# Assignment 3

**Course name: PROBABILITY AND STOCHASTIC PROCESSES (AI5030)**

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# Question 1

A sample of size $n(\geq 2)$ is drawn from a population of $N \geq 3)$ units using $PPSWR$ sampling scheme, where $p_i$ is the probability of selecting $i^{th}$ unit in a draw, $0 < p_i <= 1$, $\forall i = 1, ..., N$ and $\sum_{i=1}^{N} p_i = 1$.
Then the inclusion probability $\pi_{i,j}$ is-

1. $1 - p_i^n - p_j^n + (p_i + p_j)^n$
2. $1 - (p_i + p_j - p_i p_j)^n$
3. $1 - (1 - p_i)^n - (1 - p_j)^n - (p_i + p_j)^n$
4. $1 - (1 - p_i)^n - (1 - p_j)^n + (1 - p_i p_j)^n$

**Answer:**
PPS stands for Probability Proportional to Size Sampling, is a two stage sampling technique where each stage uses Simple Random Sampling. So, overall PPS is same as Simple Random Sampling. PPSWR stands for PPS With Replacement of the sample units which is same as Simple Random Sampling With Replacement.
In case of Simple Random Sampling, each unit of the population has an equal and independent chance of being selected in the sample.
Let,
a population of size=N and a sample of size =n. So, in case of sampling with replacement, Total number of possible samples chosen with replacement=$N^n$
According to the assumption of SRS, Probability of selecting one unit from the population= $\frac{1}{N}$
So, Probability of selecting a sample of size n from the population of size N= $(\frac{1}{N})^n$.

Let, $\pi_i$= First Order Inclusion Probability = Probability that $i^{th}$ unit will be selected in the sample

$\pi_{i,j}$=Second Order Inclusion Probability = Probability that $i^{th}$ and $j^{th}$ units will be selected in the sample

Let Events,
$A_i$= $i^{th}$ unit will be selected in the First sampling unit $A_j$= $j^{th}$ unit will be selected in the Second sampling unit
According to the question, $p_i$ = Probability of selecting $i^{th}$ unit in a draw
$p_j$ = Probability of selecting $j^{th}$ unit in a draw
and, $p_i$=$p_j$ = $\frac{1}{N}$
Also,
P($\bar{A}_i$)=$(1-p_i)^n$
P($\bar{A}_j$)=$(1-p_j)^n$
$P(\bar{A}_i \cap \bar{A}_j) = (1 - p_i p_j)^n$

Applying the Inclusion-Exclusion Principle of Probability for combination with replacement case, we solve in the following way:
(Inclusion Probability)= 1-(Exclusion Probability)

$\pi_i$= P(A$_i$) $= 1 - P(\bar{A}_i)$ $=1 - (\frac{N-1}{N})^n$ $=1 - (1 - \frac{1}{N})^n$

Now,
$\pi_i$= P(A$_i \cap A_j$) $= 1-$P($\overline{A_i \cap A_j}$) =1-$P(\bar{A}_i \cup \bar{A}_j)$ =1-(P($\bar{A}_i$) $+ P(\bar{A}_j) - P(\bar{A}_i \cap \bar{A}_j)$) =1-(1-p$_i$)$^n - (1 - p_j)^n + (1 - p_i p_j)^n$ $=1 - ((1-\frac{1}{N})^n + (1-\frac{1}{N})^n - (1-\frac{2}{N})^n)$ $=1 - (2(1-\frac{1}{N})^n - (1-\frac{2}{N})^n)$ **Hence, the correct option is : 4** $1 - (1-p_i)^n - (1-p_j)^n + (1-p_i p_j)^n$