Assignment 1

Course name: PROBABILITY AND STOCHASTIC PROCESSES (AI5030)

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Question 1

Let $X_1, X_2, ...$ be a sequence of independent normally distributed random variables with mean 1 and variance 1. Let N be a Poisson random variable with mean 2, independent of $X_1, X_2, ...$. Then the variance of $X_1 + X_2 + ... + X_{N+1}$ is -

1. **3**
2. **4**
3. **5**
4. **9**

Answer:
Given:

$$X_1, X_2, ... \sim \mathcal{N}(\mu, \sigma^2) \sim \mathcal{N}(1, 1)$$

$$N \sim \mathscr{P}oisson(\lambda) \sim \mathscr{P}oisson(2)$$

We know that when two Random Variables, say X and Y, are independent and a and b are constants then:

1. $E(aX + bY) = aE(X) + bE(Y)$
2. $Var(aX + bY) = a^2Var(X) + b^2Var(Y) + 2abCov(X, Y)$
   where, $Cov(X, Y)$ represents the covariance between X and Y.
   For independent random variables, $Cov(X, Y) = 0$.
   So, the equation becomes-

$$Var(aX + bY) = a^2Var(X) + b^2Var(Y)$$

Let, $Y = X_1 + X_2 + ... + X_{N+1}$

Theorem 1. ***Sum of Gaussian Normal Random Variable is also a Gaussian Random Variable.***

Since all $Xi's$ are independent and are gaussian normal random variables then Y is also a random variable that follows gaussian normal distribution (using the above theorem).

$$\begin{aligned}\therefore Var(Y) &= Var(X_1 + X_2 + ... + X_{N+1}) \\ &= Var(\sum_{i=1}^{N+1} X_i) \\ &= \sum_{i=1}^{N+1} Var(X_i) \\ &= N + 1\end{aligned} \tag{1}$$

In this case, we have two distributions Y, a gaussian normal distribution and N, a poisson distribution which are dependent on each other. So, we need to find expectation in one of the two ways:

1. *Find expectation of Y making N as a constant,i.e.,$E[Y|N]$,conditional expectation of Y given N.*
2. *Find expectation of N making Y as a constant,i.e.,$E[N|Y]$,conditional expectation of N given Y.*

$$\begin{aligned}\therefore Var(Y) &= E[Y|N] \\ &= E(N + 1) \\ &= E(N) + 1 \\ &= 2 + 1 \\ &= 3 \qquad Ans.\end{aligned} \tag{2}$$

Hence, the correct option is : 1 3.