

1 章 数と式の計算

§2 いろいろな数と式 (p.33~p.34)

練習問題 2-A

1.

$$(1) \text{ 与式} = \frac{4y}{3x^2z^3}$$

$$\begin{aligned} (2) \text{ 与式} &= \frac{x+y}{x-y} - \frac{x^2+y^2}{(x+y)(x-y)} \\ &= \frac{(x+y)^2 - (x^2+y^2)}{(x+y)(x-y)} \\ &= \frac{x^2+2xy+y^2-x^2-y^2}{(x+y)(x-y)} \\ &= \frac{2xy}{(x+y)(x-y)} \end{aligned}$$

$$\begin{aligned} (3) \text{ 与式} &= \frac{y(x+y) - y^2 + x^2}{xy(x+y)} \\ &= \frac{xy + y^2 - y^2 + x^2}{xy(x+y)} \\ &= \frac{x(x+y)}{xy(x+y)} \\ &= \frac{1}{y} \end{aligned}$$

$$\begin{aligned} (4) \text{ 与式} &= \frac{(a-2)(a-3)}{(a-3)(a-4)} \times \frac{(a+4)(a-4)}{(a+2)(a-2)} \times \frac{a+2}{a+4} \\ &= 1 \end{aligned}$$

$$\begin{aligned} (5) \text{ 与式} &= \frac{\left(x - \frac{2}{x+1}\right) \times (x+1)}{\left(x+1 - \frac{4}{x+1}\right) \times (x+1)} \\ &= \frac{x(x+1) - 2}{(x+1)^2 - 4} \\ &= \frac{x^2 + x - 2}{x^2 + 2x + 1 - 4} \\ &= \frac{(x-1)(x+2)}{(x-1)(x+3)} \\ &= \frac{x+2}{x+3} \end{aligned}$$

$$\begin{aligned} (5) \text{ 与式} &= \frac{\left(\frac{1}{x-1} + 1\right) \times (x+1)(x-1)}{\left(\frac{1}{x+1} - 1\right) \times (x+1)(x-1)} \\ &= \frac{(x+1) + (x+1)(x-1)}{(x-1) - (x+1)(x-1)} \end{aligned}$$

$$= \frac{x+1 + (x^2-1)}{x-1 - (x^2-1)}$$

$$= \frac{x^2+x}{-x^2+x}$$

$$= \frac{x(x+1)}{-x(x-1)}$$

$$= -\frac{x+1}{x-1}$$

2.

$$\begin{aligned} (1) \text{ 与式} &= \frac{\sqrt{2}+1}{\sqrt{2}-1} + \frac{\sqrt{2}-1}{\sqrt{2}+1} \\ &= \frac{(\sqrt{2}+1)^2 + (\sqrt{2}-1)^2}{(\sqrt{2}-1)(\sqrt{2}+1)} \\ &= \frac{2+2\sqrt{2}+1+2-2\sqrt{2}+1}{2-1} \\ &= 6 \end{aligned}$$

$$\begin{aligned} (2) \text{ 与式} &= \frac{(\sqrt{2}+1)(\sqrt{2}-1)}{(\sqrt{2}-1)(\sqrt{2}+1)} \\ &= 1 \end{aligned}$$

$$\begin{aligned} (3) \text{ 与式} &= (x^2 + 2xy + y^2) - 2xy \\ &= (x+y)^2 - 2xy \\ &= 6^2 - 2 \cdot 1 \\ &= 36 - 2 = 34 \end{aligned}$$

$$\begin{aligned} (4) \text{ 与式} &= \frac{x^2+y^2}{xy} \\ &= \frac{34}{1} = 34 \end{aligned}$$

3.

$$(1) 1 + \sqrt{5} = X \text{ とおく.}$$

$$\begin{aligned} \text{与式} &= (X + \sqrt{6})(X - \sqrt{6}) \\ &= X^2 - (\sqrt{6})^2 \\ &= (1 + \sqrt{5})^2 - 6 \\ &= 1 + 2\sqrt{5} + 5 - 6 \\ &= 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} (2) \text{ 与式} &= \frac{(1+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} - \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} \\ &= \frac{2+\sqrt{3}+2\sqrt{3}+3}{4-3} - \frac{3-2\sqrt{3}+1}{3-1} \end{aligned}$$

$$= \frac{5+3\sqrt{3}}{1} - \frac{4-2\sqrt{3}}{2}$$

$$= 5+3\sqrt{3}-2+\sqrt{3}$$

$$= 3+4\sqrt{3}$$

$$\begin{aligned} (3) \text{ 与式} &= (\sqrt{3}i + \sqrt{2})(3\sqrt{2}i - 2\sqrt{3}) \\ &= 3\sqrt{6} \cdot i^2 - 2 \cdot 3i + 3 \cdot 2i - 2\sqrt{6} \\ &= -3\sqrt{6} - 6i + 6i - 2\sqrt{6} \\ &= -5\sqrt{6} \end{aligned}$$

$$\begin{aligned} (4) \text{ 与式} &= \frac{3+\sqrt{2}i}{3-\sqrt{2}i} + \frac{3-\sqrt{2}i}{3+\sqrt{2}i} \\ &= \frac{(3+\sqrt{2}i)^2 + (3-\sqrt{2}i)^2}{(3-\sqrt{2}i)(3+\sqrt{2}i)} \\ &= \frac{9+6\sqrt{2}i+2i^2+9-6\sqrt{2}i+2i^2}{9-2i^2} \\ &= \frac{18+4 \cdot (-1)}{9+2} \end{aligned}$$

$$= \frac{14}{11}$$

$$\begin{aligned} (5) \text{ 与式} &= 1^3 - 3 \cdot 1^2 \cdot i + 3 \cdot 1 \cdot i^2 - i^3 \\ &= 1 - 3i - 3 - i^2 \cdot i \\ &= -2 - 3i + i \\ &= -2 - 2i \end{aligned}$$

$$\begin{aligned} (6) \text{ 与式} &= \frac{(\sqrt{3}+i)^2 - (\sqrt{3}-i)^2}{(\sqrt{3}-i)(\sqrt{3}+i)} \\ &= \frac{\{(\sqrt{3}+i) + (\sqrt{3}-i)\}\{(\sqrt{3}+i) - (\sqrt{3}-i)\}}{3-i^2} \\ &= \frac{2\sqrt{3} \cdot 2i}{3-(-1)} \\ &= \frac{4\sqrt{3}i}{4} \\ &= \sqrt{3}i \end{aligned}$$

4.

$$\begin{aligned} (1) \text{ 与式} &= |2\sqrt{5}-2| + |\sqrt{5}-3| \\ &= (2\sqrt{5}-2) - (\sqrt{5}-3) \\ &= 2\sqrt{5}-2-\sqrt{5}+3 \\ &= 1+\sqrt{5} \end{aligned}$$

$$\begin{aligned} (2) \text{ 与式} &= \sqrt{(\pi-3)^2} + \sqrt{(\pi-4)^2} \\ &= |\pi-3| + |\pi-4| \\ &= (\pi-3) - (\pi-4) \\ &= \pi-3-\pi+4 \\ &= 1 \end{aligned}$$

$$\begin{aligned} (3) \text{ 与式} &= \frac{|\sqrt{5}-2|}{|-3+\sqrt{5}|} \\ &= \frac{\sqrt{5}-2}{-(-3+\sqrt{5})} \\ &= \frac{\sqrt{5}-2}{3-\sqrt{5}} \\ &= \frac{(\sqrt{5}-2)(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} \\ &= \frac{3\sqrt{5}+5-6-2\sqrt{5}}{9-5} \\ &= \frac{\sqrt{5}-1}{4} \end{aligned}$$

$$\begin{aligned} (4) \text{ 与式} &= \sqrt{(-2)^2 + (\sqrt{3})^2} - \sqrt{(-\sqrt{6})^2 + (-1)^2} \\ &= \sqrt{4+3} - \sqrt{6+1} \\ &= \sqrt{7} - \sqrt{7} = 0 \end{aligned}$$

練習問題 2-B

1.

$$\begin{aligned} (1) \text{ 与式} &= \frac{2a^2}{(2a+1)(2a-1)} + \frac{a-1}{-(2a-1)} \\ &= \frac{-2a^2 + (a-1)(2a+1)}{-(2a+1)(2a-1)} \\ &= \frac{-2a^2 + 2a^2 + a - 2a - 1}{-(2a+1)(2a-1)} \\ &= \frac{-a-1}{-(2a+1)(2a-1)} \\ &= \frac{a+1}{(2a+1)(2a-1)} \end{aligned}$$

$$\begin{aligned} (2) \text{ 与式} &= \frac{1(a-1)-1(a+1)}{(a+1)(a-1)} + \frac{2}{a^2+1} + \frac{4}{a^4+1} \\ &= \frac{-2}{a^2-1} + \frac{2}{a^2+1} + \frac{4}{a^4+1} \\ &= \frac{-2(a^2+1)+2(a^2-1)}{(a^2-1)(a^2+1)} + \frac{4}{a^4+1} \\ &= \frac{-4}{a^4-1} + \frac{4}{a^4+1} \\ &= \frac{-4(a^4+1)+4(a^4-1)}{(a^4-1)(a^4+1)} \\ &= -\frac{8}{a^8-1} \end{aligned}$$

$$\begin{aligned}
 (3) \text{ 与式} &= \frac{\frac{a^2}{(a+1)(a-1)} - 1}{\frac{1}{a+1} - \frac{1}{a-1}} \\
 &= \frac{\left(\frac{a^2}{(a+1)(a-1)} - 1\right) \times (a+1)(a-1)}{\left(\frac{1}{a+1} - \frac{1}{a-1}\right) \times (a+1)(a-1)} \\
 &= \frac{a^2 - (a+1)(a-1)}{(a-1) - (a+1)} \\
 &= \frac{a^2 - a^2 + 1}{a-1-a-1} \\
 &= \frac{1}{-2} = -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 (4) \text{ 与式} &= \frac{x^2 \times \left(1 - \frac{1}{x}\right)}{\left(x + \frac{1}{1 - \frac{1}{x}}\right) \times \left(1 - \frac{1}{x}\right)} \\
 &= \frac{x^2 - x}{x - 1 + 1} \\
 &= \frac{x(x-1)}{x} \\
 &= x - 1
 \end{aligned}$$

$$\begin{aligned}
 (5) \text{ 与式} &= \frac{x \times (x+1)}{\left(1 - \frac{1}{x+1}\right) \times (x+1)} \\
 &\quad - \frac{(x+2) \times (x+1)}{\left(1 + \frac{1}{x+1}\right) \times (x+1)} \\
 &= \frac{x(x+1)}{x+1-1} - \frac{(x+2)(x+1)}{x+1+1} \\
 &= (x+1) - (x+1) \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 (6) \text{ 与式} &= \frac{2a^2}{\frac{a \times a}{\left(1 - \frac{1}{a}\right) \times a} - \frac{a \times a}{\left(1 + \frac{1}{a}\right) \times a}} \\
 &= \frac{2a^2}{\frac{a^2}{a-1} - \frac{a^2}{a+1}} \\
 &= \frac{2a^2 \times (a-1)(a+1)}{\left(\frac{a^2}{a-1} - \frac{a^2}{a+1}\right) \times (a-1)(a+1)} \\
 &= \frac{2a^2(a-1)(a+1)}{a^2(a+1) - a^2(a-1)} \\
 &= \frac{2(a-1)(a+1)}{(a+1) - (a-1)} \\
 &= \frac{2(a-1)(a+1)}{2}
 \end{aligned}$$

$$= (a-1)(a+1)$$

2.

$$\begin{aligned}
 (1) \text{ 与式} &= \frac{1(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} \\
 &\quad - \frac{1(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} \\
 &\quad + \frac{1(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} \\
 &= (\sqrt{2}+1) - (\sqrt{3}+\sqrt{2}) + (2+\sqrt{3}) \\
 &= \sqrt{2}+1-\sqrt{3}-\sqrt{2}+2+\sqrt{3} \\
 &= 3
 \end{aligned}$$

$$\begin{aligned}
 (2) \text{ 与式} &= \frac{1\{(1-\sqrt{2})-\sqrt{3}\}}{\{(1-\sqrt{2})+\sqrt{3}\}\{(1-\sqrt{2})-\sqrt{3}\}} \\
 &= \frac{1-\sqrt{2}-\sqrt{3}}{(1-\sqrt{2})^2-3} \\
 &= \frac{1-\sqrt{2}-\sqrt{3}}{1-2\sqrt{2}+2-3} \\
 &= \frac{(1-\sqrt{2}-\sqrt{3}) \cdot \sqrt{2}}{-2\sqrt{2} \cdot \sqrt{2}} \\
 &= \frac{\sqrt{2}-2-\sqrt{6}}{-4} \\
 &= \frac{2-\sqrt{2}+\sqrt{6}}{4}
 \end{aligned}$$

3.

$x = \sqrt{2a-1}$ を、 $\sqrt{a^2-x^2}$ に代入すると

$$\begin{aligned}
 \sqrt{a^2-x^2} &= \sqrt{a^2 - (\sqrt{2a-1})^2} \\
 &= \sqrt{a^2 - (2a-1)} \\
 &= \sqrt{a^2 - 2a + 1} \\
 &= \sqrt{(a-1)^2} \\
 &= |a-1|
 \end{aligned}$$

i) $a-1 \geq 0$, すなわち, $a \geq 1$ のとき

$$|a-1| = a-1$$

ii) $a-1 < 0$, すなわち, $a < 1$

$$a \geq \frac{1}{2} \text{であるから, } \frac{1}{2} \leq a < 1$$

$$|a-1| = -(a-1) = -a+1$$

よって

$$\begin{cases} a \geq 1 \text{ のとき} & a - 1 \\ \frac{1}{2} \leq a < 1 \text{ のとき} & 1 - a \end{cases}$$

4.

$\alpha = a + bi, \beta = c + di$ とおく.

$$\begin{aligned} (1) \text{ 左辺} &= (a + bi)\overline{(a + bi)} \\ &= (a + bi)(a - bi) \\ &= a^2 - (bi)^2 \\ &= a^2 + b^2 \end{aligned}$$

$$\begin{aligned} \text{右辺} &= \left(\sqrt{a^2 + b^2}\right)^2 \\ &= a^2 + b^2 \end{aligned}$$

よって, 左辺=右辺

$$\begin{aligned} (2) \text{ 左辺} &= \overline{(a + bi) + (c + di)} \\ &= \overline{(a + c) + (b + d)i} \\ &= (a + c) - (b + d)i \\ \text{右辺} &= \overline{a + bi} + \overline{c + di} \\ &= a - bi + c - di \\ &= (a + c) - (b + d)i \end{aligned}$$

よって, 左辺=右辺