

## Minimum weight design by GENOPT/BIGBOSOR4 of an externally pressurized circumferentially corrugated cylindrical shell and verification by STAGS

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The results reported here are analogous to those reported in a previous paper on minimum weight design of elastic axially compressed prismatic complexly corrugated panels and shells in which the corrugations, each of which is a little cylindrical segment, run in the longitudinal direction. Here the corrugations, each of which is a little toroidal segment, run in the circumferential direction. The complexly corrugated shell of revolution is loaded by uniform external lateral normal pressure. There is no axial load component equal to  $pr/2$ , and the shell of revolution is free to expand or contract in the axial direction as the external pressure is applied. As before, the GENOPT/BIGBOSOR4 system is used to build the model and to perform the optimization. The minimum weight of the circumferentially corrugated shell is determined in the presence of the following behavioral constraints: 1. The shell shall not buckle locally, 2. The shell shall not buckle in a general mode that is symmetric at the plane of symmetry, 3. The shell shall not buckle in a general mode that is anti-symmetric at the plane of symmetry, 4. The maximum effective stress in the wall of the shell shall be less than a specified value. “Corners” at junctions between adjacent toroidal segments are eliminated by the automatic insertion of small “smoothing” toroidal segments. The circumferentially corrugated shells are optimized with and without the inclusion of nonlinear geometric effects, and it is found that these nonlinear effects are significant. Optimized designs are extremely sensitive to small changes in certain of the decision variables, making it difficult to find “global” optimum designs, especially when nonlinear theory is used. Several optimized designs determined with GENOPT/BIGBOSOR4 are verified by comparison with predictions from the general-purpose computer program STAGS. The agreement between the predictions of GENOPT/BIGBOSOR4 and STAGS is always good when linear theory is used, good for local buckling when either linear or nonlinear theory is used and fair for general buckling when nonlinear theory is used. There is always good agreement between BIGBOSOR4 and STAGS for the prediction of axisymmetric pre-buckling maximum effective stress.

### Section 1. INTRODUCTION AND SUMMARY

#### Introduction:

References [1 – 9] treat cylindrical shells in which the corrugations are longitudinal. References [10 – 13] treat shells in which the corrugations are circumferential.

Figures A1 – A4 in the Appendix 2 (except for Fig. A2b) are taken from [1]. Fig. A1 is included to demonstrate the difference in geometry involved in the investigation reported in [1] in contrast to that reported here, a typical example of which is displayed in Fig. 1. Figures A2, A2b and A3 are included to identify the decision variables used in [1], **which are the same as those used here** except that now PHIBIG is never a decision variable and always equal to 0.01 (essentially zero); As shown in Fig. A2b,  $YPLATE(i)$ ,  $i = 1, 2, \dots, NSEG+1$ , are now radii

from the axis of revolution to the reference surface of the shell of revolution at junctions between major segments and at the ends of the shell; SUBWID( $i$ ),  $i = 1, 2, \dots, \text{NSEG}$ , are now distances measured along the axis of revolution. The parameter, WIDTH/2 in [1] now becomes half the axial length of the shell of revolution specified by the End User. This axial length is still called “WIDTH” in this paper in order to emphasize the analogy between the system treated here and that treated in [1].

Figure A2b demonstrates visually the transformation from the longitudinally complexly corrugated prismatic panel to the circumferentially complexly corrugated shell of revolution. The thickness decision variables, THICK( $i$ ),  $i = 1, 2, \dots, \text{NSEG}$ , and the half-angle decision variables, PHISEG( $i$ ),  $i = 1, 2, \dots, \text{NSEG}$ , are identified in Fig. A3. As in [1] all the specific cases in this paper have uniform thickness along the entire axial length of the circumferentially corrugated shell of revolution, that is, THICK(2), THICK(3), .., THICK(NSEG) are linked to THICK(1). NSEG is the number of major segments along WIDTH/2. Figure A4 is included in Appendix 2 to demonstrate the meaning of “smoothing segment”. What is called “x-coordinate” in Fig. A4 is now the coordinate along the axis of revolution of the circumferentially corrugated shell, as shown in Fig. A2b.

This paper includes a subset of the results that exist in [14]. Examples of the wall profiles of externally (laterally) normally pressurized circumferentially corrugated shells of revolution investigated here are shown in Figs. 1, 3, 5, 7, 20, 27 and in several other figures.

A computer program system called "GENOPT/BIGBOSOR4" is used to obtain the optimum designs. GENOPT is a system of programs that can optimize anything [15] and BIGBOSOR4 [16], the successor to BOSOR4 [17], can analyze shells of revolution or prismatic shells [18]. The gradient-based optimizer used in GENOPT is called "ADS" [19, 20]. The ADS software was created by Vanderplaats and his colleagues in the 1980s.

BIGBOSOR4 [16] is so named because it will handle complex shells of revolution with far more segments (295 segments) than will the original BOSOR4 program [17] from which it is derived. In addition, BIGBOSOR4 will handle prismatic shells [18], not just shells of revolution. Also, BIGBOSOR4 will handle shells of revolution that are connected to ground by multiple rings of springs (struts) [31].

In GENOPT the optimizer, ADS [19, 20], is "hard-wired" in a so-called "0-5-7" mode: the "modified method of steepest descent". Constraint gradients are computed by finite differences of the behaviors of the "perturbed" design and the behaviors at the "current" design, in which the word, "behaviors", means modal vibration or stress or buckling, etc. A "perturbed" design is a design in which the value of one decision variable has been increased by a small amount from its “current” value [usually five per cent but in this generic “michelin” application, for which there exists extreme sensitivity of the buckling loads to small changes in YPLATE( $i$ ), one per cent]. A constraint-gradient matrix is automatically generated by computation of the change in each behavior caused by the perturbation of each decision variable. The values of the "behavioral" constraints and the constraint-gradient matrix are inputs to the ADS optimizer by means of which a new "current" design is obtained by the modified method of steepest descent. The interaction of the BIGBOSOR4 computer program with the ADS optimizer is entirely automated. The program user need specify only the number of design iterations per execution of the GENOPT processor called “OPTIMIZE”.

For some of the cases described in this paper the gradient-based ADS software does not easily lead to a “global” optimum design. Design iterations sometimes repeatedly lead to local optima with less favorable objectives (WEIGHTs) that are not in the neighborhood of the “global” optimum. Haftka and his colleagues have

developed the “Response Surface” method [21 – 26] that often avoids this deficiency. **The “Response Surface” method has NOT been introduced into GENOPT.**

In the previous paragraph "global" is in quotes because the strategy used in GENOPT cannot rigorously determine a true global optimum design. Instead, GENOPT attempts to find a design for which the objective is likely to be near that of a true global optimum by starting from many different points in design space during a single execution of a GENOPT processor called “SUPEROPT” (Fig. 2a). Each new "starting" point is established randomly in a manner consistent with equality and inequality constraints. The GENOPT processor called “AUTOCHANGE” determines each new “starting” point in design space. In Fig. 2a each new "starting" point corresponds to a "spike" in the plot of objective versus design iterations processed during a single execution of SUPEROPT.

The GENOPT/BIGBOSOR4 system of computer programs has previously been used to obtain optimum designs of several structures that are comprised mainly of shells of revolution [16 and 27 – 32] or prismatic shells [1 and 18 and 33 – 35].

In the GENOPT system [15] there are two types of cases: **generic** and **specific**. In this work the generic case is called “**michelin**”, and the specific cases are called “**mich8**”, “**mich8u**”, “**mich1**” and “**mich1b**”. The specific cases are all members of the generic class, “**michelin**”. There are two types of user: the **GENOPT user** and the **END user**. The GENOPT user creates the software by means of which specific cases in the generic class can be solved. The END user sets up and runs specific cases such as “mich8”, “mich8u”, “mich1” and “mich1b”. In the work reported here the GENOPT user and the END user are the same person: the author of this paper.

The name for the generic case, **michelin**, was chosen because the circumferentially complexly corrugated shell (Fig. 1) could potentially resemble the torso of the well-known Michelin Tire Man, a familiar icon of the Michelin Tire Company.

## Summary:

**Sections 2 and 3** give general information about GENOPT. **Section 4** contains a statement of the problem, a list and description of the behavioral constraints (buckling, stress), identification of variables that can be decision variables during optimization, boundary conditions, loading (**uniform external lateral normal pressure**), and an example. **Section 5** provides information about the generic case called “michelin”. **Section 6** describes how, in general, optimum designs are arrived at. **Section 7** describes the STAGS computer program (with reference to Appendix 1). **Section 8** gives numerical results from BIGBOSOR4 and STAGS for four specific cases: “mich8”, “mich8u”, “mich1” and “mich1b”. **Section 9** lists some difficulties encountered that are associated with the analysis and optimization of circumferentially corrugated shells of revolution. **Section 10** lists conclusions.

## Section 2. INFORMATION ABOUT GENOPT

Information about GENOPT is provided in some of the papers cited in the Introduction. Also, there are several files provided on the “**shellbuckling.com**” website cited in [14]. The files listed next are contained in the compressed "tar" file, general.info.tar.gz, which is contained in the bigger compressed “tar” file,

.../genopt/case/span/**micelin.tar.gz**, that is part of the very big compressed “tar” file that can be downloaded from the “Downloads” page of the “**shellbuckling.com**” website [3]. The individual files contained in the “general.info.tar.gz” file are as follows:

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#### FILES GIVING GENERAL INFORMATION ABOUT GENOPT

-rw-r--r--	1	dave	staff	5720	Nov	24	2011	genopt.abstract
-rw-r--r--	1	dave	staff	2024	Nov	24	2011	genopt.commands
-rw-r--r--	1	dave	staff	3234	Nov	24	2011	genopt.files
-rw-r--r--	1	dave	staff	93262	Nov	24	2011	genopt.getting.started
-rw-r--r--	1	dave	staff	8909	Nov	24	2011	genopt.information
-rw-r--r--	1	dave	staff	987	Nov	24	2011	genopt.information.files
-rw-r--r--	1	dave	staff	2507	Nov	24	2011	genopt.programs
-rw-r--r--	1	dave	staff	251587	Nov	24	2011	genopt.runstream
-rw-r--r--	1	dave	staff	11183	Nov	24	2011	genopt.summary
-rw-r--r--	1	dave	staff	1077	Nov	24	2011	genopt.variable.roles

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In addition there exists a file in [14], ...genopt/doc/genopt.news, that contains a log of updates to the GENOPT system.

### Section 3. AN EXAMPLE OF A GENOPT/BIGBOSOR4 RUN STREAM

A rather detailed example of a GENOPT/BIGBOSOR4 run stream executed during past work (for a generic case called “tank” [14, 31]) is contained in the file called “**genopt.runstream**” that is one of the files contained in the compressed “tar” file, general.info.tar.gz, as just listed. The run stream involves execution of the various GENOPT processors, which are listed near the beginning of the file, genopt.runstream, and repeated here for convenience (with the generic name, “tank”, changed here to the generic name, “micelin”, that pertains to this paper rather than to the “tank” configuration described in [31]):

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**gentext** (to be executed in a directory called “genoptcase”) The GENOPT user and the GENOPT process, gentext, generate a prompt file, micelin.PRO (included as part of the file, .../genopt/case/span/micelin.tar.gz, of [14]) and an information file, micelin.DEF, that contains a glossary (Table A1 in this paper). (Input data for “gentext” are stored in a file called “micelin.INP”. ) Skeletal libraries, behavior.new and struct.new, are automatically created by gentext. These skeletal libraries are to be “fleshed out” by the GENOPT user. (See the files, behavior.micelin, struct.micelin and bosdec.micelin that are contained in the compressed tar file, .../genopt/case/span/micelin.tar.gz [14].) Before the next GENOPT command, “**genprograms**”, can be given by the GENOPT user, the GENOPT user must, for the generic case called “**micelin**” that is the subject of this particular paper, first type the following:

---

```
cd .../<working directory>
cp .../genopt/case/micelin/micelin.tar.gz <working directory>
gunzip micelin.tar.gz
tar xvf micelin.tar
cp micelin.INP .../genoptcase/.
cp behavior.micelin .../genoptcase/behavior.new
cp struct.micelin .../genoptcase/struct.new
```

```
cp bosdec.michelin ../bosdec/sources/bosdec.src
cp ../bigbosor4/sources/addbosor4.src ../bosdec/sources/addbosor4.src
cd ../genoptcase
```

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NOTE: The FORTRAN source file called “bosdec.src” generates a valid input file for BIGBOSOR4. “bosdec.src” must be created by the GENOPT user if it does not already exist for the generic class, **michelin**.

**genprograms** The GENOPT user (via the GENOPT system) generates (makes) executables: begin, decide, mainsetup, optimize, change, chooseplot, and diplot that pertain to the generic case called “**michelin**”.

**begin** The End user provides starting data for a specific case: For example, one input file for “begin” is called “mich8.BEG” (Table A2 in this paper and included in the file, ../genopt/case/michelin/michelin.tar.gz, of [14]), in which the specific case name is “mich8”.

**decide** The End user chooses decision variables, bounds, linked variables, and inequality constraints: For example, the input file for “decide” is called “mich8.DEC” (Table A4 in this paper and included in the file, ../genopt/case/michelin/michelin.tar.gz, of [14]).

**mainsetup** The End user sets up strategy parameters: For example, the input file for “mainsetup” is called “mich8.OPT” (Table A5 in this paper and included in the file, ../genopt/case/michelin/michelin.tar.gz, of [14]).

**optimize** The End user performs optimization (Figs. 2a,b), or analyzes a fixed design (Figs. 3, 5, 7 and others and Tables 1 – 5), or performs design sensitivity with respect to selected decision variables (Figs. 17 and 18).

**change** The End user changes/archives variables: For example, the input file for “change” is called “mich8.CHG” (Table A3 in this paper and included in the file, ../genopt/case/michelin/michelin.tar.gz, of [14]).

**autochange** New values for decision variables are randomly computed by GENOPT: (See the “spikes” in the plot in Fig. 2a). Each “spike” represents a new “starting” design created by “autochange”.

**superopt** The End user finds a “global” optimum design via several automated sequential sets of operations: (**n** executions of optimize followed by one execution of autochange, in which **n** is provided by the End user) with the same sequence (**n** x optimize, autochange) repeated many times: (Fig. 2a).

**superduperopt** GENOPT automatically executes **superopt** “**k**” times, in which “**k**” is provided by the End user.

**chooseplot** The End user chooses which variables/margins to plot vs. design iterations: For example, the input file for “chooseplot” is called “mich8.CPL” (included in the file, ../genopt/case/michelin/michelin.tar.gz, of [14]).

**diplot** The End user plots variables vs. design iterations: (Figs. 2a,b). For example, the “postscript” file for a plot of the objective vs. design iterations for the specific case, “mich8”, is called “mich8.5.ps”. The plot appears on the End user’s screen via the command: “gv mich8.5.ps”, in which “gv” means “ghost view”.

**insert** The GENOPT user adds parameters to the generic problem.

**cleanspec** The End user cleans up SPECific case files (e.g. “mich8” files).

**cleangen** The GENOPT user cleans up GENeric case files (e.g. “michelin” files).

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Table A9 (Appendix 2) lists a run stream involving executions of GENOPT/BIGBOSOR4, followed by executions of “stand-alone” BIGBOSOR4, followed by executions of STAGS for a design that has previously been optimized. The main purpose of Table A9 is to demonstrate how figures such as Figs. 3a,b,c and Figs. 7a,b are generated by the application of “stand-alone” BIGBOSOR4 and how figures such as Fig. 4 and Fig. 8 are generated by the application of STAGS.

## Section 4. SOME GEOMETRICAL AND OTHER DETAILS

### Statement of the problem

The statement of the “michelin” problem is analogous to the statement of the “span9” problem described in [1]. We wish to minimize the weight of an elastic **circumferentially** complexly corrugated shell of revolution made of aluminum subject to certain behavioral constraints on local buckling, general buckling and effective stress. In the case of the circumferentially corrugated shell it is necessary to establish a minimum allowable radius of the shell of revolution. Otherwise, during optimization cycles, the average radius would continue decreasing endlessly because the minimum-weight design would be a shell with vanishing average radius. In the work leading to this paper optimum designs were established with the limitation that the minimum radius of the circumferentially corrugated shell of revolution **be 30 inches or more**.

### Some details about the analogy between the generic cases “michelin” and “span9”

In the generic case called “span9” there are longitudinal corrugations (little cylindrical shell segments), whereas in the generic case called “michelin” there are circumferential corrugations (little toroidal shell segments). It turns out that much of the software pertaining to “michelin” can be generated from the analogous “span9” software simply by exchanging the axial and radial coordinates in the “bosdec” subroutines in “span9” and renaming the appropriate files “michelin” instead of “span9”. There is a further significant difference in the two generic cases. The “span9” analysis is entirely linear, since the analysis of prismatic panels and shells by BIGBOSOR4 is restricted to linear analysis. However, in exchanging axial and radial coordinates we are transforming the problem from analyses of prismatic panels (longitudinal corrugations) to analyses of shells of revolution (circumferential corrugations). While BIGBOSOR4 is restricted to the linear analysis of prismatic panels, it can handle both linear and nonlinear geometric analyses of elastic shells of revolution, of which a circumferentially corrugated shell is an example.

## Types of analysis of shells of revolution permitted by BIGBOSOR4

Pertaining to buckling and stress, BIGBOSOR4 processes the following types of analyses, specified via a control index called “INDIC”:

1. **INDIC = 4** means a **linear analysis** of buckling of a non-axisymmetrically or axisymmetrically loaded shell of revolution. BIGBOSOR4 first obtains an equilibrium state via linear Fourier superposition of circumferential harmonics of a non-axisymmetrically or axisymmetrically loaded shell of revolution (INDIC=3). The BIGBOSOR4 user selects the circumferential coordinate of a particular meridian as the “worst” meridian (where the user estimates that buckling is most likely to occur first), and BIGBOSOR4 uses the pre-buckled distribution of stress resultants along that “worst” meridian in a linear bifurcation buckling analysis based on the assumption that the circumferential distribution of that pre-buckled state is axisymmetric (a conservative assumption). This paper deals with a shell of revolution under uniform external lateral normal pressure. Therefore, the pre-buckled state is axisymmetric; there is only one Fourier circumferential component of the loading, that corresponding to  $n = 0$  circumferential waves. The distribution of stress resultants along any user-selected meridian is the same as that along any other meridian. Bifurcation buckling can occur with any number of circumferential waves. BIGBOSOR4 computes buckling load factors over a range of circumferential waves,  $NMINB < n < NMAXB$  in increments of  $NINCRB$  in which  $NMINB$ ,  $NMAXB$  and  $NINCRB$  are End-user-selected values. (In Table A2, which lists input data for the GENOPT processor called “BEGIN”,  $NMINB$  for general buckling is called “MLOWGS”;  $NMINB$  for local buckling is called “MLOWL”;  $NMAXB$  for general buckling is called “MHIGHGS”;  $NMAXB$  for local buckling is called “MHIGHL”.) The circumferential wave number increment,  $NINCRB$ , is set “internally”:  $NINCRB = 1$  for general buckling and  $NINCRB = 5$  for local buckling.

2. **INDIC = 3** means a **linear** analysis of (pre-buckling) equilibrium of a non-axisymmetrically or axisymmetrically loaded shell of revolution. As described above, BIGBOSOR4 uses Fourier superposition of circumferential harmonics. This is the “pre-buckling” phase of the INDIC = 4 analysis type.

3. **INDIC = 2** means **modal vibration analysis** of an axisymmetrically loaded shell of revolution. (This analysis type option was not used during the production of this paper.)

4. **INDIC = 1** means buckling of an axisymmetrically loaded shell of revolution. **Nonlinear geometric theory** is used to obtain the pre-buckling axisymmetric stress state at the design load, and BIGBOSOR4 computes bifurcation buckling load factors over a user-specified range of circumferential wavenumbers,  $NMINB < n < NMAXB$  in increments of  $NINCRB$  in which  $NMINB$ ,  $NMAXB$  and  $NINCRB$  are user-selected values. The nonlinearly established axisymmetric pre-buckled state is computed only once. The eigenvalue problem solved for each number of circumferential waves,  $n$ , is  $K1(n) \times q - \lambda(n) \times K2(n, load) \times q = 0$ , in which  $K1(n)$  is the stiffness matrix of the **unloaded** shell,  $q$  is the eigenvector,  $\lambda(n)$  is the eigenvalue, and  $K2(n, load)$  is the load-geometric matrix. Because  $K1(n)$  is the stiffness matrix of the unloaded shell the INDIC = 1 analysis option is very like a linear bifurcation buckling analysis. Call it “quasi-linear” bifurcation buckling analysis because the axisymmetric pre-buckled state is computed from nonlinear geometric theory.

5. **INDIC = 0** means **nonlinearly** determined equilibrium states of an axisymmetrically loaded shell of revolution for a user-specified series of load steps. There is no bifurcation buckling analysis.

6. **INDIC = -1** means bifurcation buckling from a **nonlinearly** determined axisymmetric pre-buckled state with an automated search for the minimum buckling load factor as a function of circumferential wavenumber and automatic re-computation of a new nonlinearly determined axisymmetric prebuckled state at the new load level indicated by the bifurcation buckling calculations (by the bifurcation buckling eigenvalue), with a repeated search for a new minimum buckling load factor as a function of circumferential wavenumber,  $n$ . The eigenvalue problem solved for each number of circumferential waves,  $n$ , is  $K1(n,load) \times q - \lambda(n) \times K2(n,load) \times q = 0$ , in which  $K1(n,load)$  is the stiffness matrix of the **loaded** shell,  $q$  is the eigenvector,  $\lambda(n)$  is the eigenvalue, and  $K2(n,load)$  is the load-geometric matrix.

7. **INDIC = -2** bifurcation buckling from a **nonlinearly** determined axisymmetric pre-buckled state. The load is increased in steps as with **INDIC = 0**. At each step the **stability determinant is computed** corresponding to a user-selected initial number of circumferential waves,  $n$ . For that user-selected  $n$  BIGBOSOR4 detects any skipped roots between two successive load steps, changes **INDIC** from  $-2$  to  $-1$ , and proceeds as if **INDIC** had always been equal to  $-1$ . This analysis type option is the most reliable one to use if nonlinear pre-buckling effects are significant because it “sneaks up” on the bifurcation buckling load by an initial load-stepping process. For a number of reasons listed in Section 9 this presumably reliable nonlinear strategy often fails during optimization cycles in the cases of circumferentially corrugated shells of revolution treated in this paper.

### Some consequences of the analogy between “michelin” and “span9”

The software, behavior.michelin, struct.michelin and bosdec.michelin, for the generic case called "michelin" is analogous to that for the generic case called "span9" [1], and the two input files for the GENOPT processor GENTEXT, michelin.INP for the generic case "michelin" and span9.INP for the generic case "span9", are identical. Therefore, the prompting files, michelin.PRO [14] and span9.PRO [14], are identical.

Since the GENTEXT input files, michelin.INP and span9.INP, are identical, some of the data names and definitions used in the generic case called "span9" are inappropriate for the generic case called "michelin". (The author of this paper apologizes for his laziness in not editing the michelin.INP file to provide more appropriate data names and definitions. He proceeded in this way in order to emphasize the analogous nature of the two generic problems: “michelin” and “span9”.) In particular, the first three data names, WIDTH, LENGTH and FACLEN, and their definitions listed in the input file, \*.BEG, for the BEGIN processor of GENOPT for a specific "michelin" case called "mich8" are as follows. (mich8.BEG is listed in Table A2.)

```
100.0  $ total width of the corrugated plate: WIDTH
 13.0  $ axial length of the corrugated plate: LENGTH
   3.0  $ fraction of LENGTH for local buckling model: FACLEN
```

In "span9" these three data names and definitions are appropriate. (Although the data value, 13.0 for LENGTH, is dubious and the data value, 3.0 for FACLEN, is inappropriate because in [1] FACLEN must be less than 1.0.) In “michelin” the first datum, 100.0, is actually **the axial length** of the shell of revolution specified by the End User. See Fig. 7a, for example. The second datum, 13.0, is now a floating point version of the value of a **strategy index**, ISTRAT. The third datum, 3.0, is now the number, MMM, of **repeating modules** used in the BIGBOSOR4 model for general instability. See Fig. 3a, for example.



The various meanings of what is now a strategy parameter, LENGTH (also called ISTRAT, but not in \*.BEG) are as follows:

---

Value of LENGTH ( ISTRAT )	various values of the BIGBOSOR4 analysis type, INDIC for <b>Local</b> buckling	for <b>general</b> buckling
1	4	4
2	4	1
3	4	-1
4	4	-2
5	1	1
6	1	-1
7	1	-2
8	-1	1
9	-1	-1
10	-1	-2
11	-2	1
12	-2	-1
13	-2	-2

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The parameter called "LENGTH" in the various input and output "michelin" files is a "leftover" from the generic case "span9", in which the complex corrugations run in the axial direction rather than circumferentially. In the "michelin" application the parameter called "LENGTH" becomes a strategy parameter often called "strategy" or "istrat" or **ISTRAT** in the following text and in many of the figures and tables.

In the work described here only four values of what is now a strategy index, LENGTH (ISTRAT) are used:  
 LENGTH = 1 for linear theory to be used for both local and general buckling (**ISTRAT = 1**)  
 LENGTH = 5 for quasi-linear theory to be used for both local and general buckling (**ISTRAT = 5**)  
 LENGTH = 7 for different types of nonlinear theory to be used for local versus general buckling (**ISTRAT= 7**),  
 LENGTH = 13 for the most conservative type of nonlinear theory to be used for both local and general buckling. (By "most conservative" is meant "most likely to work well".) (**ISTRAT = 13**)

In SUBROUTINE BOSDEC2 of the library called: /home/progs/bosdec/sources/bosdec.michelin there exists the new statement:

$$MMM = FACLEN + 0.0001$$

in which MMM is the number of repeating sections, each of length = WIDTH+ that occur in the shell of revolution (e.g. "mich8") that is being analyzed by BIGBOSOR4 for the prediction of general instability. See Fig. 3a, for example. The plus sign is added (WIDTH+) because the unpredictable automated insertion and removal of "smoothing" segments during optimization cycles gives rise to changes of the axial length of the shell of revolution by small, unpredictable amounts. Later an iterative method is described by means of which WIDTH+ approaches WIDTH.

The BIGBOSOR4 model used for **local** buckling depends on the value of INDIC. For INDIC = 4 (linear buckling) SUBROUTINE BOSDEC2 is called with MMM = 1. For INDIC < 4 (nonlinear buckling with the use

of various strategies as listed above) SUBROUTINE BOSDEC is called. There is no parameter, MMM, used in BOSDEC. (Both SUBROUTINE BOSDEC and SUBROUTINE BOSDEC2 are included in a library called “BOSDEC”.) SUBROUTINE BOSDEC produces an input file for BIGBOSOR4 for a "half-module model", that is, a model that includes the shell segments from the bottom of the shell of revolution to its first plane of symmetry, an axial distance equal to  $\text{length} = \text{WIDTH}/2$ . (For the specific case, mich8,  $\text{WIDTH}/2$  is approximately equal to 50 inches. See Fig. 10, for example.)

In the input file, mich8.BEG (Table A2), there exists the following line:

-20.0    \$ total axial load (e.g. lb): TOTLOD( 1)

In "michelin" this input datum actually means “uniform lateral normal pressure” (e.g. psi, negative for external pressure), not the total axial load as defined in Table A2 as is appropriate for the old "span9" generic case but not appropriate for the “michelin” case.

There are now no behavioral constraints corresponding to "classical" buckling of each corrugation under the  $i$ th load set, that is, CYLBUK( $i,j$ ),  $j=1,2,\dots,NSEGS$ , are not computed by the “michelin” software. In the “michelin” software the quantities, CYLBUK( $i,j$ ),  $j=1,2,\dots,NSEGS$ , are set equal to very high values so that they do not affect the evolution of the design during optimization cycles. Entries for CYLBUK( $i,j$ ) are listed in Tables 1 – 5 under the heading, “PARAMETERS WHICH DESCRIBE BEHAVIOR (e.g. stress, buckling load)”, but these entries are not repeated under the heading, “MARGINS CORRESPONDING TO CURRENT DESIGN” because their values have been set so high. Think of the CYLBUK( $i,j$ ) as “dummy” behaviors.

The behavioral constraint for maximum effective stress is computed in SUBROUTINE BOSDEC from a "half-length" (“half-module”) BIGBOSOR4 model (as if  $MMM = 0.5$ ), that is, an axial length of the shell of revolution from the bottom to its first symmetry plane, which occurs at axial station,  $\text{WIDTH}/2$ . See Fig. 10, for example.

The material is isotropic, with material properties listed near the beginning of Table A2. (The material properties used in this work represent aluminum.) There is only one layer in the wall of the corrugated panel. The thickness is constant within each major segment of the corrugated shell of revolution, and in all the work that led to this paper the thicknesses THICK( $i$ ),  $i = 2, 3, \dots, NSEG$ , are all linked to THICK(1), which is the thickness of the first major segment. Therefore, the entire shell of revolution is of constant thickness equal to THICK(1).

The circumferentially corrugated shell of revolution exists in an (axial, radial) coordinate system shown in Figs. 3a-3c, Figs. 7a,b, Fig. 10 and in Fig. A2b, for example.

## Behavioral Constraints

The circumferentially corrugated shell of revolution is optimized subject to the following "behavioral" constraints:

1. **Local buckling** is computed in SUBROUTINE BEHX1, which is one of the subroutines in the file, behavior.michelin. The FORTRAN file, behavior.michelin, is included in the long compressed “tar” file,

.../genopt/case/michelin/**michelin.tar.gz**, of [14]. See the local buckling modes shown in Figs. 7a,b for linear local buckling and in Fig. 10 for nonlinear local buckling. The local buckling modes such as those shown in Figs. 7a,b and Fig. 10 are computed by BIGBOSOR4.

**2. General buckling that is symmetric both at the bottom of the BIGBOSOR4 model and symmetric at the top of the BIGBOSOR4 model** (the symmetry plane at axial station = MMM x WIDTH, Fig. 3a). General "symsymgenbuck" buckling is computed in SUBROUTINE BEHX2, which is one of the subroutines in the file, behavior.michelin [14]. See the buckling mode shown in Fig. 3a, for example. The symmetric general buckling mode such as that shown in Fig. 3a is computed by BIGBOSOR4.

**3. General buckling that is symmetric at the bottom of the BIGBOSOR4 model and anti-symmetric at the top of the BIGBOSOR4 model** (the symmetry plane at axial station = MMM x WIDTH). General "symantigenbuck" buckling is computed in SUBROUTINE BEHX3, which is one of the subroutines in the file, behavior.michelin [14]. See the buckling mode shown in Fig. 3b, for example. The "symantigenbuck" general buckling mode such as that shown in Fig. 3b is computed by BIGBOSOR4.

**4. General buckling that is anti-symmetric at the bottom of the BIGBOSOR4 model and anti-symmetric at the top of the BIGBOSOR4 model** (the symmetry plane at axial station = MMM x WIDTH). General "antiantigenbuck" buckling is also computed in SUBROUTINE BEHX3 [14]. See the buckling mode shown in Fig. 3c, for example. The "antiantigenbuck" general buckling mode such as that shown in Fig. 3c is computed by BIGBOSOR4. What is called "antisymmetric general buckling. BUKASY(1)" in Tables 1 – 5, is the behavioral constraint corresponding to the smaller buckling load factor from the two computations in SUBROUTINE BEHX3: that for "symantigenbuck" and that for "antiantigenbuck".

**5. There no longer exists an active behavioral constraint that corresponds to CYLBUK(i,j)**, which was active in the analysis presented in [1]. In [1] CYLBUK(i,j) is the buckling load factor under load set i of the jth major segment as if this major segment were a complete (360-degree) uniformly axially compressed cylindrical shell. The buckling load factor in [1] is computed in SUBROUTINE BEHX4 from the classical formula for elastic buckling of a uniformly axially compressed perfect cylindrical shell with given radius/thickness, r/t, reduced by a "knockdown" factor that depends on r/t. This "knockdown" factor is obtained by interpolation from a curve labeled "a design recommendation" given in Fig. 5.18 of the book by Brush and Almroth [36]. This "knocked down" buckling load factor of the uniformly axially compressed little cylindrical segment no. j is no longer appropriate for the circumferentially corrugated shell of revolution, in which each major shell segment is a sector of a toroidal shell, not a cylindrical shell. However, SUBROUTINE BEHX4 still exists. In it all of the CYLBUK(i,j), j=1,NSEG are set equal to a very high value so that the CYLBUK(i,j) no longer constrain the evolution of the design during optimization cycles. However, CYLBUK(i,j) are still listed in the output file, See the section entitled "PARAMETERS WHICH DESCRIBE BEHAVIOR" in each of Tables 1 – 5, in which Behaviors 4 – 11 have the high values established in SUBROUTINE BEHX4. Because of these high values CYLBUK(i,j) do not affect the evolution of the design during optimization cycles.

**6. Maximum allowable effective (vonMises) stress.** The effective stress is that generated by the uniform external lateral normal pressure. The effective stress includes both membrane and bending deformations. **Note that there is no axial component of the loading equal to  $p r/2$**  that would exist if the shell of revolution were subjected to uniform hydrostatic compression rather than just uniform lateral normal pressure. The maximum stress is computed in SUBROUTINE BEHX5, which is one of the subroutines in the file, behavior.michelin [14]. During loading the shell is free to expand or contract in the axial direction. See Fig. 10, for example.

## Decision Variable Candidates

The problem variables, established by the GENOPT user, are listed in Tables A1 and A2. The decision variable candidates (variables with Role Type No. 1 listed in Table A1 and indicated in Figs. A2, A2b and A3) are:

1. The thickness, THICK, assumed to be constant over each major segment, but can be different in each major segment (THICK(i),  $i = 1, 2, \dots, \text{NSEG}$ ). See the middle frame of Fig. A3. There are NSEG major segments over the half-width, WIDTH/2, of the panel. All of the results presented for specific cases in this paper are for circumferentially corrugated shells of revolution in which the thickness of the entire WIDTH (axial length) of the shell is uniform, that is, the decision variables, THICK(j),  $j = 2, 3, \dots, \text{NSEG}$ , are all linked to THICK(1) with the linking constants all equal to 1.0.
2. The projected “width” (axial length), SUBWID(i), of each major segment (SUBWID(i),  $i = 1, 2, \dots, \text{NSEG}$ ). This is the “width” (axial length) of the  $i$ th major shell segment projected onto the axis of revolution. (See Fig. A2, Fig. A2b, and the bottom frame of Fig. A3). In this paper the axis of revolution is always shown as vertical rather than horizontal. (See Fig. A2b, for example.)
3. The half-angles, PHISEG(i),  $i = 1, 2, \dots, \text{NSEG}$ , (degrees) of the major “cylindrical” (toroidal) corrugation segments. (See Fig. A2, Fig. A2b, and the bottom frame of Fig. A3.) "Half-angle" (in degrees) means half the angle subtended by a single “cylindrical” (toroidal) segment of the “longitudinally” (circumferentially) corrugated shell. The major segments (and additional “smoothing” segments, if any) are joined circumferentially edge-to-edge, as shown in Fig. 2c, for example.
4. The half-angles, PHISUB(i),  $i = 1, 2, \dots, \text{NSEG}$ , (degrees), of the axial sub-segments, if any. **There are no sub-segments permitted in the configurations explored in this paper.** (Notice that in Tables 1 – 5 and in Table A2 PHISUB(i),  $i = 1, 2, \dots, \text{NSEG}$  are listed. Think of them as dummy variables that play no role in the calculations for circumferentially corrugated shells of revolution.)
5. The distances, YPLATE(i),  $i = 1, 2, \dots, \text{NSEG} + 1$ , from the axis of revolution to the shell reference surface at the two ends of the circumferentially corrugated shell and at the junctions between major segments. The YPLATE(i), for a special case (called “mich8”) in which there are 8 major segments, are indicated in Figs. A2 and A2b. The definition of YPLATE(i), “distance from the axis of revolution to the shell reference surface”, applies if the overall arching angle, PHIBIG, is zero (infinite overall arching radius of curvature). PHIBIG = 0.01 degrees (essentially zero) holds for all of the cases presented in this paper.
6. An overall arching half-angle, PHIBIG (degrees), that spans the axial distance, WIDTH/2, of the circumferentially corrugated shell. The top of the arch (or bottom of the “valley”) is located at axial station = WIDTH/2. (See Figs. A2 and A2b.) Figure A3, taken from [1], shows starting designs of corrugated shells with three values of PHIBIG: Top frame: PHIBIG = 0.1 degree (essentially zero), Middle frame: PHIBIG = 10 degrees and Bottom frame: PHIBIG = 60 degrees. **In the work reported here PHIBIG is never chosen as a decision variable and is always set equal to a very, very small value, 0.01 degrees (essentially zero). This means that all of the so-called “shells of revolution” described in this paper are, in an average sense, like circumferentially complexly corrugated CYLINDRICAL shells.**

## Boundary Conditions

There are two sets of boundary conditions that affect the behavior of the externally (laterally) normally pressurized, circumferentially corrugated shells of revolution:

1. Boundary conditions that govern the axisymmetric **pre-buckling state** of the shell of revolution
2. Boundary conditions that govern the non-axisymmetric **bifurcation buckling state** of the shell.

**Boundary conditions in the BIGBOSOR4 models:** The boundary conditions that govern the axisymmetric pre-buckling state of the circumferentially corrugated shell of revolution are illustrated in Fig. 10. The uniformly externally pressurized, circumferentially corrugated surface is permitted to expand or contract in the axial direction. At the top of the BIGBOSOR4 model shown in Fig. 10 symmetry conditions are applied. At the bottom of the BIGBOSOR4 model “moving” symmetry conditions are applied: symmetry conditions except that axial displacement is permitted. **There is no axial resultant equal to  $pr/2$  applied to the shell;** the external normal pressure exists only on the outer corrugated surface. (The normal pressure is actually applied to the reference surface of the shell, which is the middle surface of the shell wall.) Three different sets of boundary conditions govern the non-axisymmetric **general** bifurcation buckling state of the shell. These are called “symsymgenbuck”, “symantigenbuck” and “antiantigenbuck”, and they are illustrated in Figs. 3a, 3b and 3c, respectively.

**Boundary conditions in the STAGS models:** In the typical STAGS model the different boundary conditions used for the pre-buckling phase of the calculations and for the bifurcation buckling phase of the calculations are handled in the STAGS input file, \*.inp, as follows, for example:

In STAGS shell unit 1 (left-hand edge of the STAGS model shown in Fig. 14b. for example):

0	4	6	4	1	\$P-1	IBLN(i),i=1,4,	IBOND	(pre-buckling phase)
101	100				\$P-2	ITRA, IROT		(pre-buckling phase)
1,	1,	6,	1		\$P-3	JBLN(i),i=1,4		(bifurcation buckling phase)

In STAGS shell unit NUNITS (right-hand edge of the STAGS model shown in Fig. 14b, for example):

6	4	4	4	1	\$P-1	IBLN(i),i=1,4,	IBOND	(pre-buckling phase)
6,	1,	1,	1		\$P-3	JBLN(i),i=1,4		(bifurcation buckling phase)

The data entries corresponding to STAGS shell unit 1 (left-hand end in Fig. 14b) have the following meanings:

**Data entries in the first record (P-1), which govern the pre-buckling phase of the computations:**

- 0 means that the pre-buckling conditions along edge no. 1 will be specified by ITRA and IROT, to be given in the next (P-2) record
- 4 means symmetry conditions are to be applied to edge no. 2 (the generator at 0 degrees)
- 6 means that this first STAGS shell unit is attached to another shell unit along edge no. 3
- 4 means symmetry conditions are to be applied to edge no. 4 (the generator at 180 degrees)
- 1 means that the boundary conditions imposed in the pre-buckling phase are different from those imposed in the bifurcation buckling phase.

**Data entries in the second record (P-2), which govern the pre-buckling phase:**

ITRA = 101 means u displacement component is free, v displacement component is fixed, w displacement component is free, in which u, v and w displacement components are **tangential** and **normal** to the shell reference surface.

IROT = 100 means rotation about the u direction is free, rotation about the v direction is fixed and rotation about the w direction is fixed.

It is emphasized that the conventions used for end conditions of the circumferentially corrugated shell of revolution are different in BIGBOSOR4 and STAGS. In BIGBOSOR4 symmetry and anti-symmetry conditions at the ends of shells of revolution are imposed in an axial-radial sense, whereas in STAGS symmetry and anti-symmetry conditions are imposed on the displacement components in the tangential and normal shell coordinate directions. This difference in boundary condition conventions used in BIGBOSOR4 and STAGS gives rise to the need for the introduction of “smoothing” segments at the ends of the shell of revolution if the deviation of the meridional slope from vertical is greater than 5 degrees. Figure 9g shows an optimized geometry for which shell end “smoothing” segments have been automatically added by the GENOPT/BIGBOSOR4 system in order that the meridional slope of the shell of revolution be vertical and that therefore the BIGBOSOR4 and STAGS end conditions coincide, that is, the imposition of displacement restraints in a (radial, axial) sense are the same as the imposition of displacement restraints in a (normal, tangential) sense..

**Data entries in the first record (P-3), which govern the bifurcation buckling phase:**

1 means anti-symmetry conditions are to be applied to edge no. 1 (the left-hand curved edge)

1 means anti-symmetry conditions are to be applied to edge no. 2 (the generator at 0 degrees)

6 means that this first STAGS shell unit is attached to another shell unit along edge no. 3

1 means anti-symmetry conditions are to be applied to edge no. 4 (the generator at 180 degrees)

The data entries corresponding to STAGS shell unit NUNITS (the last shell unit, that is, the shell unit adjacent to the right-hand end in Fig. 14b) have the following meanings:

**Data entries in the first record (P-1), which govern the pre-buckling phase of the computations:**

6 means that this last STAGS shell unit is attached to another shell unit along edge no. 1

4 means symmetry conditions are to be applied to edge no. 2 (the generator at 0 degrees)

4 means symmetry conditions are to be applied to edge no. 3 (the right-hand curved edge)

4 means symmetry conditions are to be applied to edge no. 4 (the generator at 180 degrees)

1 means that the boundary conditions imposed in the pre-buckling phase are different from those imposed in the bifurcation buckling phase.

**Data entries in the second record (P-3), which govern the bifurcation buckling phase:**

6 means that this last STAGS shell unit is attached to another shell unit along edge no. 1

1 means anti-symmetry conditions are to be applied to edge no. 2 (the generator at 0 degrees)

1 means anti-symmetry conditions are to be applied to edge no. 3 (the right-hand curved edge)

1 means anti-symmetry conditions are to be applied to edge no. 4 (the generator at 180 degrees)

The numbering scheme of the four edges of a quadrilateral finite element (or of an entire STAGS model that has 4 edges) is given in Figs. 4.1 and 4.2 on pages 4-6 and 4-7 of the 1996 version of the STAGS user's manual [42].

## An Example

Typical values of design variables for an **optimized design** of a circumferentially corrugated shell of revolution with 8 major segments over the axial length,  $WIDTH/2 = 50$  inches (the specific case called “**mich8**”) are listed in Table 2, an edited form of which is given next for the convenience of the reader.

### OPTIMIZED VALUES OF DESIGN VARIABLES FOR THE BEST FEASIBLE DESIGN

VAR. CURRENT

NO.	VALUE	DEFINITION
1	4.1420E-02	wall thickness of the major segment: THICK(1 )
2	4.1420E-02	wall thickness of the major segment: THICK(2 )
3	4.1420E-02	wall thickness of the major segment: THICK(3 )
4	4.1420E-02	wall thickness of the major segment: THICK(4 )
5	4.1420E-02	wall thickness of the major segment: THICK(5 )
6	4.1420E-02	wall thickness of the major segment: THICK(6 )
7	4.1420E-02	wall thickness of the major segment: THICK(7 )
8	4.1420E-02	wall thickness of the major segment: THICK(8 )
9	2.7410E+00	projected width (x-width) of major segment: SUBWID(1 )
10	8.5650E+00	projected width (x-width) of major segment: SUBWID(2 )
11	5.9980E+00	projected width (x-width) of major segment: SUBWID(3 )
12	8.9330E+00	projected width (x-width) of major segment: SUBWID(4 )
13	5.6410E+00	projected width (x-width) of major segment: SUBWID(5 )
14	8.4400E+00	projected width (x-width) of major segment: SUBWID(6 )
15	5.7090E+00	projected width (x-width) of major segment: SUBWID(7 )
16	3.9480E+00	projected width (x-width) of major segment: SUBWID(8 )
17	3.6230E+01	half-angle (deg.) of major corrugation: PHISEG(1 )
18	5.9770E+01	half-angle (deg.) of major corrugation: PHISEG(2 )
19	4.0020E+01	half-angle (deg.) of major corrugation: PHISEG(3 )
20	6.5680E+01	half-angle (deg.) of major corrugation: PHISEG(4 )
21	4.2940E+01	half-angle (deg.) of major corrugation: PHISEG(5 )
22	7.2920E+01	half-angle (deg.) of major corrugation: PHISEG(6 )
23	4.7380E+01	half-angle (deg.) of major corrugation: PHISEG(7 )
24	3.4330E+01	half-angle (deg.) of major corrugation: PHISEG(8 )
25	7.0000E+01	half-angle (deg.) of sub-corrugation: PHISUB(1 )
26	7.0000E+01	half-angle (deg.) of sub-corrugation: PHISUB(2 )
27	7.0000E+01	half-angle (deg.) of sub-corrugation: PHISUB(3 )
28	7.0000E+01	half-angle (deg.) of sub-corrugation: PHISUB(4 )
29	7.0000E+01	half-angle (deg.) of sub-corrugation: PHISUB(5 )
30	7.0000E+01	half-angle (deg.) of sub-corrugation: PHISUB(6 )
31	7.0000E+01	half-angle (deg.) of sub-corrugation: PHISUB(7 )

```

32  7.0000E+01  half-angle (deg.) of sub-corrugation: PHISUB(8 )
33  3.5820E+01  vertical y above (x,y,z) origin if PHIBIG=0: YPLATE(1 )
34  3.3470E+01  vertical y above (x,y,z) origin if PHIBIG=0: YPLATE(2 )
35  3.3450E+01  vertical y above (x,y,z) origin if PHIBIG=0: YPLATE(3 )
36  3.3470E+01  vertical y above (x,y,z) origin if PHIBIG=0: YPLATE(4 )
37  3.3520E+01  vertical y above (x,y,z) origin if PHIBIG=0: YPLATE(5 )
38  3.3450E+01  vertical y above (x,y,z) origin if PHIBIG=0: YPLATE(6 )
39  3.3450E+01  vertical y above (x,y,z) origin if PHIBIG=0: YPLATE(7 )
40  3.3520E+01  vertical y above (x,y,z) origin if PHIBIG=0: YPLATE(8 )
41  3.0770E+01  vertical y above (x,y,z) origin if PHIBIG=0: YPLATE(9 )

42  1.0000E-02  half-angle (deg.) of overall arching: PHIBIG

```

Notice that sub-segment half-angles, PHISUB(i), i=1,8, are listed above (variable numbers 25 – 32) even though there are never any sub-segments in “michelin” configurations. Think of PHISUB(i) as dummy variables. The wall thicknesses of all 8 major segments are the same in this particular example because THICK(2), THICK(3), THICK(4), THICK(5), THICK(6), THICK(7), THICK(8) are all linked to THICK(1) via the GENOPT processor, DECIDE. (This linking of THICK(2) through THICK(NSEG) to THICK(1) holds for all of the specific cases described in this paper. Hence, the **entire paper pertains to circumferentially corrugated shells of revolution of uniform thickness.**)

Typical **behavioral constraints** (from which the behavioral design margins are computed) for the optimized “mich8” design just listed are as follows:

```

***** RESULTS FOR LOAD SET NO. 1 *****
PARAMETERS THAT DESCRIBE BEHAVIOR(e.g. stress,buckling load)
BEH. CURRENT
NO.  VALUE  DEFINITION
1  1.590E+00  local buckling load factor: LOCBUK(1 )(STAGS = 1.6364)
2  1.423E+00  symmetric general buckling: BUKSYM(1 )(no STAGS model)
3  1.397E+00  antisymmetric general buckling: BUKASY(1 )(STAGS=1.2259)
4  1.000E+10  classical buckling load factor: CYLBUK(1 ,1 )
5  1.000E+10  classical buckling load factor: CYLBUK(1 ,2 )
6  1.000E+10  classical buckling load factor: CYLBUK(1 ,3 )
7  1.000E+10  classical buckling load factor: CYLBUK(1 ,4 )
8  1.000E+10  classical buckling load factor: CYLBUK(1 ,5 )
9  1.000E+10  classical buckling load factor: CYLBUK(1 ,6 )
10 1.000E+10  classical buckling load factor: CYLBUK(1 ,7 )
11 1.000E+10  classical buckling load factor: CYLBUK(1 ,8 )
12 5.266E+04  maximum effective stress: STRESS(1 ) (STAGS=51859 psi)

```

The first array index in the behavioral variables, LOCBUK, BUKSYM, BUKASY, CYLBUK and STRESS, is the load case number. **In this work there exists only a single load case: uniform external normal pressure.** The second array index in the behavioral variable, CYLBUK, is the major segment number. The “CYLBUK” behavioral variables play no role in the calculations, and they do not influence the evolution of the design



during optimization cycles because they have been set to a very high value in SUBROUTINE BEHX4 [14]. The third “behavior”:

3      **1.397E+00**      antisymmetric general buckling: BUKASY(1) (**STAGS=1.2259**)  
represents the more critical (lower bifurcation buckling load factor) of the “symantigenbuck” and “antiantigenbuck” models.

Typical **design margins** for the “mich8” optimized design are:

\*\*\*\*\* RESULTS FOR LOAD SET NO. 1 \*\*\*\*\*

**MARGINS CORRESPONDING TO CURRENT DESIGN (F.S.= FACTOR OF SAFETY)**

MARGIN CURRENT

NO.	VALUE	DEFINITION
1	<b>5.250E-02</b>	$6.05 - 0.10 * V(9) - 0.10 * V(10) - 0.10 * V(11) - 0.10 * V(12) + \text{etc.}$
2	<b>2.500E-03</b>	$-3.99 + 0.10 * V(9) + 0.10 * V(10) + 0.10 * V(11) + 0.10 * V(12) + \text{etc.}$
3	<b>-6.525E-03</b>	$(\text{LOCBUK}(1) / \text{LOCBUKA}(1)) / \text{LOCBUKF}(1) - 1; \text{F.S.} = 1.60$
4	<b>1.609E-02</b>	$(\text{BUKSYM}(1) / \text{BUKSYMA}(1)) / \text{BUKSYMF}(1) - 1; \text{F.S.} = 1.40$
5	<b>-2.217E-03</b>	$(\text{BUKASY}(1) / \text{BUKASYA}(1)) / \text{BUKASYF}(1) - 1; \text{F.S.} = 1.40$
6	2.661E-01	$(\text{STRESSA}(1) / \text{STRESS}(1)) / \text{STRESSF}(1) - 1; \text{F.S.} = 1.50$

The critical margins (margins near zero) are listed in **bold face**. Margins 3 – 5 correspond to Behaviors 1 – 3 and Margin 6 corresponds to Behavior 12 in the list with the heading, “**PARAMETERS THAT DESCRIBE BEHAVIOR**”. Behaviors 4 – 11 are not represented as margins because these behaviors have been set (in SUBROUTINE BEHX4) to a very high value (1.000E+10) and therefore are not included in the list of “active” margins considered by GENOPT during optimization cycles. In the lists of “behavior” and “margins” the following conventions apply:

**Behavioral variable names (edited from Table 2):**

**LOCBUK** means local buckling load factor, computed by BIGBOSOR4 in SUBROUTINE BEHX1.

**BUKSYM** means symmetric general buckling (buckling mode is symmetric about the symmetry plane), computed by BIGBOSOR4 in SUBROUTINE BEHX2.

**BUKASY** means anti-symmetric general buckling (the critical buckling mode is anti-symmetric about the symmetry plane), computed by BIGBOSOR4 in SUBROUTINE BEHX3 as the lower of the two buckling load factors, “symantigenbuck” and “antiantigenbuck”.

**CYLBUK** (not used for circumferentially corrugated shells) means knocked down “classical” buckling load factor of a uniformly axially compressed cylindrical shell. (Used in [1])

**STRESS** means maximum effective stress, computed by BIGBOSOR4 in SUBROUTINE BEHX5.

**"A", "F", and "F.S.":**

An "A" added to a behavioral variable name means "allowable"

An "F" added to a behavioral variable name means "factor of safety"

"F.S." means "factor of safety".

### Indices, i,j:

LOCBUK(i), BUKSYM(i), BUKASY(i) buckling of various types: i = load case number

CYLBUK(i,j) = “classical” buckling of cylindrical shell: i = load case; j = cylindrical major segment number

STRESS(i) = maximum effective stress: i = load case number

The format of the definitions of the margins listed above is generated automatically by the GENTEXT processor of GENOPT.

### Two Inequality Constraints

The first two “mich8” margins listed above (associated with the heading, “**MARGINS CORRESPONDING TO CURRENT DESIGN**”):

- |   |                  |  |
|---|------------------|--|
| 1 | <b>5.250E-02</b> | $6.05 - 0.10 * V(9) - 0.10 * V(10) - 0.10 * V(11) - 0.10 * V(12) + \text{etc.}$  |
| 2 | <b>2.500E-03</b> | $-3.99 + 0.10 * V(9) + 0.10 * V(10) + 0.10 * V(11) + 0.10 * V(12) + \text{etc.}$ |

require an explanation. These two margins arise from two inequality constraints generated by the End user in the GENOPT processor called “DECIDE”. The variables, V(9), V(10), V(11), V(12), etc.), are respectively the projected widths, SUBWID(1), SUBWID(2), SUBWID(3), SUBWID(4), etc.), of the 8 major segments listed above under the heading, “**VALUES OF DESIGN VARIABLES FOR THE BEST FEASIBLE DESIGN**”. These 8 variables are decision variables, and the sum of them should always be very close to the axial length, WIDTH/2, of the circumferentially corrugated shell of revolution. In order to accomplish this, two inequality constraints are introduced during execution of the GENOPT processor called "DECIDE":

1. The overall axial length, WIDTH/2=50 inches, of the corrugated shell should be less than 50.5 inches.
2. The overall axial length, WIDTH/2=50 inches, of the corrugated shell should be greater than 49.5 inches.

In other words,

$$50.5 > V(9) + V(10) + V(11) + V(12) + V(13) + V(14) + V(15) + V(16)$$

and

$$49.5 < V(9) + V(10) + V(11) + V(12) + V(13) + V(14) + V(15) + V(16)$$

During the development of the application in [1] it was found that the numerical behavior was improved by multiplying both sides of the two inequality constraints by 0.1. Therefore, the two new (equivalent) inequality constraints used in this “michelin” paper are:

$$5.05 > 0.1 * V(9) + 0.1 * V(10) + 0.1 * V(11) + 0.1 * V(12) + 0.1 * V(13) + 0.1 * V(14) + 0.1 * V(15) + 0.1 * V(16)$$

$$4.95 < 0.1 * V(9) + 0.1 * V(10) + 0.1 * V(11) + 0.1 * V(12) + 0.1 * V(13) + 0.1 * V(14) + 0.1 * V(15) + 0.1 * V(16)$$

Item No. 603 of the file, ../genopt/execute/URPROMPT.DAT (a permanent GENOPT file, independent of the generic case, that contains prompting phrases and "help" paragraphs for the End user) includes the following lines:

603.1 Identify the type of inequality expression (1 or 2)

603.2

The inequality relations have either of the two forms:

Type 1 is:  $1.0 > f(v_1, v_2, v_3, \dots)$

Type 2 is:  $1.0 < f(v_1, v_2, v_3, \dots)$

in which the expression  $f(v_1, v_2, v_3, \dots)$  has the form:

$f(v_1, v_2, v_3, \dots) = C_0 + C_1*v_1^{D_1} + C_2*v_2^{D_2} + C_3*v_3^{D_3} +$   
+etc (up to max. of 45 terms).

In this application we always use inequality constraints of Type 2:  $1.0 < f(v_1, v_2, v_3, \dots)$ . Therefore, we multiply both sides of the first inequality constraint by -1.0, obtaining

$$-5.05 < -0.1*V(9) - 0.1*V(10) - 0.1*V(11) - 0.1*V(12) - 0.1*V(13) - 0.1*V(14) - 0.1*V(15) - 0.1*V(16)$$

Adding 5.05 to both sides of this first inequality gives:

$$0.0 < 5.05 - 0.1*V(9) - 0.1*V(10) - 0.1*V(11) - 0.1*V(12) - 0.1*V(13) - 0.1*V(14) - 0.1*V(15) - 0.1*V(16)$$

and then adding 1.0 to both sides of this inequality gives:

$$1.0 < 6.05 - 0.1*V(9) - 0.1*V(10) - 0.1*V(11) - 0.1*V(12) - 0.1*V(13) - 0.1*V(14) - 0.1*V(15) - 0.1*V(16)$$

which is in the proper “Type 2” form identified above, that is:  $1.0 < f(v_1, v_2, v_3, \dots)$ .

In an analogous manner, subtracting 4.95 from both sides of the second inequality listed above and then adding 1.0 to both sides of this second inequality gives:

$$1.0 < -3.95 + 0.1*V(9) + 0.1*V(10) + 0.1*V(11) + 0.1*V(12) + 0.1*V(13) + 0.1*V(14) + 0.1*V(15) + 0.1*V(16)$$

These two inequalities, valid for the particular specific case called “mich8”, are:

$$1.0 < 6.05 - 0.1*V(9) - 0.1*V(10) - 0.1*V(11) - 0.1*V(12) - 0.1*V(13) - \text{etc.}$$

$$1.0 < -3.95 + 0.1*V(9) + 0.1*V(10) + 0.1*V(11) + 0.1*V(12) + 0.1*V(13) + \text{etc.}$$

They correspond to the first two margins listed above in the section entitled “**MARGINS CORRESPONDING TO CURRENT DESIGN (F.S.= FACTOR OF SAFETY)**”.

## Loading

In this problem there is **only one load case: uniform external lateral normal pressure**. The uniform normal pressure is negative for external pressure, units = psi, for example. It is assumed in this “michelin” model that the applied uniform external lateral normal pressure is not associated with any axial loading. The

circumferentially corrugated panel is free to expand or contract in the axial direction when external normal pressure is applied. An example of this axial expansion is shown in Fig. 10, where this axial expansion under the normal lateral pressure is called “ENDUVS”. Also, there is no axial loading of the “ $pr/2$ ” type that would be present if the circumferentially corrugated shell of revolution were hydrostatically compressed.

## Section 5. INFORMATION ABOUT THE GENERIC CASE CALLED "michelin"

In the GENOPT universe there are two types of cases:

(1) A **generic** case (called "**michelin**" in this paper)

(2) **Specific** cases that fit within the generic set. These specific cases are here called “**mich8**”, “**mich8u**”, “**mich1**” and “**mich1b**”. The numeral “8” or “1” denotes the number of major toroidal segments over half the shell axial length: WIDTH/2. The ending letter “u” in the name, “mich8u” denotes “uniform major segments”.

Corresponding to each of the two classes of case, **generic** and **specific**, there are possibly different users. The role of the **GENOPT user** is to create the software for setting up the GENERIC environment ("michelin"). The **End user** exercises the GENERIC environment, "michelin", for SPECIFIC cases, such as the cases called "mich8", "mich8u", "mich1" and "mich1b". In the work reported here the GENOPT user and End user are the same person, the author.

The following files pertain to the GENERIC case called "**michelin**". The files whose names are listed next are contained in the compressed "tar" file, **michelin.tar.gz**, which is contained in the directory, `.../genopt/case/michelin`, which is part of the very big compressed “tar” file that can be downloaded from the “Downloads” page of the “**shellbuckling.com**” website [14]. Some of the files contained in the “**michelin.tar.gz**” file are the following:

---

### FILES RELATED TO THE GENOPT USER'S GENERIC CASE = “michelin” PHASE OF THIS PROJECT

May 6 09:10	addbosor4.src	(most of BIGBOSOR4)
May 18 16:58	behavior.michelin	(“fleshed-out” version of behavior.new)
May 17 07:59	bosdec.michelin	(generates valid BIGBOSOR4 input file)
May 12 05:37	bosdec.michelin.symsym	(no axial expansion is allowed under p)
Apr 30 11:19	mich1.tar.gz	(all results from “mich1” are stored)
May 9 07:46	mich1b.tar.gz	(all results from “mich1b” are stored)
May 12 04:20	mich8.symsym.tar.gz	(results from “mich8symsym” are stored)
May 18 18:57	mich8.tar.gz	(all results from “mich8” are stored)
Apr 25 18:19	mich8u.tar.gz	(all results from “mich8u” are stored)
Mar 8 07:35	michelin.INP	(valid input file for GENTEXT)
Mar 18 22:35	michelin.readme	(early runstream information is given)
Feb 20 03:39	struct.michelin	(“fleshed-out” version of struct.new)

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Skeletal forms of the files, **behavior.michelin** and **struct.michelin**, are created automatically by GENOPT during the GENOPT user's long interactive session associated with the GENOPT command, GENTEXT. It is

the responsibility of the GENOPT user to "flesh out" these skeletal libraries (originally called **behavior.new** and **struct.new**) so that they compute any "behaviors" (vibration, buckling, stress, etc.) that may constrain the design during optimization cycles. In the present application of GENOPT all behaviors are computed by BIGBOSOR4. The "fleshed out" versions are archived in the files, **behavior.michelin** and **struct.michelin** (included in the file, .../genopt/case/span/michelin.tar.gz, of [14]). The FORTRAN file, **behavior.michelin**, contains several subroutines, some written entirely by GENTEXT and others written partly by GENTEXT (the skeletal version) and partly by the GENOPT user (the "fleshed-out" version). The file, **behavior.michelin**, includes SUBROUTINES BEHX1 (computes local buckling with the use of BIGBOSOR4), SUBROUTINE BEHX2 (computes general symmetric buckling ("symsymgenbuck") with the use of BIGBOSOR4), SUBROUTINE BEHX3 (computes general anti-symmetric buckling (both "symantigenbuck" and "antiantigenbuck") with the use of BIGBOSOR4), SUBROUTINE BEHX4 (now essentially a dummy routine but in [1] computed knocked-down "classical" buckling of every segment from the formula,  $K \times 0.605 \times E \times t/r$ , in which K is a knockdown factor interpolated from the curve labeled "A design recommendation" plotted in Fig. 5.18 of [36]), SUBROUTINE BEHX5 (computes, from BIGBOSOR4, the maximum extreme fiber effective stress, including both membrane and bending deformations, in the circumferentially corrugated shell of revolution), and OBJECT (computes the objective function, which in the "michelin" formulation is the weight of an axial length of the shell of revolution called "WIDTH/2").

In the generic "michelin" formulation, the FORTRAN library called **struct.michelin** is almost entirely automatically written by the GENOPT processor, GENTEXT. Only three lines, having to do with opening, rewinding and closing files, are added by the GENOPT user to the automatically generated skeletal **struct.new** version.

The file, **bosdec.michelin** (included in the file, .../genopt/case/span/michelin.tar.gz, of [14]), is entirely written by the GENOPT user. The purpose of the FORTRAN library, **bosdec.michelin**, is to generate valid input files for BIGBOSOR4 [16], which is the shell-of-revolution/prismatic shell analyzer that computes the various vibration, buckling, and stress behaviors that might constrain the design during optimization cycles. There are two "bosdec" subroutines in the **bosdec.michelin** library: SUBROUTINE BOSDEC and SUBROUTINE BOSDEC2. SUBROUTINE BOSDEC is used to generate "half-module" BIGBOSOR4 models of the type shown in Fig. 10, Fig. 11, Fig. 32 and Fig. 41; SUBROUTINE BOSDEC2 is used to generate single-module models such as shown in Figs. 7a and 7b and multi-module models such as shown in Figs. 3a and 13a. SUBROUTINE BOSDEC2 is also used to generate valid input files, \*.inpi,  $i=2,3,4,5$ , for STAGS. These STAGS input files lead to STAGS models such as those shown in Fig 8 for local buckling and that shown in Fig. 4 for general buckling. A "half-module" portion of a complete "mich8" shell is indicated at the bottom of Fig. 3a, for example. The "half-module" portion is indicated in Fig. 3a as the part of the shell that is of axial length equal to WIDTH/2 (50 inches in the "mich8" and "mich8u" cases). In the specific cases called "mich8" and "mich8u" the "half-module" contains 8 major segments. Figures 10, 11, 20 and 27 show "half-modules" of the "mich8" and "mich8u" configurations. Figures 27 and 41 show "half-module" models of the "mich1" and "mich1b" configurations, respectively. Figure 28a shows the relationship of a "half-module" and "one full module" of the "mich1" configuration to the entire "mich1" shell. Figure 37 shows the relationship of a "half-module" and "one full module" of the "mich1b" configuration to the entire "mich1b" shell.

If a GENOPT user wants to optimize something that does not involve or use BIGBOSOR4, then the "bosdec" and "addbosor4" files play no role. However, dummy "bosdec" and "addbosor4" libraries must still be provided in order that a successful compilation will follow via the GENOPT user's command, "genprograms". By far,

most of the bosdec.michelin software used for the generation of results reported in this paper was copied from the analogous software, bosdec.span9, described in [1].

The file, **michelin.INP**, (included in the file, .../genopt/case/span/michelin.tar.gz, of [14]), is created automatically by the GENOPT processor, GENTEXT. **michelin.INP** contains an image of the GENOPT user's long interactive GENTEXT session.

In [1] there are separate “bosdec” files pertaining to the analysis of configurations without “smoothing” segments and with “smoothing” segments. An example of a “smoothing” segment taken from [1] is shown in Fig. A4. In the “michelin” application there exists only a single “bosdec” file pertaining to configurations that include “smoothing” segments. “Smoothing” segments are automatically added to the BIGBOSOR4 model if there exist “corners” (discontinuities in meridional slope) between adjacent major segments of more than 10 degrees and deviations of the meridional slope of the circumferentially corrugated shell of revolution from vertical of more than 5 degrees at the bottom and top. Figure 2c shows where “smoothing” segments have been automatically inserted in the “mich8” configuration optimized with the use of linear theory in BIGBOSOR4. Fig. 9f and Fig. 9g show where “smoothing” segments have been automatically inserted in the “mich8” configuration optimized with the use of nonlinear theory in BIGBOSOR4.

## Section 6. FINDING OPTIMUM DESIGNS

The following points pertain to optimization of specific cases that belong to the generic **michelin** system:

**1.** The GENOPT processor called SUPERDUPEROPT may be executed. SUPERDUPEROPT is a script by means of which several sequential executions of SUPEROPT are automatically conducted. The End user is asked to provide two input data:

- a. The number of sequential executions of OPTIMIZE for each execution of AUTOCHANGE.
- b. The number of sequential executions of SUPEROPT to be conducted during SUPERDUPEROPT.

Each execution of SUPEROPT requires from 2 to 24 hours of computer time on the author's very fast workstation, depending on the number of major segments plus “smoothing” segments in the BIGBOSOR4 model.

**2.** In this study, GENOPT often has difficulty finding a "global" optimum design. The SUPEROPT process often converges several times to objectives (WEIGHTs) that are significantly higher than that corresponding to the "best" design, that is, a design with the lowest objective (WEIGHT) that is either "FEASIBLE" or "ALMOST FEASIBLE". See Fig. 2a, for example, in which some results for the specific case called “mich8” are shown.

**3.** In optimizations done with the use of linear theory sometimes a good “global” optimum design can be obtained by first executing SUPEROPT with use of the so-called “652” strategy [1] with 5 iterations per execution of OPTIMIZE (Fig. 2a) followed by an execution of SUPEROPT with use of the so-called “3062” strategy with 12 iterations per execution of OPTIMIZE (Fig. 2b). In the “652” strategy there are 6 executions of OPTIMIZE per execution of AUTOCHANGE, the “move” index, IMOVE, is set equal to 5, and the IAUTOF index is set equal to 2. The End User specifies the number of iterations per execution of OPTIMIZE, the

“move” index IMOVE, and the “autochange” index IAUTOF, during the interactive execution of the GENOPT processor, MAINSETUP. An example of a valid input file for MAINSETUP is listed in Table A5. The End User specifies the number of executions of OPTIMIZE per execution of AUTOCHANGE during launching of a long SUPEROPT execution or an even longer SUPEDUPEROPT execution. The strategy parameter called “IMOVE” governs the move limits of the decision variables during each design iteration: IMOVE = 1 means the move limit of any decision variable is a maximum of 10 per cent of that decision variable per design iteration; IMOVE = 5 means the move limits are 5 per cent per iteration; IMOVE = 4 means the move limits are 2 per cent per iteration; IMOVE = 3 means the move limits are 1 per cent per iteration. IMOVE = 6 means the move limits are 0.5 per cent per iteration. The strategy parameter, IAUTOF, governs how a new “starting” design (“spikes” in Fig. 2a) is arrived at during execution of the GENOPT processor called “AUTOCHANGE”, as described in the permanent GENOPT prompting file called “URPROMPT.DAT” [14]. IAUTOF = 2 was used for all the cases treated in this paper.

**4.** Finding “global” optimum designs in the “michelin” generic case is much more difficult than in the “span9” generic case reported in [1] for two reasons:

**a.** Specific cases, such as “mich8”, “mich8u”, “mich1” and “mich1b”, are associated with maximum behavioral constraint gradients that are much higher than those experienced during the development of [1]. For “michelin” configurations, often the maximum behavioral constraint gradients have values such as 40 to 50, whereas the maximum behavioral constraint gradients experienced during the development of [1] are something like 10. Figure 17 demonstrates that a 3 percent reduction in the decision variable YPLATE(1) (from 35 inches to 34 inches) is associated with an 80 per cent reduction in the general buckling load factors associated with BUKSYM and BUKASY. Figures 23a and 23b of [1] exhibit a similar reduction in the buckling load factor for local buckling, LOCBUK, but this same reduction is associated with an 18 per cent change in one or more of the “YPLATE” decision variables rather than a mere 3 per cent change.

**b.** In the “michelin” generic case nonlinear geometric behavior plays a significant role, whereas the “span9” generic case is formulated so that only linear behavior exists. The nonlinear geometric behavior in the “michelin” configurations often causes a nonlinear optimization execution to abort during SUPEROPT, usually fairly early in an execution of SUPEROPT. Examples are displayed in Figs. 9a, 9b and 9d. These early “bombs” in the executions of SUPEROPT prevent a reasonably full exploration of the design space. Therefore, it is not known whether or not the nonlinearly optimized final designs accepted for presentation in this paper are close to “global” optima.

Plots can be obtained of the objective, variables, and design margins versus design iterations corresponding to each execution of SUPEROPT. Examples of such plots are given for linear optimization in Figs 2a,b and for nonlinear optimization in Figs. 9a-d.

**5.** The best overall strategy for finding optimum designs of specific cases with multiple major segments over the axial distance WIDTH/2, such as the specific case called “mich8”, which has 8 major segments over WIDTH/2, seems to be as follows:

**a.** First find the optimum design of a similar specific case with as few decision variables as possible. The fewer the number of decision variables the easier it is to obtain a “global” optimum design. The specific case called “mich8u” is a good choice. Since “mich8u” is a specific case with **uniform** corrugations, that is, all of the 8 major segments have the same geometry except they alternate convex surface outward and convex surface

inward, the only decision variables are the thickness of Major Segment No. 1 [THICK(1) = Variable No. 1 in Table 3b] and the half-angle of Major Segment No. 1 [PHISEG(1) = Variable No. 17 in Table 3b]. The variables, PHISEG(j), j=2, NSEG-1, are all linked to PHISEG(1) with a linking coefficient equal to 2.0, and PHISEG(NSEG) is linked to PHISEG(1) with a linking coefficient equal to 1.0. The variables, SUBWID(i), i=1, NSEG, are all fixed because of the restrictions on PHISEG(i) and because the total axial length of the 8 major segments must equal WIDTH/2, in which WIDTH is specified by the End User. None of the YPLATE(i), i=1, NSEG+1, is a decision variable because the corrugations are uniform, and the values listed in Table 3b are established so that the minimum radius of the uniformly corrugated shell of revolution exceeds 30 inches only by a small amount. (Fig. 27).

**b.** During optimization of the relatively simple “mich8u” configuration use the following strategy:

**(i).** First optimize mich8u using linear theory (ISTRAT = 1, that is, INDIC=4) in BIGBOSOR4. Use SUPEROPT.

**(ii).** Using the best ALMOST FEASIBLE or FEASIBLE design obtained from **(i)** as a starting design, execute SUPEROPT again, this time using the mildest form of nonlinear theory (ISTRAT = 5, that is INDIC =1 for both local buckling and general buckling) in BIGBOSOR4.

**(iii).** Using the best ALMOST FEASIBLE or FEASIBLE design obtained from **(ii)** as a starting design, execute SUPEROPT again, this time using a bolder form of nonlinear theory (ISTRAT = 7, that is, INDIC = 1 for local buckling and INDIC = -2 for general buckling) in BIGBOSOR4.

**(iv).** Using the best ALMOST FEASIBLE or FEASIBLE design obtained from **(iii)** as a starting design, execute SUPEROPT again, this time using the boldest form of nonlinear theory (ISTRAT = 13, that is, INDIC = -2 for both local and general buckling) in BIGBOSOR4.

**c.** Now optimize the configuration in which all the major segments have different geometry, as is so for the **nonuniformly** corrugated specific case called “**mich8**”. Establish the initial starting design via the “BEGIN” processor as the same as the final design obtained after completion of the 4-step process set forth in Item **5b**. (See Table A2, for example, although the initial starting design specified there is somewhat different from the latest optimized design of “mich8u” listed in Table 3b because certain corrections were made after Table A2 was created, and the author did not want to re-run “mich8” and therefore have to re-do many figures.) Again, follow the 4-step process set forth in **5b**.

For more on difficulties encountered during the long process of finding optimum designs, especially difficulties associated with optimization in the presence of nonlinear geometric effects, please read Section 9, “SOME DIFFICULTIES”.

## Section 7. ABOUT THE STAGS COMPUTER PROGRAM

The capabilities and properties of the STAGS computer program [37 – 42] are summarized in Appendix 1. The “official” version of STAGS, used for the STAGS models treated in this paper, was developed by Almroth, Brogan, Rankin and others over the past 40 years. For the most part, the details of the STAGS finite element model faithfully represent the modeling idealizations employed in the GENOPT/BIGBOSOR4 model. Only the standard features in STAGS need be used; the development of STAGS user-written subroutines was not required for the specific cases explored in the work leading to this paper. STAGS pre and post processors called “StagsViewer” have been created by Michael Jacoby. These are described briefly in [1].



## Section 8 NUMERICAL RESULTS

Four specific cases are explored: **mich8**, **mich8u**, **mich1** and **mich1b**. Tables 1 and 2 and Figs. 1 – 22 pertain to **mich8**; Tables 3a and 3b and Figs. 23 – 26 pertain to **mich8u**; Figure 27 pertains to both **mich8** and **mich8u**; Table 4 and Table 6 and Figs. 28 – 36 pertain to **mich1**; Table 5 and Figs. 37 – 42 pertain to **mich1b**.

The tables in Appendix 2 pertain to **mich8**. Their purpose is to provide details that will, in the future, help a researcher regenerate some of the results given in this paper and to generate additional analogous results for different configurations from those presented here.

There are captions and annotations associated with each figure that enable the reader to understand its purpose without having frequently to search the text for relevant explanations.

In most of the specific cases results for general buckling of the optimized design are presented first, followed by results for local buckling. For each type of buckling results from BIGBOSOR4 are presented first, followed by results from STAGS. General buckling models involve a relatively long section of the circumferentially corrugated shell of revolution compared to local buckling models. For example, compare the general buckling models shown in Figs. 3a-c and Fig. 4 with the local buckling models shown in Figs. 7a,b and Fig. 8.

STAGS models for general buckling, such as that shown in Fig. 4, have close to one million degrees of freedom. STAGS models for local buckling, such as that shown in Fig. 8, have about 1.6 million degrees of freedom. The local buckling STAGS models, although spanning a shorter axial section of the circumferentially corrugated shell of revolution, have more degrees of freedom than the longer general buckling models because the critical local buckling modes are associated with many circumferential waves over 180 degrees of circumference. (See Fig. 8, for example.) Therefore, for adequate convergence of the local buckling model with respect to mesh density, many STAGS finite elements are required over 180 degrees of circumference. **The STAGS 480 finite element is used in all of the STAGS models.**

In general there is very good agreement between predictions from STAGS and BIGBOSOR4, both for local buckling and general buckling, when linear theory is used to develop the optimized designs. There is good agreement between predictions of STAGS and BIGBOSOR4 for local buckling and maximum effective stress even when nonlinear theory is used to develop the optimized designs. **There is only fair agreement between predictions of STAGS and BIGBOSOR4 for general buckling when nonlinear theory is used to develop the optimized designs, with STAGS predicting somewhat lower general buckling load factors than BIGBOSOR4, a discrepancy of about 9 to 17 per cent.** It is not known for certain why this discrepancy exists. However, it is proposed that the discrepancy arises because BIGBOSOR4 is based on an approximate “classical” (legacy) “moderately large” meridional rotation theory (sine and cosine of the meridional rotation replaced, respectively, by the meridional rotation and unity), whereas STAGS is based on an exact “co-rotational” theory [41] in which the rigid body component of an arbitrarily large displacement stores no energy.

More results are given for the specific case called “**mich8**” than for the other specific cases; Table 1 and Figs. 2 – 8 give predictions from linear theory; Table 2 and Figs. 9 – 13 given predictions from nonlinear geometric theory; Figures 15 – 22 demonstrate, for an optimized design, the extreme sensitivity of the general buckling load factor to the introduction of nonlinear geometric effects (Figs. 15 and 16) and the extreme sensitivity of the

general buckling load factor obtained from linear theory to small changes in certain of the decision variables (Figs. 17 – 22).

**It is emphasized that optimization was performed with GENOPT/BIGBOSOR4.** Predictions from the general-purpose STAGS finite element computer program were obtained for optimized designs previously generated by GENOPT/BIGBOSOR4. **STAGS models are NOT used in the optimization loop.**

## 8.1 RESULTS PERTAINING TO THE SPECIFIC CASE CALLED “mich8”

The optimum design obtained with the use of **LINEAR** theory is listed in Table 1. The optimum design obtained with the use of **NONLINEAR** geometric theory is listed in Table 2.

Figure 1 is a STAGS model of the “mich8” configuration (except that “smoothing” segments are absent). While all the STAGS models used in this study subtend 180 degrees of circumference, the much simpler BIGBOSOR4 models that are used for optimization and analysis are for complete (360 degrees) circumferentially corrugated shells of revolution.

**mich8 results based on LINEAR theory:** Figures 2a and 2b are discussed in Items 2 and 3 in Section 6. Figure 2c shows a discretized BIGBOSOR4 model of the linearly optimized design. In Fig. 2c the radial coordinate is s-t-r-e-t-c-h-e-d relative to the axial coordinate. The meaning of “smoothing” segment is clarified by one of the figures in [1], reproduced in this paper as Fig. A4. The discretization depicted in Fig. 2c is typical of all the BIGBOSOR4 models treated in this paper. During optimization cycles, as the profile of the circumferentially corrugated shell evolves, different numbers of “smoothing” segments are required, depending on the meridional slope discontinuities between adjacent major segments and the deviation from vertical of the meridional slope at the bottom and top of the BIGBOSOR4 model. In this particular instance, for the linearly optimized shell, no “smoothing” segments happen to be required between Major Segments 7 and 8, nor at the bottom or top of the BIGBOSOR4 model. Figures 3a-c show the BIGBOSOR4 prediction of **general** buckling for the linearly optimized design, and Fig. 4 shows the STAGS model that yields the lowest linear **general** buckling load factor. Figures 5 and 6 show BIGBOSOR4 and STAGS predictions of **general** buckling of a much longer circumferentially corrugated shell with the same linearly optimized wall profile. These figures demonstrate that the **general** buckling model that is one third as long (MMM=3 rather than MMM=9) is long enough to predict the general buckling behavior of a very long optimized circumferentially corrugated shell with sufficient accuracy. Figures 7a,b and 8 show linear **local** buckling from BIGBOSOR4 and STAGS models of the linearly optimized “mich8” configuration. Of particular interest is that, according to BIGBOSOR4, there exist two minima of **local** buckling load factor as a function of number of circumferential waves (Fig. 7b). These minima have nearly identical **local** buckling load factors, as demonstrated in Fig. 7b. From Fig. 8 it appears that the STAGS model predicts **local** buckling with the higher number of circumferential waves, although there are so many circumferential waves it is hard to tell.

**mich8 results based on NONLINEAR geometric theory:** Figures 9a-d are discussed in Item 4b in Section 6. A significant difficulty with optimization in which nonlinear geometric theory is used in the BIGBOSOR4 model is that executions of SUPEROPT usually “bomb” fairly early, preventing a “complete” enough exploration of design space to develop anything like what might be termed a “global” optimized design. SUPEROPT “bombs” with diagnostics such as those included in Figs. 9a, b, d. SUPEROPT aborts early because of the terminations of computations in BIGBOSOR4 for known reasons, not from bugs in the

SUPEROPT process. **These early “bombings” of SUPEROPT lead to what the author of this paper regards as the weakest feature of the research presented here.** Figure 9e shows a nonlinearly optimized design that is not acceptable because the smallest radius of the circumferentially corrugated shell of revolution is somewhat less than 30 inches, violating the requirement set forth in the sub-section entitled “Statement of the problem” in Section 4. An acceptable nonlinearly optimized design, depicted in Figs. 9f,g was developed by slightly increasing the lower bounds of YPLATE(i), i=1,2,...,NSEG+1, in the input file, mich8.DEC, for the GENOPT processor called “DECIDE”. An example of input data required for mich8.DEC is listed in Table A4. The decision variables, YPLATE(i), i=1,2,...,NSEG+1, are variable numbers 33 – 41 listed in Table A4. Figure 10 shows the axisymmetric **pre-buckling deformation** of the acceptable re-optimized design shown in Figs. 9f,g. Fig. 11 shows the critical nonlinear **local buckling** mode of the re-optimized shell from BIGBOSOR4, and Fig. 12 shows the critical nonlinear **local buckling** mode from STAGS. Figures 13a,b,c show three nonlinear **general buckling** modes of the re-optimized design from BIGBOSOR4, and Figs. 14a,b show two nonlinear **general buckling** modes from STAGS. Tables A6 – A9 (Appendix 2) pertain to the nonlinearly optimized “mich8” configuration determined by GENOPT/BIGBOSOR4. Of particular importance are:

**Table A7a**, which demonstrates the type of BIGBOSOR4 computations that occur when the BIGBOSOR4 analysis indicator, INDIC = -2,

**Table A7b**, which demonstrates the difficulty that BIGBOSOR4 has in determining certain buckling load factors, and

**Table A9**, which lists a run stream appropriate for a situation in which a nonlinearly optimized design has already been determined and archived, and the user wants to generate plots from BIGBOSOR4 such as those shown in Figs. 9e, 10, 11, 13a,b,c and bifurcation buckling load factors and plots from STAGS such as those shown in Figs. 12, 14a and 14b.

**Summary of predictions from BIGBOSOR4 and STAGS for LINEARLY and NONLINEARLY optimized “mich8” shells:** The data listed here are taken from Tables 1 and 2. There is excellent agreement between BIGBOSOR4 and STAGS for local buckling, general buckling, and maximum effective stress when **linear** theory is used to optimize the “mich8” configuration and when STAGS is subsequently used in a **linear** mode to compute the behavior of the “mich8” shell previously **linearly optimized** by GENOPT/BIGBOSOR4. There is excellent agreement between BIGBOSOR4 and STAGS for local buckling and maximum effective stress when **nonlinear** theory is used to optimize the “mich8” configuration and when STAGS is subsequently used in a **nonlinear** mode to compute the behavior of the “mich8” configuration previously **nonlinearly optimized** by GENOPT/BIGBOSOR4. **However, there is only fair agreement between BIGBOSOR4 and STAGS for the prediction of nonlinear general buckling, and the nonlinear prediction by BIGBOSOR4 is unconservative.**

**From Table 1: Predictions from the “mich8” configuration optimized with the use of LINEAR theory (The linearly optimized WEIGHT of a 50-inch-long portion = 46.14 lb):**

PARAMETERS WHICH DESCRIBE BEHAVIOR (e.g. stress, buckling load)

BEH. NO.	CURRENT VALUE	DEFINITION
1	<b>1.592E+00</b>	local buckling load factor: LOCBUK(1 ) ( <b>STAGS = 1.621</b> )
2	<b>1.419E+00</b>	symmetric general buckling: BUKSYM(1 )(no STAGS model)
3	<b>1.408E+00</b>	antisymmetric general buckling: BUKASY(1 )( <b>STAGS=1.451</b> )
12	<b>5.406E+04</b>	maximum effective stress: STRESS(1 ) ( <b>STAGS = 53120 psi</b> )

Note: “No STAGS model” is indicated for BUKSYM because the “symsymgenbuck” general buckling STAGS model permits two rigid body modes of deformation: translation normal to the axis of revolution and rotation about the axis of revolution.

**From Table 2: Predictions from the “mich8” configuration optimized with the use of NONLINEAR theory (The nonlinearly optimized WEIGHT of a 50-inch-long portion = 51.83 lb):**

PARAMETERS WHICH DESCRIBE BEHAVIOR (e.g. stress, buckling load)		
BEH. NO.	CURRENT VALUE	DEFINITION
1	<b>1.590E+00</b>	local buckling load factor: LOCBUK(1 ) ( <b>STAGS = 1.6364</b> )
2	<b>1.423E+00</b>	symmetric general buckling: BUKSYM(1 ) (no STAGS model)
3	<b>1.397E+00</b>	antisymmetric general buckling: BUKASY(1 ) ( <b>STAGS=1.2259</b> )
12	<b>5.266E+04</b>	maximum effective stress: STRESS(1 ) ( <b>STAGS=51859 psi</b> )

**mich8 results that demonstrate the extreme sensitivity of general buckling load factors:** Figures 15 and 16 demonstrate the extreme sensitivity of the critical general buckling load factor to the introduction of nonlinear geometric effects in the analysis of a configuration optimized with the use of linear theory. The BIGBOSOR4 and STAGS models display the same degree of sensitivity. Figures 17 – 22 show the extreme sensitivity of the linearly optimized design to small changes in the values of certain of the decision variables. Again, the BIGBOSOR4 and STAGS models display the same degree of sensitivity.

## 8.2 RESULTS PERTAINING TO THE SPECIFIC CASE CALLED “mich8u”

The final optimum design obtained with the use of nonlinear geometric theory is listed in Table 3b.

In this sub-section and in the following sub-sections pertaining to the remaining specific cases, mich1 and mich1b, results corresponding to the optimum designs obtained from the application of **nonlinear** geometric theory are presented. As a general practice in each specific case, optimization was first performed with the use of linear theory, followed by subsequent optimizations, first with the use of the strategy ISTRAT = 5, then with use of the strategy, ISTRAT = 7, and finally with use of the strategy, ISTRAT = 13. Optimizations are performed in this order because SUPEROPT will not “bomb” during a linear optimization and is increasingly likely to “bomb” with increasing values of ISTRAT. In this way we attempt to “sneak up” on a final “global” nonlinearly optimized design obtained with the use of the most conservative nonlinear geometric theory: that employed when ISTRAT = 13, for which the BIGBOSOR4 analysis indicator, INDIC = -2, for both local and general buckling.

The “u” in the name, “**mich8u**”, signifies “**uniformly corrugated shell**”. Figures 23a,b,c show nonlinear **general buckling** predicted by BIGBOSOR4, and Fig. 24 shows the critical nonlinear **general buckling** mode and load factor from STAGS. The STAGS prediction of nonlinear general buckling is about 10 per cent lower than the most critical general buckling load factor obtained from BIGBOSOR4 (Fig. 23b). Figures 25 and 26 pertain to nonlinear **local buckling** of the nonlinearly optimized, uniformly corrugated shell of revolution.

Figure 27 shows the optimized wall profiles for three cases:

1. The “mich8” profile obtained from optimization with the use of linear theory (Table 1)

2. The “mich8” profile obtained from optimization with the use of nonlinear geometric theory (Table 2)
3. The “mich8u” profile obtained from optimization with the use of nonlinear geometric theory (Table 3b)

Of course, not only the optimized profiles differ from each other, but also the optimized shell wall thicknesses differ from each other. These thicknesses are listed in Tables 1, 2 and 3b.

### 8.3 RESULTS PERTAINING TO THE SPECIFIC CASE CALLED “mich1”

The optimum design obtained with the use of nonlinear geometric theory is listed in Table 4.

#### General comments:

The “1” in the name, “**mich1**”, signifies that there is only **one major segment** spanning the axial length, WIDTH/2. (See Figs. 28a and Fig. 32.) In this relatively simple case the End-User-specified axial length, WIDTH = 9.2 inches. This specification of WIDTH = 9.2 inches in the file, mich1.BEG, is obtained via an iterative process leading to an optimized design in which 9 full modules (major segments plus “smoothing” segments) correspond to an axial length of shell very close to 100 inches, as shown in Figs. 28a. The iterative process is needed because each half-module contains not only the single major segment of axial length=SUBWID(1), but also at least one “smoothing” segment, such as that indicated in Fig. 32. (There could also be a “smoothing” segment at the bottom of the BIGBOSOR4 model, depending on the deviation of the meridional slope from vertical there.) The total axial length of a single half-module (WIDTH/2 + x) is not known before optimization. The iterative process involves linearly optimizing mich1 several times. After each iteration (optimization via SUPEROPT) the following data entries in the mich1.BEG file

```
9.200      $ total width of the corrugated panel: WIDTH
4.600      $ projected width (x-width) of major segment: SUBWID(1)
```

can be adjusted upward or downward so that the full module axial length, WIDTH + 2x indicated in Fig. 28a, is closer to  $100/9 = 11.11111$  inches than it was in the previous iteration. Note that in the specific case, mich1, the quantity SUBWID(1) is always equal to WIDTH/2. During the iterative process, for each linearly optimized design, that is, for each iteration, the End User can determine, via the following commands, how close 9 full modules comes to spanning an axial length of WIDTH = 100 inches:

```
mainsetup (set analysis type ITYPE = 2 and NPRINT = 2)
optimize (analysis of the fixed, latest linearly optimized design)
cp mich1.BEHX2 to a working directory for BIGBOSOR4 runs.
cd <working directory for BIGBOSOR4 runs>
mv mich1.BEHX2 mich1.ALL (mich1.BEHX2 is the BIGBOSOR4 input file for
“symsymgenbuck” general buckling.)
bigbosorall (execute BIGBOSOR4 for case name mich1.)
bosorplot (obtain a plot file of the BIGBOSOR4 model.)
gv metafile.ps (show the BIGBOSOR4 model on the monitor.)
```

The plot generated by the “gv” (“ghost view”) command for the current iteration will inform the End User whether SUBWID(1) and WIDTH should be increased or decreased and approximately by how much in order that twice the axial length of the major segment plus the “smoothing” segment be close enough to 11.11111 inches (100/9 inches). In the work reported here three iterations or less were required.

**General buckling of the nonlinearly optimized mich1:** Figures 28 – 31 pertain to nonlinear general buckling. Figures 28a,b,c show predictions from BIGBOSOR4 and Fig. 29 shows predictions from STAGS for a “short” general buckling model, that is, a general buckling model with only 9 full modules spanning a total axial length of 100 inches. (See the bottom of Fig. 28a for what is meant by “one full module”.) That “short” 9-module BIGBOSOR4 model (MMM = 9) was used for nonlinear optimization. A question arises: Is this “short” 9-module general buckling model of sufficient axial length to generate a reasonably accurate prediction of general buckling of a much longer, perhaps even an infinitely long circumferentially corrugated shell? Figures 30 and 31, pertaining to a 27-module “long” general buckling model (MMM = 27), seem to answer this question affirmatively. For the “short” model with 9 full modules BIGBOSOR4 predicts the critical (lowest) general buckling load factor, 1.4144 (Fig. 28c), and STAGS predicts the critical general buckling load factor, 1.3008 (Fig. 29). The same design (that optimized with use of the “short” 9-module model) was analyzed with use of a “long” model with 27 full modules. For this “long” model BIGBOSOR4 predicts the critical general buckling load factor, 1.4499 (Fig. 30), and STAGS predicts the critical general buckling load factor, 1.2565 (Fig. 31). There is only fair agreement between the predictions of BIGBOSOR4 and STAGS for nonlinear general buckling of the nonlinearly optimized design.

**Local buckling of the nonlinearly optimized mich1:** Figures 32 and 33 show predictions from BIGBOSOR4 and STAGS. The BIGBOSOR4 model is a “half-module” model, whereas the STAGS model is a “full-module” model. This difference arises from the fact that “smoothing” segments are not included in the STAGS “half-module” model, the input data for which are generated in SUBROUTINE BEHX1. SUBROUTINE BEHX1 in [1] was written before the concept of “smoothing” segments was introduced for STAGS models, and it was never updated to incorporate “smoothing” segments in STAGS models. **There is good agreement between BIGBOSOR4 and STAGS with respect to both nonlinear local buckling load factor and nonlinear local buckling mode shape.**

**Nonlinear axisymmetric pre-buckling load-displacement behavior of mich1:** Figures 34 – 36 pertain to this sub-sub-section. The BIGBOSOR4 “half-module” models displayed in Figs. 34 and 35 are used to generate the BIGBOSOR4 predictions. The end conditions are symmetry at the top of the BIGBOSOR4 model and “moving” symmetry at the bottom of the BIGBOSOR4 model. “Moving” symmetry means symmetry conditions are imposed except that under uniform external lateral normal pressure the shell of revolution is free to expand or contract in the axial direction. No axial load is applied at the ends of the shell.

There are two distinct types of nonlinear axisymmetric pre-buckling behavior under the uniform external normal pressure:

**a.** The shell **lengthens** and the load-end-displacement curve exhibits a **stiffening** characteristic with increasing external pressure. This type of nonlinear axisymmetric pre-buckling behavior occurs when  $YPLATE(1) > YPLATE(2)$ , in which  $YPLATE(i)$  = the radius from the axis of revolution to the shell reference surface at the  $i$ th end of the major segment. (See Fig. 34.)

**b.** The shell **shortens** and the load-end-displacement curve exhibits a **softening** characteristic with increasing external pressure. This type of axisymmetric pre-buckling behavior occurs when  $YPLATE(2) > YPLATE(1)$ . (See Fig. 35.)

Figure 36 shows pre-buckling load-end-displacement curves predicted by BIGBOSOR4 and STAGS for optimized “mich1” circumferentially corrugated shells of revolution with  $YPLATE(2) < YPLATE(1)$  (the

**stiffening** curves) and with  $YPLATE(2) > YPLATE(1)$  (the **softening** curves). There is good agreement between the predictions from BIGBOSOR4 and STAGS for the nonlinear axisymmetric pre-buckling behavior..

## 8.4 RESULTS PERTAINING TO THE SPECIFIC CASE CALLED “mich1b”

The optimum design obtained with the use of nonlinear geometric theory is listed in Table 5.

The general comments at the beginning of the sub-section on mich1 apply also to the specific case, mich1b. The specific case, mich1b, differs from mich1 in that the one major segment in the half-module model shown in Fig. 41 curves inward rather than outward, as shown in Fig. 32, which pertains to the mich1 case. Because of this, the “smoothing” segment also has the opposite meridional curvature, as is demonstrated in Figs. 41 and 32.

**General buckling of the nonlinearly optimized mich1b:** Figures 37 – 40 pertain to nonlinear general buckling. Compared to the mich1 case, there is a more significant difference in the general buckling load factor obtained from the “short” 9-module general buckling model (Figs. 37 and 38) and from the “long” 27-module general buckling model with the same nonlinearly optimized profile (Figs. 39 and 40). From the annotations in Fig. 39 it is seen that the “short” general buckling BIGBOSOR4 model yields a critical general buckling load factor close to 1.4, whereas the “long” general buckling BIGBOSOR4 model yields a critical general buckling load factor close to 1.3. The STAGS “short” and “long” general buckling models yield buckling load factors, 1.3327 and 1.1712, respectively. Because of the significant differences in the general buckling predictions from the “mmm9” and “mmm27” BIGBOSOR4 models, the “mich1b” shell was **re-optimized** with use of the “mmm27” general buckling model in the optimization loop. Dimensions and predictions for the **re-optimized** “mich1b” model are listed in the bottom part of Table 5, the part starting with the text entitled “NOTES”.

**Local buckling of the nonlinearly optimized mich1b:** Figures 41 and 42 show predictions from BIGBOSOR4 and STAGS before the re-optimization referred to in the previous paragraph. **There is good agreement between BIGBOSOR4 and STAGS with respect to both nonlinear local buckling load factor and nonlinear local buckling mode shape.**

## Section 9. SOME DIFFICULTIES

Several characteristics associated with the optimization of circumferentially corrugated shells of revolution were encountered during this work that make it difficult to find “global” optimum designs:

**1.** Certain of the behavioral constraints have very high gradients, as listed in Table 6, for example. High behavioral constraint gradients exist when one or more of the design margins for an optimized design are extremely sensitive to perturbations in certain of the decision variables. For example, Figs. 17 and 18 (linearly optimized “mich8” configuration) demonstrate extreme sensitivity of the design margins for general buckling, BUKSYM and BUKASY, with respect to perturbations in  $YPLATE(1)$  and  $YPLATE(9)$ . From Table 6 (nonlinearly optimized “mich1” configuration) we see that the “G R A D I E N T S OF C O N S T R A I N T S” associated with  $YPLATE(1)$  (decision variable number 3) and  $YPLATE(2)$  (decision variable number 4) are much higher than those associated with  $THICK(1)$  and  $PHISEG(1)$ . The constraint gradients associated with  $THICK(1)$  and  $PHISEG(1)$  are of a size that has typically been encountered in the past experience of the author.

2. In cases with significant nonlinear axisymmetric pre-buckling behavior, such as that exhibited in Fig. 36, optimization computer executions of SUPEROPT (or OPTIMIZE) often “bomb” early in the execution because of the failure of BIGBOSOR4 to determine a converged nonlinear bifurcation buckling load factor, both for local buckling and for general buckling. Failure to determine a converged nonlinear **local** bifurcation buckling load factor is usually caused by a “clustering” of eigenvalues in the neighborhood of the critical (lowest) eigenvalue. Failure to determine a converged **general** bifurcation buckling load factor is often caused by axisymmetric collapse of a non-optimum design at a load factor lower than the lowest buckling load factor corresponding to non-axisymmetric bifurcation buckling.

3. There may be closely spaced critical **local** bifurcation buckling load factors associated with widely separated critical numbers of circumferential waves, such as is demonstrated for linear bifurcation buckling in Figs. 7a and 7b. In nonlinear analyses of configurations that exhibit this behavior, BIGBOSOR4 in its rather elaborate search for a minimum local bifurcation buckling load factor, may arrive at critical number of circumferential waves that represents a local minimum, not the absolute minimum.

4. There may be closely spaced critical **general** bifurcation buckling load factors associated with widely separated numbers of circumferential waves, such as is demonstrated in the following list of buckling load factors for bifurcation buckling with use of the ISTRAT = 5 strategy (BIGBOSOR4 analysis indicator, INDIC = 1):

GENERAL BUCKLING LOAD FACTORS AND MODES (n)

symmetry at the bottom; symmetry at the top.

1.1075E+02(	0)	
1.2681E+02(	1)	
<b>2.0828E+00(</b>	<b>2)</b>	<b>←first minimum eigenvalue versus circ. waves, n</b>
9.1414E+00(	3)	
1.5711E+01(	4)	
1.3734E+01(	5)	
<b>2.8937E+00(</b>	<b>6)</b>	<b>←second minimum eigenvalue versus circ. waves, n</b>
2.9735E+00(	7)	
3.2638E+00(	8)	
2.8698E+00(	9)	
<b>2.0653E+00(</b>	<b>10)</b>	<b>←third minimum eigenvalue versus circ. waves, n</b>

Critical buckling load factor, BUKSYM= 2.0653E+00

Critical number of circumferential waves, NWVCRT= 10

The general buckling load factors for n = 2 circumferential waves (2.0828) and for n = 10 circumferential waves (2.0653) are very close. This characteristic gives rise to the following difficulties:

a. In this example the third minimum buckling load factor (the most critical one) is at the end of the End-user-specified range of circumferential waves for general buckling: n from 0 to 10. If this happens when the BIGBOSOR4 analysis indicator, INDIC = -2, BIGBOSOR4 will bomb with the diagnostic: “WAVENUMBER N EXCEEDS RANGE NMINB.LE.N.LE.NMAXB”

The INDIC = -2 strategy involves a search over n in the neighborhood of the critical value of n, which in this example is at the top of the specified range of n: n = 10. When, in its search over n, BIGBOSOR4 attempts to



determine a nonlinear buckling load factor corresponding to  $n = 11$ , BIGBOSOR4 “bombs” and the optimization process in SUPERPT (or OPTIMIZE) is therefore abruptly terminated.

**b.** The “current” (unperturbed) design and the “perturbed” design may have very different critical buckling modes. In this particular example, for the unperturbed design, the buckling load factor corresponding to  $n = 2$  circumferential waves happens to be lower than that corresponding to  $n = 10$  circumferential waves, whereas the opposite holds for the perturbed design, as listed above. As a result of this difference in behavior exhibited by the unperturbed and perturbed designs, the constraint gradient associated with the particular decision variable that is perturbed is inappropriate. As a result the design may be driven in the wrong direction in design space.

**5.** In the case of circumferentially corrugated shells of revolution subjected to uniform normal pressure it is possible that general and/or local bifurcation buckling occur both with uniform **external** pressure and uniform **internal** pressure. This phenomenon causes difficulty whenever SUBROUTINE EBAND2 in BIGBOSOR4 converges to a **negative** eigenvalue (root) or whenever SUBROUTINE EBAND2 has difficulty shifting sufficiently far away from a **negative** root to allow convergence to the lowest **positive** root. A configuration such as that shown in Fig. 34 has a stiffening load-end-axial-displacement curve with increasing **external** pressure. (See Fig. 36.) Under **internal** pressure, however, the configuration shown in Fig. 34 has a softening load-end-axial-displacement curve, and therefore axisymmetric collapse is possible. The same observation applies to Fig. 10. Also, under uniform **internal** pressure critical general bifurcation buckling with  $n = 1$  circumferential wave is possible. Figures 43 – 45 show an example of the behavior of the nonlinearly optimized “mich8” shell (Table 2) under INTERNAL pressure of +2.0 psi rather than EXTERNAL pressure of –20.0 psi. Figure 43 shows that under INTERNAL pressure the “mich8” shell contracts in the axial direction, which is the opposite behavior from that displayed in Fig. 10 that corresponds to the application of external pressure. Figure 44 shows the BIGBOSOR4 prediction of the critical general buckling mode and load factor of the “mich8” shell under INTERNAL pressure. Figure 45 shows the STAGS prediction of the same. There is reasonably good agreement between BIGBOSOR4 and STAGS. The most important thing to notice from Fig. 44 is that the critical buckling load factor of the **internally** pressurized “mich8” shell is much, much smaller than that of the externally pressurized shell for the same absolute value of pressure. The very low critical buckling load factor obtained for the **internally** pressurized “mich8” shell is what causes SUBROUTINE EBAND2 in BIGBOSOR4 occasionally to accept a negative root or to have difficulty shifting away from a negative root in the calculations corresponding to  $n = 1$  circumferential wave. (A negative root is an indication that bifurcation buckling occurs with applied loads of the opposite signs than those specified in the BIGBOSOR4 input file.)

**6.** In order to minimize the number of occasions on which SUPEROPT (or OPTIMIZE) “bombs” during execution, the author has introduced into BIGBOSOR4 several modifications to the strategy used there to determine the nonlinear critical buckling load factor in cases for which the analysis indicator,  $INDIC = -2$ . However, **“bombs” of SUPEROPT and OPTIMIZE computer runs still occur frequently enough to prevent the reliable determination of “global” optimum designs for cases in which nonlinear geometric effects are included and are significant.** The recent modifications to BIGBOSOR4 are identified in a general sense in Appendix 3.

## Section 10. CONCLUSIONS

1. There is **good agreement** between the predictions of all types of bifurcation buckling from BIGBOSOR4 and STAGS **when linear theory is used**. (Table 1, Fig. 4, Fig. 8)
2. There is **good agreement** between the predictions of **LOCAL bifurcation buckling** from BIGBOSOR4 and STAGS **when either linear or nonlinear geometric theory is used**. (Table 2, Fig. 12, Fig. 26, Fig. 33, Fig. 42)
3. There is **only fair agreement** between the predictions of **GENERAL bifurcation buckling** from BIGBOSOR4 and STAGS **when nonlinear geometric theory is used**. STAGS predicts general buckling loads that are from 9 to 17 per cent lower than those predicted by BIGBOSOR4, perhaps because STAGS uses an “exact” co-rotational formulation whereas BIGBOSOR4 uses an approximate (legacy) nonlinear formulation in which the sine of the meridional rotation is replaced by the meridional rotation in the nonlinear governing equations. (Table 2, Figs. 14a and 14b, Fig. 24, Fig. 31, Fig. 40)
4. The maximum axisymmetric **pre-buckling effective stress** predicted by BIGBOSOR4 **agrees well with that from STAGS when either linear or nonlinear geometric theory is used** (Tables 1, 2, 3b, 4, 5).
5. **Optimization** by GENOPT/BIGBOSOR4 in the presence of nonlinear pre-buckling behavior **often fails early** during the execution of the GENOPT processor called “SUPEROPT”, either because the shell collapses axisymmetrically in the pre-buckling stage during an optimization cycle, or because the BIGBOSOR4 bifurcation buckling computations enter a potentially infinite loop in which the nonlinear buckling load factor oscillates between two widely separated values, or because BIGBOSOR4 accepts a negative root or has difficulty shifting away from a negative root.. **These unfortunate situations severely hamper the search for a “global” optimum design obtained with the use of nonlinear geometric theory**. (Figs. 9a, 9b and 9d)
6. The buckling load factors of optimized circumferentially corrugated shells of revolution are **extremely sensitive to small changes in certain of the decision variables** [especially YPLATE(i)]. This property leads to very high gradients of behavioral constraints that make it difficult to find “global” optimum designs (Figs. 17 – 22, Table 6).
7. When optimizing, one should include nonlinear geometric effects during the ultimate stages of optimization (Figs. 15 and 16).
8. The **axisymmetric nonlinear pre-buckling behavior predicted by BIGBOSOR4 agrees well with that predicted by STAGS** (Fig. 36), at least in one case (“mich1”).
9. The software for the analysis of circumferentially complexly corrugated shells of revolution was rather easily developed by relatively simple modification of the software for the analysis of longitudinally complexly corrugated prismatic panels [1].

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## APPENDIX 1

### ABOUT STAGS (STructural Analysis of General Shells)

In this paper optimum designs obtained by GENOPT/SPAN9 are evaluated later via STAGS models. **NOTE: STAGS is not used inside the optimization loop.**

STAGS [37 - 42] is a finite element code for the **general-purpose nonlinear analysis of stiffened shell structures of arbitrary shape and complexity**. Its capabilities include stress, stability, vibration, and transient analyses with both material and geometric nonlinearities permitted in all analysis types. STAGS includes enhancements, such as a higher order thick shell element, more advanced nonlinear solution strategies, and more comprehensive post-processing features such as a link with the STAGS postprocessor, STAPL.

Research and development of STAGS by Brogan, Almroth, Rankin, Stanley, Cabiness, Stehlin and others, formerly of the Computational Mechanics Department of the Lockheed Palo Alto Research Laboratory, was under continuous sponsorship from U.S. government agencies and internal Lockheed funding for more than 40 years. During this time particular emphasis was placed on improvement of the capability to solve difficult nonlinear problems such as the prediction of the behavior of axially compressed stiffened panels loaded far into their locally post-buckled states. STAGS has been extensively used worldwide for the evaluation of stiffened panels and shells loaded well into their locally post-buckled states.

**A large rotation algorithm that is independent of the finite element library** was incorporated into STAGS. With this algorithm there is no artificial stiffening due to large rotations. The finite elements in the STAGS

library do not store energy under arbitrary rigid-body motion and the first and second variations of the strain energy are consistent. These properties lead to quadratic convergence during Newton iterations.

Solution control in nonlinear problems includes specification of load levels or use of the **advanced Riks-Crisfield path parameter** that enables traversal of limit points into the post-buckling regime. Two load systems with different histories (Load Sets A and B) can be defined and controlled separately during the solution process. Flexible restart procedures permit switching from one strategy to another during an analysis. This includes shifts from bifurcation buckling to nonlinear collapse analyses and back and shifts from static to transient and transient to static analyses with modified boundary conditions and loading. STAGS provides solutions to the generalized eigenvalue problem for **buckling and vibration from a linear or nonlinear stress state**.

Quadric surfaces can be modeled with minimal user input as individual substructures called "**shell units**" in which the analytic geometry is represented exactly. "Shell units" can be connected along edges or internal grid lines with partial or complete compatibility. In this way complex structures can be assembled from relatively simple units. Alternatively, a structure of arbitrary shape can be modeled with use of "element units".

Geometric imperfections can be generated automatically in a variety of ways, thereby permitting imperfection-sensitivity studies to be performed. For example, **imperfections can be generated by superposition of several buckling modes determined from previous STAGS analyses of a given case**.

A variety of material models is available, including both plasticity and creep. STAGS handles isotropic and anisotropic materials, including composites consisting of up to 60 layers of arbitrary orientation. Four plasticity models are available, including isotropic strain hardening, the White Besseling (mechanical sublayer model), kinematic strain hardening, and deformation theory.

Two independent load sets, each composed from simple parts that may be specified with minimal input, define a spatial variation of loading. Any number of point loads, prescribed displacements, line loads, surface tractions, thermal loads, and "live" pressure (hydrostatic pressure which remains normal to the shell surface throughout large deformations) can be combined to make a load set. For transient analysis the user may select from a menu of loading histories, or a general temporal variation may be specified in a user-written subroutine.

Boundary conditions (B.C.) may be imposed either by reference to certain standard conditions or by the use of single- and multi-point constraints. Simple support, symmetry, anti-symmetry, clamped, or user-defined B.C. can be defined on a "shell unit" edge. Single-point constraints which allow individual freedoms to be free, fixed, or a prescribed non-zero value may be applied to grid lines and surfaces in "shell units" or "element units". A useful feature for buckling analysis allows these constraints to differ for the prestress and eigenvalue analyses. Lagrangian constraint equations containing up to 100 terms may be defined to impose multi-point constraints.

STAGS has a variety of finite elements suitable for the analysis of stiffened plates and shells. Simple four node quadrilateral plate elements with a cubic lateral displacement field (called "410" and "411" elements) are effective and efficient for the prediction of post-buckling thin shell response. A linear (410) or quadratic (411) membrane interpolation can be selected. For thicker shells in which transverse shear deformation is important, STAGS provides the Assumed Natural Strain (ANS) nine node element (called "480" element). A two-node beam element compatible with the four-node quadrilateral plate element is provided to simulate stiffeners and

beam assemblies. Other finite elements included in STAGS are described in the STAGS literature [37 – 42].

## APPENDIX 2

Figures A1 – A4 and Tables A1 – A9 are included in this appendix.

The purposes of Figs. A1 – A4 are to relate the present paper to the different but analogous configurations studied in [1] and to explain how the problem studied in [1] (longitudinally corrugated prismatic panels) is transformed into the problem studied here (circumferentially corrugated shells of revolution, Fig. A2b). This transformation is explained in the caption of Fig. A2 and in Fig. A2b.

Table A1 presents a glossary of variables used in the generic “michelin” case. This table is identical to Table 1 in [1]. As explained in the main text of the present paper, a few of the variable names, their one-line definitions and their “help” paragraphs are inappropriate for the circumferentially corrugated shells of revolution that are the subject of the present paper.

Tables A2 – A5 list input data for the four GENOPT processors, BEGIN (Table A2), CHANGE (Table A3), DECIDE (Table A4) and MAINSETUP (Table A5). Table A6 lists a somewhat abridged version of the GENOPT/BIGBOSOR4 output contained in the mich8.OPM file corresponding to ISTRAT = 13. (ISTRAT = 13 means that the BIGBOSOR4 analysis type indicator, INDIC = -2 for both local and general buckling.) The presentation of the mildly abridged mich8.OPM file listed in Table A6 is less abridged than that listed in Table 2. Tables A7a and A7b give some details about how the BIGBOSOR4 nonlinear analysis is performed when the BIGBOSOR4 analysis indicator, INDIC = -2. Table A8 lists a typical input file, \*.bin, for a nonlinear buckling analysis to be performed by STAGS. An example of a typical STAGS input file, \*.inp, is too long to include in this paper.

Table A9 lists a run stream appropriate for a situation in which a nonlinearly optimized design (“mich8”) has already been determined by GENOPT/BIGBOSOR4, this optimized design has been archived in an input file for the GENOPT processor called “CHANGE”, and one wants to obtain:

1. plots from BIGBOSOR4 of the discretized model (Fig. 9e), the pre-buckled state of the shell (Fig. 10), the local buckling mode (Fig. 11), and the three general buckling modes, “symsymgenbuck” (Fig. 13a), “symantigenbuck” (Fig. 13b), and “antiantigenbuck” (Fig. 13c).
2. bifurcation buckling load factors and plots from STAGS of local buckling (Fig. 12), “symantigenbuck” general buckling (Fig. 14a), and “antiantigenbuck” general buckling (Fig. 14b).

## APPENDIX 3

This is Item No. 42 in the file, /home/progs/bigbosor4/doc/bigbosor4.news:

42. February – May 2014

The strategy used in connection with INDIC = -2 was modified as a result of trying to optimize circumferentially corrugated



shells of revolution subject to external lateral normal pressure (the project called "michelin"). The modifications are identified by bracketed "comment" records: "BEG FEB 2014 - - "END FEB 2014", "BEG MAR 2014 - - "END MAR 2014", "BEG APR 2014 - - "END APR 2014", "BEG MAY 2014 - - "END MAY 2014". The new coding occurs mainly in B4MAIN and EBAND2. The purpose is to avoid:

1. BIGBOSOR4 getting into an infinite loop
2. BIGBOSOR4 bombing during an INDIC=-2 execution.

The "infinite loop" problem seems to have been solved, but the "bombing" problem, although occurring less frequently than before, still happens occasionally, especially in the following situations:

1. There exist negative eigenvalues with smaller load factors than those corresponding to the lowest positive eigenvalue.
2. There exist a cluster of positive eigenvalues near the critical (smallest) positive eigenvalue.
3. The critical bifurcation buckling load factor is in a region where nonlinear effects are significant, especially when the critical bifurcation point is close to the load at which the shell collapses axisymmetrically.

NOTE: HUGEBSOR4 was not updated.