Optimum Detection of Binary Signals in Rayleigh Fading Channels with Imperfect Channel Estimates

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Abstract— **The optimum detection of binary antipodal signals in additive white Gaussian noise (AWGN) channels with Gaussian channel estimation error has been studied in prior work. In this paper, we present the optimum detector based on the maximum-likelihood criterion for binary orthogonal signals in the presence of Gaussian distributed channel estimation error and AWGN. It is shown that the optimum detector is a linear combination of the optimum coherent and optimum noncoherent detectors. We derive the exact closed-form expression of the average bit error probability of the proposed optimum detector in Rayleigh fading channels with AWGN. It is found that if the variance of channel estimation error for a given average SNR is greater than a threshold, then orthogonal signalling outperforms antipodal modulation, and the analytical expression of this threshold is derived.**

I. Introduction

The structure of optimum detectors at the receiver of communication systems has been investigated in the literature [1], [2]. For coherent reception, when the channel information is precisely known at the receiver, the optimum detection in additive white Gaussian noise (AWGN) channels can be obtained by passing the received signals through a matched filter in the case of binary antipodal signalling or through a pair of matched filters in the case of binary orthogonal signalling. For noncoherent reception, when it is not possible for the receiver to perform channel estimation, and thus no channel estimate is available at the receiver, the optimum detector in AWGN channels is the envelope detector or square-law detector. The performance of these detectors have been studied in both non-faded and fading channels [1]-[4].

In practice channel information cannot be estimated without error at the receiver. Given the statistics of the channel estimation error, we can derive the optimum detector which utilizes these statistics to achieve a superior performance. Therefore, the structure and performance of this optimum detector depends on the statistics of the channel estimation error.

In [5], the authors have designed the optimum detector of binary signals for AWGN channels in the presence of phase error where the phase error has a Tikhonov distribution. This type of error emerges from using a phased-locked loop to estimate the phase of the channel. They have shown that the optimum detector in that case, which is known as the partially coherent detector, is a linear combination of the optimum coherent detector and optimum noncoherent detector. Moreover, it was shown that the optimum detector simplifies to the coherent detector in antipodal signalling. The authors derived the average bit error probability (BEP) of the detector over an AWGN channel in integral forms. Partially coherent communication has also been studied in [2].

Another important type of error is Gaussian distributed channel estimation error which arises from using the minimum mean-square error (MMSE) estimator for channel estimation in the Bayesian linear model [6], such as pilot-symbol-aided channel estimation [7]. This model is also valid when pilot and data signal are separated in frequency or time such that the magnitude of the correlation coefficient between them is no longer unity [8]. The optimum detection of binary antipodal signals with Gaussian channel estimation error based on the maximum-likelihood (ML) criterion is studied in [9] when the receiver is equipped with multiple antennas. It is shown that the optimum detector for single antenna reception case in AWGN channels is a matched filter that is matched to the channel estimate.

Optimum detection in the presence of Gaussian distributed channel estimation error is not known in the literature for orthogonal signalling case. In this paper, we derive the structure of the optimum detector for orthogonal binary signals with Gaussian channel estimation error and single antenna reception based on the ML criterion in AWGN channels. It will be seen that the optimum detector is a linear combination of the optimum coherent detector for purely coherent reception, i.e. the matched filter, and the optimum noncoherent detector for totally noncoherent reception, i.e. the square-law detector.

We derive an exact closed-form expression for the average BEP of the proposed optimum detector in Rayleigh fading channels with AWGN. It is well-known that for perfectly coherent reception the optimum coherent detector for antipodal modulation always leads to a better performance compared with orthogonal signalling. However, we show that in the presence of channel estimation error orthogonal signalling may outperform antipodal modulation if the variance of the channel estimation error for a given average SNR is greater than a threshold, and we find this threshold analytically.

The remainder of the paper is organized as follows: Section II describes the system model. In section III, the structure of the optimum detector in the presence of Gaussian distributed channel estimation error is presented for orthogonal signalling, and in section IV, we derive the exact closed-form expression

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of the average BEP of the proposed optimum detector in Rayleigh fading channels. Numerical results are provided in section V, and section VI concludes the paper.

II. System Model

We consider transmitting binary signals over flat fading channels where the received baseband signal r is given by

$$r = hs + n \tag{1}$$

and s is the transmitted binary signal, h denotes the fading channel, and n represents complex AWGN. The fading channel h is modeled with a flat Rayleigh fading model with variance 2 without loss of generality, i.e. $h \sim CN(0, 2)$ where $CN(m, 2\sigma^2)$ denotes a complex normal distribution for a complex random variable with mean m and variance $2\sigma^2$.

To detect the transmitted bits, the channel h must be estimated at the receiver. In practice channel information cannot be estimated without error. In this paper, we assume that channel estimation error e is Gaussian distributed and can be written as [6], [8]

$$e = h - \hat{h} \tag{2}$$

where $\hat{h}$ is the channel estimate. Channel estimation error e is assumed to be circularly symmetric Gaussian with variance $2\sigma_e^2$, i.e. $e \sim CN\left(0, 2\sigma_e^2\right)$. Channel estimate $\hat{h}$ is also circularly symmetric Gaussian distributed with variance $2\sigma_{\hat{h}}^2$, i.e. $\hat{h} \sim CN(0, 2\sigma_{\hat{h}}^2)$. Channel estimation error e and channel estimate $\hat{h}$ are assumed to be mutually independent which is true according to the Gans' model [8]. This assumption is also valid for MMSE estimation in which the estimate and the error are orthogonal [6]. Note that since $\hat{h}$ and e are independent, from (2) we have $\sigma_e^2 = 1 - \sigma_{\hat{h}}^2$, and $\hat{h}$ and h are jointly circularly symmetric Gaussian distributed with the correlation coefficient ρ which can be written as

$$\rho = \frac{E\left(h^* \hat{h}\right)}{\sqrt{2}\left(\sqrt{2}\sigma_{\hat{h}}\right)} = \sigma_{\hat{h}} = \sqrt{1 - \sigma_e^2} \tag{3}$$

For perfectly coherent reception we have $\rho = 1$ or $\sigma_e^2 = 0$, and if the reception is noncoherent then $\rho = 0$ or $\sigma_e^2 = 1$.

Note that when channel is estimated by using pilot symbols, e is independent from n since e depends only on the noise during transmission of pilot symbols which is independent of n (the AWGN during transmission of data signals). Therefore, throughout this paper we assume that e and n are independent.

III. Optimum Detection

In this section, we present the structure of the optimum detectors based on the ML criterion in AWGN channels in the presence of Gaussian distributed channel estimation errors for binary orthogonal and antipodal signals.

A. Orthogonal Signalling

In orthogonal signalling, e.g. orthogonal frequency-shift keying, the transmitted signal vector is either $\boldsymbol{s}_1 = \left(\sqrt{E_b}, 0\right)$ or $\boldsymbol{s}_2 = \left(0, \sqrt{E_b}\right)$, where E_b is the energy per bit. The received vector $\boldsymbol{r} = (r_1, r_2)$ if $\boldsymbol{s}_1$ is transmitted can be written as

$$\boldsymbol{r} = h\boldsymbol{s}_1 + \boldsymbol{n} = \left(\hat{h} + e\right)\boldsymbol{s}_1 + \boldsymbol{n} = \left(\sqrt{E_b}\hat{h} + \sqrt{E_b}e + n_1, n_2\right) \tag{4}$$

and if $\boldsymbol{s}_2$ is transmitted, the vector $\boldsymbol{r}$ can be expressed as

$$\boldsymbol{r} = h\boldsymbol{s}_2 + \boldsymbol{n} = \left(\hat{h} + e\right)\boldsymbol{s}_2 + \boldsymbol{n} = \left(n_1, \sqrt{E_b}\hat{h} + \sqrt{E_b}e + n_2\right) \tag{5}$$

where $\boldsymbol{n}$ is the complex AWGN vector, n_1 and n_2 are the first and second elements of vector $\boldsymbol{n}$, and $n_1, n_2 \sim CN(0, 2\sigma_n^2)$.

By combining mutually independent Gaussian channel estimation error e with n_1 and n_2, the likelihood functions from (4) and (5) can be written as

$$p\left(\boldsymbol{r}|\hat{h}, \boldsymbol{s}_1\right) = A \exp\left[-\left(\frac{\left|r_1 - \sqrt{E_b}\hat{h}\right|^2}{2\left(E_b\sigma_e^2 + \sigma_n^2\right)} + \frac{|r_2|^2}{2\sigma_n^2}\right)\right] \tag{6}$$

and

$$p\left(\boldsymbol{r}|\hat{h}, \boldsymbol{s}_2\right) = A \exp\left[-\left(\frac{|r_1|^2}{2\sigma_n^2} + \frac{\left|r_2 - \sqrt{E_b}\hat{h}\right|^2}{2\left(E_b\sigma_e^2 + \sigma_n^2\right)}\right)\right] \tag{7}$$

where $A = \left((2\pi)^2 \sigma_n^2 \left(E_b\sigma_e^2 + \sigma_n^2\right)\right)^{-1}$.

Based on the ML criterion, the decision rule is

$$p\left(\boldsymbol{r}|\hat{h}, \boldsymbol{s}_1\right) \underset{\hat{\boldsymbol{s}}=\boldsymbol{s}_2}{\overset{\hat{\boldsymbol{s}}=\boldsymbol{s}_1}{\gtrless}} p\left(\boldsymbol{r}|\hat{h}, \boldsymbol{s}_2\right) \tag{8}$$

From (6) and (7), the ML decision rule in (8) can be expressed as

$$\frac{\left|r_1 - \sqrt{E_b}\hat{h}\right|^2}{E_b\sigma_e^2 + \sigma_n^2} + \frac{|r_2|^2}{\sigma_n^2} \underset{\hat{\boldsymbol{s}}=\boldsymbol{s}_2}{\overset{\hat{\boldsymbol{s}}=\boldsymbol{s}_1}{\lessgtr}} \frac{|r_1|^2}{\sigma_n^2} + \frac{\left|r_2 - \sqrt{E_b}\hat{h}\right|^2}{E_b\sigma_e^2 + \sigma_n^2} \tag{9}$$

which can be simplified to

$$\sqrt{E_b}\sigma_e^2 |r_1|^2 + \sigma_n^2 \Re\left(\hat{h}^* r_1\right) \underset{\hat{\boldsymbol{s}}=\boldsymbol{s}_2}{\overset{\hat{\boldsymbol{s}}=\boldsymbol{s}_1}{\gtrless}} \sqrt{E_b}\sigma_e^2 |r_2|^2 + \sigma_n^2 \Re\left(\hat{h}^* r_2\right) \tag{10}$$

where $\Re(\cdot)$ indicates the real part of a complex number.

Therefore, the optimum detector for binary orthogonal signals is not just a coherent receiver that treats the estimated channel as the true channel. In other words, the optimum detector in (10) is a linear combination of the matched filter and the square-law detector, the optimum detectors for purely coherent and totally noncoherent receptions, respectively. As σ_e^2 increases, the weight of the square-law detector in the optimum detector of (10) increases as well.

In the limiting case of purely coherent reception ($\sigma_e^2 = 0$), $e = 0$ and thus $\hat{h} = h$, and the decision rule (10) reduces to

$$\Re(h^* r_1) \underset{\hat{\boldsymbol{s}}=\boldsymbol{s}_2}{\overset{\hat{\boldsymbol{s}}=\boldsymbol{s}_1}{\gtrless}} \Re(h^* r_2) \tag{11}$$

which is the matched filter, as expected.

For the opposite case of noncoherent reception ($\sigma_e^2 = 1$), $\sigma_{\hat{h}}^2 = 0$ and thus $\hat{h} = 0$, and the detector in (10) simplifies to

$$|r_1|^2 \underset{\hat{\boldsymbol{s}}=\boldsymbol{s}_2}{\overset{\hat{\boldsymbol{s}}=\boldsymbol{s}_1}{\gtrless}} |r_2|^2 \tag{12}$$

which is the square-law detector, as expected.

B. Antipodal Signalling

For binary antipodal signalling, the transmitted signal is either $s_1 = \sqrt{E_b}$ or $s_1 = -\sqrt{E_b}$, and the received signal is

$$r = hs_i + n = \hat{h}s_i + es_i + n, \quad i = 1, 2 \tag{13}$$

where complex AWGN n is distributed as $n \sim CN(0, 2\sigma_n^2)$.

The ML decision rule can again be obtained by treating the channel estimation error e as an additional Gaussian noise term independent from AWGN. From [9, eq. (16)], the decision rule for single antenna reception case can be expressed as

$$\Re\left(\hat{h}^* r\right) \underset{\hat{s}=s_2}{\overset{\hat{s}=s_1}{\gtrless}} 0 \tag{14}$$

Therefore, in contrast to the optimum detector for binary orthogonal signalling case in (10), the optimum detector for binary antipodal modulation in (14) is just a matched filter that is matched to the channel estimate.

IV. Performance Analysis

In this section, we investigate the average BEP of the optimum detectors (10) and (14) in Rayleigh fading channels with AWGN when the transmitted signals are equiprobable. First, we derive the exact closed-form expression of the average BEP of the optimum detector (10) for orthogonal signalling and then present the average BEP of the optimum detector (14) derived in [9] for antipodal modulation.

A. Orthogonal Signalling

To find the BEP of the optimum detector for this case, first note that the decision rule in (10) can be simplified to

$$\left| r_1 + \frac{\sigma_n^2 \hat{h}}{\sqrt{E_b}\sigma_e^2} \right|^2 \underset{\hat{\boldsymbol{s}}=\boldsymbol{s}_2}{\overset{\hat{\boldsymbol{s}}=\boldsymbol{s}_1}{\gtrless}} \left| r_2 + \frac{\sigma_n^2 \hat{h}}{\sqrt{E_b}\sigma_e^2} \right|^2 \tag{15}$$

The BEP when transmitted signals are equiprobable is equal to the BEP when $\boldsymbol{s}_1$ is transmitted. So, from (15) we have

$$\begin{aligned} P_b\left(E|\hat{h}\right) &= P_b\left(E|\hat{h}, \boldsymbol{s}_1\right) \\ &= P\left(\left| r_2 + \frac{\sigma_n^2 \hat{h}}{\sqrt{E_b}\sigma_e^2} \right| > \left| r_1 + \frac{\sigma_n^2 \hat{h}}{\sqrt{E_b}\sigma_e^2} \right| |\hat{h}, \boldsymbol{s}_1\right) \end{aligned} \tag{16}$$

If $\boldsymbol{s}_1$ is transmitted, from (4) the first element of $\boldsymbol{r}$, r_1, is distributed as

$$r_1|\hat{h}, \boldsymbol{s}_1 \sim CN\left(\sqrt{E_b}\hat{h}, 2\left(E_b\sigma_e^2 + \sigma_n^2\right)\right) \tag{17}$$

and for r_2, the second element of $\boldsymbol{r}$, we have

$$r_2|\hat{h}, \boldsymbol{s}_1 \sim CN\left(0, 2\sigma_n^2\right) \tag{18}$$

Now, we define r_1' and r_2' as follows

$$r_1' = r_1 + \frac{\sigma_n^2 \hat{h}}{\sqrt{E_b}\sigma_e^2}; \quad r_2' = r_2 + \frac{\sigma_n^2 \hat{h}}{\sqrt{E_b}\sigma_e^2} \tag{19}$$

So, from (17)-(19), we have

$$r_1'|\hat{h}, \boldsymbol{s}_1 \sim CN\left(\frac{\hat{h}\left(E_b\sigma_e^2 + \sigma_n^2\right)}{\sqrt{E_b}\sigma_e^2}, 2\left(E_b\sigma_e^2 + \sigma_n^2\right)\right) \tag{20}$$

and

$$r_2'|\hat{h}, \boldsymbol{s}_1 \sim CN\left(\frac{\sigma_n^2 \hat{h}}{\sqrt{E_b}\sigma_e^2}, 2\sigma_n^2\right) \tag{21}$$

Note that random variables $r_1'|\hat{h}, \boldsymbol{s}_1$ and $r_2'|\hat{h}, \boldsymbol{s}_1$ are independent since from (4), the former depends on n_1 and e while the latter depends on n_2.

From (19), the conditional error probability in (16) can be written as

$$P_b\left(E|\hat{h}\right) = P_b\left(E|\hat{h}, \boldsymbol{s}_1\right) = P\left(|r_2'| > |r_1'|\,|\hat{h}, \boldsymbol{s}_1\right) \tag{22}$$

Since from (20) and (21), independent random variables $r_1'|\hat{h}, \boldsymbol{s}_1$ and $r_1'|\hat{h}, \boldsymbol{s}_1$ are complex Gaussian distributed with nonzero mean, the absolute value of these variables, i.e. $|r_1'|\,|\hat{h}, \boldsymbol{s}_1$ and $|r_2'|\,|\hat{h}, \boldsymbol{s}_1$, are Rician distributed and mutually independent. The probability that one Rice random variable exceeds another is derived in [3, App. A] when they are independent. Therefore, by using the results in [3, App. A], the conditional BEP in (22) can be simplified to

$$\begin{aligned} P_b(E|\hat{h}) = &\frac{\zeta^2}{1+\zeta^2}\left(1 - Q\left(\beta\sqrt{\gamma_o}, \alpha\sqrt{\gamma_o}\right)\right) \\ &+ \frac{1}{1+\zeta^2} Q\left(\alpha\sqrt{\gamma_o}, \beta\sqrt{\gamma_o}\right) \end{aligned} \tag{23}$$

where $Q(x, y)$ is the first-order Marcum's Q-function defined as

$$Q(x, y) = \int_y^\infty t \exp\left(-\frac{t^2 + x^2}{2}\right) I_0(xt) dt \tag{24}$$

and $I_0(.)$ denotes the zeroth-order modified Bessel function of the first kind, and γ_o in (23) is defined as

$$\gamma_o = \frac{E_b\left|\hat{h}\right|^2}{E_b\sigma_e^2 + 2\sigma_n^2} \tag{25}$$

and α, β and ζ in (23) are defined as

$$\alpha = \frac{1}{\bar{\gamma}_b\sigma_e^2}; \quad \beta = \frac{\bar{\gamma}_b\sigma_e^2 + 1}{\bar{\gamma}_b\sigma_e^2}; \quad \zeta = \sqrt{\bar{\gamma}_b\sigma_e^2 + 1} \tag{26}$$

where the received SNR per bit γ_b is defined as $\gamma_b = \frac{E_b|h|^2}{2\sigma_n^2}$ and thus, the average SNR per bit $\bar{\gamma}_b$ is

$$\bar{\gamma}_b = \frac{E_b}{2\sigma_n^2} E\left(|h|^2\right) = \frac{E_b}{\sigma_n^2} \tag{27}$$

To compute the average BEP, we should average the conditional BEP in (23) over the PDF of γ_o. Since $\hat{h}$ is circularly symmetric Gaussian distributed, γ_o in (25) is exponentially

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distributed [10, p. 190], and its PDF is

$$f_{\gamma_o}(\gamma_o)=\frac{1}{\bar{\gamma}_o}\exp\left(-\frac{\gamma_o}{\bar{\gamma}_o}\right),\quad \gamma_o\geq 0 \tag{28}$$

where

$$\bar{\gamma}_o=\frac{2E_b\left(1-\sigma_e^2\right)}{E_b\sigma_e^2+2\sigma_n^2}=\frac{2\bar{\gamma}_b\left(1-\sigma_e^2\right)}{\bar{\gamma}_b\sigma_e^2+2} \tag{29}$$

From (23) and (28), the average BEP can be expressed as

$$\begin{aligned}P_b(E)=&\int_0^\infty P_b(E|\gamma_o)f_{\gamma_o}(\gamma_o)d\gamma_o\\=&\frac{\zeta^2}{1+\zeta^2}+\frac{1}{1+\zeta^2}\int_0^\infty\frac{1}{\bar{\gamma}_o}\exp\left(-\frac{\gamma_o}{\bar{\gamma}_o}\right)\\&\times\left(Q\left(\alpha\sqrt{\gamma_o},\beta\sqrt{\gamma_o}\right)-\zeta^2Q\left(\beta\sqrt{\gamma_o},\alpha\sqrt{\gamma_o}\right)\right)d\gamma_o\end{aligned} \tag{30}$$

By using the integral results in [4, eqs. (5.50)-(5.53)], the average BEP expression in (30) can be computed as

$$P_b(E)=\frac{1}{2}\left(1-\frac{\frac{(1+\zeta^2)(\beta^2-\alpha^2)\bar{\gamma}_o}{2}+\zeta^2-1}{(1+\zeta^2)\sqrt{\left(1+(\alpha^2+\beta^2)\frac{\bar{\gamma}_o}{2}\right)^2-\alpha^2\beta^2\bar{\gamma}_o^2}}\right) \tag{31}$$

By substituting (26) and (29) in (31), the average BEP of the optimum detector of binary orthogonal signals can be expressed as

$$P_b(E)=\frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}_b\left(\bar{\gamma}_b\sigma_e^2+2-2\sigma_e^2\right)}{\left(\bar{\gamma}_b+2\right)\left(\bar{\gamma}_b\sigma_e^2+2\right)}}\right) \tag{32}$$

In the limiting case of $\sigma_e^2=0$ (perfectly coherent reception), expression (32) reduces to

$$P_b(E)=\frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}_b}{2+\bar{\gamma}_b}}\right) \tag{33}$$

which is a well-known expression for the average BEP of the coherent optimum detector of binary orthogonal signals when the receiver estimates the channel perfectly [1, eq. (14.3-8)].

For the other extreme case of $\sigma_e^2=1$ (totally noncoherent reception), expression (32) simplifies to

$$P_b(E)=\frac{1}{2}\left(1-\frac{\bar{\gamma}_b}{2+\bar{\gamma}_b}\right)=\frac{1}{2+\bar{\gamma}_b} \tag{34}$$

which is identical to [1, eq. (14.3-12)], the average BEP of the optimum noncoherent detector (the square-law detector) when there is no channel information available at the receiver.

B. Antipodal Signalling

By using [9, eq. (38)] for single antenna reception case, the average BEP of the optimum detector in (14) for equiprobable antipodal binary signals can be expressed as

$$P_b(E)=\frac{1}{2}\left(1-\sqrt{\frac{\left(1-\sigma_e^2\right)\bar{\gamma}_b}{1+\bar{\gamma}_b}}\right) \tag{35}$$

where $\bar{\gamma}_b$ is defined in (27).

In the extreme case of $\sigma_e^2=0$ (purely coherent reception), expression (35) simplifies to [1, eq. (14.3-7)]. At the other extreme case of $\sigma_e^2=1$ (totally noncoherent reception), average BEP expression in (35) reduces to $P_b(E)=\frac{1}{2}$ as expected since for antipodal signalling when there is no channel information available at the receiver, there is no basis for the decision, which can also be seen from (14), since when $\sigma_e^2=1$, $\sigma_{\hat{h}}^2=0$ and $\hat{h}=0$.

C. Performance Comparison

It well-known that for ideal coherent reception when channels are estimated without error, the optimum coherent detector of antipodal signalling has a better performance than that of orthogonal modulation [1].

In this part, we show that in the presence of channel estimation error when the variance of the error for a given average SNR is greater than a threshold, the average BEP of optimum detector for orthogonal modulation in (32) is less than that of antipodal modulation in (35), and derive this threshold analytically. This result can be used to find out which type of modulation, antipodal or orthogonal, provides a better performance for a given channel estimation error variance, σ_e^2, and average SNR, $\bar{\gamma}_b$.

By comparing (32) and (35), we find that for a given $\bar{\gamma}_b$ orthogonal signalling has smaller average BEP than antipodal signalling if σ_e^2 satisfies

$$\sigma_e^2>\frac{1}{1+0.5\bar{\gamma}_b}, \tag{36}$$

otherwise, antipodal modulation has smaller average BEP.

Expression (36) can also be rearranged to obtain the cross-over value of $\bar{\gamma}_b$ for a given σ_e^2. From (36), orthogonal modulation results in better performance if for a given σ_e^2, the average SNR $\bar{\gamma}_b$ is large enough such that

$$\bar{\gamma}_b>\frac{2\left(1-\sigma_e^2\right)}{\sigma_e^2}, \tag{37}$$

otherwise, antipodal signalling will do better in terms of average BEP.

V. Numerical Results

In this section, we present simulation results of the average BEP of the optimum detector proposed in the last section for binary orthogonal signalling.

Figs. 1-3 show the average BEP of the optimum detector of binary orthogonal signals for $\sigma_e^2=0.01$, $\sigma_e^2=0.1$ and $\sigma_e^2=0.2$, respectively, versus average SNR $\bar{\gamma}_b$, by both theory (expression (32)) and simulation. We can see that the analytical result matches precisely the Monte Carlo simulation, which verifies the derived theoretical result in (32). The average BEP of optimum coherent and noncoherent detectors of binary orthogonal signals in (33) and (34), as well as the average BEP of the optimum detector of binary antipodal signals in (35) have also been added for comparison.

It is clear from Figs. 1-3 that the optimum detector (10) results in performance gain in comparison with noncoherent

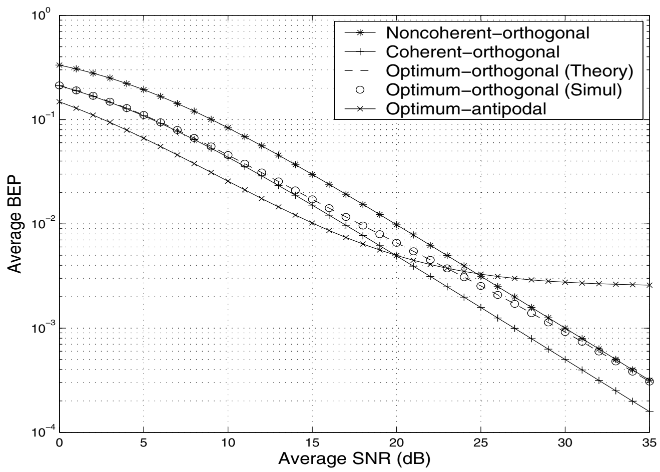

Fig. 1. Average BEP versus the average SNR for $\sigma_e^2 = 0.01$.

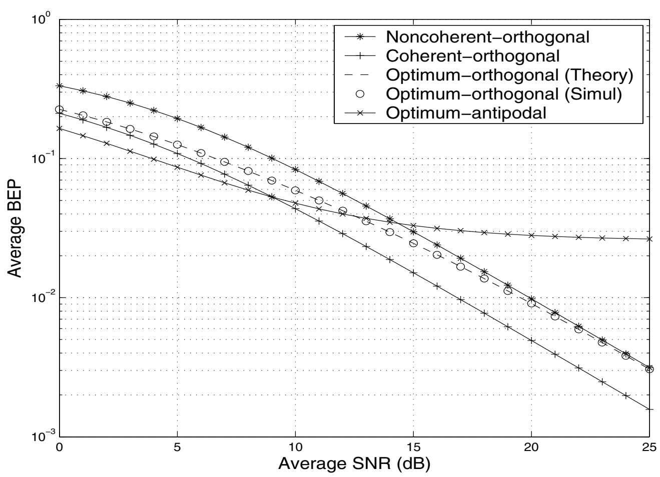

Fig. 2. Average BEP versus the average SNR for $\sigma_e^2 = 0.1$.

detector for orthogonal modulation. The gain is greater than 1 dB for $\sigma_e^2 = 0.01$ when $\bar{\gamma}_b < 26$ dB, for $\sigma_e^2 = 0.1$ when $\bar{\gamma}_b < 16$ dB, and for $\sigma_e^2 = 0.2$ when $\bar{\gamma}_b < 13$ dB. Therefore, as σ_e^2 decreases there is a larger range of $\bar{\gamma}_b$ where the proposed optimum detector for orthogonal modulation leads to considerable performance gain compared to the noncoherent detector.

From Figs. 1-3, it is evident that the optimum detector for orthogonal signalling outperforms the antipodal one for $\sigma_e^2 = 0.01$ when $\bar{\gamma}_b > 22.97$ dB, for $\sigma_e^2 = 0.1$ when $\bar{\gamma}_b > 12.55$ dB, and for $\sigma_e^2 = 0.2$ when $\bar{\gamma}_b > 9.03$ dB. These results can also be derived from (37). Therefore, as σ_e^2 increases, there is a larger range of $\bar{\gamma}_b$ where orthogonal signaling has better performance than antipodal modulation, using the optimum detector proposed.

VI. Conclusion

In this paper, we derived the structure of the optimum detector for binary orthogonal signals in AWGN channels in the presence of Gaussian channel estimation error for single

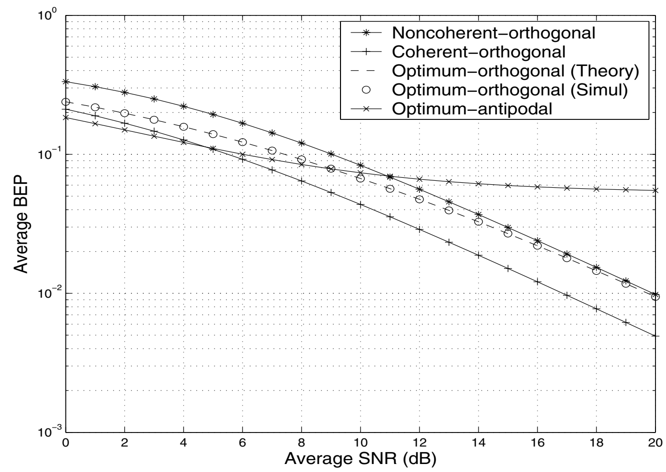

Fig. 3. Average BEP versus the average SNR for $\sigma_e^2 = 0.2$.

antenna reception based on the ML criterion. It was discovered that the optimum detector is a linear combination of the matched filter and the square-law detector, and the weight of the square-law detector increases as the variance of the channel estimation error increases. We examined the performance of the proposed optimum detector in Rayleigh fading channels with AWGN and obtained the exact closed-form expression of the average BEP as a function of the average SNR and the variance of channel estimation error. It was found that in the presence of channel estimation error the optimum detector of orthogonal modulation outperforms the optimum detector of antipodal signalling if the variance of channel estimation error for a given average SNR is greater than a threshold, and we derived this threshold analytically. It was observed that as the variance of channel estimation error increases there is a larger range of average SNR where the optimum detector for orthogonal modulation has a better performance than the optimum detector for antipodal signalling.

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