State Feedback: The CSU Method

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- Standard Approach
  - First-Order Differential Equation
$$\dot{y} = -ay + bu$$
  - Assign State Variable
$$x_1 = y$$
  - State Equations
$$\dot{x}_1 = -ax_1 + bu$$
$$y = x_1$$
  - State Feedback
$$u = -kx_1$$
$$\dot{x}_1 = -ax_1 - bkx_1 = (-a - bk)x_1$$
    * Use PLACE command in MATLAB to find k
- CSU Approach
  - Assign state for unknown disturbance
$$x_1 = y$$
$$x_2 = -ay$$
  - State Equations
$$\dot{x}_1 = x_2 + bu$$
$$\dot{x}_2 = (-a\dot{y}) = h$$
  - Add state to represent unknown
$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} b \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ h \end{bmatrix} = Ax + Bu + w$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x = Cx$$

- Ideal plant (perfect integrator)
  * let f represent total unknown and disturbance

$$u = \frac{-\hat{f} + u_0}{b}$$

$$\dot{y} = f + b\frac{-\hat{f} + u_0}{b} = \left(f - \hat{f}\right) + u_0 = u_0$$

- State Approximations
  * as $z \to x$, $z_1 \to y$, $z_2 \to f$

$$\dot{z} = Az + Bu + L\left(y - \hat{y}\right)$$

$$\hat{y} = Cz$$

  * let $L = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} u + \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} (y - \hat{y})$$

  * Since $\hat{y} = Cz$

$$L\hat{y} = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$\begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} (y - \hat{y}) = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} y - \begin{bmatrix} \beta_1 & 0 \\ \beta_2 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} -\beta_1 & 1 \\ -\beta_2 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} b & \beta_1 \\ 0 & \beta_2 \end{bmatrix} \begin{bmatrix} u \\ y \end{bmatrix}$$

- Controller Bandwidth

$$\dot{y} = u_0 = k_p(r - y)$$

  * Closed loop transfer function (low-pass filter corner frequency at k_p)
  * Refer to k_p as ω_c and tune by frequency

$$\frac{k_p/s}{1 + k_p/s} = \frac{k_p}{s + k_p}$$

- Observer Bandwidth
  * Choose β_1 and β_2 so that A matrix contains correct eigenvalues

$$\lambda(s) = |sI - A| = \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -\beta_1 & 1 \\ -\beta_2 & 0 \end{bmatrix} \right| = \begin{vmatrix} s + \beta_1 & -1 \\ \beta_2 & s \end{vmatrix}$$

$$\lambda(s) = s\left(s + \beta_1\right) + \beta_2 = s^2 + \beta_1 s + \beta_2$$

* Set both eigenvalues to ω_o

$$\omega_o = \lambda_1 = \lambda_2$$

$$\lambda(s) = (s + \omega_o)^2 = s^2 + 2\omega_o + \omega_o^2$$

$$\beta_1 = 2\omega_o$$

$$\beta_2 = \omega_o^2$$