School of Mathematics and Statistics

MAST30012 Discrete Mathematics, Assignment 2, Semester 2, 2020

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Submission deadline is 23:59pm Monday 21 September.

Check Canvas for details of the late submission policy.
Submit your assignment online using Canvas as a *single* pdf *including this cover page* (more details to follow).

- Full working must be shown in your solutions.
- Marks will be deducted for incomplete working, insufficient justification or incorrect notation.

Q1: Give a bijective proof of the following identity

$$\sum_{k\geq 0}\binom{n}{2k} = 2^{n-1}$$

using the following steps:

(a) Use subsets of the set $[n]$ to define a set Ω_L representing the left-hand side.

(b) Define a set Ω_R representing the right-hand side as a certain set of tuples with entries from the set $\{0, 1\}$.

(c) Define a bijective function $\Gamma : \Omega_R \to \Omega_L$. (Note the direction of the function must be right-to-left).

(d) Show that your function in (c) is well defined. (There is no need to prove it is a bijection).

Your definitions in (a) and (b) *must* use the specification form of a set definition (see week 1 lecture notes).

Q2: Select $n^2 + 1$ distinct points in (or on the boundary of) the unit square. Prove that at least two points are no more than a distance $\sqrt{2}/n$ apart.

End of Assignment.

1. (a) Let $\Omega_L = \bigcup_{k \geq 0} B^n_{2k}$

where $B^n_{2k} = \{ S : S \subseteq [n] \ \& \ |S| = 2k \}$

$$|\Omega_L| = \left| \bigcup_{k\geq 0} B^n_{2k} \right|$$
$$= \sum_{k \geq 0} |B^n_{2k}| \quad \text{by Add th}^{\underline{m}}$$
$$= \sum_{k\geq 0} \binom{n}{2k}$$

(b) Let $\Omega_R = \{ (a_1, a_2, \cdots, a_{n-1}) : \forall i \in [n-1] \ a_i \in \{0,1\} \}$

(since $\Omega_R = \{0,1\}^{n-1}$ $\quad |\Omega_R| = |\{0,1\}^{n-1}|$
$$= |\{0,1\}|^{n-1} \quad \text{by Mul th}^{\underline{m}}$$
$$= 2^{n-1}$$

(c) Define $T : \Omega_R \to \Omega_L$ by

Let $(a_1, \cdots, a_{n-1}) \in \Omega_R$.

$$T(a_1, \cdots, a_{n-1}) = \begin{cases} \{ i \in [n-1] : a_i = 1 \} & \text{if } \sum_{i=1}^{n-1} a_i \text{ is even} \\ \{ i \in [n-1] : a_i = 1 \} \cup \{n\} & \text{if } \sum_{i=1}^{n-1} a_i \text{ is odd} \end{cases}$$

(d) Check the well defineness of T

① if $\sum a_i$ is even, we have an even number of occurences of 1

$\Rightarrow |\{i \in [n-1] : a_i = 1\}|$ is even as Ω_L required

Also $\{i \in [n-1] : a_i = 1\} \subseteq [n-1] \subseteq [n]$ as required

② if $\sum a_i$ is odd, we have an odd number of occurences of 1

$\Rightarrow |\{i \in [n-1] : a_i = 1\}|$ is odd

$\Rightarrow |\{i \in [n-1] : a_i = 1\} \cup \{n\}| = |\{i \in [n-1] : a_i = 1\}| + |\{n\}|$ by Add th^{m}

$= |\{i \in [n-1] : a_i = 1\}| + 1$ which is even

(sum of odd numbers is even)

This is as Ω_L required.

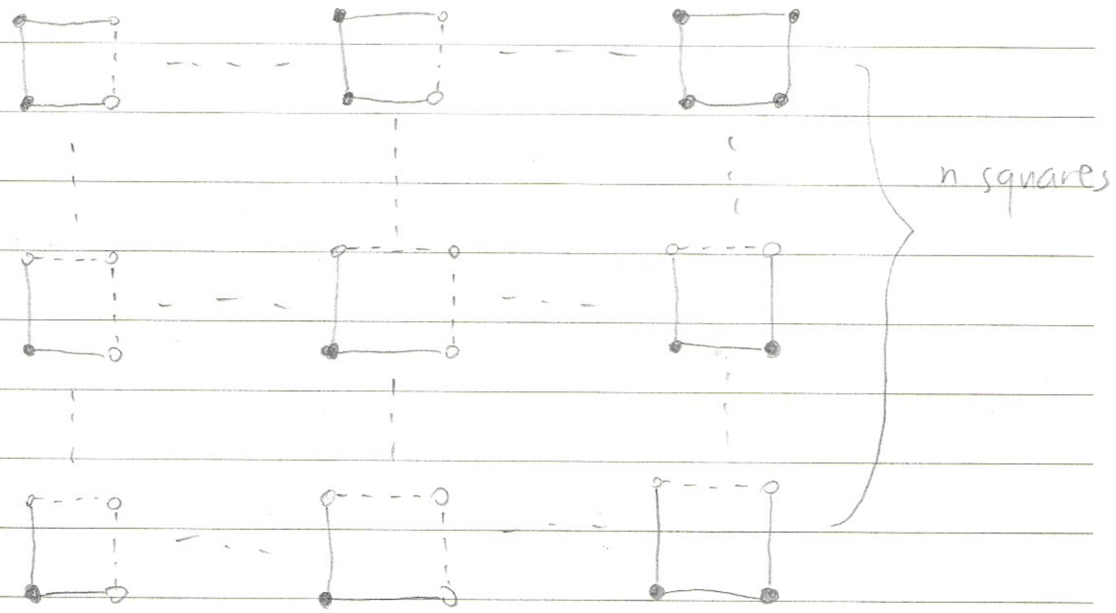
Also $(\{i \in [n-1] : a_i = 1\} \cup \{n\}) \subseteq [n]$ because

$\{i \in [n-1] : a_i = 1\} \subseteq [n]$ and $\{n\} \subseteq [n]$

This is also as Ω_L required

2. - Divide this unit square into n^2 smaller squares, each with a side length of $1/n$.
- The maximum distance between two points in the same square is then $\sqrt{2}/n$.
- We are placing n^2+1 distincts points into these n^2 squares. Hence by PHP, at least two points are in the same square. This means at least two points have their distance in between smaller or equal to $\sqrt{2}/n$.

Allocate boundaries to unique squares like below:



•, — means the point or line belongs to the square.
○, - - - means the point or line doesn't belong to the square.