Assignment 2

Course name: PROBABILITY AND STOCHASTIC PROCESSES (AI5030)

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Question 1

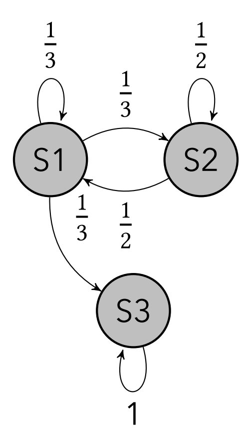
Suppose $\{X_n\}$ is a Markov Chain with 3 states and a transition probability matrix

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Then which of the following statement is true?

1. $\{X_n\}$ **is irreducible.**
2. $\{X_n\}$ **is recurrent.**
3. $\{X_n\}$ **does not admit a stationary probability distribution.**
4. $\{X_n\}$ **has an absorbing state.**

Answer:
The State-Transition Graph associated to above given Transition Probability Matrix is -



Definition 1. *A Markov Chain is said to be Irreducible if every state is reachable from every other state.*

Definition 2. *The state 'i' is said to be recurrent if starting from state 'i' wherever one can reach and there is a way to return back to same state state 'i'. Otherwise, state 'i' is said to be transient state. If all the states are Recurrent then the Markov Chain is said to be recurrent Markov Chain.*

Definition 3. *A state 'i' of a Markov Chain is called Absorbing State if $P_{i,i} = 1$ in the Transition Probability Matrix. A Markov Chain is called Absorbing Markov Chain if it contains at least one absorbing state.*

Theorem 4. *Absorbing Markov Chains have stationary distributions with non-zero elements only in the absorbing states.*

Analysis of Option 1 :

From state S1, we can reach the state S3, but from state S3, we can't reach state S1 again.

The Markov Chain is reducible, and Option 1 is wrong.

Analysis of Option 2 :

From state S2, we can reach state S1 and from state S1, we can reach state S3, but from state S3, there is no way to reach state S2. The return to state S2 via state S3 is not possible. So, state S2 is transient,i.e.,Non-Recurrent.

Hence, Option 2 is also wrong.

Analysis of Option 4 :

Clearly, we have $P_{3,3} = 1$. State S3 is absorbing state. So, Option 4 is correct.

Analysis of Option 3 :

From theorem 4 stated above, we can come to this conclusion that the given Markov Chain is an absorbing one and it should also have stationary distributions.

Hence, the correct option is : 4 $\{X_n\}$ has an absorbing state.