DIGITAL COMMUNICATION
EED 350
Project Report
ON
Optimum Detection of Binary Signals in Rayleigh Fading Channels with Imperfect Channel Estimates

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Objective: The objective of our project is to make an optimum detector using (MATLAB and Simulink) based on the maximum likelihood criterion for binary orthogonal signals in the presence of Gaussian distributed channel estimation and AWGN (Additive White Gaussian Noise) and then compare it with the optimum detector for antipodal signals.

INTRODUCTION :- [1]

In our daily life, we generally deal with two kind of optimum detectors at the receiver side of communication systems. One for the coherent reception and another kind for non coherent reception.

In case of coherent reception, the channel information is precisely known at the receiver, the optimum detector in AWGN channels can be made by passing the received signals through a pair of matched filters in case of binary orthogonal signalling.For noncoherent reception, when it is not possible for the receiver to perform channel estimation, and thus no channel estimate is available at the receiver, the optimum detector in AWGN channels is the envelope detector or square-law detector.

The structure and performance of an optimum detector depends on the statistics of the channel estimation error. We generally consider AWGN error, but one more important type of error is Gaussian distributed channel estimation error that arises from using the minimum mean square error**(MMSE)** estimator for channel estimation in the Bayesian linear model, such as pilot-symbol-aided channel estimation.
In this project, we have made a structure of the optimum detector for orthogonal binary signals with Gaussian channel estimation error and single antenna reception

based on the ML criterion in AWGN channels.

Optimum detector is a linear combination of the optimum coherent detector for purely coherent reception, i.e. the matched filter, and the optimum noncoherent detector for totally non-coherent reception, i.e. the square-law detector. We made an exact closed-form expression for the average BEP of the proposed optimum detector in Rayleigh fading channels with AWGN.

NOTE:-

The ***sections*** further are divided as:-

(ii) Model Information
(iii) Performance Analysis
(iv) Code Explanation
(v) Results
(vi) Individual Conclusion
(vii) Frequently Asked Questions
(viii) Contribution

Model Information: [2]

We are transmitting a binary signals over a fading channels and the received signal r is given by: r = hs + n ,where

s- transmitted binary signal
h -denotes the fading channel
n - Additive White Gaussian Noise

The fading channel h is modeled using flat Rayleigh fading model having variance 2. The receiver must know the channel h for detecting the transmitted bits. The estimation error e is Gaussian distributed and can be written as:-
 e = h − $\widehat{h}$,where $\widehat{h}$ is the channel estimate.

Channel estimation error e and channel estimate ˆh are assumed to be mutually independent according to the Gans model. Since ˆh and e are independent ,Gaussian distributed with the correlation coefficient ρ which can be written as

$$\rho = \frac{E\left(h^{*}\hat{h}\right)}{\sqrt{2}\left(\sqrt{2}\sigma_{\hat{h}}\right)} = \sigma_{\hat{h}} = \sqrt{1-\sigma_e^2}$$

- 2(i)

For perfectly coherent reception we have ρ = 1 or σ^2 e = 0, and if the reception is non-coherent then ρ = 0 or σ^2e=1.

(i) For Orthogonal Signalling:

For orthogonal signalling, the transmitted signal is either s1 =(√Eb, 0) or s2 =(0,√Eb). The received vector r = (r1, r2) if s1 is transmitted can be written as

$$r = hs_1 + n = \left(\hat{h} + e\right) s_1 + n = \left(\sqrt{E_b}\hat{h} + \sqrt{E_b}e + n_1, n_2\right)$$

-2(ii)

And if s2 was transmitted, it can be written as

$$r = hs_2 + n = \left(\hat{h} + e\right) s_2 + n = \left(n_1, \sqrt{E_b}\hat{h} + \sqrt{E_b}e + n_2\right)$$

-2(iii)

Here n is the complex AWGN vector and n1 and n2 are the first and the second elements of vector n.
If we combine Gaussian channel estimation error e with n1 and n2, the likelihood functions from the above equations can be written as

$$p\left(r|\hat{h}, s_1\right)=A \exp\left[-\left(\frac{\left|r_1 - \sqrt{E_b}\hat{h}\right|^2}{2\left(E_b\sigma_e^2+\sigma_n^2\right)} + \frac{|r_2|^2}{2\sigma_n^2}\right)\right]$$

and

$$p\left(r|\hat{h}, s_2\right)=A \exp\left[-\left(\frac{|r_1|^2}{2\sigma_n^2} + \frac{\left|r_2 - \sqrt{E_b}\hat{h}\right|^2}{2\left(E_b\sigma_e^2+\sigma_n^2\right)}\right)\right]$$

where $A = \left((2\pi)^2 \sigma_n^2 \left(E_b\sigma_e^2 + \sigma_n^2\right)\right)^{-1}$.

-2(iv)

So the decision rule based on ML criterion can be written as

$$p\left(r|\hat{h}, s_1\right) \underset{\hat{s}=s_2}{\overset{\hat{s}=s_1}{\gtrless}} p\left(r|\hat{h}, s_2\right)$$

-2(v)

The ML decision rule above can also be expressed as

$$\sqrt{E_b}\sigma_e^2 |r_1|^2+\sigma_n^2\Re\left(\hat{h}^* r_1\right) \underset{\hat{s}=s_2}{\overset{\hat{s}=s_1}{\gtrless}} \sqrt{E_b}\sigma_e^2 |r_2|^2+\sigma_n^2\Re\left(\hat{h}^* r_2\right)$$

-2(vi)

where R(·) indicates the real part of a complex number.
Therefore, the optimum detector for binary orthogonal signals is a linear combination of the matched filter and the square-law detector.

For the case of purely coherent reception (σ^2 e = 0), e = 0 and thus $\widehat{h}$ = h, and the decision rule can be written as

$$\Re(h^* r_1) \underset{\hat{s}=s_2}{\overset{\hat{s}=s_1}{\gtrless}} \Re(h^* r_2)$$

-2(vii)

which is the matched filter.

Now for the opposite case of noncoherent reception (σ^2 e = 1), (σ^2 $\widehat{h}$ = 0) and thus $\widehat{h}$ = 0, the detector is

$$|r_1|^2 \underset{\hat{s}=s_2}{\overset{\hat{s}=s_1}{\gtrless}} |r_2|^2$$

-2(viii)

which is the square-law detector.

(ii) For Antipodal SIgnalling:-

In case of binary anti-podal signalling we know the transmitted signal can be either S1 i.e root(Eb) or S2 i.e - root(Eb), and the received signal is given as:-

$$r=h*s + n = (\widehat{h})*s + e*s +n \text{ where } i = 1 \text{ or } 2$$

where complex AWGN n is distributed as n ~ CN(0,2* σ^2 n).

The Maximum Likelihood decision rule for anti-podal signal is calculated by taking e as an additional Gaussian noise term independent from AWGN. The decision rule for single antenna reception case can be expressed as:-

$$\Re\left(\hat{h}^* r\right) \underset{\hat{s}=s_2}{\overset{\hat{s}=s_1}{\gtrless}} 0 \qquad \text{-2(ix)}$$

As we can notice, the expression of optimum detector for anti-podal signals came out to be simply matched filters.

III Performance Analysis :- (Our Understanding) [3]

We derived the average BEP of the optimum detectors and in Rayleigh fading channels with AWGN when the transmitted signals are equiprobable.

(i) For Orthogonal SIgnalling:-

The average BEP of the optimum detector (partial combination of coherent and non coherent reception) comes out to be:-

$$P_b(E)=\frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}_b\left(\bar{\gamma}_b\sigma_e^2+2-2\sigma_e^2\right)}{\left(\bar{\gamma}_b+2\right)\left(\bar{\gamma}_b\sigma_e^2+2\right)}}\right) \quad \text{- 3(i)}$$

The average BEP of the optimum detector of binary orthogonal signals in the limiting case, i.e *perfectly coherent reception* comes out to be (taking $\sigma^2 e = 0$) :-

$$P_b(E)=\frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}_b}{2+\bar{\gamma}_b}}\right) \quad \text{-3(ii)}$$

For the other extreme case of totally non coherent reception (taking $\sigma^2 e = 1$) comes out to be :-

$$P_b(E)=\frac{1}{2}\left(1-\frac{\bar{\gamma}_b}{2+\bar{\gamma}_b}\right)=\frac{1}{2+\bar{\gamma}_b} \quad \text{-3(iii)}$$

(ii) For Anti-podal Signalling:-

The average BEP of the optimum detector for equiprobable antipodal binary signals can be expressed as:-

$$P_b(E) = \frac{1}{2}\left(1 - \sqrt{\frac{\left(1-\sigma_e^2\right)\bar{\gamma}_b}{1+\bar{\gamma}_b}}\right) \qquad \text{-3(iv)}$$

POINTS to Consider:-

(i) For purely coherent reception (σ^2 e=0) , the above expression simplifies to something similar to the case of perfectly coherent reception in orthogonal signalling.

(ii) For purely non coherent reception (σ^2 e=1), the above expression reduces to simply,

P(E)= ½ -3(v)

The above expression was expected as we know there is <u>no channel information available at the receiver for antipodal signalling</u> when there is no channel information available at the receiver, there is no basis for the decision, since when, sigmae^2=0, then sigmah^2=0 and hcap=0.

Code Explanation:- [4]

Rayleigh Fading was channel using the function rayleighchan. It had 1/1000000 is the sample time of the input signal, Maximum Doppler Shifts of 50, delayVector = 1.0e-004 * [0 0.0400 0.0800 0.1200], gain Vector = [0 -3 -6 -9].

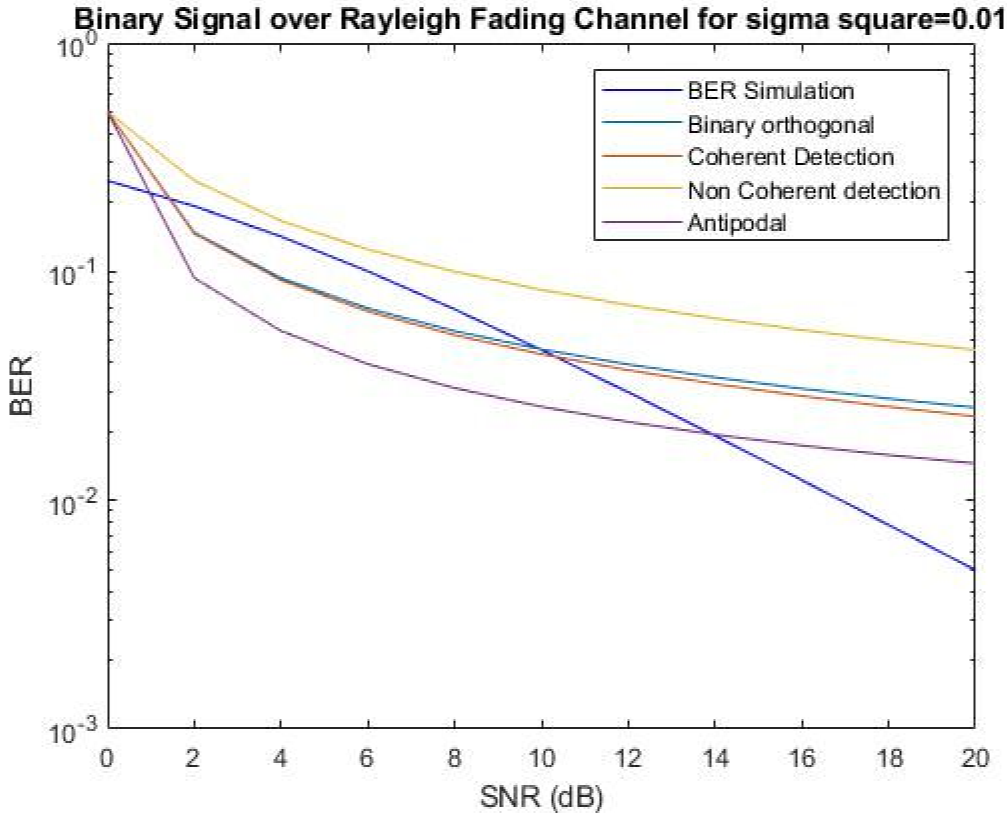
Random Binary signals was formed using randi function which was modulated using the Modulator.

The channel effects was applied to the generated binary signals which resulted in the faded signal. AWGN channel was created using the AWGN Channel with noise method used as SNR.

Varying over various values of SNR, Gaussian noise was added to the faded signal and finally was demodulated at the receiver's end. The signal which was transmitted was found and compared with the original signal sent and BER was calculated.

RESULT :- [5]

(i) Experimental Result



The results are quite similar to that obtained on the paper. This is plotted for a particular value of (sige)^2. Similarly, we can plot for different values of (sige)^2.

(ii) Theoretical Results:-

Concluding Performance Comparison between Orthogonal Signalling and Anti-Podal Signalling.

(1) We observed that in the presence of channel estimation error when the variance of the error for a given average SNR is greater than a threshold, the average BEP of the optimum detector for orthogonal modulation as shown in 3(i) is less than that of antipodal modulation in 3(iv).

(2) So, for a given value of gamma, orthogonal signalling has smaller average BEP than antipodal signalling if sigmae^2 satisfies :-

$$\sigma_e^2 > \frac{1}{1 + 0.5\bar{\gamma}_b}$$

-5(i)

And for a given value of sigmae^2, orthogonal signalling has smaller average BEP than antipodal signalling if:-

$$\bar{\gamma}_b > \frac{2\left(1 - \sigma_e^2\right)}{\sigma_e^2}$$

-5(ii)

Conclusion:- [6]

By Keshav Chandak

In this project, we made a structure of the optimum detector for binary orthogonal signals in AWGN channels in the presence of Gaussian channel estimation error for single antenna reception based on the ML criterion using *MATLAB with Simulink.*

I observed for coherent reception, when the channel information is precisely known at the receiver, the optimum detection in additive white Gaussian noise (AWGN) channels can be obtained by passing the received signals through a matched filter in the case of binary antipodal signalling or through a pair of matched filters in the case of binary orthogonal signalling.

But that is not the case with non coherent reception, as here, it is not possible for the receiver to perform channel estimation.We use square law detector in this case.

In our paper, We also considered Rayleigh fading. As fading results in loss of signal power without reducing the noise power, it is an important parameter to consider to calculate the BEP for both, Orthogonal and Anti-podal Signalling.

The noise we considered is Gaussian mean distributed noise in AWGN channel. We mainly consider Orthogonal and Anti podal signals as they are highly used. Both has major advantages and dis-advantages. In Gaussian mean distrbuted noise, the optimum detector is a linear combination of coherent and non coherent detector, but in case of Anti-podal signals, it is just a Matched filter for coherent and a Square law detector in case of Non coherent.

We get a smaller BEP in case of orthogonal signalling than anti-podal signalling when sigmae^2 is greater than a certain threshold, for given gammab. Similarly, for a given sigmae^2, we will smaller BEP for the same when it is less than a certain threshold as derived in the Result section.

By Kshitiz Srivastava

Rayleigh fading is caused by multipath reception. The mobile antenna receives a large reflected and scattered waves. Because this the instantaneous received power seen by a moving antenna becomes a random variable, dependent on the location of the antenna.

Rayleigh fading model is used when analysing and prediction radio wave propagation performance for areas such as cellular communications in a well built up urban environment where there are many reflections from buildings.

Fading is an important effect to consider which can cause poor performance in a communication system because it can result in a loss of signal power without reducing the power of the noise.

Orthogonal signals are used extensively in the communications industry. Signals can be used to send and receive separate information channels on each orthogonal signal with minimal interference between them. Similarly antipodal signals are quite important for communication purpose.

We use the Matched Filter detection which is optimal because it maximizes the SNR of received signal and also use square law detector for non coherent signals.

The weight of the square-law detector increases as the variance of the channel estimation error increases.

The optimum detector of orthogonal modulation works better than the optimum detector of antipodal signalling if the variance of channel estimation error for a given average SNR is greater than a threshold value.

By Shobhit Garg

We usually deal with two kind of optimum detectors at the receiver side of communication systems. One for the coherent reception and another kind for non coherent reception.It is shown that the optimum detector is a linear combination of the optimum coherent and optimum non-coherent detectors.

Coherent systems need carrier phase information at the receiver and they use matched filters to detect and decide what data was sent , while noncoherent systems do not need carrier phase information and use methods like square law to recover the data.

In this paper, we present the optimum detector based on the maximum likelihood criterion for binary orthogonal signals in the presence of Gaussian distributed channel estimation error and AWGN.

We inspected that the execution of the proposed ideal indicator in Rayleigh fading channels with AWGN and got the correct expression of the average BEP as an element of the average SNR and the difference of channel estimation mistake.

In presence of channel estimation error , the optimum detector of orthogonal modulation is better than the optimum detector of antipodal signalling if the variance of channel estimation error for a given average SNR is greater than a threshold value that we already derived .

It was also seen that as the variance of channel estimation error increments there is a larger range of average SNR where the optimum detector for orthogonal modulation has a better performance than the optimum detector for antipodal signalling.

Frequently Asked Questions:- [7]

Q1. What is AWGN?

Additive white Gaussian noise (AWGN) is a basic noise model used in Information theory to mimic the effect of many random processes that occur in nature. The modifiers denote specific characteristics:

- ***Additive*** because it is added to any noise that might be intrinsic to the information system.
- ***White*** refers to the idea that it has uniform power across the frequency band for the information system. It is an analogy to the color white which has uniform emissions at all frequencies in the visible spectrum.
- ***Gaussian*** because it has a normal distribution in the time domain with an average time domain value of zero.

Q2. What do you mean when you say Fading Channel and Rayleigh Fading Channel?

FADING Channel

The presence of reflectors in the environment surrounding a transmitter and receiver create multiple paths that a transmitted signal can traverse. As a result, the receiver sees the superposition of multiple copies of the transmitted signal, each traversing a different path. Each signal copy will experience differences in attenuation, delay and phase shift while travelling from the source to the receiver. This can result in either constructive or destructive interference, amplifying or attenuating the signal power seen at the receiver.

Rayleigh Fading Channel

Rayleigh fading models assume that the magnitude of a signal that has passed through such a transmission medium will vary randomly, or fade according to a Rayleigh distribution.

Rayleigh fading is a reasonable model when there are many objects in the environment that scatter the radio signal before it arrives at the receiver. The central limit theorem holds that, if there is sufficiently much scatter, the channel impulse response will be well-modelled as a Gaussian process irrespective of the distribution of the individual components. If there is no dominant component to the scatter, then such a process will have zero mean and phase evenly distributed between 0 and 2π radians. The envelope of the channel response will therefore be Rayleigh distributed.

Q3. Explain Matched filter.

Matched Filter

The Matched filter is the optimal linear filter for maximizing the signal-to-noise ratio (SNR) in the presence of additive stochastic noise. It is used to detect presence of known signal with the unknown signal.

Q4. Explain Optimum Detection and its types.

Optimum Detection

The generic signal detection problem that corresponds to receiving a signal r(t) over a noisy channel. After using the matched filter or correlation receiver etc, it produces a vector r. This contains all the relevant information of the received signal. Optimum detection makes a decision of which transmitted signal was transmitted at that time interval based on vector r such that probability of correct decision is maximized.

Coherent and Non Coherent Detection

Coherency in signal processing is similar to correlation in statistics. In statistics two random variables are correlated if there exists a linear relationship between the two. Perfectly coherent signals are signals such that one is the response of a linear system to the other signal. Hence there exists a linear system such that one signal is the input and the other signal is its output.

Coherence is between 0 and 1. The higher the coherency is, the better one can explain the spectral content of the first signal by analyzing the spectral content of the other signal since a linear system is completely characterized by the system's associated Frequency Response Function .

The basic difference between Coherent systems and non Coherent systems is that coherent systems need carrier phase information at the receiver and they use matched

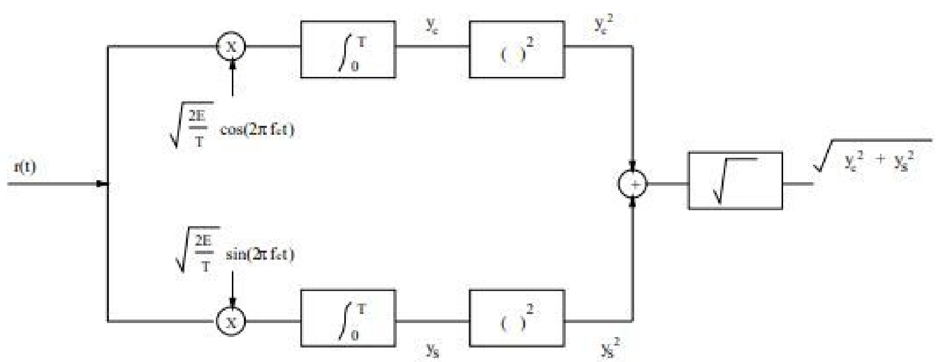
filters to detect and decide what data was sent , while noncoherent systems do not need carrier phase information and use methods like square law to recover the data.
The performance of these detectors have been studied in both non-faded and fading channels.

For Coherent reception, when the channel information is known at the receiver. The optimum detection in additive white Gaussian noise (AWGN) channels can be obtained by passing the received signals through a matched filter.

It requires expensive and complex carrier recovery circuit. But it has better bit error rate of detection.

With noncoherent detection, the carrier phase is not recovered at the receiver. Information is transmitted in amplitude and frequency only, not in the phase of the carrier

$$r(t) = \sqrt{\frac{2E}{T}} \cos\Phi \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\Phi \sin(2\pi f_c t) + n(t)$$



In the absence of noise $n(t)$,

$$y_c = \sqrt{E}\cos\Phi, \;\; y_s = \sqrt{E}\sin\Phi$$

$$y_c^2 + y_s^2 = E \quad \text{the energy in the waveform}$$

Fig. Square-Law Correlation Detector

Non Coherent systems do not need carrier phase information and use methods like square law to recover the transmitted data at receiver end.

• It does not require expensive and complex carrier recovery circuit, but has poorer bit error rate of detection.

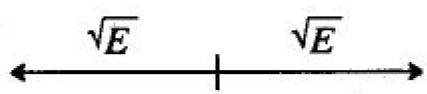
Q5. What’s the difference between Anti-podal and orthogonal signals?

Antipodal Signals :-

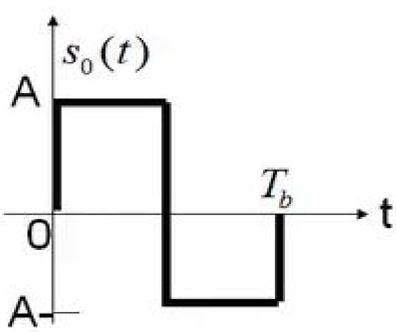
The signals that are 180 degree opposite to each other are called as antipodal signals. E.g: One signal can have value +1 and other will be equal to -1. Or one can be + $\sqrt{E_b}$ and other could be - $\sqrt{E_b}$

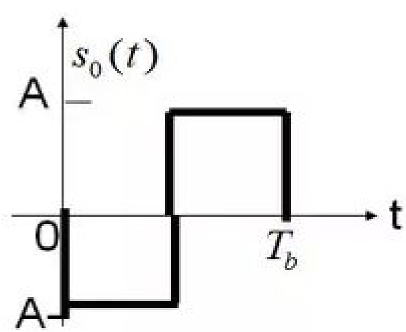
This signal shown in left, telecommunication used it in digital systems like BPSK (Binary Phase Shift Keying); the idea from this signal in telecom receiver to know if we get 0 or we get 1. If the value are closer to +E the receiver understand this as 1 and if the value are closer to -E the receiver understand it as 0.

BINARY ANTIPODAL SIGNALING



There’s another way to represent anti-podal signals too. One of the examples is given below:-



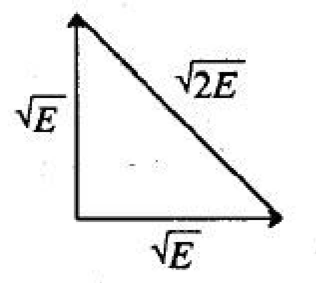


NOTE: The anti-podal signals shown in the first figure is the general form and the second one is just another form.

Orthogonal signals

The definition of orthogonality in linear algebra is that two vectors are orthogonal, if their inner product is zero or Orthogonal means having exactly 90 degree shift between those 2 signals.

BINARY OTHOGONAL SIGNALING



Contribution:- [8]

Digital communication assignment was a great experience for us. It helped in analyzing how the signals (orthogonal and anti podal) are transmitted in Gaussian Mean distributed error in AWGN channel.

We would like to express our special thanks to our Prof. Vijay Kumar Chakka Sir to let us work in a group that not only helped us understand how to implement ideas of others in MATLAB code but also improved our skills and ways to solve a particular problem.

Contribution of Group Members:-

Keshav and Shobhit, went through various articles related to the topics taught in the classes of Digital Communication from IEEE website and introduced many new concepts in the group. Kshitiz and Shobhit derived the algorithms for creating a Fading channel raysnr.m whereas Keshav and Kshitiz made the function mynn.m that makes major prediction for the received data on the basis of neural networks. All of us did hard coding to find the proper equations. We all in the end, combined the results and succeeded in getting the final model information and the threshold values as given in the paper.

We would also like to thank our lab instructors Venu Gopala Kotha and Srikanth Goli Sir for clarifying our doubts.