cycle by subsequent application of uniform external pressure in Load Set A.

A "**thick-apex**" optimum design of the unstiffened, imperfect shell is found which survives the design pressure in the presence of either axisymmetric or non-axisymmetric buckling modal imperfections or off-center residual dents.

The optimum design of the **isogrid-stiffened** shell derived in Section 8.1 is evaluated in Section 10 by means of STAGS models that include elastic-plastic material, axisymmetric and non-axisymmetric buckling modal imperfections, and imperfections in the form of off-center residual dents.

It is emphasized that STAGS is not used within the optimization "loop", but only AFTER the optimum design has been obtained by GENOPT.

3.0 ABOUT GENOPT (GENeral OPTimization)

The purpose of GENOPT [2] is to enable an engineer or researcher to create a user-friendly system of computer programs for analyzing and/or optimizing anything. **One of the main advantages of GENOPT is that it provides a way in which an engineer or researcher can extend an EXISTING ANALYSIS CAPABILITY to a capability to obtain OPTIMUM DESIGNS based on that existing analysis capability.** The application of GENOPT is not limited to the field of structural mechanics. In [2] the purpose, properties, and operational details of GENOPT are described. The reader is urged to read [2] and [7] in order to obtain a more complete understanding of the work described in this report.

In **Sub-section 3.1** the GENOPT processors, GENTEXT, GENPROGRAMS, BEGIN, DECIDE, MAINSETUP, OPTIMIZE, SUPEROPT, CHOOSEPLOT, CHANGE, and AUTOCHANGE are briefly described. In **Sub-section 3.2** the concept is introduced that GENOPT automatically creates FORTRAN code some of which is complete and some of which is in **skeletal** form, to be "**fleshed out**" by the GENOPT user for application to a particular generic system chosen by the GENOPT user. In **Sub-section 3.3** the terms "design gradients", "design constraints", and "design margins" are defined. In **Sub-section 3.4** two types of user, the GENOPT user and the "end" user, are identified. **Sub-section 3.5** is long. In **Sub-section 3.5.1** the optimization problem for the generic case, "**equivellipse**" (equivalent ellipsoidal shell of revolution) is formulated with respect to configuration, boundary conditions, shell wall construction, loading, and imperfections (Sub-section 3.5.1.1); the "behaviors" such as stress, displacement, buckling, and collapse that govern the evolution of the design during optimization cycles are defined (Sub-section 3.5.1.2); and the fact that the GENOPT user has to create "user-friendly" variable names, one-line definitions, and "help" paragraphs is explained (Sub-section 3.5.1.3). In **Sub-section 3.5.2** the seven roles that GENOPT-user-established variables play are defined. In **Sub-section 3.5.3** an example of part of a "GENTEXT" interactive session is given in which the GENOPT user creates variable names, one-line definitions, and "help" paragraphs for the generic case, "equivellipse". In **Sub-section 3.5.4** the completed and skeletal FORTRAN libraries automatically created by GENOPT are briefly described. In **Sub-sections 3.6 and 3.7** details are presented corresponding to two parts of the "GENTEXT" interactive "equivellipse" session;

exactly what FORTRAN coding GENOPT automatically generates is listed in several tables and suggestions are given relating to the provision of “user-friendly” “help” paragraphs (Sub-section 3.7.2), automatic creation by GENOPT of the skeletal “behavior” subroutines BEHXi (Sub-section 3.7.3), and whether the GENOPT user should “flesh out” the “behavior” subroutines BEHXi or the “STRUCT” subroutine (Sub-section 3.7.4). Finally, in **Sub-section 3.8** some details are provided relative to “fleshing out” SUBROUTINE STRUCT.

3.1 GENOPT processors

GENOPT is executed via the following commands:

GENOPTLOG	(The GENOPT command set is activated.)
GENTEXT	(The GENOPT user responds interactively to prompts by GENOPT in order to provide names, definitions, and roles of variables to be used during execution of the user-friendly system of programs described next.)
GENPROGRAMS	(GENOPT compiles and creates executable processors called "BEGIN", "DECIDE", "OPTIMIZE", "CHOOSEPLOT", "CHANGE", "AUTOCHANGE", described next.)

During the interactive execution of "GENTEXT" by the GENOPT user, GENOPT creates a system of computer programs consisting of the following independently executable processors:

BEGIN	(The user supplies the starting design, material properties, loads, allowables, factors of safety, etc.)
DECIDE	(The user chooses decision variables, lower and upper bounds, linked variables, and inequality constraints.)
MAINSETUP	(The user chooses strategy parameters, which design constraints to ignore during program execution, and analysis type: 1. optimization or 2. analysis of a fixed design or 3. design sensitivity.)
OPTIMIZE or SUPEROPT	(The program system performs the analysis type specified by the user in MAINSTUP. SUPEROPT is described in [15] and briefly on the second page of Table 34.)
CHOOSEPLOT	(The user chooses which quantities to plot vs design iterations or vs the value of a user-selected design sensitivity variable.)
DIPLOT	(The user obtains postscript files, *.ps, which can be used to obtain plots of objective, margins, decision variables vs design iterations or vs a user-selected design sensitivity variable.)

CHANGE	(The user changes selected problem variables. CHANGE is most often used as a device by means which to save a previously obtained optimum design.)
AUTOCHANGE	(The program system changes all decision variables randomly, in a manner consistent with user-specified bounds, linking constraints, and inequality constraints. AUTOCHANGE and OPTIMIZE are executed repeatedly as part of the SUPEROPT script [15] described briefly on the second page of Table 34.)

3.2 Some software is written by GENOPT, some software is written by the GENOPT user

Certain parts of some of these processors (BEGIN, OPTIMIZE, CHANGE) are written in FORTRAN by the GENOPT program system during the interactive "GENTEXT" execution [2,7]. For example, certain subroutines called by the processor, OPTIMIZE (which is named “MAIN” internally), are partly written by GENOPT. These subroutines are named SUBROUTINE STRUCT and SUBROUTINEs BEHXi, i = 1,2,3… and SUBROUTINE OBJECT. (See [7]). SUBROUTINEs BEHXi, i = 1,2,3… are called by SUBROUTINE STRUCT. As created by GENOPT, these subroutines are "**skeletons**"; they have automatically generated argument lists, automatically generated labeled common blocks, and automatically generated "RETURN" and END" statements. They must be supplemented (“**fleshed out**”) by the GENOPT user, whose responsibility it is to add the coding that computes the behavior (for example, stress, buckling, vibration, clearance, etc.) of the generic class of items to be optimized.

The labeled common blocks generated automatically during the GENTEXT interactive session contain all the variables that define the class of objects to be optimized. The body of each of the "skeletal" subroutines STRUCT and/or BEHXi must be “**fleshed out**” by the GENOPT user. See [2,7,16] for examples of how this is done. Also see Tables a30 and a31 in the appendix. In the present application, which has the generic name, “**equivellipse**” (meaning “equivalent ellipsoidal shell”), only SUBROUTINE STRUCT is "fleshed out" by the GENOPT user; the SUBROUTINEs BEHXi, i = 1,2,3,..14, are not modified by the GENOPT user but are left as automatically created by GENOPT. In other applications [2 - 6] SUBROUTINEs BEHXi are “fleshed out” by the GENOPT user ([2] and Table a31, for example) in order to compute the “behavior”, that is, stress, buckling, etc. SUBROUTINE OBJECT, which is part of the “behavior.new” library, computes the objective function, which in this application is the weight of the ellipsoidal or “equivalent” ellipsoidal shell. In this generic case, “**equivellipse**”, the “fleshed out” version of SUBROUTINE STRUCT is very long. A complete list of it is provided in the appendix (Table a16). Complete lists of the skeletal SUBROUTINE STRUCT (Table a14) and SUBROUTINEs BEHXi, i = 1,2,3… and SUBROUTINE OBJECT (Table a13) are also given in the appendix for the generic case called “**equivellipse**”.

3.3 Design gradients, design constraints, design margins

During each optimization cycle, SUBROUTINE STRUCT is called to evaluate the "current" design and each "perturbed" design. A "perturbed" design is the same as the "current" design except that one of the decision variables has been perturbed a small amount (usually 5 per cent) in order to obtain gradients of the responses or “behaviors” (stress, buckling, collapse, apex deflection), which are needed by the optimization software, ADS [8,9]. ADS is embedded in the GENOPT system. The gradients are obtained by simple finite difference:

gradient=[(response for perturbed design) - (response for current design)]/DX (1)

in which DX is the change (perturbation) in one of the decision variables. “Response”, which is a “behavior” such as stress, buckling, collapse, apex deflection, etc., is closely related to what in the optimization literature is called a “design constraint”. Design constraints usually have one of the following three forms:

Form 1: (not used in this report; see Table 16)

Form 2: (design constraint) = [(response)/(allowable response)]/(factor of safety) (2)
Form 2 is used for responses such as **bifurcation buckling** or **nonlinear collapse** that must be greater than a user-specified minimum allowable amount.

Form 3: (design constraint) = [(allowable response)/(response)]/(factor of safety) (3)
Form 3 is used for maximum **stress** or maximum **deflection** because the response (stress or deflection) must be less than a user-specified maximum allowable amount.

Design “margins” are given by

(design margin) = (design constraint) – 1.0 (4)

In an academic sense a design is considered feasible only if all design constraints exceed unity or design margins exceed zero. However, for certain practical reasons GENOPT accepts a design as “FEASIBLE” provided that no margin is less than –0.01. GENOPT accepts a design as “ALMOST FEASIBLE” provided that no margin is less than –0.05. GENOPT accepts a design as “MILDLY UNFEASIBLE” provided that no margin is less than –0.1. Which quality of design is accepted by GENOPT is governed by a user-selected index, IDESIGN, an input datum prompted for during the MAINSETUP interactive session. (See Item 725 in Table a24).

3.4 Two types of user

There are two types of user referred to in [2] and [7] and in this report:

1. the GENOPT user

2. the "end" user or simply “the user”.

The roles of the two types of user are defined in [2] and briefly in [7]. In brief, the GENOPT user decides what **generic class of objects** is to be optimized and, using GENTEXT and GENPROGRAMS, "sets up" the processors just listed (BEGIN, DECIDE, etc.) for the "end" user to use for a **specific member** of that generic class. The "end" user establishes, for the specific member of the generic class: 1. the starting design (BEGIN), 2. decision variables and bounds (DECIDE), and 3. analysis type and strategy for the specific object to be optimized (MAINSETUP). Then the "end" user performs the optimization (OPTIMIZE or SUPEROPT). Often the GENOPT user and the "end" user are the same person. Two types of user are identified here and in [2] because the activities required of each differ.

3.5 What the GENOPT user creates and what GENOPT creates

3.5.1 Formulation of the optimization problem for the generic case, "equivellipse"

Before working with the computer the GENOPT user must formulate the optimization problem.

3.5.1.1 Configuration, boundary conditions, loading

The GENOPT user must decide (in this particular application of GENOPT, "equivellipse") what shell of revolution or class of shells of revolution is to be optimized:

1. shape of the shell (ellipsoidal or **"equivalent" ellipsoidal** or torispherical)
2. boundary conditions (symmetry conditions at the equator)
3. shell wall (internally isogrid stiffened, isotropic elastic material)
4. imperfections (axisymmetric linear buckling modal imperfections)
5. loading (uniform external pressure)

In the generic case, **equivellipse**, the GENOPT user (the writer) decided to consider the imperfections as part of the "environment", that is, part of the loading. Therefore, the GENOPT user decided to introduce multiple load sets even though there is only one type of physical loading: uniform external pressure. The GENOPT user established the following load sets:

Load Set 1 = uniform external pressure and +mode 1 axisymmetric buckling modal imperfection and +mode 2 axisymmetric buckling modal imperfection, in which "mode 1" means the first axisymmetric linear buckling eigenvector and "mode 2" means the second axisymmetric linear buckling eigenvector. The "mode 1" and "mode 2" imperfections are applied one at a time, each in its own turn. The GENOPT user established "behavioral" variables (also called "response" variables in this report such as variables for collapse, bifurcation buckling, stress, and displacement), pertaining to the "mode 1" environment that are different from those behavioral variables pertaining to the "mode 2" environment. For example, the GENOPT user decided to name the behavioral variable for collapse in the presence of a "mode 1" imperfection shape CLAPS1 and the behavioral variable for collapse in the presence of a "mode 2" imperfection shape CLAPS2. The GENOPT user established analogous "1" and "2" names for the behavioral variables for bifurcation buckling, stress, and displacement. These GENOPT-user-established names are listed in Table 2, which will be discussed later.

Load Set 2 = uniform external pressure and –mode 1 axisymmetric buckling modal imperfection and –mode 2 axisymmetric buckling modal imperfection. . Hence, Load Set 1 and Load Set 2 are the same except for the sign of the “mode 1” and the sign of the “mode 2” axisymmetric linear buckling modal imperfection shapes.

In cases for which the optimization included the effect of the first four axisymmetric linear buckling imperfection shapes, that is, mode 1, mode 2, mode 3, and mode 4, the GENOPT user established four load sets. Load Sets 1 and 2 pertain to the presence of plus and minus mode 1 and mode 2 imperfections as just listed. Load Sets 3 and 4 are as follows:

Load Set 3 = uniform external pressure and +mode 3 axisymmetric buckling modal imperfection and +mode 4 axisymmetric buckling modal imperfection. In Load Set 3 the digit “1” in the behavioral variable name denotes “odd-numbered imperfection mode shape” and the digit “2” in the behavioral variable name denotes “even-numbered imperfection mode shape”.

Load Set 4 = uniform external pressure and –mode 3 axisymmetric buckling modal imperfection and –mode 4 axisymmetric buckling modal imperfection.

3.5.1.2 Which behaviors (stress, displacement, buckling, collapse) constrain the design

The GENOPT user must decide what behaviors may constrain the design:

1. stress (maximum effective stress in the skin and in the stiffeners of the imperfect shell)
2. displacement (normal displacement at the apex of the imperfect shell)
3. buckling (local buckling of skin and stiffeners, general buckling of the imperfect shell)
4. nonlinear collapse (axisymmetric collapse)

3.5.1.3 The GENOPT user creates variable names, definitions, and “help” paragraphs

The GENOPT user must identify all variables in the problem, create user-friendly names for these variables, create user-friendly one-line definitions for each of the variables, and create supporting "HELP" paragraphs for some or all of the variables. The variable names (8 characters or less) and one-line definitions (60 characters or less) are especially important because they appear in the output data and therefore should be easy to understand by the “end” user, who may not be as familiar with the jargon of the field as is the GENOPT user. The GENOPT-user-established variable names, one-line definitions, and "HELP" paragraphs are what make the system of processors created by GENOPT "user-friendly" if the GENOPT user has done his or her job well. (NOTE: In the “equivellipse” example provided here the GENOPT user (the writer) did **not** do his job very well because he did not provide enough “help” paragraphs. He was a bit sloppy about this because he also planned to be the “end” user. See Sub-section 3.7.2 for examples of where additional “help” should probably have been provided.)

3.5.2 Various roles that variables governing the generic problem play

As described in [2], GENOPT requires that each of the variables be categorized as performing one of the seven roles listed in Table 1. Variables that perform roles 4, 5, and 6 are always "bundled" together. For example, if the GENOPT user selects a variable with Role No. 4 to have the name, "SKNST1", meaning "maximum stress in the shell skin, mode 1 axisymmetric imperfection shape", then the next variable name he or she must provide during the GENTEXT interactive session is an “allowable”. The “allowable” that corresponds to the behavior called “SKNST1” might well have the name, "SKNST1A", meaning "**allowable** stress in the shell skin, mode 1 axisymmetric imperfection shape". The third variable in the same "Role 4,5,6" bundle might well have the name, "SKNST1F", meaning "**factor of safety** for maximum stress in the shell skin, mode 1 axisymmetric imperfection shape". All "responses" (also called "behaviors" in this report) such as stress, buckling, displacement, etc. are treated in this manner. GENOPT requires the GENOPT user to provide all input relating to variables with Roles 1 and 2 before variables with Role 3. All variables with Role 3 must be provided before variables with Roles 4, 5, and 6. All "Role 4,5,6" bundles must be provided before the objective, which is the only Role 7 variable.

Table 2, taken from the GENOPT output file called "equivellipse.DEF" (Table a2), lists the variable names and one-line definitions established by the GENOPT user for the generic case explored here: the “equivalent” ellipsoidal shell. Table 2 also identifies the role number of each variable and whether or not the variable is an array. Notice especially the sequence or “bundle”, (Role 4,5,6) = (response, allowable response, factor of safety), that appears repeatedly for each type of response (each type of behavior). During the GENTEXT interactive session the GENOPT user (the writer in this case) “invented” the variable names, such as “CLAPS” for “collapse”, “SKNBK” for “local skin buckling”, “STFBK” for “stiffener buckling”, “SKNST” for “maximum effective stress in the skin”, “STFST” for “maximum stress in the stiffeners”, and “WAPEX” for “maximum normal displacement **w** at the apex of the shell”. The GENOPT user also supplied the phrases (one-line definitions of the variables) that follow the equals symbols on the right-hand half of Table 2. In this case Table 2 lists **seven** “Role 4,5,6” bundles pertaining to the behavior in the presence of an axisymmetric “mode 1” buckling modal imperfection shape (names ending with the digit “1”, such as CLAPS1) and **seven** “Role 4,5,6” bundles pertaining to the behavior in the presence of an axisymmetric “mode 2” buckling modal imperfection shape (names ending with the digit “2”, such as CLAPS2). The last variable listed in Table 2, the only Role 7 variable, is called by the GENOPT user WEIGHT. WEIGHT is the objective.

3.5.3 The “GENTEXT” interactive session

In order to establish user-friendly variable names, one-line definitions, and “help” paragraphs, the GENOPT user executes GENTEXT. **Table 3** lists the GENOPT user’s input (bold face) for the **first part** of the interactive GENTEXT session pertaining to the equivalent ellipsoidal shell, that is, the generic class called by the GENOPT user “**equivellipse**”. The complete input file for GENTEXT for “equivellipse” is listed in the appendix (Table a1). This complete file is called “**equivellipse.INP**”. If, during a rather long GENTEXT interactive session, the GENOPT user makes a mistake, he or she can terminate the GENTEXT session, suitably edit the end of the *.INP file where the mistake occurs, and re-execute GENTEXT, indicating that he/she is restarting a partly completed interactive session. GENTEXT will read input from the existing *.INP file until the end of that file, then return to the interactive mode of execution. Hence, the GENOPT user does not interactively have to repeat all his/her input up to the point where he/she made the mistake. This mode of operation is a characteristic of all the computer programs created by the writer over the years.

Table a1-b in the appendix reproduces **what the GENOPT user actually sees on his/her computer screen during an interactive GENTEXT session**. The GENOPT user’s responses are in bold face. Some comments in connection with this table are:
1. The lines that contain the string, “PART 1 …” and “PART 2 …” and “PART 3 …” do not appear on the computer screen.
2. In the middle of the first page of Table a1-b GENTEXT informs the GENOPT user that one of his/her tasks is “To complete subroutines BEHX1, BEHX2, BEHX3,…BEHXn which calculate equivellipse behavior for a given design;” Actually, GENOPT is more general than implied by that statement, since the GENOPT user may choose to “flesh out” SUBROUTINE STRUCT either instead of or in addition to “fleshing out” (completing) the “behavior” subroutines, BEHXi. In the generic case “equivellipse” the GENOPT user decided to “flesh out” SUBROUTINE STRUCT instead of completing the skeletal BEHXi routines automatically created by GENOPT.
3. On the second page of Table a1-b there occurs the instruction, “Hit RETURN”. Do that.
4. On pages 3 and 4 and later in Table a1-b there occur the lines, “(lines skipped to save space)”. Table a1-b would be very long if all those “lines skipped” were included. The material that has been skipped is analogous to the material included in the table.
5. GENTEXT echoes some of the GENOPT user’s input data. For example, when the GENOPT user typed, “this is explanatory text”, GENOPT echoed that phrase in the next line.
6. Where, in Table a1-b, the GENOPT user responded, “this is explanatory text” and “one more line”, in the actual case listed in Table a1 the GENOPT user typed the multi-lined INTRODUCTORY EXPLANATORY TEXT listed near the beginning of Table a1 that begins with the string, “`OPTIMUM DESIGN OF ISOGRID-STIFFENED ELLIPSOIDAL HEAD`”.

Table 4 is the part of the glossary produced by GENTEXT corresponding to the GENOPT user’s partial interactive input for the generic case “equivellipse” reproduced in Table 3. The complete glossary becomes part of the file called “equivellipse.DEF”. The complete glossary is listed in Table 2. This complete glossary is produced by GENTEXT after the GENOPT user finishes the GENTEXT interactive session. The entire input data file, “equivellipse.INP”, is reproduced in the appendix (Table a1). Also, the entire file, “equivellipse.DEF”, appears in the appendix (Table a2).

Table 5 is the part of the prompting file, “equivellipse.PRO”, produced by GENTEXT corresponding to the GENOPT user’s partial interactive input for the generic case “equivellipse” listed in Table 3. The complete prompting file, “equivellipse.PRO”, appears in Table 6. The prompting file is arranged automatically by GENTEXT. This file contains the following information.

1. prompting numbers, such as **5.0, 10.1, 10.2, 15.1, 20.1, 20.2**, etc.

2. the GENOPT-user-selected variable names:

npoint, xinput, ainput, binput, nodes, xlimit, THKSKN, HIGHST, SPACNG, THSTIF, THKCYL, RADCYL, etc.

3. corresponding one-line definitions of the variables created by the GENOPT user and just listed. NOTE: GENOPT automatically adds the string, ": <variable name>", to the one-line definition supplied by the GENOPT user, resulting in the following modified one-line definitions corresponding to **npoint**, **xinput**, **ainput**, etc:

number of x-coordinates: npoint
x-coordinates for ends of segments: xinput
length of semi-major axis: ainput
length of semi-minor axis of ellipse: binput
number of nodal points per segment: nodes
max. x-coordinate for x-coordinate callouts: xlimit
skin thickness at xinput: THKSKN
height of isogrid members at xinput: HIGHST
spacing of the isogrid members: SPACNG
thickness of an isogrid stiffening member: THSTIF
thickness of the cylindrical shell: THKCYL
radius of the cylindrical shell: RADCYL
etc.

The GENOPT user's one-line definitions are in bold face in the items just listed. The one-line definitions with the added string, ":<variable name>", are what is seen by the "end" user.

4. "help" paragraphs created by the GENOPT user, such as:

10.2
"**The ellipse is simulated by a number of shell segments (try 10)
each of which has constant meridional curvature (toroidal).
npoint is the number of x-coordinates corresponding to the
ends of the toroidal segments that make up the equivalent
ellipse. You might try to simulate the ellipse by using 10
toroidal segments. Then the value of npoint would be 11
npoint includes the apex of the ellipse (x = 0) and the equator
of the ellipse (x = a, in which a = semimajor axis length).**"

as a "help" paragraph for the variable, **npoint**

and

20.2

“**Please make sure to include x = 0 and x = a (equator) when you provide values for xinput**.”

as a “help” paragraph for the variable, **xinput**

and

25.2

“**ainput is the maximum "x=dimension" of the ellipse.**
The equation for the ellipse is x^2/a^2 + y^2/b^2 = 1.0”

as a “help” paragraph for the variable, **ainput**

and

30.2

“**binput is the y-dimension of the ellipse, the equation for which is x^2/a^2 + y^2/b^2 = 1.0**.”

as a “help” paragraph for the variable, **binput**

and

35.2

“**If you have about 10 segments, use a number less than 31.**
Use an odd number, greater than or equal to 11”

as a “help” paragraph for the variable, **nodes**

and

40.2

“**xlimit has two functions:**
1. a delimiter for the definition of callouts:
for x < xlimit callouts are x-coordinates.
for x > xlimit callouts are y-coordinates.
Set xlimit equal to about a/2, where a = length of the semi-major axis of the ellipse.
2. a delimiter for the boundary between Region 1 and Region 2, Design margins for maximum stress and minimum buckling load in the shell skin and in the isogrid stiffeners can be computed in two regions,
Region 1: 0 < x < xlimit, and
Region 2: xlimit < x < semi-major axis.”

as a “help” paragraph for the variable, **xlimit**

and

45.2
“**xinput is the vector of x-coordinate callouts for thickness of the shell skin and height of the isogrid stiffeners**.”

as a “help” paragraph for the variable, **THKSKN**

and

50.2
“**xinput is the vector of x-coordinate callouts for thickness of the shell skin and height of the isogrid stiffeners**.”

as a “help” paragraph for the variable, **HIGHST**.

and

55.2
“**SPACNG = altitude of the equilateral triangle between adjacent isogrid members, measured to middle surfaces of isogrid members.
SPACNG = (length of side of triangle)*sqrt(3)/2.
SPACNG is constant over the entire shell**.”

as a “help” paragraph for the variable, **SPACNG**

and

60.2
“**THSTIF is constant over the entire shell**.”

as a “help” paragraph for the variable, **THSTIF**.

GENOPT automatically provides the prompting numbers, such as **50.1** for the one line definition of the variable **HIGHST** and **50.2** for the corresponding “help” paragraph.

3.5.4 Completed and “skeletal” FORTRAN libraries created by GENOPT

Table 7 lists the names of FORTRAN libraries created by GENOPT during the GENTEXT interactive session. BEGIN.NEW, STOGET.NEW, and CHANGE.NEW are entirely written by GENOPT. Lists of these three "*.NEW" FORTRAN libraries are given in the appendix (Tables a3, a4, and a5, respectively). The GENOPT user should not modify them in any way. As previously mentioned, STRUCT.NEW and BEHAVIOR.NEW, as created automatically by GENOPT, are **skeletal** libraries either or both of which must be "**fleshed out**" by the GENOPT user. These **skeletal** libraries are listed in the appendix (Tables a14 and a13, respectively). In this particular generic case, called "equivellipse" by the GENOPT user, only the **skeletal** library, **struct.new** (Table a14), is "**fleshed out**" by the GENOPT user (Table a16). The **skeletal** library, **behavior.new** (Table a13) remains unmodified in this case. Table a31 in the appendix provides an example from the GENOPT "literature" in which the **skeletal** behavior.new library is "**fleshed out**" by the GENOPT user for a generic case called "cylinder". Another case in which the behavior.new library is "fleshed out" is described in [2].

3.6 What GENOPT creates corresponding to the GENTEXT input listed in Table 3

Tables 8 – 14 pertain to this sub-section.

Table 8 lists the names and functions of several files, "equivellipse".xxx, automatically created by GENOPT during the GENTEXT interactive session. GENOPT uses these files:

1. to provide information to the user (equivellipse.PRO, equivellipse.DEF), and

2. to save the interactively provided input data (equivellipse.DAT, equivellipse.INP), and

3. to create the FORTRAN libraries listed in Table 7 and the FORTRAN files listed in Table 8 (equivellipse.NEW, equivellipse.COM, equivellipse.WRI, equivellipse.REA, equivellipse.SET, equivellipse.CON, equivellipse.SUB, equivellipse.CHA).

Corresponding to the GENOPT-user-provided input listed in Table 3, GENOPT automatically creates the FORTRAN fragments listed in Tables 9 – 14.

The file, equivellipse.CON, and the skeletal libraries, STRUCT.NEW (Table a14) and BEHAVIOR.NEW (Table a13), are not "worked on" by GENOPT until the GENOPT user starts defining the "bundles" of variables with Roles 4, 5, and 6 (Role 4 = behavioral variable, Role 5 = allowable variable, Role 6 = factor-of-safety variable). These Role 4, 5, and 6 variable names are used to construct behavioral constraints [Eqs.(2,3)]. This is described in the next sub-section.

The entire files that exist after the GENOPT user has completed the "GENTEXT" interactive session and that are identified in Table 8 are listed in the appendix (except for equivellipse.PRO, a complete list of which appears in Table 6, and equivellipse.DAT, which contains the same information as equivellipse.INP). The "equivellipse.xxx" files indicated in Table 8 appear in the appendix as follows: equivellipse.CHA = Table a7, equivellipse.COM = Table a6, equivellipse.CON = Table a12, equivellipse.DEF = Table a2, equivellipse.INP = Table a1, equivellipse.NEW = Table a10, equivellipse.REA = Table a8, equivellipse.SET = Table a11, equivellipse.SUB = Table a28, equivellipse.WRI = Table a9.

3.7 What GENOPT creates corresponding to the GENTEXT input listed in Table 15

3.7.1 General information

Tables 15 – 26 pertain to this sub-section, which is analogous to the previous sub-section.

Table 15 lists the GENOPT user's input (**bold face**) during the interactive GENTEXT session relating to

1. buckling of an isogrid member (STFBK1) in the presence of a "mode 1" axisymmetric imperfection and

2. stress in the skin of the stiffened shell (SKNST1) in the presence of a "mode 1" axisymmetric imperfection.

Table 16 identifies three types of behavioral constraints from which the GENOPT user can choose one type for each "bundle" of Role 4, 5, and 6 variables. Only Types 2 and 3 (called Form 2 and Form 3 in sub-section 3.3) are used in the application described in this report.

Table 17 is analogous to Table 9. Table 18 is analogous to Table 4. Table 19 is analogous to Table 5. Table 21 is analogous to Table 10. Table 22 is analogous to Table 11. Table 23 is analogous to Table 12. Table 24 is analogous to Table 13.

3.7.2 There should be more "help" paragraphs

Note that there are no "help" paragraphs in Table 19. In order to make the "equivellipse" optimization system more user-friendly, the GENOPT user (the writer) should have included some "help" paragraphs as described next.

3.7.2.1 Additional "help" paragraph option 1

In the GENTEXT interactive session immediately following where the GENOPT user provides input for the Role 3 variable, "uniform external pressure: PRESS" (see Table a1 and Table 6), he should have included a general "help" paragraph concerning the Role 4,5,6 "bundles". Without any additional "help" paragraph Table a1 now contains the following lines (GENOPT user's responses in **bold face**):

```
      1  $ Type of prompt: 0="help" paragraph, 1=one-line prompt
PRESS    $ Name of a variable in the users program (defined below)
      3  $ Role of the variable in the users program
uniform external pressure      $ (one line definition of PRESS)
n        $ Do you want to include a "help" paragraph?
n        $ Any more variables for role type  3 ?            $
      1  $ Type of prompt: 0="help" paragraph, 1=one-line prompt
```

```
  CLAPS1    $ Name of a variable in the users program (defined below)
```

During the interactive GENTEXT session the GENOPT user **should have included** something like the following material immediately after he answers the GENOPT prompt, "`Any more variables for role type 3 ?`" The GENOPT user's additional "**should have included**" responses to GENTEXT prompting are in **bold face**:

```
      0    $ Type of prompt: 0="help" paragraph, 1=one-line prompt
  Next, you supply input for the allowables
  y         $ Any more lines in the "help" paragraph?
  for every load case, followed by the factors
  y         $ Any more lines in the "help" paragraph?
  of safety for every load case. See Table 35
  y         $ Any more lines in the "help" paragraph?
  in the report "sdm50.report.pdf" for an example.
  y         $ Any more lines in the "help" paragraph?
  See Section 3.5.2 in "sdm50.report.pdf" for the
  y         $ Any more lines in the "help" paragraph?
  meanings of the behavioral array names and
  y         $ Any more lines in the "help" paragraph?
  subscripts. See Tables 31 and 32 in "sdm50.report.pdf"
  y         $ Any more lines in the "help" paragraph?
  for how the margins appear.
  n         $ Any more lines in the "help" paragraph?
```

The next two lines in the interactive GENTEXT input would remain as before:

```
      1   $ Type of prompt: 0="help" paragraph, 1=one-line prompt
  CLAPS1    $ Name of a variable in the users program (defined below)
```

The new "help" paragraph would have appeared in a modified equivellipse.PRO file (Table 6) as follows:

```
 115.0
     Next, you supply input for the allowables
     for every load case, followed by the factors
     of safety for every load case. See Table 35
     in the report "sdm50.report.pdf" for an example.
     See Section 3.5.2 in "sdm50.report.pdf" for the
     meanings of the behavioral array names and
     subscripts. See Tables 31 and 32 in "sdm50.report.pdf"
     for how the margins appear.
```

and the prompting numbers now given as 115.0, 120.1, 125.1, etc. would all have been increased by 5 .

3.7.2.2 Additional "help" paragraph option 2

Without any additional "help" paragraph Table 15 currently includes the following (GENOPT users responses in **bold face**):

```
y        $ Any more variables for role type  4 ?             $160
      1  $ Type of prompt: 0="help" paragraph, 1=one-line prompt
STFBK1   $ Name of a variable in the users program (defined below)
      4  $ Role of the variable in the users program
n        $ Do you want to reset the number of columns in STFBK1 ?
buckling load factor, isogrid member,mode 1 $ one-line def.,STFBK1
n        $ Do you want to include a "help" paragraph?
```

The GENOPT user **should have included** a “help” paragraph immediately following his answer to the GENOPT prompt, “`$ Any more variables for role type  4 ?`” He **should have included** something like the following responses to GENOPT prompting. The GENOPT user’s additional “**should have included**” responses to GENTEXT prompting are in **bold face:**

```
      0   $ Type of prompt: 0="help" paragraph, 1=one-line prompt
Next, you will be asked to supply allowables
y         $ Any more lines in the “help” paragraph?
for STFBK1 (STFBK1A) for every load case, followed
y         $ Any more lines in the “help” paragraph?
by factors of safety (STFBK1F) for every load case.
y         $ Any more lines in the “help” paragraph?
“STFBK” means “isogrid-stiffener buckling”. The
y         $ Any more lines in the “help” paragraph?
buckling load factor is computed as described
y         $ Any more lines in the “help” paragraph?
in Table 27 of the report, “sdm50.report.pdf”.
y         $ Any more lines in the “help” paragraph?
The digit, “1”, in the name STFBK1 means “isogrid-
y         $ Any more lines in the “help” paragraph?
stiffener buckling in the presence of an imperfect
y         $ Any more lines in the “help” paragraph?
shell with a plus or minus “mode 1” (or “mode 3”)
y         $ Any more lines in the “help” paragraph?
axisymmetric imperfection shape. Please see Section
y         $ Any more lines in the “help” paragraph?
3.5.2 of “sdm50.report.pdf” for more information about
y         $ Any more lines in the “help” paragraph?
the naming of behavioral variables, allowables, and
y         $ Any more lines in the “help” paragraph?
factors of safety.
n         $ Any more lines in the “help” paragraph?
```

The new “help” paragraph would have appeared in a modified equivellipse.PRO file (Table 6) as follows:

```
165.0
     Next, you will be asked to supply allowables
     for STFBK1 (STFBK1A) for every load case, followed
```

```
by factors of safety (STFBK1F) for every load case.
“STFBK” means “isogrid-stiffener buckling”. The
buckling load factor is computed as described
in Table 27 of the report, “sdm50.report.pdf”.
The digit, “1”, in the name STFBK1 means “isogrid-
stiffener buckling in the presence of an imperfect
shell with a plus or minus “mode 1” (or “mode 3”)
axisymmetric imperfection shape. Please see Section
3.5.2 of “sdm50.report.pdf” for more information about
the naming of behavioral variables, allowables, and
factors of safety.
```

The next several lines in the interactive GENTEXT input would remain as before:

```
      1   $ Type of prompt: 0="help" paragraph, 1=one-line prompt
STFBK1    $ Name of a variable in the users program (defined below)
      4   $ Role of the variable in the users program
n         $ Do you want to reset the number of columns in STFBK1 ?
buckling load factor, isogrid member,mode 1 $ one-line def.,STFBK1
n         $ Do you want to include a "help" paragraph?
```

and the prompting numbers now given as 165.0, 170.1, 175.1, etc. would all have been increased by 5.

With the GENOPT user’s “**should have included**” material, the modified Table 19 (and Table 6) would have the following entries:

```
165.0
    Next, you will be asked to supply allowables
    for STFBK1 (STFBK1A) for every load case, followed
    by factors of safety (STFBK1F) for every load case.
    “STFBK” means “isogrid-stiffener buckling”. The
    buckling load factor is computed as described
    in Table 27 of the report, “sdm50.report.pdf”.
    The digit, “1”, in the name STFBK1 means “isogrid-
    stiffener buckling in the presence of an imperfect
    shell with a plus or minus “mode 1” (or “mode 3”)
    axisymmetric imperfection shape. Please see Section
    3.5.2 of “sdm50.report.pdf” for more information about
    the naming of behavioral variables, allowables, and
    factors of safety.

 170.0 buckling load factor, isogrid member, mode 1: STFBK1
 175.1 allowable for isogrid stiffener buckling (Use 1.): STFBK1A
 180.1 factor of safety for isogrid stiffener buckling: STFBK1F
 185.0 maximum stress in the shell skin, mode 1: SKNST1
 190.1 allowable stress for the shell skin: SKNST1A
 195.1 factor of safety for skin stress: SKNST1F
```

3.7.2.3 Additional "help" paragraph option 3

The GENOPT user could have added the same "help" paragraph as that listed under the prompting number 165.0 above at a slightly different point in the interactive GENTEXT session. Without any additional "help" paragraph two of the existing lines in Table 15 are:

```
buckling load factor, isogrid member,mode 1 $ one-line def.,STFBK1
 n       $ Do you want to include a "help" paragraph?
```

The GENOPT user could have answered the GENOPT prompt,

```
"Do you want to include a "help" paragraph?"
```

with a "**y**" instead of a "**n**", then provided the same "help" paragraph as that listed above. In that case the new Table 19 (and Table 6) would have the following entries:

```
165.0 buckling load factor, isogrid member, mode 1: STFBK1
165.2
     Next, you will be asked to supply allowables
     for STFBK1 (STFBK1A) for every load case, followed
     by factors of safety (STFBK1F) for every load case.
     "STFBK" means "isogrid-stiffener buckling". The
     buckling load factor is computed as described
     in Table 27 of the report, "sdm50.report.pdf".
     The digit, "1", in the name STFBK1 means "isogrid-
     stiffener buckling in the presence of an imperfect
     shell with a plus or minus "mode 1" (or "mode 3")
     axisymmetric imperfection shape. Please see Section
     3.5.2 of "sdm50.report.pdf" for more information about
     the naming of behavioral variables, allowables, and
     factors of safety.

 170.1 allowable for isogrid stiffener buckling (Use 1.): STFBK1A
 175.1 factor of safety for isogrid stiffener buckling: STFBK1F
 180.0 maximum stress in the shell skin, mode 1: SKNST1
 185.1 allowable stress for the shell skin: SKNST1A
 190.1 factor of safety for skin stress: SKNST1F
```

This last choice, **Additional "help" paragraph option 3**, is not as good as the previous two choices, **Additional "help" paragraph option 1** and **Additional "help" paragraph option 2**, because the user of the BEGIN processor would never see the "help" paragraph with **Additional "help" paragraph option 3**. The "help" paragraph would be listed in the new version of Table 6 (the new equivellipse.PRO file) but would not appear on the screen as the user provides input for BEGIN in an interactive mode. That is because BEGIN does not prompt for the behavior such as STFBK1 but only for its allowable STFBK1A and for its factor of safety STFBK1F. **The behavior itself cannot be prompted for because its value is unknown, of course. The behavior such as STFBK1 is what is calculated during the execution of OPTIMIZE.**

The discussion above about additional "help" paragraphs applies in an analogous way to each of the 14 Role 4,5,6 "bundles" listed in Table 2 and the corresponding prompting file, equivellipse.PRO, listed in Table 6.

3.7.3 "Behavior" subroutines, constraints, and margins

Table 20 lists a GENOPT-created FORTRAN fragment: the part of the equivellipse.CON file that is directly related to the GENTEXT input listed in Table 15. The complete equivellipse.CON file **is automatically created by GENOPT and is automatically inserted later in the skeletal library STRUCT.NEW by GENOPT**. The complete GENOPT-created equivellipse.CON file corresponding to the entire GENTEXT input file, equivellipse.INP, is listed in the appendix (Table a12).

The particular GENOPT-created FORTRAN fragment listed in Table 20 is generated automatically by GENOPT specifically in association with the GENOPT user's input listed in Table 15. In this particular example this is where SUBROUTINEs BEHX4 and BEHX5 are called. For example, in connection with SUBROUTINE BEHX4, see the GENOPT-created statement,

```
 IF (IBEHV(4  ).EQ.0) CALL BEHX4
1     (IFILE8,NPRINX,IMODX,IFAST,ILOADX,J,
1     'buckling load factor, isogrid member, mode 1')
```

which pertains to isogrid stiffener buckling in the presence of a "mode 1" axisymmetric linear buckling modal imperfection shape.

(The indices, IBEHV(i), are all pre-set to zero. For example, the index, IBEHV(4), is either 0 or 1. This index and the other IBEHV(i) are determined by the "end" user for each load set during the MAINSETUP interactive session. See Table 37. If the "end" user has, for some reason, decided that for load set ILOADX he or she does NOT want to let isogrid member buckling in the presence of a "mode 1" imperfection constrain the design, then he or she can respond appropriately during the MAINSETUP interactive session. In that case IBEHV(4) will be set to 1 instead of 0. Usually the "end" user will want to have all the design constraints calculated. That is so in this report. However, there may be complicated cases in which, for one reason or another, the "end" user wants to omit certain design constraints. See [7] for examples.)

The following two GENOPT-created statements are where the wording (WORDCX) is constructed for the design constraint corresponding to buckling of an isogrid stiffening member (STFBK1, STFBK1A, STFBK1F) and where the value of this design constraint is computed (CALL CONX):

```
 WORDCX='(STFBK1('//CIX//','//CJX//')/STFBK1A('//CIX//','//CJX//
1  ')) / STFBK1F('//CIX//','//CJX//')'
 CALL CONX(STFBK1(ILOADX,J),STFBK1A(ILOADX,J),STFBK1F(ILOADX,J)
1,'buckling load factor, isogrid member, mode 1',
1 'allowable for isogrid stiffener buckling (Use 1.)',
```

```
1 'factor of safety for isogrid stiffener buckling',
1 2,INUMTT,IMODX,CONMAX,ICONSX,IPOINC,CONSTX,WORDCX,
1 WORDMX,PCWORD,CPLOTX,ICARX)
```

The ICONSXth design constraint, CONSTX, is calculated in SUBROUTINE CONX. SUBROUTINE CONX is located in the library, …/genopt/sources/main.src and is invariable, that is, CONX is not created or modified by GENOPT. It is the "CALL CONX(…)" statement that is automatically created by GENOPT, not SUBROUTINE CONX.

As explained above, in Tables 16, and as listed in Tables 31 and 32, **design margins** are automatically constructed by GENOPT using the Role 4,5,6 variable names such as STFBK1, STFBK1A, and STFBK1F (local isogrid stiffener buckling) as follows (from Table 31):

```
Mar.
No.      Margin                    definition of margin
 5   1.919E+00 (STFBK1(1,1)/STFBK1A(1,1))/STFBK1F(1,1)-1;F.S.=1.00
```

and using Role 4,5,6 variable names such as SKNST1, SKNST1A, and SKNST1F (local skin stress) as follows (from Table 32):

```
Mar.
No.      Margin                    definition of margin
 8   4.979E-02 (SKNST1A(2,2)/SKNST1(2,2))/SKNST1F(2,2)-1;F.S.=1.00
```

In the FORTRAN code fragments listed above (especially the fragment that defines **WORDCX**) and in the definitions of the margins just listed, there are two array indexes. These and other symbols are defined in the next two paragraphs.

The Role 5 variables with names ending in the letter "A" in this particular generic example are **allowables**. The Role 6 variables with the names ending in the letter "F" in this particular generic example are the **factors of safety**. In the two-dimensional arrays such as STFBK1A(i, j) the integer i denotes the **load set number** (called **ILOADX** in the FORTRAN code fragments listed above), and the integer j denotes the **region number** (called **J** in the FORTRAN code fragments listed above). Region 1 is the meridional domain between the pole and **xlimit** (Tables 3 and 4 and Item 40 in Tables 5 and 6), and Region 2 is the meridional domain between **xlimit** and the equator of the shell (Fig. 2).

The variable names are all defined in Table 2. If a shell is to be optimized in the presence of the first two axisymmetric buckling modal imperfections, mode 1 and mode 2, then the number "1" in the variable name, such as CLAPS1A, indicates "mode 1 axisymmetric imperfection" and the number "2" in the variable name indicates "mode 2 axisymmetric imperfection". For example, the name, **SKNBK1A(2,1)**, means "local **SK**i**N** **B**uc**K**ling, axisymmetric mode **1** imperfection shape, **A**llowable, load set **2** (which is for a –mode 1 imperfection shape), region **1** of the meridian of the equivalent ellipsoidal shell". If a shell is to be optimized in the presence of the first four axisymmetric buckling modal imperfections, mode 1, mode 2, mode 3, and mode 4, then the number "1" in the variable name indicates "odd-numbered modal axisymmetric imperfection" (mode 1 or mode 3) and the number "2" in the variable name indicates "even-numbered modal axisymmetric imperfection" (mode 2 or mode 4).

Table 25 lists the part of the **skeletal** BEHAVIOR.NEW library specifically associated with the GENOPT user's input listed in Table 15. The skeletal BEHAVIOR.NEW library is entirely created **automatically** by GENOPT. Listed in Table 25 are the **skeletal** subroutines SUBROUTINE BEHX4, which is associated with buckling of an isogrid member (STFBK1) in the presence of a "mode 1" axisymmetric imperfection, and SUBROUTINE BEHX5, which is associated with maximum stress in the shell skin (SKNST1) in the presence of a "mode 1" axisymmetric imperfection.

Were the GENOPT user to "**flesh out**" either or both of the two **skeletal** SUBROUTINE BEHXi, i = 4 or 5 in Table 25, he or she would insert his or her new material after the line that reads, "INSERT SUBROUTINE STATEMENTS HERE". Presumably this theoretical GENOPT user would find formulas or find existing or create computer code that would compute the "buckling load factor, isogrid member, mode 1", STFBK1(ILOADX, JCOL), in SUBROUTINE BEHX4 and the "maximum stress in the shell skin, mode 1", SKNST1(ILOADX, JCOL), in SUBROUTINE BEHX5. The array indices, (ILOADX, JCOL) mean (load set number, region number).

The entire **skeletal** BEHAVIOR.NEW library corresponding to the entire input file, equivellipse.INP, is listed in the appendix (Table a13). This **skeletal** BEHAVIOR.NEW library exists upon the GENOPT user's completion of the "GENTEXT" interactive session. As noted in the footnote in Table 25, the GENOPT user (the writer) in this case decided to do all the calculations in SUBROUTINE STRUCT rather than "**flesh out**" SUBROUTINEs BEHXi, i = 1, 14. Listed in Table a13 there are 14 **skeletal** "behavioral" subroutines. In this "**equivellipse**" application of GENOPT the skeletal "behavioral" subroutines remain unmodified by the GENOPT user, that is, **not "fleshed out"**.

The first **seven** BEHXi, i = 1,7, correspond to the first seven bundles of Role 4,5,6 variables listed in Table 2 (behaviors in the presence of an axisymmetric mode 1 imperfection shape associated with variable names that contain the digit "1", such as CLAPS1). The second **seven** BEHXi, i = 8,14, correspond to the second seven bundles of Role 4,5,6 variables listed in Table 2 (behaviors in the presence of an axisymmetric mode 2 imperfection shape associated with names that contain the digit "2", such as CLAPS2).

3.7.4 Should the BEHXi routines be "fleshed out" or should STRUCT be "fleshed out"?

In this particular application of GENOPT, the GENOPT user decided **not** to "flesh out" any of the BEHXi, i=1,14 subroutines, but instead to perform all the computations in SUBROUTINE STRUCT. The reason for this decision is that the "behaviors" (such as buckling, stress, displacement) are computed by BIGBOSOR4, and **one execution of BIGBOSOR4 yields more than one "behavior"**. For example, one execution of BIGBOSOR4 generates skin and stiffener stresses and buckling load factors for both Region 1 and Region 2. It would take much more computer time if BIGBOSOR4 had to be re-executed inside each of the 14 BEHXi subroutines to yield a particular “behavior” that is the “responsibility” of that particular BEHXi subroutine.

The question arises: **In general, for what kinds of problems should the GENOPT user choose to “flesh out” the BEHXi routines as opposed to or in addition to “fleshing out” SUBROUTINE STRUCT?** In general, if the computation of a behavior is from a relatively simple formula or subroutine or system of subroutines that is independent of other subroutines used for the computation of other behaviors, then it is probably best to “flesh out” whatever BEHXi routine is “responsible” for computing that behavior. A detailed example of this strategy is presented in [2]. On the other hand, if the computation of the behavior occurs within an elaborate system of subroutines that computes several different behaviors, such as is the case with BIGBOSOR4, then it is probably best to compute that or those behavior(s) in SUBROUTINE STRUCT or in a subroutine or subroutines called by SUBROUTINE STRUCT. The GENOPT user must then make sure that in SUBROUTINE STRUCT the value of the behavior is copied to a variable or an array with the correct, **GENOPT-user-established name**.

For example, the very complicated “**fleshed out**” version of SUBROUTINE STRUCT applicable to the generic case, **equivellipse**, is listed in Table a16. Within this “**fleshed out**” SUBROUTINE STRUCT certain quantities, `bskin1, bstif1, sknmx1, stfmx1, bskin2, bstif2, sknmx2, stfmx2,` and `ENDUV` are computed. (See Tables 26, 30 and Tables 42 – 45). These quantities are are defined as:

`bskin1` = local buckling load factor of the shell skin in Region 1
`bstif1` = local buckling of a meridionally oriented isogrid member in Region 1
`sknmx1` = maximum effective stress of the shell skin in Region 1
`stfmx1` = maximum isogrid stiffener stress in Region 1
`bskin2` = local buckling load factor of the shell skin in Region 2
`bstif2` = local buckling of a meridionally oriented isogrid member in Region 2
`sknmx2` = maximum effective stress of the shell skin in Region 2
`stfmx2` = maximum isogrid stiffener stress in Region 2
`ENDUV` = axial displacement of the apex of the shell

At some point in SUBROUTINE STRUCT (before any of the BEHXi subroutines are called) the following statements appear:

```
SKNBK1(ILOADX,1) = bskin1
STFBK1(ILOADX,1) = bstif1
SKNST1(ILOADX,1) = sknmx1
STFST1(ILOADX,1) = stfmx1
SKNBK1(ILOADX,2) = bskin2
STFBK1(ILOADX,2) = bstif2
SKNST1(ILOADX,2) = sknmx2
STFST1(ILOADX,2) = stfmx2
WAPEX1(ILOADX) = ABS(ENDUV)
```

The names, SKNBK1, STFBK1, SKNST1, STFST1, WAPEX1, are **the GENOPT-user-established names** listed in Table 2 for several of the behaviors generated in connection with a “mode 1” axisymmetric buckling modal imperfection shape. There are analogous statements that occur later in SUBROUTINE STRUCT involving SKNBK2, STFBK2, SKNST2, STFST2, WAPEX2, which are the GENOPT-user-established names listed in Table 2 for several of the behaviors generated in connection with a “mode 2” axisymmetric buckling modal imperfection shape. (`ILOADX` is the load set number, and the second array index is the region number).

Whenever SUBROUTINE STRUCT is “fleshed out” rather than SUBROUTINES BEHXi, the variables or arrays with the **GENOPT-user-established names** must be filled with the correct behavior values before any of the BEHXi subroutines are called by SUBROUTINE STRUCT.

3.8 “Fleshing out” SUBROUTINE STRUCT

The **skeletal** SUBOUTINE STRUCT, produced entirely by GENOPT after the GENOPT user’s completion of the GENTEXT interactive session, is listed in the appendix (Table a14). The appendix also provides another example of a **skeletal** version of SUBROUTINE STRUCT for a different generic case, “cylinder”. (See Table a30). The **skeletal** SUBROUTINE STRUCT, as produced automatically by GENTEXT, is part of the library called **struct.new**. It resides in the same directory where GENTEXT has been executed. It is the library, **struct.new**, that should be “**fleshed out**” in this particular generic case, “equivellipse”.

It has been mentioned that in this particular application of GENOPT (the application for which the generic case name is “**equivellipse**”) the GENOPT user (the writer in this case) must “**flesh out**” SUBROUTINE STRUCT (called **struct.new** as generated by GENTEXT). Table 26 lists a small section of SUBROUTINE STRUCT that has been written by the GENOPT user, not produced automatically by GENOPT. In this small section of SUBROUTINE STRUCT (**struct.new** library) local buckling and local stress quantities are computed as follows:

1. local skin buckling (between isogrid members: SKNBK1) and local isogrid stiffener buckling (buckling along a meridionally oriented side of the equilateral triangle formed by adjacent isogrid members: STFBK1) are computed for Load Set Number ILOADX for Region 1 (pole to x = xlimit) and Region 2 (x = xlimit to equator) in the presence of an axisymmetric “mode 1” linear buckling modal imperfection with amplitude Wimp.

2. maximum extreme fiber effective stress in the shell skin (SKNST1) and maximum longitudinal extreme fiber stress in a meridionally oriented isogrid member (STFST1) in the presence of a “mode 1” axisymmetric linear buckling modal imperfection with amplitude Wimp.

(Analogous quantities, SKNBK2, STFBK2, SKNST2, and STFST2, are computed in the presence of a “mode 2” axisymmetric linear buckling modal imperfection with amplitude Wimp in another analogous section of SUBROUTINE STRUCT via coding not listed in Table 26 but analogous to that listed in Table 26. See Table a16 in the appendix.)

BIGBOSOR4 is used to compute these quantities for each segment in the multi-segment BIGBOSOR4 model (Fig. 2). This is accomplished via the calls to SUBROUTINES BOSDEC, B4READ, and B4MAIN, as listed in Table 26. SUBROUTINE BOSDEC, **which must be written by the GENOPT user**, generates a valid input file for the BIGBOSOR4 preprocessor, SUBROUTINE B4READ. A complete list of SUBROUTINE BOSDEC for the generic case “equivellipse” is provided in the appendix (Table a15). Guidelines for how to go about generating a valid SUBROUTINE BOSDEC are provided for a different (simpler) generic case, “cylinder”, in Table a29 of the appendix.

Part of the output from the BIGBOSOR4 mainprocessor, B4MAIN, consists of the four arrays:

1. minimum local buckling load of the triangular piece of skin between adjacent isogrid stiffeners in shell segment number iseg: BUCMIN(iseg),

2. minimum local buckling load of the most critical isogrid stiffening member in shell segment number iseg: BUCMNS(iseg),

3. maximum extreme fiber effective stress in the shell skin in shell segment number iseg: SKNMAX(iseg), and

4. maximum extreme fiber longitudinal stress in the most critical isogrid member in shell segment number iseg: STFMXS(iseg).

IMPORTANT NOTE: **It is assumed that the most critical buckling load and stress in an isogrid member are associated with an isogrid member that runs in the meridional direction. This assumption is based on the fact that the imperfect shell is axisymmetric, since both the perfect shell and the imperfection shape are axisymmetric. This assumption lies behind the new FORTRAN coding in BIGBOSOR4 listed in Table 27, in which the most critical isogrid member is treated as if it were a stringer (meridionally oriented stiffener).**

Table 26 lists only a small section of SUBROUTINE STRUCT. The entire SUBROUTINE STRUCT is provided in the appendix (Table a16, a very, very long table!). Also listed in the appendix is the **skeletal** SUBROUTINE STRUCT (Table a14), which is produced entirely automatically by GENOPT upon the GENOPT user’s completion of the GENTEXT interactive session. The skeletal “STRUCT” library is called “**struct.new**”.

The “**fleshed out**” version of SUBROUTINE STRUCT must be part of the library called **struct.new**. The **“fleshed out”** version of **struct.new** must reside in the same directory (e.g. **genoptcase**) in which GENTEXT was executed, and it must be there, along with the possibly “fleshed out” file, **behavior.new**, before the GENOPT processor, GENPROGRAMS, is executed. Also, the correct version of the GENOPT-user-written SUBROUTINE BOSDEC must exist in a certain directory (not the directory, “**genoptcase**”, but a different directory, …/bosdec/sources, that is specified in the file, …/genopt/doc/getting.started, before GENPROGRAMS is executed.

NOTE: IT IS VERY IMPORTANT THAT THE GENOPT USER SAVE THE “FLESHED OUT” FILE, struct.new. SAVE IT WITH THE USE SOME OTHER NAME, SUCH AS struct.equivellipse. THE “FLESHED OUT” FILE SHOULD BE SAVED BY THE GENOPT USER BECAUSE struct.new IS OVER-WRITTEN WHENEVER GENTEXT IS

RE-EXECUTED. THIS WARNING APPLIES ALSO TO A POSSIBLY “FLESHED OUT” VERSION OF behavior.new. The GENOPT user must also save valid versions of SUBROUTING BOSDEC. For example, in the generic case, “equivellipse”, bosdec.src should be saved in a file called “bosdec.equivellipse”, or some such name.

The reader should consult Table a30 in the appendix. Table a30 gives more information about “fleshing out” SUBROUTINE STRUCT and provides complete lists for a simple generic case called “cylinder”, including “diff” files that show the difference between skeletal and “fleshed out” versions, etc.

Additional guidance for both the GENOPT user and the "end" user is provided in [16], a file called "**getting.started**" that describes how to set up GENOPT at a facility different from that used by the author and how to solve problems with GENOPT. “**getting.started**” is located in the directory, …/genopt/doc .

4.0 ABOUT BIGBOSOR4 AND STAGS

4.1 About BIGBOSOR4 (BIG Buckling Of Shells Of Revolution, 4th version of BOSOR)

BOSOR4 [10-12] (or its latest version, BIGBOSOR4 [7]) is a program for the static and dynamic analysis of any shell of revolution. Input files valid for BOSOR4 are also valid for BIGBOSOR4. BIGBOSOR4 is essentially the same as BOSOR4 except that BIGBOSOR4 will solve bigger problems (more shell segments, more degrees of freedom). The shell can be loaded axisymmetrically or non-axisymmetrically by line loads, distributed loads, temperature, and acceleration. BOSOR4 (BIGBOSOR4) computes static equilibrium states, axisymmetric and nonaxisymmetric bifurcation buckling, nonlinear axisymmetric collapse, modal vibration, and linear response to lateral and axial base excitation. In BOSOR4 (BIGBOSOR4) a complex, branched, stiffened elastic shell of revolution is modeled as an assemblage of shell segments or branches, each with its own geometry (flat, conical, cylindrical, spherical, toroidal), loading, wall construction, and linear elastic material properties. Laminated composite wall construction can be handled. The user of BOSOR4 (BIGBOSOR4) provides input data in an interactive mode on a segment-by-segment basis. These input data are automatically stored in a fully annotated file, one input datum and a phrase defining it on each record of the file. (See Table A.7 in the appendix of [7] for an example of such a file. Also, see Table a18 in the appendix of this report.) The meridian of each shell segment is discretized [12]. Variation in the circumferential coordinate direction is assumed to be trigonometric. For more details about BOSOR4 see [10-12] and [7].

In BOSOR4 (and BIGBOSOR4) the type of analysis to be performed is controlled by an index called INDIC, as follows:

INDIC= -2: stability determinant for axisymmetric or for non-axisymmetric nonlinear bifurcation buckling calculated for increasing load, nonlinear axisymmetric prebuckling analysis

INDIC= -1: axisymmetric or non-axisymmetric bifurcation buckling with nonlinear axisymmetric prebuckling analysis

INDIC= 0: nonlinear axisymmetric stress (and axisymmetric collapse) analysis

INDIC= 1: bifurcation buckling with "linear" axisymmetric prebuckling analysis. (Actually the prebuckling analysis is the same as for INDIC = -1. However, the portion of the applied load that affects the stiffness matrix is never changed during a case. Linear behavior is exhibited provided that the user applies a load that is very small compared to the design load.)

INDIC= 2: axisymmetric or non-axisymmetric modal vibration with axisymmetric nonlinear prestress

INDIC= 3: linear axisymmetric and linear non-axisymmetric stress analysis

INDIC= 4: bifurcation buckling with linear non-axisymmetric prebuckling. The INDIC=4 branch is a combination of the INDIC=3 and INDIC=1 branches. INDIC=3 computations are first executed followed by INDIC=1 computations. The user selects the circumferential coordinate, theta, of the meridian and BOSOR4 (BIGBOSOR4) uses the prebuckled stress state along that meridian in a bifurcation buckling analysis that is identical to that in the INDIC=1 branch, that is, the fact that the prebuckled state may be non-axisymmetric is ignored. This is a conservative approximation provided that the user has chosen the meridian (the value of the circumferential coordinate, theta) for which the prebuckled state is the most destabilizing. Note that BOSOR4 (and BIGBOSOR4) cannot handle buckling of shells with significant in-plane shear resultant, Nxy.

BOSOR4 (BIGBOSOR4) will also compute peak response to loads that vary either harmonically or randomly in time. Buckling under harmonic or random axial or lateral base excitation can also be calculated.

As described in [7], BOSOR4 was modified (advanced to BIGBOSOR4) in order to work in the context of automated optimization.

In 2005 BIGBOSOR4 was further modified (Table 27) to compute local skin buckling between meridionally oriented stiffeners ("stringers") and to compute approximately local buckling between stiffeners that form an isogrid. Also, BIGBOSOR4 was modified to compute the maximum stress in a stringer or isogrid that has been "smeared out" and to compute approximately the local buckling load factor of an isogrid stiffening member that is assumed to run in the meridional coordinate direction. In the case of isogrid stiffening the isogrid members must have rectangular cross sections only the height of which varies within any single shell segment. BIGBOSOR4 was further modified to permit axisymmetric imperfection shapes that are to be provided by the user for every nodal point along the meridian of each shell segment. These modifications are described in [17]. They are required for optimization of the isogrid-stiffened, imperfect ellipsoidal shell and imperfect "equivalent" ellipsoidal shell.

Table 27 lists the modifications made to SUBROUTINEs WALLCF, CFB1, and PLOCAL in BIGBOSOR4 to compute the maximum stiffener stress and minimum stiffener and skin buckling load factors in an isogrid-stiffened shell segment. Comments are added in the FORTRAN coding to explain the variables and equations. The formula for local buckling of the triangular piece of skin between adjacent isogrid stiffeners is from Gerard and Becker [18]. This triangular piece of skin is assumed to be flat and uniformly compressed in both the meridional and circumferential coordinate directions. The formula for local buckling of an isogrid member that comprises one side of the equilateral triangle formed by adjacent isogrid stiffeners is from Roark [19]. **It is assumed that the most critical isogrid stiffening member with regard to both buckling and stress runs in the meridional coordinate direction. Therefore, in Table 27, the most critical isogrid member is treated as if it were a stringer (a meridionally oriented stiffener).**

4.2 ABOUT STAGS (STructural Analysis of General Shells)

In this report optimum designs obtained by GENOPT are evaluated later via STAGS models. **NOTE: STAGS is not used inside the optimization loop**.

STAGS [20 – 23] is a finite element code for the **general-purpose nonlinear analysis of stiffened shell structures of arbitrary shape and complexity**. Its capabilities include stress, stability, vibration, and transient analyses with both material and geometric nonlinearities permitted in all analysis types. STAGS includes enhancements, such as a higher order thick shell element, more advanced nonlinear solution strategies, and more comprehensive post-processing features such as a link with the STAGS postprocessor, STAPL.

Research and development of STAGS by Brogan, Almroth, Rankin, Stanley, Cabiness, Stehlin and others of the Computational Mechanics Department of the Lockheed Palo Alto Research Laboratory has been under continuous sponsorship from U.S. government agencies and internal Lockheed funding for the past 40 years. During this time particular emphasis has been placed on improvement of the capability to solve difficult nonlinear problems such as the prediction of the behavior of axially compressed stiffened panels loaded far into their locally postbuckled states. STAGS has been extensively used worldwide for the evaluation of stiffened panels and shells loaded well into their locally postbuckled states.

A large rotation algorithm that is independent of the finite element library has been incorporated into STAGS. With this algorithm there is no artificial stiffening due to large rotations. The finite elements in the STAGS library do not store energy under arbitrary rigid-body motion and the first and second variations of the strain energy are consistent. These properties lead to quadratic convergence during Newton iterations.

Solution control in nonlinear problems includes specification of load levels or use of the **advanced Riks-Crisfield path parameter** that enables traversal of limit points into the post-buckling regime. Two load systems with different histories (Load Sets A and B) can be defined and controlled separately during the solution process. Flexible restart procedures permit switching from one strategy to another during an analysis. This includes shifts from bifurcation buckling to nonlinear collapse analyses and back and shifts from static to transient and transient to static analyses with modified boundary conditions and loading. STAGS provides solutions to the generalized eigenvalue problem for **buckling** and **vibration from a linear or nonlinear stress state.**

Quadric surfaces can be modeled with minimal user input as individual substructures called "**shell units**" in which the analytic geometry is represented exactly. "Shell units" can be connected along edges or internal grid lines with partial or complete compatibility. In this way complex structures can be assembled from relatively simple units. Alternatively, a structure of arbitrary shape can be modeled with use of "element units".

Geometric imperfections can be generated automatically in a variety of ways, thereby permitting imperfection-sensitivity studies to be performed. For example, **imperfections can be generated by superposition of several buckling modes determined from previous STAGS analyses of a given case**.

A variety of material models is available, including both plasticity and creep. STAGS handles isotropic and anisotropic materials, including composites consisting of up to 60 layers of arbitrary orientation. Four plasticity models are available, including isotropic strain hardening, the White Besseling (mechanical sublayer model), kinematic strain hardening, and deformation theory.

Two independent load sets, each composed from simple parts that may be specified with minimal input, define a spatial variation of loading. Any number of point loads, prescribed displacements, line loads, surface tractions, thermal loads, and "live" pressure (hydrostatic pressure which remains normal to the shell surface throughout large deformations) can be combined to make a load set. For transient analysis the user may select from a menu of loading histories, or a general temporal variation may be specified in a user-written subroutine.

Boundary conditions (B.C.) may be imposed either by reference to certain standard conditions or by the use of single- and multi-point constraints. Simple support, symmetry, antisymmetry, clamped, or user-defined B.C. can be defined on a "shell unit" edge. Single-point constraints which allow individual freedoms to be free, fixed, or a prescribed non-zero value may be applied to grid lines and surfaces in "shell units" or "element units". A useful feature for buckling analysis allows these constraints to differ for the prestress and eigenvalue analyses. Langrangian constraint equations containing up to 100 terms may be defined to impose multi-point constraints.

STAGS has a variety of finite elements suitable for the analysis of stiffened plates and shells. Simple four node quadrilateral plate elements with a cubic lateral displacement field (called "410" and "411" elements) are effective and efficient for the prediction of postbuckling thin shell response. A linear (410) or quadratic (411) membrane interpolation can be selected. For thicker shells in which transverse shear deformation is important, STAGS provides the Assumed Natural Strain (ANS) nine node element (called "480" element). A two node beam element compatible with the four node quadrilateral plate element is provided to simulate stiffeners and beam assemblies. Other finite elements included in STAGS are described in the STAGS literature [20 – 23].

5.0 “TRUE” ELLIPSOIDAL SHELL versus “EQUIVALENT” ELLIPSOIDAL SHELL

5.1 “True” ellipsoidal shell

Figure 1 shows predictions of elastic collapse of an optimized **true** unstiffened 2:1 titanium ellipsoidal shell under uniform external pressure. The ellipsoidal shell has a semi-major axis length of 24.75 inches and semi-minor axis length of 12.375 inches. The inner surface is the reference surface, which has the ellipsoindal profile. The design external pressure is 460 psi and the minimum allowable pressure at which the shell collapses axisymmetrically is 550 psi. The unstiffened ellipsoidal shell has thickness that varies along the meridian. The decision variables of the optimization problem are the values of wall thickness at 13 stations on the meridian including that at the pole and that at the equator. The shell is optimized in the presence of any one of four possible initial buckling modal imperfections, each with amplitude, Wimp = 0.2 inch. The four imperfections are all in the shape of linear axisymmetric bifurcation buckling modes as follows:

Imperfection no. 1: positive first axisymmetric eigenmode, called “+mode 1”
Imperfection no. 2: positive second axisymmetric eigenmode, called “+mode 2”
Imperfection no. 3: negative first axisymmetric eigenmode, called “–mode 1”
Imperfection no. 4: negative second axisymmetric eigenmode, called “–mode 2”

The optimization is conducted in such a way that, according to predictions by BIGBOSOR4, the final optimum design will survive (not exhibit any significantly negative margins) in the presence of any **one** of the four imperfection shapes just listed. The four curves in Fig. 1 with labels, “GENOPT results…”, correspond to the predictions by BIGBOSOR4 of nonlinear axisymmetric collapse of the optimized **true** ellipsoidal pressure vessel head in the presence of each of the 4 axisymmetric linear bifurcation buckling modal imperfection shapes, +mode 1, –mode 1, +mode 2, –mode2, taken one at a time corresponding to each curve.

The overall dimensions of the shell, the external uniform design pressure loading, the allowable maximum external pressure for collapse, and the four axisymmetric linear bifurcation buckling modal imperfection shapes taken one at a time also govern the behavior and optimization of the “**equivalent**” ellipsoidal shells that are the subject of most of this report.

Figure 1 also shows the prediction from STAGS [20-23] for the optimized **true** unstiffened ellipsoidal shell with Imperfection No. 3: “–mode 1”. There is a huge difference between the BIGBOSOR4 (GENOPT) and STAGS predictions for the pressure-carrying capability of this optimized axisymmetrically imperfect unstiffened shell. The predictions from BIGBOSOR4 are unacceptably unconservative. This result is caused by finite element “lockup” in the BIGBOSOR4 model. **BOSOR4 [10-12] and BIGBOSOR4 [7] should be applied only to shells for which the meridional radius of curvature is constant within each perfect shell segment of a multi-segment model of a shell of revolution**. For a **true** perfect ellipsoidal shell the meridional radius of curvature of the reference surface decreases monotonically from the pole to the equator.

5.2 "Equivalent" ellipsoidal shell

The BIGBOSOR4 finite element "lockup" problem is essentially solved by representation of the "true" ellipsoidal shell as an "**equivalent**" ellipsoidal shell consisting of a shallow spherical cap plus multiple toroidal segments connected in series, each segment of which has **constant meridional radius of curvature** and each segment of which closely approximates the local meridional shape of the "true" ellipsoidal shell at the location of that segment. In the present analysis the "equivalent" ellipsoidal shell consists of 12 shell segments: a spherical cap (Segment 1) and 11 toroidal segments (Segments 2 – 12) the radial (x-coordinate) end points of which are located as listed in **Table 28**. The input data required by BIGBOSOR4 for each shell segment are the (x,y) coordinates of the two end points of that segment, (x_1,y_1) and (x_2,y_2), and the (x,y) coordinates of the center of meridional curvature, (x_3,y_3), of that segment. The coordinates, (x_1,y_1) and (x_2,y_2), lie on the profile of the true ellipsoid.

Table 29 lists how the location, (x_3,y_3), of the center of meridional curvature of the "equivalent" toroidal segment is derived for a typical toroidal segment. Figure 2 shows the meridional profile of the 12-segment "equivalent" ellipsoidal shell. The (x,y) coordinates of the end points of each toroidal shell segment, (x_1,y_1) and (x_2,y_2), lie on the meridional profile of the true ellipsoidal shell, of course.

6.0 ANALYSES INCLUDED IN THE "FLESHED OUT" SUBROUTINE STRUCT

As previously mentioned, in this work only the "**skeletal**" SUBROUTINE STRUCT (part of the library called **struct.new** in the **genoptcase** directory immediately following the GENOPT user's execution of GENTEXT) automatically produced during the interactive GENTEXT session of GENOPT (Table a14) was "**fleshed out**" (Table a16) by the GENOPT user (the writer). The 14 **skeletal** "behavioral" subroutines, SUBROUTINE BEHXi, i = 1,14, remain unchanged from the versions created automatically by GENTEXT. (See Table a13 in the appendix). **Much of the effort in this project was spent on creating a final correct version of SUBROUTINE STRUCT**. The long final ("**fleshed out**") version of SUBROUTINE STRUCT (part of the **struct.new** library) is listed in the appendix (Table a16). For each "current" (unperturbed) design and for each perturbed design virtually all the computations take place in SUBROUTINE STRUCT.

SUBROUTINE STRUCT consists essentially of seven analyses. These analyses have the following two most significant elements:

1. A call to SUBROUTINE BOSDEC (Table a15) creates a file that is valid as input to BIGBOSOR4 (or BOSOR4). This is done for each of the seven analyses in SUBROUTINE STRUCT. An example has already been provided for one of the seven analyses in Table 26. For any application that involves the use of BIGBOSOR4 (or BOSOR4), the GENOPT user **must** create a SUBROUTINE BOSDEC, that is, a subroutine that when executed creates a valid input file, *.ALL, for BIGBOSOR4 (or BOSOR4). A list of SUBROUTINE BOSDEC valid for the "equivalent" ellipsoidal shell is given in the appendix (Table a15). An example of a valid *.ALL file as created by BOSDEC is also listed in the appendix (Table a17). This file does not have the

usual complete annotations for each input datum typical of BOSOR4 (BIGBOSOR4) input files; it is none-the-less valid. The "complete annotation" format can easily be produced by execution of "**bigbosorall**" followed by execution of "**cleanup**". An example of the valid *.ALL file in the "complete annotation" format after execution of "bigbosorall" followed by execution of "cleanup" is also listed in the appendix (Table a18).

2. Calls to BIGBOSOR4 software in SUBROUTINE STRUCT perform typical BIGBOSOR4 computations for:

a. axisymmetric linear bifurcation buckling analysis (INDIC = 1) with a very small applied pressure: one thousanth of the design pressure, or p = 0.460 psi, in order to ensure linear behavior. This is **analysis number 1**. The purpose is to obtain axisymmetric buckling modes that are used in Items b, c, d as imperfection shapes.

b. nonlinear axisymmetric stress analysis of the shell with an axisymmetric linear bifurcation buckling modal imperfection (INDIC = 0) with the applied pressure equal to the design pressure, p = 460 psi. This is **analysis number 2** (axisymmetric mode 1 imperfection shape) and **analysis number 3** (axisymmetric mode 2 imperfection shape).

c. nonlinear axisymmetric collapse analysis of the shell with an axisymmetric linear bifurcation buckling modal imperfection (INDIC = 0) for a series of monotonically increasing pressure p until axisymmetric collapse occurs or until p reaches a maximum value of 920 psi. This is **analysis number 4** (axisymmetric mode 1 imperfection shape) and **analysis number 5** (axisymmetric mode 2 imperfection shape). The upper limit on applied external pressure, p = 920 psi, is arbitrarily set by the GENOPT user to be equal to twice the design pressure, p = 460 psi.

d. partially nonlinear bifurcation buckling analysis of the shell with an axisymmetric linear bifurcation buckling modal imperfection (INDIC = 1) with pressure in Load Set B (Load Set B means "not eigenvalue load": affects only the stiffness matrix), p = 460 psi, and "dp" the pressure increment in Load Set A (Load Set A means "yes eigenvalue load": affects only the load-geometric matrix), dp = 0.46 psi. This is **analysis number 6** (axisymmetric mode 1 imperfection shape) and **analysis number 7** (axisymmetric mode 2 imperfection shape). The analysis is called "partially nonlinear" because the static response to the applied design pressure, p = 460 psi, treated as "Load Set B", a load that affects the stiffness matrix but not the load-geometric matrix, is obtained from nonlinear theory but the eigenvalue is obtained from the linear equation,

$$\mathbf{K_1}(\text{at p=460 psi}) \times \mathbf{q} = (\text{eigenvalue})\mathbf{K_2}(\text{dp}) \times \mathbf{q} \qquad (5)$$

in which $\mathbf{q}$ is the eigenvector, $\mathbf{K_1}$ is the stiffness matrix of the structure as loaded by p = 460 psi, and $\mathbf{K_2}$ is the load-geometric matrix which is proportional to the pressure increment, dp. The "nonlinear" bifurcation buckling pressure, p(critical) is given by

$$\text{p(critical)} = 460 + (\text{eigenvalue}) \times \text{dp} \quad \text{psi} \qquad (6)$$

If, for the current design, the applied external pressure, p = 460 psi, happens to exceed the pressure at which the shell collapses axisymmetrically, there is a strategy to avoid numerical problems. (That is one of the reasons SUBROUTINE STRUCT is so long: the need for strategies to avoid numerical problems in the face of nonlinear behavior).

Table 30 presents a summary of results from each of the seven analyses for the case of an optimized titanium axisymmetrically imperfect, internally isogrid-stiffened, "equivalent" ellipsoidal shell with skin thickness and isogrid stiffener height that vary along the meridian. The amplitude of the linear axisymmetric bifurcation buckling modal imperfection is Wimp = 0.2 inch. The inner surface of the shell skin is the reference surface, has the "equivalent" ellipsoidal shape, and has semi-major axis of length 24.75 inches and semi-minor axis of length 12.375 inches. The decision variables are the skin thicknesses, THKSKN(i), and isogrid stiffener heights, HIGHST(i), at the 13 radial (x-coordinate, xinput(i)) locations listed in Table 28, plus the isogrid stiffener spacing, SPACNG, (constant along the meridian) and the isogrid stiffener thickness, THSTIF, (constant along the meridian). The optimum design to which the results in Table 30 correspond is listed in Table 33 under the heading, **"isogrid-stiffened, imperfect"**.

The computations listed in Table 30 are for Load Set 1 (+mode 1 and + mode 2 imperfections). They are repeated for Load Set 2, which corresponds to use of the negatives of the first and second axisymmetric bifurcation buckling modal imperfection shapes, that is, with the use of –mode 1 and –mode 2 linear axisymmetric bifurcation buckling modal imperfection shapes.

A complete list of the output file, eqellipse.OPM, corresponding to the **optimized imperfect isogrid-stiffened equivalent ellipsoidal shell** generated with use of the ITYPE = 2 choice of analysis type (ITYPE = 2 means "fixed design", not optimization; see Table 37) appears in the appendix (Table a19).

If the shell is optimized with the use of the **four** lowest axisymmetric buckling modal imperfections rather than only the **two** lowest axisymmetric buckling modal imperfections, then the seven analyses are conducted, not only for +mode 1 and +mode 2 axisymmetric imperfection shapes (Load Set 1) and for –mode 1 and –mode 2 axisymmetric imperfection shapes (Load Set 2), but also for +mode 3 and +mode 4 axisymmetric imperfection shapes (Load Set 3) and for –mode 3 and –mode 4 axisymmetric imperfection shapes (Load Set 4). The results from such an extensive "4-mode" treatment are presented for axisymmetrically imperfect **isogrid-stiffened** and for axisymmetrically imperfect **unstiffened** optimized equivalent ellipsoidal shells in Sub-sections 8.1.8 and 8.2.8, respectively. The "4-mode" (4 load set) optimization of the isogrid-stiffened equivalent ellipsoidal shell requires a run time of about eight days on the very efficient LINUX-based workstation on which the work reported here was performed.

Table 31 lists the design margins corresponding to Load Set 1 (+mode 1 and +mode 2 axisymmetric linear bifurcation buckling modal imperfection shapes) and Table 32 lists the design margins corresponding to Load Set 2 (–mode 1 and –mode 2 axisymmetric linear bifurcation buckling modal imperfection shapes). The GENOPT-user-selected variable names that appear in the margins, such as "CLAPS1", "CLAPS1A", "CLAPS1F", are defined in Table 2. The optimum design is deemed by GENOPT to be acceptable even though there are several small negative margins because the design is "ALMOST FEASIBLE", that is, all margins are greater than –0.05. If small negative margins were not permitted many, many executions of SUPEROPT might be required to find a “global” minimum-weight design that is either FEASIBLE or ALMOST FEASIBLE. It is best, therefore, to allow small negative margins and to compensate for them, if one really thinks it is necessary, by raising the factors of safety by a correspondingly small amount. For GENOPT to accept a design as FEASIBLE rather than ALMOST FEASIBLE all margins must be greater than –0.01. For GENOPT to accept a design that is “MILDLY UNFEASIBLE” all margins must be greater than –0.10. (See Item no. 725 in Table a24).

In Table 31 Margin No. 1 is developed from Analysis No. 4 (nonlinear axisymmetric collapse); Margin No. 2 is developed from Analysis No. 6 (“nonlinear” bifurcation buckling); Margin Nos. 3-11 are developed from Analysis No. 2 (nonlinear axisymmetric stress). Margins 1 – 11 are developed with the use of the axisymmetric +mode 1 linear bifurcation buckling modal imperfection shape.

Margin No. 12 is developed from Analysis No. 5 (nonlinear axisymmetric collapse); Margin No. 13 is developed from Analysis No. 7 (“nonlinear” bifurcation buckling); Margin Nos. 14 – 22 are developed from Analysis No. 3 (nonlinear axisymmetric stress). Margins 12 – 22 are developed with use of the axisymmetric +mode 2 linear bifurcation buckling modal imperfection shape.

The margins in Table 32 are developed in an analogous manner. They are based on –mode 1 and –mode 2 axisymmetric linear bifurcation buckling modal imperfection shapes (Load Set 2).

7.0 NUMERICAL RESULTS FOR SEVERAL CASES

The isotropic titanium material has a modulus, E = 16 x 10^6 psi, Poisson ratio = 0.25, weight density = 0.16 lb/in^3. The amplitude of the linear axisymmetric buckling modal imperfection shapes is Wimp = (+ or –) 0.2 inch, unless otherwise noted. All of these cases have the “equivalent” ellipsoidal shape with semi-major and semi-minor axes lengths, 24.75 inches and 12.375 inches, respectively. The reference surface is always the inner surface of the shell skin. It is this surface that has the shape of the “equivalent” ellipsoid. The value of “xlimit” (radial, that is, x-coordinate) where Regions 1 and 2 join is xlimit = 17.63477 inches, which is the same as the location of the junction between shell segments 6 and 7 (Fig. 2). As listed in Tables 31 and 32, Region 1 is 0 < x < xlimit, and Region 2 is xlimit < x < 24.75 inches.

Optimum designs from several cases are listed in Table 33. The values of xinput are the same as those listed in Table 28: xinput are the radial coordinate locations of the ends of the 12 segments of the “equivalent” ellipsoidal shell. For an isogrid-stiffened shell the decision variables are the shell skin thicknesses, THKSKN(i), at the 13 “xinput” points, plus the heights, HIGHST(i), (dimension measured normal to the shell reference surface) of the isogrid stiffeners at the same 13 points, plus the isogrid spacing, SPACNG, (altitude of the equilateral triangle formed by three adjacent isogrid stiffeners), which is constant over the entire shell, plus the thickness, THSTIF, of each isogrid member, which is constant over the entire shell. This makes a total of 28 decision variables for the isogrid-stiffened shell. The isogrid members have rectangular cross sections. The shell skin thickness and height (depth) of the isogrid stiffening system vary linearly with meridional arc length between the shell segment ends. The isogrid member spacing and isogrid member thickness are constant over the entire shell. The isogrid is attached to the inner surface of the shell skin, which is selected as the shell reference surface.

IMPORTANT NOTE: Note that in this research no attempt was made to determine if a doubly-curved shell of this type could actually be manufactured with isogrid stiffening of the type specified in this report.

Perfect and imperfect, isogrid-stiffened and unstiffened "equivalent" ellipsoidal shells were optimized with GENOPT, that is, based on BIGBOSOR4 models. Some of the optimum designs were obtained only after multiple executions of SUPEROPT. **All of the results listed in Table 33 correspond to the use of only two axisymmetric linear bifurcation buckling modal imperfection shapes, mode 1 and mode 2, with the plus and minus version of each imperfection shape included in the model.** The amplitude of each imperfection shape is Wimp = 0.2 inches. A single execution of SUPEROPT for the "two-mode" isogrid-stiffened imperfect shell requires approximately 95 hours of CPU time on the efficient LINUX workstation on which this work was done.

The optimum designs obtained with GENOPT were evaluated with the use of STAGS, a general-purpose, nonlinear, finite element code [20-24].

Table 34 lists a typical run stream for obtaining an optimum design with GENOPT. This table forms part of the *.DEF file (called "equivellipse.DEF" for the generic case, "equivellipse"). The complete equivellipse.DEF file is listed in the appendix (Table a2). The best way to optimize something is to use the "global" optimization scheme launched by the command SUPEROPT. (See the second page of Table 34).

Table 47 is analogous to Table 33 in that it pertains to all the four optimum designs. Table 47 lists the maximum extreme fiber stresses predicted by BIGBOSOR4, BOSOR5 [25], and STAGS for the optimized stiffened and unstiffened, perfect and imperfect equivalent ellipsoidal shells the dimensions of which are listed in Table 33.

8.0 DETAILS FOR EACH OF THE FOUR CASES LISTED IN TABLE 33

Sub-section 8.1: Tables 30, 31, 32, and 35 – 55 and Figures 3 – 68 pertain to the **isogrid-stiffened, imperfect** equivalent ellipsoidal shell. **Sub-section 8.2:** Tables 56 – 66 and Figs. 69 – 114 pertain to the **unstiffened, imperfect** equivalent ellipsoidal shell. **Sub-section 8.3:** Tables 67 – 71 and Figs. 115 – 128 pertain to the **isogrid-stiffened, "perfect"** equivalent ellipsoidal shell. **Sub-section 8.4:** Tables 72 – 76 and Figs. 129 – 142 pertain to the **unstiffened, "perfect"** equivalent ellipsoidal shell. "Perfect" is in quotes because the "perfect" shells were actually optimized in the presence of very, very small +mode 1 and +mode 2 axisymmetric buckling modal imperfections. (imperfection amplitude, Wimp = 0.0001 inch).

8.1 Details pertaining to the isogrid-stiffened, imperfect equivalent ellipsoidal shell

Tables 30, 31, 32, and 35 – 55 and Figures 3 – 68 pertain to this sub-section.

8.1.1 Input data

Table 35 lists the input data for the BEGIN processor for the specific case, “eqellipse”, which is a member of the GENOPT user’s generic class, “equivellipse”. (Both the generic case name, “equivellipse”, and the specific case name, “eqellipse”, stand for “equivalent ellipsoidal shell”.) The name of the input file for BEGIN is “eqellipse.BEG”. The decision variables in this specific case are the “skin thickness at xinput: THKSKN”(i), i = 1,13, the “height of isogrid members at xinput: HIGHST”(i), i = 1,13, the “spacing of the isogrid members: SPACNG”, and “thickness of an isogrid stiffening member: THSTIF”. Note that the names of these decision variables, THKSKN, HIGHST, SPACNG, and THSTIF, are the GENOPT-user-selected variable names, and the one-line definitions, “skin thickness at xinput”, etc. are those chosen by the GENOPT user (Table 2).

“NCASES” is the number of load sets: two in this example, Load Set 1 for the shells with +mode 1 and +mode 2 axisymmetric linear bifurcation buckling modal imperfection shapes and Load Set 2 for the shells with –mode 1 and –mode 2 axisymmetric linear bifurcation buckling modal imperfection shapes. If the user had set NCASES equal to 4, Load Sets 1 and 2 would have been as just defined, Load Set 3 would have been for shells with +mode 3 and + mode 4 axisymmetric linear buckling modal imperfection shapes, and Load Set 4 would have been for shells with –mode 3 and –mode 4 axisymmetric linear bifurcation buckling modal imperfection shapes.

The indexes, JSKNBK1 and JSKNBK2, in Table 35 are the number of **regions** for computing local stress and local buckling of the shell skin and isogrid stiffeners. See the next paragraph and Item No. 40.2 in Table 5, which is the “help” paragraph associated with the variable **xlimit**, for the meaning of “**regions**” as used in this context.

The GENOPT-user-specified variables with names ending in the letter “A” are **allowables**. The variables with the names ending in the letter “F” are the **factors of safety**. In the two-dimensional arrays such as SKNBK1A(i, j) the integer i denotes the **load set number** and the integer j denotes the **region number** (Fig. 2). Region 1 is the meridional domain between the pole and xlimit=17.63477 inches, and Region 2 is the meridional domain between xlimit=17.63477 inches and the equator.

The variable names are all defined in Table 2. If a shell is to be optimized in the presence of the first two axisymmetric buckling modal imperfections, mode 1 and mode 2, then the number “1” in the variable name, such as CLAPS1A, indicates “in the presence of the mode 1 axisymmetric imperfection” and the number “2” in the variable name indicates “in the presence of the mode 2 axisymmetric imperfection”. For example, the name, **SKNBK1A(2,1)**, means “local **SK**i**N** **B**uc**K**ling, axisymmetric mode **1** imperfection shape, **A**llowable, load set **2** (which is for a –mode 1 imperfection shape), region **1** of the meridian of the equivalent ellipsoidal shell". If a shell is to be optimized in the presence of the first four axisymmetric buckling modal imperfections, mode 1, mode 2, mode 3, and mode 4, then the number "1" in the variable name indicates "odd-numbered modal axisymmetric imperfection" (mode 1 or mode 3) and the number "2" in the variable name indicates "even-numbered modal axisymmetric imperfection" (mode 2 or mode 4).

Note from Table 35 that the Role 3 variable, PRESS, and each of the "behavioral" allowables (Role 5 variable) and factors of safety (Role 6 variable) are provided in **loops over the number of load sets**. Role 4 variables are not prompted for in BEGIN because they are unknown values, such as collapse load factor (CLAPS1), general buckling load factor (GENBK1), local skin buckling load factor (SKNBK1), local stiffener buckling load factor (STFBK1) maximum stress in the shell skin (SKNST1), maximum stress in the stiffeners (STFST1), and apex displacement (WAPEX1). **The user provides only the allowables and factors of safety corresponding to these "behaviors".** Where Region 1 and Region 2 apply (local stress and buckling in shell skin and stiffener in Regions 1 and 2) the inner loop is the loop over load set number, and the outer loop is the loop over region number. The allowables and factors of safety corresponding to shells with mode 2 axisymmetric buckling modal imperfection shapes (CLAPS2A, CLAPS2F, etc.) are provided by the user after all the "mode 1" quantities have been provided.

Table a47 in the appendix lists **what the "end" user actually sees on his or her computer screen** during the interactive "BEGIN" session corresponding to Table 35. The "end" user's responses are in bold face. Some comments on Table a47 are:
1. The first line, "GENOPT = /home/progs/genopt" refers to the location of the GENOPT material on the writer's computer where this work was done. At another user's facility the string, "/home/progs" would be replaced by wherever the GENOPT material is located at that facility.
2. In response to the query, "Are you correcting, adding to, or using an existing file?", the "end" user responds "n" in Table a47. The "n" response leads to an interactive session. If the "end" user wanted to use as input the existing eqellipse.BEG file listed in Table 35, he would have responded "y" to this prompt. GENOPT would then have read the input data from the eqellipse.BEG file instead of requiring interactive input from the "end" user.
3. On the computer screen GENOPT echoes the "end" user's data entries. These echoes are not listed in Table a47 in order to save space.
4. GENOPT does not require any input from the "end" user for the "behavioral" variables (collapse, buckling, apex deflection, etc) such as CLAPS1, GENBK1, SKNBK1, WAPEX1, CLAPS2, STFST2, WAPEX2, etc. No values yet exist for the "behavioral" variables. Values for them are computed later in SUBROUTINE STRUCT (or possibly in the "behavior" subroutines BEHXi if the GENOPT user has decided on that strategy). During the "BEGIN" interactive session GENOPT only requires values for the allowable and the factor of safety that correspond to each "behavioral" variable. For example, the several lines on page 3 of Table a47 that begin with the line, "DEFINITION OF THE ROW INDEX OF THE ARRAY, CLAPS1 =" and end with the second line that reads, "collapse pressure with imperfection mode 1: CLAPS1" do not require any response from the "end" user. The "end" user is first required to respond when the prompt, "allowable pressure for axisymmetric collapse: CLAPS1A(1)=" appears on the computer screen. The same holds for the rather long series of lines having to do with the "behavioral" variable, SKNBK1. GENOPT presents all those "SKNBK1" lines on the screen but waits for a response from the end user only after presentation of the prompt, "allowable buckling load factor: SKNBK1A(1, 1)=".

5. Where there are two-dimensional arrays for allowable and factor of safety, GENOPT requires responses from the user over the "load case loop" as the inner loop and over the "region loop" as the outer loop.
6. Where the line, "(many lines skipped to save space)", occurs the input is analogous to that which is included in Table a47.

As explained in Sub-section 3.7.3 and in Tables 16 and 31 and 32, **design margins** are automatically constructed by GENOPT using the Role 4,5,6 variable names such as STFBK1, STFBK1A, and STFBK1F (local isogrid stiffener buckling) as follows (from Table 31):

(STFBK1(1,1)/STFBK1A(1,1))/STFBK1F(1,1)-1; F.S.= 1.00

or using Role 4,5,6 variable names such as SKNST1, SKNST1A, and SKNST1F (local skin effective stress) as follows (from Table 32):

(SKNST1A(2,2)/SKNST1(2,2))/SKNST1F(2,2)-1; F.S.= 1.00

Table 36 lists the input file, eqellipse.DEC, for the DECIDE processor for the specific case, "eqellipse". The writer has added the names, THKSKN(1), THKSKN(2), etc. on the right-hand side in Table 36 so that the reader knows the correspondence between decision variable number and name of that decision variable. (Actual *.DEC files do not include the decision variable names). "Escape" variables are those variables that when increased drive the design toward the feasible region. Typically a wall thickness is an escape variable because a thicker wall almost always leads to higher buckling loads, lower stresses, and smaller deformations. "Escape" variables are needed for instances during optimization cycles when a design is so far in the unfeasible region that it is impractical to rely on ADS to drive it toward the feasible region. In such instances, instead of using ADS, the main processor, OPTIMIZE, drives the design toward the feasible region by cyclically increasing all of the user-selected or default-selected escape variables by 10 per cent per cycle until ADS resumes control of the optimization process. The default selection chooses a decision variable as an escape variable if the string, "thick" occurs in the one-line definition of that decision variable.

Table 37 lists the input file, eqellipse.OPT, for the MAINSETUP/OPTIMIZE processors for the specific case, "eqellipse". The response to the prompt, "Choose an analysis you DON'T want (1, 2,…), IBEHAV" is repeated NCASES times, that is, for the number of load sets, in this example 2 load sets. See Sub-section 3.7.3 for more on IBEHAV (spelled IBEHV there). IDESIGN = 2, the preferred choice, means that an "ALMOST FEASIBLE" design will be accepted by GENOPT, that is, a design for which all margins are greater than –0.05. IMOVE = 1, the preferred choice, means that the move limit for each decision variable during an optimization cycle is ten per cent. These and other inputs are explained via "help" paragraphs in the file, /home/progs/genopt/execute/URPROMPT.DAT. The complete file, URPROMPT.DAT, is listed in the appendix (Table a24). Unlike the GENOPT-generated prompting file, called "equivellipse.PRO" for the generic case "equivellipse", the prompting file called URPROMPT.DAT, remains the same for all GENOPT generic cases. This prompting file is used during the execution of the GENOPT processors called CHANGE, DECIDE, MAINSETUP, and CHOOSEPLOT.

8.1.2 Optimization

Figure 3 shows the objective vs design iterations for the last of a series of four executions of SUPEROPT (Table 39). Each "spike" in the plot corresponds to a new starting design, obtained randomly as described in [15]. The presence of the three "dense" or "quiet" regions starting approximately at Iteration Numbers 150, 325, and 440 in this particular case, is explained in Section 9 on p. 10 of [24] as follows:

"During a SUPEROPT run the 'starting' design is set equal to the best design determined so far at or near Design Iteration Numbers. 150, 300, and 450, and the maximum permitted 'move limits' are reduced temporarily from 0.1 to 0.02 at or near each of these same three Iteration Numbers. (See Items 730 and 740 in Table a24 for more information about the index, IMOVE). This new strategy helps PANDA2 'close in' on a FEASIBLE or ALMOST FEASIBLE local minimum-weight design. The 'move limits' are re-expanded to 0.1 at the next execution of AUTOCHANGE." (For "next execution of AUTOCHANGE" see the second page of Table 34).

The optimum design is listed in Table 33 under the heading, "**isogrid-stiffened, imperfect**".

Table 38 lists an input file, eqellipse.CHG, for the processor called CHANGE. In this instance **CHANGE is used as a means to preserve the optimum design** electronically so that in the future that same optimum design can easily be re-established by execution of BEGIN with use of the file listed in Table 35 followed immediately by execution of CHANGE with use of the file listed in Table 38. **Over the years the writer has found that it is always a good idea to use CHANGE to "save" previously obtained optimum designs.**

Table 39 lists the run stream used to produce, in this particular case, the optimum design of the isogrid-stiffened imperfect equivalent ellipsoidal shell and to produce the plots from BIGBOSOR4 of axisymmetric linear bifurcation buckling modes 1 and 2 corresponding to the optimum design. The plots of +mode 1 and +mode 2 axisymmetric buckling modes are displayed in Figs. 4 and 5. The –mode 1 and –mode 2 imperfection shapes are simply the negatives of the deformed shapes exhibited in Figs. 4 and 5, respectively.

In Table 39 notice that each execution of SUPEROPT is followed immediately by executions of CHOOSEPLOT and DIPLOT before SUPEROPT is executed again. After successful completion of a SUPEROPT run, execution of CHOOSEPLOT causes the total number of design iterations to be reset to zero so that the next execution of SUPEROPT begins at Design Iteration Number zero. If the user does not execute CHOOSEPLOT after successful completion of a SUPEROPT run, but instead tries immediately to execute SUPEROPT again, the second SUPEROPT run will terminate after only a few design iterations because the total number of design iterations (a number greater than 465, probably between 468 and 474) will exceed the number that automatically "tells" SUPEROPT to stop.

PART 2 of Table 39 briefly instructs the user how to run BIGBOSOR4 using one of the files (eqellipse.ALL1) as input data. The long footnote at the bottom of Table 30 gives more details about how to use independent BIGBOSOR4 runs to obtain results for each of the seven analyses listed in the body of Table 30. The reader should study the footnote of Table 30 carefully.

8.1.3 Design margins

Tables 31 and 32 list the design margins for the **optimized isogrid-stiffened imperfect equivalent ellipsoidal shell**. In each table a line is skipped between Margins 11 and 12 to emphasize that the first 11 margins correspond to the shell with the axisymmetric mode 1 imperfection shape and Margins 12 – 22 correspond to the shell with the axisymmetric mode 2 imperfection shape. (This skipped line is not present in the actual *.OPM file from which the margins listed in Tables 31 and 32 are abstracted; see Table a19). For this particular case general nonlinear bifurcation buckling margins (GENBK) and local skin buckling margins (SKNBK) are not critical or nearly critical, as can be seen from inspection of Tables 31 and 32. The –mode 1 and –mode 2 imperfections (Load Set 2 = Table 32 = imperfection causes flattening near the apex of the shell as seen in Figs. 14 and 15) generate more critical and near-critical margins than do the +mode 1 and +mode 2 imperfections (Load Set 1 = Table 31 = imperfection causes local bulging near the apex of the shell as seen in Figs. 12 and 13). The isogrid stiffener local buckling margins for –mode 1 and –mode 2 in Region 1 [STFBK1(2,1) and STFBK2(2,1)] are near-critical (Table 32, margins 5 and 16). Several of the skin and isogrid stiffener stress margins (SKNST and STFST) are critical or near-critical. The collapse margin corresponding to the –mode 1 imperfection shape [CLAPS1(2)] is critical. (Design margin no. 1 in Table 32 is near zero). Margin 12 in Table 32 is identical to Margin 12 in Table 31 (CLAPS2). This is not a coincidence but indicates that no axisymmetric collapse occurs for an external pressure less than 920 psi, which is twice the design pressure, p = 460 psi, and which is the maximum external pressure used in the collapse analyses conducted in SUBROUTINE STRUCT. (See Analysis No. 5 in Table 30, for example).

Figures 4 and 5 display the axisymmetric buckling modes, +mode 1 in Fig. 4 and +mode 2 in Fig. 5, obtained for the optimized design by BIGBOSOR4 as described in Part 2 of Table 39, in the long footnote in Table 30, and in the next paragraph.

When the GENOPT processor called "OPTIMIZE" is executed **with the use of "type of analysis", ITYPE = 2 (fixed design, Table 37)**, several files are produced, the name of each file starting with the string, "eqellipse" (the user-selected name for the specific case). Most of these files have names of the form, eqellipse.ALLxxx. Each of these *.ALL* files is valid input for BIGBOSOR4 (or BOSOR4). One can use each of these files to obtain results directly from BIGBOSOR4 (or BOSOR4). For example, the file called "eqellipse.ALL1" can be used to obtain the axisymmetric linear buckling modes shown in Figs. 4 and 5 by means of the statements listed in Part 2 of Table 39 and as described in the long footnote to Table 30. The other *.ALL* files produced by OPTIMIZE **running in the ITYPE=2 mode** are described in the file, /home/progs/genopt/case/torisph/readme.equivellipse. (Note: the string, "/home/progs/genopt" points to the location of the GENOPT material stored on the writer's computer. To find the same file at your facility, replace the string, "/home/progs", with whatever string corresponds to the appropriate location on your computer.) The processor, OPTIMIZE, **running in the ITYPE=2 mode** also produces a file called "eqellipse.STAGS". The eqellipse.STAGS file corresponding to the optimized isogrid-stiffened shell is listed in Table a23 of the appendix. This file, with its name changed to **WALLTHICK.STAGS**, is used in connection with STAGS models described in the next and in other sub-sections. (See Table 40 for more on WALLTHICK.STAGS).

8.1.4 Evaluation of the optimum design with the use of STAGS

We have an optimum design derived from GENOPT by means of repeated executions of SUPEROPT/CHOOSEPLOT/DIPLOT, and we wish now to check this optimum design through the use of a general-purpose finite element computer program. **Here we choose STAGS [20 – 24] to evaluate the optimum design obtained by GENOPT.** We choose STAGS because STAGS is very good at solving difficult nonlinear shell problems and because the developer of STAGS, Dr. Charles Rankin, is close by.

Table 40, many pages long, instructs the user how to generate STAGS input files and how to obtain results. Because the thickness of the optimized shell varies over the surface of the shell, STAGS requires that a user-written subroutine called "WALL" or that a user-written subroutine called "USRFAB" be available. Versions of SUBROUTINE WALL are listed in Tables a20 – a22 and Tables a32 and a33. Versions of SUBROUTINE USRFAB are listed in Tables a34 – a36.

In this section only SUBROUTINE WALL is used. Four copies of SUBROUTINE WALL are listed in the appendix of [26]:

1. a skeletal ("template") version of WALL that is provided with the STAGS software (Table a20 of [26]),

2. a complete version of WALL valid for elastic material (Table a21 of [26]),

3. a complete version of WALL valid for elastic-plastic material (Table a22 of [26]), and

4. a complete version of WALL valid for a 180-degree STAGS "soccerball" model of the equivalent ellipsoidal shell (Table a32 of [26]).

There are also several versions of USRFAB listed in the appendix (Tables a34 – a36).

The complete versions of WALL or USRFAB require that a file called "**WALLTHICK.STAGS**" be available. WALLTHICK.STAGS contains the meridional distributions of shell skin thickness, isogrid height, and certain other information. A copy of WALLTHICK.STAGS (called "eqellipse.STAGS" for the specific case, "eqellipse", because it is generated by execution of the mainprocessor, OPTIMIZE for the specific case called "eqellipse") is listed in the appendix (Table a23). Lists of the two input files required for STAGS, eqellipse.bin and eqellipse.inp, are included on pages 3 – 8 of Table 40. From the list of eqellipse.inp (pages 4 – 8 of Table 40) one sees that the STAGS finite element called "410" is used in the 360-degree STAGS model (Fig. a1). (The favored STAGS 480 finite element does not work in connection with the 360-degree STAGS model displayed in Fig. a1, probably because of the greatly elongated finite elements shown in the insert at the top of Fig. a1).

Also included in Table 40 are input and output for the sequence of STAGS runs and "post-processing" by the STAGS processors, STAPL and XYTRANS, needed to obtain the maximum pressure-carrying capability of the optimized isogrid-stiffened equivalent ellipsoidal shell with axisymmetric or non-axisymmetric linear buckling modal imperfections.

STAGS is used:

1. to obtain the axisymmetric mode 1 and mode 2 and non-axisymmetric linear bifurcation buckling modal imperfection shapes from an INDIC=1 STAGS analysis (pp. 3 – 11 of Table 40 and Table 41, and Figs. 6 – 10),

2. to obtain plots of the axisymmetric and the non-axisymmetric linear buckling modes (pp. 11, 12 of Table 40 and Figs. 6 – 10),

3. to modify the eqellipse.bin and eqellipse.inp files in order to run nonlinear equilibrium and nonlinear bifurcation buckling analyses (INDIC = 3) of the optimized shell with plus and minus mode 1 and mode 2 initial axisymmetric imperfection shapes and other non-axisymmetric buckling modal imperfection shapes (pp. 12 – 17 of Table 40 and Figs. 16 and 17),

4. to obtain the maximum pressure-carrying capabilities of the imperfect shells (pp. 17 – 19 of Table 40 and Figs. 16 and 17), and

5. to obtain "x,y" plots of external pressure versus normal deflection of the apex of the shell for various axisymmetric and non-axisymmetric linear buckling modal imperfection shapes (pp. 18, 19 of Table 40 and Figs. 16 and 17).

A typical 12-shell-unit 360-degree STAGS model of the equivalent ellipsoidal shell is displayed in the appendix (Fig. a1). The 12 shell units shown in Fig. a1 correspond exactly to the 12 shell segments in the BIGBOSOR4 model displayed in Fig. 2.

Table 41 lists a small part of the STAGS output file, eqellipse.out2, corresponding to a linear bifurcation buckling STAGS execution (INDIC=1). In this case the fundamental buckling mode, displayed in Fig. 6, is axisymmetric. It is analogous to the "+mode 1" axisymmetric imperfection shape predicted by BIGBOSOR4 and depicted in Fig. 4. Axisymmetric "+mode 2" corresponds to the sixth eigenvalue computed by STAGS (Fig. 9). It is analogous to the "+mode 2" axisymmetric imperfection shape predicted by BIGBOSOR4 and depicted in Fig. 5.

STAGS eigenvalues 2 – 5 correspond to non-axisymmetric eigenvectors (buckling modes), as displayed in Figs. 7 and 8. Note from Table 41 that in the 360-degree STAGS model the STAGS eigenvalues corresponding to non-axisymmetric buckling modes occur in pairs. The corresponding eigenmodes are the same in each member of the pair except that one mode is rotated circumferentially with respect to the other in the pair. Figure 9 shows the axisymmetric "+mode 2" from STAGS, and Fig. 10 shows the non-axisymmetric mode corresponding to the next higher eigenvalue.

NOTE: BIGBOSOR4 is capable of computing linear and nonlinear buckling modal eigenvalues and eigenvectors (buckling modes) corresponding to non-axisymmetric buckling modes as well as axisymmetric buckling modes. However, no comparison is made here for linear bifurcation buckling between the predictions of STAGS and BIGBOSOR4 for linear non-axisymmetric buckling modes because non-axisymmetric initial linear buckling modal imperfections cannot be used in the GENOPT models described in this report. BIGBOSOR4 (AND BOSOR4) CANNOT HANDLE PREDICTION OF THE BEHAVIOR OF SHELLS OF REVOLUTION WITH NON-AXISYMMETRIC INITIAL IMPERFECTION SHAPES.

8.1.5 Some predictions from BIGBOSOR4 (GENOPT), BOSOR5, and STAGS

Figure 11 demonstrates that there is excellent agreement between BIGBOSOR4 and STAGS for the axisymmetric linear bifurcation buckling modes 1 and 2. This agreement leads to the reasonably good agreement between STAGS and BIGBOSOR4 predictions of maximum pressure-carrying capability of the optimized axisymmetrically imperfect isogrid-stiffened equivalent ellipsoidal shells, as displayed in Fig. 16.

Figures 12 – 15 show the axisymmetric deformed states (according to BOSOR5 [25] with the use of elastic material) of the optimized isogrid-stiffened equivalent ellipsoidal shells with +mode 1 (Fig. 12), +mode 2 (Fig. 13), –mode 1 (Fig. 14), and –mode 2 (Fig. 15) axisymmetric linear buckling modal imperfections with amplitude, Wimp = 0.2 inch. The axisymmetrically imperfect shells are loaded by the external design pressure, p = 460 psi.

These axisymmetrically deformed states predicted by BIGBOSOR4 give rise to the maximum local stress and minimum buckling load factors of shell skin and isogrid stiffeners listed in Tables 42 – 45 for the shells with the axisymmetric +mode 1 imperfection (Table 42), +mode 2 imperfection (Table 43), –mode 1 imperfection (Table 44), and –mode 2 imperfection (Table 45). The results in Tables 42 – 45 are produced by BIGBOSOR4. They are included in the output file, *.OPM, which is produced by the GENOPT mainprocessor, “OPTIMIZE”, run in the ITYPE=2 mode and with NPRINT = 2 in the *.OPT file (Table a19). In Tables 42 – 45 the local maximum effective stress in the shell skin (SKNMAX) in each of the 12 segments of the equivalent ellipsoidal shell (Fig. 2) are computed taking into account the comparison of the effective stresses at both the inner and outer extreme fibers of the skin. Output for the inner fiber effective stress and outer fiber effective stress are not available as separate quantities in the data produced by BIGBOSOR4 as processed by GENOPT. Similarly, the maximum absolute value of the stress (STFMXS) along the axis of a **MERIDIONALLY ORIENTED** isogrid stiffener (assumed here to be the most critical orientation of an isogrid member) is obtained from comparison of the stresses both at the root and tip of the stiffener. The stiffener root and tip stresses are not available as separate quantities in the output produced by BIGBOSOR4 as processed by GENOPT. There are no “rings” (circumferentially running stiffeners independent of the isogrid). Hence the (nonexistent) ring buckling load (BUCMNR) is arbitrarily set equal to a very high value and the maximum stress in the (nonexistent) ring (STFMXR) is arbitrarily set equal to zero so that these irrelevant quantities will not produce critical margins that affect the evolution of the design during optimization cycles.

Figure 16 shows plots of load-apex-deflection curves from GENOPT (BIGBOSOR4, elastic material), STAGS (elastic material), and BOSOR5 (elastic-plastic material)[25] for the optimized isogrid-stiffened imperfect equivalent ellipsoidal shell. There is reasonably good agreement between BIGBOSOR4 and STAGS predictions of maximum pressure-carrying capabilities of the elastic shells. Note that BIGBOSOR4 and BOSOR5 cannot obtain solutions “beyond” the maximum pressure because the BOSOR programs do not have the Riks nonlinear continuation algorithm [22], whereas STAGS does [22]. The most critical case is that with the –mode 1 axisymmetric buckling modal imperfection. There is no effect of plasticity below or at the design pressure, 460 psi. Plastic flow (BOSOR5) affects the behavior only above the design pressure, p = 460 psi. Sample input data for the BOSOR5 preprocessor (BOSORREAD) and the BOSOR5 mainprocessor (BOSORMAIN) are listed in the appendix (Tables a25 and a26, respectively).

Figure 17 demonstrates that for this particular optimized isogrid-stiffened imperfect shell the axisymmetric –mode 1 imperfection is more critical than any of the non-axisymmetric linear buckling modal imperfection shapes displayed in Figs. 7, 8, and 10 (last three traces in Fig. 17). **As we will see in Sub-section 8.2 this is not true for the optimized UNSTIFFENED imperfect equivalent ellipsoidal shell.**

Figures 18 and 19 display a spurious nonlinear bifurcation buckling mode from the 360-degree STAGS model that affects the region near the pole of the optimized isogrid-stiffened imperfect equivalent ellipsoidal shell. The presence of this spurious nonlinear bifurcation (eigenvalue) on the primary nonlinear equilibrium path at a pressure close to 300 psi (well below the design pressure, p = 460 psi) does not seem to affect the prediction by STAGS of nonlinear behavior of the shell in this case.

Section 10.0 gives STAGS results for the same optimized isogrid-stiffened shell including plastic flow in the shells with both axisymmetric and non-axisymmetric imperfections.

8.1.6 Predictions of extreme fiber stress

Figures 20 – 35 show the distributions of meridional stress, sigma1 or sxx, at the inner fiber of the isogrid “layer” (layer number 1 in the STAGS model) and the inner and outer fiber effective stress, seff, in the shell skin (layer number 2 in the STAGS model) generated for the axisymmetrically imperfect shells with a +mode 1 imperfection (Figs. 20 – 23), a +mode 2 imperfection (Figs. 24 – 27), a –mode 1 imperfection (Figs. 28 – 31), and a –mode 2 imperfection (Figs. 32 – 35). Compare the +mode 1 STAGS predictions with those from GENOPT (BIGBOSOR4) listed in Table 42. Compare the +mode 2 STAGS predictions with those from GENOPT (BIGBOSOR4) listed in Table 43. Compare the –mode 1 STAGS predictions with those from GENOPT (BIGBOSOR4) listed in Table 44. Compare the –mode 2 STAGS predictions with those from GENOPT (BIGBOSOR4) listed in Table 45.

Note that the STAGS predictions for meridional inner fiber stress, sigma1 or sxx, in the isogrid "layer" may not agree very well with those from BIGBOSOR4, especially in the immediate neighborhood of the pole. This is to be expected. It is because the isogrid "layer" in the STAGS model is treated as an isotropic material with Poisson's ratio equal 1/3 both in the computation of overall shell wall stiffness (the 6 x 6 integrated constitutive matrix C_{ij}) and in the computation of stress at a point in that layer. In contrast, in the BIGBOSOR4 model, although the same "smeared" stiffener model is used for computation of the overall stiffness C_{ij} of the shell wall, the maximum stress in a single isogrid member is calculated **as if that member were oriented in the meridional coordinate direction** and as if it were not affected by any other neighboring isogrid members. Therefore, in the STAGS model the meridional stress at a point in the isogrid isotropic "layer" is computed from

$$\text{sigma1(meridional)} = [E/(1-nu^2)](eps_1 + nu*eps_2) \qquad (7)$$

in which E is the elastic modulus, nu is the Poisson ratio, eps_1 is the strain in the meridional direction and eps_2 is the strain in the circumferential direction at any point through the thickness of the shell wall layer. In contrast, in the BIGBOSOR4 model the stress at a point in the meridionally oriented isogrid stiffener is computed from

$$\text{sigma1(meridional)} = E*eps_1 \qquad (8)$$

At the pole, where $eps_1 = eps_2$, STAGS predicts

$$\text{sigma(meridional)} = E*eps_1/(1-nu) \qquad (9)$$

which for nu = 1/3 is 50 per cent greater than the value, $E*eps_1$ predicted by the BIGBOSOR4 model.

The value predicted by the BIGBOSOR4 model is the correct value to use for optimization of an actual isogrid-stiffened shell for the isogrid members that are oriented in the meridional coordinate direction, WHICH IS ASSUMED HERE TO BE THE MOST CRITICAL DIRECTION for axisymmetrically deformed, perfect or axisymmetrically imperfect, equivalent ellipsoidal shells.

The maximum stresses predicted from the axisymmetric (360-degree) STAGS models displayed in Fig. a1 of the appendix and in Figs. 20 – 35 may not represent converged values. Therefore, a much more refined 12-shell-unit STAGS model was set up. The STAGS input data for this refined model are listed in Table 46a. The model subtends only 10 degrees of circumference. Symmetry conditions are applied along the two meridionally oriented edges. These symmetry conditions simulate axisymmetric deformations.

Table 46b lists the first eight eigenvalues from both the STAGS "10-degree" model and the first eight eigenvalues from BIGBOSOR4 all of which correspond to axisymmetric mode shapes. (See the BIGBOSOR4 predictions listed under the heading, Analysis no. 1, in Table 30). In this narrow 10-degree "slice" STAGS model with symmetry conditions applied on the two meridionally oriented edges, all the eigenvalues correspond to axisymmetric buckling modes. Compare eigenvalues 1 and 2 in Table 46b with eigenvalues 1 and 6 in Table 41, which applies to the 360-degree STAGS model.

Figures 36 and 37 show +mode 1 and +mode 2 linear bifurcation buckling modes from the 10-degree refined STAGS model. These figures are analogous to Figs. 6 and 9, respectively. Figure 38 displays nonlinear load-apex-deflection curves from GENOPT (BIGBOSOR4) and from STAGS for the 360-degree STAGS model and for the 10-degree STAGS model. Figures 39 – 46 show the inner fiber meridional stress in the isogrid "layer" for the +mode 1 imperfection (Figs. 39, 40), for the +mode 2 imperfection (Figs. 41, 42), for the –mode 1 imperfection (Figs. 43, 44), and for the –mode 2 imperfection (Figs. 45, 46). Compare the stresses displayed in these figures with those from the 360-degree STAGS models shown in Figs. 20 and 21 for the +mode 1 imperfection, in Figs. 24 and 25 for the +mode 2 imperfection, in Figs. 28 and 29 for the –mode 1 imperfection, and in Figs. 32 and 33 for the –mode 2 imperfection. Compare with those from GENOPT (BIGBOSOR4) listed in Tables 42 – 45, respectively.

Figure 47 shows the extreme fiber meridional stress in the isogrid "layer" for the optimized isogrid-stiffened imperfect equivalent ellipsoidal shell at the design external pressure, p=460 psi. The shell has a –mode 1 axisymmetric linear bifurcation buckling modal imperfection with amplitude, Wimp = –0.2 inch. The BOSOR5 (with the use of elastic material) and STAGS predictions are from similar models in that the meridional stress is computed from a model in which the isogrid "layer" is treated as an isotropic layer with smeared stiffeners [Eq.(7)] and in both models the material remains elastic. Most of the BOSOR5 data points correspond to the extreme inner fiber of the isogrid "layer". The BOSOR5 data points corresponding to the root of the isogrid "layer" (the extreme outer fiber of the isogrid "layer") are plotted only over part of the meridian, from about 8 inches to about 27 inches of the meridional reference surface arc length. No STAGS data points are plotted for the root of the isogrid "layer". The BIGBOSOR4 points are taken directly from Table 44 (maximum STFMXS in each shell segment, except for the sign of the stress; only absolute values are listed in Table 44). As has been mentioned previously, the BIGBOSOR4 data entries in Table 44 represent the maximum absolute values of inner and outer fiber meridional stresses in each shell segment; inner and outer fiber meridional stresses are not both listed. (Outer fiber of the isogrid "layer" is the same location as the root of the isogrid "layer"). The BOSOR5 (with use of elastic material) and STAGS predictions are in very good agreement because they are based on the same type of model [Eq.(7)]. The BIGBOSOR4 predictions differ because the maximum axial stress in a meridionally oriented isogrid member is computed from a different formula [Eq.(8)] in BIGBOSOR4, as previously explained, than that used for computation of the meridional stress from the BOSOR5 and STAGS models in which the isotropic stiffeners are replaced by a uniform homogenous isotropic material (smeared isotropic "layer" model).

Figure 48 is analogous to Fig. 47. It corresponds to the same shell with a –mode 2 imperfection shape instead of a –mode 1 imperfection shape.

Note that at the pole the meridional stress from both the BOSOR5 (elastic) and STAGS models is approximately 1.5 = 1/(1 – nu) times the meridional stress from the BIGBOSOR4 model. That difference is as it should be for a Poisson's ratio equal to 1/3, as previously explained [Eq.(9)]. In Fig. 47 the STAGS prediction is a bit low, possibly due to the relative crudeness of the 360-degree STAGS model or possibly due to the extremely elongated shape of the finite elements nearest the pole, as seen in Fig. 19 and in the insert at the top of Fig. a1.

The most critical stresses from the BIGBOSOR4, STAGS, and BOSOR5 models for all four axisymmetric imperfection shapes (+mode 1, +mode 2, –mode 1, and –mode 2, are listed in the two columns in Table 47 headed, **"isogrid-stiffened, imperfect"**.

See Section 10.0 and Figs. 254 – 276 for more results relating to the optimized isogrid-stiffened shell for various STAGS analyses in which the effect of plastic flow is included.

Table 47 is analogous to Table 33 in that it pertains to all the four optimum designs: Table 47 lists the maximum extreme fiber stresses predicted by BIGBOSOR4, BOSOR5 [25], and STAGS for the optimized stiffened and unstiffened, perfect and imperfect equivalent ellipsoidal shells, the optimum designs of which are listed in Table 33.

8.1.7 Use of plus and minus axisymmetric modes 1 – 4 (4 load sets)

Table 48 lists the input file, eqellipse.BEG, for the BEGIN processor. Compare with Table 35. Now there are 4 load sets. (NCASES = 4 rather than 2). Therefore, there are 4 entries for each allowable and for each factor of safety. In order to re-establish the previously determined optimum design, the processor CHANGE (with Table 38 input) was executed immediately following the execution of BEGIN in order to retrieve the optimum design obtained with the "2-mode-imperfection" model. This "2-mode" optimum design is listed in Table 33 under the heading, "isogrid-stiffened, imperfect". The input data for the execution of CHANGE for resurrecting the "2-mode" optimum design appear in Table 38.

The input data for the DECIDE processor are the same as for the "2-mode" case. Hence, Table 36 is valid for this new case.

Table 49 lists the input file, eqellipse.OPT, for MAINSETUP/OPTIMIZE. Now there are 4 entries for IBEHAV, one for each of the 4 load sets. In Table 37, which pertains to the "2-mode" case, there are only two entries for IBEHAV, one for each of the 2 load sets.

Figures 49 and 50 display the axisymmetric mode 3 and mode 4 linear bifurcation buckling modes predicted by BIGBOSOR4 for the same design as that obtained from optimization with the use of plus and minus mode 1 and mode 2 only. This design is listed in Table 33 under the heading, "isogrid-stiffened, imperfect". The mode 1 and mode 2 linear bifurcation buckling modes are shown in Figs. 4 and 5.

Tables 50 lists the margins corresponding to the +mode 3 and +mode 4 axisymmetric imperfection shapes (Load Set 3), and Tables 51 lists the margins corresponding to the –mode 3 and –mode 4 axisymmetric imperfection shapes (Load Set 4). **Note that a few of the margins are significantly negative, indicating the need to do some optimization.**

There is nomenclature in Tables 50 and 51 that differs in one respect from that in the analogous tables, Tables 31 and 32. Items a – e are defined in the upper part of each table. In Tables 31 and

32 Item c is defined as

c= Imperfection mode number, (1 or 2 in the cases explored here)

In contrast, in Tables 50 and 51 Item c is defined as

c= Imperfection mode number, [1= odd (mode 3); 2= even (mode 4)]

This is because, in the "4-mode" case, the names, CLAPS1, GENBK1, SKNBK1, etc. refer in Load Set 3 to the shell with an axisymmetric +mode 3 imperfection shape, and the names, CLAPS2, GENBK2, SKNBK2, etc. refer in Load Set 3 to the shell with an axisymmetric +mode 4 imperfection shape. The same holds for Load Set 4, for which the shell has either an axisymmetric –mode 3 or –mode 4 imperfection shape.

8.1.8 Optimization with the use of plus and minus modes 1 – 4

Figure 51 shows the evolution of the objective during the initial phase of a SUPEROPT run in which the move limit of the decision variables has been set by the user via the move limit index, IMOVE, to a number significantly less than the preferred value of 0.1. (See Table 49 and Items 730 and 740 in Table a24 for information about IMOVE. The writer thinks that in this case IMOVE was set equal to 4, but he cannot recall for certain. IMOVE may have been set to 3). The starting design is the previously determined optimum design obtained with the use of only two axisymmetric buckling modal imperfections, mode 1 and mode 2.

Figure 52 shows the evolution of the most critical design margins during the same design iterations. In this case there are about 20 design margins that are critical or nearly critical.

Figure 53 displays the evolution of the objective for a complete SUPEROPT run with the decision variable move limit index, IMOVE, set in Table 49 equal to its preferred value, IMOVE = 1, which means the move limit is 0.1 except in the three "quiet" regions starting near design iterations 150, 300, and 450 where the move limit is **temporarily** reset automatically by GENOPT to 0.02, as explained in Sub-section 8.1.2. The complete SUPEROPT computer run required about 8 days on the efficient LINUX workstation on which this work was done.

Listed in Table 52 are the optimum design, objective, and design margins for the four load sets (+mode 1 and +mode 2 imperfection shapes = load set 1; –mode 1 and –mode 2 imperfection shapes = load set 2; +mode 3 and +mode 4 imperfection shapes = load set 3; –mode 3 and –mode 4 imperfection shapes = load set 4). The objective (weight of the isogrid-stiffened equivalent ellipsoidal shell) exceeds that optimized with the use of only plus and minus axisymmetric mode 1 and mode 2 imperfection shapes (2 load sets). Compare the "4-mode" optimum design and objective listed in Table 52 with the "2-mode" optimum design and objective listed in Table 33 under the heading, "isogrid-stiffened, imperfect". Objective in Table 52 = 91.51 lb; Objective in Table 33 = 86.10 lb.

Figures 54 – 57 show the four axisymmetric linear bifurcation buckling modal imperfection shapes, modes 1 – 4, as predicted by BIGBOSOR4 for the new, somewhat heavier, "4-mode" optimum design. It appears that what was Mode 2 in the “2-mode” optimum design (Fig. 5) is now Mode 3 in the “4-mode” optimum design (Fig. 56).

Tables 53 – 55 list output from STAGS (greatly abridged) corresponding to three executions of STAGS with different “eigenvalue shifts” to obtain linear buckling eigenvalues and mode shapes. (STAGS analysis type index, INDIC=1). In each STAGS run the “eigenvalue shift” was changed. “Eigenvalue shift” is an input datum near the end of the STAGS input data file, eqellipse.bin, a sample of which occurs at the bottom of page 3 and the top of page 4 of Table 40. Tables 53 – 55 are edited to add brief descriptions of the STAGS buckling mode shapes. The STAGS linear bifurcation buckling mode shapes are displayed in Figs. 58 – 68. Figures 58, 62, 63, and 67 are analogous to the axisymmetric mode 1, mode 2, mode 3, and mode 4 linear axisymmetric buckling modal imperfection shapes produced by BIGBOSOR4 and shown in Figs. 54 – 57, respectively.

No nonlinear load-deflection curves or fringe plots of stress were obtained for the “4-mode” optimum design. They would be similar to those displayed in Figs. 16 – 48 for the “2-mode” optimum design. **In this case it appears to be unnecessary to use a “4-mode” model for optimization. The “2-mode” optimum design is strong enough to survive the design pressure, p = 460 psi, in the presence of either axisymmetric or non-axisymmetric linear buckling modal imperfections, as is demonstrated in Fig. 17.**

8.2 Details pertaining to the unstiffened, imperfect equivalent ellipsoidal shell

Tables 56 – 66 and Figs. 69 – 114 pertain to the **unstiffened, imperfect** equivalent ellipsoidal shell as discussed in this section. (There is more in Section 9.0). As will be seen in this sub-section, the optimized unstiffened shell analyzed in this sub-section is severely under-designed, in the author’s opinion, because the collapse pressure of this optimized design turns out to be extremely sensitive to **non-axisymmetric** buckling modal imperfections, imperfection types not present during the optimization cycles. Non-axisymmetric buckling modal imperfections are not present in the GENOPT model used here because the GENOPT model is based on BIGBOSOR4 and **BIGBOSOR4 can handle only axisymmetric** imperfections. This inadequacy is eliminated in a simple way, as described later in Section 9.0.

Although the shell is termed “**unstiffened**”, in this section (as in Section 9.0) the shell is modeled as if it has two layers: an isogrid “layer” and the shell skin. To represent an unstiffened shell the height of the isogrid is set equal to a very, very small value, HIGHST(i) = 0.000001 inch, and the thickness of each isogrid member is also set equal to a very, very small number, 0.00001 inch (Table 56). Therefore, the presence of the isogrid layer has no effect on the behavior of the shell. In STAGS models the isogrid layer is Layer No. 1 and the shell skin is Layer No. 2.

8.2.1 Input data

Table 56 lists the input data for the BEGIN processor for the specific case, “eqellipse”, now signifying the **imperfect unstiffened** equivalent ellipsoidal shell, which is a member of the generic class, “equivellipse”. The name of the input file for BEGIN is “eqellipse.BEG”. The decision variables in this specific case are the “skin thickness at xinput: THKSKN”(i), i = 1,13. **The unstiffened shell is simulated with the use of a tiny isogrid, the dimensions of which are not decision variables and so remain fixed during optimization cycles.**

Table 57 lists the input file, eqellipse.DEC, for the DECIDE processor for the specific case, “eqellipse”. The correspondence between decision variable number (datum supplied by the user in response to the prompt, “Choose a decision variable”) and the definition of the variable chosen is listed near the top of Table 52, for example. (Also, the writer added the names, THKSKN(1), etc. on the right-hand side in Table 57.) **In this case of the unstiffened equivalent ellipsoidal shell only the thirteen thicknesses at the thirteen callout points, xinput (Table 28), are decision variables.**

Table 58 lists the input file, eqellipse.OPT, for the MAINSETUP/OPTIMIZE processors for the specific case, “eqellipse”. The response to the prompt, “Choose an analysis you DON’T want (1, 2,…), IBEHAV” is repeated NCASES times, that is, for the number of load sets, in this example 2 load sets, the first corresponding to the +mode 1 and +mode 2 axisymmetric imperfection shapes and the second corresponding to the –mode 1 and –mode 2 axisymmetric imperfection shapes. IDESIGN = 2, the preferred choice, means that an “ALMOST FEASIBLE” design will be accepted by GENOPT, that is, a design for which all margins are greater that –0.05. In this example IDESIGN = 3 for the first two executions of SUPEROPT, then changed to IDESIGN = 2 for the remaining three executions of SUPEROPT. IDESIGN = 3 means that GENOPT will accept “MILDLY UNFEASIBLE” designs, that is designs for which all design margins are greater than –0.1. The decision variable move limit indicator, IMOVE = 1 is the preferred choice for that input datum. IMOVE and IDESIGN are defined in the URPROMPT.DAT file, which is listed in the appendix. (See Items 725, 730, and 740 in Table a24).

8.2.2 Optimization

Figures 69 - 73 show the objective vs design iterations for five executions of SUPEROPT. Each “spike” in each plot corresponds to a new starting design, obtained randomly as described in [15]. The presence of the three “dense”, “quiet” regions starting approximately after Iteration Numbers 150, 300, and 450, is explained in sub-section 8.1.2. The optimum design at the end of the fourth execution of SUPEROPT (Fig. 72) is the same as that at the end of the fifth execution of SUPEROPT (Fig. 73). Figures 69 – 73 exhibit a much “jumpier” characteristic than the analogous figures for the isogrid-stiffened shell (Figs. 3 and 53), probably because the axisymmetric mode 1 and mode 2 imperfection shapes in the unstiffened shell are much more sensitive to small design changes than is so in the isogrid-stiffened shell.

The optimum design is listed in Table 33 under the heading, “**unstiffened, imperfect**”. The unstiffened shell is optimized in the presence of plus and minus axisymmetric linear bifurcation buckling mode 1 and mode 2, each mode taken separately and each mode with amplitude, Wimp = (+ and –) 0.2 inch.

8.2.3 Design margins

Tables 59a and 59b list the design margins for the optimized, "2-mode" imperfect, unstiffened equivalent ellipsoidal shell. In this case the margins relating to local buckling of the skin between "fictitious" isogrid stiffeners (SKNBK1 and SKNBK2), the margins relating to local buckling of the "fictitious" isogrid stiffeners (STFBK1 and STFBK2), and the margins relating to stress along the axis of a meridionally oriented "fictitious" isogrid stiffener (STFST1 and STFST2) are irrelevant because the shell is stiffened only by a tiny (ineffectual) isogrid. These margins are computed because the "unstiffened" shell actually does have a tiny isogrid (Table 56) and so "goes through" all the same computations it would were it stiffened by a significant isogrid. The dimensions of the tiny isogrid are established in the input for BEGIN (Table 56) so that the local buckling margins would not be critical and therefore would not affect the evolution of the optimum design.

NOTE: the stiffener spacing, SPACNG, should have been made smaller because the margins for local buckling of the flat, triangular piece of shell skin between adjacent isogrid members are uncomfortably close to being critical and therefore doubtless had some effect on the evolution of the design during optimization cycles. The stress margins associated with the isogrid stiffeners (STFST1 and STFST2) could have been made much larger if a different material had been selected for the isogrid than for the shell skin, a material with a very high maximum allowable stress or a material with a very low elastic modulus combined with a maximum allowable stress that corresponds to a very high maximum allowable meridional strain.

8.2.4 Some predictions from BIGBOSOR4 (GENOPT), BOSOR5, and STAGS

Figures 74 and 75 show the axisymmetric linear bifurcation buckling modes 1 and 2, respectively, of the **optimized unstiffened equivalent ellipsoidal shell** as predicted by BIGBOSOR4. Comparing Fig. 74 with Fig. 4, which pertains to the optimized isogrid-stiffened shell, we see that the characteristic meridional wavelength of the axisymmetric mode 1 buckle in the unstiffened shell is much shorter than that in the isogrid-stiffened shell. The shape of the mode 1 buckle in the neighborhood of the pole (Fig. 74) is almost identical to that of the mode 2 buckle in the neighborhood of the pole (Fig. 75), and the shape of the mode 1 buckle far from the pole is similar to the negative of the mode 2 buckle far from the pole.

The fact that both the mode 1 and mode 2 axisymmetric imperfection shapes exhibit rather small deflections everywhere except in the neighborhood of the pole has a significant consequence with respect to the suitability of the optimum design of the axisymmetrically imperfect unstiffened equivalent ellipsoidal shell developed with use of the decision variables and their lower bounds as listed in Table 57. It is found from STAGS models that the pressure-carrying capability of the shell optimized in the presence of only **axisymmetric** linear buckling modal imperfection shapes turns out to be especially sensitive to **non-axisymmetric** linear buckling modal imperfections, imperfections that have shapes for which the maximum buckling modal normal deflection is maximum some distance away from the pole rather than at the pole. We will see this later in connection with STAGS predictions. A simple way to avoid this difficulty is to force the spherical cap (Shell Segment 1 in Fig. 2) to be relatively thick. Results for "**thick apex**" optimized unstiffened shells are presented later in Section 9.0 in Figs. 143 – 254 and Tables 77 – 95.

Table 60, abridged output from STAGS (part of the eqellipse.out2 file), lists the lowest 8 eigenvalues and identifies the mode shapes. The linear bifurcation buckling modes from STAGS are displayed in Figs. 76 – 81. Figures 76 and 78 show the two axisymmetric modes that are analogous to the axisymmetric mode 1 and mode 2 imperfection shapes predicted by BIGBOSOR4 and displayed in Figs. 74 and 75, respectively.

Figure 82 shows the axisymmetrically deformed profile of the optimized imperfect unstiffened equivalent ellipsoidal shell with a –mode 1 imperfection shape with amplitude, Wimp = –0.2 inch, loaded by the external design pressure, p = 460 psi. This axisymmetric deformation, predicted by both BIGBOSOR4 and BOSOR5 (with the use of elastic material), gives rise to the maximum effective stresses in the shell skin in each of the 12 shell segments (Fig. 2). These maximum effective stresses are listed in Table 61. For each shell segment the maximum effective stress in that segment is the maximum of the two extreme fiber stresses, that at the inner fiber and that at the outer fiber of the shell skin.

Figure 83 displays load-apex-deflection curves predicted by GENOPT (BIGBOSOR4, elastic material), BOSOR5 (elastic-plastic material), and STAGS (elastic material) for the optimized imperfect unstiffened equivalent ellipsoidal shell. There is good agreement between the three models for the shell with the axisymmetric –mode 1 imperfection shape (Fig. 82), but the discrepancy between BIGBOSOR4 and STAGS predictions is greater for the other load-deflection curves, probably because the STAGS models are too coarse to represent accurately the relatively short-meridional-wavelength axisymmetric mode shapes displayed in Figs. 74 and 75.

The first and second traces in Figure 84 are the same as the first and fifth traces in Fig. 83. The third trace in Fig. 84 (right-side-up triangles) is the load-apex-deflection curve for the unstiffened shell with:

1. an axisymmetric –mode 1 imperfection with amplitude, Wimp = –0.2 inch, plus

2. the non-axisymmetric nonlinear bifurcation buckling mode shown in Fig. 85 with a very small amplitude, Wimp(n=6) = 0.001 inch.

The maximum pressure-carrying capability of the axisymmetrically imperfect shell with a tiny non-axisymmetric n=6 "trigger" component is slightly below the external design pressure, p=460 psi. This situation is different from that for the optimized isogrid-stiffened shell. In that case the nonlinear non-axisymmetric bifurcation buckling loads are all significantly higher than the most critical axisymmetric collapse load.

In Fig. 84 the critical pressures predicted by BIGBOSOR4 from the INDIC = 1 and INDIC = -2 types of analysis differ because the INDIC = 1 analysis is governed by Eq.(5) whereas the INDIC = -2 analysis includes a changing stiffness matrix, $\mathbf{K_1}$, as the applied external pressure is increased in steps. With externally pressurized ellipsoidal shells that are "flatter" than spherical the stiffness matrix, $\mathbf{K_1}$, "softens" as the external pressure is increased. Therefore, nonlinear bifurcation buckling loads predicted by BIGBOSOR4 with INDIC = -2 will always be lower than those predicted with INDIC = 1, as long as the pressure used for computation of the stiffness matrix, $\mathbf{K_1}$, in the INDIC = 1 analysis is less than the nonlinear bifurcation buckling pressure in the INDIC = 1 analysis.

8.2.5 Predictions of extreme fiber stress

Figures 86 - 93 show the distributions of effective stress in the inner and outer fiber in the shell skin (layer number 2 in the STAGS model) generated for the axisymmetrically imperfect shells with a +mode 1 imperfection (Figs. 86 and 87), a +mode 2 imperfection (Figs. 88 and 89), a –mode 1 imperfection (Figs. 90 and 91), and a –mode 2 imperfection (Figs. 92 and 93). Compare the +mode 1 STAGS predictions with those from GENOPT (BIGBOSOR4) listed in the first part of Table 61. Compare the +mode 2 STAGS predictions with those from GENOPT (BIGBOSOR4) listed in the second part of Table 61. Compare the –mode 1 STAGS predictions with those from GENOPT (BIGBOSOR4) listed in the third part of Table 61. Compare the –mode 2 STAGS predictions with those from GENOPT (BIGBOSOR4) listed in the fourth part of Table 61.

The most critical stresses from the BIGBOSOR4, STAGS, and BOSOR5 models for all four axisymmetric imperfection shapes (+mode 1, +mode 2, –mode 1, and –mode 2, are listed in the column in Table 47 headed, **"unstiffened, imperfect"**.

8.2.6 The inadequacy of the optimized unstiffened axisymmetrically imperfect shell

Figure 94 (a very important figure!) displays load-apex-deflection curves for axisymmetric –mode 1, –mode 2, +mode 3, and –mode 3 imperfection shapes and non-axisymmetric imperfection shapes corresponding to the linear bifurcation buckling modes shown in Fig. 77 (n=1 circumferential wave), Fig. 79 (n=2 circumferential waves), and Fig. 81 (n=3 circumferential waves). The traces 1, 2, 3, and 4 in Fig. 94 are the same as Traces 1, 5, 3, and 7 in Fig. 83. Traces 5, 6, and 9 in Fig. 94 correspond to load-apex-deflection curves corresponding to the presence of non-axisymmetric imperfections of the shapes displayed in Figs. 77, 79, and 81, respectively. The amplitude of each of the non-axisymmetric imperfections is the same as that of the axisymmetric imperfections: Wimp = 0.2 inch. **Note that the sensitivities of the maximum pressure-carrying capability to NON-AXISYMMETRIC imperfections are much greater than those to axisymmetric imperfections, especially greater than those corresponding to the mode 1 and mode 2 axisymmetric imperfections in the presence of which the shell was designed. The non-axisymmetric buckling modal imperfection with n=1 circumferential wave is the most harmful imperfection , given its amplitude, Wimp = 0.2 inch.**

8.2.7 Important Note!

The most significant results in Fig. 94 are the extremely low maximum pressure-carrying capabilities of the **non-axisymmetrically imperfect unstiffened shells** optimized with the use of only the two axisymmetric linear buckling modal imperfection shapes, mode 1 and mode 2. The lowest collapse load of a non-axisymmetrically imperfect shell is at an external pressure of about 215 psi (trace with Xs), less than half the specified design pressure, p = 460 psi. **The non-axisymmetric imperfections are far more harmful than the axisymmetric mode 1 and mode 2 imperfections, indicating that the optimized unstiffened shell is probably an impractical design. A small dent anywhere on the surface of the shell except in the immediate neighborhood of the pole will probably cause collapse at a pressure far below the design pressure, p = 460 psi.**

What appears to be occurring here is that the optimization "tailors" the axisymmetric mode shapes, mode 1 and mode 2, so that these modes exhibit relatively small deflections away from the pole, where the contribution of wall thickness to the total weight of the shell is most significant. This characteristic of the axisymmetric linear buckling modal imperfection shapes of the optimized unstiffened shell can be seen in Figs. 74 and 75.

The local nature of the axisymmetric buckling modal imperfection shapes displayed in Figs. 74 and 75 is a consequence of a certain peculiarity of the meridional distribution of shell wall thickness that evolves during the optimization process. In the optimized shells there is a thick and rather local circumferential band centered at xinput(3)= 5.66645 inches (Table 28). This locally thickened circumferential band is exhibited in Table 33 (the column with the heading "**unstiffened, imperfect**"). The thick band acts very much as a ring stiffener that isolates the apex region from the rest of the shell. Essentially we have a shallow axisymmetrically imperfect spherical cap clamped at its edge and connected to an almost perfect remainder of the shell. If, during the fabrication and handling process, an off-center dent with depth, Wimp. approaching 0.2 inch, is somehow produced in the remainder of the shell, there is a high probability that under uniform external pressure the dented shell will collapse at a pressure significantly below the design pressure, p = 460 psi.

The following questions arise:

1. Would the axisymmetric buckling modal imperfection shapes depicted in Figs. 74 and 75 be typical of imperfections in actual fabricated optimized unstiffened shells?

2. Is it safe to use only the axisymmetric mode 1 and mode 2 linear buckling modal imperfection shapes in designing and evaluating the unstiffened shell?

The answers to both questions, in the writer's opinion, is, "Almost certainly not."

In the writer's opinion the method as described up to this point to obtain optimum designs of UNSTIFFENED ellipsoidal shells is faulty. This weakness in the approach does not seem to exist in the case of isogrid-stiffened shells, as is seen from Fig. 17. Hence, the method is most likely a reasonable one for that class of shells. The method as described so far probably will not work well if a shell is only weakly stiffened.

HOWEVER, THERE IS A SIMPLE "FIX". Since the inadequacy of the optimum design of

the unstiffened imperfect shell is related to the shapes of the axisymmetric buckling modal imperfections displayed in Figs. 74 and 75 (significant local deviation from perfect only in the immediate neighborhood of the apex of the shell), one should obtain optimum designs of unstiffened shells in which the apex region is forced to remain thick enough during optimization cycles so that the maximum axisymmetric buckling modal deflection will not occur there. This has been done, and the results are presented in Section 9.0 in Figs. 143 – 253 and Tables 77 – 95.

Figures 95 and 96 show the post-collapse deformation of the optimized unstiffened shell with the n=1 non-axisymmetric imperfection (Fig. 95) and for the shell with the n=2 non-axisymmetric imperfection (Fig. 96).

8.2.8 Optimization with the use of plus and minus modes 1 – 4

Table 62 lists the GENOPT input data for the BEGIN processor (the eqellipse.BEG file). The input data for the DECIDE processor are the same as that listed in Table 57. The input for the MAINSETUP/OPTIMIZE processors appears in Table 63.

Table 64 lists the most negative margins for Load Set 3 (+mode 3 and +mode 4 axisymmetric buckling modal imperfection shapes) and for Load Set 4 (–mode 3 and –mode 4 axisymmetric buckling modal imperfection shapes). The unstiffened shell is the same as the optimized unstiffened shell obtained with use of only the plus and minus mode 1 and mode 2 imperfection shapes, that is, the design listed in Table 33 under the heading, "**unstiffened, imperfect**". If one is to obtain a FEASIBLE or ALMOST FEASIBLE design, one must re-optimize in the presence of the four most critical axisymmetric buckling modal imperfection shapes, modes 1 – 4.

Figure 97 shows the evolution of the objective during one execution of SUPEROPT. This computer run required about four days on the efficient LINUX workstation where this work was done.

Table 65 lists the new "4-mode" optimum design and the associated design margins. Note that the local thick circumferential band, THKSKN(3), is still present, indicating that the axisymmetric buckling modes will still resemble those displayed in Figs. 74 and 75 and that therefore the collapse pressure of the optimized shell will still be extremely sensitive to non-axisymmetric buckling modal imperfection shapes.

Figures 98 – 101 display the axisymmetric linear buckling modes 1 – 4 predicted by BIGBOSOR4 for the re-optimized unstiffened equivalent ellipsoidal shell. Indeed, these imperfection shapes are localized primarily in the neighborhood of the apex of the shell.

Tables 66a and 66b list abridged and edited output from the STAGS file, eqellipse.out2,
for the re-optimized unstiffened shell (INDIC=1 STAGS run). Figures 102 – 108 show axisymmetric and non-axisymmetric linear bifurcation buckling modes. Axisymmetric buckling modes (predicted by STAGS) that are analogous to the GENOPT modes 1 – 4 (predicted by BIGBOSOR4) are shown in Figs. 102, 105, 107, and 108, respectively. These four axisymmetric

STAGS modes are analogous to the BIGBOSOR4 modes displayed in Figs. 98 – 101, respectively.

Figure 109, displaying load-apex-deflection curves for the re-optimized unstiffened shell, is analogous to Figs. 83 and 94 for the shell optimized with use of only plus and minus axisymmetric modes 1 and 2. Two points must be emphasized with regard to Fig. 109:

1. The discrepancy between BIGBOSOR4 and STAGS predictions is greater than is exhibited for the "2-mode" optimized shell in Fig. 83, and

2. The maximum pressure-carrying capability of the new "4-mode" optimum design is unacceptably low and is just as sensitive to non-axisymmetric linear buckling modal imperfections as is the "2-mode" optimum design (Fig. 94).

Figure 110 shows the nonlinear bifurcation buckling mode shape corresponding to the critical (lowest) eigenvalue for the re-optimized unstiffened shell with a +mode 1 axisymmetric linear buckling modal imperfection. This non-axisymmetric (n=9 circumferential waves) imperfection shape is used as a "trigger" [very small amplitude, Wimp(n=9) = 0.001 inch] to produce the load-apex-deflection curve corresponding to the third trace in Fig. 109 (right-side-up triangles).

Figure 111 explains why the BIGBOSOR4 and STAGS predictions for axisymmetric collapse in Fig. 109 disagree with each other: the linear bifurcation buckling mode shapes from BIGBOSOR4 and STAGS differ significantly, at least that is so for the mode 2 axisymmetric mode shape. As demonstrated in the next three figures, the discrepancy seems to be caused by the fact that the axisymmetric linear buckling mode 2 shown in Fig. 105 has not converged with respect to mesh density in the STAGS model.

Figure 112 gives a comparison between the BIGBOSOR4 prediction and STAGS predictions for the axisymmetric mode 2 imperfection shape. The first two traces in Fig. 112 are the same as those in Fig. 111. The fourth trace in Fig. 112 corresponds to the mode shape shown in Fig. 113 and the third trace in Fig. 112 corresponds to the mode shape shown in Fig. 114. In the writer's experience this mode shape convergence behavior is unusual. The axisymmetric (360-degree) STAGS model shown in Fig. 105 should be refined enough to reproduce the prediction by BIGBOSOR4 depicted in Fig. 112, as it has about eight 410 finite elements per half-wave of the axisymmetric buckling pattern. The refined 10-degree STAGS model with the 480 finite element (Fig. 114) matches the BIGBOSOR4 prediction much better than does that obtained with use of the 410 finite element (Fig. 113). Unfortunately, nonlinear (INDIC = 3) STAGS runs based on the 480 finite element fail to converge at pressures well below the design pressure, possibly because of the writer's failure to suppress the "drilling" degrees of freedom in the STAGS model and possibly because of the greatly elongated shape of the finite elements in the immediate neighborhood of the pole, as shown in Figs. 19 and a1, for example.

8.3 Details pertaining to the isogrid-stiffened, "perfect" equivalent ellipsoidal shell

Tables 67 – 71 and Figs. 115 – 128 pertain to the **isogrid-stiffened, "perfect"** equivalent ellipsoidal shell. "Perfect" is in quotes because in fact a very small axisymmetric initial imperfection amplitude is used: Wimp = 0.0001 inch in the eqellperf.BEG file. (For the perfect shells the user has chosen the specific case name, "eqellperf" instead of "eqellipse").

8.3.1 Input data

Table 67 lists the GENOPT input data for the BEGIN processor (the eqellperf.BEG file). The input data for the DECIDE processor are listed in Table 68. The input for the MAINSETUP/OPTIMIZE processors appears in Table 69. In this case there is only a single load set, that corresponding to +mode 1 and +mode 2 axisymmetric imperfection shapes. Again, the amplitude of the initial imperfection is very small, Wimp = 0.0001 inch. There is no need to process a second load set, that with –mode 1 and –mode 2, because the design margins would have essentially the same values as those for the first load set because the amplitude of the imperfection is so small.

8.3.2 Optimization

Figures 115 and 116 show the objective vs design iterations for two executions of SUPEROPT. Each "spike" in each plot corresponds to a new starting design, obtained randomly as described in [15] and on the second page of Table 34. The presence of the three "dense", "quiet" regions starting approximately after Iteration Numbers 150, 300, and 450, is explained in sub-section 8.1.2. The optimum design is listed in Table 33 under the heading, "**isogrid-stiffened, perfect**".

8.3.3 Design margins

Table 70 lists the design margins for the optimized isogrid-stiffened "perfect" equivalent ellipsoidal shell. Note that Margins 12 – 22 are essentially the same as Margins 1 – 11 because there are only very, very small-amplitude mode 1 and mode 2 initial axisymmetric buckling modal imperfections: Wimp = 0.0001 inch. The critical margins are CLAPSi (axisymmetric collapse corresponding to the mode i imperfection shape), STFBKi(1,1) (local isogrid stiffener buckling in Region 1 corresponding to the mode i imperfection shape), SKNSTi(1,2) (skin effective stress in Region 2 corresponding to the mode i imperfection shape), and STFSTi(1,1) and STFSTi(1,2) (isogrid stiffener meridional stress in Regions 1 and 2 corresponding to the mode i imperfection shape). Almost critical are GENBKi (general nonlinear bifurcation buckling corresponding to the mode i imperfection shape) and SKNSTi(1,1) (skin effective stress in Region 1 corresponding to the mode i imperfection shape).

8.3.4 Some predictions from BIGBOSOR4 (GENOPT), BOSOR5, and STAGS

Figures 117 and 118 show the axisymmetric linear bifurcation buckling modes 1 and 2, respectively, of the optimized isogrid-stiffened "perfect" equivalent ellipsoidal shell as predicted by BIGBOSOR4. The shape of the mode 1 buckle is similar to that of the mode 1 buckle of the optimized isogrid-stiffened imperfect shell depicted in Fig. 4. In contrast the shape of the mode 2 buckle does not at all resemble that of the mode 2 buckle of the optimized isogrid-stiffened imperfect shell displayed in Fig. 5.

The linear bifurcation buckling modes from STAGS for the optimized isogrid-stiffened “perfect” shell are displayed in Figs. 119 – 121. This is the only case in which the fundamental (lowest) eigenvalue corresponds to a non-axisymmetric mode (Fig. 119). Figures 120 and 121 show the axisymmetric mode 1 and mode 2 linear bifurcation buckling shapes according to STAGS. These two axisymmetric buckling mode shapes are analogous to those predicted by BIGBOSOR4 and displayed in Figs.117 and 118.

Figure 122 shows the BIGBOSOR4-predicted axisymmetrically deformed profile of the optimized isogrid-stiffened “perfect” equivalent ellipsoidal shell loaded by the external design pressure, p = 460 psi. This axisymmetric deformation gives rise to the effective stress in the skin listed in Table 71.

Figure 123, which is analogous to Fig. 84, displays load-apex-deflection curves predicted by GENOPT (BIGBOSOR4, elastic material), BOSOR5 (elastic-plastic material), and STAGS (elastic material) for the optimized isogrid-stiffened “perfect” equivalent ellipsoidal shell. There is good agreement between the three models. The second-to-last trace in Fig. 123 (diamonds with internal cross) is the load-apex-deflection curve for the isogrid-stiffened “perfect” shell with the non-axisymmetric nonlinear bifurcation buckling modal imperfection shown in Fig. 124 with a very small amplitude, Wimp(n=3) = 0.001 inch (non-axisymmetric “trigger” with n=3 circumferential waves).

The maximum pressure-carrying capability of “perfect” shell with a tiny non-axisymmetric n=3 nonlinear buckling modal imperfection is slightly below the nonlinear bifurcation buckling pressure predicted by both STAGS and by BIGBOSOR4 with INDIC = -2 type of analysis. The critical pressures predicted by BIGBOSOR4 from the INDIC = 1 and INDIC = -2 types of analysis differ because the INDIC = 1 analysis is governed by Eq.(5) whereas the INDIC = -2 analysis includes a changing stiffness matrix, $\mathbf{K_1}$, as the applied external pressure is increased in steps. With externally pressurized ellipsoidal shells that are “flatter” than spherical the stiffness matrix, $\mathbf{K_1}$, “softens” as the pressure is increased. Therefore, nonlinear bifurcation buckling loads predicted with INDIC = -2 will always be lower than those predicted with INDIC = 1 as long as the pressure used for computation of the stiffness matrix, $\mathbf{K_1}$, corresponding to the INDIC = 1 analysis is less than the nonlinear bifurcation buckling pressure.

8.3.5 Predictions of extreme fiber stress

Figures 125 - 128 show the distributions of inner fiber meridional stress in the isogrid “layer” (layer no. 1 in the STAGS model, Figs. 125 and 126) and effective stress in the inner and outer fiber in the shell skin (layer number 2 in the STAGS model, Figs. 127 and 128) generated for the “perfect” optimized isogrid-stiffened shell. Compare the STAGS predictions with those from BIGBOSOR4 listed in Table 71.

The most critical stresses from the BIGBOSOR4, STAGS, and BOSOR5 are listed in the two columns in Table 47 headed, **“isogrid-stiffened, perfect”**.

8.4 Details pertaining to the unstiffened, “perfect” equivalent ellipsoidal shell

Tables 72 – 76 and Figs. 129 – 142 pertain to the **unstiffened, “perfect”** equivalent ellipsoidal shell. “Perfect” is in quotes because in fact a very small axisymmetric initial imperfection amplitude is used: Wimp = 0.0001 inch.

8.4.1 Input data

Table 72 lists the GENOPT input data for the BEGIN processor (the eqellperf.BEG file). The input data for the DECIDE processor are listed in Table 73. The input for the MAINSETUP/OPTIMIZE processors appears in Table 74. As with the previous case, the isogrid-stiffened “perfect” shell, there is only a single load set.

8.4.2 Optimization

Figure 129 shows the objective vs design iterations for one execution of SUPEROPT. Each “spike” in each plot corresponds to a new starting design, obtained randomly as described in [15] and in the footnote of Table 34. The optimum design is listed in Table 33 under the heading, “**unstiffened, perfect**”.

8.4.3 Design margins

Table 75 lists the design margins for the **optimized unstiffened “perfect” equivalent ellipsoidal shell.** Note that Margins 12 – 22 are essentially the same as Margins 1 – 11 because there are only very, very small-amplitude mode 1 and mode 2 initial axisymmetric buckling modal imperfections: Wimp = 0.0001 inch. The critical margins are CLAPSi (axisymmetric collapse corresponding to the mode i imperfection shape), GENBKi (general nonlinear bifurcation buckling corresponding to the mode i imperfection shape), and SKNSTi(1,2) (skin effective stress in Region 2 corresponding to the mode i imperfection shape). The CLAPSi and GENBKi margins are almost the lowest they are allowed to be for an “ALMOST FEASIBLE” design to be accepted by GENOPT. The minimum allowable margin for an “ALMOST FEASIBLE” optimum design is –0.05. The negative values of the CLAPSi and GENBKi margins are probably the reason that in the three “dense” or “quiet” regions in Fig. 129, starting approximately at Iteration no. 150, Iteration no. 305, and Iteration no. 455, the objective increases rather than holding fairly constant, as is the case for the isogrid-stiffened “perfect” shell analogous results for which are shown in Figs. 115 and 116.

8.4.4 Some predictions from BIGBOSOR4 (GENOPT), BOSOR5, and STAGS

Figures 130 and 131 show the axisymmetric linear bifurcation buckling modes 1 and 2, respectively, of the optimized unstiffened “perfect” equivalent ellipsoidal shell as predicted by

BIGBOSOR4. The shapes of the mode 1 and mode 2 axisymmetric buckling modes are very different from those of the mode 1 and mode 2 axisymmetric buckling modes of the optimized unstiffened imperfect shell depicted in Figs 74 and 75.

The linear bifurcation buckling modes from STAGS for the optimized unstiffened "perfect" shell are displayed in Figs. 132 – 137. Figures 132 and 137 show the axisymmetric mode 1 and mode 2 linear bifurcation buckling shapes according to STAGS. These modes are analogous to the axisymmetric buckling modes predicted by BIGBOSOR4 and shown in Figs. 130 and 131, respectively.

Figure 138 shows the axisymmetrically deformed profile of the optimized unstiffened "perfect" equivalent ellipsoidal shell loaded by the external design pressure, p = 460 psi. This axisymmetric deformation gives rise to the effective stress in the skin listed in Table 76.

Figure 139, analogous to Figs. 84 and 123, displays load-apex-deflection curves predicted by GENOPT (BIGBOSOR4), BOSOR5(elastic-plastic material), and STAGS for the optimized unstiffened "perfect" equivalent ellipsoidal shell. There is good agreement between the three models. The second-to-last trace in Fig. 139 (upside-down triangles) is the load-apex-deflection curve for the unstiffened "perfect" shell with the non-axisymmetric nonlinear bifurcation buckling modal imperfection "trigger" shown in Fig. 140 with a very small amplitude, Wimp(n=3) = 0.001 inch.

The maximum pressure-carrying capability of "perfect" shell with a tiny non-axisymmetric nonlinear buckling modal imperfection with n=3 circumferential waves (Fig. 140) is slightly below the nonlinear bifurcation buckling pressure predicted by BIGBOSOR4 and 4.35 per cent below the design pressure. (See Margin No. 2 in Table 75, the margin for nonlinear general buckling, GENBK1).

8.4.5 Predictions of extreme fiber stress

Figures 141 and 142 show the distributions of effective stress in the inner and outer fiber in the shell skin (layer number 2 in the STAGS model) generated for the "perfect" optimized unstiffened shell. Compare the STAGS predictions with those from BIGBOSOR4 listed in Table 76.

The most critical stresses from the BIGBOSOR4, STAGS, and BOSOR5 are listed in the column in Table 47 headed, **"unstiffened, perfect"**.

9.0 OPTIMIZATION AND ANALYSIS OF IMPERFECT UNSTIFFENED EQUIVALENT ELLIPSOIDAL SHELLS WITH A THICK APEX

In the previous section we learned that the optimum design of an axisymmetrically imperfect **unstiffened** equivalent ellipsoidal shell would fail at an external pressure far lower than the design pressure, p = 460 psi, if the shell happened to have, instead of the axisymmetric linear buckling modal imperfections in the presence of which it was optimized, a **non-axisymmetric** linear buckling modal imperfection of approximately the same amplitude, Wimp = 0.2 inch (Fig. 94). The worst non-axisymmetric imperfection has n=1 circumferential waves (trace 5, the trace with the X symbol, in Fig. 94).

In the presence of only **axisymmetric** linear buckling modal imperfections and if the wall thickness in the neighborhood of the shell apex is given a **low lower bound**, the meridional thickness distribution of the optimized shell evolves during optimization cycles so that the axisymmetric linear buckling modes (the imperfection shapes) have profiles such as those displayed in Figs. 74 and 75 for the unstiffened shell optimized with plus and minus mode 1 and mode 2 axisymmetric imperfection shapes and in Figs. 98 – 101 for the unstiffened shell optimized with plus and minus mode 1, mode 2, mode 3, and mode 4 axisymmetric imperfection shapes. **Note that with these axisymmetric buckling modal imperfection shapes there is significant deviation of the profile of the imperfect shell from that of the perfect shell only in a rather small neighborhood of the apex of the shell. This property of the optimized, axisymmetrically imperfect shell causes the collapse pressure of the optimized shell to be especially sensitive to NON-AXISYMMETRIC linear buckling modal imperfections of the type displayed in Fig. 77 (n=1 circumferential wave).**

The local nature of the axisymmetric buckling modal imperfection shapes displayed in Figs. 74 and 75 and in Figs. 98 – 101 is a consequence of a certain peculiarity of the meridional distribution of shell wall thickness that evolves during the optimization process. In the optimized shells there is a thick and rather local circumferential band centered at xinput(3)= 5.66645 inches (Table 28). This locally thickened circumferential band is exhibited both in Table 33 (the column with the heading "**unstiffened, imperfect**": the "2-mode" optimum design) and on the first page of Table 65 (the "4-mode" optimum design). **The thick band acts very much as a ring stiffener that isolates the apex region from the rest of the shell.** Essentially we have a shallow axisymmetrically imperfect spherical cap clamped at its edge and connected to a fairly perfect remainder of the shell. If, during the fabrication and handling process, an off-center dent is somehow produced in the remainder of the shell, there is a high probability that under uniform external pressure the dented shell will collapse at a pressure significantly below the design pressure, p = 460 psi, especially if the off-center dent has a depth approaching Wimp = 0.2 inch.

The purpose of this section is to demonstrate a formulation of the optimization problem by means of which the optimum design of the unstiffened imperfect shell will survive the design pressure of 460 psi even if there exist non-axisymmetric linear buckling modal imperfections or non-axisymmetric imperfections in the form of off-center residual dents generated either by a single normal inward-directed concentrated load or by a number of normal inward-directed concentrated loads applied along a circumferential line and distributed over half of the circumference of the shell as cos(theta), where theta is the circumferential coordinate.

Tables 77 – 95 and Figs. 143 – 253 pertain to this section. By "**thick apex**", a phrase used in the heading of this section, is meant the uniform thickness, **t(apex)**, of the spherical cap, that is, the uniform thickness of Shell Segment No. 1 in the BIGBOSOR4 model shown in Fig. 2. Optimization is performed with use of the starting design listed in Table 56 and with the 12 decision variables listed in Table 77: THKSKN(1) and THKSKN(3), THKSKN(4),…, THKSKN(13). The thickness at the junction between Shell Segment 1 and Shell Segment 2, THKSKN(2), is linked to the thickness at the apex of the shell, THKSKN(1), with a linking constant equal to unity, that is, THKSKN(2) = THKSKN(1). In other words, the thickness of the spherical cap (Shell Segment 1 in Fig. 2) is uniform and equal to **t(apex)**. See Table 77, which lists the input file, *.DEC, for the "DECIDE" processor where the linking relationship, THKSKN(2) = THKSKN(1), is set up by the user.

This section has the following three sub-sections:

1. **sub-section 9.1:** optimization and analysis of the "**thick-apex**" shell with the lower bound of **t(apex)** = THKSKN(1) set equal to **0.4 inch** and the amplitude of the axisymmetric linear buckling modal imperfection, **Wimp = 0.2 inch** (Figs. 143 – 200, Tables 77 – 88),

2. **sub-section 9.2:** optimization and analysis of the "**thick-apex**" shell with the lower bound of **t(apex)** = THKSKN(1) set equal to **0.4 inch** and the amplitude of the axisymmetric linear buckling modal imperfection, **Wimp = 0.1 inch** (Figs. 201 – 225, Tables 89 – 91), and

3. **sub-section 9.3:** optimization and analysis of the "**thick-apex**" shell with the lower bound of **t(apex)** = THKSKN(1) set equal to **0.6 inch** and the amplitude of the axisymmetric linear buckling modal imperfection, **Wimp = 0.2 inch** (Figs. 226 – 253, Tables 92 – 95).

In each of these three sub-sections the STAGS computer program [20 – 24] is used to determined the **elastic-plastic** collapse loads of the optimized shells with axisymmetric and non-axisymmetric **linear buckling modal imperfection** shapes and with imperfection shapes in the form of **off-center residual dents** produced by an elastic-plastic load cycle in what in STAGS jargon is called "Load Set B". The external uniform pressure is applied in Load Set A. For shells with an imperfection in the form of an off-center residual dent the Load Set B cycle is first applied with zero external uniform pressure (zero loads in Load Set A). Then, with zero loads in Load Set B, the pressure-carrying capacity of the dented shell is determined by application of Load Set A, the uniform external pressure. By "**off-center residual dent**" is meant a dent the maximum depth of which occurs at some distance from the axis of revolution of the perfect shell.

The purpose of computing collapse pressures of shells with residual dents is to compare the harmfulness of a residual dent with the harmfulness of the "worst" (most harmful) linear buckling modal imperfection. In Fig. 94 it is demonstrated that the most harmful linear buckling modal imperfection is the non-axisymmetric buckling modal imperfection shape with n=1 circumferential wave.

Although the shell is termed "**unstiffened**", in this section (as in Section 8.2) the shell is modeled as if it has two layers: an isogrid "layer" and the shell skin. As in Section 8.2, the height of the isogrid is set equal to a very, very small value, HIGHST(i) = 0.000001 inch, and the thickness of each isogrid member is also set equal to a very, very small number, 0.00001 inch (Table 56). Therefore, the presence of the isogrid layer has no effect on the behavior of the shell. In STAGS models the isogrid layer is Layer No. 1 and the shell skin is Layer No. 2.

9.1 “Thick-apex” unstiffened shell with lower bound of t(apex) = 0.4 inch and Wimp = 0.2 inch

The purpose of this sub-section is to find and evaluate an optimum design that may or may not survive the external design pressure, p = 460 psi provided that the amplitude of any imperfection, **axisymmetric or non-axisymmetric**, does not exceed 0.2 inch.

Figures 143 – 200 and Tables 77 – 88 pertain to this sub-section. First, **in sub-section 9.1.1** a “**thick-apex**” optimum design is found by GENOPT via two executions of SUPEROPT (Tables 77 and 78 and Figs. 143 and 144). Next, **in sub-section 9.1.2** linear buckling modes by BIGBOSOR4 and STAGS are presented for the optimized design (Tables 79 and 80 and Figs. 145 – 151). Then, **in sub-section 9.1.3** extreme fiber effective stress distributions from BIGBOSOR4 and STAGS are presented for the optimized design (Table 81 and Figs. 152 – 160). Then, **in sub-section 9.1.4** collapse of the shells with axisymmetric mode 1 and mode 2 imperfection shapes are determined for the optimized design by BIGBOSOR4 for elastic material and by STAGS for either axisymmetric or non-axisymmetric linear buckling modal imperfections and for either elastic or elastic-plastic material (Figs 161 – 163). Finally, **in sub-section 9.1.5** a rather long series of STAGS models is used to determine the pressure-carrying capabilities of the optimized shell with **residual dents** of various depths, one residual dent for each collapse analysis (Tables 82 – 88 and Figs. 164 – 200). In this final long sub-section the STAGS 180-degree “soccerball” model of the optimized equivalent ellipsoidal shell is introduced (Fig. 169, Figs. a2 – a13). Each residual dent is produced by application of a load cycle that is included in the *.inp and *.bin files as what is called in STAGS jargon, “Load Set B”. The external uniform pressure is subsequently introduced in Load Set A. This external pressure is applied to each of the dented shells that exist after completion of a Load Set B load cycle, that is, following the load cycle in which a residual dent is generated.

9.1.1 Optimization

The optimization was performed with the eqellipse.BEG file (input for the “BEGIN” processor) the same as that listed in Table 56. The input file, eqellipse.DEC, for the “DECIDE” processor is listed in Table 77. This new input file differs in two ways from the original eqellipse.DEC file listed in Table 57:

1. The lower bound of **t(apex)**, the wall thickness at the apex of the shell, THKSKN(1), is set equal **to 0.4 inch rather than 0.1 inch** as it is in Table 57.

2. The thickness, THKSKN(2) at the junction between Shell Segment 1 and Shell Segment 2 (Fig. 2) is linked to the thickness at the apex, THKSKN(1). Therefore, during optimization cycles the **thickness of the spherical cap (Shell Segment 1 in Fig 2) will remain uniform.**

By these two simple changes in the formulation of the optimization problem we hope to obtain an optimum design that will not exhibit the extreme sensitivity to non-axisymmetric buckling modal imperfections that is exhibited in Fig. 94 for the optimized design obtained with use of the original input file for the “DECIDE” processor listed in Table 57.

Figures 143 and 144 show the evolution of the objective during the two executions of SUPEROPT used to obtain the new “**thick-apex**” optimum design. The optimized shell with the **thick apex** is significantly heavier than that listed for the unstiffened imperfect shell in Table 33. Table 78 lists the new optimum design and the design margins. Critical margins are listed in bold face. Compare the value of the objective, weight = 127.1 lb, with the objective listed in Table 33 for the “**unstiffened, imperfect**” shell: weight = 96.461 lb.

Note that the evolution of the objective during optimization cycles exhibited in Figs. 143 and 144 is much less “jumpy” than that exhibited in Figs. 69 – 72, which pertain to the evolution of the objective leading to the previously obtained optimum design listed in Table 33 under the heading, “unstiffened, imperfect”. The reason for this difference is not known, but probably has something to do with the sensitivity of axisymmetric mode 1 and mode 2 imperfection shapes in the neighborhood of the apex of the shell to changes in the design from iteration to iteration during the optimization process. With the increased lower bound of the thickness of the shell wall in the neighborhood of the apex (Table 77), there is most likely a decreased sensitivity of imperfection shape in the apex region to design changes during optimization cycles.

The “**thick apex**” shell is much heavier than the optimized unstiffened shell listed in Table 33 because the mode 1 and mode 2 axisymmetric buckling modal imperfection shapes for the optimized “**thick apex**” shell (Figs. 145 and 146) exhibit much greater amplitude in the region away from the neighborhood of the apex of the shell (the shell remainder) than is exhibited in Figs. 74 and 75. This much greater imperfection amplitude in the shell remainder gives rise to the need for greater thickness in the shell remainder in order to avoid unacceptably high extreme fiber stresses there and an unacceptably low overall collapse pressure than would result from mode 1 and mode 2 imperfection shapes of the type shown in Figs. 74 and 75. An increase of the wall thickness in the shell remainder leads to an increase in shell weight that is much more significant than an increase of the wall thickness only in the neighborhood of the shell apex.

Notice from the first part of Table 78 that there still exists a locally thick circumferential band, THKSKN(3) = 0.60418 inch, in the optimized unstiffened shell. Therefore, the apex region of this optimized shell is still somewhat isolated from the remainder, although much less so than for the previously obtained optimum design of the unstiffened imperfect shell listed in Table 33. The presence of this thick band makes the pressure-carrying capacity of this optimized shell significantly more sensitive to initial **non-axisymmetric** imperfections than to the axisymmetric mode 1 and mode 2 imperfections under which it was designed. Therefore, the optimized shell is still under-designed, as we shall see (Fig. 161), but much less so than the previously obtained optimized unstiffened imperfect shell developed with use of the much smaller lower bound of thickness in the neighborhood of the apex. (Compare Table 57 with Table 77 and Fig. 161 with Fig. 94).

The rest of Section 9.1 is devoted to a study of the optimum design listed in Table 78.

9.1.2 Linear buckling from BIGBOSOR4 and from STAGS

Figures 145 – 151 and Tables 79 and 80 pertain to this sub-section. Figures 145 and 146 display the axisymmetric linear buckling modes 1 and 2 predicted by BIGBOSOR4 for the optimized design. Compare these axisymmetic buckling modes with those shown in Figs. 74 and 75. Note that for this optimum design the maximum normal deflection in both axisymmetric linear buckling modes 1 and 2 still occurs at the apex of the unstiffened shell. **However, there is much more buckling modal deformation away from the apex than is exhibited in Figs. 74 and 75.** When each of the two axisymmetric buckling modes is used as an imperfection shape, the remainder of the shell is now far from perfect.

Table 79 lists the file, WALLTHICK.STAGS, "called" by the user-written STAGS subroutines, wall.F and usrfab.F. (See Tables 40 and a20 – a22 and a34 – a36).

Table 80 lists an abridged and edited version of the STAGS output file, eqellipse.out2, corresponding to a linear bifurcation buckling analysis (INDIC = 1 in the eqellipse.bin file). Figures 147 – 151 show the linear bifurcation buckling modes predicted by STAGS for the new optimized design listed in Table 78. The most harmful linear buckling modal imperfection shape is that displayed in Fig. 148, the buckling mode with n=1 circumferential wave. (See the fourth from last and third from last traces in Fig. 161). It is this n=1 imperfection shape that will be simulated by the creation of off-center residual dents as described in Sub-section 9.1.5.

9.1.3 Extreme fiber distributions of effective stress in the shell skin

Table 81 and Figs. 152 – 160 pertain to this sub-section. Compare Table 81 with Table 61 and Figs. 153 – 160 with Figs. 86 – 93. Also, compare the STAGS predictions (Figs. 153 – 160) with those from BIGBOSOR4 (Table 81).

9.1.4 Collapse of the optimized shell with linear buckling modal imperfections

Figures 161 – 163 pertain to this sub-section. Compare Fig. 161 with Fig. 94. The optimized shell is still under-designed but much less so than the previously optimized unstiffened imperfect shell (Table 33) to which Fig. 94 applies. As stated in the caption of Fig. 163, the results shown in Fig. 161 compared to those shown in Fig. 94 suggest that one should set a higher lower bound for the thickness of the spherical cap, say 0.6 inch rather than 0.4 inch, and then re-optimize the unstiffened shell. The results of such a study are described in sub-section 9.3.

9.1.5 Collapse pressures of the optimized shell with various off-center residual dents

Tables 82 – 88 and Tables a20 – a22 and a34 – a37 and Figs. 164 – 200 and Figs. a1 – a13 pertain to this long sub-section. The STAGS computer program is used for all the analyses in this sub-section. Plasticity is included in all the STAGS models. Each off-center residual dent is produced by a Load Set B load cycle. Following completion of this Load Set B load cycle, Load Set A, which contains the uniform external pressure, is applied to the shell with a residual dent. Collapse pressures are determined for each shell with a residual dent.

This sub-section is subdivided into the following parts. **Sub-section 9.1.5.1** shows results from 360-degree STAGS models (Fig. a1). The residual dent is produced by a single concentrated load applied at a location that corresponds to where the non-axisymmetric n=1 linear buckling mode shown in Fig. 148 has a maximum inward normal displacement nearest the axis of revolution. The "single concentrated load" is produced by uniform normal inward-directed pressure applied over a single finite element. Tables 82 – 86 and Figs. 164 – 168 pertain to this sub-section. **Sub-section 9.1.5.2** introduces the STAGS 180-degree "soccerball" model of the optimized shell. Figure 169 and Figs. a2 – a13 pertain to this sub-section. **Sub-section 9.1.5.3** shows results obtained with use of the 180-degree "soccerball" model for the shell with a residual dent produced by a single concentrated load comprised of uniform normal inward-directed pressure applied over a single finite element. Figures 170 – 176 pertain to this sub-section. **Sub-section 9.1.5.4** shows results obtained with use of the 180-degree "soccerball" model for the shell with a residual dent produced by a vector of concentrated normal inward-directed loads or a vector of imposed normal inward directed displacements distributed as cos(theta) along a circumferential line from circumferential coordinate theta = 0 to 90 degrees at the radius from the axis of revolution indicated in Figs. 148 and 190 and 191. Tables 87 and 88 and Figs. 177 – 200 pertain to this sub-section.

9.1.5.1 360-degree STAGS model of the optimized shell with a residual dent produced by a single concentrated load (normal pressure applied to a single finite element)

The purpose of this sub-section is to determine the collapse load of the optimized shell with an "off-center" residual dent compared to the collapse load of the optimized shell with a non-axisymmetric linear buckling modal imperfection with n=1 circumferential wave (Fig. 148). The n=1 imperfection shape shown in Fig. 148 is the most harmful of the buckling modal imperfection shapes, as is demonstrated in Fig. 161. Therefore, it is assumed that a residual dent at the location indicated in Fig. 148 would be more harmful than a residual dent of the same amplitude at any other location in the shell.

Table 79, Tables 82 – 86, and Table a22 and Figs. 164 – 168, Figs. 175 and 176, and Fig. a1 pertain to this sub-section. The residual dent is produced by a single concentrated **load** applied as normal inward-directed pressure over a single finite element at the location indicated in Fig. 148. The dent-production phase of the nonlinear elastic-plastic STAGS analysis consists of a Load Set B (PB) load cycle.

9.1.5.1.1 Results obtained with the use of SUBROUTINE WALL

Table 82 lists the input file, eqellipse.inp, for the 360-degree STAGS model displayed in Fig. a1. This STAGS input file is valid for use with SUBROUTINE WALL (Table a22 with Table 79). The STAGS "GCP" input style is selected, that is, the index, NGCP = 1. The dent-producing normal inward-directed concentrated load in Load Set B is listed in Shell Unit 4: ISYS = 2; P, LT, LD, LI, LJ = -1.0, 5, 3, 1, 2 . For each of the 12 shell units the index, **IWALL, is set equal to zero**, which tells STAGS to use SUBROUTINE WALL rather than SUBROUTINE USRFAB. The input file, eqellipse.bin, is not listed here. It is similar to the *.bin file listed in

Table a41, which applies to dent production via applied normal displacement rather than via applied normal load. Different values for STLD(2), STEP(2), and FACM(2) are used for this case: applied normal **load**.

Figure 164 shows the state of the shell at the "top" of the Load Set B load cycle, that is, at the maximum value of the Load Set B load factor, PB = 72966.9. In this case (with the use of SUBROUTINE WALL) STAGS was unable to find converged nonlinear solutions for PB greater than 72966.9. As of this writing the reason for this anomaly is unknown. (As we shall see, the anomaly is eliminated when SUBROUTINE USRFAB is used instead of SUBROUTINE WALL).

Table 83 lists the input file, eqellipse.bin, valid for the unloading phase of the Load Set B load cycle, and Fig. 165 shows the state of the shell after the concentrated load in Load Set B has been removed. The **residual dent** has a depth of 0.1413 inch, not deep enough to represent an imperfection with amplitude, Wimp = 0.2 inch, the amplitude of the plus and minus axisymmetric mode 1 and mode 2 imperfections used during the GENOPT optimization process. Therefore, one cannot compare the harmfulness of the residual dent displayed in Fig. 165 relative to the harmfulness of a linear buckling modal imperfection shape such as that displayed in Fig. 148.

The load-deflection curve corresponding to the Load Set B load cycle described in the previous two paragraphs is displayed as Trace No. 1 in Fig. 175. No collapse pressure for the shell with this residual dent of depth 0.1413 inch is computed because the residual dent is not deep enough.

9.1.5.1.2 Results obtained with the use of SUBROUTINE USRFAB

Table 84 lists in input file, eqellipse.inp, valid for use with SUBROUTINE USRFAB (Table a35). This file is the same as that listed in Table 82 (360-degree STAGS model displayed in Fig. a1) except that for each shell unit the index, **IWALL, is set equal to –1**, which tells STAGS to use SUBROUTINE USRFAB rather than SUBROUTINE WALL.

Figure 166 shows the state of the shell at the "top" of the Load Set B load cycle. This time, with the use of SUBROUTINE USRFAB rather than SUBROUTINE WALL, STAGS was able to obtain converged nonlinear equilibrium solutions for the entire range of load factor, PB, specified by the STAGS user (the writer). Compare the depth of the dent at PB = 90000.0 (depth = 1.510 inches) with the depth of the dent displayed in Fig. 164 (depth = 0.9573 inch).

Figure 167 shows the residual dent after completion of the Load Set B load cycle. Compare with Fig. 165. This time the residual dent of depth 0.2480 inch is deeper than the amplitude, Wimp = 0.2 inch, of the linear buckling modal imperfection in the presence of which the shell was optimized. Therefore, the harmfulness of such a dent relative to that of the "worst" linear buckling modal imperfection shape can be ascertained.

The load-deflection curve corresponding to the Load Set B load cycle described in the previous two paragraphs is displayed as Trace No. 2 in Fig. 175. Trace No. 3 in Fig. 175 pertains to a “one-layer” STAGS model of the shell (*.inp file not shown). Since the isogrid “layer” is tiny (Tables 56 and 79) the points in Trace No. 3 lie on top of those in Trace No. 2.

After completion of the Load Set B load cycle, Load Set A is applied to the shell with the residual dent shown in Fig. 167. Load Set A contains the uniform external pressure. The eqellipse.inp file remains the same, and the appropriate eqellipse.bin files for two consecutive nonlinear STAGS runs with non-zero Load Set A are listed in Tables 85 and 86. Please read the captions of these tables carefully. They identify the strategies chosen by the STAGS user to obtain the nonlinear solutions. Figure 168 shows the post-collapse state of the externally pressurized shell.

The pressure-deflection curve for the shell with the residual dent shown in Fig. 167 is the fifth trace in Fig. 176. The dented shell collapses at a pressure slightly below the design pressure, p = 460 psi. The linear buckling modal imperfection with n=1 circumferential wave of amplitude, Wimp = 0.2 inch (smaller than the amplitude, 0.248 inch, of the residual dent in Fig. 167) leads to a significantly lower collapse pressure, as demonstrated by the first four traces in Fig. 176. **The residual dent produced by a concentrated normal inward-directed load is not as harmful an imperfection as the linear buckling modal imperfection with n=1 circumferential wave.**

9.1.5.2 STAGS 180-degree “soccerball” model of the optimized shell

Figure 169 and Figs. a2 – a13 and Tables a32 and a36 – a39 pertain to this sub-section. The STAGS “soccerball” model avoids the singularity at the pole associated with the 360-degree STAGS model shown in Fig. a1. Also, there are no oddly shaped finite elements in the neighborhood of the pole, such as those displayed at the top of Fig. a1. Those oddly shaped elements in the 360-degree STAGS model prevent the productive use of the STAGS 480 finite element for this particular geometry, even for elastic material properties. (The writer prefers the STAGS 480 finite element because it seldom produces spurious behavior).

Figure 169 shows the STAGS 180-degree “soccerball” finite element model. The entire model contains 50 shell units (Table a37): six shell units for the “soccerball” spherical cap (Shell Segment 1 in the BIGBOSOR4 model shown in Fig. 2 and Shell unit 1 in the 360-degree STAGS model shown in Fig. a1), and four shell units for each of what are called “Shell Segments 2 –12” in Fig. 169 and in Fig. a2. In this report the term “Shell Segment” used in connection with STAGS models denotes one of the twelve shell segments displayed in Fig. 2 or in Fig. a1. In the 360-degree STAGS model shown in Fig. a1 “Shell Segment” and “Shell Unit” have the same meaning.

Figure 169 shows what is called the STAGS “refined” model (with 480 finite elements). Most of the results in this report are obtained with use of what is called here the STAGS “crude” model displayed in Figs. a2 and a3. The “crude” model has twice the nodal point spacing as the “refined” model. The differences in the “soccerball.inp” file between the “refined” and “crude” models are listed in Table a38. Whenever the STAGS “soccerball” model is used STAGS must be compiled with the subroutines listed in Tables a32, a36, and a39. Directions for re-compiling

STAGS in the presence of user-written (or user-modified) subroutines are given in Items 6 and 7 on page 2 of Table 40.

9.1.5.3 STAGS 180-degree "soccerball" model of the optimized shell with an off-center residual dent produced by a single concentrated load

Figures 170 – 176 pertain to this sub-section. Figures 170 – 174 display results from the "refined" model. The residual dent is centered at the same radius from the axis of revolution as is indicated in Fig. 148. Figure 170 shows the dented shell at the "top" of the Load Set B load cycle. Compare Fig. 170 with Fig. 166. Figure 171 shows the shell with the residual dent after completion of the Load Set B load cycle. This Load Set B load cycle is represented by the last trace in Fig. 175 (upside-down triangles). Figures 172 and 173 show, respectively, the outer fiber and inner fiber residual plastic meridional strain distributions in the unstiffened shell. Figure 174 shows the shell with the residual dent as loaded by uniform normal pressure (Load Set A) slightly above the design pressure, PA = 1.0. The corresponding pressure-deflection curve for the dented shell is the last trace in Fig. 176.

It is seen from Fig. 176 that the residual dents produced by a single concentrated load at the location indicated in Fig. 148 are significantly less harmful than a non-axisymmetric linear buckling modal imperfection with n=1 circumferential wave and with amplitude, Wimp = 0.2 inch.

9.1.5.4 STAGS 180-degree "soccerball" model of the optimized shell with a residual dent produced by a "cos(theta)" distribution of normal loads or imposed displacements

The purpose of this sub-section is to determine whether a residual dent with a "cos(theta)" shape from circumferential coordinate, theta = 0 to 90 degrees is more harmful than a residual dent generated from a single normal inward-directed concentrated load. The center of the residual dent is located as indicated in Fig. 148.

Tables 87 and 88 and Figs. 177 – 200 pertain to this sub-section. The STAGS 180-degree "soccerball" model is used to generate all the results presented in this sub-section.

Figures 177 – 179 show linear buckling modes obtained from the "refined" 180-degree "soccerball" model. Of special interest is the non-axisymmetric linear buckling mode with n=1 circumferential wave shown in Fig. 179. This is the imperfection shape that, given the amplitude, Wimp = 0.2 inch, is associated with the lowest collapse pressure, as is exhibited by the first four traces in Fig. 176 and by the fourth from last and the third from last traces in Fig. 161. The state of the collapsed shell with the negative of the linear buckling mode shape shown in Fig. 179 is similar to that displayed in Figs. 163 and 192.

The negative of the n=1 linear buckling mode shape shown in Fig. 179 has a "valley" that varies circumferentially as cos(theta) and the deepest point of which is located at theta = 0 degrees as indicated in Fig. 148. It is logical to predict that a residual dent with its deepest point at this location and produced by a vector of normal inward-directed concentrated loads or imposed normal inward-directed displacements that vary as cos(theta) from circumferential coordinate, theta = 0 to 90 degrees (Figs. 190 and 191) will be more harmful than a residual dent of the same depth and centered at the same location but with a shape produced by a single concentrated normal inward-directed load (traces 5 – 8 in Fig. 176). It is logical to predict this "extra" harmfulness because the "cos(theta)" residual dent more closely resembles the n=1 linear buckling modal imperfection shape **in the local region where the dented shell collapses** under uniform external pressure (Figs. 163, 192). The purpose of this sub-section is to confirm this prediction.

9.1.5.4.1 Residual dent produced by a vector of normal inward-directed concentrated LOADS that vary as cos(theta) from theta = 0 to 90 degrees along the circumference at the junction between Shell Segment 3 and Shell Segment 4 (Figs 2, 169, 190, 191)

Table 87 and Figs. 180 - 192 pertain to this sub-section.

Table 87 lists the series of STAGS runs (*.bin files and abridged *.out2 files) that are used to produce the Load Set B load cycles shown in Fig. 180 (STAGS Runs 1 – 8) and the Load Set A STAGS Runs 9 and 11 that correspond to the third trace in Fig. 188. For all STAGS runs the *.inp file is the same as that listed in Tables a37 except:

1. the STAGS 480 finite element is used for all the shell units,

2. the input data listed for Shell units 1 – 18 in Table a40 are used instead of the input data for Shell units 1 - 18 in Table a37, and

3. the input data for Load Set B (ISYS = 2) in Shell units 11 and 12 in Table a40 are modified: the index, LT, in the Q-3 records for Load Set B must be changed from LT = –1 to LT = 1 for the application of loads rather than the application of imposed displacements.

Figure 180 shows the results from three Load Set B load cycles. There are residual dents of three different depths at load Step 37, Step 45, and Step 50.

Figures 181 – 186 display the states of the shell subjected to Load Set B when the dent is loaded (Figs. 181, 183, 185) and at the ends of the three Load Set B load cycles when the shell is unloaded and the residual dents exist (Figs. 182, 184, 186). Note that the residual dents shown in Figs. 182, 184, and 186 more closely resemble **locally** the non-axisymmetric linear buckling modal n=1 imperfection shape in Figs. 179, 190, and 191 than does the residual dent generated by a single concentrated load (Fig. 171).

Figure 187 shows the outer fiber residual plastic meridional strain at the end of STAGS Run 8, that is, at Load Step 50 (Fig. 186).

Figure 188 shows collapse pressures of the optimized unstiffened shell with a non-axisymmetric linear buckling modal imperfection with n=1 circumferential wave and amplitude, Wimp = 0.2

inch (fourth trace), with a residual dent of depth 0.297 inch produced by a concentrated load (first trace), with a residual dent of depth 0.2343 inch produced by a "cos(theta)" vector of concentrated loads (third trace), and with a residual dent of depth 0.2043 inch produced by a "cos(theta)" vector of concentrated loads (second trace). **It is seen that, as we expected, the "cos(theta)" residual dent is significantly more harmful than the residual dent produced by a single concentrated load. In fact, from traces 2 and 4 we see that the "cos(theta)" residual dent is just as harmful as the n=1 linear buckling modal imperfection of essentially the same amplitude.**

Figure 189 shows the post-collapse state of the shell corresponding to the third trace in Fig. 188.

Figures 190 and 191 show the non-axisymmetric linear buckling modal imperfection shape with n=1 circumferential wave predicted by the STAGS "crude" 180-degree "soccerball" model. "Cos(theta)" residual dents were produced only at the location of the first "valley" (or "ridge") in the mode shape, as indicated. This first valley occurs close to the junction between Shell Segments 3 and 4 (Figs. 2, 169, 191). We could have performed additional STAGS nonlinear analyses corresponding to residual dents produced at ridges and valleys farther from the axis of revolution. We did not do so because we already know that this optimized unstiffened shell is somewhat under-designed.

In a later sub-section (Sub-section 9.3) collapse pressures are computed by STAGS for shells with residual dents centered at three different radii from the axis of revolution (one dent for each collapse analysis). This extra work is performed because the optimized shell described in Sub-section 9.3 comes acceptably close to surviving the design pressure, p = 460 psi, in the presence of any axisymmetric or non-axisymmetric imperfection the amplitude of which is at least 0.2 inch.

Figure 192 displays the mode of collapse of the shell with the non-axisymmetric n=1 linear buckling modal imperfection shown in Figs. 190 and 191. Compare with Fig. 163, which applies to the same optimized shell analyzed with use of the STAGS 360-degree finite element model.

9.1.5.4.2 Residual dent produced by a vector of normal inward-directed IMPOSED DISPLACEMENTS w that vary as cos(theta) from theta = 0 to 90 degrees along the circumference at the junction between Shell Segment 3 and Shell Segment 4 (Figs 2, 169, 190, 191)

Table 88 and Figs. 193 - 200 pertain to this sub-section.

Table 88 lists the series of STAGS runs (*.bin files and abridged *.out2 files) which are used to produce the Load Set B load cycles shown in Fig. 193 (STAGS Runs 1, 2, 5, and 6) and the Load Set A STAGS Runs 3 and 4 that correspond to the third trace in Fig. 200 and the Load Set A STAGS Runs 7 and 8 that correspond to the fourth trace in Fig. 200. For the **loading part of the Load Set B** load cycles (STAGS Runs 1 and 5) the *.inp file is the same as that listed in Table a37 except:

1. the STAGS 480 finite element is used for all the shell units,

2. the input data listed for Shell units 1 – 18 in Table a40 are used instead of the input data for Shell units 1 - 18 in Table a37.

For the **unloading part of the Load Set B** load cycles (STAGS Runs 2 and 6) and for the Load Set A STAGS runs (Runs 3 and 4 and 7 and 8) Load Set B in the *.inp file must be removed; the input data for Shell units 11 and 12 listed in Table a43 must be used instead of the corresponding input data for Shell units 11 and 12 listed in Table a40.

Figure 193 shows the Load Set B load cycles that produce residual dents of two different depths, 0.1777 inch and 0.2615 inch. Figures 194 – 197 show the dents from the Load Set B loaded (Figs. 194 and 196) and unloaded (Figs. 195 and 197) shells. Compare Fig. 197 with Fig. 186, in which the residual dent is produced by “cos(theta)” applied normal inward-directed loads rather than by “cos(theta)” imposed normal inward-directed displacements.

Figures 198 and 199 show, respectively, the residual inner fiber and outer fiber meridional plastic strain associated with the residual dent displayed in Fig. 197. Compare Fig. 199 with Fig. 187.

Figure 200, analogous to Fig. 188, shows collapse pressures of the optimized unstiffened shell with a non-axisymmetric linear buckling modal imperfection with n=1 circumferential wave and amplitude, Wimp = 0.2 inch (fifth trace), with a residual dent of depth 0.297 inch produced by a concentrated load (first trace), with a residual dent of depth 0.2343 inch produced by a “cos(theta)” vector of concentrated **loads** (second trace), with a residual dent of depth 0.2615 inch produced by a “cos(theta)” vector of **imposed normal displacements** (fourth trace), and with a residual dent of depth 0.1777 inch produced by a “cos(theta)” vector of imposed normal displacements (third trace). Traces 2 and 4 in Fig 200 exhibit the same collapse pressure. However, the amplitude, 0.2343 inch, of the residual dent produced by a “cos(theta)” vector of concentrated **loads** (second trace) is significantly less than the amplitude, 0.2615 inch, of the residual dent produced by a “cos(theta)” vector of **imposed normal displacements** (fourth trace). Therefore, the “cos(theta)” **applied load distribution** produces a more harmful residual dent than that produced by the “cos(theta)” **imposed normal displacements**, given an amplitude of the dent.

9.2 “Thick-apex” unstiffened shell with lower bound of t(apex) = 0.4 inch and Wimp = 0.1 inch

The purpose of this sub-section is to find and evaluate an optimum design that will most likely survive the external design pressure, p = 460 psi provided that the amplitude of any imperfection, **axisymmetric or non-axisymmetric**, does not exceed **Wimp = 0.1 inch**.

Figures 201 – 225 and Tables 89 – 91 pertain to this sub-section. First, **in sub-section 9.2.1** a “**thick-apex**” optimum design is found by GENOPT via one execution of SUPEROPT (Table 89 and Fig. 201). Next, **in sub-section 9.2.2** linear buckling modes by BIGBOSOR4 and STAGS are presented for the optimized design (Table 90 and Figs. 202 – 207). Then, **in sub-section**

9.2.3 extreme fiber effective stress distributions from BIGBOSOR4 are presented for the optimized design (Table 91 and Fig. 208). Then, **in sub-section 9.2.4** elastic-plastic collapse of the shells with either axisymmetric or non-axisymmetric (n=1 circumferential wave) linear buckling modal imperfection shapes are determined for the optimized design by STAGS (Figs 209 – 215). Finally, **in sub-section 9.2.5** elastic-plastic STAGS models are used to determine the pressure-carrying capabilities of the optimized shell with **residual dents** of various depths, one residual dent for each collapse analysis (Figs. 216 – 225). A summary conclusion is given **in sub-section 9.2.6**.

9.2.1 Optimization

The optimization was performed with the eqellipse.BEG file (input for the "BEGIN" processor) the same as that listed in Table 56 except the "amplitude of the axisymmetric imperfection: WIMP" is set equal to **0.1 inch** instead of 0.2 inch. The input file, eqellipse.DEC, for the "DECIDE" processor is the same as that listed in Table 77. (See sub-section 9.1.1 for a description of the formulation of the optimization problem).

Figure 201 shows the evolution of the objective during the one execution of SUPEROPT used to obtain the new optimum design. The optimized shell with the **thick apex** is heavier than that listed for the unstiffened imperfect shell in Table 33 and lighter than that listed in Table 78.

Table 89 lists the new optimum design and the design margins. Critical margins are listed in bold face. Compare the value of the objective, weight = 110.5 lb, with the objective listed in Table 33, weight = 96.461 lb, and the objective listed in Table 78, weight = 127.1 lb. Note from Table 89 that there is no locally thick circumferential band, THKSKN(3).

The rest of Section 9.2 is devoted to a study of the optimum design listed in Table 89.

9.2.2 Linear buckling from BIGBOSOR4 and from STAGS

Figures 202 - 207 pertain to this sub-section. Figures 202 and 203 display the axisymmetric linear buckling modes 1 and 2 predicted by BIGBOSOR4 for the optimized design. Compare these axisymmetic buckling modes with those shown in Figs. 74 and 75 and with those shown in Figs. 145 and 146. Note that for this optimum design, for which the amplitude of the axisymmetric buckling modal imperfection shapes, Wimp = 0.1 inch, the maximum normal deflection in both axisymmetric linear buckling modes 1 and 2 occurs away from the apex of the unstiffened shell.

Table 90 lists the file, WALLTHICK.STAGS, "called" by the user-written STAGS subroutine, usrfab.F. (See Tables 40 and a36). **Only SUBROUTINE USRFAB and only the STAGS 180-degree "soccerball" model are used to generate the results in sub-section 9.2 and in the remaining sections and sub-sections of this report. Previous experience during this study has demonstrated conclusively that this is the preferred STAGS model to be used for cases of the type explored in this report. The STAGS finite element E480 is the preferred choice.**

Figures 204 – 207 show the linear bifurcation buckling modes predicted by STAGS. The most harmful linear buckling modal imperfection shape is that displayed in Figs. 205 and 206, the buckling mode with n=1 circumferential wave. (See the fifth trace in Fig. 209 and the first trace in Fig. 211). Compare the STAGS non-axisymmetric n=1 buckling mode displayed in Figs. 205 and 206 with that shown in Figs. 190 and 191. Residual dents are produced by a "cos(theta)" distribution of concentrated **loads** applied along the circumference at Row No. 2 of Shell Segment No. 5 (sub-section 9.2.5).

NOTE: Probably for completeness the effect of a "cos(theta)" residual dent should also have been determined for a dent closer to the axis of revolution than Row No. 2 of Shell Segment No. 5, that is, a dent centered at a radius from the axis of revolution equal to the location of the **first** "valley" (or "ridge") in the n=1 linear buckling mode shape displayed in Fig. 205. This was not done, however, and , as they say in academia, is left as an exercise for the student.

9.2.3 Extreme fiber distributions of effective stress in the shell skin

Figure 208 and Table 91 pertain to this sub-section. In this case STAGS was not used to obtain extreme fiber stresses. Compare Table 91 with Table 81.

9.2.4 Collapse of the optimized shell with linear buckling modal imperfections

Figures 209 – 215 pertain to this sub-section. Compare Fig. 209 with Fig. 161. This "**thick-apex**" shell is only very slightly under-designed. The lowest collapse pressure depicted in Fig. 211 is equal to 445 psi, which is only 3.3 per cent lower than the design pressure, p = 460 psi. A negative margin of less than 5 per cent is accepted by GENOPT for what is called in GENOPT jargon an "ALMOST FEASIBLE" design. Note that there are negative margins listed in Table 89 that are less than –0.04. The margin for axisymmetric collapse in the presence of a –mode 2 linear buckling modal imperfection shape is equal to –0.03065 (Margin No. 12 listed on the last page of Table 89).

Figure 210 shows the post-collapse state of the optimized shell with the non-axisymmetric linear buckling modal imperfection shape displayed in Fig. 205 with amplitude, Wimp = 0.1 inch.

Figure 211 is a "zoomed" version of most of the data shown in Fig. 209.

Figures 212 – 215 show the nonlinear bifurcation buckling modes from STAGS that are used as imperfection "triggers" (imperfections with a very small amplitude, Wimp = 0.001 inch) in order to obtain the post-collapse portions of the load-deflection curves shown in Fig. 211, that is, the traces identified by the legends that include the string, "trigger".

9.2.5 Collapse pressures of the optimized shell with various "cos(theta)" residual dents

Figures 216 – 225 pertain to this sub-section. Figure 216 is analogous to Fig. 180. Residual dents of various depths with approximately a “cos(theta)” distribution are generated by the application in Load Set B of a line of normal inward-directed concentrated **loads** applied along the circumference at a radius from the axis of revolution that corresponds to the first **major** ridge in the non-axisymmetric linear buckling mode with n=1 circumferential wave (Fig. 205). This n=1 buckling mode and the location of the circumferential line of application of the concentrated loads are depicted in Fig. 205. The “cos(theta)” load distribution is applied from circumferential coordinate theta = 0 to 90 degrees.

Figure 216 shows several Load Set B load cycles that produce residual dents of various depths. Note that several of the curves on the unloading portion of the Load Set B load cycle are **non-monotonic** (the curves labeled “Run 2”, “Run 3”, “Run 4”, and “Run 5”). What is going on in those cases is described in the captions of Figs. 219 – 222.

Figure 217 displays the pressure-deflection curves (Load Set A) for the imperfect shells with the off-center residual dents produced by the Load Set B load cycles shown in Fig. 216. Note that the imperfect shell with the residual dent of depth very close to 0.1 inch barely carries the design pressure, p = 460 psi. This is as it should be, since the optimum design was obtained in the presence of axisymmetric linear buckling modal imperfections with amplitude, Wimp = 0.1 inch.

Figures 218 – 225 show the states of the shells during the Load Set B production of the residual dents (Figs. 218 and 219, 221, 223 and 224) and in the Load Set A post-collapse phase (Figs. 220, 222, and 225). The unexpected “off-axis” residual dent and mode of collapse shown especially in Figs. 219 and 220 are explained in the figure captions.

9.2.6 Conclusion from the results obtained in Section 9.2

The optimum design listed in Table 89 will most likely survive the external design pressure, p = 460 psi provided that the amplitude of any imperfection, axisymmetric or non-axisymmetric, does not exceed 0.1 inch.

9.3 “Thick-apex” unstiffened shell with lower bound of t(apex) = 0.6 inch and Wimp = 0.2 inch

The purpose of this sub-section is to find and evaluate an optimum design that will most likely survive the external design pressure, p = 460 psi provided that the amplitude of any imperfection, **axisymmetric or non-axisymmetric**, does not exceed 0.2 inch.

Figures 226 – 253 and Tables 92 – 95 pertain to this sub-section. First, **in sub-section 9.3.1** the uniform thickness of the optimized “**thick-apex**” shell in sub-section 9.1 (Table 78) is arbitrarily increased from 0.4 inch to 0.6 inch, and design margins and axisymmetric linear buckling mode 1 and mode 2 are computed from GENOPT and BIGBOSOR4 (Table 92, Figs. 226 and 227) without any optimization. Next, **in sub-section 9.3.2** a new “**thick-apex**” optimum design is found by GENOPT via one execution of SUPEROPT (Table 93 and Fig. 228). Then, **in sub-section 9.3.3** linear buckling modes by BIGBOSOR4 and STAGS are presented for the optimized design (Tables 94 and 95 and Figs. 229 – 236). Then, **in sub-section 9.3.4** elastic-plastic collapse of the shells with either axisymmetric or non-axisymmetric (n=1 circumferential wave) linear buckling modal imperfection shapes are determined for the optimized design by STAGS (Figs 237 – 239). Finally, **in sub-section 9.3.5** elastic-plastic STAGS models are used to determine the pressure-carrying capabilities of the optimized shell with **residual dents** of various depths centered at three different radial coordinates, one residual dent for each collapse analysis (Figs. 240 – 253). The STAGS 180-degree "soccerball" model with the 480 finite element is used in all the STAGS runs.

9.3.1 Same design as that listed in Table 78 except t(apex) = 0.6 inch instead of 0.4 inch

Table 92 lists the design and the design margins. Note that several of the design margins are significantly negative even though all we did was increase the thickness, **t(apex)**, of the spherical cap (Shell Segment No. 1 in Fig. 2) from 0.4 inch to 0.6 inch. Earlier, in sub-section 8.1.1, we wrote the following:

"'Escape' variables are those variables that when increased drive the design toward the feasible region. Typically a wall thickness is an escape variable because a thicker wall almost always leads to higher buckling loads, lower stresses, and smaller deformations."

Here is one of those rare examples in which increasing a wall thickness has the opposite effect. This non-intuitive result follows from the difference in the axisymmetric linear buckling modal imperfection shapes for the shell with **t(apex)** = 0.6 inch (Figs. 226 and 227) and for the shell with **t(apex)** = 0.4 inch (Figs. 145 and 146). The axisymmetric buckling modal imperfection shapes for the shell with the thicker apex exhibit maximum deflections in the region away from the apex, deflections that are significantly larger than those displayed in the same region for the shell with the thinner apex. Therefore, under the uniform external pressure the axisymmetrically imperfect shell with the thicker apex may collapse (CLAPS) earlier, experience nonlinear bifurcation buckling (GENBK) earlier, and exhibit higher maximum shell skin effective stress (SKNST) than the imperfect shell with the thinner apex.

The same phenomenon occurs for other optimum designs. If one changes the optimum design of the unstiffened, imperfect shell listed in Table 33 by increasing THKSKN(1) and THKSKN(2) from 0.2269 inch and 0.1575 inch, respectively, to 0.4 inch, most of the design margins become significantly negative for the same reason as that given in the previous paragraph.

Figures 226 and 227 show the axisymmetric mode 1 and mode 2 linear buckling modes predicted by BIGBOSOR4 for the non-optimized shell the dimensions of which are listed in Table 92.

9.3.2 Optimization

A new optimum design is found for the unstiffened, imperfect shell by a single execution of SUPEROPT. The evolution of the objective is shown in Fig. 228. The input for the "BEGIN" processor, eqellipse.BEG, is the same as that listed in Table 56. The input for the "DECIDE" processor, eqellipse.DEC, is the same as that listed in Table 77 **except the lower bound of**

THKSKN(1) is equal to 0.6 inch instead of 0.4 inch. The starting design is the optimum design listed in Table 78, corresponding to which the objective is equal to 127.1 lb.

The optimum design, design objective, and design margins are listed in Table 93. Note that the objective has increased from 127.1 lb to 132.5 lb, not nearly as dramatic a change as that from the unstiffened, imperfect optimum design listed in Table 33 (weight = 96.461 lb) to that listed in Table 78 (127.1 lb). Note from Table 93 that there is no locally thick circumferential band, THKSKN(3).

9.3.3 Linear buckling of the optimum design listed in Table 93

Figures 229 and 230 show the axisymmetic buckling modal impefection shapes, mode 1 and mode 2, predicted by BIGBOSOR4. The maximum axisymmetric buckling modal displacements occur away from the shell apex. Table 94 lists the WALLTHICK.STAGS file for the optimum design. This file is "called" by the version of SUBROUTINE USRFAB listed in Table a36 of the appendix, which is the version of USRFAB that applies to the STAGS 180-degree "soccerball" model of the optimized shell (Figs. 169 and Figs. a2 – a13). The results from the linear buckling STAGS run (INDIC = 1) are listed in Table 95 and the buckling modes predicted by STAGS are shown in Figs. 231 – 236.

As before, the "worst" (most harmful) imperfection shape is the non-axisymmetric linear buckling modal imperfection shape with n=1 circumferential wave (Fig. 232 and the last trace in Fig. 237).

Figure 232 shows where the "cos(theta)" concentrated normal inward-directed load distributions will be applied for the production of residual dents: Case 1 = Row No. 2, Shell Segment No. 3; Case 2 = Row No. 3, Shell Segment No. 5; Case 3 = Row No. 4, Shell Segment No. 7. (See Fig. 233).

9.3.4 Collapse of the optimized shell with buckling modal imperfections

Figure 237 shows pressure-deflection curves for the shells with plus and minus "mode 1" and "mode 2" axisymmetric buckling modal imperfections and for the shell with the non-axisymmetric n=1 buckling modal imperfection shape displayed in Fig. 232. The shell with the n=1 buckling modal imperfection shape collapses at a pressure slightly below the design pressure, p = 460 psi. However, its collapse pressure is close enough to the design pressure to qualify the shell as ALMOST FEASIBLE (all design margins greater than –0.05). The shell was optimized such that ALMOST FEASIBLE designs were accepted by GENOPT.

Figure 237 also shows the pressures at which nonlinear bifurcation buckling occurs according to GENOPT (BIGBOSOR4) and according to STAGS. One of the nonlinear bifurcation buckling mode shapes is displayed in Fig. 238. No post-nonlinear-bifurcation-buckling analyses were conducted for this optimum design. Hence, there are no traces in Fig. 237 with legends that contain the string, "trigger", such as exist in Fig. 211, for example. This is left as another exercise for the student.

Figure 239 shows the post-collapse state of the shell with the non-axisymmetric n=1 buckling modal imperfection (Fig. 232). The maximum inward normal deflection occurs close to Row No. 2 of Shell Segment No. 3, which is indicated in Fig. 232 and which is one of the locations where a residual "cos(theta)" dent is produced as described in the next sub-section.

9.3.5 Collapse of the optimized shell with residual "cos(theta)" dents produced by imposed loads

Figures 240 – 253 pertain to this sub-section. There are three "cases". (See Figs. 232 and 233):

Case 1: residual dent centered at theta = 0, Row No. 2 of Shell Segment No. 3
Case 2: residual dent centered at theta = 0, Row No. 3 of Shell Segment No. 5
Case 3: residual dent centered at theta = 0, Row No. 4 of Shell Segment No. 7

In all three cases the dents are generated by "cos(theta)" normal inward-directed **loads**, not by imposed displacements. Each dent is produced by a Load Set B load cycle. Collapse pressures are subsequently computed by application of the uniform external normal pressure (Load Set A) to each shell with its residual dent.

9.3.5.1 Case 1:

Figure 240 is analogous to Figs. 180 and 216. Load Set B is applied along Row No. 2 of Shell Segment No. 3 in order to produce residual dents of four depths. Figure 241 shows the collapse pressures for each of the four dented shells. The shell with the residual dent of depth 0.2226 inch collapses at a pressure just slightly above the design pressure, p = 460 psi. Figures 242, 243, and 244 show the states of the shell with the smallest residual dent just before unloading Load Set B (Fig. 242), at the completion of the Load Set B load cycle (Fig. 243), and after the application of Load Set A to the shell with the residual dent of depth 0.198 inch (Fig. 244).

9.3.5.2 Case 2:

Figure 245 is analogous to Fig. 240. Load Set B is applied along Row No. 3 of Shell Segment No. 5 in order to produce residual dents of two depths. Figure 246 shows the collapse pressures for each of the two dented shells. The shell with the residual dent of depth 0.204 inch collapses at a pressure above the design pressure, p = 460 psi. Figures 247, 248, and 249 show the states of the shell with the smaller of the two residual dents just before unloading Load Set B (Fig. 247), at the completion of the Load Set B load cycle (Fig. 248), and after the application of Load Set A to the shell with the residual dent of depth 0.2 inch (Fig. 249).

9.3.5.3 Case 3:

Figure 250 is analogous to Fig. 245. Load Set B is applied along Row No. 4 of Shell Segment No. 7 in order to produce a residual dent of depth slightly above 0.2 inch. Figure 251 shows the collapse pressure for the dented shell. The shell with the residual dent of depth 0.215 inch collapses at a pressure significantly above the design pressure, p = 460 psi. Figure 252 shows the residual dent at the completion of the Load Set B load cycle, and Fig. 253 shows the post-collapse state of the dented shell after the application of Load Set A.

9.3.6 Conclusion from the results obtained in Section 9.3

The shell optimized with a lower bound of apex thickness, t(apex) = 0.6 inch, will most likely survive at the design pressure, p = 460 psi, in the presence of either axisymmetric or non-axisymmetric imperfections with amplitude less than or equal to Wimp = 0.2 inch. The linear buckling modal imperfection with n=1 circumferential wave (last trace in Fig. 237) is a somewhat more harmful imperfection shape than the off-center residual "cos(theta)" dents collapse pressures for which are displayed as the third trace in Fig. 241 (Case 1) and the second trace in Fig. 246 (Case 2). The far-off-center residual dent (Case 3) is not as harmful as the residual dents produced closer to the axis of revolution (Cases 1 and 2).

10.0 ELASTIC-PLASTIC ANALYSIS WITH USE OF THE STAGS 180-DEGREE "SOCCERBALL" MODEL OF THE PRESSURE-CARRYING CAPABILITY OF THE OPTIMIZED ISOGRID-STIFFENED SHELL WITH LINEAR BUCKLING MODAL IMPERFECTIONS OR WITH RESIDUAL DENTS

Figures 254 – 276 pertain to this section. The various elastic-plastic STAGS models all pertain to the optimum design listed in the two columns of Table 33 headed, **"isogrid-stiffened, imperfect"**. First, **in sub-section 10.1** the pressure-carrying capacities of both elastic and elastic-plastic shells are determined for almost perfect and imperfect shells in which the imperfections are n=0 and n=1 **linear buckling modal imperfections** with amplitude, Wimp = 0.2 inch (Figs. 254 – 262). Then, **in sub-section 10.2** the pressure-carrying capacities of elastic-plastic imperfect shells are determined for shells with **off-center residual dents** produced by Load Set B load cycles (Figs. 263 – 276).

10.1 Elastic-plastic collapse of the optimized isogrid-stiffened shell with n=0 and n=1 buckling modal imperfections with amplitude, Wimp = 0.2 inch

Figures 254 – 262 pertain to this sub-section. The geometry of the optimized isogrid-stiffened shell is listed in Table 33 in the two columns under the heading, "**isogrid-stiffened, imperfect**". The STAGS 180-degree "soccerball" model is used to generate most of the results. The user-written SUBROUTINE USRFAB is employed with the associated file, WALLTHICK.STAGS, that is listed in Table a23. For the 360-degree models the version of SUBROUTINE USRFAB listed in Table a35 is used if the material is elastic-plastic. For the 180-degree "soccerball" models the version of SUBROUTINE USRFAB listed in Table a36 is used if the material is elastic-plastic. For elastic material the index, iplas, near the end of SUBROUTINE USRFAB must be changed from 1 to 0.

Figure 254 shows pressure-apex-deflection curves corresponding to STAGS 360-degree models (Fig. a1) and STAGS 180-degree "soccerball" models (Figs. 169, a2 - a13). The "worst" (most harmful) imperfection is the first non-axisymmetric linear buckling modal imperfection with n=1 circumferential wave. (See the second-to-last curve in Fig. 254, the curve pointed to by the arrow from the box containing the text, "Load Step 12…"). The elastic-plastic collapse of this imperfect shell occurs at a pressure slightly in excess of the design pressure, p = 460 psi.

Figures 255 and 256 show the inner and outer fiber meridional plastic strains in the isogrid-stiffened shell at a pressure slightly above the design pressure (Load Step 12). The meridional plastic strains have the localized distribution shapes because the n=1 linear buckling modal imperfection has the shape displayed in Figs. 258 and 259.

Figures 257 – 262 show the linear buckling modes for the optimized isogrid-stiffened shell. The first six curves in Fig. 254 corresponding to the legends that contain the string, "n=0", are for the shell with the **negative** of the axisymmetric linear buckling modal imperfection shape displayed in Fig. 257. The next six curves in Fig. 254 corresponding to the legends that contain the string, "n=1", are for the shell with the non-axisymmetric linear buckling modal imperfection shape displayed in Figs. 258 and 259. The last curve in Fig. 254, the curve with the legend that contains the string, "2nd n=1", is for the shell with the non-axisymmetric buckling modal imperfection shape displayed in Fig. 262.

In the next sub-section the behavior of the optimized isogrid-stiffened shell with "cos(theta)" residual dents at two radial locations (one residual dent at one location for each collapse analysis) is explored. The first location is at Row 2 of Shell Segment 2, as indicated in Figs. 258 and 259. The second location is at Row 5 of Shell Segment 4, as indicated in Fig. 262. These residual dents **locally** resemble the n=1 linear buckling modal imperfection shapes displayed in Figs. 258 and 262, respectively.

10.2 Collapse of the optimized isogrid-stiffened shell with "cos(theta)" residual dents

Figures 263 – 276 pertain to this sub-section. Figures 263 – 269 and Figs. 275 and 276 pertain to the case in which residual dents are centered at Row 2 of Shell Segment 2 (Figs. 258 and 259). Figures 270 – 275 pertain to the case in which residual dents are centered at Row 5 of Shell Segment 4.

10.2.1 Residual dent centered at Row 2 in Shell Segment 2

Figure 263 is analogous to Fig 240, which applies to the unstiffened optimized shell. Residual "cos(theta)" dents of three depths are produced by three Load Set B load cycles in which the dents are generated by the application of a cos(theta) distribution of normal inward-directed **loads** rather than normal inward-directed imposed displacements.

Figure 264 shows the state of the shell under the maximum Load Set B load factor, PB. Figure 265 shows the residual dent after unloading from Step 54 (Run 2 in Fig. 263). Collapse of the shell with this deep residual dent is not computed because the depth of this residual dent far exceeds the amplitude, Wimp = 0.2 inch, of the axisymmetric buckling modal imperfections in the presence of which the isogrid-stiffened shell was optimized. Figure 266 shows the smaller residual dent remaining after unloading from an earlier load step in Fig. 263 (Run 3); Figure 267 shows an even smaller residual dent remaining after unloading from a yet earlier load step in Fig. 263 (Run 4). This smallest residual dent is taken to be the imperfection present in the collapse analysis under Load Set A, the uniform external pressure, results from which are displayed as the **first** trace in Fig. 275.

Figure 268, analogous to Fig. 193, shows the Load Set B load cycle in which a “cos(theta)” residual dent is produced by a cos(theta) distribution of normal inward-directed imposed **displacements** from circumferential coordinate theta = 0 to 90 degrees along Row 2 of Shell Segment 2. Figure 269 shows the residual dent. This residual dent at Load Step 55 is taken to be the imperfection present in the collapse analysis under Load Set A, the uniform external pressure, results from which are displayed as the **second** trace in Fig. 275. The collapse pressure is lower than that corresponding to the first trace in Fig. 275 because the residual dent is deeper.

Figure 276 shows the state of the shell at a load step in the post-collapse phase of the Load Set A analysis.

10.2.2 Residual dent centered at Row 5 in Shell Segment 4

Figure 270, analogous to Fig. 268, shows two Load Set B load cycles in which “cos(theta)” residual dents of two depths are produced by a cos(theta) distribution of normal inward-directed imposed **displacements** from circumferential coordinate theta = 0 to 90 degrees along Row 5 of Shell Segment 4.

Figure 271 shows the larger residual dent at Load Step 158, and Figs. 272 and 273 display the corresponding residual meridional inner and outer fiber plastic strains, respectively. The residual dent at Step 158 is taken to be the imperfection present in the collapse analysis under Load Set A, the uniform external pressure, results from which are displayed as the **third** trace in Fig. 275.

Figure 274 shows the extreme fiber meridional and circumferential plastic strain components corresponding to the STAGS Run 1 + Run 3 + Run 4 of the Load Set B load cycle shown in Fig. 270. As of this writing it is not known why so many tiny load steps are required by STAGS for this particular case.

10.3 Conclusion from the results obtained in Section 10.0

The optimized isogrid-stiffened shell, the design of which is listed in Columns 2 and 3 of Table 33, will most likely survive at the design pressure, p = 460 psi, in the presence of either axisymmetric or non-axisymmetric imperfections with amplitude less than or equal to Wimp = 0.2 inch. For the optimized isogrid-stiffened shell off-center residual dents produced by “cos(theta)” Load Set B load cycles are not as harmful (Fig. 275) as the linear buckling modal imperfection with n=1 circumferential wave (second-to-last trace in Fig. 254), given the amplitude of the imperfection.

11.0 CONCLUSIONS

1. GENOPT can be used in combination with BIGBOSOR4 to obtain minimum-weight designs of isogrid-stiffened or unstiffened perfect or imperfect ellipsoidal shells provided that the “**equivalent” ellipsoid model** (Table 29, Fig. 2) is used and provided that the **imperfection shapes are axisymmetric**.

2. **UNSTIFFENED imperfect shells should be optimized with relatively high lower bounds set on the thicknesses of the shell wall in the neighborhood of the shell apex** (Table 77). In the case of the unstiffened, imperfect shell, if the lower bound of the wall thickness in the neighborhood of the shell apex is set too low the thickness distribution in the neighborhood of the shell apex evolves during optimization cycles in a way that essentially isolates the apex from the remainder of the shell. This produces optimum designs the collapse pressures of which are **especially sensitive to non-axisymmetric imperfection shapes**, **a type of imperfection that cannot be modeled with BIGBOSOR4 and therefore cannot be accommodated during the GENOPT optimization process.**

3. It is generally found from STAGS models that for optimized shells the **“worst” imperfections are non-axisymmetric linear buckling modal imperfections with n=1 circumferential wave** (Figs. 94, 109, 161, 176, 209, 211, 237 and 254).

4. For the optimized **unstiffened imperfect** shell it is found that imperfections in the form of **off-center residual dents** produced by a distribution of normal inward-directed concentrated loads that vary as the cos(theta) over a circumferential line from circumferential coordinate, theta = 0 to 90 degrees applied at a radius from the axis of revolution that corresponds to the first pronounced “valley” or “ridge” in the non-axisymmetric linear buckling mode shape with n=1 circumferential wave are approximately as harmful as the n=1 linear buckling modal imperfection shapes (Figs. 188, 190, 205, 211, 217, 232, 237, 241).

5. **Off-center residual dents produced by a single normal inward-directed concentrated load are significantly less harmful than off-center “cos(theta)” residual dents** (Figs. 176, 188, 200).

6. **A STAGS “soccerball” model (Figs. 169, a2 – a13) of the optimized shells is better than a 360-degree STAGS model based on polar coordinates** (Fig. a1) because the STAGS 480 finite element, which fails when used for nonlinear analysis in connection with the 360-degree model, works well when used in connection with the “soccerball” model, which has no singularity at the apex of the shell. Also, spurious buckling modes such as that shown in Figs 18 and 19 are avoided when the STAGS “soccerball” model is used.

7. The STAGS user should **rely on user-written SUBROUTINE USRFAB** rather than on user-

written SUBROUTINE WALL for providing wall properties that vary within a shell unit (Fig. 175).

8. Sections 3.0 – 6.0 of this report contain enough detail about how GENOPT works so that the **reader can use it as a guide for setting up user-friendly optimization software** for other structural or even non-structural applications.

9. The material about STAGS models is extensive enough so that the **reader should be able to set up other STAGS models** without too much trouble.

12.0 ACKNOWLEDGMENTS

Dr. Charles Rankin, the principal developer of the STAGS computer program, helped the author avoid certain pitfalls in setting up the STAGS models. Ms. Liz Ling helped the author transfer files from place to place whenever that was necessary.

13.0 REFERENCES

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