Application of Utility Maximization to Artificial Intelligence

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I. Abstract

Within artificial intelligence (AI), robotics, and game design there are often situations where an agent, an autonomous entity that makes decisions, modeled by either a finite state machine [1] or hierarchical state machine like a behavior tree [2] needs to make a decision from a finite set of certain decisions (decisions that lead to a specific outcome). For instance, say an autonomous car for a rideshare company is picking up and dropping off clientele at a fixed price. Their goal is to maximize their profit. However, they are just picking up and dropping off clients in order. This method does not take into account the utility [3] of each of their actions. Such as, the driver could be dropping off nearby clients first and then picking up all nearby clients to cut down on drive time. In this case, the action with the highest utility would be dropping off nearby clients followed by picking up nearby clients. However, the agent must also consider the amount of time each client is waiting to be picked up, since if a client waits too long, they might not use the service again. We would like to suggest an algorithmic approach to the above problem (not choosing an action without considering the utility of the action) that is based upon the economics concept of utility maximization. This algorithmic approach would determine which action an agent should take from a set of actions assigned with a given utility. In other words, an agent should choose an interaction that would produce the highest utility for their circumstances.

II. Problem Definition

Given a list of actions, it is not a trivial task for a state-driven agent to decide on which action to perform. As a result of this, our goal is to design a generalized algorithmic approach to this problem that is based on the concept of utility maximization.

In this section we will formally define key terminologies used and the problem definition for our algorithmic approach. Before we can formally define the problem, it is important that we first define several key terms.

Table 1: Key Terminology

Name	Description
Utility (u)	The utility gained such that $u \in \mathbb{R}$. Each u is identified by a unique string.
Utilities (U)	A set of u such that $U \subset \mathbb{R}^n$ where $n \in \mathbb{z}$.
Actions (A)	The set of all actions that can be taken.
Action (a)	A single action that can be taken such that $a \in A$.
State (s)	Represents a and the U gained from completing that specific action.
Agent (s_a)	A s instance which represents the beginning state of the agent making the decision.
Decision Tree (T)	A tree of s from which utility can be maximized. Each node in the tree can have 0 to ∞ children. This tree indicates all of the future decisions the agent can make in a finite amount of time.
Path (P)	An ordered set of s for paths such that $P \subset T$. A path consists of one chosen state from each level of T.
Utility Function $w(P)$	$w(P) = \sum_{s \in P} \sum_{U \in s} \sum_{u \in U} u$

With the key terminology defined, we can confidently outline our problem. Given an ***Agent (s_a)*** and a ***Decision Tree (T)***, what decision ***Path (P)*** can be made such that the utility of the agent is maximized based on the ***Utility Function f(P)***? Our goal is to create an algorithm that will find the path from the root of a tree to a leaf that maximizes the Utility of the Agent at the end of the decision tree.

In order to help visualize this problem, we have provided a visualization of example input in Table 2. In this example, the goal is to find the path that maximizes the utility of the agent, where the utility function is the summation of all U for a given P. In this example each node in the tree can have multiple parents

Table 2: Tree visualization

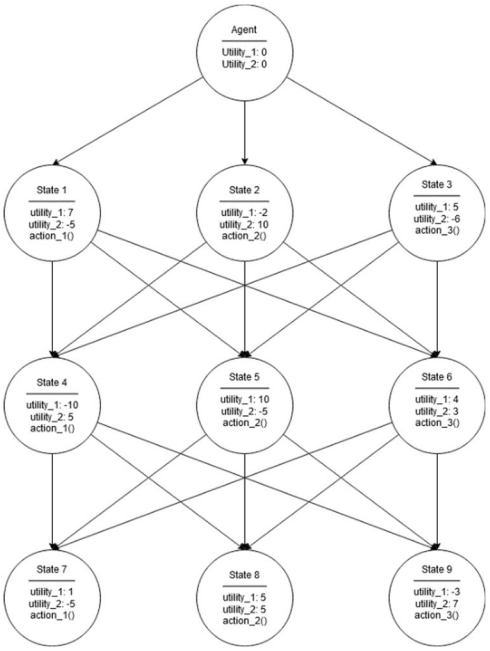
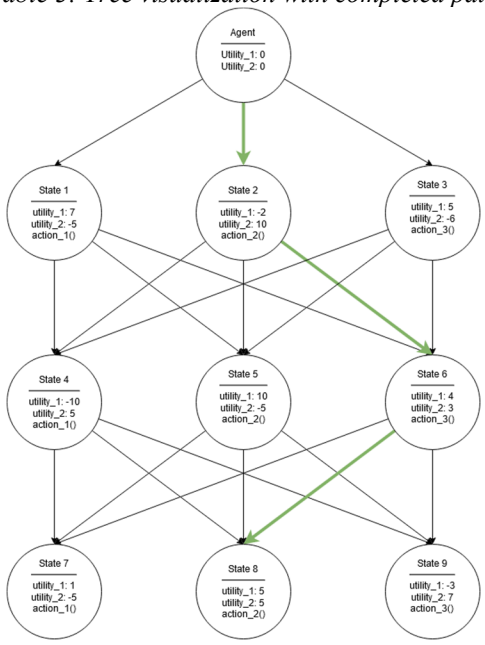


Table 3: Tree visualization with completed path



Based on table 3 the path in which the summation of utility is greatest is {"$State$ 2", "$State$ 6", "$State$ 8"} where $f(\{$"$State$ 2", "$State$ 6", "$State$ 8"$\})$ returns 25.

III. Literature Review

Coming up with a qualitative algorithm for decision making is not a new concept. This is one of the areas studied by decision theory, an interdisciplinary topic related with game theory [4] that is the study of an agent's choice(s) [5]. In order for us to establish a generalized algorithmic approach to qualitative decision making for a state driven AI, it is important that we take into account theories in both classical and contemporary decision theory.

Classical decision theory was proposed by mathematicians John von Neummann and Oskar Morgenstern in their 1944 book *Theory of Games and Economic Behavior* in which they defined the Von Neumann-Morgenstern (VNM) utility theorem [6]. In the VNM utility theorem, von Neummann and Morgenstern showed that when an agent is faced with a set of outcomes of different choices, they will behave in such a way such as to maximize the expected value of some function defined over potential outcomes at some point in the future [7]. Moreover, if the conditions for the VNM utility theorem are met, the agent is said to be rational and thus abide by the VNM utility theorem. The VNM utility theorem was the first basis to the expected utility hypothesis. Considered to first be introduced by Daniel Bernoulli in 1738 as a resolution to the St. Petersburg paradox [8], expected utility hypothesis is concerned with choosing an action when the decision maker is uncertain of the consequence from a chosen action. The VNM utility theorem was pivotal in establishing the classical decision theory. Leonard Savage shortly after proposed an alternate framework for looking at utility. Savage argued that expected utility is subjective to the agent [6]. As a result of this, an agent acts in their own interest whereas the VNM utility theorem does not treat an agent as being capable of making subjective decisions.

For our algorithm, we will pull logic and approaches from a combination of both the VNM utility theorem and Savage's subjective expected utility theorem to calculate the utility at each state within the decision tree. However, we will not be taking into account the probability that an action is successful: like both the VNM utility theorem [7] and Savage's subjective expected utility theorem [9]. We will also pull concepts from John von Neumann's minimax theorem. The most popular implementation of the minimax theorem is the minimax search algorithm which is a backtracking algorithm used to find a path in a tree, typically in two player board games [10]. Finally, we will also pull inspiration from both Dijkstra's shortest path algorithm and Bellman-Fords shortest path algorithm so that we have a perspective on popular optimization algorithms [11]. By building off of the VNM utility theorem, Savage's subjective expected utility theorem, von Neumann's minimax theorem, and popular optimization algorithms we have a solid theoretical foundation in classical decision theory from which to base our algorithm.

Contemporary decision theory is similar to classical decision. However, the introduction of more

advanced physical hardware has made decision theory more complicated in recent years. One of the primary differences between classical decision theory and contemporary decision theory is the introduction of machine learning and neural networks.

Machine learning takes a different route than classical decision theory when it comes to creating a model that represents an agent's decisions. In a 2014 article titled "On combining machine learning with decision making" published in Machine Learning, the authors created a machine learning framework that took into account utility through operational costs [12]. This allowed them to create a predictive model that better represented human decision-making when it came to modeling decisions making. However, in this instance decision-making was only applied to a single decision instead of a sequential set of decisions. Moreover, the goal was to both reflect human decision-making behavior as well as maximize utility. Moreover, their approach had to be retrained to deal with new decisions. Ultimately, since our approach for our project is for certain decisions, we do need not take into account probabilistic models and thus an approach through machine learning is excessive.

Neural networks, like machine learning, also take a different approach than classical decision theory. Like machine learning, neural networks are being used to determine the probabilistic outcome of a decision [13]. Through Bayesian statistics, neural networks are capable of determining sequential decisions that maximize utility [13]. Again, since our approach deals with certain decisions instead of uncertain decisions, the need to take into account the probability is unnecessary. Machine learning and neural networks have the power to model what decision should be taken based upon utility and human behavior incredibly accurately [12], [13], however they are specifically geared towards modeling decisions that take into account probability of uncertain decisions. As a result, our approach will not be dealing with contemporary decision theory, as it is excessive to utilize machine learning or a neural network where it is not necessary.

It's important to look at Classical and Contemporary Decision Theory in order to determine a generalized approach to determining the best way to implement our specific algorithm. In the case of the Classical decision theories, we have the VNM utility theorem and Savage's expected utility theorem. We will be referencing the VNM utility theorem because it provides insight on how an agent can maximize a specific value, given a set of outcomes. Additionally, we will reference Savage's subjective expected utility theorem since it deals with the addition of the agent's own interests with the expected outcome based on maximizing utility.

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IV. Our Algorithm

1. Our Proposed Algorithm

The problem we are trying to solve is an optimization problem. Because of the complexity of decision making, utilizing a brute force or similar approach would be an incredibly inefficient method to our specified problem. Given that the number of potential paths that could result from an agent making decisions, checking every single combination would result in an incredible amount of overhead for even a simple decision tree. As a result of this, we have opted to utilize a greedy approach to determine the path that maximizes an agent's utility. In order to do this, we are looking at two very famous greedy algorithms, Dijkstra's shortest path algorithm and the Bellman-Ford shortest path algorithm, as a starting point. Most importantly, we are taking inspiration from the technique of ***relaxation*** that both Dijkstra's and Bellman-Ford algorithms utilize [11]. However, we are modifying the relaxation step to favor larger path values over smaller path values. This is done since we are looking for the path that produces the largest utility value and not the smallest utility value. Furthermore, we are taking inspiration of the ***initialization*** step present in both afore mentioned. Again, it has been modified to favor larger path values over smaller path values. By taking these two steps as inspiration and modifying them to favor the paths with the largest utility values, it allowed us to quickly formulate the rest of the algorithm and find the decision path with the greatest resulting utility.

2. Algorithm Pseudo-Code

Before, we formally define our proposed algorithm we will define the relaxation principle. In our algorithm we will maintain an attribute $s.w$ which is lower bound on the weight of the maximum utility path from s_a to a leaf s in T and we will call the ***maximum utility estimate***. We will initialize the maximum utility estimates using the following procedure

INITIALIZE-SINGLE-SOURCE (T, s_a)

1 **for** each $s \in T.S$
2 $s.w = -\infty$
3 $s.\pi = NIL$
4 $s_a.w = 0$

Once initialization is complete, for all $s \in S$, $s_a.w = 0$, $s.\pi = NIL$, and for all $s \in S - \{s_a\}$, $s.w = -\infty$. With initialization defined, we can define the procedure for relaxation where $u \in T.S$. In the relaxation procedure we will update the maximum utility estimates for a State m. Furthermore, we are using our previously defined utility function $w(P)$ in our relaxation.

RELAX (m, s)

1 **if** $s.w < m.w + w(m, s)$
2 $s.w = m.w + w(m, s)$
3 $s.\pi = m$

With relaxation and initialization defined, we can move onto the core algorithm. Similar to Dijkstra's algorithm we will maintain a set of nodes and repeatedly select nodes to relax. However, unlike Dijkstra's algorithm, our algorithm can handle negative estimates. This is a result of our algorithm using a tree instead of graph making repeating cycles impossible. Our algorithm will select nodes from a maximum priority queue Q, relax the node's maximum utility estimate using the utility function, and then add those to set F. Once Q is empty, we find the maximum path in F and then the algorithm is done.

FIND-MAX-UTILITY-PATH (T, s_a)

1 INITIALIZE-SINGLE-SOURCE (T, s_a)
2 $F \leftarrow \emptyset$
3 $Q \leftarrow T.S$
4 **while** $Q \neq \emptyset$
5 **do** $m \leftarrow$EXTRACT-MAX (Q)
6 $F \leftarrow F \cup \{m\}$
7 **for** each $s \in Children[m]$
8 RELAX(m, s)
9 **return** FIND-MAX-PATH (F)

Finally, we need to define the procedure CONSTRUCT-PATH (F). This procedure will find the leaf State with the highest maximum utility estimate, such that $m \in F$. Then we will back track up the tree using the temporary state p such that $p \in F$. As a we backtrack up the tree, we will put States into the path variable P such that $P \in T$. Finally, we return P.

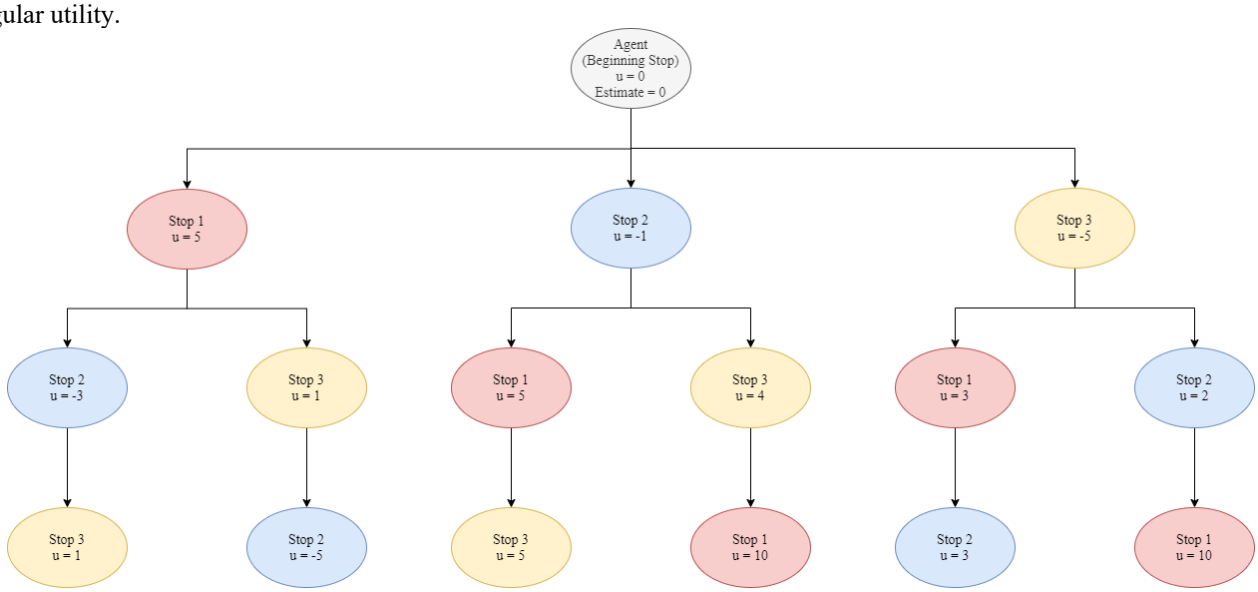
CONSTRUCT-PATH (F)

1 $m \leftarrow F[0]$
2 **for** each $s \in F$ with no children
3 **if** $m.w > s.w; m = s$
4 $P \leftarrow NIL$
5 $p = m$
6 **while** $p.\pi \neq NIL$
7 $P \leftarrow p$
8 $p = p.\pi$
9 **return** P

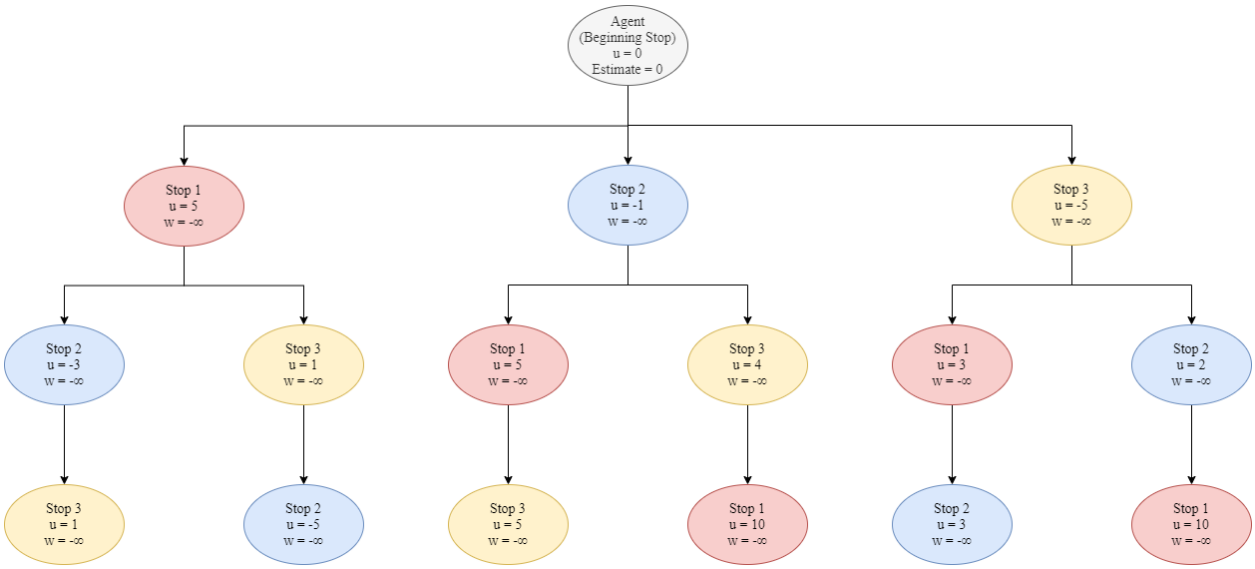
3. Use Case

For this algorithm, we will use an example of ride sharing to demonstrate how the driver can choose the order of each of their stops to yield the highest utility. For this example, we will use this tree to depict all the possible actions and the respective utility gained from taking those actions. The utility in this example is found based around the time it takes to reach

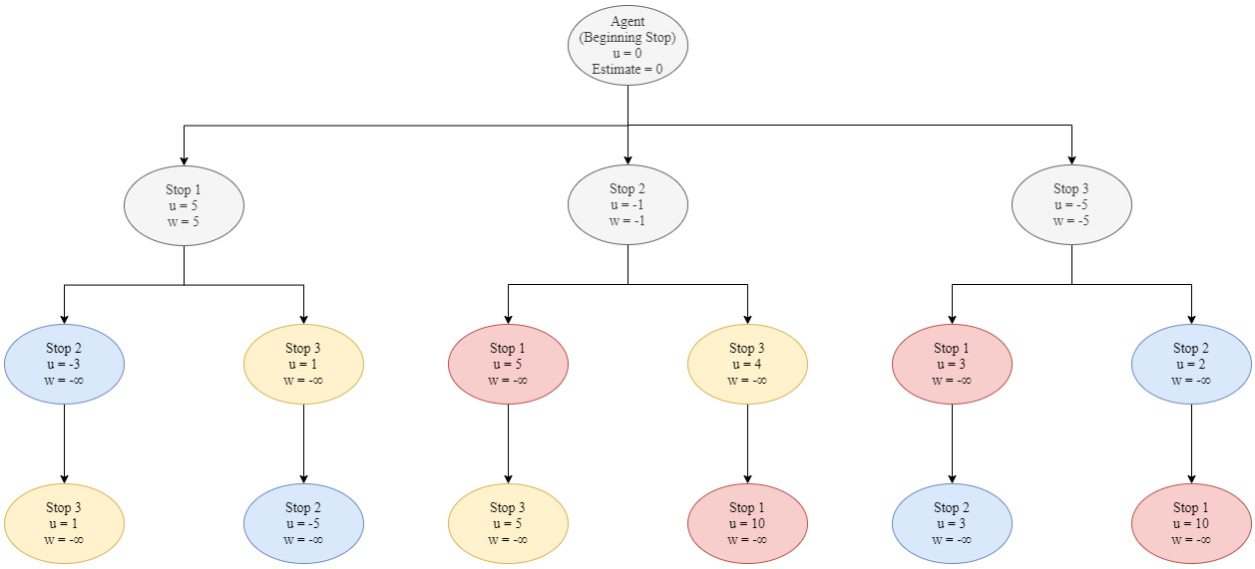
each stop. Thus, less time results in higher utility. Moreover, in this example the set of utilities is only made up of one singular utility.



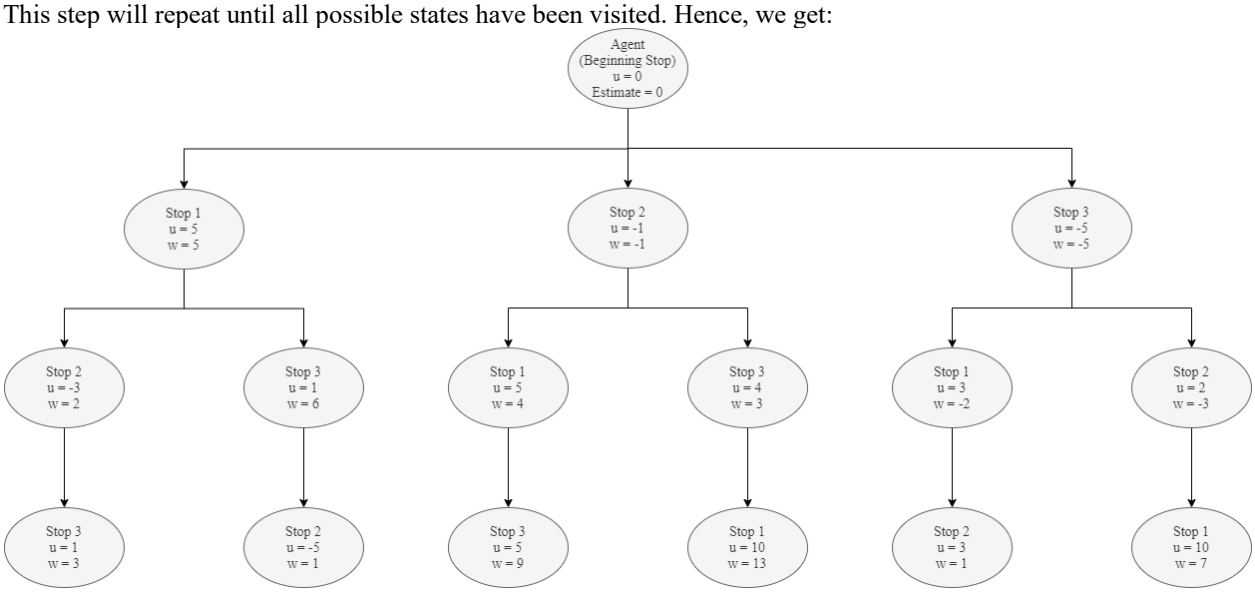
We first set the weight of all the state's, excluding the agent (s_a), as –∞ with the INITIALIZE-SINGLE-SOURCE (T, s_a) function.



Then, we "relax" the first three instances of stop 1, stop 2, and stop 3 by setting their weights equal to the sum of the stop's utility and the utility of all that stop's parent states. This is done through the utilization of RELAX(m, s). Thus, we have:



This step will repeat until all possible states have been visited. Hence, we get:



We then will utilize CONSTRUCT-PATH (F) to determine the path with the highest utility. With this, we will find all the last stops and compare their weights to determine which stop has the maximum weight (this stop being p). Then, we will find all the parent stops of p and store them into the maximum path P. Then, we shall receive the path of stops that garner the maximum utility.

4. Cost Analysis

There are several cases where our algorithm may operate in the best case, but we have concentrated on the worst-case analysis, primarily. The time-cost complexity analysis for our algorithm closely mirrors that of Dijkstra's shortest path algorithm. The final path consists of a single State from each level and back-traces from the bottom of the tree to the top. To complete our cost analysis, we have considered the cost for each of the major methods which we have for algorithm. The schema is such that the algorithm steps are on the left side and the complexity is on the right side for individual relevant steps.

Cost for the Initialization Step:

INITIALIZE-SINGLE-SOURCE ($\boldsymbol{T}, \boldsymbol{s_a}$)

1 **for** each $s \in T.S$ $\theta(|S|)$

2	$s.w = -\infty$	$\theta(\|S\|)$
3	$s.\pi = NIL$	$\theta(\|S\|)$
4	$s_a.w = 0$	$\theta(1)$

Explanation: Line 1 i.e. grabbing each state from the set of states – has complexity $\theta(|S|)$, where $|S|$ is the number of states or nodes in our graph. We initialize all the state's weights to minimum i.e. $-\infty$, and initializes the parent of each node to NIL – both having time complexity $\theta(|S|)$. Finally, the weight of the root node i.e. Agent is initialized to 0 which has a complexity of $\theta(1)$.

Cost for Relaxation Step:

RELAX ($\boldsymbol{m, s}$)

1	**if** $s.w < m.w + w(m,s)$	0 to $\theta(1)$
2	$s.w = m.w + w(m,s)$	0 *to* $\theta(1)$
3	$s.\pi = m$	0 *to* $\theta(1)$

Explanation: The relax step checks whether the accumulated utility of an extracted vertex(node) plus the additional utility attainable by going to state s will be greater than the accumulated utility up until s. This is an $\theta(1)$ operation and it will do so for all the edges.

Cost for Proposed Algorithm:

FIND-MAX-UTILITY-PATH ($\boldsymbol{T, s_a}$)

1	INITIALIZE-SINGLE-SOURCE (T, s_a)	$\theta(\|S\|)$
2	$F \leftarrow \emptyset$	$\theta(1)$
3	$Q \leftarrow T.S$	$\theta(\|S\|)$
4	**while** $Q \neq \emptyset$	$\theta(\|S\|)$
5	**do** $m \leftarrow$EXTRACT-MAX (Q)	$\theta(\|S\| \cdot$ *time complexity of* EXTRACT-MAX)
6	$F \leftarrow F \cup \{m\}$	$\theta(\|S\|)$
7	**for** each $s \in Children[m]$	$\theta(\|S\|)$
8	RELAX(m, s)	$\theta(\|$Children in T$\| \cdot$ *time complexity of* INCREASE-KEY)
9	**return** CONSTRUCT-PATH (F)	θ(h)

Explanation: The first step is the initialization step which we have described earlier. F is the queue which will hold all the vertices or states as we extract each one from Q. The priority Q is used to store as a max-heap all the states. Initially it will take $\theta(|S|)$ time, since all the states are initially $-\infty$ (there need not be any max-heapify operation involved at this stage). Line 4 is the while loop that runs $|S|$ times i.e. as long as Q is not empty. Line 5 is the extract maximum utility state until now, which runs $|S|$ times. The complexity for extracting and adjusting the priority queue to preserve its property of priority is $\theta(1)$ for array, $\theta(log|S|)$ for max-heap and $\theta(log|S|)$ for a Fibonacci heap implementations of the priority queue Q. Line 6 just unionizes the extracted vertex(state) m into the set F which runs $\theta(|S|)$. In line 7, it runs $Children\ in\ T$ times i.e. it will run the number of times equal to the number of outgoing children of m. The RELAX(m, s) method will take $\theta(|R|)$ to relax each edge. Finally, we return the CONSTRUCT-PATH (F) which takes θ(h) time.

Cost for Construct Path:

CONSTRUCT-PATH ($\boldsymbol{F}$)

1	$m \leftarrow F[0]$	$\theta(1)$
2	**for** each $s \in F$ with no children	
3	**if** $m.w > s.w; m = s$	
4	$P \leftarrow NIL$	$\theta\ (1)$
5	$p = m$	$\theta\ (1)$
6	**while** $p.\pi \neq NIL$	$\theta\ (h)$
7	$P \leftarrow p$	$\theta\ (h)$
8	$p = p.\pi$	$\theta\ (h)$
9	**return** P	$\theta\ (1)$

Explanation: To find maximum utility path, we first assign in line 1 the first state in F to m. Now, in line 2 and 3, we try to find the maximum utility path-weight amongst the leaf states. The time complexity for those two steps will depend on the number of leaf states we have. In line 4, we create the path P and assign it a null value which takes $\theta(1)$ time. Line 5 initially has the first highest utility vertex from the leaf nodes. Line 6 starts to iteratively back-trace the path by tracing back the parent of each current child until it reaches the root state. This should have a time-complexity of $\theta(h)$, where h is the height of the tree.

Therefore, the algorithm has a time complexity of our proposed algorithm is $\theta(|S| + 1 + |S| + |S| + |S| \cdot$ $time\ complexity\ of$ EXTRACT-MAX + |S| + |S| · |Children[m]| + $|Children\ in\ T| * time\ complexity\ of$ INCREASE)-KEY + h · time complexity of EXTRACT-MAX+ $|Children\ in\ T| * time\ complexity\ of$ INCREASE-KEY.

This can be simplified to $\theta(|S| \cdot$ EXTRACT-MAX+ $|Children\ in\ T| \cdot$ INCREASE-KEY).

5. Proof of Correctness

Here we prove that our algorithm is correct. However, before we do that let us declare the state attribute $s.\delta$ to be the maximum utility path summation from s_a to m. It is also to be noted that $s.w$ continues to be the maximum utility path summation estimate.

Lemma: For each $s \in F, s.w = s.\delta$; this is similar to Lemma 24.17 in *Introduction to Algorithms* by Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein [11].

Proof by Induction: *Base case*, $|F| = 1$: $|F| = 1$ only when $F = \{s_a\}$ and $s.w = s.\delta = 0$.

Inductive Hypothesis (IH)*:* Let m be the last state added to F. Let $F' = F \cup \{m\}$. Thus, $m \notin F$ and for each $s \in F'$, $s.w = s.\delta$.

By the IH, for every state in F' that is not m, the correct utility summation value is present. In order to prove by induction, we need to show that $m.w = m.\delta$.

Suppose there exists a maximum utility path Q from s_a to m such that $w(Q) < m.w$.

Q starts in F' and at some leaves F'. Let xy be the first edge in Q such that it leaves F'. Let F_x be the s_a to x path of Q. Thus, $w(Q_x) + w(xy) \leq w(Q)$.

Since $x.w$ is the length of the maximum path from s_a to x by IH, $x.w \leq w(Q_x)$ which gives us $x.w + w(xy) \leq w(Q_x)$.

Since y is adjacent to x, $y.w$ must have been updated by our algorithm, so $y.w \leq x.w + w(xy)$.

Finally, since m was picked by our algorithm, m must have the greatest utility summation value such that $m.w \leq y.w$. Combining these inequalities in reverse order gives us $x.w < x.w$ which is a contradiction so there is no path Q with greater maximum utility thus $m.w = m.\delta$.

Thus, through induction, there exists a path within the tree that maximizes the utility.

V. REPORT

1. Performance metrics

In regards to how we measured our algoritms performance, we decided to measure based on the accuracy and running-time of our algorithm when finding the max utility path for a given decision tree. With these metrics, we were able to determine in what conditions our the implementation of our algorithm worked best in.

When measuring the accuracy of the implmentation of our algorithm, we confirmed that the core class of our implementation, MaxUtilityPath, worked as intended. To test this, we checked to ensure the implmentation would compute the correct max utility path for a variety of different decision trees. The first decision tree struture we tested was one that allowed each state to have multiple parent states (*see Table 4 below*). Afterwards, we tested a tree that only allowed one parent state for each child state (*see Table 5 below*).

To test the running-time of our algorithm, we measured the amount of time that our *findMaxUtilityPath(State agent)* method, the core method of our implmentation, took to execute. We did this by using our randomally generated ride share dataset (described in the dataset description below) and changing the amount of locations the ride share driver needed to visit. With this, we were able to see how the run-time of our algorithm was affected with an increasing number of paths and states.

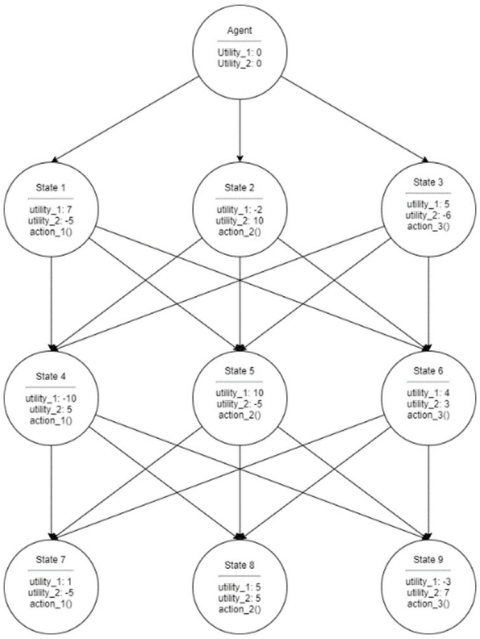
2. Description of datasets

We used two datasets to show that our implementaition of the algorithm was able to successfully find the maximum utility of path for a given tree. In our first dataset, we used a generic decision tree with arbitrary values. For the second dataset, we used an example of a ride sharing service, in which we generated a decision tree for every possible combination of locations that the driver will visit. These two examples will show how this algorithm can work on decision trees with varing structures.

To model a decision tree that meets the conditions defined in our problem definition, we created a State class and Action interface. The State class has attributes for each state's name, action, utility array, and child states. The Action interface represents an action an agent can take. To build the decision tree, we would first assign a State object a name, action, and utility array (The utilities in the utility array must correspond in all States in the decision tree). Any children of that state are added with the *addChild(State child)* method: which adds that child State object to a list of children states in the parent State object. With this, we have a decision tree of states, where each state has an action, set of utilities, and a list of references to any children of that state.

For our first dataset, our goal was to exemplify how our Utility Function worked on a generic descision tree that allowed for multiple parents to each child state. We used the following decision tree to test this:

Table 4: Tree visualization



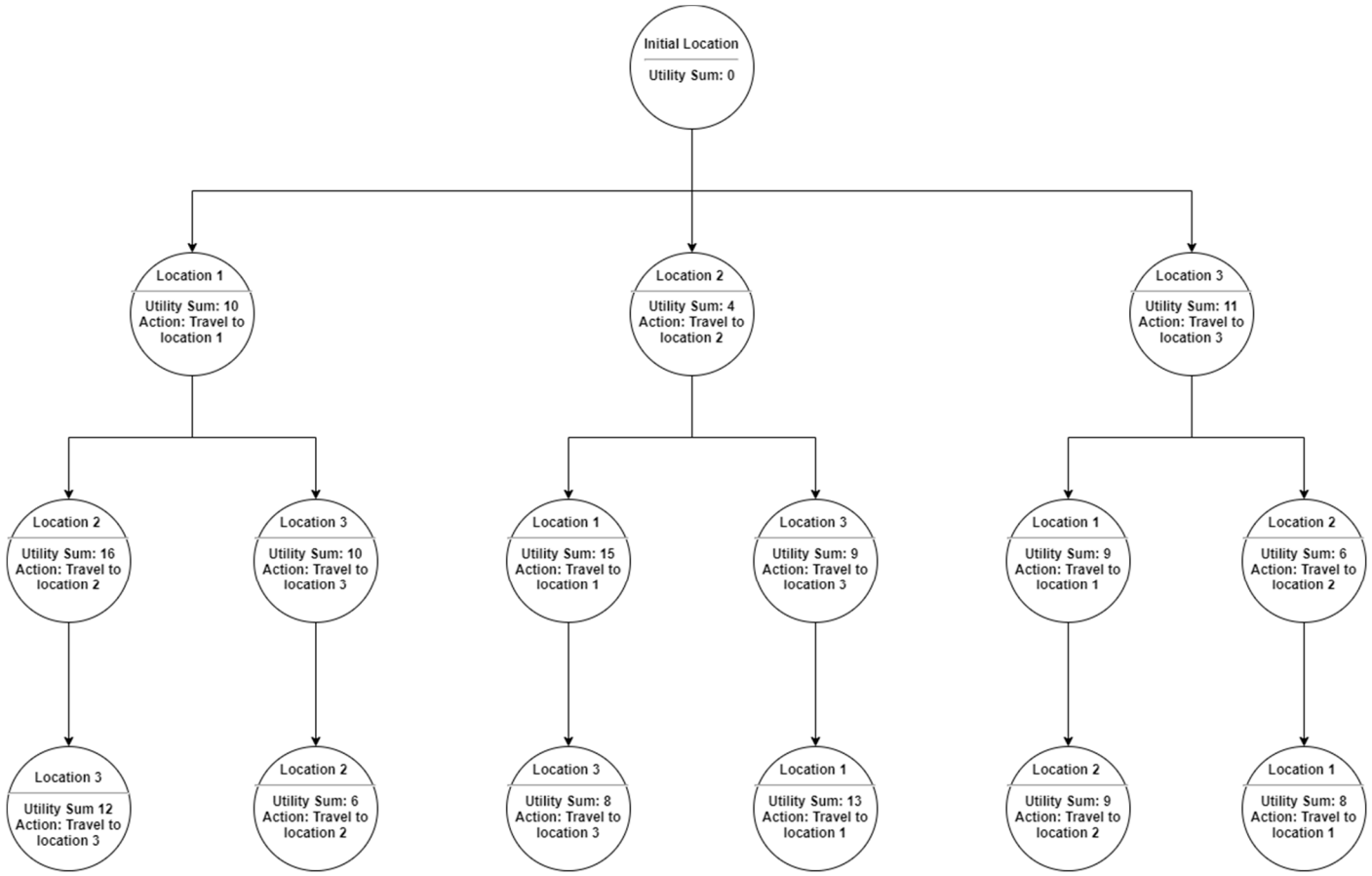
The path in which the utility is greatest is {"Agent", "$State$ 2", "$State$ 6", "$State$ 8"}. Therefore, when we use the *findMaxUtilityPath(State agent)* method, we should get an ordered list that represents the path where utility is the greatest.

In our ride share example, we took a different approach to building the decision tree. This example considers a situation where a ride share driver must go to n locations in an order that maximizes their utility. Also, the utility values for each state will be determined by multiple factors, such as: the order the locations are reached in and the time it takes to get from one specific location to another. This means that in every possible order of n locations, each location will have a unique set of utility values. Thus, all states in the decision tree are unique.

The number of possible paths in the tree will be all permutations of the n locations. In other words, there will be $n!$ possible paths in the tree. This means that even a small amount of locations will generate an enormous number of paths that a driver could take. For instance, 10 locations would have 3,628,800 possible combinations of locations the driver could visit. With this in mind, we tried to keep the number of locations lower so that is clear to display that our algorithm will pick the path with the highest utility.

In our implementation of this example, we created a RideShare class to construct the decision tree. We had the root state of the decision tree be the driver's initial position and the utility values for each visited location as randomized integer. With these parameters, the class would generate a decision tree. Below is an example of a decision tree generated by our RideShare class:

Table 5: Tree visualization



In this case, the path in the decision tree that would maximize the driver's utility would be : {"Initial Location", "$Location$ 1", "$Location$ 2", "$Location$ 3"}. Therefore, when we use the *findMaxUtilityPath(State initialLocation)* method in the MaxUtilityPath class, we should get the sum of the utilities contained in these locations.

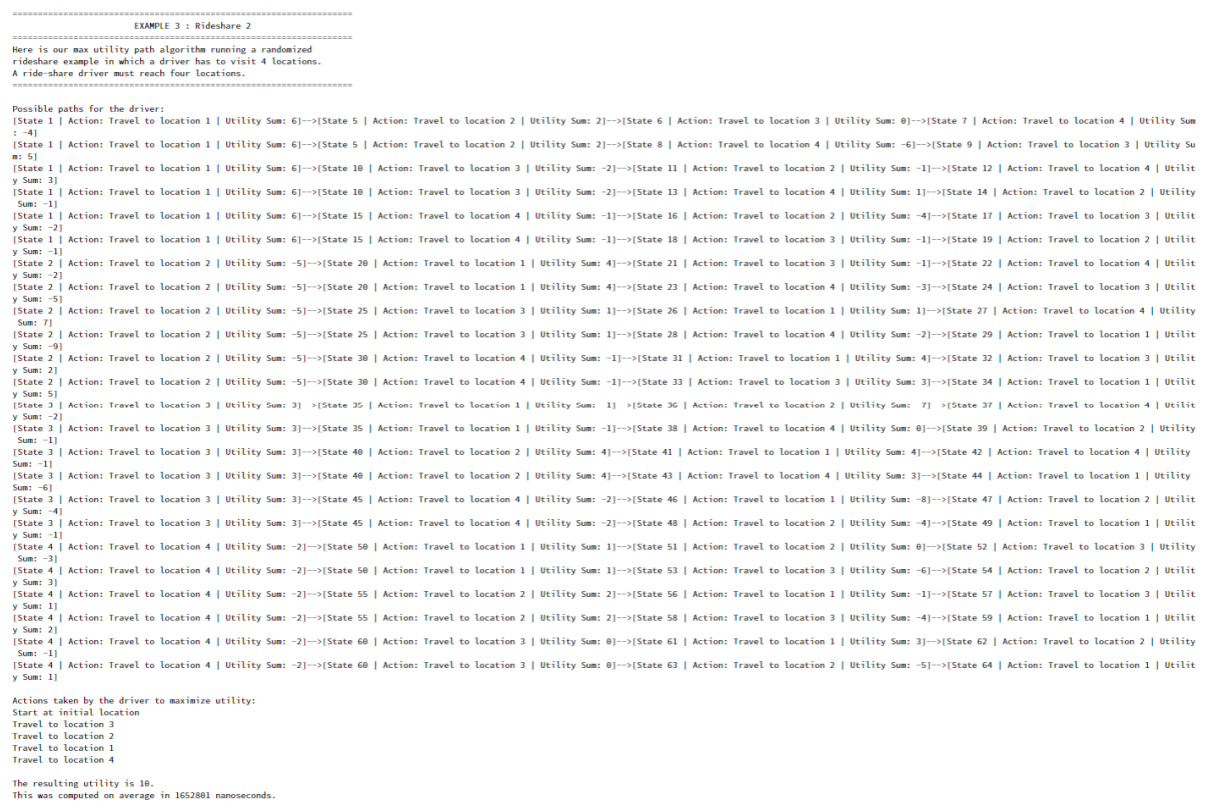
3. Screenshots of Test Runs

Below we have included screenshots of three of our tests runs (*see Table 6 and Table 7*) of our implementation running on a 19.10 Ubuntu Linux machine with an Intel(R) Core(TM) i7-3632QM CPU @ 2.20GHz, 8 GB DDR4 2666 MHz, and a 750GB 5400 RPM SATA hard drive. The code was compiled with version 1.8.0_252 of the Java compiler and the machine was dedicated to the task.

Table 6: Screenshot of First 2 Test Runs

```
kyle@probatus:~/MaxUtility/MaxUtility/src$ java main
=====================================================================
                        EXAMPLE 1 : Table 2
=====================================================================
Here is our max utility path algorithm running on the
example provided in table 2 of our paper.
=====================================================================

This actions are taken by visiting:
Start at empty state.
Visit State 2
Visit State 6
Visit State 8

The resulting utility is 25.
This was computed in 56005133 nanoseconds.

=====================================================================
                        EXAMPLE 2 : Rideshare
=====================================================================
Here is our max utility path algorithm running a randomized
rideshare example in which a driver has to visit 3 locations.
A ride-share driver must reach three locations.
=====================================================================

Possible paths for the driver:
[State 1 | Action: Travel to location 1 | Utility Sum: 10]-->[State 4 | Action: Travel to location 2 | Utility Sum: 16]-->[State 5 | Action: Travel to location 3 | Utility Sum: 12]
[State 1 | Action: Travel to location 1 | Utility Sum: 10]-->[State 6 | Action: Travel to location 3 | Utility Sum: 10]-->[State 7 | Action: Travel to location 2 | Utility Sum: 6]
[State 2 | Action: Travel to location 2 | Utility Sum: 4]-->[State 8 | Action: Travel to location 1 | Utility Sum: 15]-->[State 9 | Action: Travel to location 3 | Utility Sum: 8]
[State 2 | Action: Travel to location 2 | Utility Sum: 4]-->[State 10 | Action: Travel to location 3 | Utility Sum: 9]-->[State 11 | Action: Travel to location 1 | Utility Sum: 13]
[State 3 | Action: Travel to location 3 | Utility Sum: 11]-->[State 12 | Action: Travel to location 1 | Utility Sum: 9]-->[State 13 | Action: Travel to location 2 | Utility Sum: 9]
[State 3 | Action: Travel to location 3 | Utility Sum: 11]-->[State 14 | Action: Travel to location 2 | Utility Sum: 6]-->[State 15 | Action: Travel to location 1 | Utility Sum: 8]

Actions taken by the driver to maximize utility:
Start at initial location
Travel to location 1
Travel to location 2
Travel to location 3

The resulting utility is 38.
This was computed on average in 451037 nanoseconds.
```

Table 7: Screenshot of 3rd Test Run



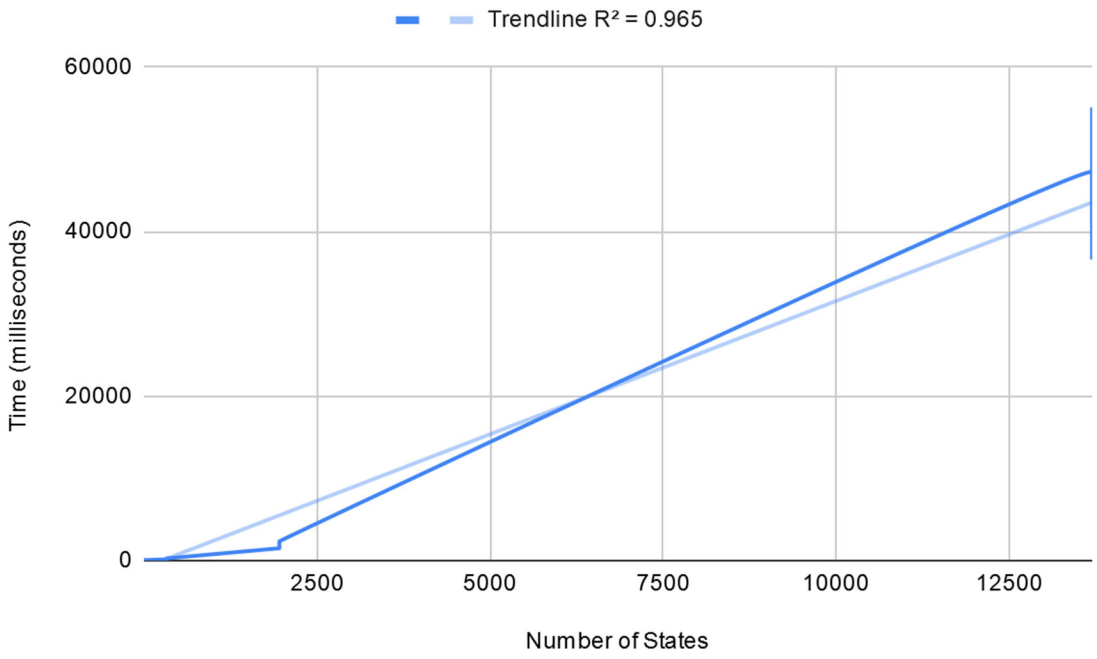
```
=====================================================================
                        EXAMPLE 3 : Rideshare 2
=====================================================================
Here is our max utility path algorithm running a randomized
rideshare example in which a driver has to visit 4 locations.
A ride-share driver must reach four locations.
=====================================================================

Possible paths for the driver:
[State 1 | Action: Travel to location 1 | Utility Sum: 6]-->[State 5 | Action: Travel to location 2 | Utility Sum: 2]-->[State 6 | Action: Travel to location 3 | Utility Sum: 0]-->[State 7 | Action: Travel to location 4 | Utility Sum
: -4]
[State 1 | Action: Travel to location 1 | Utility Sum: 6]-->[State 5 | Action: Travel to location 2 | Utility Sum: 2]-->[State 8 | Action: Travel to location 4 | Utility Sum: -6]-->[State 9 | Action: Travel to location 3 | Utility Su
m: 5]
[State 1 | Action: Travel to location 1 | Utility Sum: 6]-->[State 10 | Action: Travel to location 3 | Utility Sum: -2]-->[State 11 | Action: Travel to location 2 | Utility Sum: -1]-->[State 12 | Action: Travel to location 4 | Utilit
y Sum: 3]
[State 1 | Action: Travel to location 1 | Utility Sum: 6]-->[State 10 | Action: Travel to location 3 | Utility Sum: -2]-->[State 13 | Action: Travel to location 4 | Utility Sum: 1]-->[State 14 | Action: Travel to location 2 | Utility
 Sum: -1]
[State 1 | Action: Travel to location 1 | Utility Sum: 6]-->[State 15 | Action: Travel to location 4 | Utility Sum: -1]-->[State 16 | Action: Travel to location 2 | Utility Sum: -4]-->[State 17 | Action: Travel to location 3 | Utilit
y Sum: -2]
[State 1 | Action: Travel to location 1 | Utility Sum: 6]-->[State 15 | Action: Travel to location 4 | Utility Sum: -1]-->[State 18 | Action: Travel to location 3 | Utility Sum: -1]-->[State 19 | Action: Travel to location 2 | Utilit
y Sum: -1]
[State 2 | Action: Travel to location 2 | Utility Sum: -5]-->[State 20 | Action: Travel to location 1 | Utility Sum: 4]-->[State 21 | Action: Travel to location 3 | Utility Sum: -1]-->[State 22 | Action: Travel to location 4 | Utilit
y Sum: -2]
[State 2 | Action: Travel to location 2 | Utility Sum: -5]-->[State 20 | Action: Travel to location 1 | Utility Sum: 4]-->[State 23 | Action: Travel to location 4 | Utility Sum: -3]-->[State 24 | Action: Travel to location 3 | Utilit
y Sum: -5]
[State 2 | Action: Travel to location 2 | Utility Sum: -5]-->[State 25 | Action: Travel to location 3 | Utility Sum: 1]-->[State 26 | Action: Travel to location 1 | Utility Sum: 1]-->[State 27 | Action: Travel to location 4 | Utility
 Sum: 7]
[State 2 | Action: Travel to location 2 | Utility Sum: -5]-->[State 25 | Action: Travel to location 3 | Utility Sum: 1]-->[State 28 | Action: Travel to location 4 | Utility Sum: -2]-->[State 29 | Action: Travel to location 1 | Utilit
y Sum: -9]
[State 2 | Action: Travel to location 2 | Utility Sum: -5]-->[State 30 | Action: Travel to location 4 | Utility Sum: -1]-->[State 31 | Action: Travel to location 1 | Utility Sum: 4]-->[State 32 | Action: Travel to location 3 | Utilit
y Sum: 2]
[State 2 | Action: Travel to location 2 | Utility Sum: -5]-->[State 30 | Action: Travel to location 4 | Utility Sum: -1]-->[State 33 | Action: Travel to location 3 | Utility Sum: 3]-->[State 34 | Action: Travel to location 1 | Utilit
y Sum: 5]
[State 3 | Action: Travel to location 3 | Utility Sum: 3]  >[State 35 | Action: Travel to location 1 | Utility Sum:  1]  >[State 36 | Action: Travel to location 2 | Utility Sum:  7]  >[State 37 | Action: Travel to location 4 | Utilit
y Sum: -2]
[State 3 | Action: Travel to location 3 | Utility Sum: 3]-->[State 35 | Action: Travel to location 1 | Utility Sum: -1]-->[State 38 | Action: Travel to location 4 | Utility Sum: 0]-->[State 39 | Action: Travel to location 2 | Utility
 Sum: -1]
[State 3 | Action: Travel to location 3 | Utility Sum: 3]-->[State 40 | Action: Travel to location 2 | Utility Sum: 4]-->[State 41 | Action: Travel to location 1 | Utility Sum: 4]-->[State 42 | Action: Travel to location 4 | Utility
Sum: -1]
[State 3 | Action: Travel to location 3 | Utility Sum: 3]-->[State 40 | Action: Travel to location 2 | Utility Sum: 4]-->[State 43 | Action: Travel to location 4 | Utility Sum: 3]-->[State 44 | Action: Travel to location 1 | Utility
Sum: -6]
[State 3 | Action: Travel to location 3 | Utility Sum: 3]-->[State 45 | Action: Travel to location 4 | Utility Sum: -2]-->[State 46 | Action: Travel to location 1 | Utility Sum: -8]-->[State 47 | Action: Travel to location 2 | Utilit
y Sum: -4]
[State 3 | Action: Travel to location 3 | Utility Sum: 3]-->[State 45 | Action: Travel to location 4 | Utility Sum: -2]-->[State 48 | Action: Travel to location 2 | Utility Sum: -4]-->[State 49 | Action: Travel to location 1 | Utilit
y Sum: -1]
[State 4 | Action: Travel to location 4 | Utility Sum: -2]-->[State 50 | Action: Travel to location 1 | Utility Sum: 1]-->[State 51 | Action: Travel to location 2 | Utility Sum: 0]-->[State 52 | Action: Travel to location 3 | Utility
 Sum: -3]
[State 4 | Action: Travel to location 4 | Utility Sum: -2]-->[State 50 | Action: Travel to location 1 | Utility Sum: 1]-->[State 53 | Action: Travel to location 3 | Utility Sum: -6]-->[State 54 | Action: Travel to location 2 | Utilit
y Sum: 3]
[State 4 | Action: Travel to location 4 | Utility Sum: -2]-->[State 55 | Action: Travel to location 2 | Utility Sum: 2]-->[State 56 | Action: Travel to location 1 | Utility Sum: -1]-->[State 57 | Action: Travel to location 3 | Utilit
y Sum: 1]
[State 4 | Action: Travel to location 4 | Utility Sum: -2]-->[State 55 | Action: Travel to location 2 | Utility Sum: 2]-->[State 58 | Action: Travel to location 3 | Utility Sum: -4]-->[State 59 | Action: Travel to location 1 | Utilit
y Sum: 2]
[State 4 | Action: Travel to location 4 | Utility Sum: -2]-->[State 60 | Action: Travel to location 3 | Utility Sum: 0]-->[State 61 | Action: Travel to location 1 | Utility Sum: 3]-->[State 62 | Action: Travel to location 2 | Utility
 Sum: -1]
[State 4 | Action: Travel to location 4 | Utility Sum: -2]-->[State 60 | Action: Travel to location 3 | Utility Sum: 0]-->[State 63 | Action: Travel to location 2 | Utility Sum: -5]-->[State 64 | Action: Travel to location 1 | Utilit
y Sum: 1]

Actions taken by the driver to maximize utility:
Start at initial location
Travel to location 3
Travel to location 2
Travel to location 1
Travel to location 4

The resulting utility is 10.
This was computed on average in 1652801 nanoseconds.
```

4. Descriptions of Results

Table 8: Average Run Times

Average Time (milliseconds per 1000 runs)	Number of States	Number of Locations
78.110682	2	1
14.25147	5	2
27.310935	16	3
154.116136	326	5
1503.014378	1957	6
47227.34895	13700	7

Table 9: Run Times and Number of States



The implementation of our algorithm was put to test with the example RideShare class. An agent (analogous to the driver of the car in this case) decides the sequence of location he is going to visit in order to maximize the utility at the end of the process. This is decision is taken based on the generation of the maximum utility path. Each state is the location that can be visited after a certain number of locations already visited. The time calculated in the graph above is through Java's nanoTime() for just finding the max-utility path after the decision tree generated.

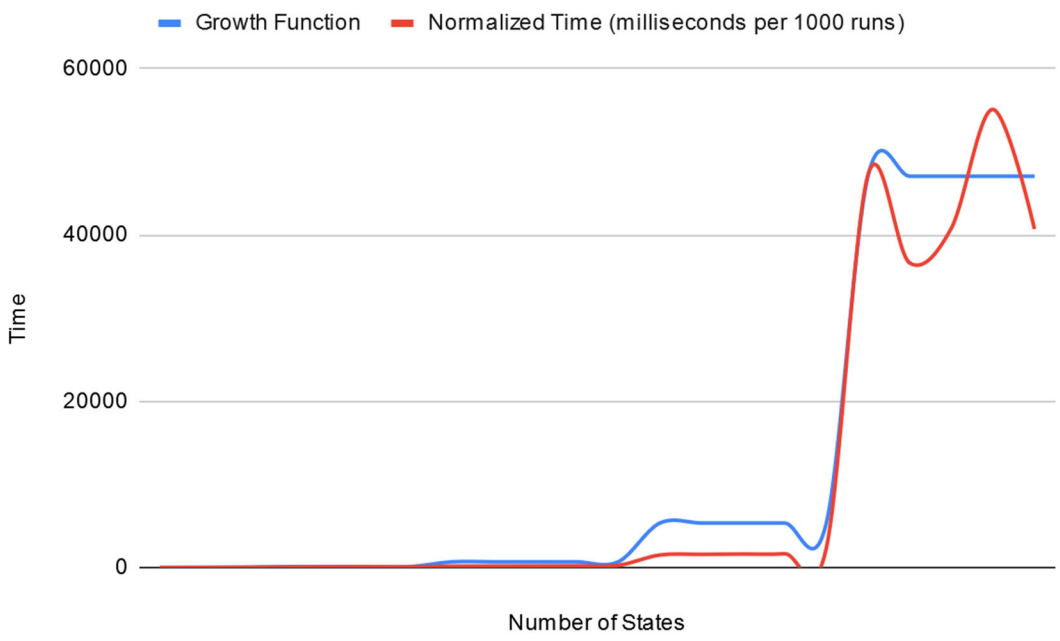
The above data table has been recorded by running simulations of the RideShare example for 1000 times for each number of locations and then averaged over to reduce the deviations. This has been done 5 times for each number of locations resulting in total of 35 runs. We have gone up to 7 locations from 1 location. The graph shows a strong correlation and the large value of R indicates an accurate representation of the algorithmic time complexity.
Parameters Calculated/used:

Time (Milliseconds): This is just the unit conversion for better perspective of the nanoseconds column, which depicts the time in nano seconds, to milli seconds.

States: This is the number of possible states for a given number of locations.

Location: This is the number of locations the driver must travel to

VI. Result Analysis



In our RideShare simulation example we have depicted the states to have only one parent. That is a driver can choose that state at most once after having travelled an arbitrary number of states. We have also used a binary Max-Heap implementation of priority queue for storing all the states in accordance with their maximum expected utility values. So, the larger the max-expected-utility, greater priority that state has.

For our algorithm, the time complexity is $\theta(|S| \cdot EXTRACT - MAX + |Children\ in\ T| \cdot INCREASE - KEY)$, where $|S|$ is the number of states and T is the decision tree or the entire utility tree. Since, we used a max-heap implementation of the priority queue, we have the time complexity for extract-max as $\theta(|S| \cdot logS)$ for all the states in T. The second part has a lower order growth as the number of children is the number of states. The increase key operation takes constant time. So, the overall time complexity of this RideShare example is $\theta(|S| \cdot logS)$. From the above graph, we can observe that the growth function of theoretical time complexity closely resembles the computed graph for the datasets after normalizing for time.

CONTRIBUTION CLARIFICATIONS

1. **Abstract**

   All members equally contributed to the abstract.

2. **Literature Review**

   All members equally contributed to the literature review including reading and writing all of the referenced material.

3. **Problem Definition**

   All members equally contributed to the problem definition.

4. **Our Algorithm**

   Soumya Mitra wrote 33% of section IV.1, 5% of IV.2, 5% of IV.3, and 90% of IV.4.
   Matthew Coe wrote 33% of section IV.1, 5% of IV.2, 90% of IV.3, and 5% of IV.4.
   Kyle Lierer wrote 33% of section 4.1, 90% of IV.2, 5% of IV.3, 5% of IV.4, and 100% of IV.5.

5. **Report**

   All members equality contributed to the report.

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