T.D. 7 : Calcul d'intégrales **CORRECTION**

Calculer les intégrales suivantes :

i) $\int_0^{\pi} x\cos x\,dx = \left[x\sin x\right]_0^{\pi} - \int_0^{\pi}\sin x\,dx = -2$

(I.P.P. $u(x) = x$; $v'(x) = \cos x$)

ii) $\int_{-1}^{0}(x+2)\,e^{x+2}\,dx = \left[(x+2)e^{x+2}\right]_{-1}^{0} - \int_{-1}^{0}e^{x+2}dx = e^2$

(I.P.P. $u(x) = x + 2$; $v'(x) = e^{x+2}$)

iii) $\int_0^{\frac{\pi}{2}} x^2\sin x\,dx = \left[-x^2\cos x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}}-2x\cos x\,dx = 2\left[x\sin x\right]_0^{\frac{\pi}{2}} - 2\int_0^{\frac{\pi}{2}}\sin x\,dx = \pi - 2$

(2 I.P.P. $u(x) = x^2$, $v'(x) = \sin x$ puis $u(x) = x$, $v'(x) = \cos(x)$)

iv) $\int_0^1 \frac{1}{\sqrt{x+1}}\,dx = \left[2\sqrt{x+1}\right]_0^1 = 2\sqrt{2} - 2$

v) $\int_0^{2\pi}(x^2+x)\,e^{2x}\,dx = \left[\frac{1}{2}(x^2+x)e^{2x}\right]_0^{2\pi} - \frac{1}{2}\int_0^{2\pi}(2x+1)e^{2x}dx$

$$= (2\pi^2+\pi)e^{4\pi} - \frac{1}{2}\left(\left[\frac{1}{2}(2x+1)e^{2x}\right]_0^{2\pi} - \int_0^{2\pi}e^{2x}dx\right) = 2\pi^2 e^{4\pi}$$

(2 I.P.P. $u(x) = x^2 + x$, $v'(x) = e^{2x}$ puis $u(x) = 2x + 1$, $v'(x) = e^{2x}$)

vi) $\int_0^t x\sqrt{1+x^2}\,dx = \left[\frac{1}{3}(1+x^2)^{\frac{3}{2}}\right]_0^t = \frac{1}{3}\left((1+t^2)^{\frac{3}{2}} - 1\right)$

vii) $\int_0^t \frac{1}{e^x+1}\,dx = \int_1^{e^t}\frac{du}{u(1+u)} = \int_1^{e^t}\left(\frac{1}{u} - \frac{1}{1+u}\right)du = \left[\ln|u| - \ln|1+u|\right]_1^{e^t} = t - \ln(1+e^t) + \ln(2)$

(changement de variable : $u = e^x$, $x = \ln(u)$, $dx = \frac{du}{u}$)

viii) $\int_1^2 \ln(4x-1)dx = \left[x\ln(4x-1)\right]_1^2 - \int_1^2\frac{4x}{4x-1}dx = 2\ln 7 - \ln 3 - \int_1^2\left(1 + \frac{1}{4x-1}\right)dx$

$$= 2\ln 7 - \ln 3 - \left[x + \frac{1}{4}\ln(4x-1)\right]_1^2 = \frac{7}{4}\ln 7 - \frac{3}{4}\ln 3 - 1$$

(I.P.P. $u(x) = \ln(4x-1)$, $v'(x) = 1$)

ix) $$\int_0^1 x\,\mathrm{e}^{x^2}\,dx=\left[\frac{1}{2}\mathrm{e}^{x^2}\right]_0^1=\frac{\mathrm{e}-1}{2}$$

x) $$\int_1^{\mathrm{e}}\frac{1}{x+x(\ln x)^2}\,dx=\int_0^1\frac{du}{1+u^2}=\left[\operatorname{Arctan} u\right]_0^1=\frac{\pi}{4}$$

(changement de variable $u=\ln(x)$, $x=\mathrm{e}^u$, $dx=\mathrm{e}^u\,du$)

xi) $$\int_0^t\ln\left(1+x^2\right)dx=\left[x\ln\left(1+x^2\right)\right]_0^t-\int_0^t\frac{2x^2}{1+x^2}dx=t\ln\left(1+t^2\right)-2\int_0^t\left(1-\frac{1}{1+x^2}\right)dx$$

$$=t\ln\left(1+t^2\right)-2\left[x-\operatorname{Arctan} x\right]_0^t=t\ln\left(1+t^2\right)-2t+2\operatorname{Arctan} t$$

(I.P.P. $u(x)=\ln(1+x^2)$, $v'(x)=1$)

xii) $$\int_0^1 x\ln\left(1+x^2\right)dx=\left[\frac{x^2}{2}\ln\left(1+x^2\right)\right]_0^1-\int_0^1\frac{x^3}{1+x^2}dx=\frac{1}{2}\ln 2-\int_0^1\left(x-\frac{x}{1+x^2}\right)dx$$

$$=\frac{1}{2}\ln 2-\left[\frac{x^2}{2}-\frac{1}{2}\ln\left(1+x^2\right)\right]_0^1=\ln 2-\frac{1}{2}$$

(I.P.P. $u(x)=\ln(1+x^2)$, $v'(x)=x$)

xiii) $$\int_{-1}^1\frac{\mathrm{e}^x}{\mathrm{e}^x+1}\,dx=\left[\ln\left(\mathrm{e}^x+1\right)\right]_{-1}^1=1$$

xiv) $$\int_0^t\frac{\mathrm{e}^{2x}}{\mathrm{e}^x+1}\,dx=\int_0^{\mathrm{e}^t}\frac{u}{1+u}\,du=\int_0^{\mathrm{e}^t}\left(1-\frac{1}{1+u}\right)du=\mathrm{e}^t-1-\ln\left(1+\mathrm{e}^t\right)+\ln 2$$

(changement de variable $u=\mathrm{e}^x$, $x=\ln(u)$, $dx=\dfrac{du}{u}$)