

Q.1 (a)

A	B	C	$(A \wedge B) \Rightarrow C) \Rightarrow ((A \Rightarrow C) \vee (B \Rightarrow C))$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

Disjunction Normal Form  $\Rightarrow$

$$(A \wedge B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge \neg B \wedge \neg C) \\ \vee (\neg A \wedge B \wedge C) \vee (\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge \neg B \wedge C) \vee (\neg A \wedge \neg B \wedge \neg C)$$

$$\textcircled{2} \textcircled{a} (\exists x (B(x) \wedge \forall y (C(y) \Rightarrow (S(x,y) \Leftrightarrow \neg S(y,y)))) \Rightarrow \exists x (B(x) \wedge \neg C(x))$$

$$T (\exists x (B(x) \wedge \forall y (C(y) \Rightarrow (S(x,y) \Leftrightarrow \neg S(y,y))))$$

$$F \neg \exists x (B(x) \wedge \neg C(x))$$

$$T B(a) \wedge \forall y (C(y) \Rightarrow (S(a,y) \Leftrightarrow \neg S(y,y))) [x|a]$$

$$T B(a)$$

$$T \forall y (C(y) \Rightarrow (S(a,y) \Leftrightarrow \neg S(y,y))) [x|a]$$

$$T C(a) \Rightarrow (S(a,a) \Leftrightarrow \neg S(a,a)) [x|a] [y|a]$$

$$F C(a)$$

$$F (B(a) \wedge \neg C(a))$$

$$F B(a)$$

$$F \neg C(a)$$

$$T S(a,a) \Leftrightarrow \neg S(a,a)$$

$$T S(a,a)$$

$$T \neg S(a,a)$$

$$F S(a,a)$$

$$F \neg S(a,a)$$

Contradiction

All branches of Tableau are closed  
so its valid.

(b)  $\neg (\exists x (A(x) \Rightarrow B(x))) \Leftrightarrow (\forall x \neg A(x) \Rightarrow \exists x B(x))$

$\neg (\exists x (A(x) \Rightarrow B(x)))$   
 $\neg \forall x A(x) \Rightarrow \exists x B(x)$

$\neg \exists x (A(x) \Rightarrow B(x))$   
 $\neg \forall x A(x) \Rightarrow \exists x B(x)$

~~$\neg \forall x A(x)$~~   
 ~~$\neg \exists x B(x)$~~

$\neg A(a) \Rightarrow B(a) [x|a]$  new

$\neg A(a)$   
 $\neg \forall x A(x)$   
 $\neg \exists x B(x)$   
 $\neg A(a) [x|a]$

This branch is closed.

Since all branches are closed  
 so tableau is closed.

$\neg \forall x A(x)$  new  
 $\neg \exists x B(x)$  new  
 $\neg A(a) [x|a]$   
 $\neg A(a) \Rightarrow B(a) [x|a]$   
 $\neg A(a)$   
 $B(a)$   
 $\neg A(a) \Rightarrow B(a) [x|a]$   
 $\neg A(a)$   
 $B(a)$

This Branch is also closed.

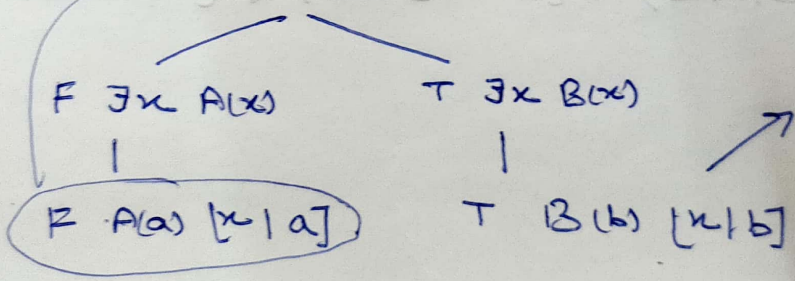


②  $\neg (\exists x A(x) \Rightarrow \exists x B(x)) \Rightarrow (\forall x (A(x) \Rightarrow B(x)))$

$\neg (\exists x A(x) \Rightarrow \exists x B(x))$   
 $\neg \forall x (A(x) \Rightarrow B(x))$

$\neg (A(a) \Rightarrow B(a)) \quad [x|a]$

$\neg (A(a) \Rightarrow B(a))$   
 $\neg (A(a) \Rightarrow B(a))$



This branch is open, so Tableau is not closed and it's not valid.

Q-3

A: Aldo passed the exam  
B: Bruno " " "  
C: Carlo " " "

Branch is also closed

- (a)  $C \wedge \neg A \wedge \neg B$
- (b)  $\neg A \wedge B \wedge C$
- (c)  $(A \wedge \neg B \wedge \neg C) \vee (\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge \neg B \wedge C)$
- (d)  $(A \wedge B \wedge \neg C) \vee (A \wedge \neg B \wedge C) \vee (\neg A \wedge B \wedge C)$
- (e)  $(A \wedge B \wedge \neg C) \vee (A \wedge \neg B \wedge C) \vee (\neg A \wedge B \wedge C) \vee (A \wedge B \wedge C)$