

3 章 積分法

§ 1 不定積分と定積分 (p.82~p.98)

問 1 C は積分定数

$$\begin{aligned} (1) \text{ 与式} &= \frac{1}{5+1} x^{5+1} + C \\ &= \frac{1}{6} x^6 + C \end{aligned}$$

$$\begin{aligned} (2) \text{ 与式} &= \int x^{-2} dx \\ &= \frac{1}{-2+1} x^{-2+1} + C \\ &= -x^{-1} + C \\ &= -\frac{1}{x} + C \end{aligned}$$

$$\begin{aligned} (3) \text{ 与式} &= \int x^{-\frac{1}{2}} dx \\ &= \frac{1}{-\frac{1}{2}+1} x^{-\frac{1}{2}+1} + C \\ &= \frac{1}{\frac{1}{2}} x^{\frac{1}{2}} + C \\ &= 2\sqrt{x} + C \end{aligned}$$

問 2 C は積分定数

$$\begin{aligned} (1) \text{ 与式} &= \int (2x^3 + 3x^2 - 2x + 5) dx \\ &= 2 \int x^3 dx + 3 \int x^2 dx - 2 \int x dx + 5 \int dx \\ &= 2 \cdot \frac{1}{4} x^4 + 3 \cdot \frac{1}{3} x^3 - 2 \cdot \frac{1}{2} x^2 + 5x + C \\ &= \frac{1}{2} x^4 + x^3 - x^2 + 5x + C \end{aligned}$$

$$\begin{aligned} (2) \text{ 与式} &= \int (3 \cos x + 4e^x) dx \\ &= 3 \int \cos x dx + 4 \int e^x dx \\ &= 3 \sin x + 4e^x + C \end{aligned}$$

$$\begin{aligned} (3) \text{ 与式} &= \int \left(6 \sin x + \frac{2}{x} \right) dx \\ &= 6 \int \sin x dx + 2 \int \frac{1}{x} dx \\ &= 6 \cdot (-\cos x) + 2 \log|x| + C \end{aligned}$$

$$= -6 \cos x + 2 \log|x| + C$$

$$\begin{aligned} (4) \text{ 与式} &= \int \left(x^2 - 2 + \frac{1}{x^2} \right) dx \\ &= \int x^2 dx - 2 \int dx + \int x^{-2} dx \\ &= \frac{1}{3} x^3 - 2x - \frac{1}{x} + C \end{aligned}$$

問 3 C は積分定数

$$\begin{aligned} (1) \int x^4 dx &= \frac{1}{5} x^5 + C \text{ より} \\ \text{与式} &= \frac{1}{4} \cdot \frac{1}{5} (4x+1)^5 + C \\ &= \frac{1}{20} (4x+1)^5 + C \end{aligned}$$

$$\begin{aligned} (2) \int \sin x dx &= -\cos x \text{ より} \\ \text{与式} &= \frac{1}{3} \cdot (-\cos 3x) + C \\ &= -\frac{1}{3} \cos 3x + C \end{aligned}$$

$$\begin{aligned} (3) \int e^x dx &= e^x + C \text{ より} \\ \text{与式} &= \frac{1}{5} e^{5x+2} + C \end{aligned}$$

問 4

$$(1) x_k = \frac{k}{n}, \quad \Delta x_k = \frac{1}{n} \quad (n=1, 2, \dots, n) \text{ より}$$

$$\begin{aligned} S_{\Delta} &= \sum_{k=1}^n \frac{k}{n} \cdot \frac{1}{n} = \frac{1}{n^2} \sum_{k=1}^n k \\ &= \frac{1}{n^2} \cdot \frac{n(n+1)}{2} \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2} \cdot \frac{n^2+n}{n^2} \\ &= \frac{1}{2} \left(1 + \frac{1}{n} \right) \end{aligned}$$

(2) $\Delta x_k \rightarrow 0$ のとき, $n \rightarrow \infty$ であるから

$$\begin{aligned}\int_0^1 x \, dx &= \lim_{n \rightarrow \infty} \frac{1}{2} \left(1 + \frac{1}{n} \right) \\ &= \frac{1}{2} (1 + 0) = \frac{1}{2}\end{aligned}$$

問 5

$$\begin{aligned}(1) \text{ 与式} &= 3 \int_0^1 x \, dx + \int_0^1 dx \\ &= 3 \cdot \frac{1}{2} + 1 \cdot (1 - 0)\end{aligned}$$

$$= \frac{3}{2} + 1 = \frac{5}{2}$$

$$\begin{aligned}(2) \text{ 与式} &= 5 \int_0^1 x^2 \, dx + 3 \int_0^1 x \, dx - 4 \int_0^1 dx \\ &= 5 \cdot \frac{1}{3} + 3 \cdot \frac{1}{2} - 4 \cdot (1 - 0) \\ &= \frac{5}{3} + \frac{3}{2} - 4 \\ &= \frac{10 + 9 - 24}{6} = -\frac{5}{6}\end{aligned}$$

問 6

$$(1) \int \cos x \, dx = \sin x + C \text{ であるから}$$

$$\begin{aligned}\text{与式} &= \left[\sin x \right]_0^{\frac{\pi}{2}} \\ &= \sin \frac{\pi}{2} - \sin 0 \\ &= 1 - 0 = 1\end{aligned}$$

$$\begin{aligned}(2) \int \sqrt[3]{x} \, dx &= \int x^{\frac{1}{3}} \, dx \\ &= \frac{3}{4} x^{\frac{1}{3}+1} + C \\ &= \frac{3}{4} x^{\frac{4}{3}} \sqrt[3]{x} + C\end{aligned}$$

であるから

$$\begin{aligned}\text{与式} &= \left[\frac{3}{4} x^{\frac{4}{3}} \sqrt[3]{x} \right]_0^1 \\ &= \frac{3}{4} \cdot 1^{\frac{4}{3}} \sqrt[3]{1} - 0 = \frac{3}{4}\end{aligned}$$

問 7

$$\begin{aligned}(1) \text{ 与式} &= 5 \int_0^2 x^3 \, dx + 3 \int_0^2 x^2 \, dx - 3 \int_0^2 x \, dx - 2 \int_0^2 dx \\ &= 5 \left[\frac{1}{4} x^4 \right]_0^2 + 3 \left[\frac{1}{3} x^3 \right]_0^2 - 3 \left[\frac{1}{2} x^2 \right]_0^2 - 2 \left[x \right]_0^2 \\ &= \frac{5}{4} \left[x^4 \right]_0^2 + \left[x^3 \right]_0^2 - \frac{3}{2} \left[x^2 \right]_0^2 - 2 \left[x \right]_0^2 \\ &= \frac{5}{4} (2^4 - 0) + (2^3 - 0) - \frac{3}{2} (2^2 - 0) - 2(2 - 0) \\ &= 20 + 8 - 6 - 4 = \mathbf{18}\end{aligned}$$

または

$$\begin{aligned}\text{与式} &= \left[\frac{5}{4} x^4 + x^3 - \frac{3}{2} x^2 - 2x \right]_0^2 \\ &= \left(\frac{5}{4} \cdot 2^4 + 2^3 - \frac{3}{2} \cdot 2^2 - 2 \cdot 2 \right) - 0 \\ &= 20 + 8 - 6 - 4 = \mathbf{18}\end{aligned}$$

$$\begin{aligned}(2) \text{ 与式} &= \int_1^4 \left(x - 2 + \frac{1}{x} \right) \\ &= \int_1^4 x \, dx - 2 \int_1^4 dx + \int_1^4 \frac{1}{x} \, dx \\ &= \left[\frac{1}{2} x^2 \right]_1^4 - \left[2x \right]_1^4 + \left[\log|x| \right]_1^4 \\ &= \left(8 - \frac{1}{2} \right) - (8 - 2) + (\log 4 - \log 1) \\ &= \frac{15}{2} - 6 + \log 2^2 \\ &= \frac{3}{2} + 2 \log 2\end{aligned}$$

または

$$\begin{aligned}\text{与式} &= \left[\frac{1}{2} x^2 - 2x + \log|x| \right]_1^4 \\ &= (8 - 8 + \log|4|) - \left(\frac{1}{2} - 2 + \log|1| \right) \\ &= \log 2^2 - \frac{1}{2} + 2 \\ &= \frac{3}{2} + 2 \log 2\end{aligned}$$

$$\begin{aligned}(3) \text{ 与式} &= 3 \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin x \, dx - 2 \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \cos x \, dx \\ &= 3 \left[-\cos x \right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} - 2 \left[\sin x \right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}\end{aligned}$$

$$\begin{aligned}
&= -3\left(\cos\frac{5\pi}{4} - \cos\frac{\pi}{4}\right) - 2\left(\sin\frac{5\pi}{4} - \sin\frac{\pi}{4}\right) \\
&= -3\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) - 2\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) \\
&= -3(-\sqrt{2}) - 2(-\sqrt{2}) \\
&= 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}
\end{aligned}$$

または

$$\begin{aligned}
\text{与式} &= \left[-3\cos x - 2\sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \\
&= \left(-3\cos\frac{5\pi}{4} - 2\sin\frac{5\pi}{4}\right) - \left(-3\cos\frac{\pi}{4} - 2\sin\frac{\pi}{4}\right) \\
&= \left(3\cdot\frac{\sqrt{2}}{2} + 2\cdot\frac{\sqrt{2}}{2}\right) - \left(-3\cdot\frac{\sqrt{2}}{2} - 2\cdot\frac{\sqrt{2}}{2}\right) \\
&= \frac{5\sqrt{2}}{2} - \left(-\frac{5\sqrt{2}}{2}\right) = 5\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
(4) \text{ 与式} &= \int_{-2}^2 e^x dx + \int_{-2}^2 e^{-x} dx \\
&= \left[e^x\right]_{-2}^2 + \left[-e^{-x}\right]_{-2}^2 \\
&= (e^2 - e^{-2}) + \{-e^{-2} - (-e^2)\} \\
&= 2e^2 - 2e^{-2} \\
&= 2\left(e^2 - \frac{1}{e^2}\right)
\end{aligned}$$

または

$$\begin{aligned}
\text{与式} &= \left[e^x - e^{-x}\right]_{-2}^2 \\
&= (e^2 - e^{-2}) - (e^{-2} - e^2) \\
&= 2e^2 - 2e^{-2} \\
&= 2\left(e^2 - \frac{1}{e^2}\right)
\end{aligned}$$

問 8

(1) x^3 , x は奇関数, x^2 , 5 は偶関数であるから

$$\begin{aligned}
\text{与式} &= 2 \int_0^1 (-3x^2 + 5) dx \\
&= 2 \left[-x^3 + 5x\right]_0^1 \\
&= 2\{(-1 + 5) - 0\} = 8
\end{aligned}$$

(2) $\sin x$ は奇関数, $\cos x$ は偶関数であるから

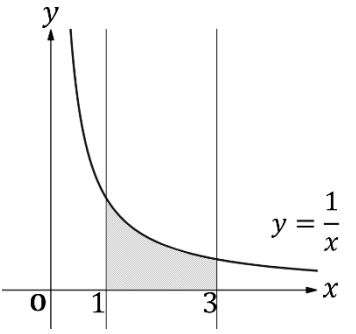
$$\begin{aligned}
\text{与式} &= 2 \int_0^{\frac{\pi}{3}} \cos x dx \\
&= 2 \left[\sin x\right]_0^{\frac{\pi}{3}}
\end{aligned}$$

$$\begin{aligned}
&= 2\left(\sin\frac{\pi}{3} - \sin 0\right) \\
&= 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}
\end{aligned}$$

問 9

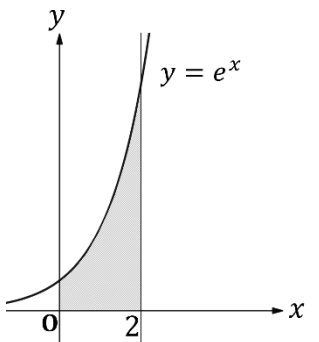
(1) 区間 $[1, 3]$ において, $\frac{1}{x} > 0$ であるから,

求める図形の面積を S とすると

$$\begin{aligned}
S &= \int_1^3 \frac{1}{x} dx \\
&= \left[\log|x|\right]_1^3 \\
&= \log|3| - \log|1| \\
&= \log 3 - 0 = \log 3
\end{aligned}$$


(2) 区間 $[0, 2]$ において, $e^x > 0$ であるから,

求める図形の面積を S とすると

$$\begin{aligned}
S &= \int_0^2 e^x dx \\
&= \left[e^x\right]_0^2 \\
&= e^2 - e^0 = e^2 - 1
\end{aligned}$$


問 10

曲線と x 軸の交点を求めると

$$\begin{aligned}
x^2 - 3x &= 0 \\
x(x - 3) &= 0
\end{aligned}$$

よって, $x = 0, 3$

区間 $[0, 3]$ において, $x^2 - 3x \leq 0$ であるから,

求める図形の面積を S とすると

$$\begin{aligned}
S &= - \int_0^3 (x^2 - 3x) dx \\
&= - \left[\frac{1}{3}x^3 - \frac{3}{2}x^2\right]_0^3
\end{aligned}$$

$$= -\left(9 - \frac{27}{2}\right)$$

$$= -\left(-\frac{9}{2}\right) = \frac{9}{2}$$

問 11 C は積分定数

$$(1) \text{ 与式} = \int \frac{\cos^2 x}{\sin^2 x} dx$$

$$= \int \frac{1 - \sin^2 x}{\sin^2 x} dx$$

$$= \int \left(\frac{1}{\sin^2 x} - 1 \right) dx$$

$$= -\cot x - x + C$$

$$(2) \text{ 与式} = \int \left(\frac{2}{\cos^2 x} + 5 \cos x \right) dx$$

$$= 2 \tan x + 5 \sin x + C$$

問 12 C は積分定数

$$(1) \text{ 与式} = \int \frac{dx}{\sqrt{4^2 - x^2}} dx$$

$$= \sin^{-1} \frac{x}{4} + C$$

$$(2) \text{ 与式} = \log |x + \sqrt{x^2 - 16}| + C$$

$$(3) \text{ 与式} = \int \frac{(x^2 + 1) + 2}{x^2 + 1} dx$$

$$= \int \left(1 + \frac{2}{x^2 + 1} \right) dx$$

$$= x + 2 \tan^{-1} x + C$$

問 13

$$(1) \text{ 与式} = \left[\log |x + \sqrt{x^2 + 7}| \right]_0^3$$

$$= \log |3 + \sqrt{3^2 + 7}| - \log |0 + \sqrt{0 + 7}|$$

$$= \log(3 + \sqrt{16}) - \log \sqrt{7}$$

$$= \log 7 - \log \sqrt{7}$$

$$= \log \frac{7}{\sqrt{7}} = \log \sqrt{7}$$

$$(2) \text{ 与式} = \int_{-\sqrt{3}}^3 \frac{dx}{x^2 + 3^2}$$

$$= \left[\frac{1}{3} \tan^{-1} \frac{x}{3} \right]_{-\sqrt{3}}^3$$

$$= \frac{1}{3} \left\{ \tan^{-1} 1 - \tan^{-1} \left(-\frac{1}{\sqrt{3}} \right) \right\}$$

$$= \frac{1}{3} \cdot \left\{ \frac{\pi}{4} - \left(-\frac{\pi}{6} \right) \right\}$$

$$= \frac{1}{3} \cdot \frac{5}{12} \pi = \frac{5}{36} \pi$$