1.1 - Introduction and overview of machine learning concepts

Exercise 1.15.

In a test of 'psychometry', the car keys and wristwatches of 5 people are given to a medium. The medium then attempts to match the wristwatch with the car key of each person. What is the expected number of correct matches that the medium will make (by chance)? What is the probability that the medium will obtain at least 1 correct match?

Answer:

Matching distribution equation:

$$P(x) = \frac{1}{x!}\left[\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}..... \pm \frac{1}{n-x!}\right]$$

$$P(No\ match) = \frac{1}{0!}\left[\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right] = 0.3664$$

$$P(1\ match) = \frac{1}{1!}\left[\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right] = 0.3747$$

$$P(2\ matches) = \frac{1}{2!}\left[\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right] = 0.1665$$

$$P(3\ matches) = \frac{1}{3!}\left[\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!}\right] = 0.0835$$

$$P(4\ matches) = \frac{1}{4!}\left[\frac{1}{0!} - \frac{1}{1!}\right] = 0$$

(since there cannot be 4 matches without 5 matches)

$$P(5\ matches) = \frac{1}{4!}\left[\frac{1}{0!}\right] = 0.0083$$

$$E(Number\ of\ correct\ matches) = (0*0.3664)+(1*0.3747)+(2*0.1665)+ (3*0.0835)+(5*0.0083) = 0.9997$$

$$P(\geq 1\ match) = 1 - P(No\ match) = 1 - 0.3664 = 0.6336$$

Exercise 1.18.

Sally is new to the area and listens to some friends discussing about another female friend. Sally knows that they are talking about either Alice or Bella but doesn't know which. From previous conversations Sally knows some independent pieces of information: She's 90% sure that Alice has a white car, but doesn't know if Bella's car is white or black. Similarly, she's 90% sure that Bella likes sushi, but doesn't know if Alice likes sushi. Sally hears from the conversation that the person being discussed hates sushi and drives a white car. What is the probability that the friends are talking about Alice? Assume maximal uncertainty @@ in the absence of any knowledge of the probabilities.

Answer:

KNOWNS:

- P(Car=White|Person=Alice) = 0.9
- P(Car= ~White | Person = Alice) = 0.1
- P(Sushi = Tr| Person=Bella) = 0.9
- P(Sushi=Fa | Person=Bella) = 0.1
- P(Car = White) = 1

PRINCIPLE OF MAXIMAL UNCERTAINTY:

- P(Person = Alice) = P(Person = Bella) = 0.5
- P(Sushi = False| Person=Alice) = 0.5
- P(Car=White|Person=Bella) = 0.5

P(Person= Alice |Sushi = False , Car= White) =

[P(Sushi = False , Car= White|Person= Alice) * P(Alice)]/P(Sushi = False , Car= White)

P(Sushi = False , Car= White) =

P(Sushi = False , Car= White | Person=Alice) *P(Person= Alice) +P(Sushi = False , Car= White | Person=Bella) *P(Person= Bella)

P(Sushi = False , Car= White) = (0.1*0.5*0.5) + (0.9*0.5*0.5) = 0.25

(0.5*0.9*0.5)/0.25

P(Person= Alice |Sushi = False , Car= White) = 0.9